Dark matter in Neutrino mass models

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Introduction

The Standard Model can not explain some mysterious

- The observation of neutrino oscillation indicates non zero mass, but in SM model, neutrino only has left-hand chirality.
- Many evidences of dark matter exit, such as galaxy rotation curves, gravitational lensing and CMB observation. What is it?
- Surely active neutrino cannot be the major component of DM, but we can construct BSMs for both.

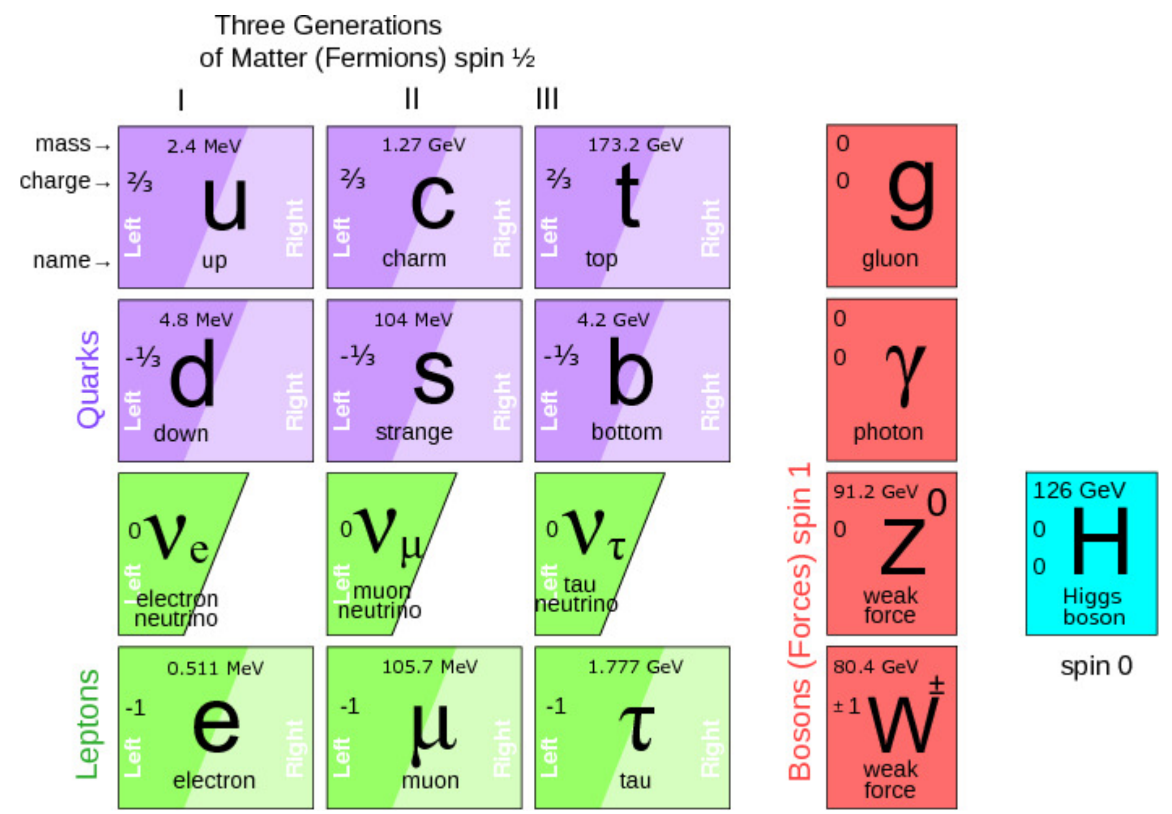


Fig: Oliver & Stefan

Neutrino Global Fit

The Global data fit 3 active neutrino oscillation with parameters:

$\theta_{12}, \theta_{23}, \theta_{13},\ \delta,\ \Delta m^2_{21},\ \Delta m^2_{32(31)}$

