



TESTING NATURALNESS

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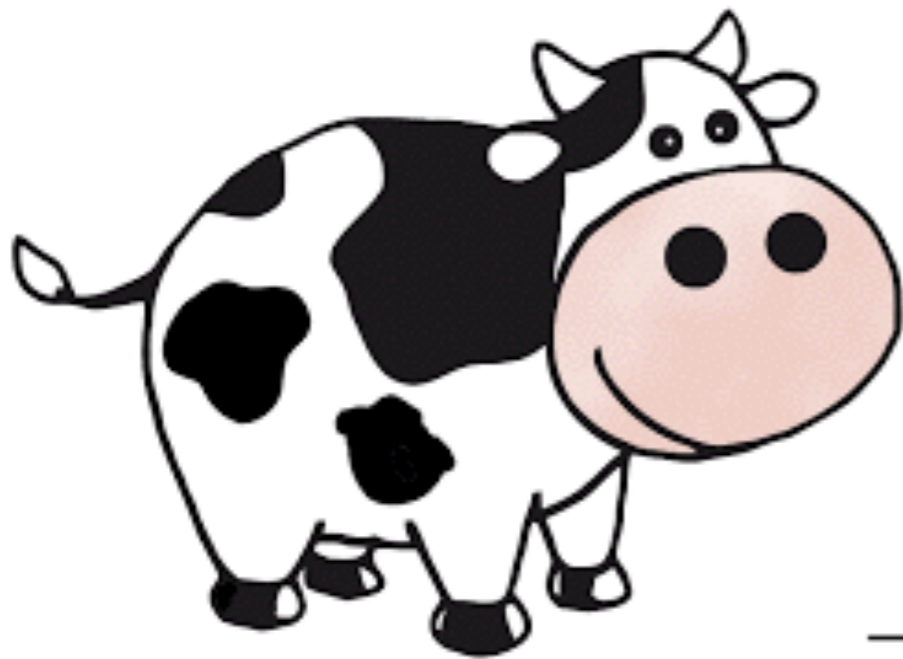
Based on

[C. Chen, J. Hajer, TL, I. Low and H. Zhang, arXiv: 1705.07743 (JHEP 2017)]



Naturalness Problem in Particle Physics

A **large discrepancy** between two energy scales expected to be close, without any protection mechanism

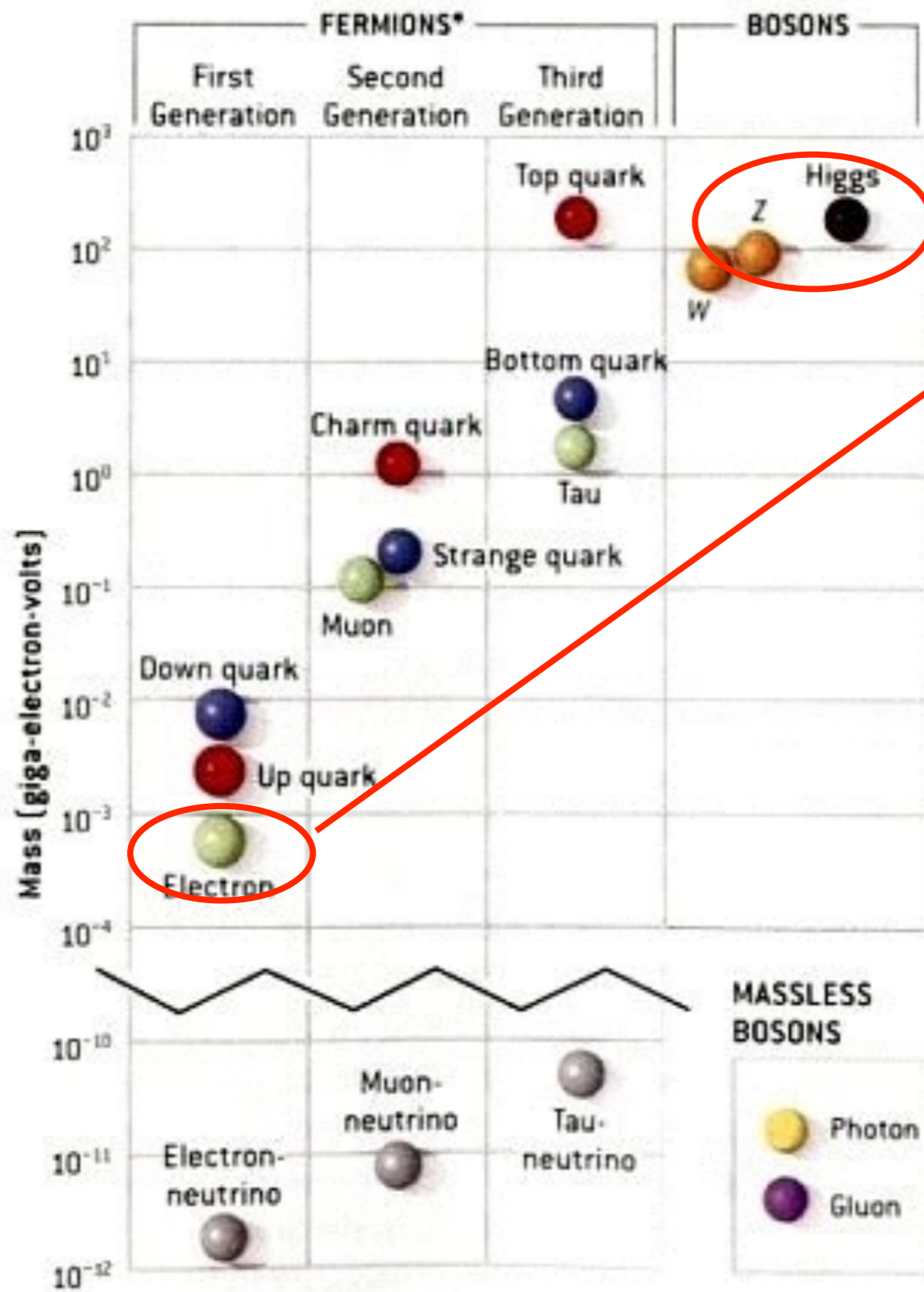


Unnatural, unless some underlying protection mechanism exists!





Naturalness Problem in Particle Physics



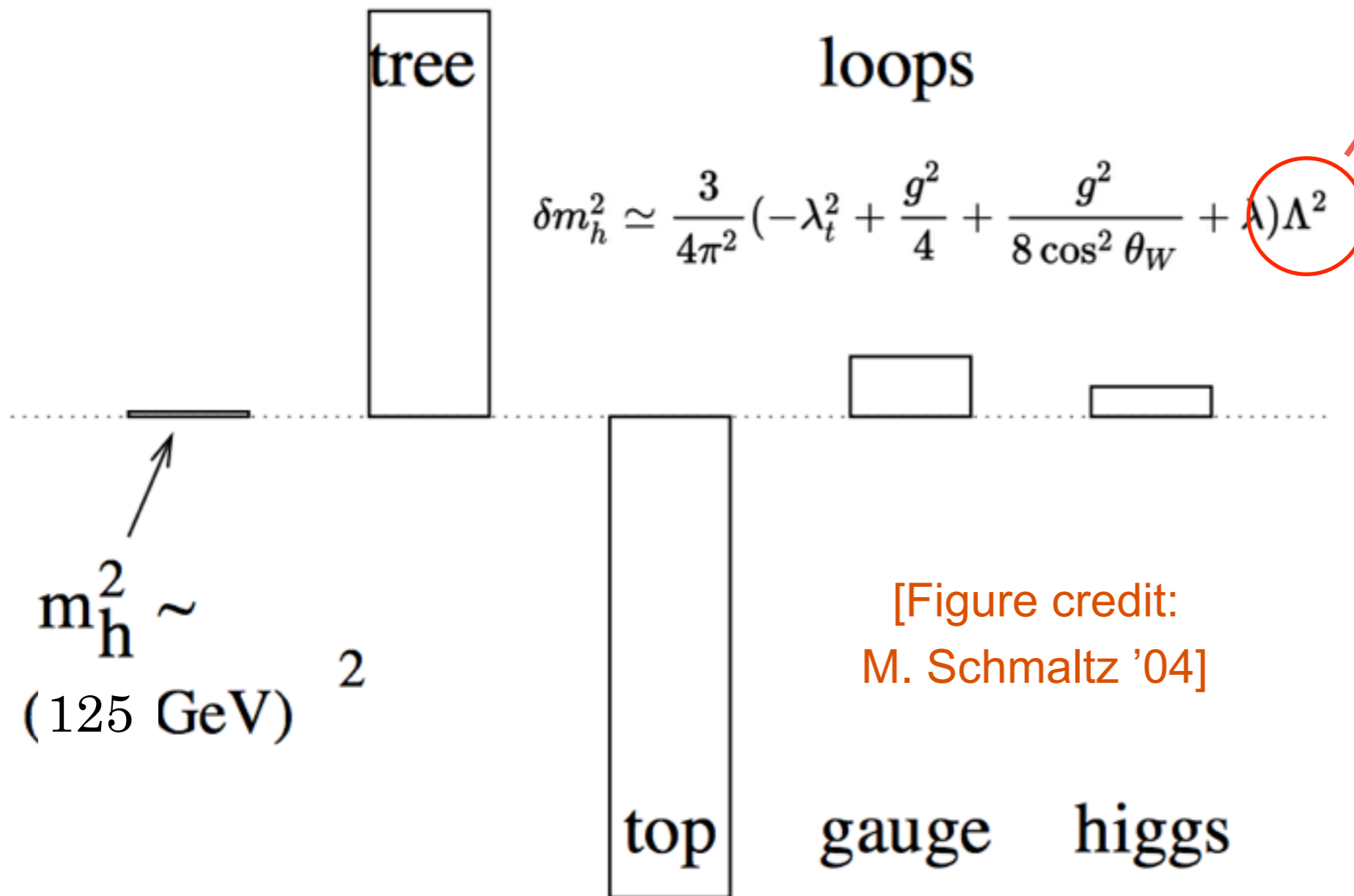
electron mass is orders smaller than the EW scale: **unnatural; but technically natural (t'hooft)**

