Nuclear Science Computing Center at CCNU

QCD phase structure from Lattice QCD

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International Mini Workshop on Composite Higgs, Dark Matter, Neutrinos, New Physics, Gauge-Higgs Unification and Related Topics

Nov. 21-24@SYSU

Symmetries of QCD in the vacuum

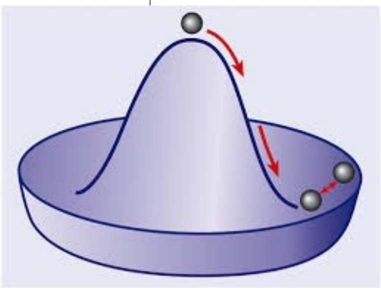
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$

Classical QCD symmetry (m_q=0)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

Quantum QCD vacuum (m_q=0)

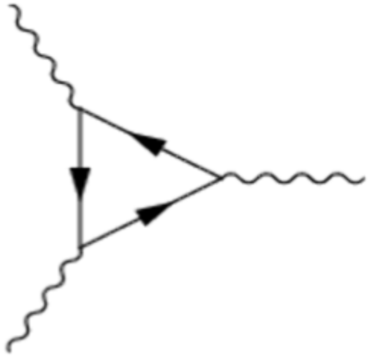
Chiral condensate:
spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

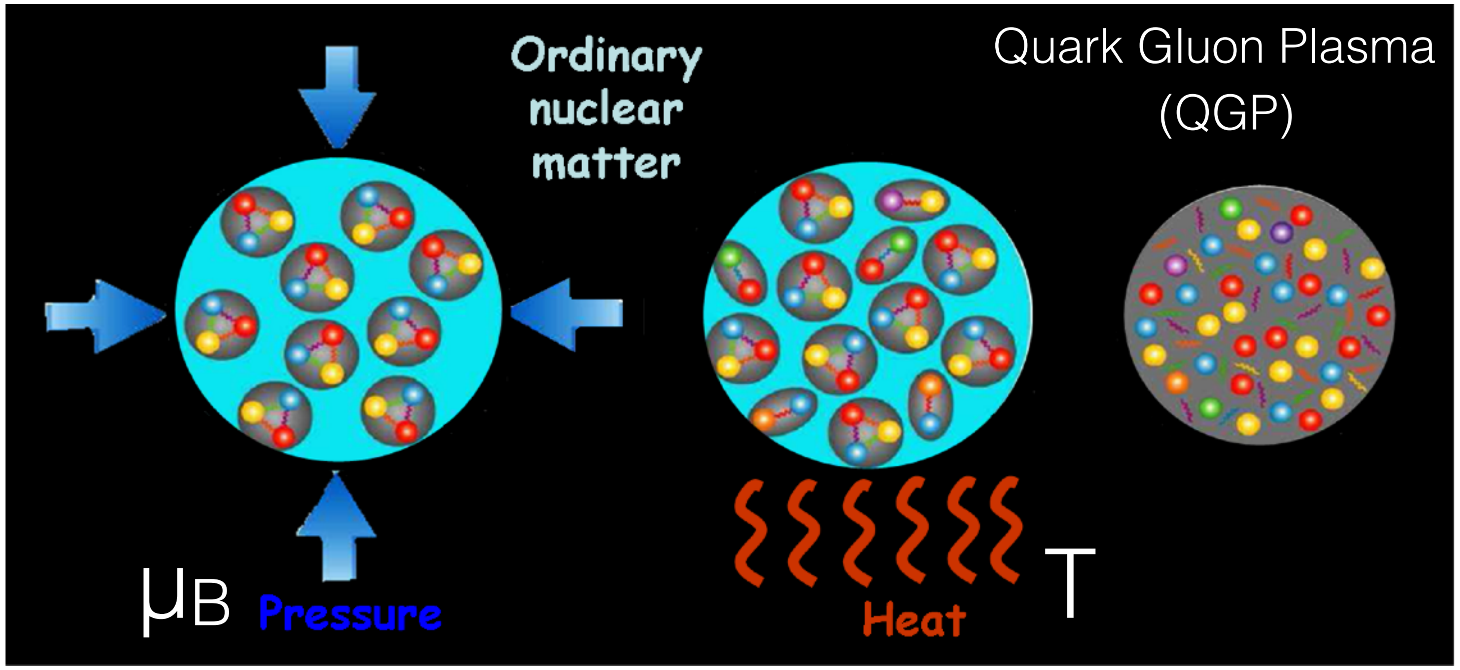
Axial anomaly:
quantum violation of $U(1)_A$

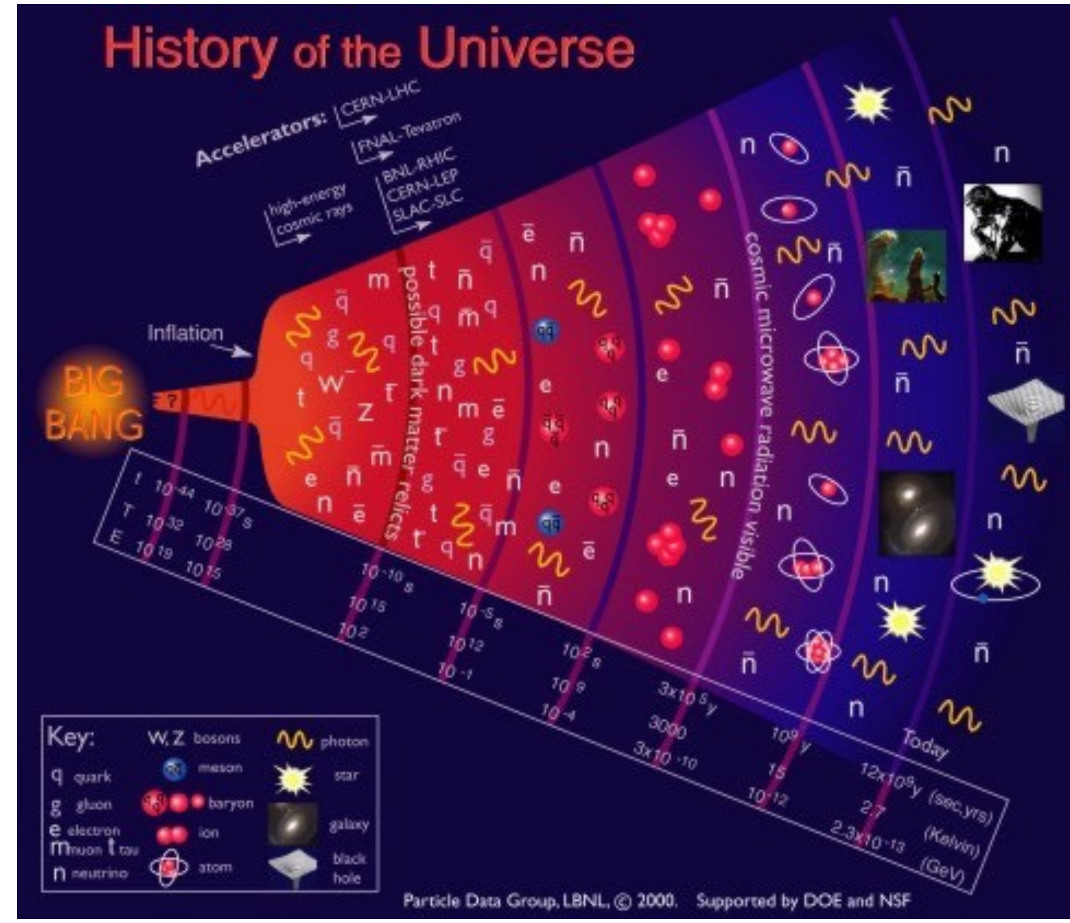
$$\partial_{\mu} j_5^{\mu} = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu}F^{\mu\nu})$$



$$SU(N_f)_V \times U(1)_V$$

Symmetry restoration in extreme conditions: QCD phase transitions





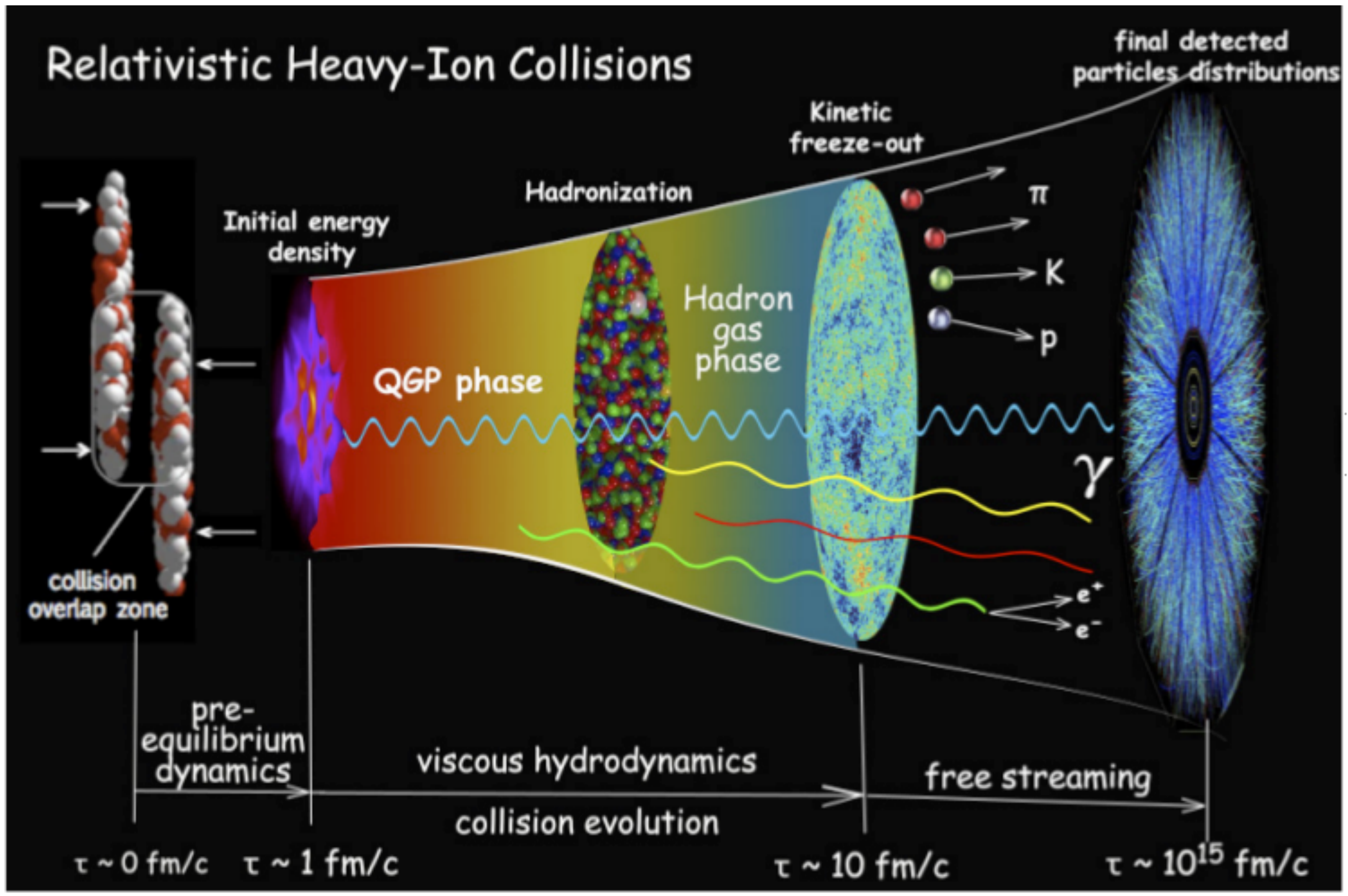
"The whole is more than sum of its parts."
Aristotle, Metaphysica 10f-1045a