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.7$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.580^{+0.017}_{-0.021}$	$0.418 \to 0.627$	$0.584^{+0.016}_{-0.020}$	$0.423 \to 0.629$
	$\theta_{23}/^\circ$	$49.6^{+1.0}_{-1.2}$	$40.3 \to 52.4$	$49.8^{+1.0}_{-1.1}$	$40.6 \to 52.5$
	$\sin^2\theta_{13}$	$0.02241^{+0.00065}_{-0.00065}$	$0.02045 \to 0.02439$	$0.02264^{+0.00066}_{-0.00066}$	$0.02068 \to 0.02463$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.13}$	$8.27 \to 9.03$
	$\delta_{\rm CP}/^\circ$	215^{+40}_{-29}	$125 \to 392$	284^{+27}_{-29}	$196 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.525^{+0.033}_{-0.032}$	$+2.427 \to +2.625$	$-2.512^{+0.034}_{-0.032}$	$-2.611 \to -2.412$

Gonzalez-Garcia et al., 2019; Capozzi et al., 2018; Forero et al. 2017

Normal or Inverted Ordering

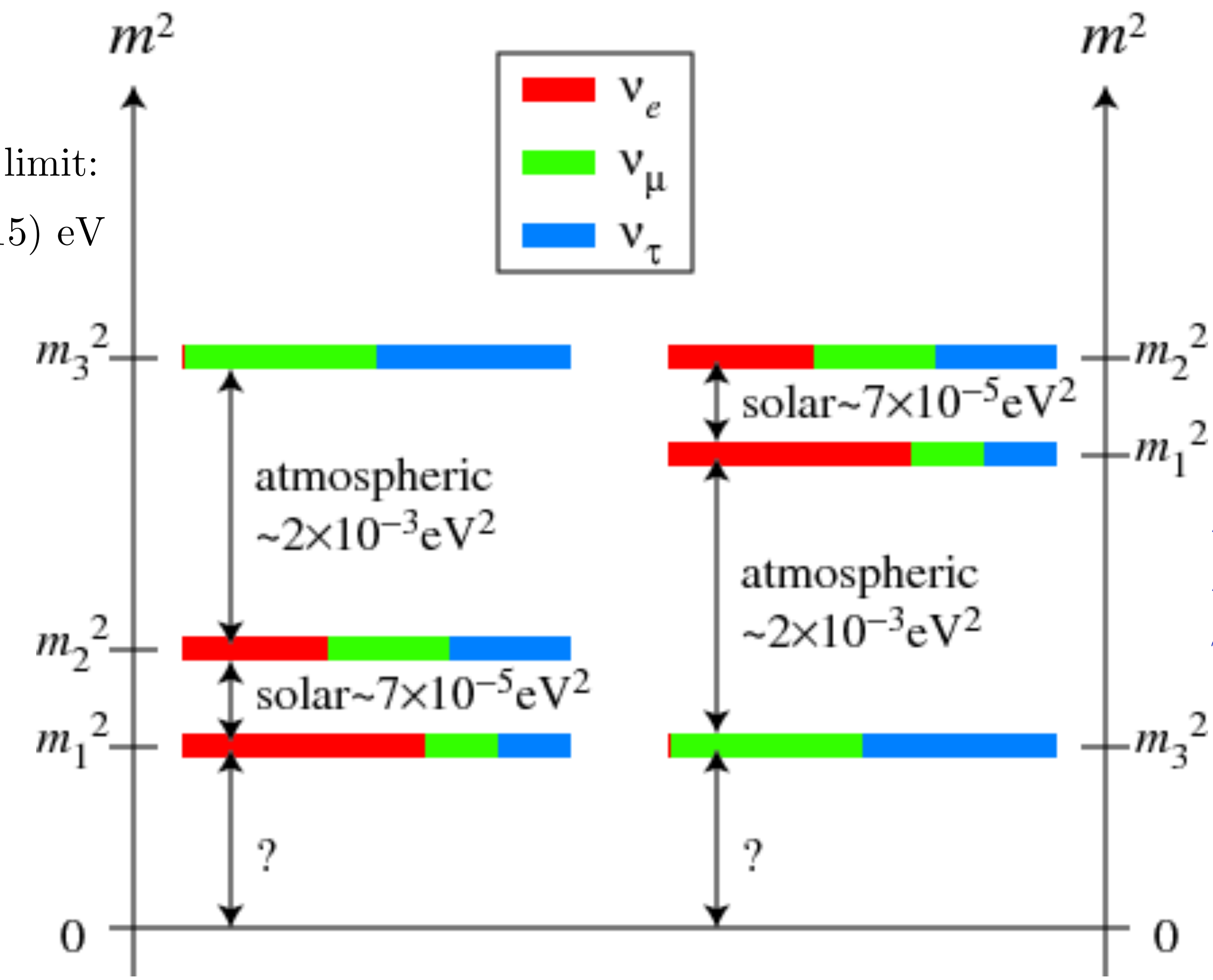
CMB data 95% C.L. limit:

$\Sigma_j m_j < (0.340 - 0.715)$ eV

$\Delta m^2_{31} = \Delta m^2_A > 0$

$\Delta m^2_{21} = \Delta m^2_\odot > 0$

$m_2 = (m_1^2 + \Delta m^2_{21})^{\frac{1}{2}}$



$\Delta m^2_{32} = \Delta m^2_A < 0$

$\Delta m^2_{21} = \Delta m^2_\odot > 0$

$m_2 = (m_3^2 + \Delta m^2_{23})^{\frac{1}{2}}$

Fig: Adv.High Energy Phys. 2016

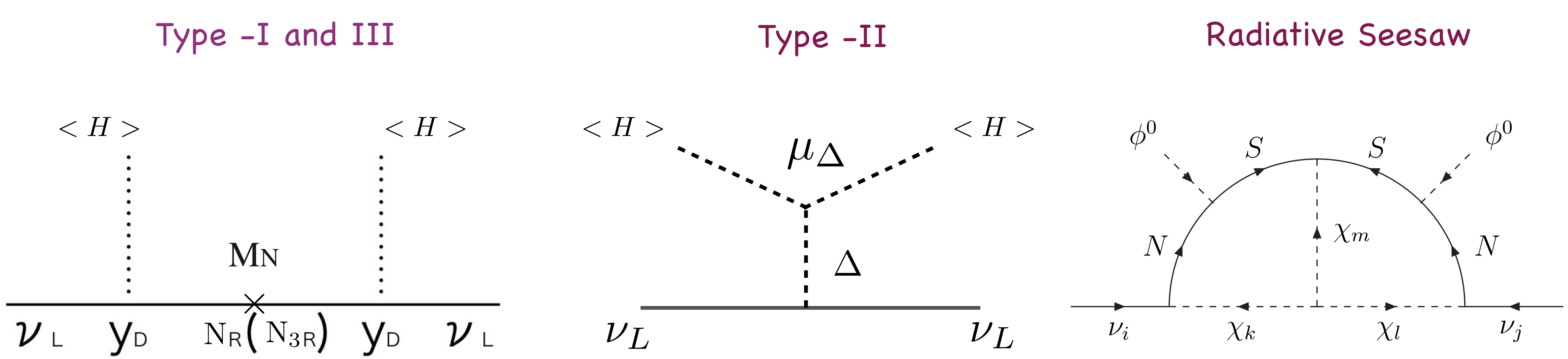
Many existing experiments, like NOvA, MINOS, JUNO etc. can not determine the sign of Δm^2_{31} or Δm^2_{32} ; At least 2 neutrinos are massive and $m_1 = 0$ seems to be allowed.

Seesaw mechanism

(a) Type I and III seesaw: heavy right-handed neutrino, as singlet for I and triplet for III; M_N is usually related to GUT.

(b) Type II seesaw: scalar triplet, $H^T i\sigma_2 \Delta^\dagger H$ term violating lepton number: $\mu_\Delta \ll 1$

(c) Radiative seesaw: exotic particles at TeV scale, testable at LHC; Natural symmetry may ensure DM.



Other ideas: Inverse, linear or double seesaw, etc.