- massless limit => chiral symm
- non-zero mass => softly symm breaking (logarithmic correction from the cutoff at quantum level)

$$m_e \sim m_e^0 \left[1 + \frac{3\alpha}{4\pi} \ln(\Lambda/m_e) \right]$$

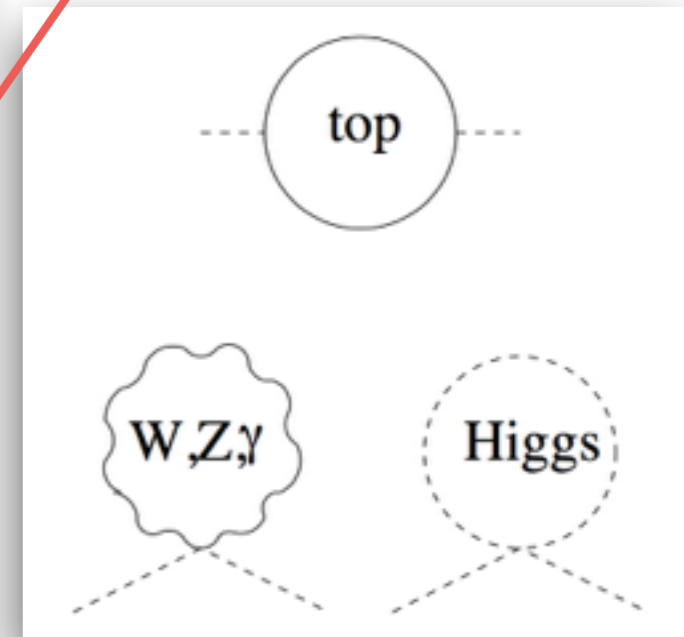


Naturalness Problem in Particle Physics



[Figure credit:
M. Schmaltz '04]

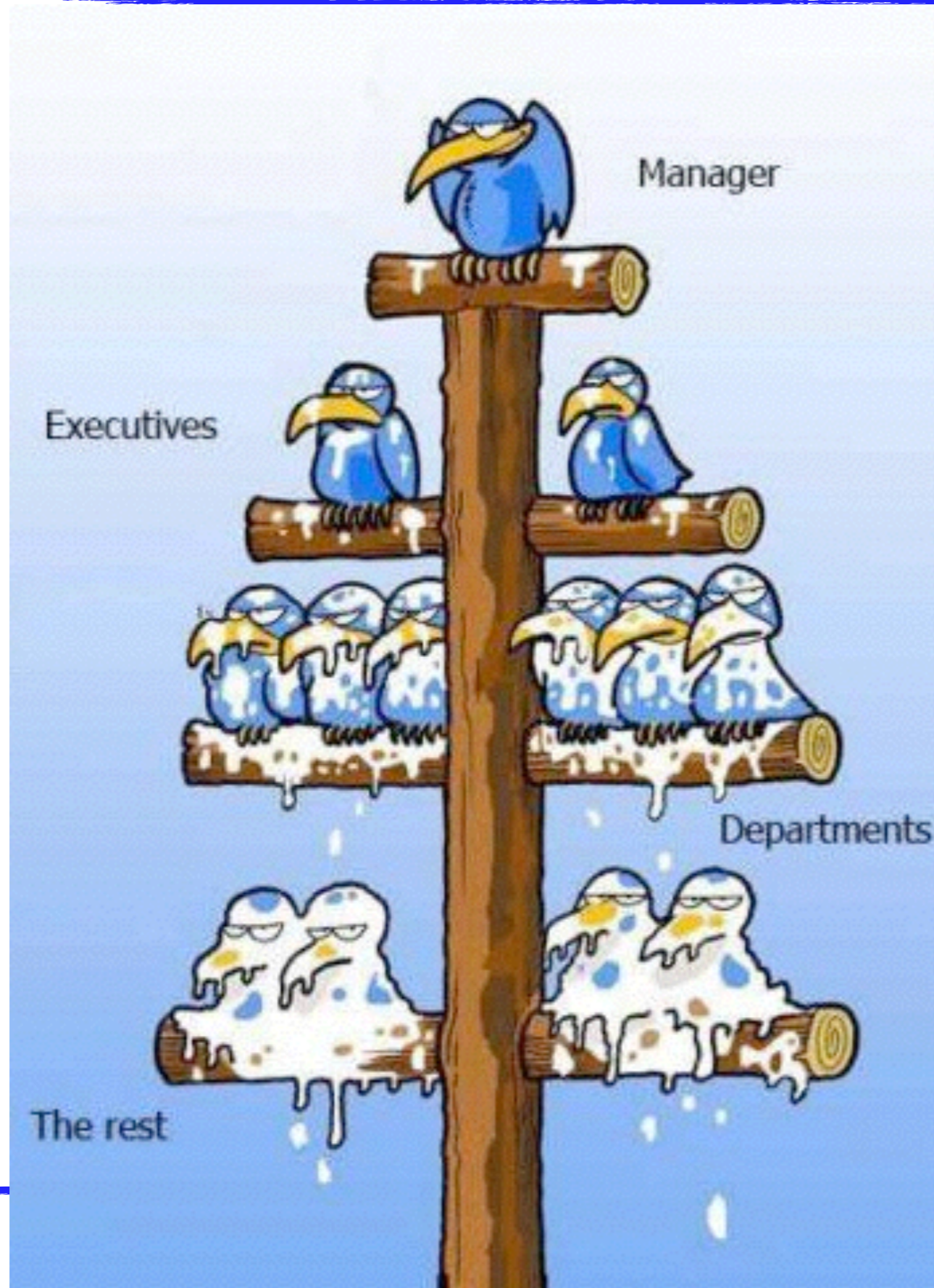
$$\sim M_{\text{Planck}}^2$$



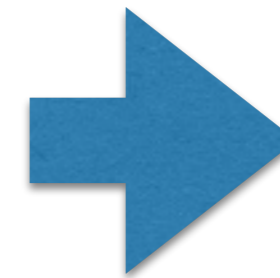
A hierarchy of 30 orders!
- Unnatural!



“Hierarchy” Problem



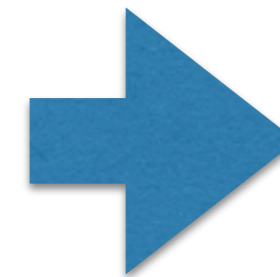
Picture credit: www



Planck scale



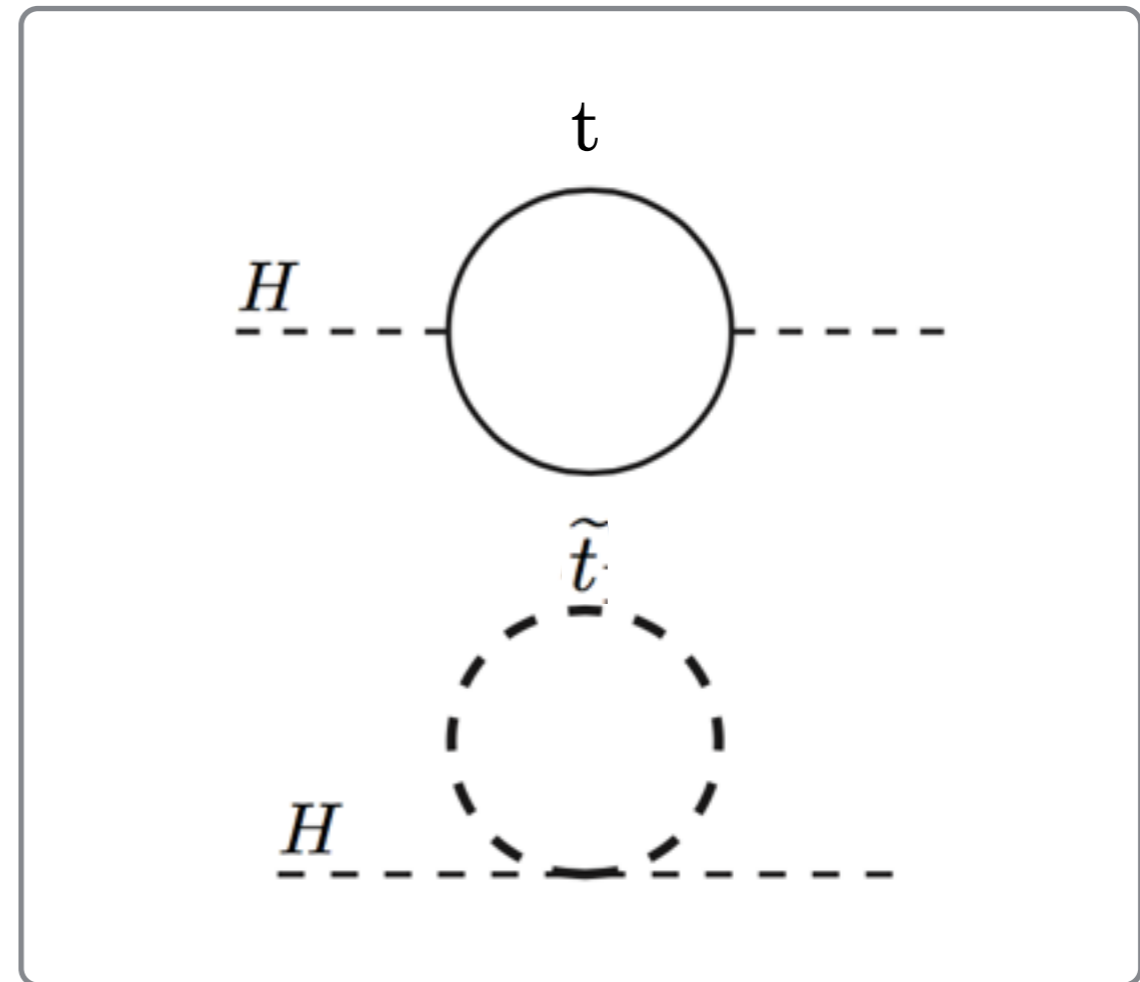
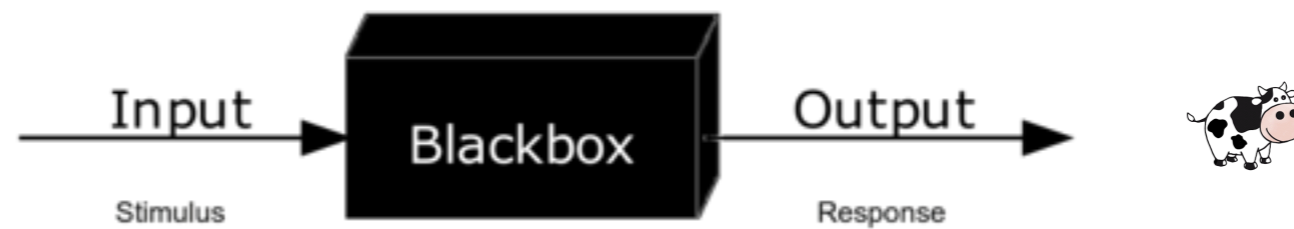
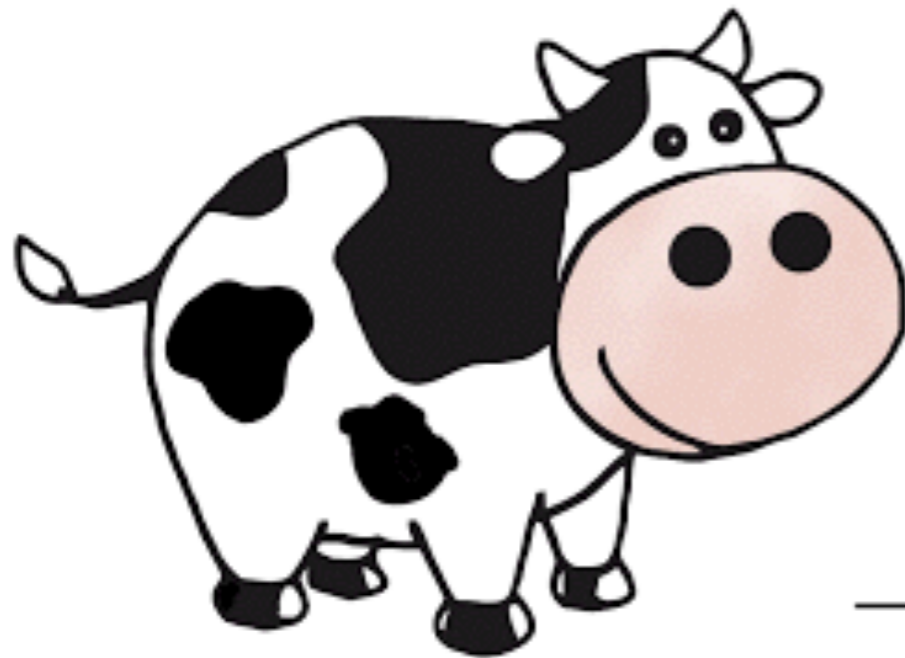
Is there a
“technically natural”
solution?