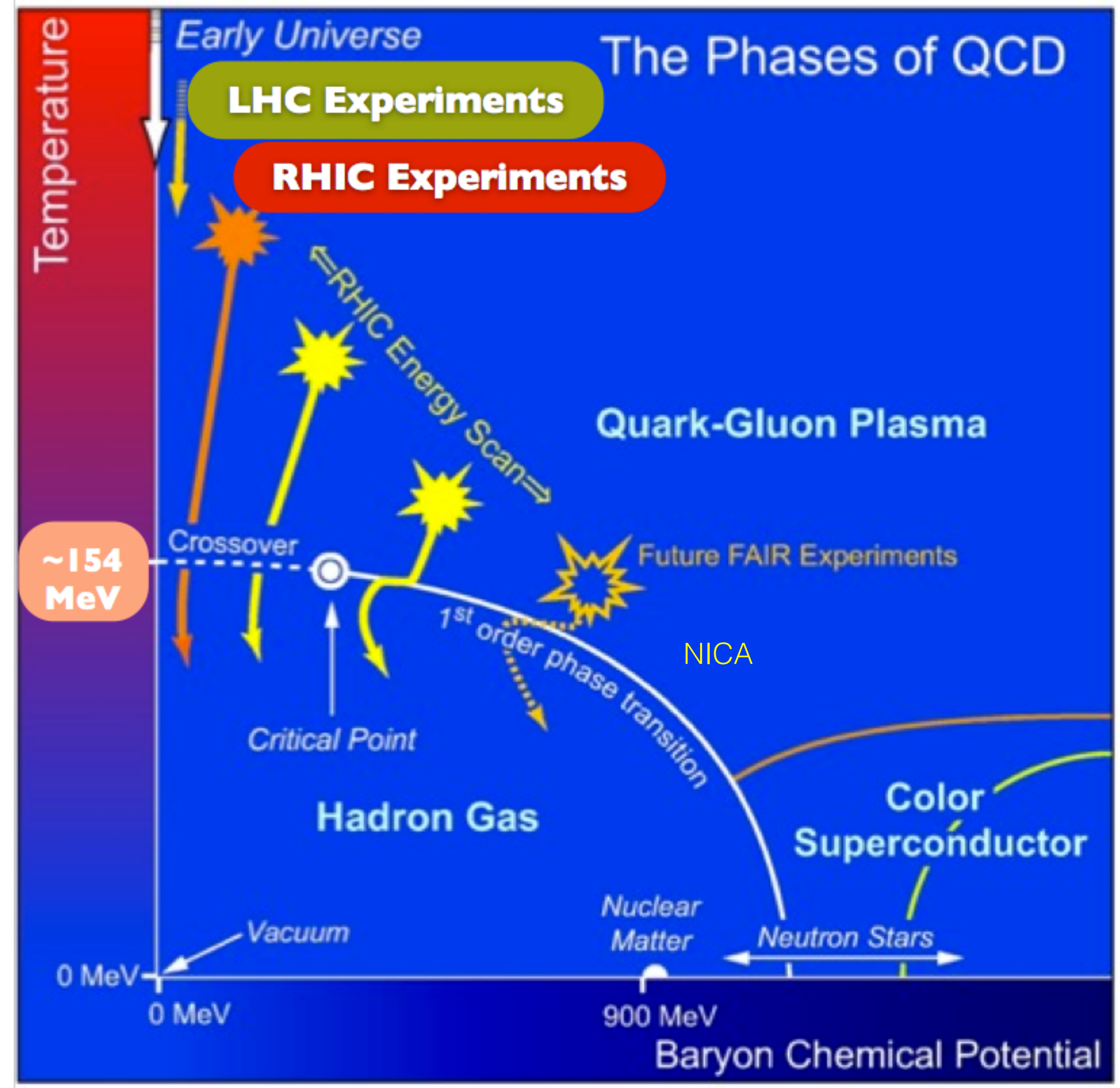
What are the phases of strong-interaction matter and what roles do they play in cosmos?

What are the T_c, orders and universality classes of (chiral & deconfinement) phase transitions?

What does QCD predict for the properties of the strong-interaction matter in extreme conditions?

Recreate QGP in Heavy Ion Collisions (HIC)…





Landau functional of QCD

Pisarski & Wilczek (1984)

Symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}^j_R q^i_L$

Chiral transformation: $\Phi \to e^{-2i\alpha_A} V_L \, \Phi \, V_R^\dagger$

$$\mathcal{L}_{eff} = \frac{1}{2}\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\,\left(\Phi^\dagger\Phi\right)^2$$

→ $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$

$$- \frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)$$

→ $SU(N_f)_L \times SU(N_f)_R$

$$- \frac{d}{2}\mathrm{tr}\,h\left(\Phi + \Phi^\dagger\right).$$

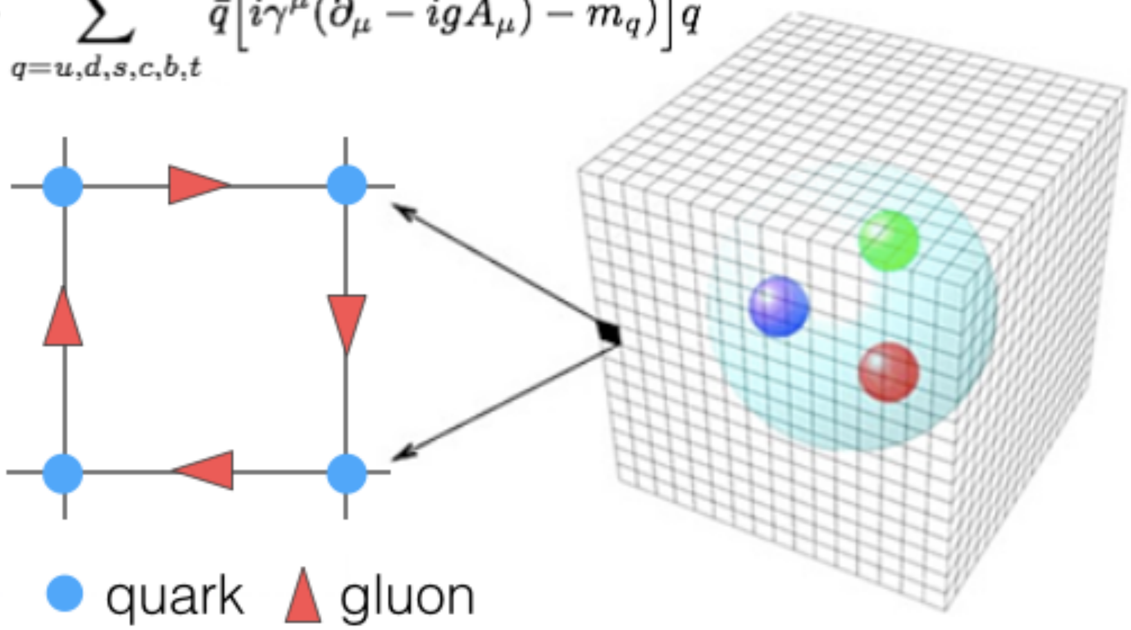
→ Quark mass term

Results on phase transitions should be eventually checked by Lattice QCD

Lattice QCD

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q \right] q$$



Discretization in Euclidean space

quarks: lattice sites
gluons: lattice links

Supercomputing the QCD matter:

structural equivalence between statistical mechanics & QFT on the lattice

$$< \mathcal{O} > = \frac{1}{Z} \int \mathcal{D}\mathcal{U}\ \mathcal{D}\psi\ \mathcal{D}\bar{\psi}\ \mathcal{O}\ e^{-S_{lat}}$$

$$S_{lat} = S_g + S_f$$

$$Z = \int \mathcal{D}\mathcal{U}\ \mathcal{D}\psi\ \mathcal{D}\bar{\psi}\ e^{-S_{lat}} = \int \mathcal{D}\mathcal{U}\ e^{-S_g} \det M_f$$

$N_c \otimes N_f \otimes N_{spin} \otimes N_d \otimes N_\sigma \otimes N_T^3 \gtrsim 10^6$

chiral condensate: $$\langle \bar{q}q \rangle = \frac{\partial \ln Z}{\partial m_q} = \frac{n_f}{4} \langle \mathrm{Tr} M^{-1} \rangle$$

NSC³

Nuclear Science Computing Center at CCNU

计算决定未来！

N: Nuclear **S**: Science **C³**: Color 3 -> QCD

“道生一，一生二，二生三，三生万物。” — 《道德经》老子 600 BC

”Tao gives birth to One, One gives birth to Two, Two gives birth to Three, Three gives birth to everything.“- Lao Tzu

38 computing nodes
(304 V100 GPUs)
Peak performance:
2 PFlops/s
(每秒2 千万亿次浮点运算)
Storage:
1 PB

Nov., 2019

TOP 500 The List.

The GREEN 500

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8 nodes in each rack

8 GPUs in each node

On 1-single GPU: $64^3 \times 16$, $72^3 \times 12$

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(每秒2 千万亿次浮点运算)

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NuclearScience
Computing CenteratCCNU

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GPU Hackathon（黑客马拉松）

- 针对人群：
  1）想优化GPU应用程序的研究团队
  2）想把CPU程序移植到GPU上的研究团队
- 指导专家：英伟达公司及业内GPU和计算机专家
- 时间：2020年4月20-24
- 地点：华中师范大学核科学计算中心
- 费用：免费

38 computing nodes
(304 V100 GPUs)
Peak performance:
2 PFlops/s
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Nov., 2019
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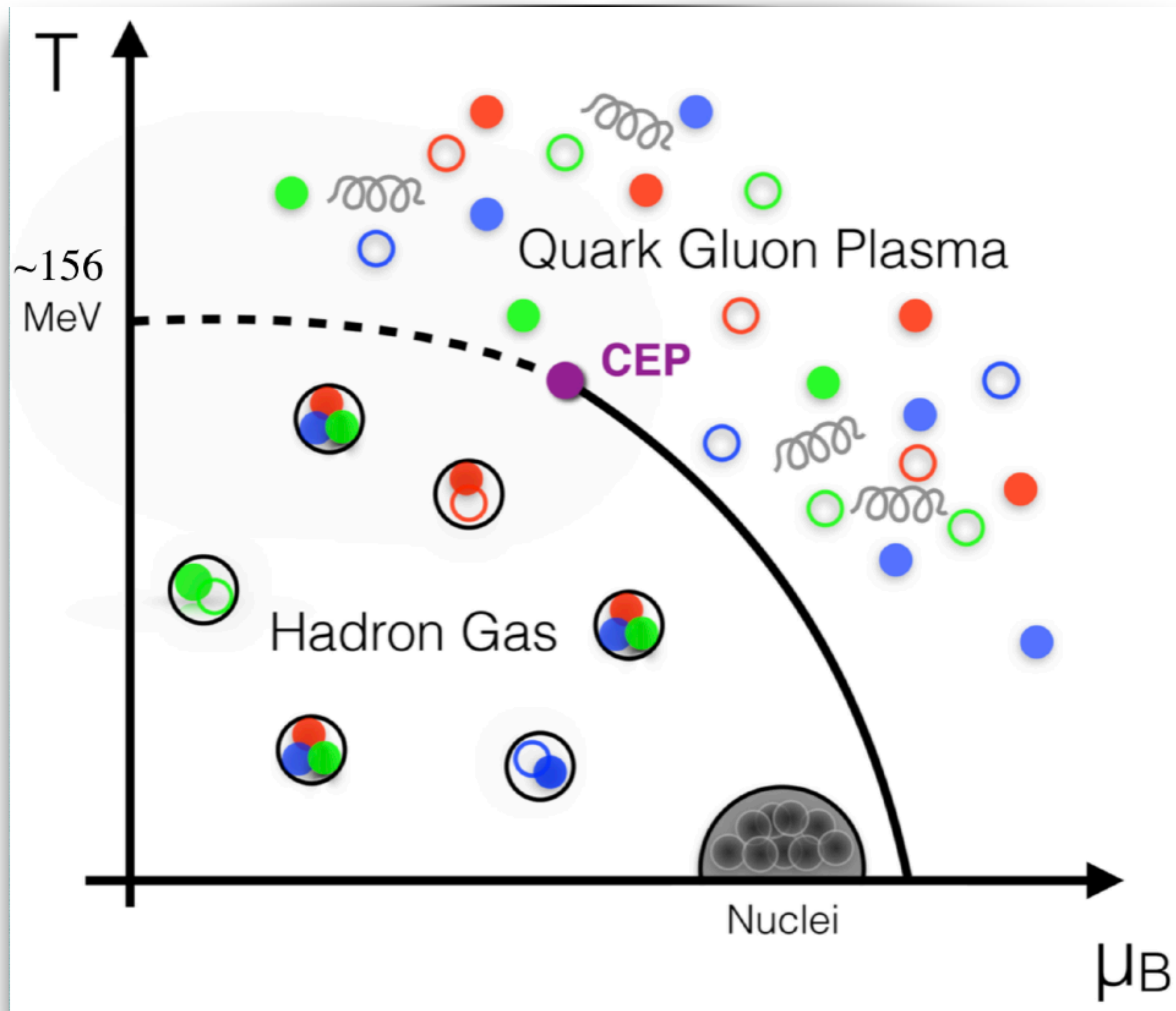
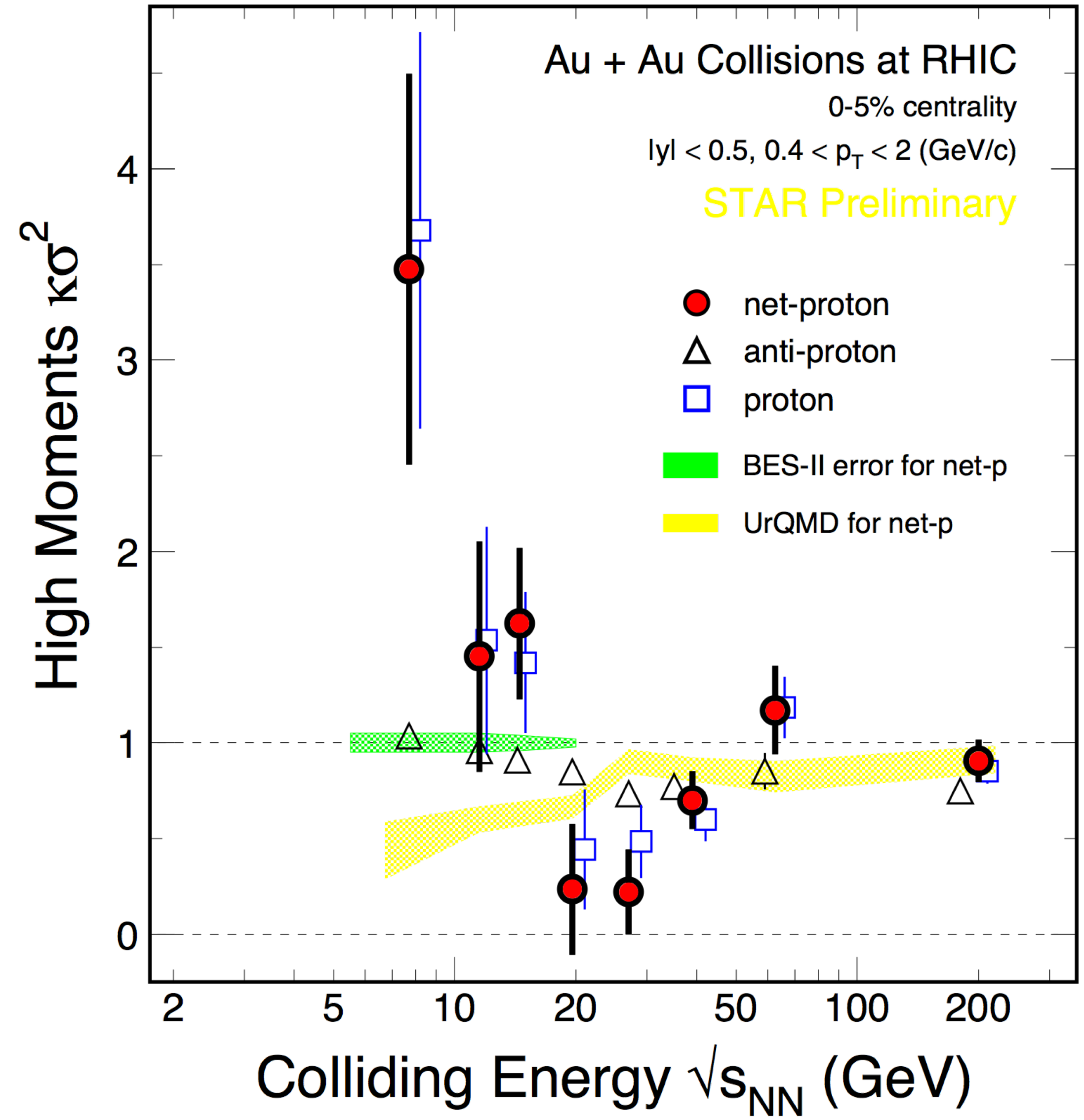
Search for QCD criticality: Critical End Point (CEP) ?