Two-loop seesaw with DM

	Fermion Fields					Scalar Fields				Inert Scalar Fields			
	L_L	e_R	$L'_{L/R}$	$\chi_{L/R}$	$N_{L/R}$	H	H'	Δ	φ	s	η	s'	η'
$SU(2)_L$	**2**	**1**	**2**	**1**	**1**	**2**	**2**	**3**	**1**	**1**	**2**	**1**	**2**
$U(1)_Y$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
$U(1)_H$	0	0	$2x$	x	y	0	$-3x$	$-3x$	$-3x$	$-2x$	x	$x+y$	$-2x+y$

Added as variation, H'
not for 2-loop seesaw

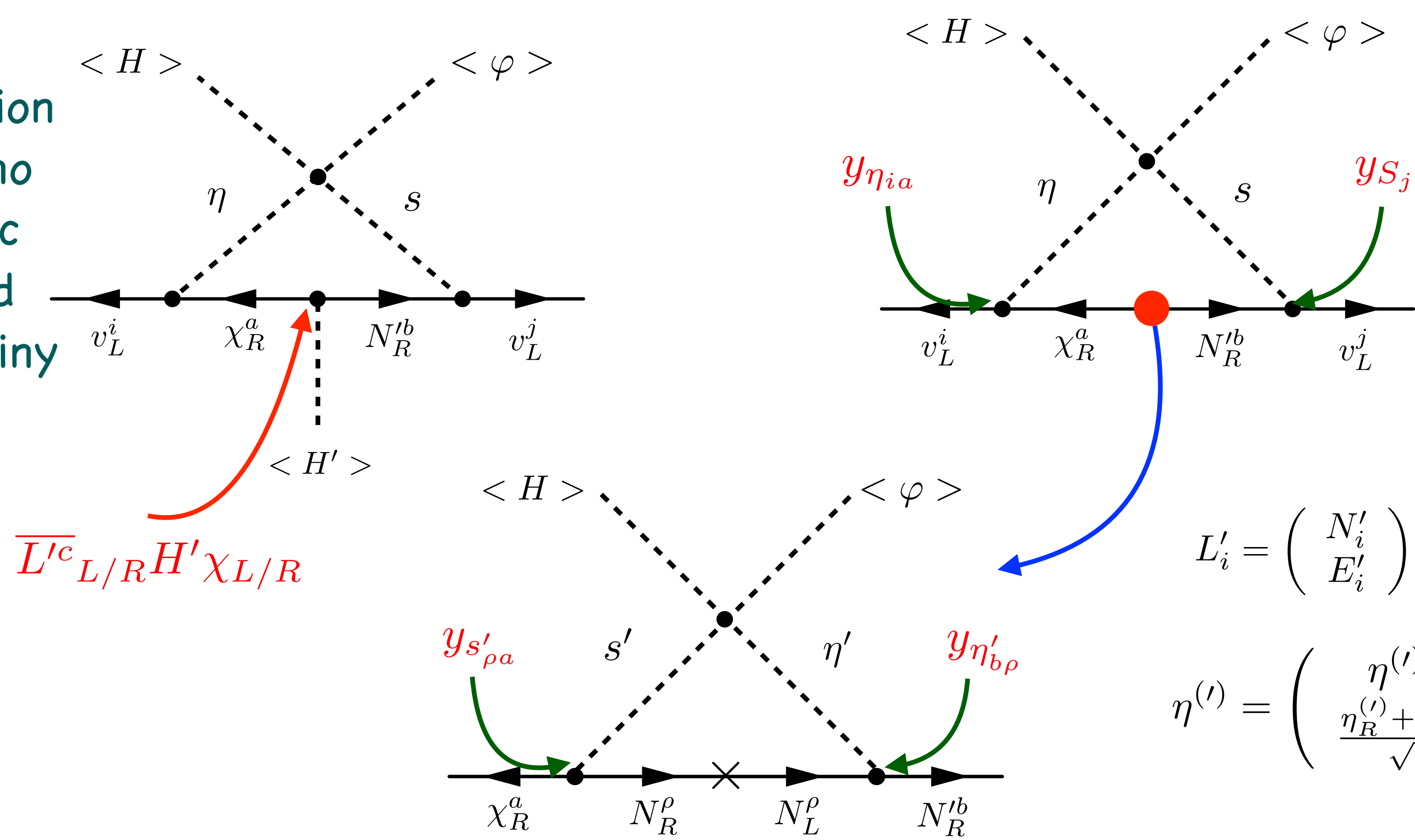
$\varphi \to e^{i3x\alpha}\varphi$ observes a $U(1)$ symmetry; However if $\langle\varphi\rangle \neq 0$, we need $\alpha = 2\pi/3$ for $x \in$ Integer, a Z_3 parity remains.