EW scale

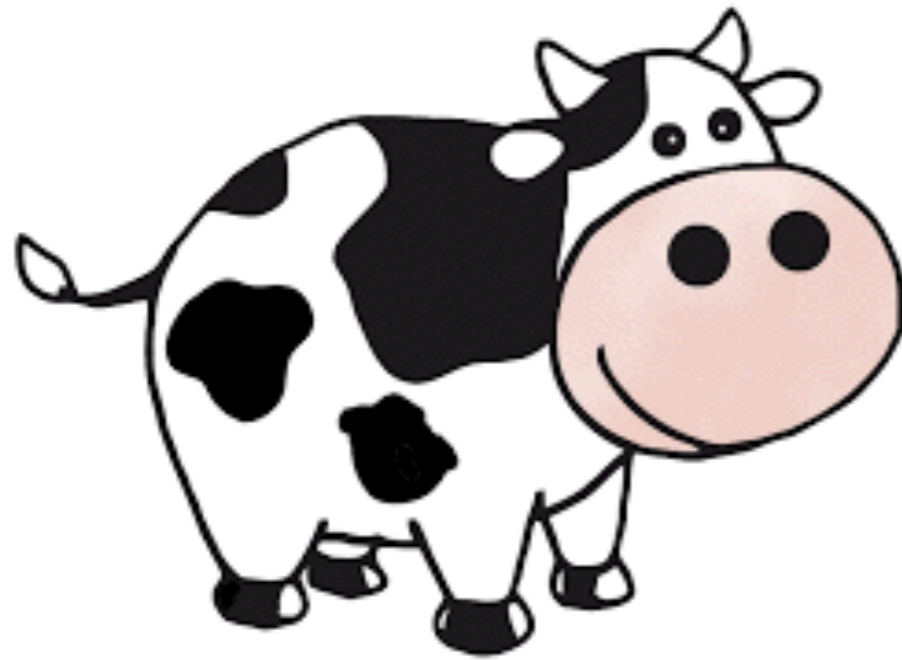


Solution I - Fermionic Symmetry (Supersymmetry)

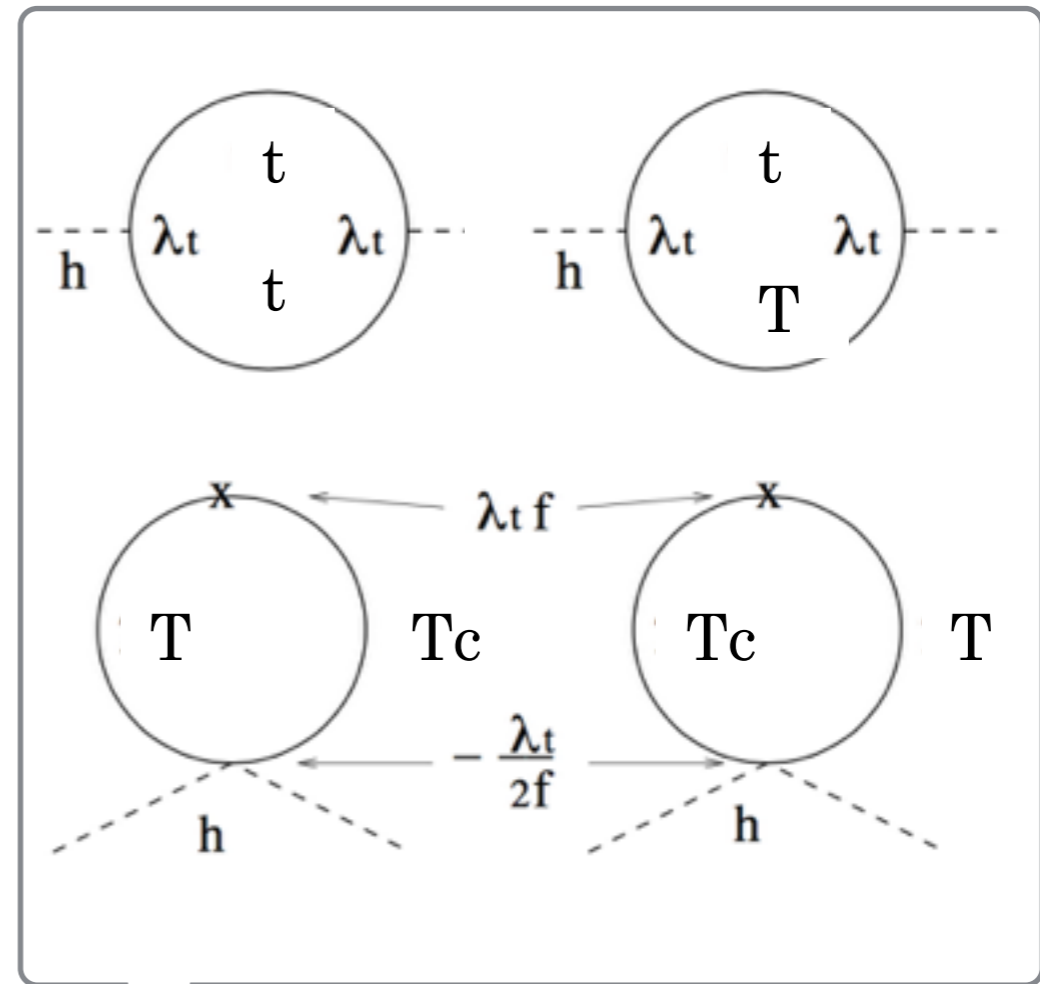
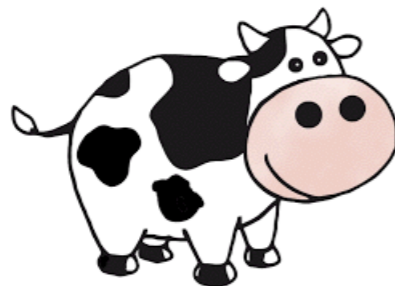




Solution II - Bosonic Symmetry (Little/Twin Higgs)



Strong dynamics





Some Wisdoms

The underlying symmetry =>

- (1) orders a list of “partner” particles
- (2) predicts a sum rule for canceling quadratic divergence in squared Higgs mass, either completely or at a leading quantum level

Motivated extensive searches for “partner” particles at, e.g., LEP, Tevatron, LHC, for decades

A must-be-done task post the discovery of any partner-like particle:

Measuring the sum rule



Simplified Model

SM + one pair of vector-like (weak isospin singlet) top partners

$$\mathcal{L}_U = u_3^c \left(c_0 f U + c_1 H q_3 + \frac{c_2}{f} H^2 U + \dots \right) \\ + U^c \left(\hat{c}_0 f U + \hat{c}_1 H q_3 + \frac{\hat{c}_2}{f} H^2 U + \dots \right) + \text{h.c. .}$$



Simplified Model

SM + one pair of vector-like (weak isospin singlet) top partners

$$\mathcal{L}_U = u_3^c \left(c_0 f U + c_1 H q_3 + \frac{c_2}{f} H^2 U + \dots \right) \\ + U^c \left(\hat{c}_0 f U + \hat{c}_1 H q_3 + \frac{\hat{c}_2}{f} H^2 U + \dots \right) + \text{h.c. .}$$

