A more attractive scheme for radion stabilization and supercooled phase transition

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Based on K. Fujikura (TITech), YN and M. Yamada (MIT), arXiv:1910.07546.

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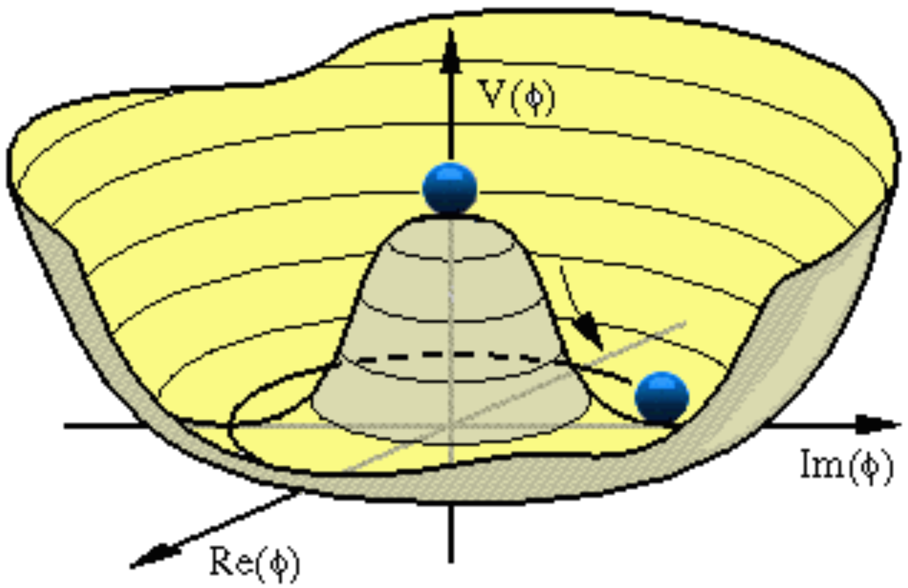
The Naturalness Problem

The Higgs potential (and the weak scale) is unstable under quantum effects.

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

~100 GeV



Severe fine-tuning is needed.

$$\mu^2 = \underbrace{m_0^2 + \delta\mu^2}_{\text{Tuning}}, \ \delta\mu^2 \sim M_{Pl}^2$$

$$\Rightarrow \frac{\mu^2}{M_{Pl}^2} \sim 10^{-32} \ !$$

New Physics Candidates

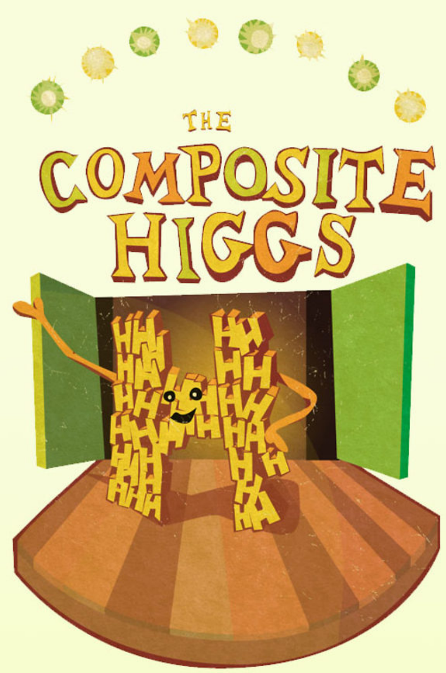
Many new physics models have been considered, motivated by **the naturalness problem**.

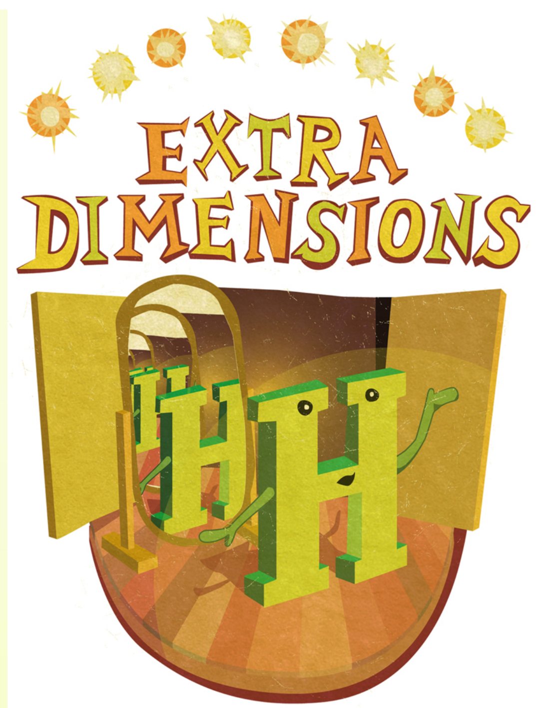
Cut off by new physics near the weak scale.

$$\mu^2 = m_0^2 + \delta\mu^2,\ \ \delta\mu^2 \sim \cancel{M_{Pl}^2}$$

~100 GeV

No tuning !





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Randall-Sundrum Model

- 5D universe bounded by two branes: Higgs located on IR (TeV) brane and gravity localized toward UV (Planck) brane.