Under Z_3 parity, the lightest particle in (χ_i, N'_i, η, s) with parity $w = e^{i2\pi/3}(w^2)$ is stable. We will focus on the mass pattern where χ_1 is our DM.

Mass in gauge basis

$$-\mathcal{L}_Y \supset y_{\eta_{ia}} \bar{L}_{L_i} \tilde{\eta} \chi_{R_a} + y_{S_{ia}} s \bar{L}_{L_i} L'_{R_a} + y_{\eta'_{ab}} \bar{L}'_{R_a} \tilde{\eta}' N_{L_b} + y'_{\eta'_{ab}} \bar{L}'_{L_a} \tilde{\eta}' N_{R_b}$$
$$+ y_{s'_{ab}} \bar{N}_{R_a} \chi^c_{R_b} s' + y'_{s'_{ab}} \bar{N}_{L_a} \chi^c_{L_b} s' + \text{h.c.}$$

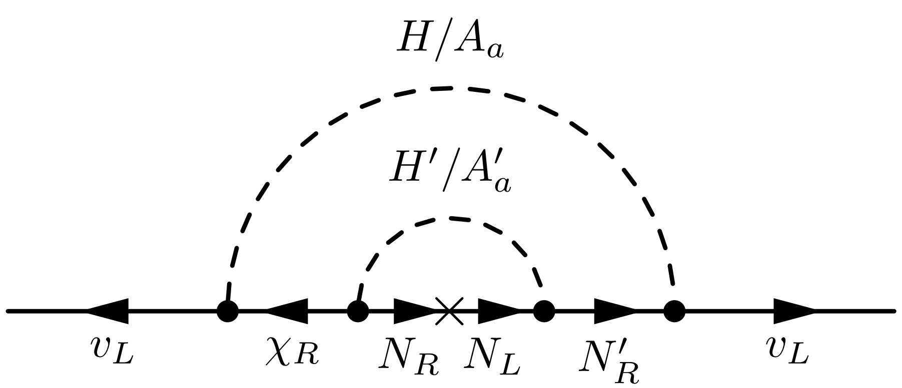
Causing a tension among neutrino mass and relic density bound unless <H'> is tiny

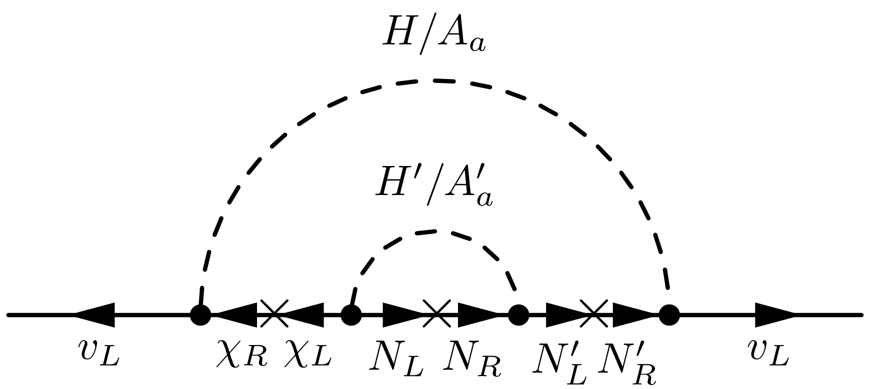


$$L'_i = \begin{pmatrix} N'_i \\ E'_i \end{pmatrix}$$

$$\eta^{(\prime)} = \begin{pmatrix} \eta^{(\prime)+} \\ \frac{\eta_R^{(\prime)} + i\eta_I^{(\prime)}}{\sqrt{2}} \end{pmatrix}$$