Model	Coset		SU(2)	c_0	c_1	c_2	\hat{c}_0	\hat{c}_1	\hat{c}_2
Toy model	$\frac{\text{SU}(3)}{\text{SU}(2)}$	[22]	1	λ_1	$-\lambda_1$	$-\lambda_1$	λ_2	0	0
Simplest	$\left(\frac{\text{SU}(3)}{\text{SU}(2)}\right)^2$	[23]	1	λ	$-\lambda$	$-\lambda$	λ	λ	$-\lambda$
Littlest Higgs	$\frac{\text{SU}(5)}{\text{SO}(5)}$	[14]	1	λ_1	$-\sqrt{2}i\lambda_1$	$-2\lambda_1$	λ_2	0	0
Custodial	$\frac{\text{SO}(9)}{\text{SO}(5)\text{SO}(4)}$	[20]	2	y_1	$\frac{i}{\sqrt{2}}y_1$	$-\frac{1}{2}y_1$	y_2	0	0
T -parity invariant	$\frac{\text{SU}(3)}{\text{SU}(2)}$	[19]	1	λ	$-\lambda$	$-\lambda$	$-\lambda$	$-\lambda$	λ
T -parity invariant	$\frac{\text{SU}(5)}{\text{SO}(5)}$	[19]	1	λ	$-\sqrt{2}i\lambda$	-2λ	$-\lambda$	$-\sqrt{2}i\lambda$	2λ
Mirror twin Higgs	$\frac{\text{SU}(4)\text{U}(1)}{\text{SU}(3)\text{U}(1)}$	[24]	1	0	$i\lambda_t$	0	λ_t	0	$-\lambda_t$



Naturalness Sum Rule - Mass Basis Before EWSB

$$\mathcal{L}_{T'} = m_{T'} T'^c T' + \lambda_{t'} H t'^c t' + \lambda_{T'} H T'^c t' + \frac{\alpha_{t'}}{2m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2m_{T'}} H^2 T'^c T' + \frac{\beta_{t'}}{6m_{T'}^2} H^3 t'^c t' + \frac{\beta_{T'}}{6m_{T'}^2} H^3 T'^c t' + \mathcal{O}(H^4) + \text{h.c.}$$

Quadratically divergent contribution to the C-W potential from top sector (one-loop level)

$$\frac{1}{16\pi^2} \Lambda^2 \text{tr} \mathcal{M}(H)^\dagger \mathcal{M}(H)$$

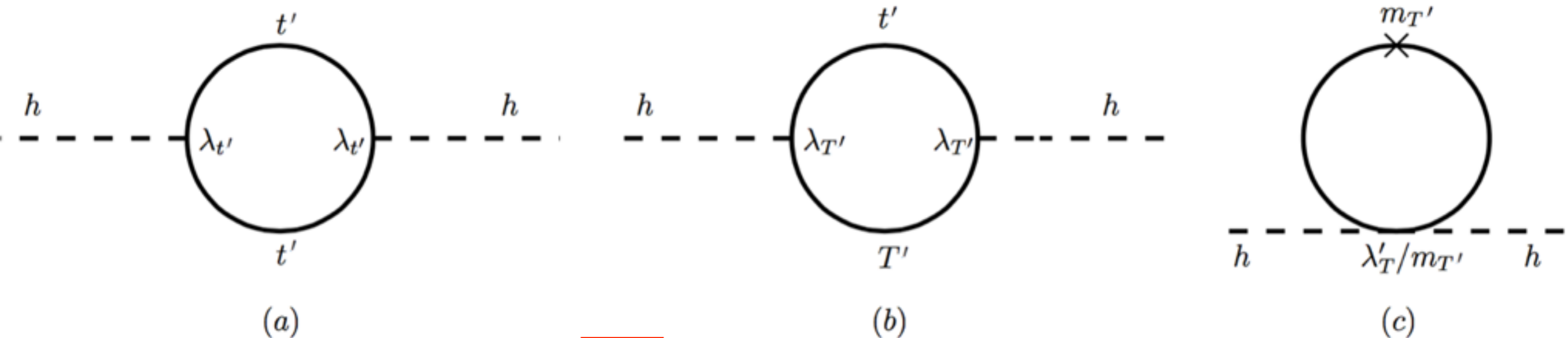
Require coefficient in H^2 to vanish \Rightarrow

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$



Testing the Sum Rule - Traditional Wisdom

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$



$$\mathcal{L}_{T'} = m_{T'} T'^c T' + \lambda_{t'} H t'^c t' + \lambda_{T'} H T'^c t' + \frac{\alpha_{t'}}{2m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2m_{T'}} H^2 T'^c T' + \frac{\beta_{t'}}{6m_{T'}^2} H^3 t'^c t' + \frac{\beta_{T'}}{6m_{T'}^2} H^3 T'^c t' + \mathcal{O}(H^4) + \text{h.c.}$$

Traditional wisdom - reconstruct the three couplings



Testing the Sum Rule -Traditional Wisdom

Top quarks and electroweak symmetry breaking in little Higgs models

Maxim Perelstein, Michael E. Peskin, and Aaron Pierce
Phys. Rev. D **69**, 075002 – Published 8 April 2004

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$

$$\alpha_{T'} = \lambda_{T'} \frac{m_T}{f}$$

How difficult!

4	Testing the Model at the LHC	16
4.1	Measuring the parameter f	16
4.2	Measuring $\lambda_{T'}$	17
4.2.1	Decays of the T quark	17
4.2.2	Production of the T quark	20



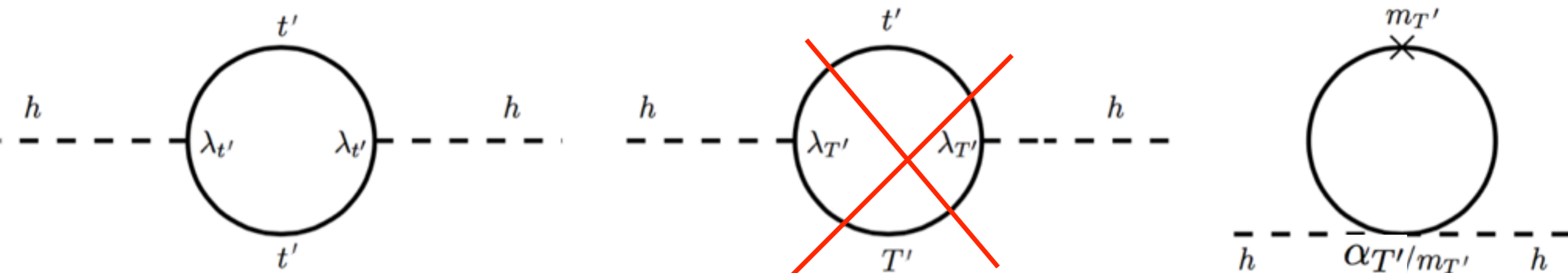
Testing the Sum Rule -Traditional Wisdom

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$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$

$$\alpha_{T'} = \lambda_{T'} \frac{m_{T'}}{f}$$



Not representative! E.g., little Higgs with T-parity



Naturalness Sum Rule - Mass Basis After EWSB

$$\mathcal{L}_T = m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c t + \frac{\lambda_T}{\sqrt{2}} h T^c t + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_T v}{\sqrt{2} m_T} h T^c T + \frac{\alpha_t}{4 m_T} h^2 t^c T + \frac{\alpha_T}{4 m_T} h^2 T^c T + \frac{b_t v}{4 m_T^2} h^2 t^c t + \frac{b_T v}{4 m_T^2} h^2 T^c t + \mathcal{O}\left(h^3, \frac{v^2}{m_T^2}\right) + \text{h.c.}$$

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$



$$a_T = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)$$

- Leading order - involves diagonal Yukawa couplings only
- Could be generalized with more top partners introduced:

$$\sum_i a_{T_i} = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_{T_i}^2}\right)$$

- No measurement of quartic coupling is needed => a more feasible guideline



Collider Strategy - Colored Top Partners

- With this guideline, we are able to study various benchmark scenarios, e.g., little higgs models without T parity
- Introduce a “naturalness parameter”