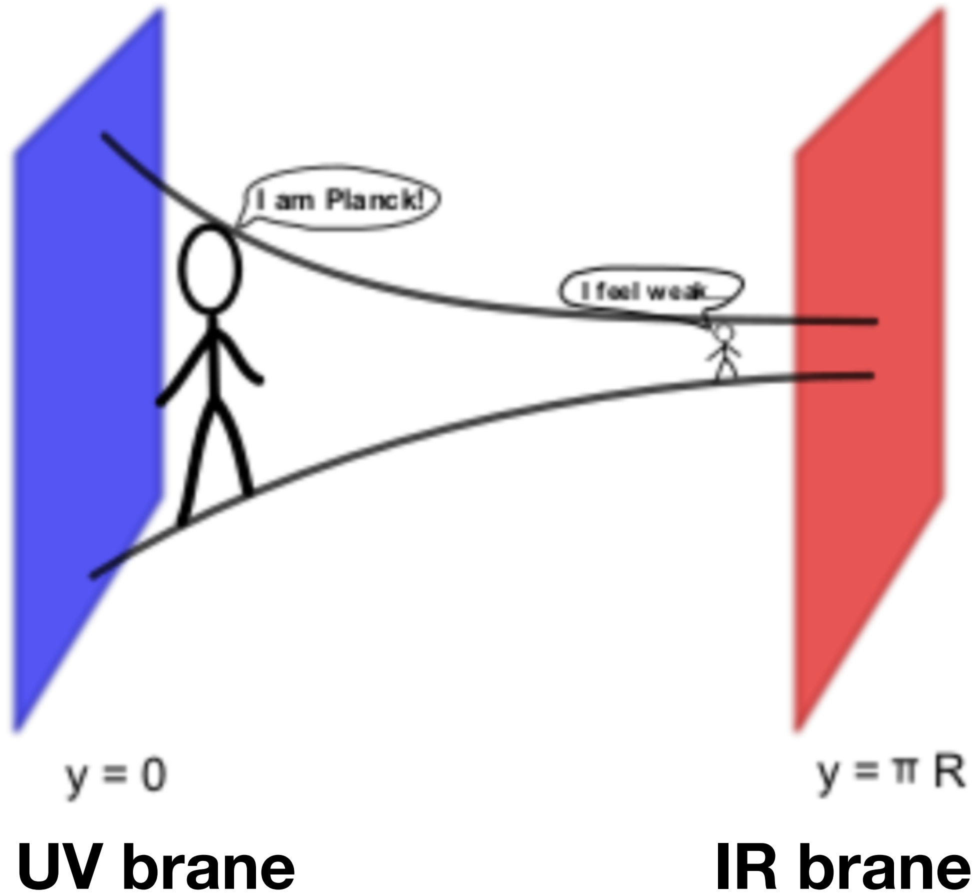
- 5th dimension highly curved

  ⟶ Anti-de-Sitter (AdS) space

Metric : $ds^2 = \boxed{e^{-k|y|}} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$

Warp factor

(4D flat : $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$)



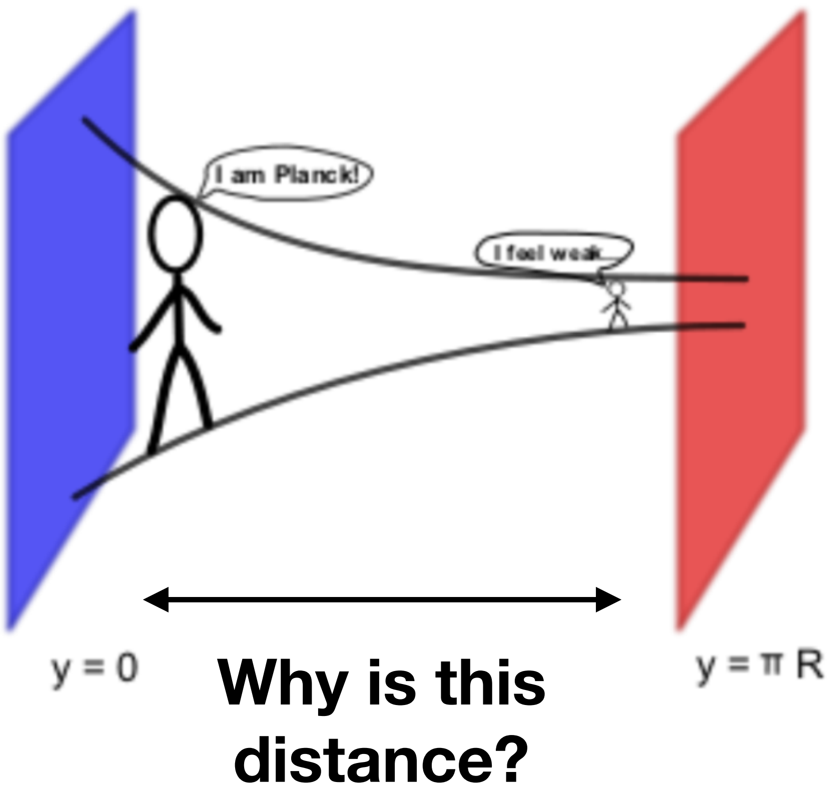
- Solution to the naturalness problem

All fundamental mass parameters on the IR brane are exponentially redshifted.

Radion

- A modulus field, called **radion**, parameterizes the distance between the IR and UV branes.

- In the original RS model, its vacuum expectation value is fixed by hand to realize an adequate redshift factor.



To solve the naturalness problem completely …

We need a mechanism that stabilizes the radion VEV without fine-tuning.

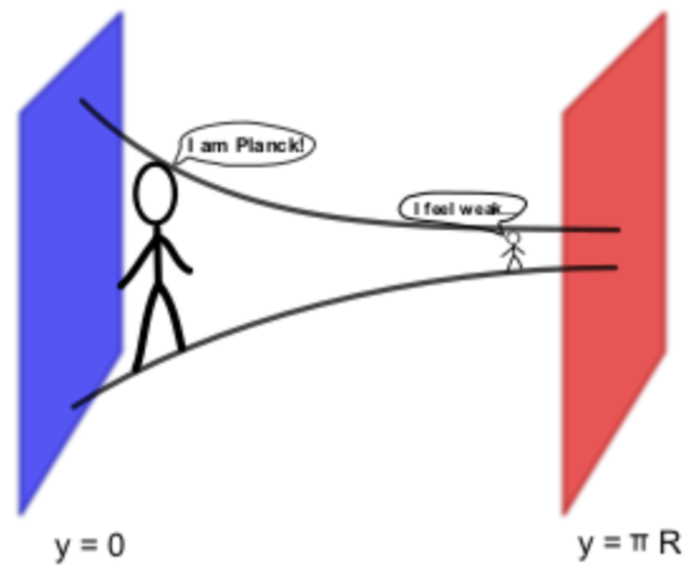
Goldberger-Wise mechanism Goldberger, Wise (1999)

introduces a bulk scalar field with brane-localized potentials.