Neutrino mass





$$\begin{bmatrix} s_R^{(')} + i s_I^{(')} \\ \eta_R^{(')} + i\eta_I^{(')} \end{bmatrix} = \begin{bmatrix} c_{\alpha^{(')}} & -s_{\alpha^{(')}} \\ s_{\alpha^{(')}} & c_{\alpha^{(')}} \end{bmatrix} \begin{bmatrix} H_1^{(')} + iA_1^{(')} \\ H_2^{(')} + iA_2^{(')} \end{bmatrix}$$

Mass Eigenstates

$$(m_\nu)_{ij} \equiv \frac{1}{(4\pi)^4}\left(y_{\eta_{ia}}[F_I + F_{II}]_{ab} y^T_{S_{bj}} + y_{S_{ja}}[F_I^T + F_{II}^T]_{ab} y^T_{\eta_{bj}}\right)$$

$$\equiv \frac{1}{(4\pi)^4}\left(y_{\eta_{ia}} G_{ab} y^T_{S_{bj}} + y_{S_{ja}} G^T_{ab} y^T_{\eta_{bj}}\right)$$

$F_I, F_{II} \propto (m^2_{H_1} - m^2_{H_2})(m'^2_{H_1} - m'^2_{H_2})$ are finite form factors.

Imposing Neutrino oscillation data

$$y_\eta = \frac{1}{2}[(V^*_{\rm MNS} D_\nu V^\dagger_{\rm MNS} + A](y^T_S)^{-1} G^{-1}$$

A: Arbitrary anti-symmetric matrix

y_η is a function of y_S and form factors up to an uncertainty.

The heavy Z'

The VEV of Δ mixes the $U(1)_Y$ and $U(1)_H$ gauge bosons. The mass eigenstates and mixing angle are:

$$m_Z^2 \approx m_{Z_0}^2 \left(1 - 4\epsilon_2^2\epsilon_3^2\right), \quad m_{Z'}^2 \approx m_{\tilde{Z}}^2 \left(1 + \epsilon_2^2\right),$$

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{bmatrix} c_Z & s_Z \\ -s_Z & c_Z \end{bmatrix} \begin{pmatrix} Z_0 \\ \tilde{Z} \end{pmatrix}, \quad \tan\theta_Z = \frac{-2\epsilon_1\epsilon_2\epsilon_3}{1 + \epsilon_2^2 - \epsilon_1^2}.$$

with $\epsilon_1 = \frac{m_{Z_0}}{m_{\tilde{Z}}}$, $\epsilon_2 = \frac{v_\Delta}{v_\varphi}$ and $\epsilon_3 = \frac{v_\Delta}{\sqrt{v_H^2 + 4v_\Delta^2}}$.

$$\rho_0 \simeq \frac{\left(1 + \frac{2v_\Delta^2}{v_H^2}\right)}{\left(1 + \frac{4v_\Delta^2}{v_H^2}\right)\left(1 - 4\epsilon_2^2\epsilon_3^2\right)} \quad \Rightarrow \quad v_\Delta \lesssim 3.5 \text{ GeV}$$

Assuming $m_{Z'} > 1$ TeV, this gives $|\tan\theta_Z| < 10^{-5}$. Thus no impact on DM annihilation or DM-nucleon scattering.

S and T parameters

In this model inert scalars (η, s) cause notable deviation to $S = -16\pi\Pi'(0)_{W_3B}$ and $T = \frac{4\pi}{m_Z^2 s_W^2 c_W^2}\left[2\Pi_{W_1W_1}(0) - \Pi_{W_3W_3}(0)\right]$.