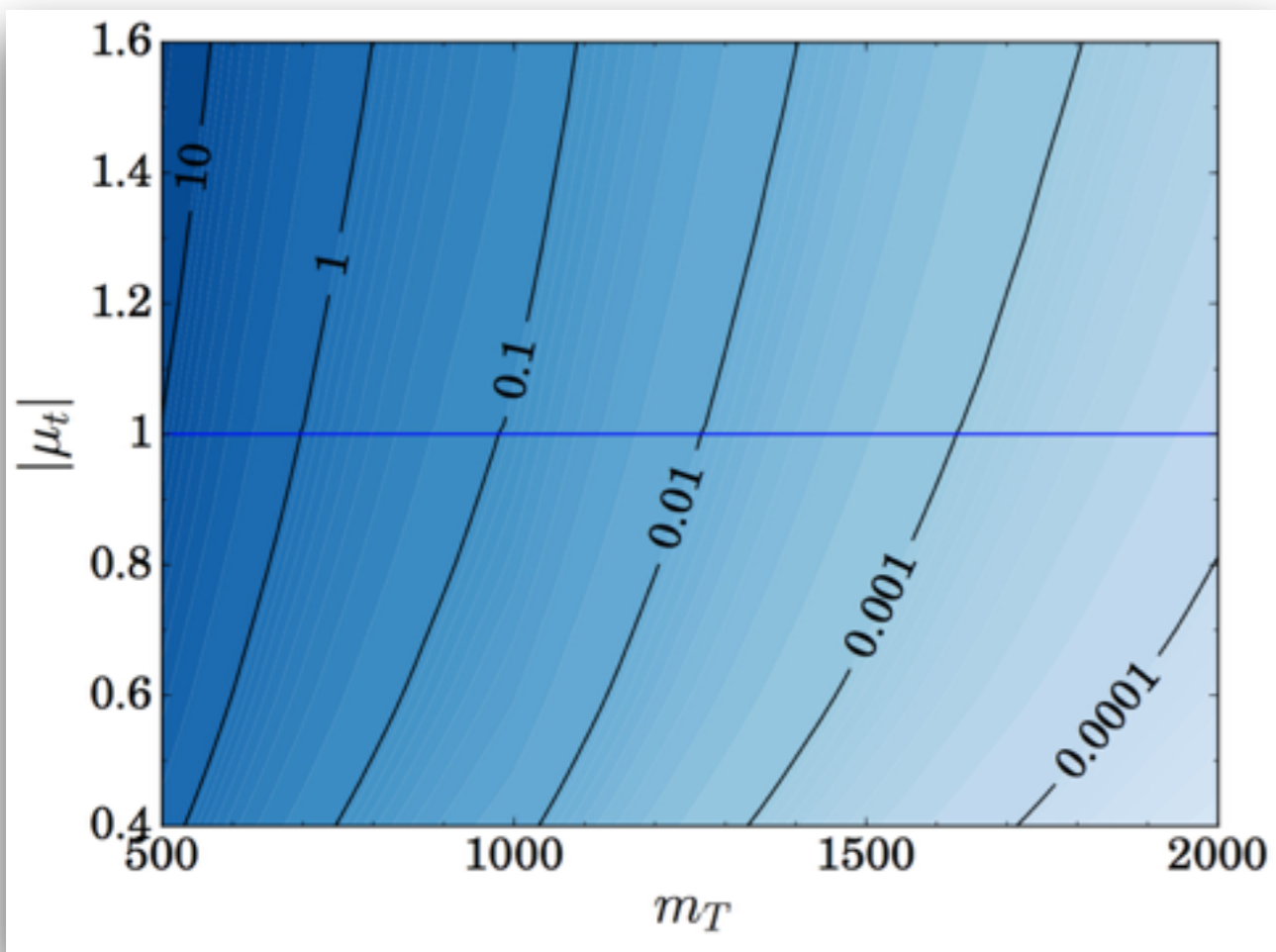
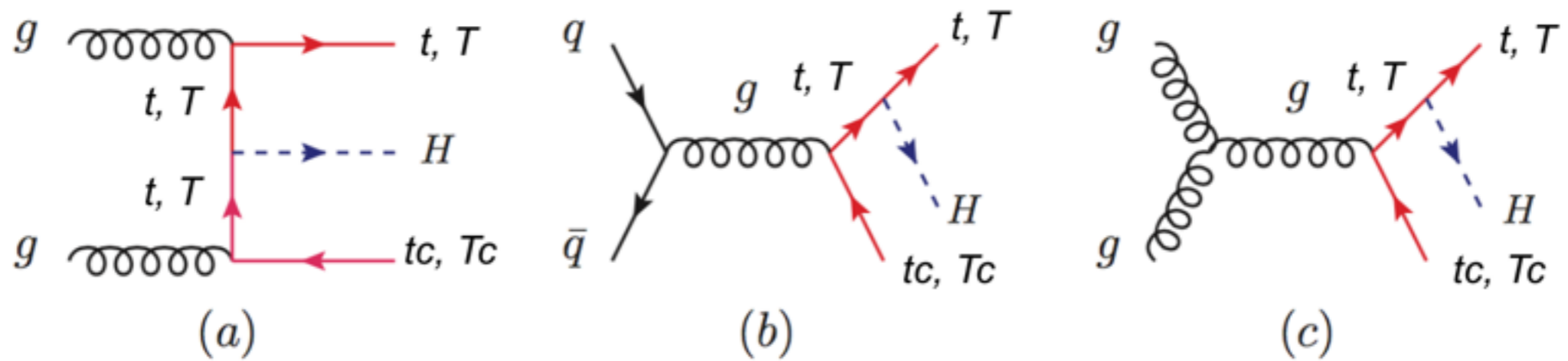
$$\mu = -\frac{\Delta m_H^2|_{\text{NP}}}{\Delta m_H^2|_{\text{SM}}} \Rightarrow \mu_t = -\frac{a_T}{\lambda_t^2} + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)$$

$$\mu|_{\text{nat}} \equiv 1$$

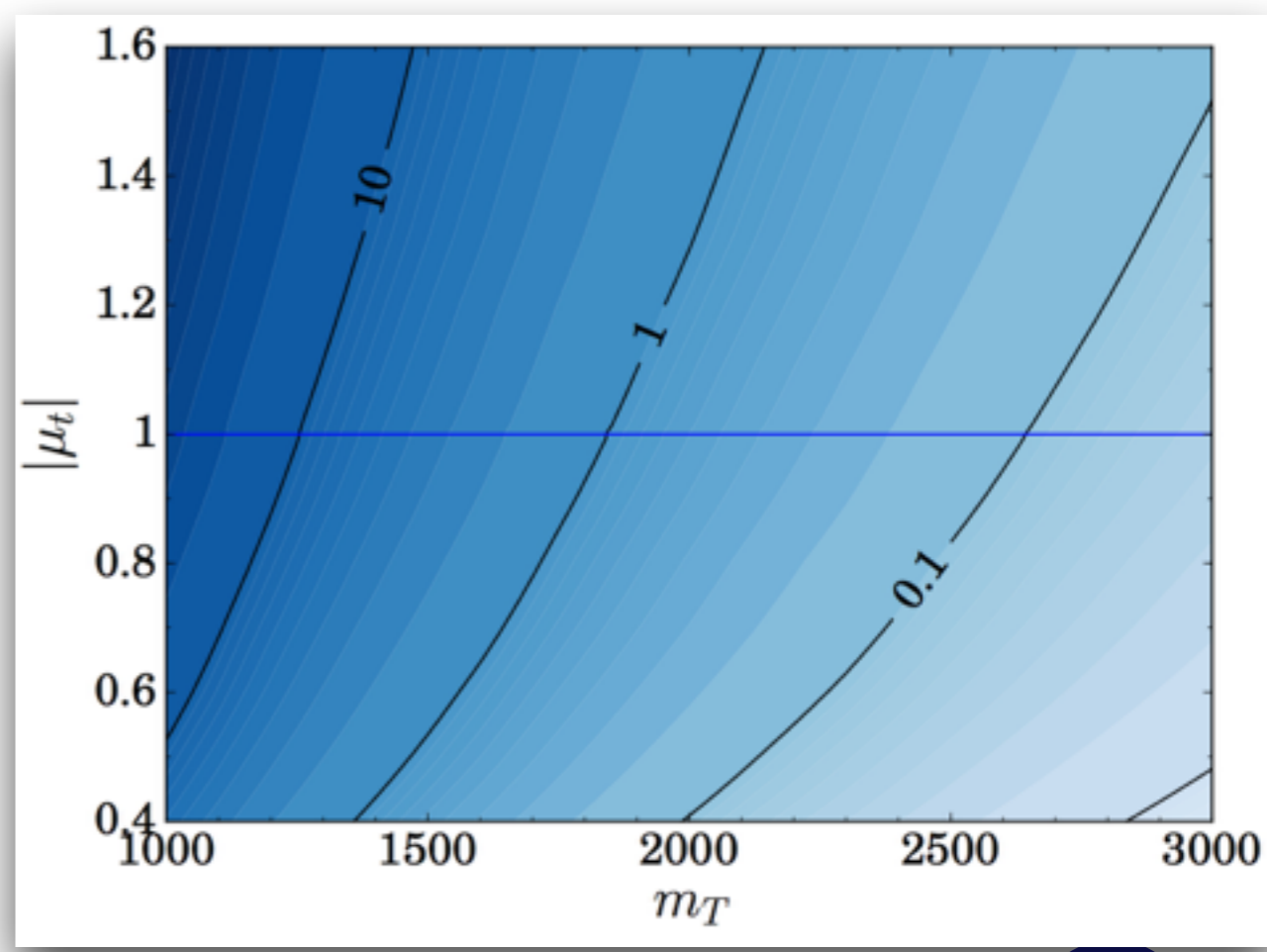
- Test the sum rule \Leftrightarrow measure the “naturalness parameter”