➡ Radion potential

Cosmological History

If the RS model is realized in nature, it must predict a consistent cosmological history of our Universe.



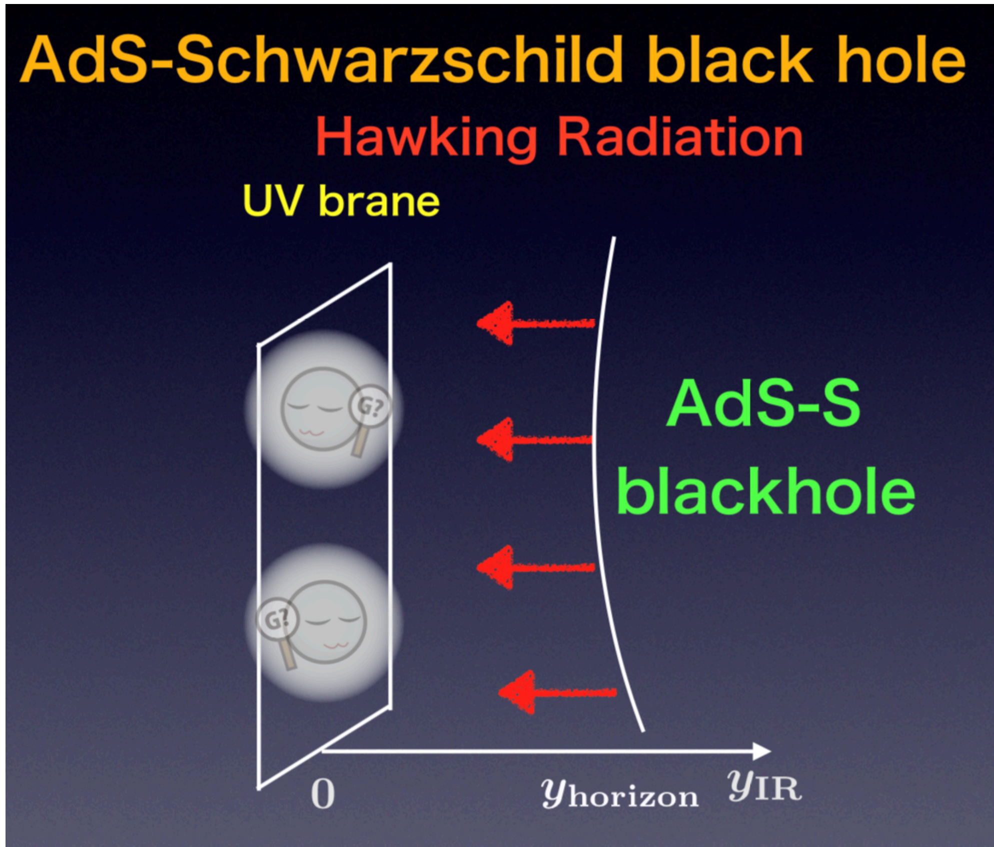
At low temperature …

The Universe is described by the compact RS model.

At high temperature …

The system is described by the de-compactified AdS-Schwarzschild (AdS-S) black hole with the IR brane replaced by an event horizon.

Creminelli, Nicolis, Rattazzi (2001)



Fujikura

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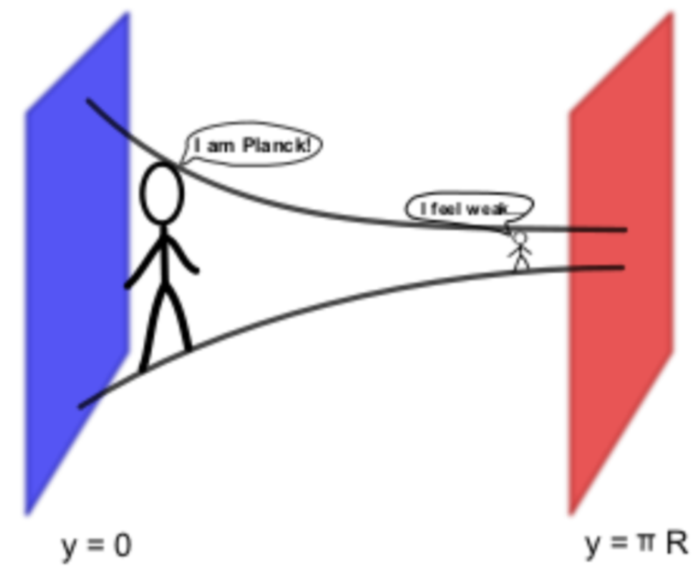