Sign problem at nonzero baryon chemical potential μB:
Taylor Expansion, Imaginary muB…

See review paper:
HTD, F. Karsch, S. Mukherjee,
"Thermodynamics of strong-interaction matter from Lattice QCD"
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

4th to 2nd order proton number fluctuations



See review paper:
X. F. Luo and N. Xu,
Nucl.Sci.Tech. 28 (2017) no.8, 112

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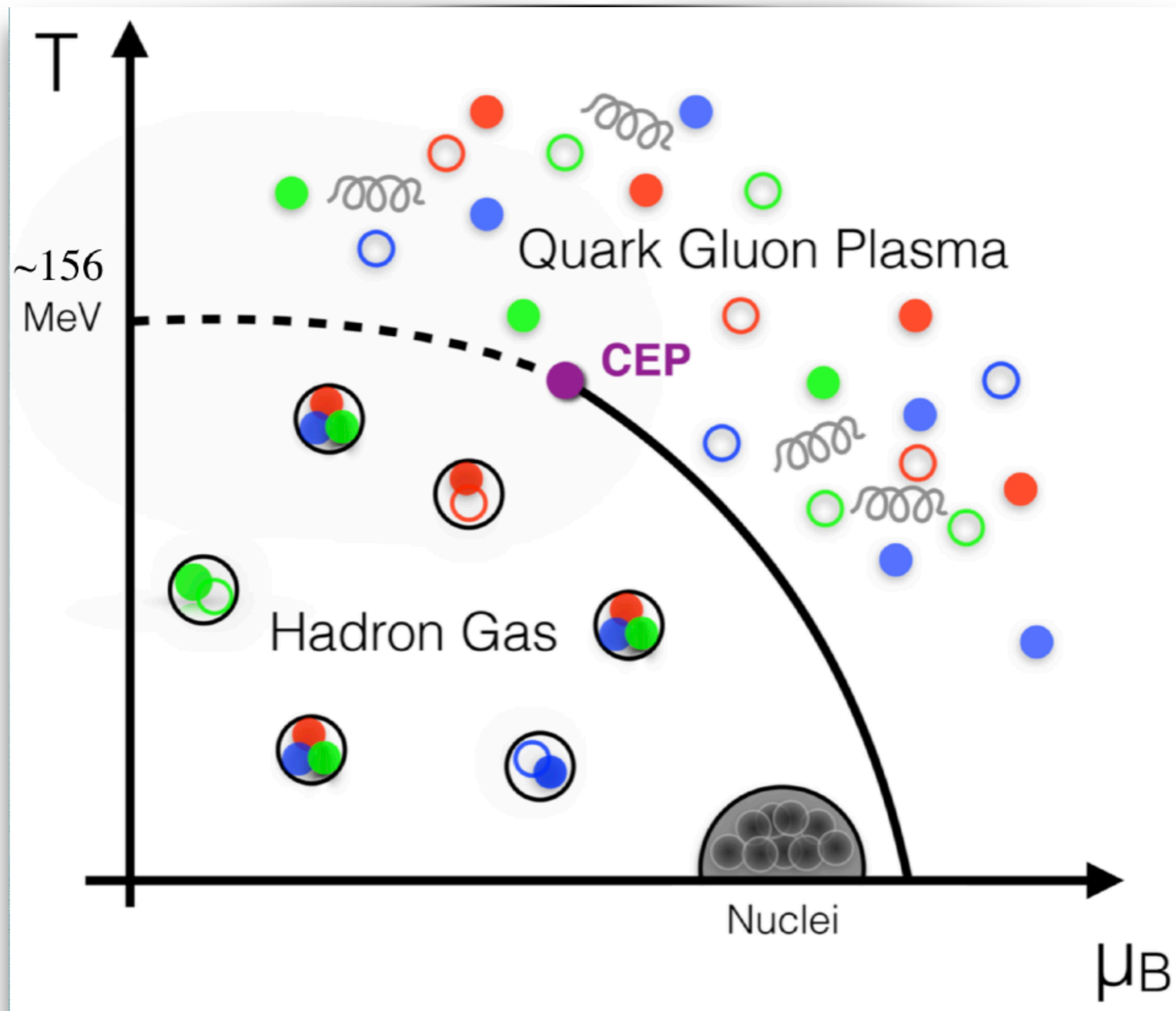

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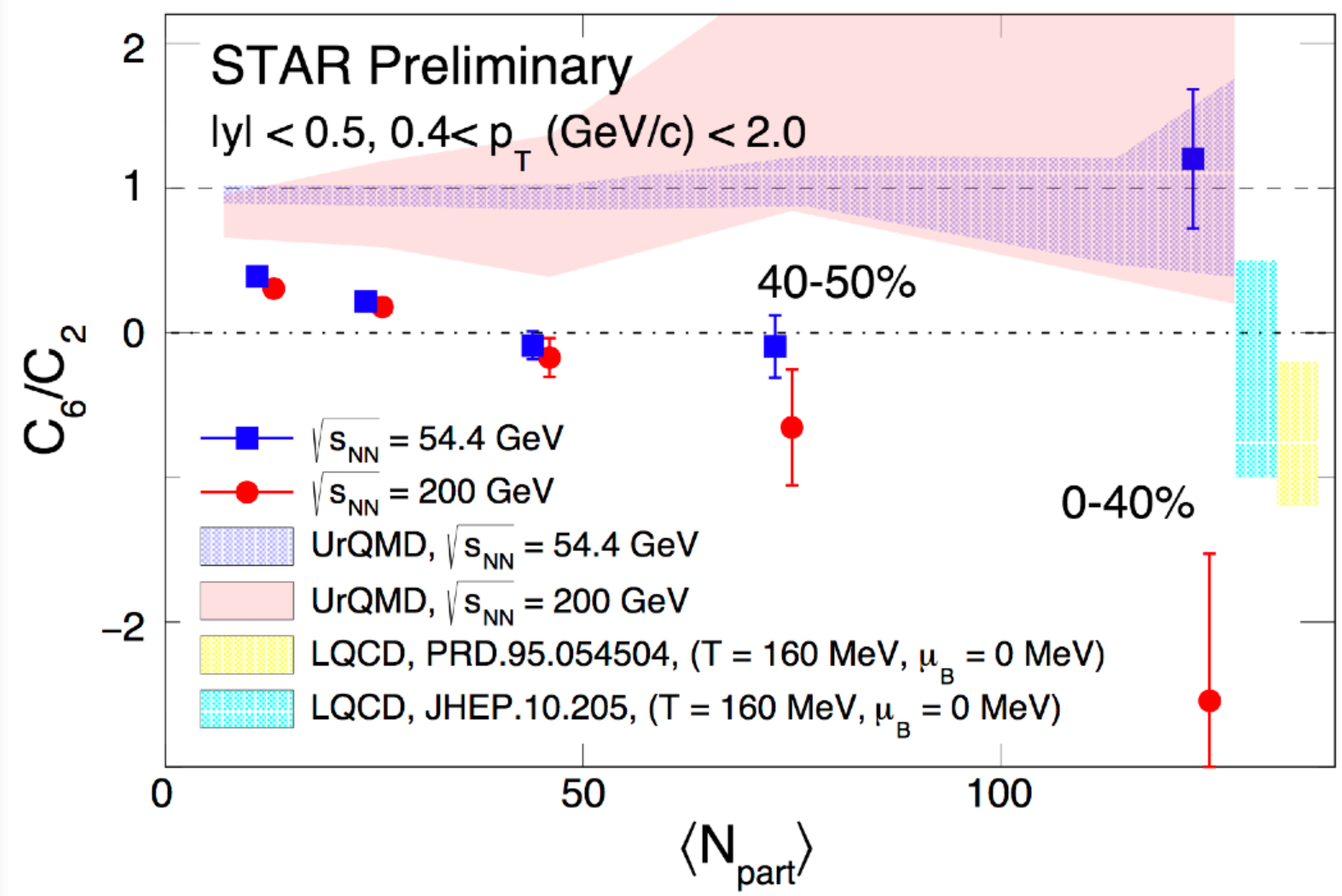
"Thermodynamics of strong-interaction matter from Lattice QCD"
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

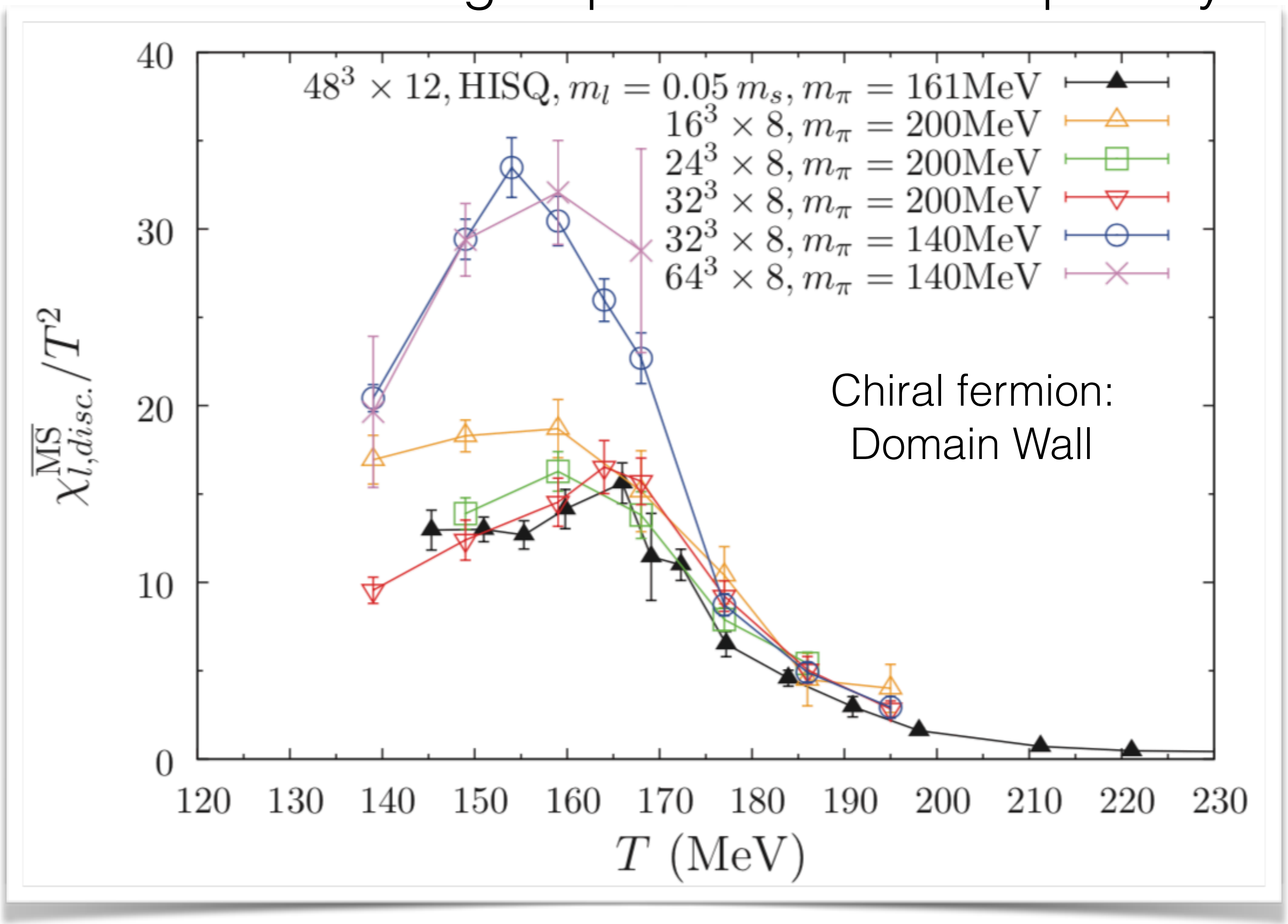
6th to 2nd order proton number fluctuations