TTh Production



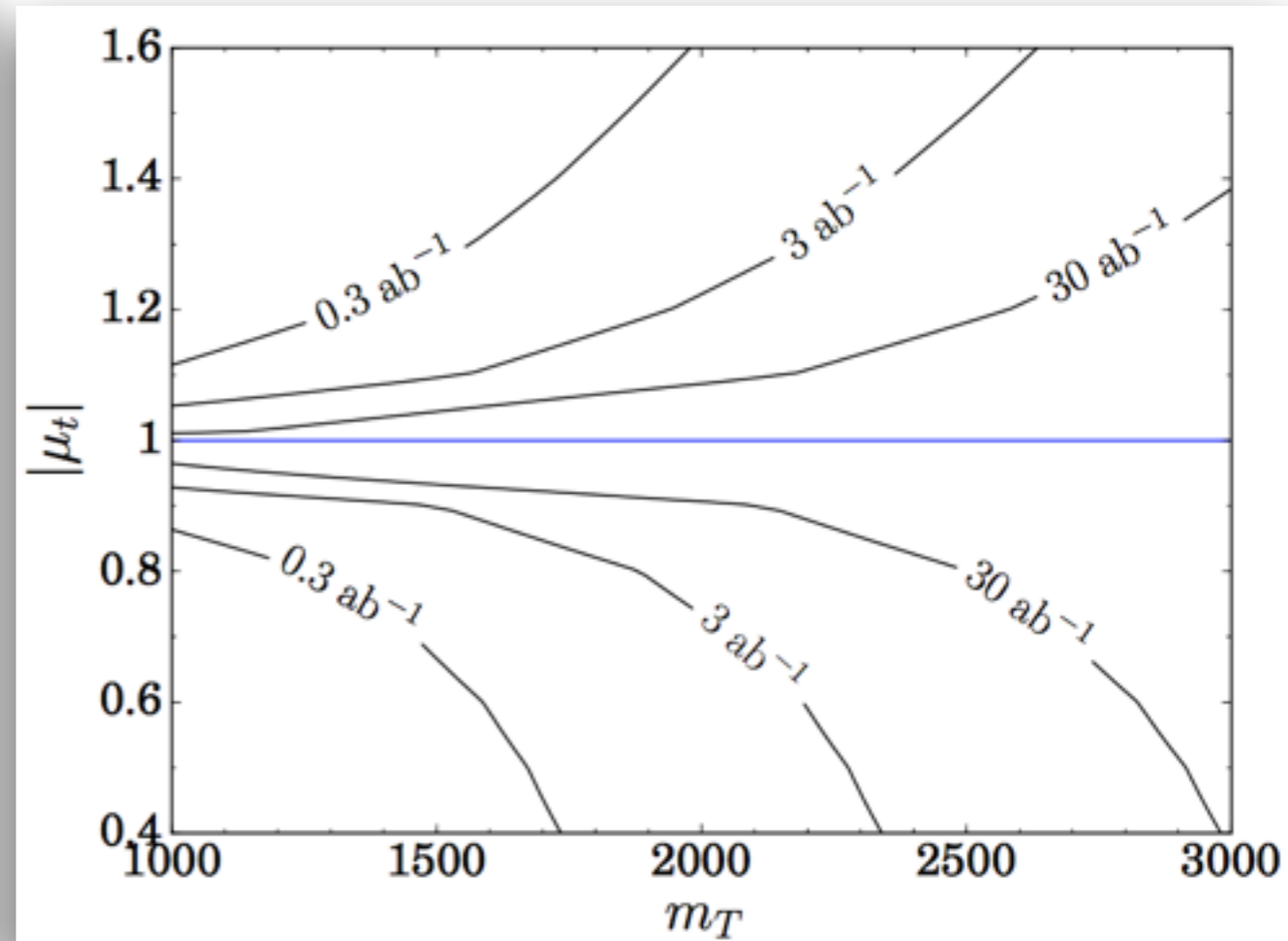
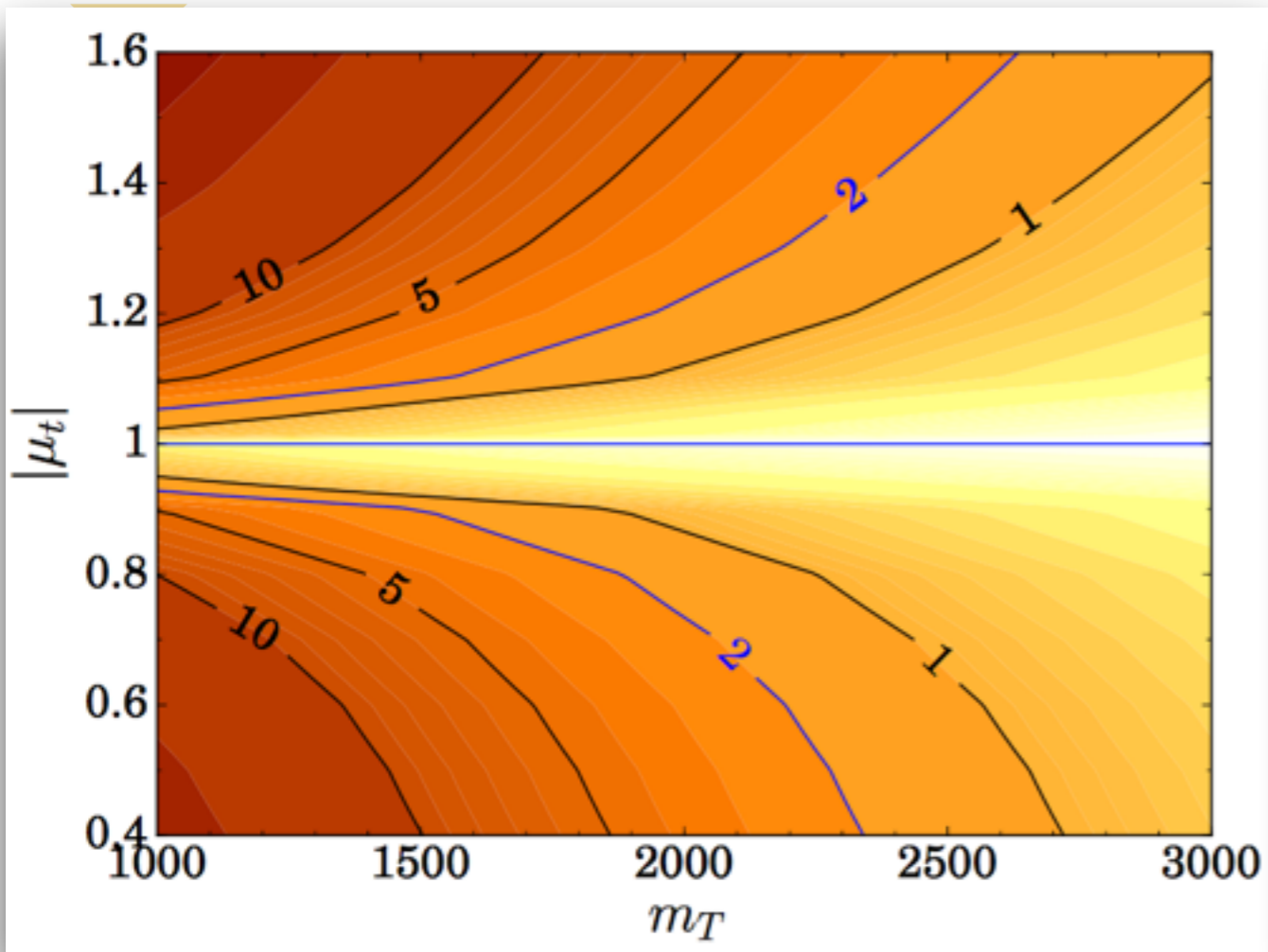
14 TeV



100 TeV



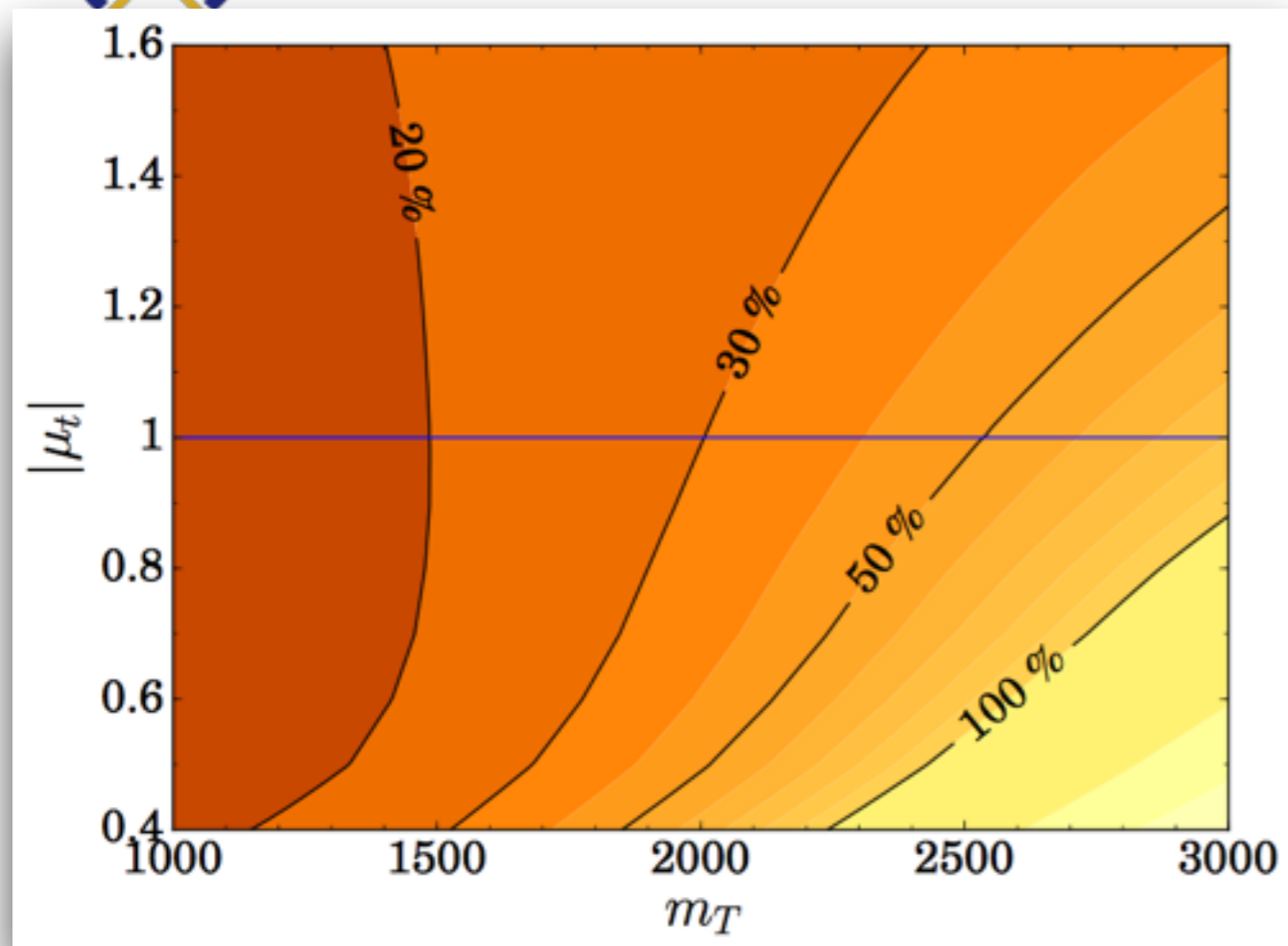
Exclusion of Unnatural Theories at 100 TeV