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Phase transition

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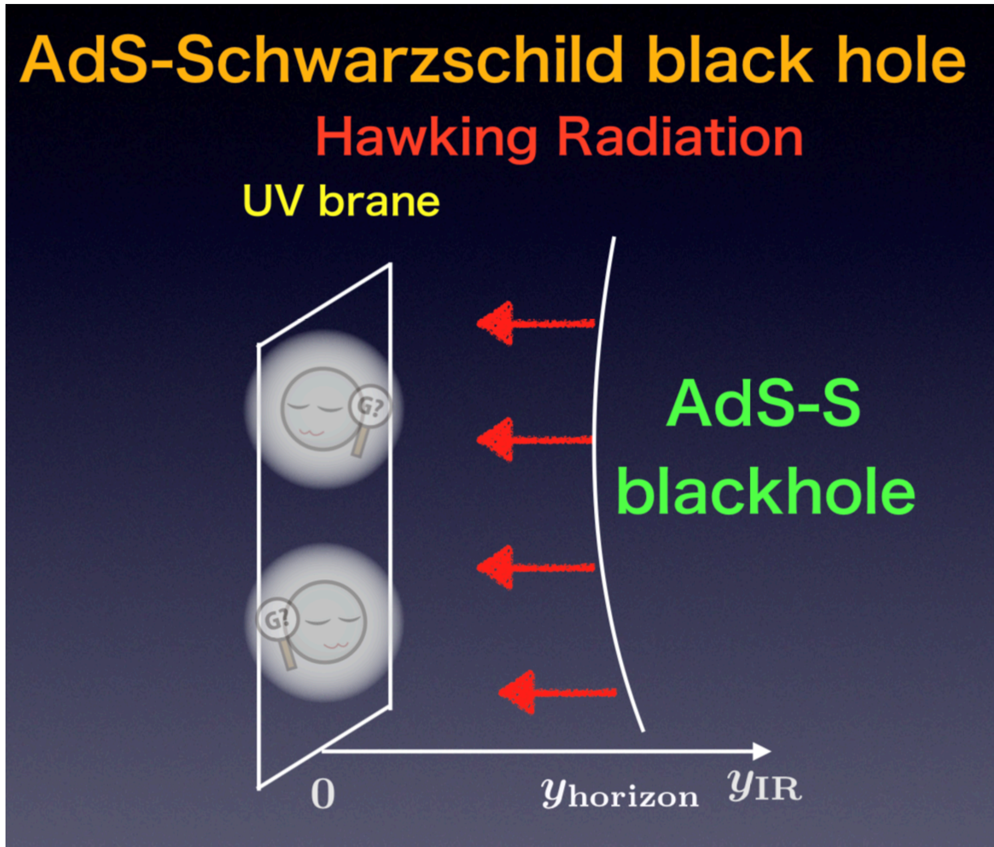


Eternal Inflation

✓ The phase transition is of the first order, takes place via a supercooling phase and proceeds via nucleation of true vacuum bubbles.

✓ In the Goldberger-Wise mechanism, the supercooling phase lasts very long and **the phase transition is never completed** in most of the region where a classical treatment of the gravity is meaningful.

Creminelli, Nicolis, Rattazzi (2001)

➡ **Eternal Inflation**

✓ Even in the remaining parameter space, the brane-localized potentials of the bulk scalar field give a non-negligible back-reaction to the gravitational action and **the analysis without including the back-reaction is not trustable**.

We propose a new mechanism of radion stabilization with no issue in completion of the phase transition !

Radion Effective Action

The geometry of the RS spacetime :

$$ds^2 = G_{AB}dx^A dx^B = e^{-2kT(x)|y|}g_{\mu\nu}dx^\mu dx^\nu - T^2(x)dy^2$$

$y \in (-1/2, 1/2)$, UV and IR branes are placed at $y = 0$ and $y = 1/2$ respectively

T(x) determines the size of the extra dimension and is a modulus field associated with a fluctuation along the extra dimension.

A pure gravity action of RS :

$$S = \int d^4x dy \left[\sqrt{G}\left(\frac{1}{2}M_5^3 R - \Lambda_{\rm bulk}\right) - \Lambda_{\rm IR}\sqrt{-g_{\rm IR}}\,\delta(y - y_{\rm IR}) - \Lambda_{\rm UV}\sqrt{-g_{\rm UV}}\,\delta(y)\right]$$

$\Lambda_{\rm bulk}$: bulk cosmological constant, $\Lambda_{\rm IR}$, $\Lambda_{\rm UV}$: IR and UV brane tensions

The RS geometry is realized when $\Lambda_{\rm bulk}|_{\rm RS}/k = \Lambda_{\rm IR}|_{\rm RS} = -\Lambda_{\rm UV}|_{\rm RS} = -6M_5^3 k$

Tunings!

But, in general…

$$\Lambda_{\rm IR} = -6M_5^3 k + \delta\Lambda_{\rm IR}, \quad \Lambda_{\rm UV} = 6M_5^3 k + \delta\Lambda_{\rm UV}$$

Radion Effective Action

The Kaluza-Klein (KK) reduction of the pure gravity action

➡ **4D effective action of radion** $\mu \equiv k e^{-kT(x)/2}$

$$S_{\text{radion}} = \int d^4x \left[\frac{3N^2}{4\pi^2} (\partial\mu(x))^2 - V(\mu) \right]$$

$$V(\mu) = \delta\Lambda_{\text{UV}} + \mu^4 \delta\Lambda_{\text{IR}}/k^4$$

The radion kinetic term is <u>not canonically normalized.</u>

$$N \equiv 2\pi(M_5/k)^{3/2}$$

Terms with higher powers of the Ricci scalar coming from <u>quantum gravity effects</u> can be neglected for

$$N \gtrsim 4 \cdot 5^{3/4}/\sqrt{3\pi} \simeq 4.4$$

Harling and G. Servant (2018)

Radion Stabilization

Introduce **a SU(N$_H$) pure Yang-Mills field** in the bulk of the extra dimension.

$$S_{\rm Yang-Mills} = \int d^5x \sqrt{G} \left(-\frac{1}{4g_5^2} F_{AB} F^{AB} \right)$$

KK decomposition and integrating over the extra dimension

➡ 4D effective action for the zero-mode gauge field

RGE of 4D gauge coupling:

Radion

$$\frac{1}{g_4^2(Q,\mu)} = \frac{\log \frac{k}{\mu}}{k g_5^2} - \frac{b_{\rm YM}}{8\pi^2} \log \left(\frac{k}{Q} \right) \quad \text{for} \quad Q \lesssim \mu$$

$b_{YM} = 11N_H/3$

Gauge coupling becomes strong at low-energies and the theory confines !

Radion Stabilization

The confinement scale is naturally at the TeV scale.

(i) $\Lambda_H(\mu) < m_{KK} = \pi\mu$

Confinement scale: $\Lambda_H(\mu) = \Lambda_{H,0}\left(\frac{\mu}{\mu_{\rm min}}\right)^n \quad n = \frac{8\pi^2}{b_{\rm YM}\cdot kg_5^2}$

(ii) $\Lambda_H(\mu) > m_{KK} = \pi\mu$

The description of the 4D effective theory breaks down.