Toshihiro Nonaka, Quark Matter 2019
Ashish Pandav, Quark Matter 2019

QCD crossover with m_π =140 MeV from Hadron phase to Quark Gluon Plasma phase

disconnected light quark chiral susceptibility



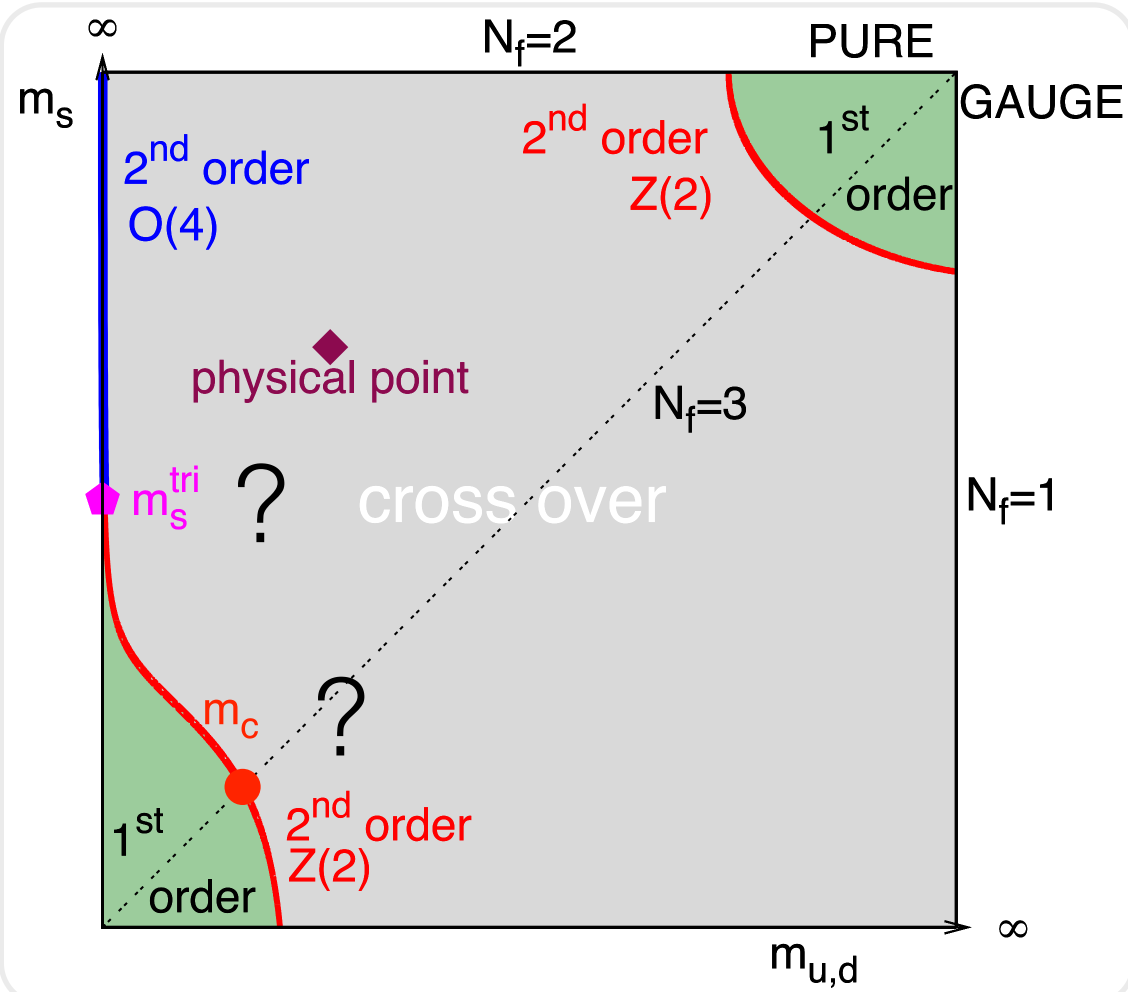
Bhattacharya, Buchoff, Christ, HTD et al.[HotQCD], Phys. Rev. Lett.,113(2014)082001

No Criticality:
Rapid crossover!

Ambiguities in the definition of **Pseudo-critical** temperature

QCD criticality relevant to the real world

Columbia plot:
QCD phase diagram in quark mass plane



- At physical point: crossover type transition

  [HotQCD] PRL,113(2014)082001, PRD 85(2012)054503
  [Wuppertal-Budapest], JHEP 1009 (2010) 073

- Critical lines of second order transition

  Pisarski & Wilczek PRD '84

  N_f=2: O(4) universality class Kogut & Sinclair, PRD '06

  N_f=3: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

- Chiral T_c^0: possible upper bound of transition T at the critical end point

  M. A. Halasz et al, PRD 58 (1998) 096007
  Y. Hatta & T. Ikeda, PRD67 (2003) 014028
  Hegde & HTD, PoS LATTICE2015 (2016) 14
  O. Kaczmarek et al., PRD83 (2011) 014504

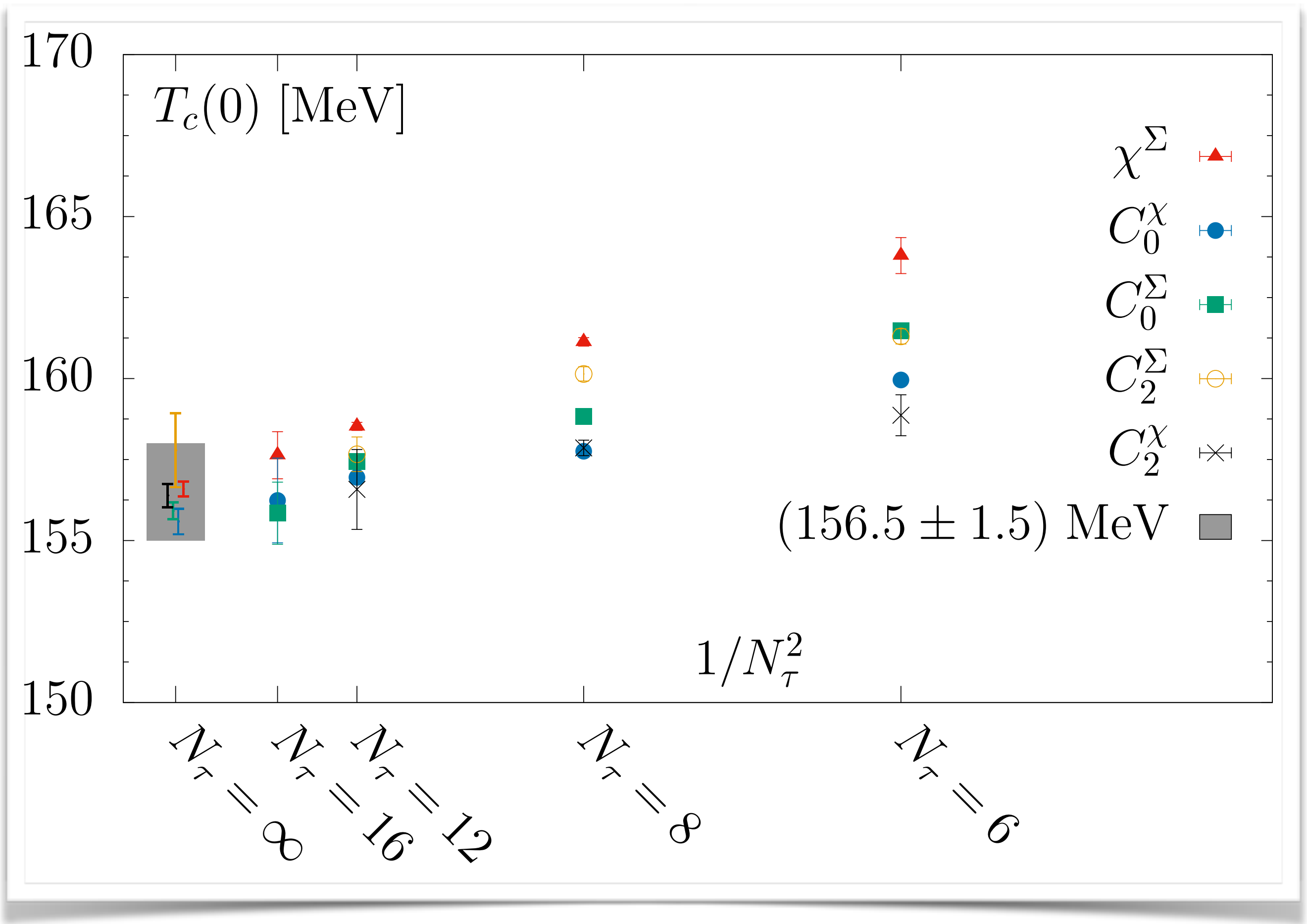
- Axial U(1) symmetry mostly remains broken at T_c

- 2nd order O(4) phase transition: more relavant

- Fundamental scale of QCD: chiral T_c^0 ?

QCD chiral crossover temperature with $m_\pi = 140$ MeV

New from



A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

Rigorous definition based on the O(4) critical behavior of QCD in the chiral limit

$$\partial_T \chi^\Sigma(T),\quad \partial_T C_0^\chi(T),\quad C_2^\chi(T) \quad \sim \quad m^{1/\delta - 1 - 1/\beta\delta} f'_\chi(z)$$

$$\partial_T^2 C_0^\Sigma(T),\quad \partial_T C_2^\Sigma(T) \quad \sim \quad m^{1/\delta - 2/\beta\delta} f''_G(z)$$

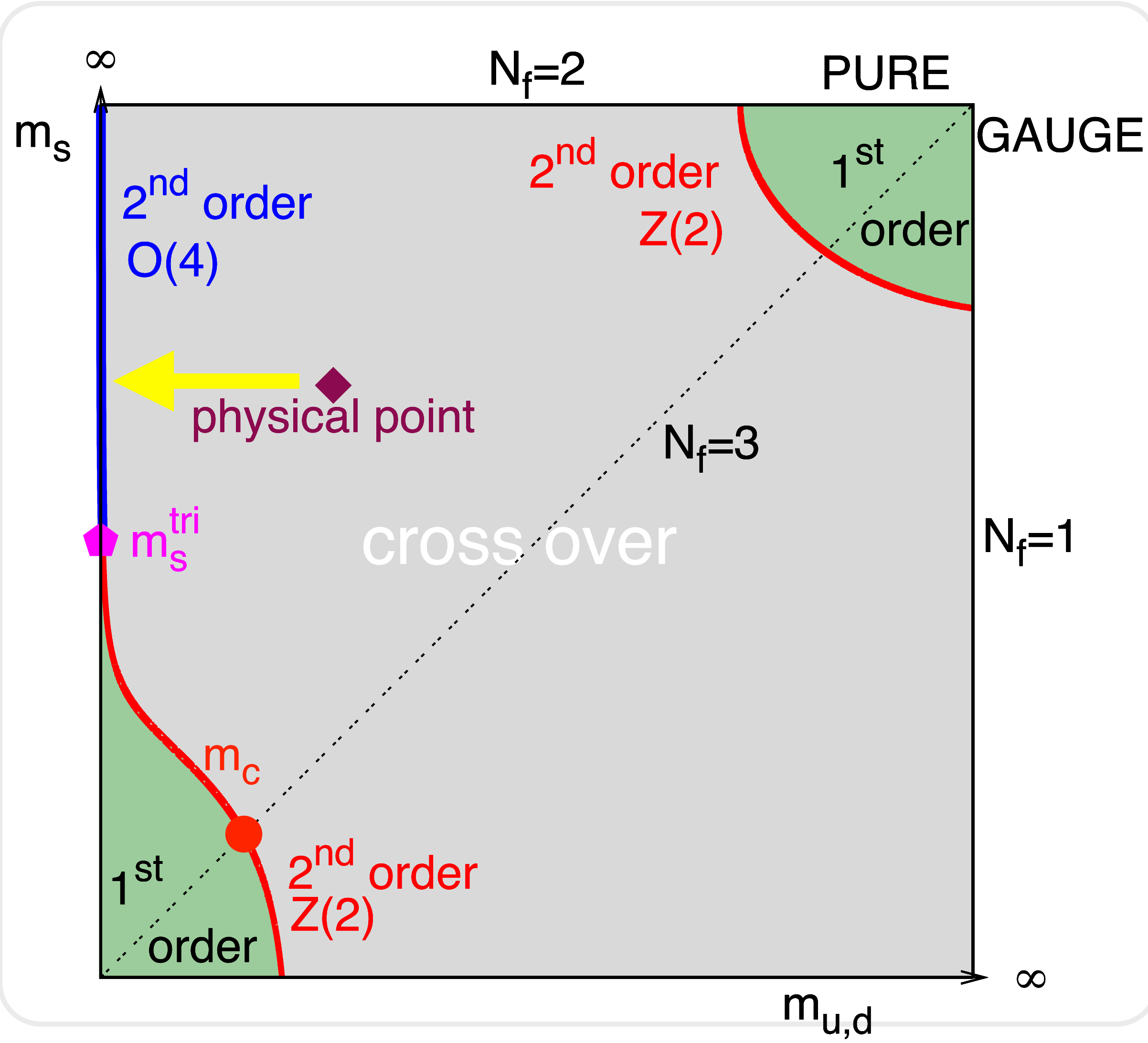
Chiral crossover temperature in the continuum limit:

$T_{pc} = 156.5(1.5)$ MeV

Chiral phase transition temperature

New from NSC³

Possible upper bound of the transition T at CEP



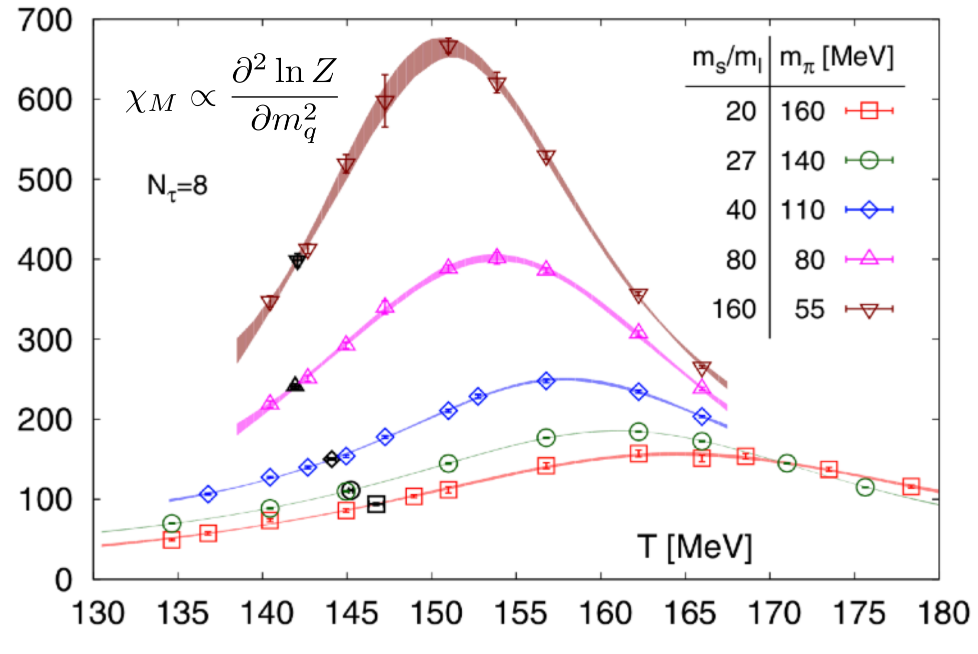
HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

- HISQ/tree action
- **N_f=2+1:**
  - ☑ Nt=6,8,12
  - ☑ $m_s^{phy}/\mathbf{m_l}$ = 20, 27, 40, 60, 80
    $m_\pi \approx$160,140,110,90,80,55 MeV
  - ☑ $7 \geq N_s/N_t \geq 4 \Leftrightarrow 5 \gtrsim m_\pi L \gtrsim 3$

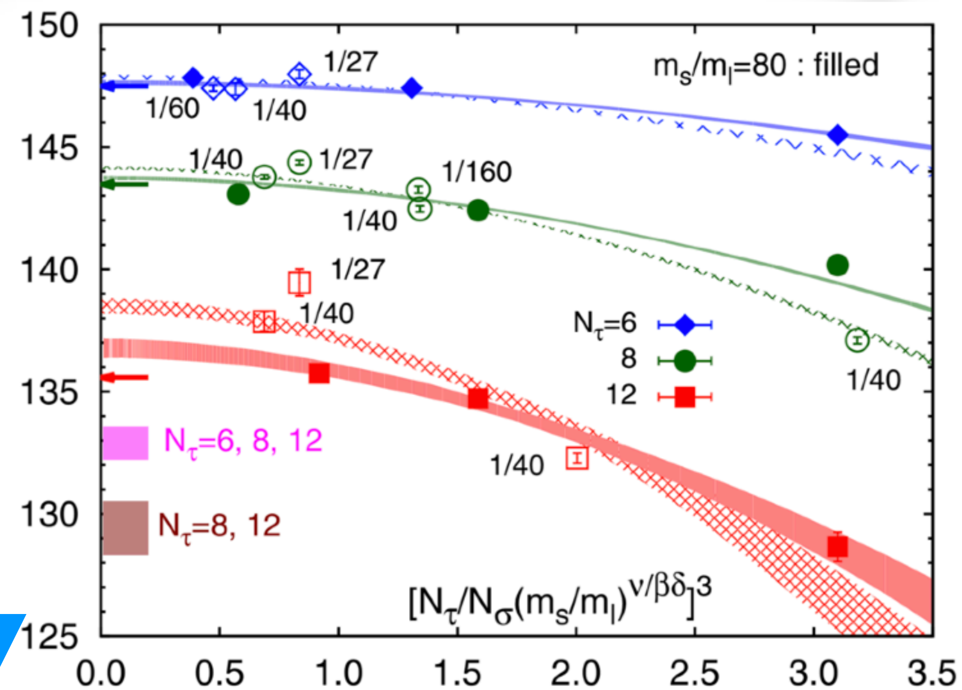
This allows us to perform
infinite volume, continuum and then chiral extrapolation!

O(4) scaling analyses: Nf=2+1, Nt=6,8 & 12 lattices, M_π down to 55 MeV thermodynamic, continuum & chiral extrapolated

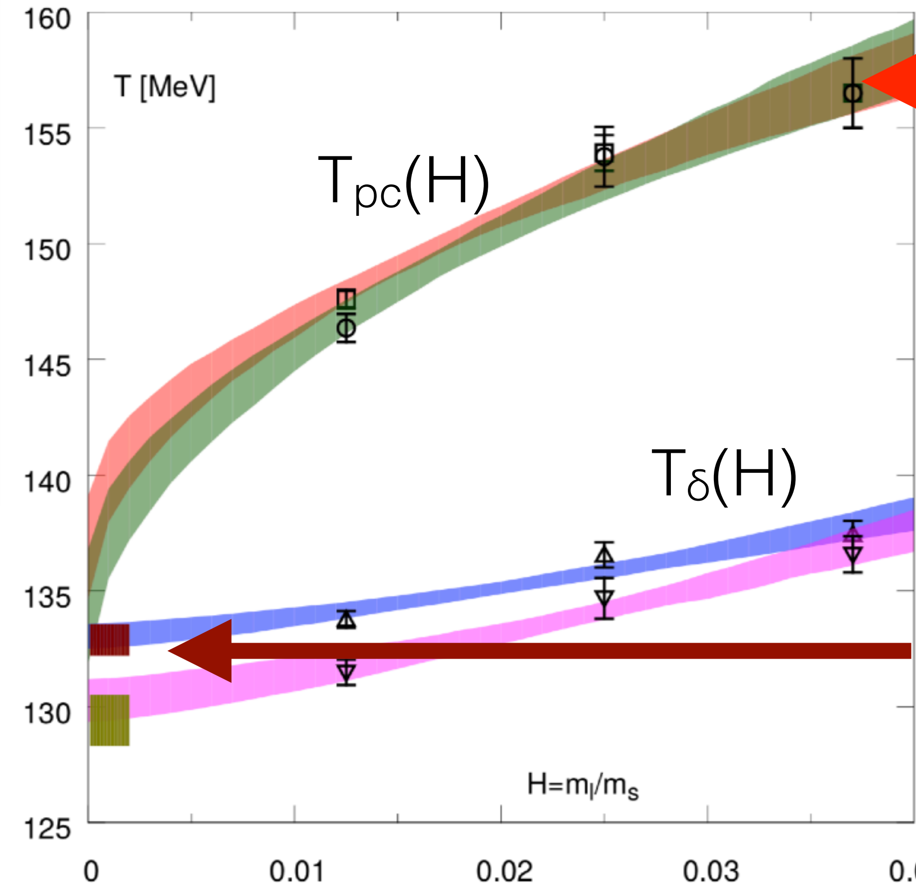
Order parameter susceptibility



Continuum extrapolation



Infinite volume & chiral extrapolation



Chiral crossover
T_{pc}^{phy} = 156.5(1.5) MeV

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

Chiral phase transition T

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

T_c^0 is ~ 25 MeV smaller than the chiral crossover T!

See QCD-inspired model calculations:
e.g. J. Berges, D. U. Jungnickel and C. Wetterich, Phys. Rev.D59, 034010 (1999)
J. Braun, B. Klein, H.-J. Pirner and A. H. Rezaeian,Phys. Rev. D73, 074010 (2006)

Explore the QCD phase diagram through fluctuations of conserved charges x=B,Q,S

HIC

mean: M_x

variance: σ_x^2

skewness: S_x

kurtosis: κ_x

hyper-skewness: S_x^h

hyper-kurtosis: κ_x^h

$$\frac{M_x(\sqrt{s})}{\sigma_x^2(\sqrt{s})} = \frac{\langle N_x \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_1^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{12}^x(T,\mu_B)$$

$$\frac{S_x(\sqrt{s})\,\sigma_x^3(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^3 \rangle}{\langle N_x \rangle} = \frac{\chi_3^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{31}^x(T,\mu_B)$$

$$\kappa_x(\sqrt{s})\,\sigma_x^2(\sqrt{s}) = \frac{\langle (\delta N_x)^4 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_4^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{42}^x(T,\mu_B)$$

$$\frac{S_x^h(\sqrt{s})\,\sigma_x^5(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^5 \rangle}{\langle N_x \rangle} = \frac{\chi_5^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{51}^x(T,\mu_B)$$

$$\kappa_x^h(\sqrt{s})\,\sigma_x^4(\sqrt{s}) = \frac{\langle (\delta N_x)^6 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_6^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{62}^x(T,\mu_B)$$

LQCD

generalized susceptibilities

$$\chi_n^x(T,\mu_B) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_x/T)^n}$$

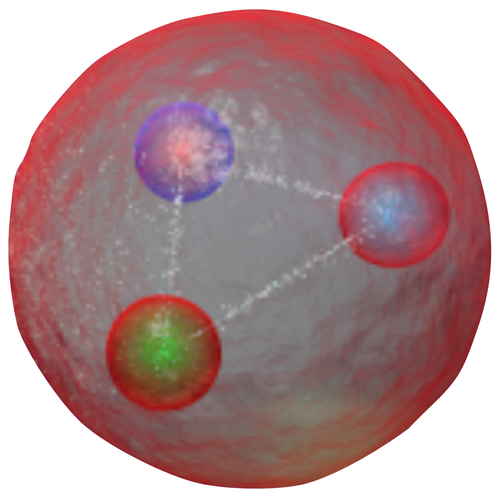
This makes the comparison between Exp. and LQCD possible

Change in Degree of freedom

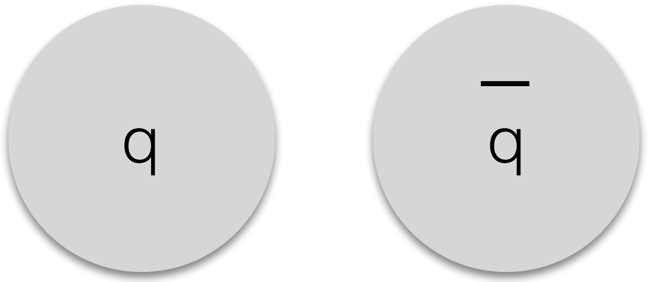
Baryon number —> B

Baryon number fluctuation —> B^2

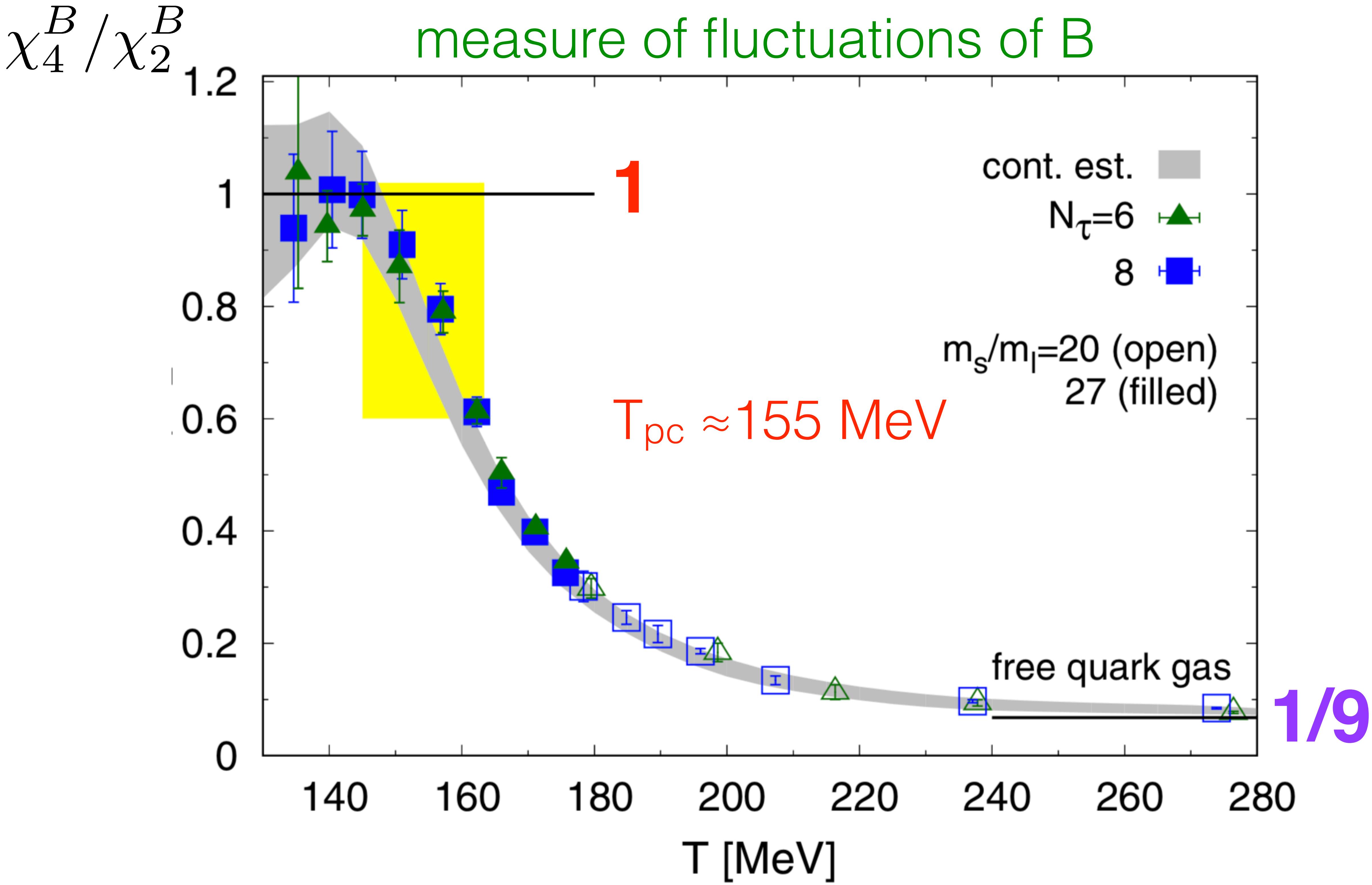
Bound state of quarks:
$B=1, -1$
$B^2=1$