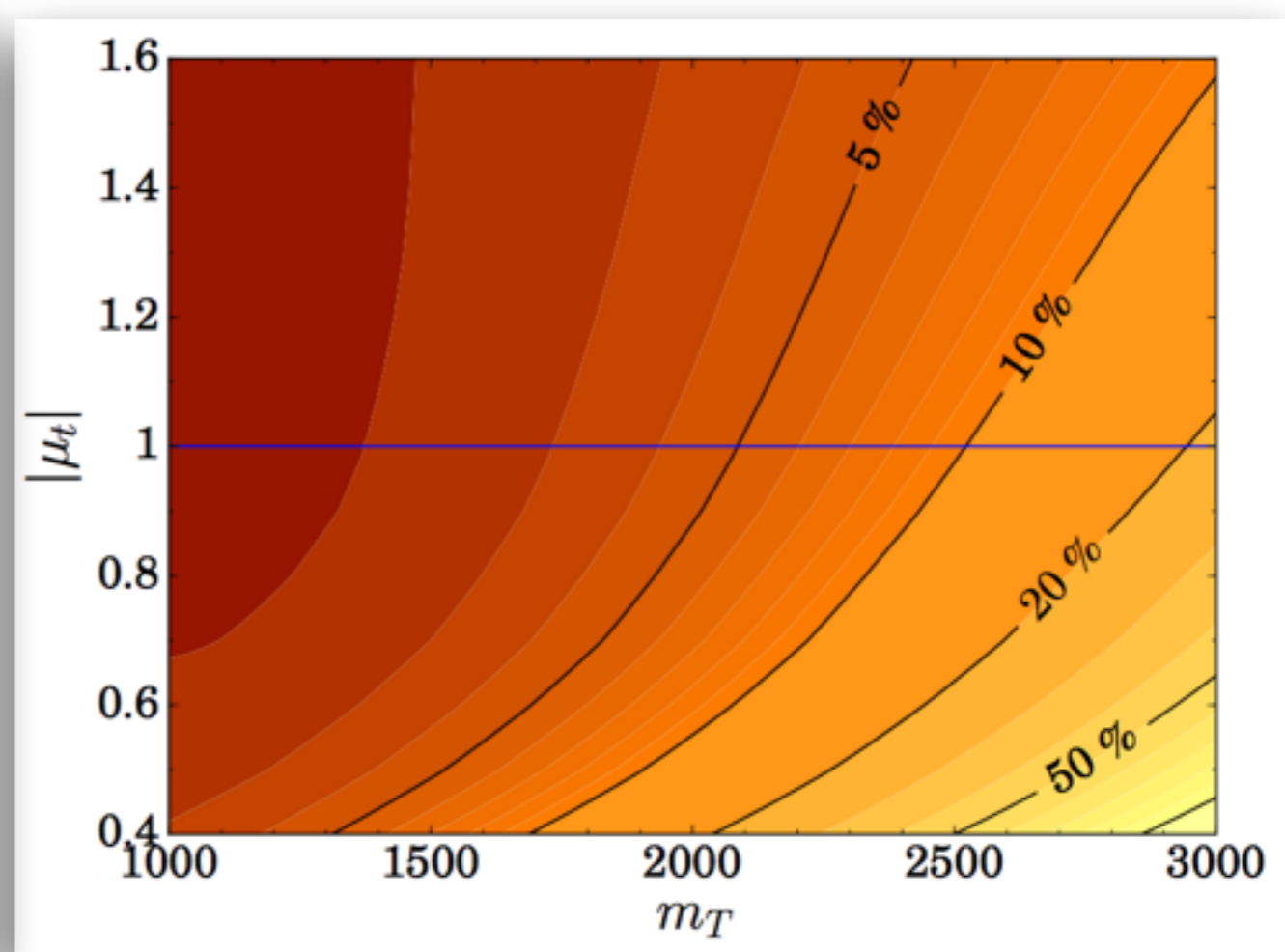
- ☐ “unnaturalness” hypothesis: exclusion of “unnatural theories” against a natural theory
- ☐ given 30/ab, 10% deviation from “naturalness”: excluded up to 2.2TeV



Precision of Measuring Naturalness Parameter at 100 TeV



$\delta_{\lambda_t} \sim 10\%$ (HL-LHC)
+ δ_{a_T} (3/ab, 100TeV)



$\delta_{\lambda_t} \sim 1\%$ (30/ab, 100TeV)
+ δ_{a_T} (30/ab, 100TeV)

☐ A measurement precision of 10% of μ could be achieved up to ~ 2.5 TeV

$$\delta\mu = \sqrt{\left(-\frac{1}{\lambda_t^2}\delta a_T\right)^2 + \left(2\frac{a_T}{\lambda_t^3}\delta\lambda_t\right)^2}$$



Questions Unaddressed

How to break the degeneracy w.r.t. the sign of the naturalness parameter?

In twin Higgs model, how to test the naturalness sum rule at colliders?

How to test the sum rule for supersymmetry at colliders, post the discovery of any superpartner-like particle?



Questions Unaddressed

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In twin Higgs model, how to test the naturalness sum rule at colliders?

How to test the sum rule for supersymmetry at colliders, post the discovery of any superpartner-like particle?

[arXiv.org > hep-ph > arXiv:1811.01961](https://arxiv.org/abs/1811.01961)

High Energy Physics – Phenomenology

Naturalness Sum Rules and Their Collider Tests

[Csaba Csáki](#), [Felipe Ferreira De Freitas](#), [Li Huang](#), [Teng Ma](#), [Maxim Perelstein](#), [Jing Shu](#)

(Submitted on 5 Nov 2018 (v1), last revised 7 Nov 2018 (this version, v2))



Summary

- ☒ The naturalness problem has driven particle physics for decades
- ☒ To establish the naturalness principle, it is crucial to measure the naturalness sum rule, post the discovery of any partner-like particle
- ☒ For a top sector with fermionic top partners, the naturalness sum rule only depends on flavor-diagonal Yukawa couplings, up to an order $O(v^2/m_T^2)$

$$a_T = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)$$

- ☒ At 100 TeV with 30/ab, a precision of 10% for the measurement of the naturalness parameter could be achieved for top partners up to ~ 2.5 TeV, for the benchmark considered in this analysis

Thank you!

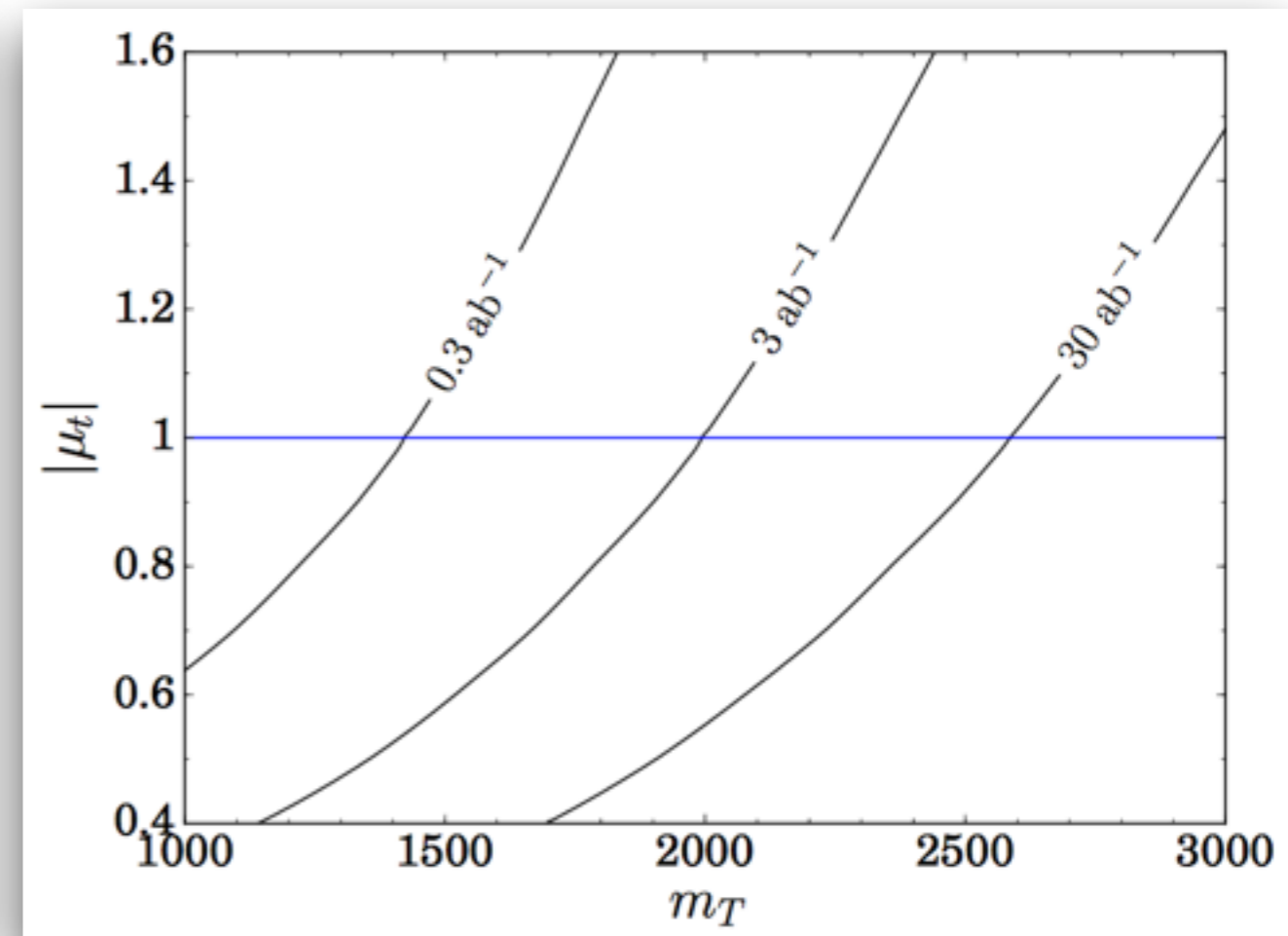
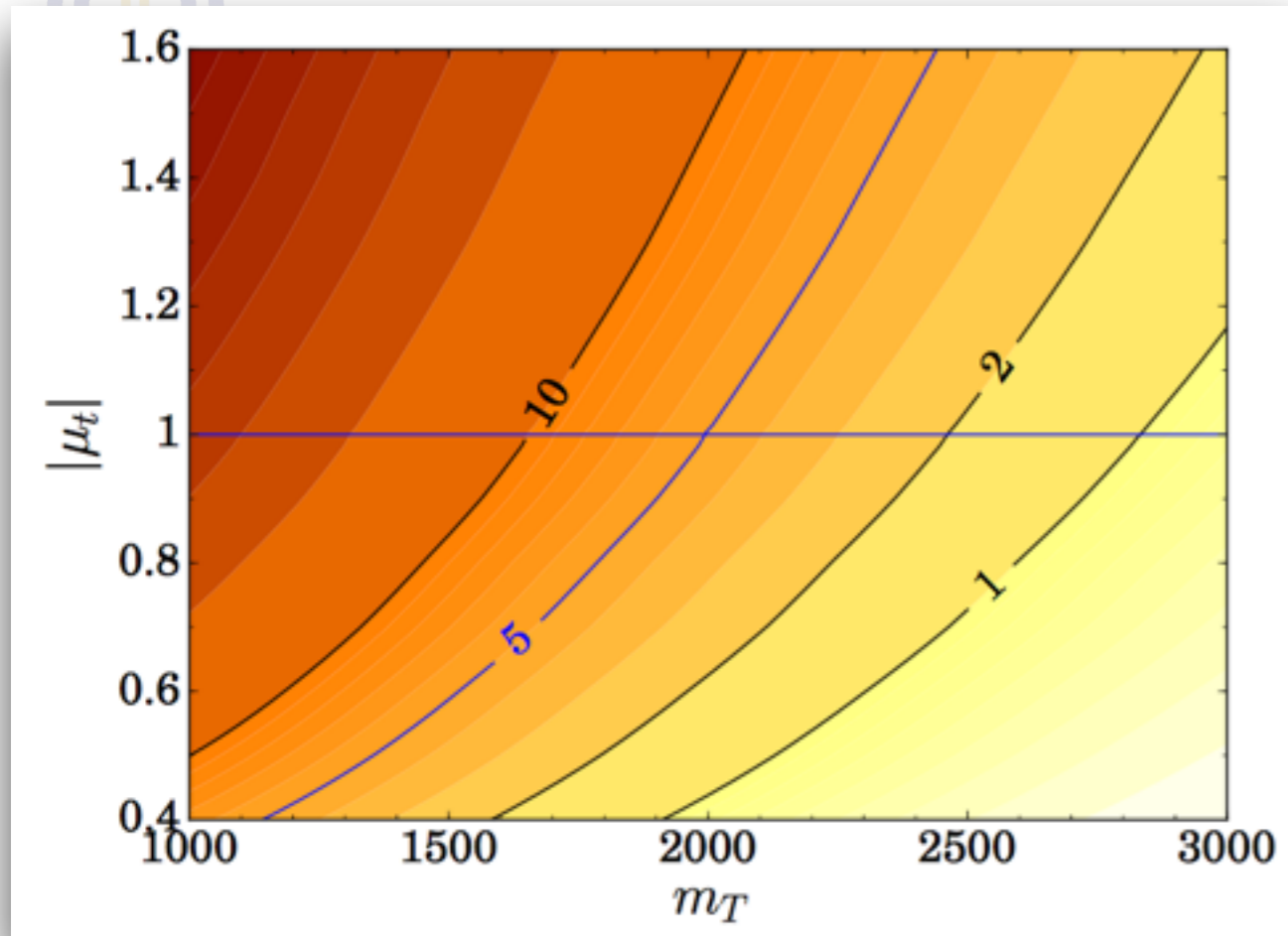