The confinement scale is independent of the radion VEV.

Confinement scale: $\Lambda_H(\mu) = \Lambda_H(\mu_c) \equiv \gamma_c\mu_c \quad \gamma_c = \pi$

Confinement scale of (i) and (ii) are the same at $\mu = \mu_c$

Radion Stabilization

The confinement generates a vacuum energy.

$$V_H = \frac{1}{4}\langle T^\mu_\mu \rangle \simeq -\frac{b_{\rm YM}}{8}\left(\Lambda_H(\mu)\right)^4$$

Radion can be stabilized by the balance between the vacuum energy and the IR brane tension.

$$V_{r,\rm eff}(\mu) = \begin{cases} V_0 + \frac{\lambda}{4}\mu^4 - \frac{b_{\rm YM}}{8}\Lambda^4_{H,0}\left(\frac{\mu}{\mu_{\rm min}}\right)^{4n} & \text{for } \mu > \mu_c\,, \\ V_0 + \frac{\lambda}{4}\mu^4 - \frac{b_{\rm YM}}{8}\gamma_c^4\mu_c^4 & \text{for } \mu < \mu_c \end{cases}$$

$n < 1$ is required.

$\lambda \equiv 4\delta\Lambda_{IR}/k^4$

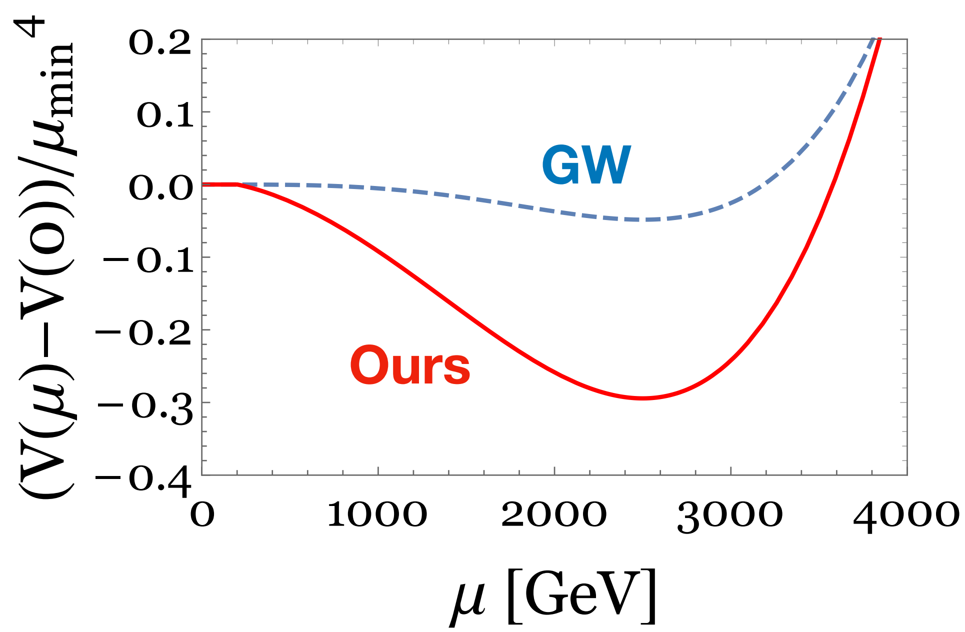
$V_0 \equiv \delta\Lambda_{UV}$ determined by the condition that the potential energy at the minimum is vanishingly small.

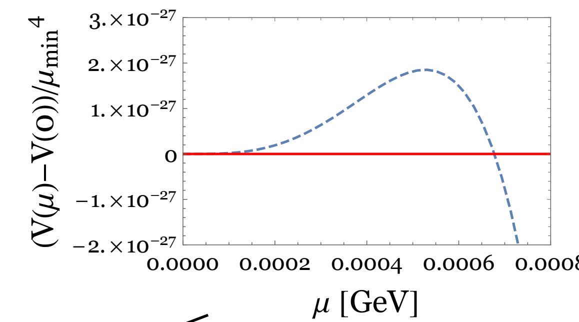
$$\Rightarrow \quad \mu_{\rm min} = \left(\frac{n b_{\rm YM}}{2\lambda}\right)^{\frac{1}{4}}\Lambda_{H,0}$$