Unbounded quarks:
$B=1/3, -1/3$
$B^2=1/9$

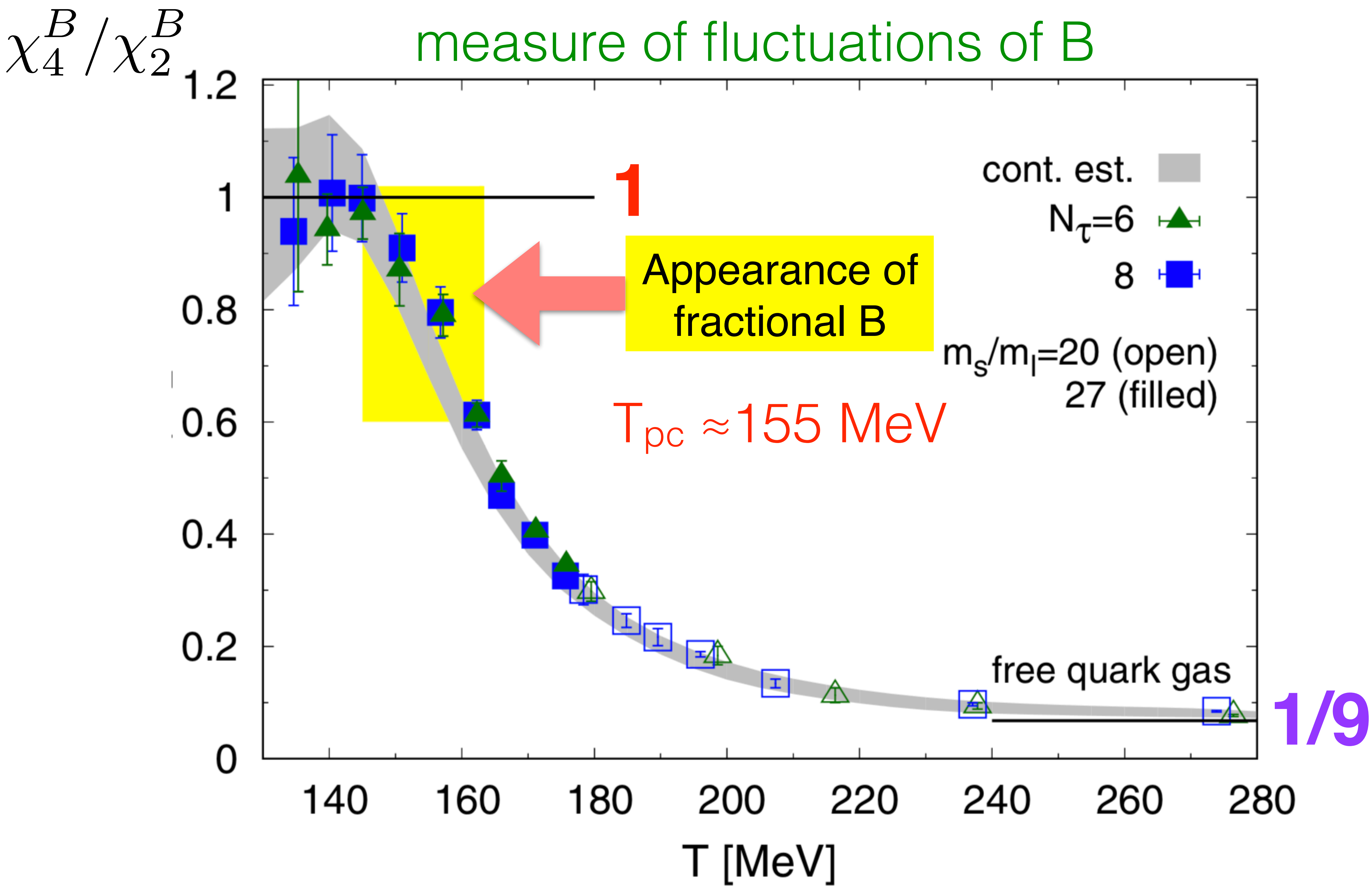


Change in Degree of freedom



Bielefeld-BNL-CCNU: PRL 111(2013) 082301, PLB 737(2014) 210

Change in Degree of freedom



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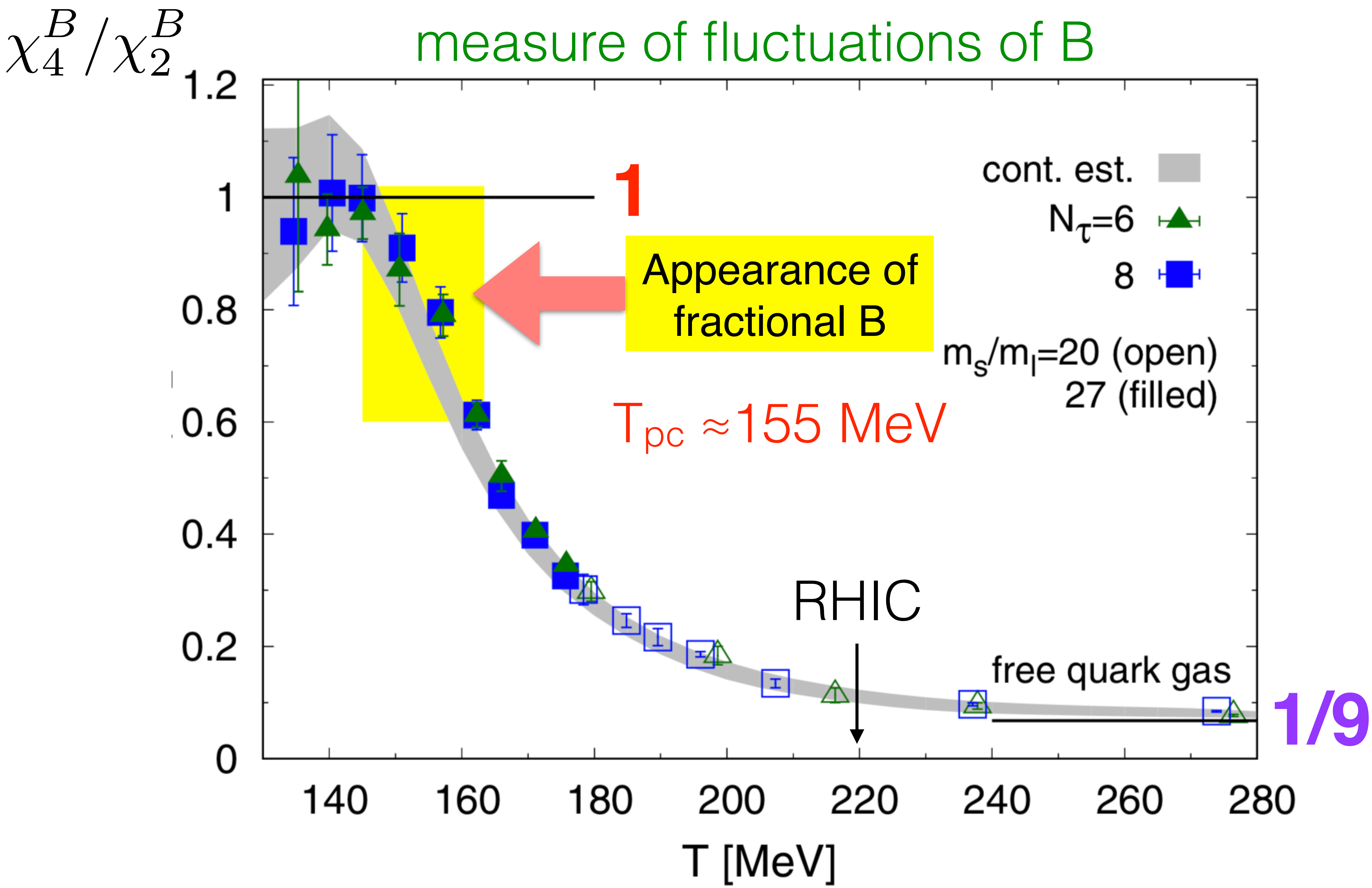
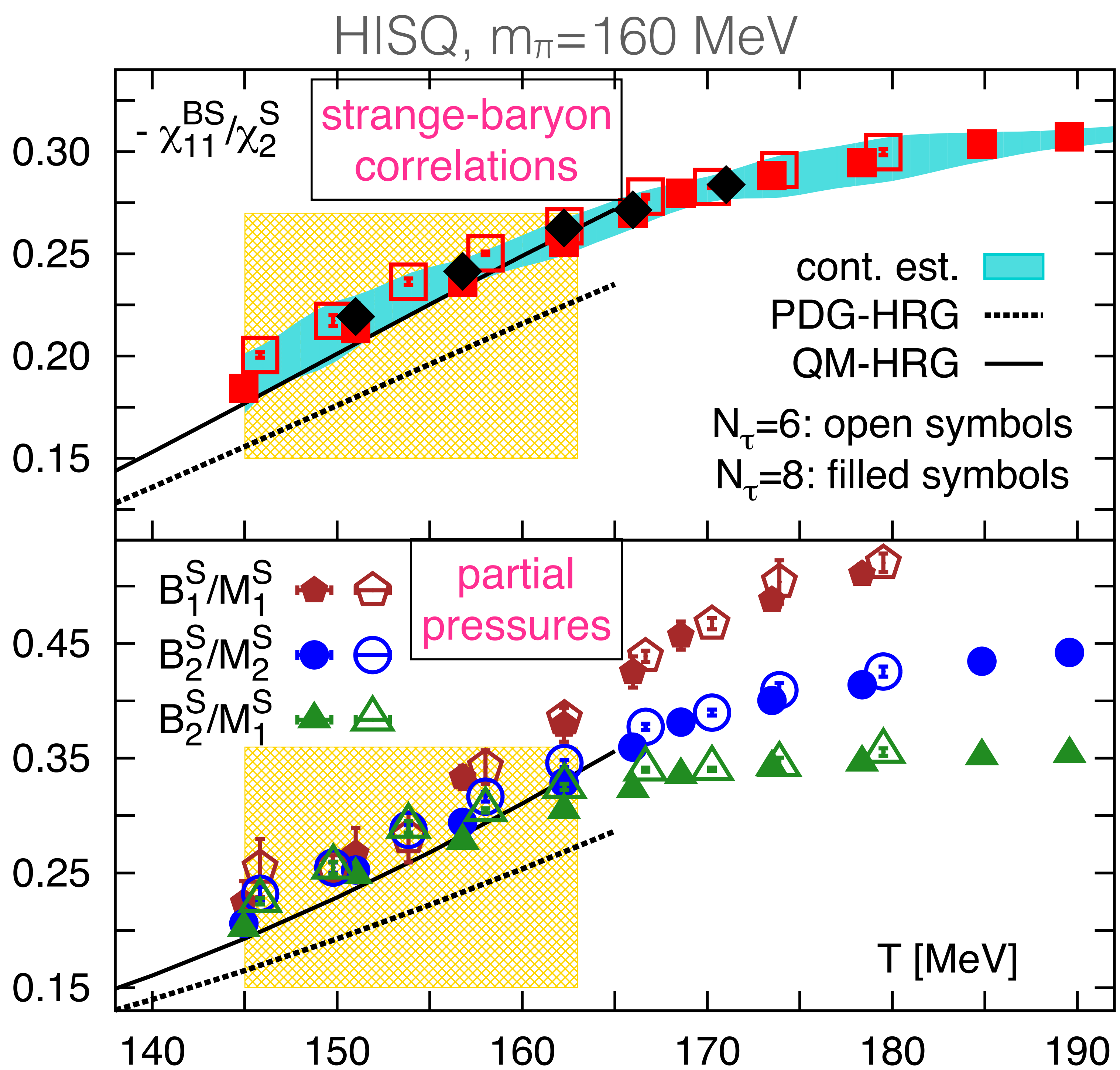
Change in Degree of freedom



Bielefeld-BNL-CCNU: PRL 111(2013) 082301, PLB 737(2014) 210

Indirect evidence of experimentally not yet observed strange states hinted from QCD thermodynamics