GRF under Grant No. 16312716

CRF under grant No. HUKST4/CRF/13G



Discovery Potential of Top Partner at 100 TeV

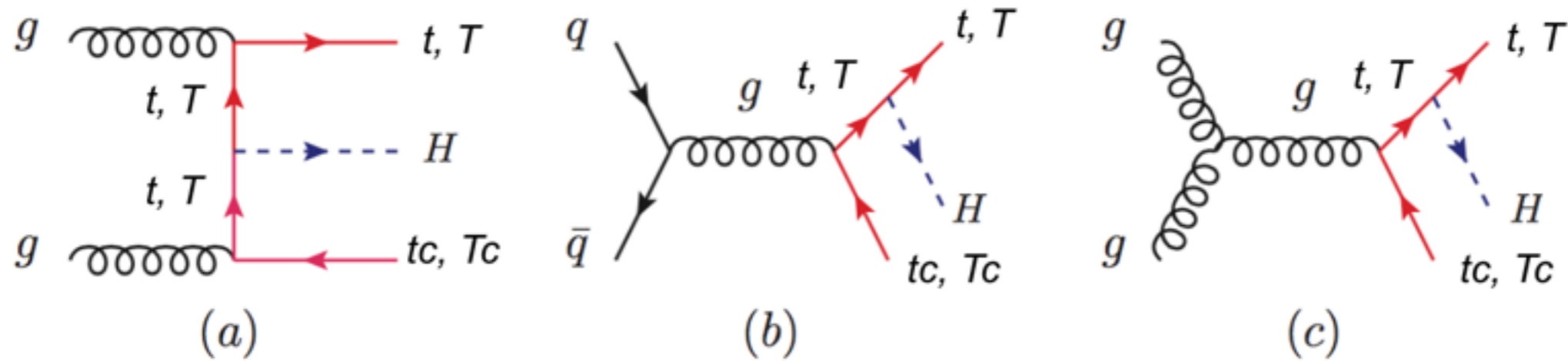


☒ Not the "Gold" channel for discovery of top partner, but show the effectiveness of the analysis



Questions Unaddressed

How to break the degeneracy of the sign of the naturalness parameter?

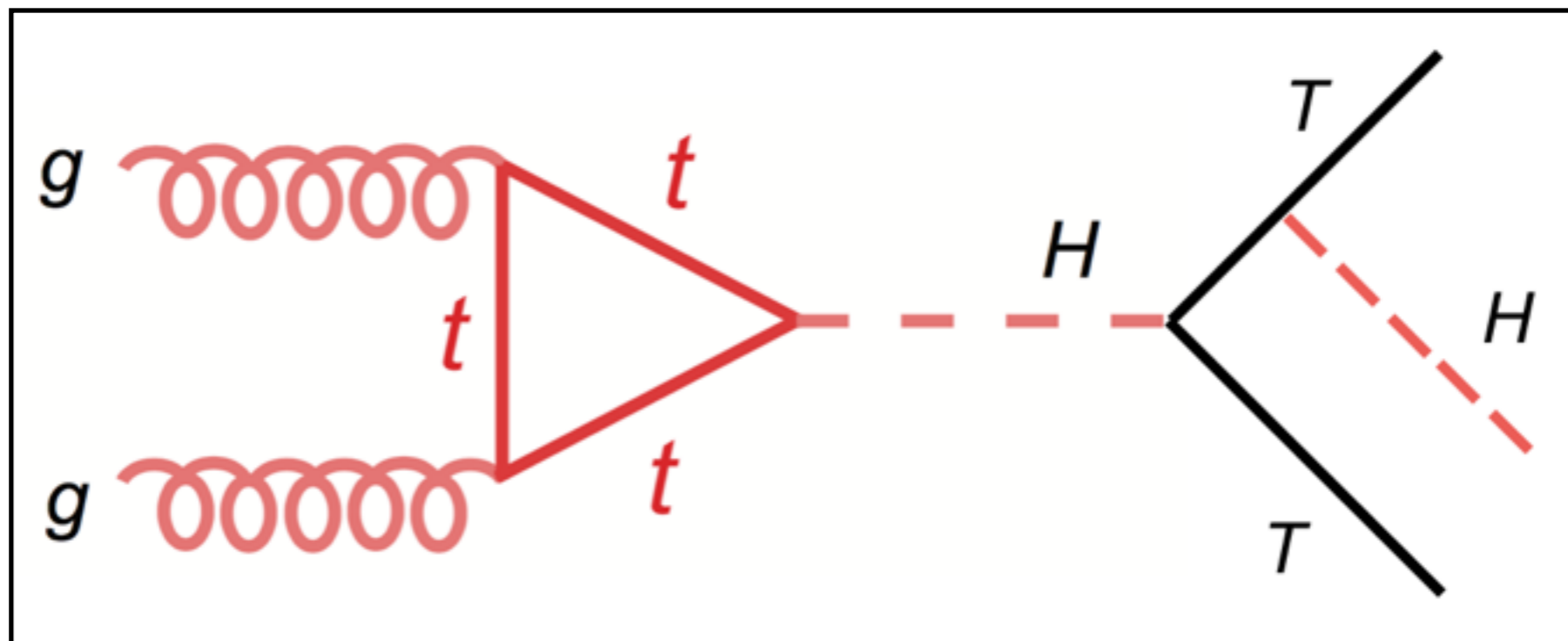




Outlook II

In twin Higgs model, how to test the naturalness sum rule at colliders?

Maybe mono-Higgs search can help





Simplified Model - Mass Basis Before EWSB

$$\begin{aligned} t'^c &= \frac{\hat{c}_0 u_3^c - c_0 U^c}{c} & t' &= q_3 \\ T'^c &= \frac{\hat{c}_0 U^c + c_0 u_3^c}{c} & T' &= U \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{T'} &= m_{T'} T'^c T' + \lambda_{t'} H t'^c t' + \lambda_{T'} H T'^c t' + \frac{\alpha_{t'}}{2m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2m_{T'}} H^2 T'^c T' \\ &\quad + \frac{\beta_{t'}}{6m_{T'}^2} H^3 t'^c t' + \frac{\beta_{T'}}{6m_{T'}^2} H^3 T'^c t' + \mathcal{O}(H^4) + \text{h.c.} \end{aligned}$$

$$m_{T'} = f c, \quad c = \sqrt{c_0^2 + \hat{c}_0^2}$$

$$\lambda_{t'} = \frac{\hat{c}_0 c_1 - c_0 \hat{c}_1}{c}, \quad \lambda_{T'} = \frac{c_0 c_1 + \hat{c}_0 \hat{c}_1}{c},$$

$$\alpha_{t'} = \hat{c}_0 c_2 - c_0 \hat{c}_2, \quad \alpha_{T'} = c_0 c_2 + \hat{c}_0 \hat{c}_2,$$

$$\beta_{t'} = (\hat{c}_0 c_3 - c_0 \hat{c}_3) c, \quad \beta_{T'} = (c_0 c_3 + \hat{c}_0 \hat{c}_3) c$$



Simplified Model - Mass Basis After EWSB

$$t^c = t'^c + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right), \quad t = t' - T' \frac{v}{m_{T'}} \lambda_{T'}^* + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right)$$
$$T^c = T'^c + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right), \quad T = T' + t' \frac{v}{m_{T'}} \lambda_{T'} + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right)$$

$$\mathcal{L}_T = m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c t + \frac{\lambda_T}{\sqrt{2}} h T^c t + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_T v}{\sqrt{2} m_T} h T^c T$$
$$+ \frac{\alpha_t}{4 m_T} h^2 t^c T + \frac{\alpha_T}{4 m_T} h^2 T^c T + \frac{b_t v}{4 m_T^2} h^2 t^c t + \frac{b_T v}{4 m_T^2} h^2 T^c t + \mathcal{O}\left(h^3, \frac{v^2}{m_T^2}\right) + \text{h.c.}$$

$$a_t = \alpha_{t'} + \lambda_{T'}^* \lambda_{t'},$$

$$b_t = \beta_{t'} - \alpha_{t'} \lambda_{T'},$$

$$a_T = \alpha_{T'} + |\lambda_{T'}|^2$$

$$b_T = \beta_{T'} - \alpha_{T'} \lambda_{T'}$$