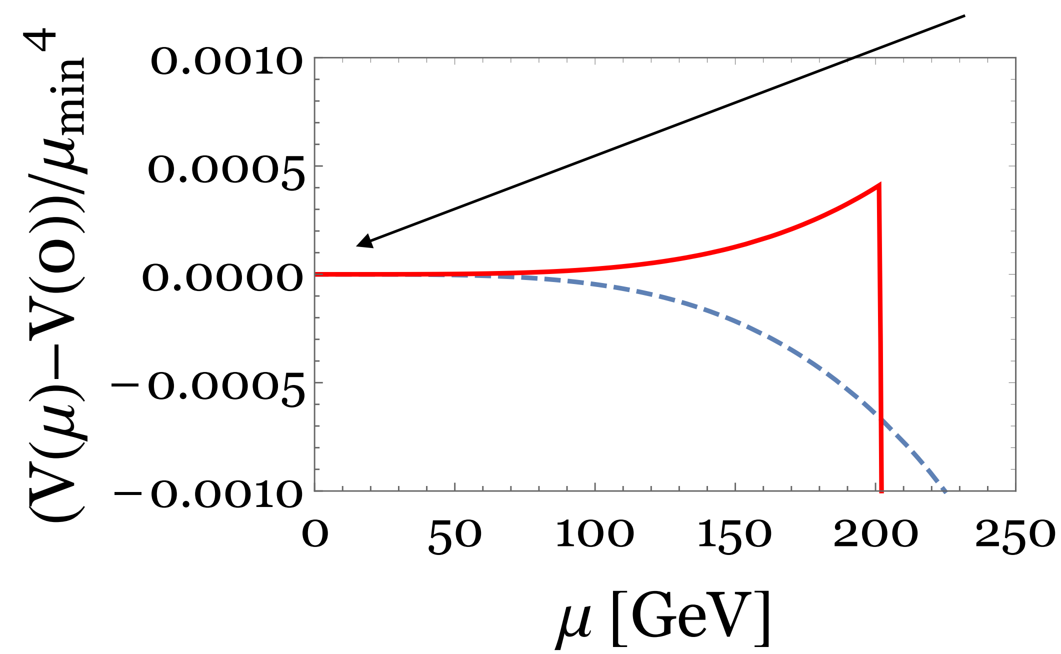
Radion Potential

μ_{min} = 2.5TeV, λ = 1, γ_C = π, α = 1, N = 5 and N_H = 3

$m_{radion} \simeq$ 2.2 TeV







- ✓ Our radion potential has **a deeper minimum** than the Goldberger-Wise.
- ✓ The origin is a local minimum.
- ✓ Not smooth at $\mu = \mu_C$, reflecting our ignorance of the precise radion potential around this point.

AdS-S Spacetime

At high temperature, the system is described by the AdS-S spacetime with the IR brane replaced by the event horizon.

$$ds^2 = k^2\rho^2\left(1-\frac{\rho_H^4}{\rho^4}\right)dt^2 - k^2\rho^2\sum_{i=1}^{3}dx_i^2 - \frac{d\rho^2}{k^2\rho^2\left(1-\frac{\rho_H^4}{\rho^4}\right)}$$

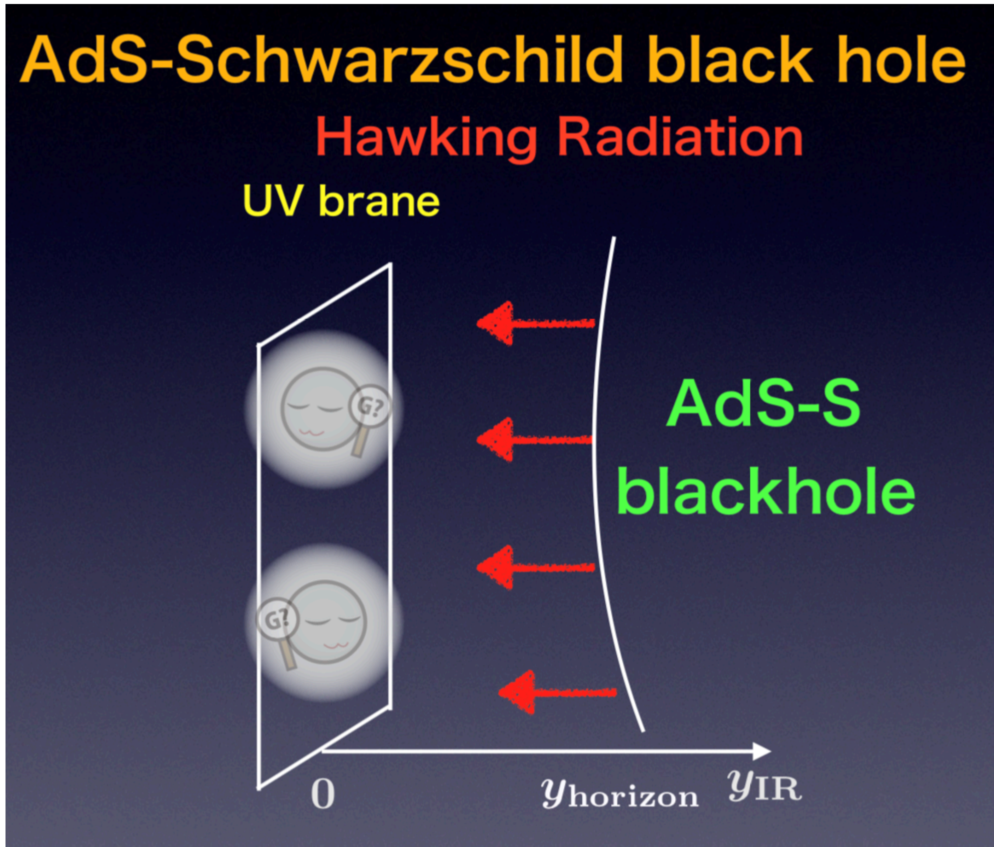
T_H ($\equiv k^2\rho_H/\pi$) : the Hawking temperature parameterized by the position of the event horizon

The free energy of the AdS-S spacetime :

$$F_{\mathrm{AdS-S}}(T_H) = \frac{3}{8}\pi^2 N^2 T_H^4 - \frac{1}{2}\pi^2 N^2 T_H^3 T$$

Creminelli, Nicolis, Rattazzi (2001)

The minimum is given by $T_H = T$.



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Phase Transition

As the temperature cools down, the phase transition from the AdS-S spacetime to the RS spacetime can take place.