PDG-HRG: Hadron Resonance Gas model calculations with spectrum from PDG

QM-HRG: Similar as PDG-HRG but with spectrum from Quark Model

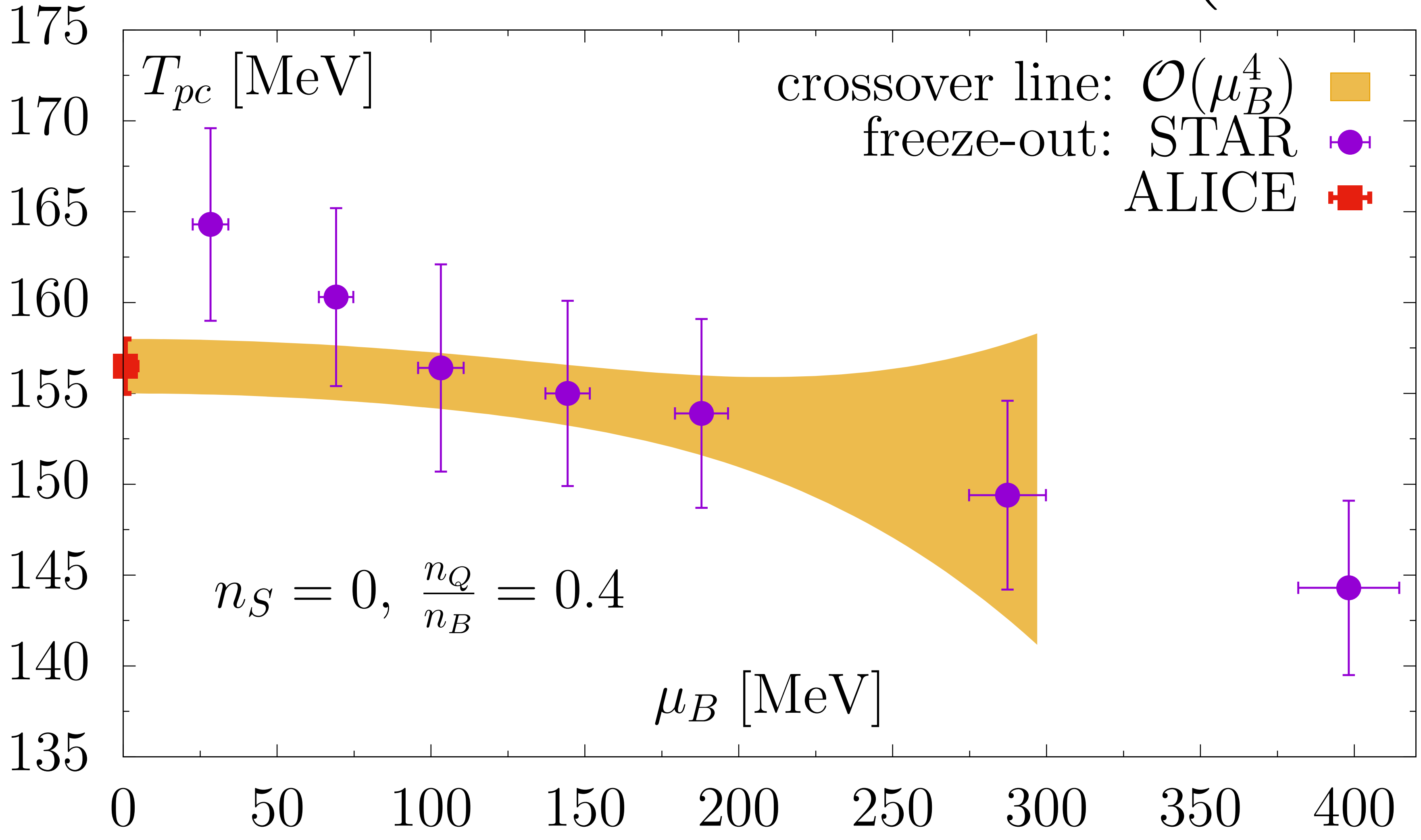
similar findings for charmed states

Bielefeld-BNL-CCNU, PLB737 (2014) 210-215

A. Bazavov, HTD et al.[Bielefeld-BNL-CCNU], Phys. Rev. Lett. 113 (2014)072001

QCD meets Experiment

Chiral crossover line: $T_{pc}(\mu_B) = T_{pc}(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4\right)$



ALICE data point:

T_f = 156.5(1.5) MeV

Andronic et al,
Nature 561 (7723) (2018) 321

STAR data points:

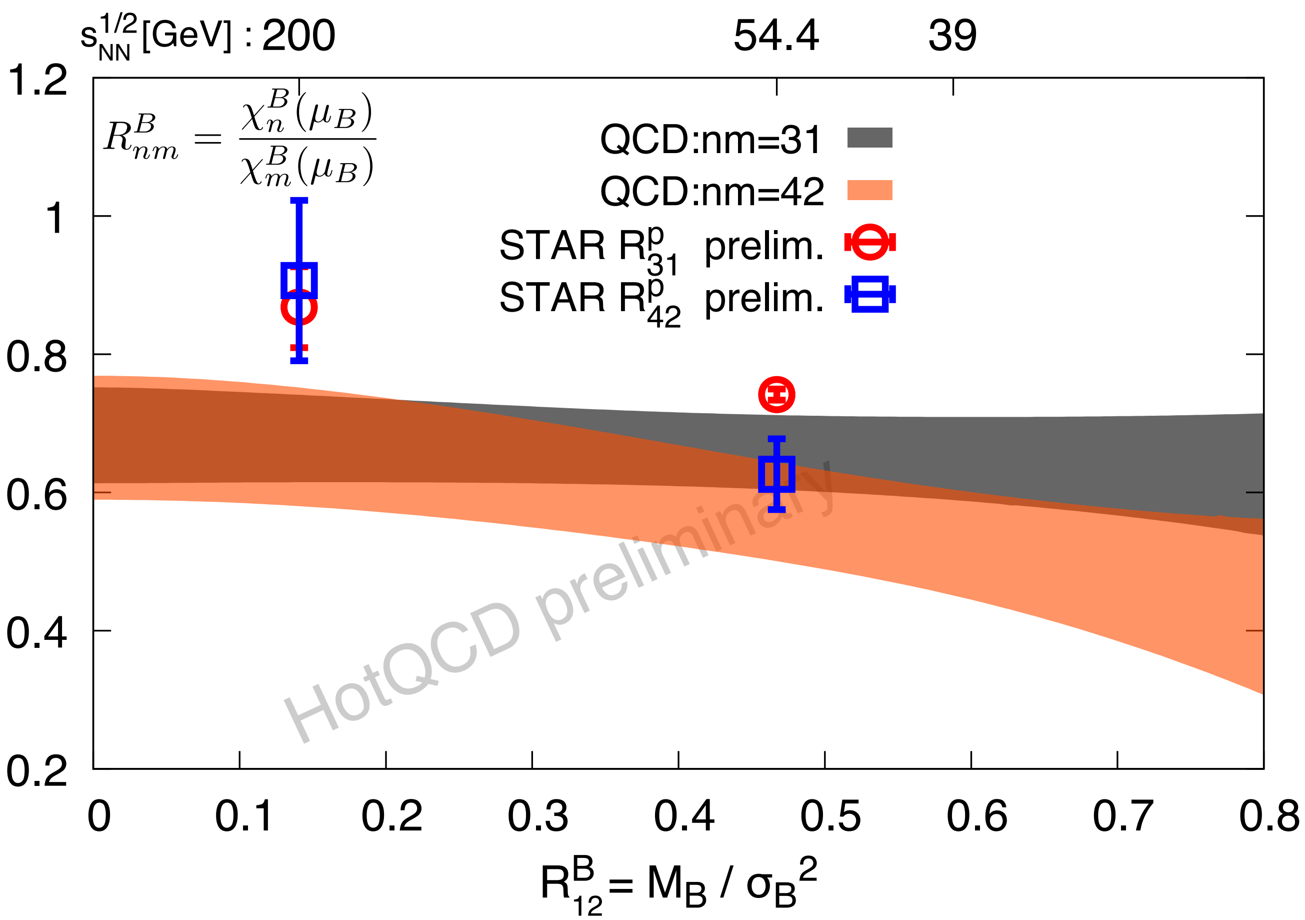
Adamczyk et al.,
Phys. Rev. C 96 (4) (2017) 044904

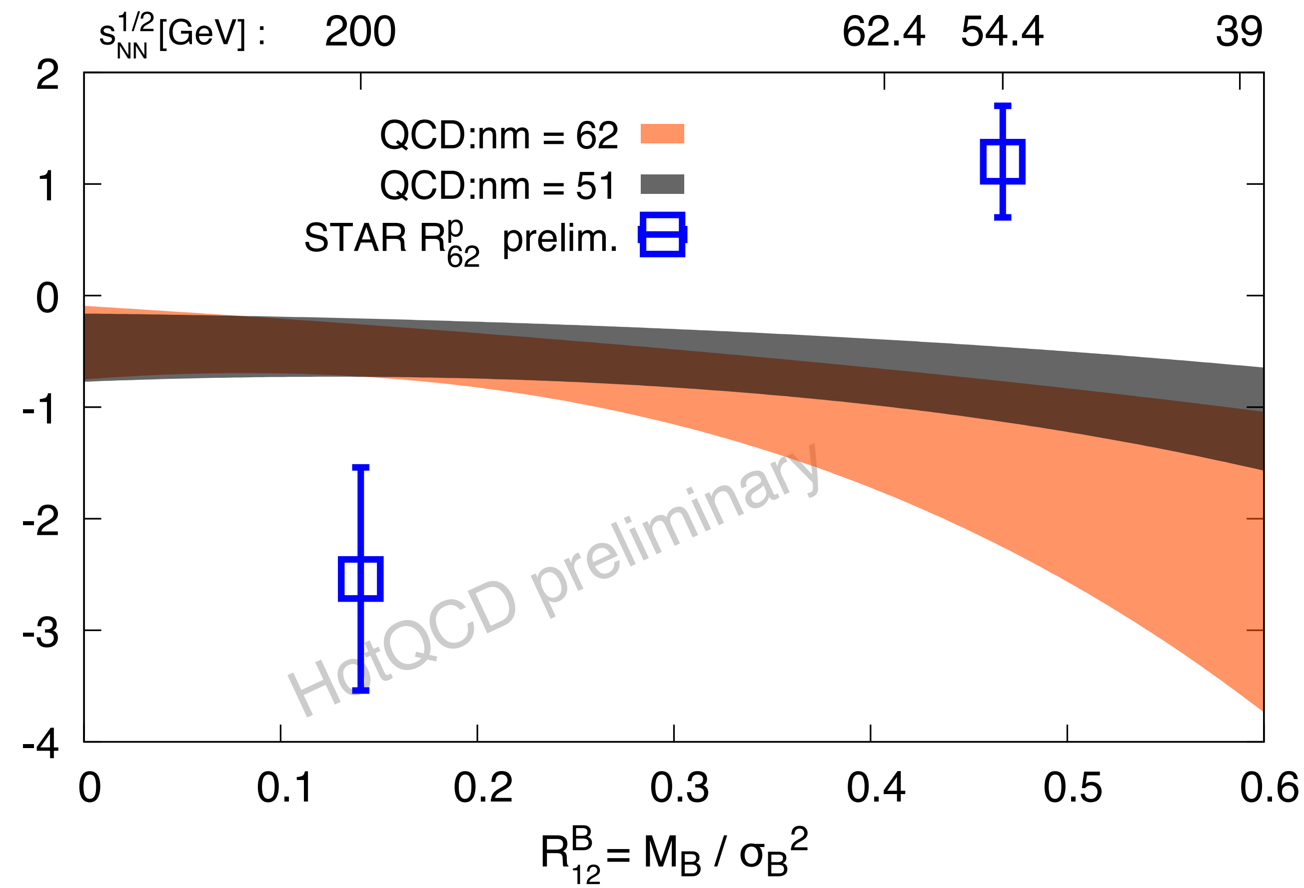
ALICE data point is in perfect agreement with QCD
STAR data at $\sqrt{s_{NN}}$=200 GeV give higher freeze out T of 164 MeV