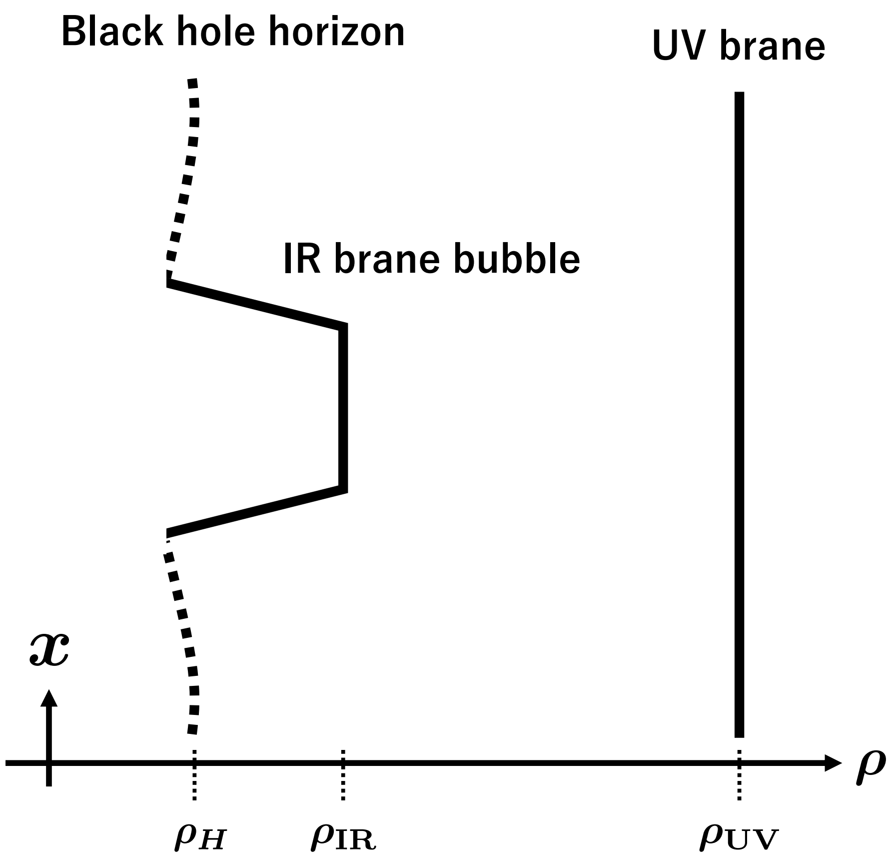
Both the AdS-S spacetime and the RS spacetime are locally stable.

→ The phase transition occurs via the decay of the false vacuum and the phase transition is expected to be of the first order.

As the temperature decreases, the event horizon moves toward $T_H = 0$.

The phase transition proceeds via the "IR brane bubble nucleation"

Spherical brane patches on the horizon appear and are combined to form the IR brane.



Transition Rate

- The rate of the phase transition per unit volume per unit time :

$$\Gamma \propto e^{-S}$$

- The phase transition can be completed only when the bubble nucleations are not diluted by the cosmic expansion :

$$\Gamma > H^4$$

- The O(4)-symmetric bounce action after canonically normalizing the radion kinetic term :

Tunneling point

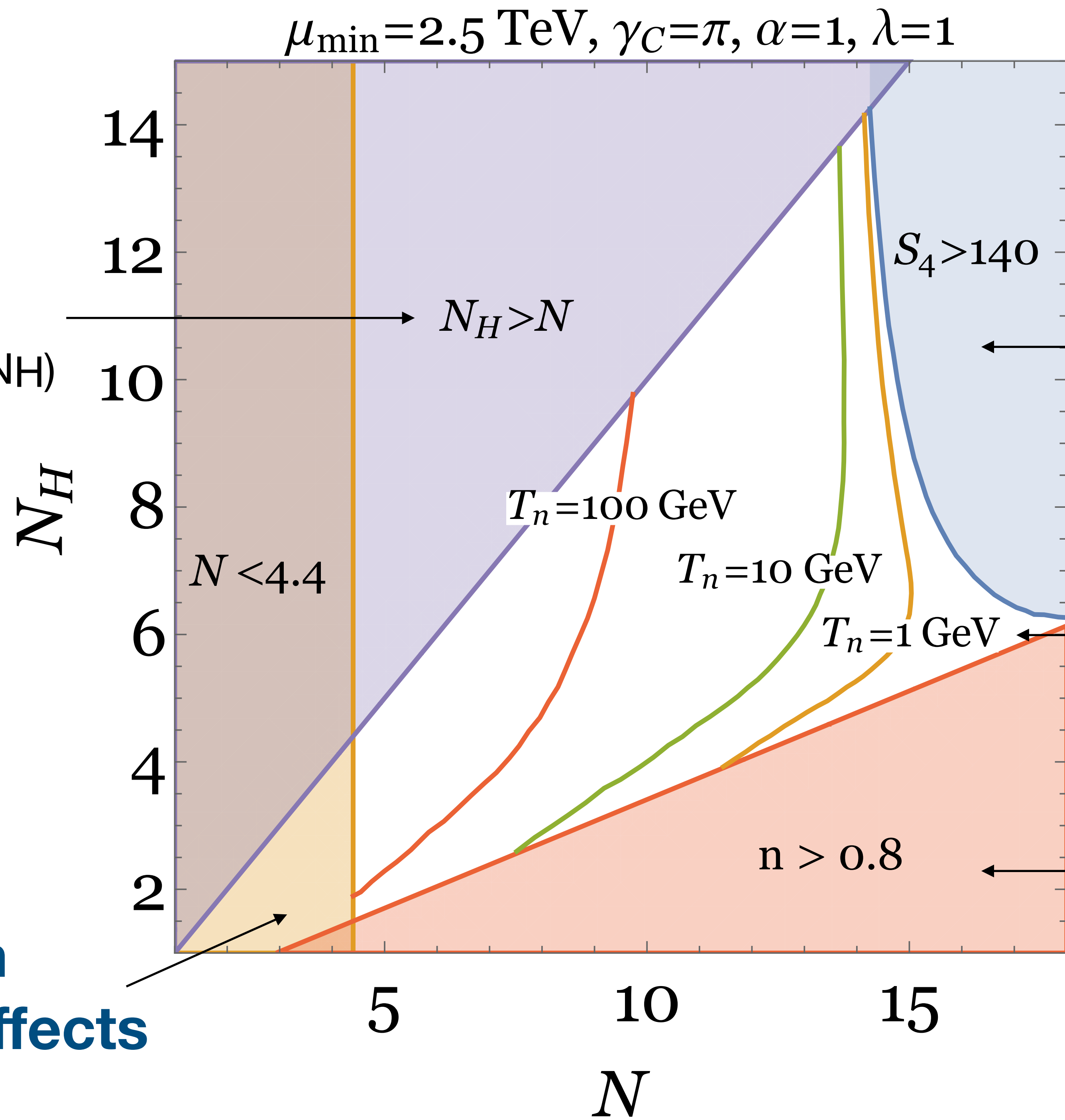
$$S_4 \sim \frac{9N^4}{8\pi^2} \frac{\mu_t^4}{V(\mu_{\min})\left(\frac{T}{T_c}\right)^4 - V(\mu_t)} \qquad \frac{\partial S_4}{\partial \mu_t} = 0$$

✓ A large N dependence.

✓ A shallower potential leads to a larger bounce action.

Transition Rate

Numerical computation of the bounce action



Finite temperature effects of SU(NH)

The phase transition is **never completed** here

Bubble nucleation temperature

QCD effects cannot be ignored

Quantum gravity effects

The phase transition can be completed for N < 16 !

Supercooling

The phase transition is completed, but there is still a supercooling phase.

The vacuum energy dominates the energy density of the Universe and **mini-inflation** takes place before the phase transition is completed.

The e-folding number of mini-inflation :

$$N_e \simeq \log\left(\frac{T_c}{T_n}\right) \qquad N_e \lesssim \log(1\,\mathrm{TeV}/1\,\mathrm{GeV}) \simeq 7$$

➡ Dilution of dark matter and baryon asymmetry if they are produced before the phase transition.

The dilution factor ~10^{-9}

We need a very large amount of dark matter and baryon asymmetry before the phase transition or need to produce them after the phase transition.

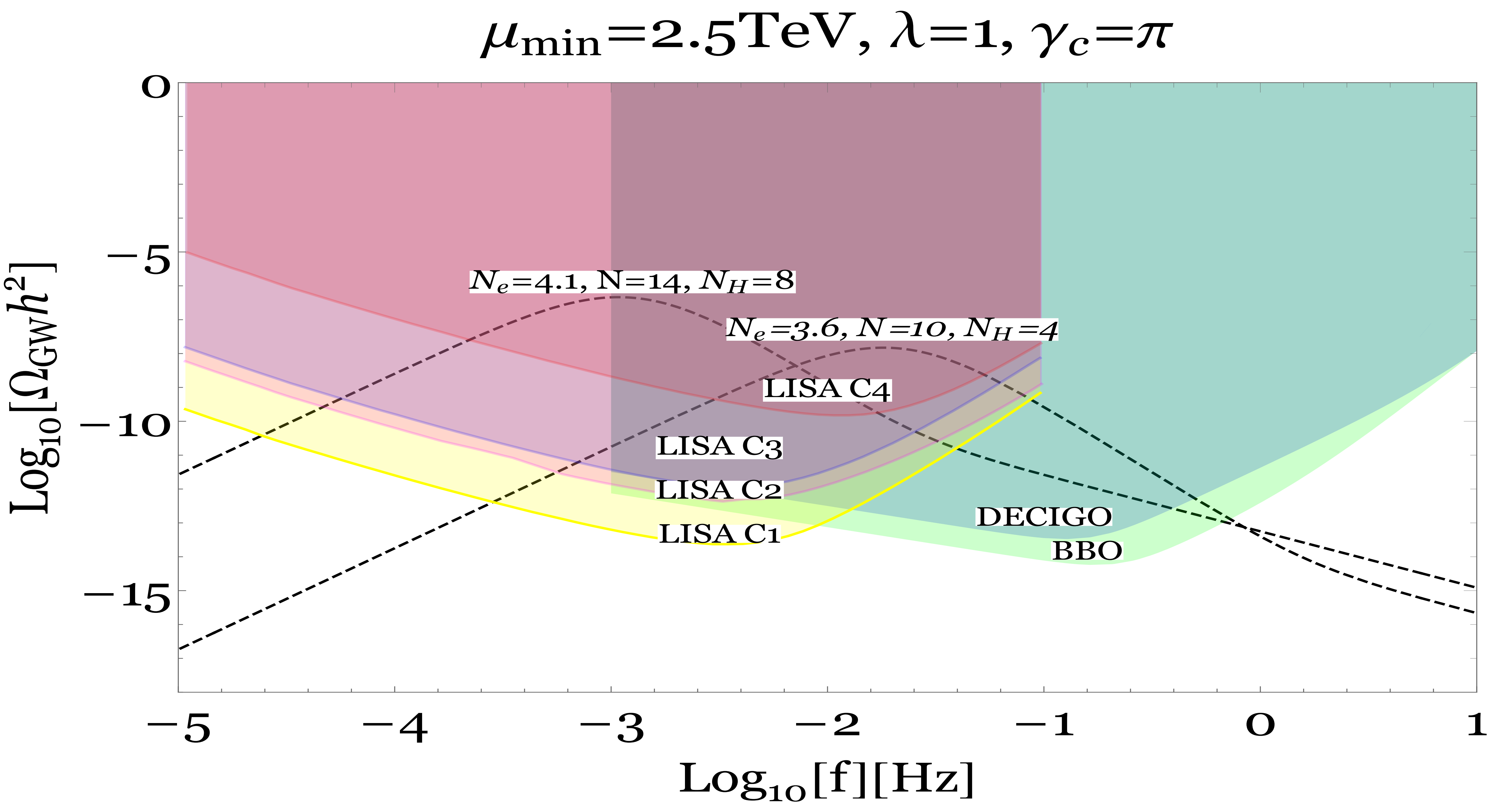
⬆

Not unlikely

QCD axion, Affleck-Dine baryogenesis, …

Gravitational Waves

A long supercooling epoch results in an almost maximal GW amplitude detected by future experiments !



Summary

✓ A new radion stabilization mechanism in the RS model, introducing a bulk $SU(N_H)$ gauge field which confines at a TeV scale.

✓ The radion potential can be stabilized by the balance between the vacuum energy from the confinement and the IR brane tension.

✓ Asymptotic freedom makes the vacuum energy irrelevant at the Planck scale and the back-reaction to the gravitational action is negligible.

✓ The phase transition from the AdS-S spacetime is of the first order.

✓ The phase transition is completed even when the 5D Planck scale is much larger than the AdS curvature scale.

✓ A mini-inflation occurs before the phase transition is completed.

✓ The produced GW can be detected by future experiments.

Thank you.