No critical point till μ_B=300 MeV

QCD meets Experiment

HTD, Quark Matter 2019 Plenary



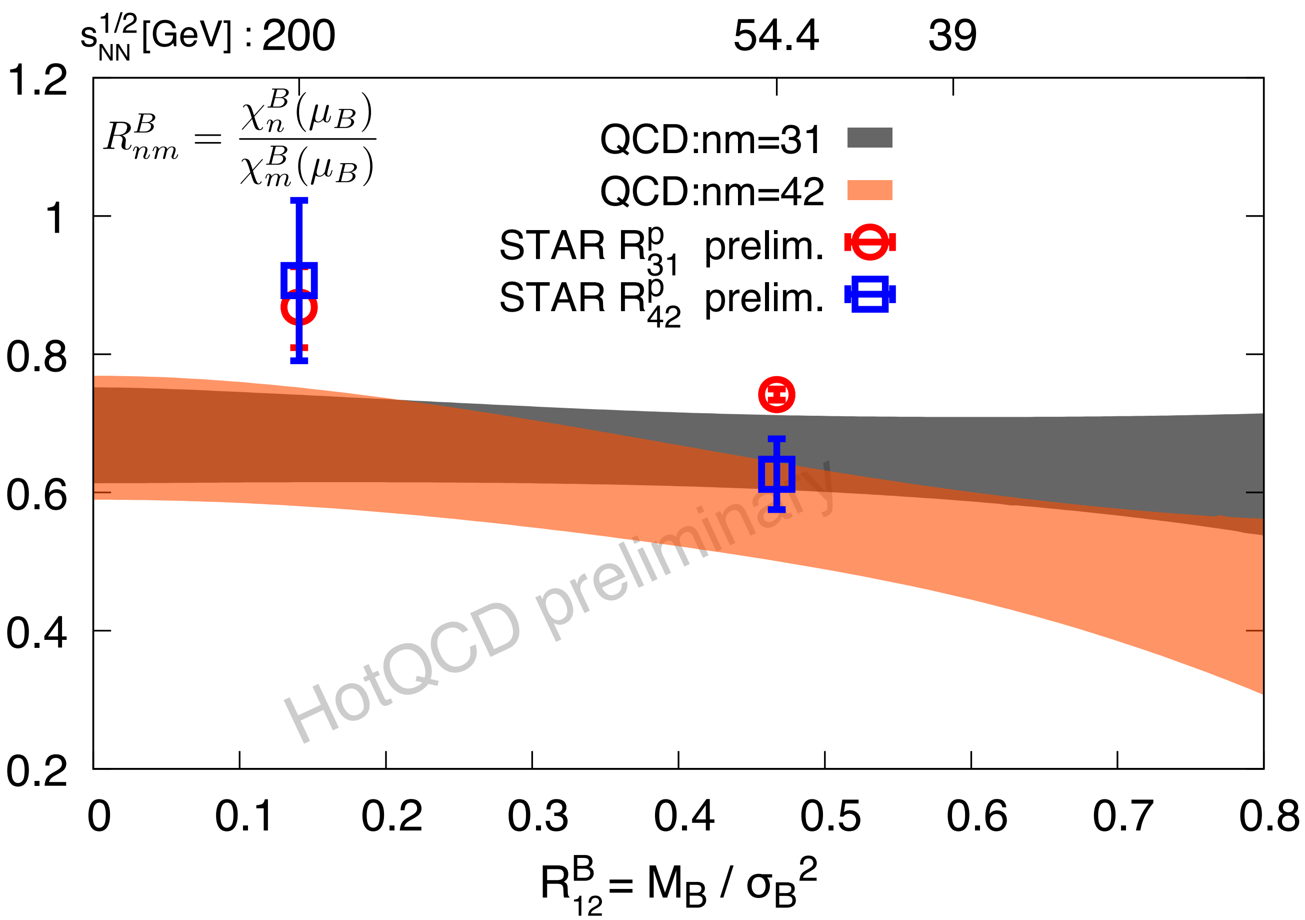


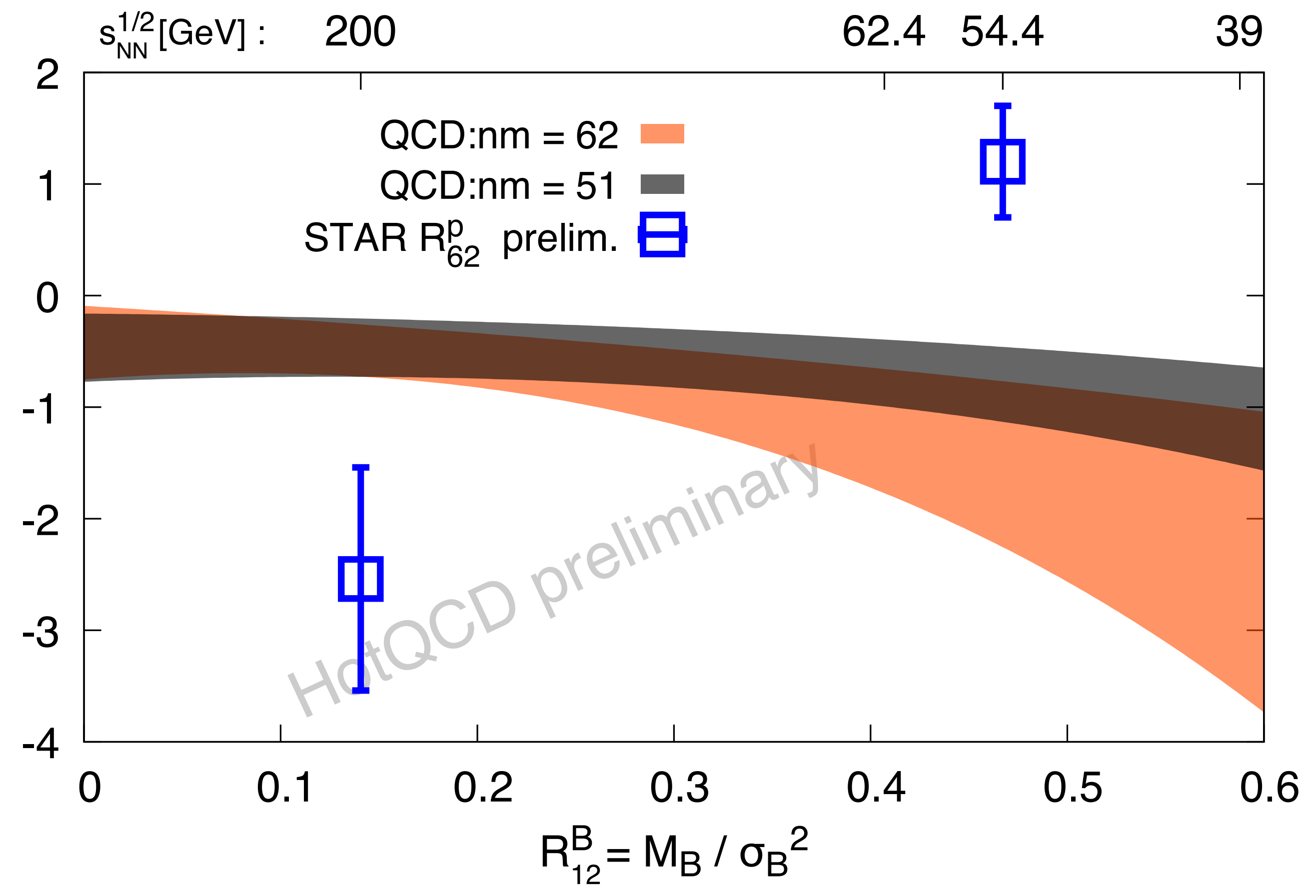
R_{31} and R_{42}:
$\sqrt{s_{NN}}$=200 GeV: deviation from QCD
$\sqrt{s_{NN}}$=54.4 GeV: consistent with QCD

R_{62}:
$\sqrt{s_{NN}}$=200 & 54.4 GeV:
deviation from QCD

QCD meets Experiment

HTD, Quark Matter 2019 Plenary





R_{31} and R_{42}:
$\sqrt{s_{NN}}$=200 GeV: deviation from QCD
$\sqrt{s_{NN}}$=54.4 GeV: consistent with QCD

R_{62}:
$\sqrt{s_{NN}}$=200 & 54.4 GeV:
deviation from QCD

???: New Physics: Proton v.s. Baryon, Non-equilibrium…

Summary

- Chiral crossover transition temperature is determined:

$$T_{pc} = 156.5(1.5)\ \mathrm{MeV}$$

A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

- The possible location of critical end point is

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002

$$T_c^{CEP} \lesssim T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

$$\mu_B^{CEP} > 300\mathrm{MeV}$$

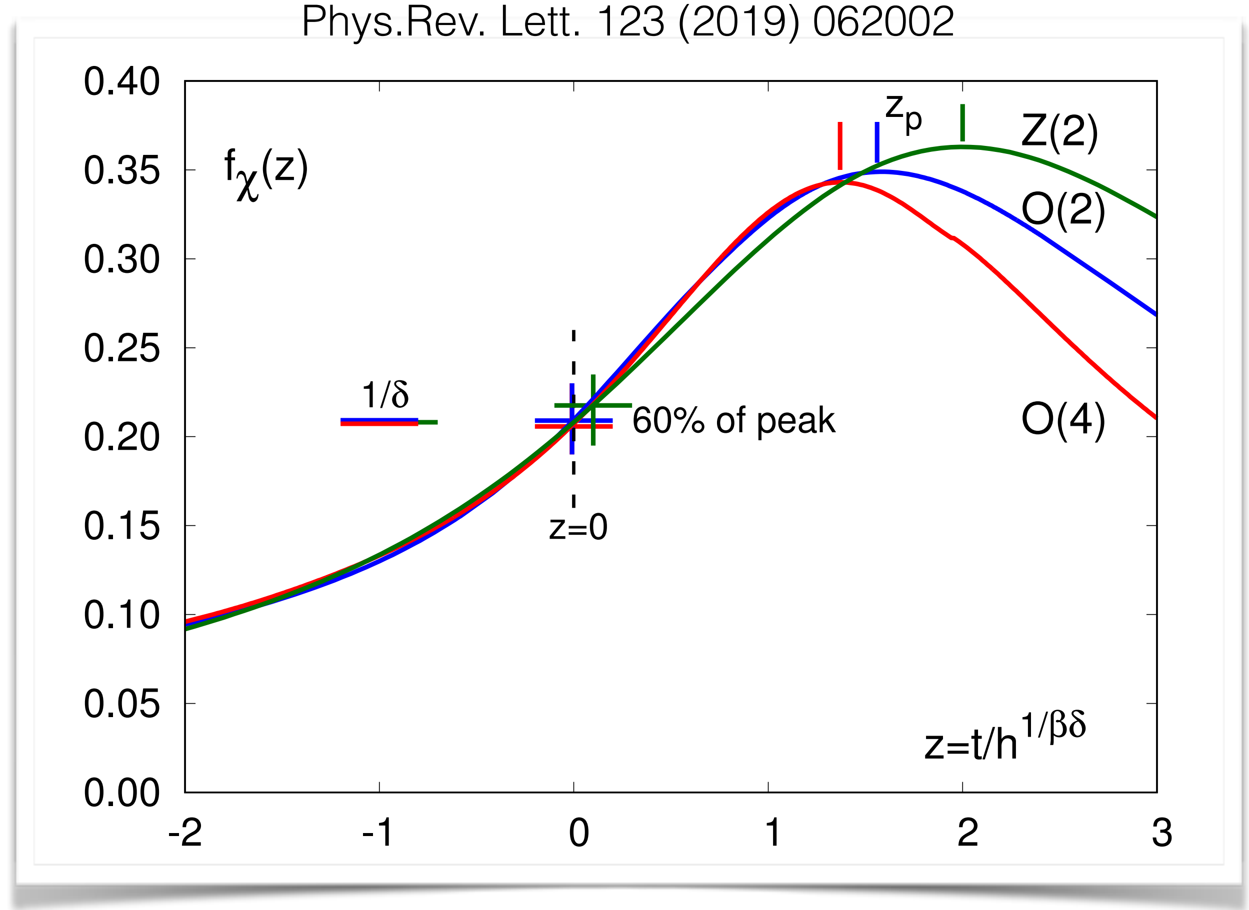
- Higher order fluctuations: QCD meets Experiment, more to explore

Thanks for your attention!

谢谢！

A novel approach to estimate T_c^0

HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002



- Pseudo-critical temperature at H

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

$$z = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0}\left(\frac{H}{h_0}\right)^{-1/\beta\delta} = z_0 \frac{T - T_c^0}{T_c^0} H^{-1/\beta\delta}$$

- Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(T_\delta, H, L)}{M(T_\delta, H, L)} = \frac{1}{\delta} \quad \Longleftrightarrow \quad z(T_\delta)=0$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max} \quad \Longleftrightarrow \quad z(T_{60})\simeq 0$$

z_p: peak location of the susceptibility

z_{60}: location of 60% of peak height from left

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

- ☑ small quark mass dependence
- ☑ small variations among universality classes

Lattice QCD simulation at μ_B =/=0

fermion sign problem

QCD:

$$Z = \mathrm{Tr}\left[e^{-(H-\mu N)/T}\right] = \int [\mathrm{d}A]\, \det[D + m_q + i\mu\gamma_4]\, e^{-S(A)}$$

complex

Toy model: Yagi, Hatsuda & Miake, "Quark-Gluon Plasma — From Big Bang to Little Bang"

$$Z = \sum_{\{\phi(x)=\pm 1\}} \mathrm{sign}(\phi)\, e^{-S(\phi)}\,; \qquad Z_0 = \sum_{\{\phi(x)=\pm 1\}} e^{-S(\phi)}$$

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O}(\phi)\, \mathrm{sign}(\phi) \rangle_0}{\langle \mathrm{sign}(\phi) \rangle_0}\,, \quad \langle \mathrm{sign}(\phi) \rangle_0 = \frac{Z}{Z_0} = e^{-(f-f_0)V/T} \ll 1$$

$f(f_0)$: free energy density corresponding to $Z(Z_0)$

$$\frac{\Delta \mathrm{sign}(\phi)}{\langle \mathrm{sign}(\phi) \rangle_0} = \frac{\sqrt{\langle \mathrm{sign}^2 \rangle_0 - \langle \mathrm{sign} \rangle_0^2}}{\sqrt{N} \langle \mathrm{sign} \rangle_0} \simeq \frac{e^{(f-f_0)V/T}}{\sqrt{N}} \ll 1$$

➡

$$N \gg e^{2(f-f_0)V/T}$$

Beam Energy Scan at RHIC

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	BES II / BES I	Weeks	μ_B (MeV)	T_{CH} (MeV)
200	350	2010		25	166
62.4	67	2010		73	165
39	39	2010		112	164
27	70	2011		156	162
19.6	**400** / 36	**2019-20** / 2011	**3**	206	160
14.5	**300** / 20	**2019-20** / 2014	**2.5**	264	156
11.5	**230** / 12	**2019-20** / 2010	**5**	315	152
9.2	**160** / 0.3	**2019-20** / 2008	**9.5**	355	140
7.7	**100** / 4	**2019-20** / 2010	**14**	420	140

Courtesy of N. Xu

Theoretically it is crucial to know:

QCD Equation of State for $\mu_B/T \lesssim 3$

The possible location of CP or window of criticality

Location of the transition and freeze out lines