$$
\begin{aligned}
\Delta S &= \frac{1}{12\pi}\left[s_\alpha^2 \ln\left(\frac{m_{H_1}^2}{m_{\eta^+}^2}\right) + c_\alpha^2 \ln\left(\frac{m_{H_2}^2}{m_{\eta^+}^2}\right) - 3c_\alpha^2 s_\alpha^2 \chi(m_{H_1}, m_{H_2})\right] \\
\Delta T &= \frac{1}{16\pi m_W^2 s_W^2}\left[s_\alpha^2 F(m_{H_1}, m_{\eta^+}) + c_\alpha^2 F(m_{H_2} m_{\eta^+}) - c_\alpha^2 s_\alpha^2 F(m_{H_1}, m_{H_2})\right]
\end{aligned}
$$

$$
\begin{aligned}
\chi(m_1, m_2) &= \frac{5\left(m_1^4 + m_2^4\right) - 22 m_1^2 m_2^2}{9\left(m_1^2 - m_2^2\right)^2} + \frac{3 m_1^2 m_2^2\left(m_1^2 + m_2^2\right) - m_1^6 - m_2^6}{3\left(m_1^2 - m_2^2\right)^3} \ln\left(\frac{m_1^2}{m_2^2}\right), \\
F(m_1, m_2) &= m_1^2 + m_2^2 - \frac{2 m_1^2 m_2^2}{m_1^2 - m_2^2} \ln\left(\frac{m_1^2}{m_2^2}\right).
\end{aligned}
$$

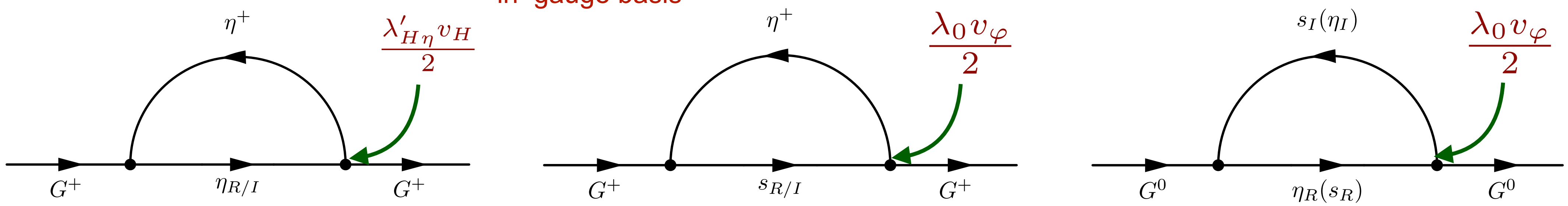
Exactly the same functions used for VLQ mixing contribution to S and T parameters in Phys. Rev. D 47, 2046 (1993)

L_i' have no impact, i.e. $\Delta S_f = \frac{2}{3\pi}(t_{3L} - t_{3R})^2 = 0$ and $\Delta T_f = 0$.

Wave-function renormalisation

Equivalence Theorem $\Rightarrow$ $\delta\rho = \delta Z_{G^+} - \delta Z_{G^0}$, where $\partial_\mu G^{+,0}$ are longitude modes of W, Z game bosons.

Self-energy diagrams in gauge basis



$$\begin{aligned} \hat{\alpha}\Delta T &= 2(\lambda'_{H\eta}\frac{v_H}{2}\sin\alpha + \lambda_0\frac{v_\varphi}{2}\cos\alpha)^2 f(m_{H_1}, m_{\eta^+}) \\ &+ 2(\lambda'_{H\eta}\frac{v_H}{2}\cos\alpha - \lambda_0\frac{v_\varphi}{2}\sin\alpha)^2 f(m_{H_2}, m_{\eta^+}) \Big\} \; \delta Z_{G^+} \\ &- \frac{1}{2}\lambda_0^2 v_\varphi^2 f(m_{H_1}, m_{H_2}) \;\; \Rightarrow \delta Z_{G^0} \end{aligned}$$

This generalises the result in Phys. Rev. D 74, 015007 (2006)

$$f(m_1, m_2) = \frac{1}{4\pi^2}\frac{dB_0(p, m_1, m_2)}{dp^2}\Big|_{p^2=0}$$

Passarino-Veltman Function

$$B_0(p, m_1, m_2) = \int \frac{d^4k}{i(4\pi^2)}\frac{1}{(k^2 - m_1^2)((k+p)^2 - m_2^2)}$$

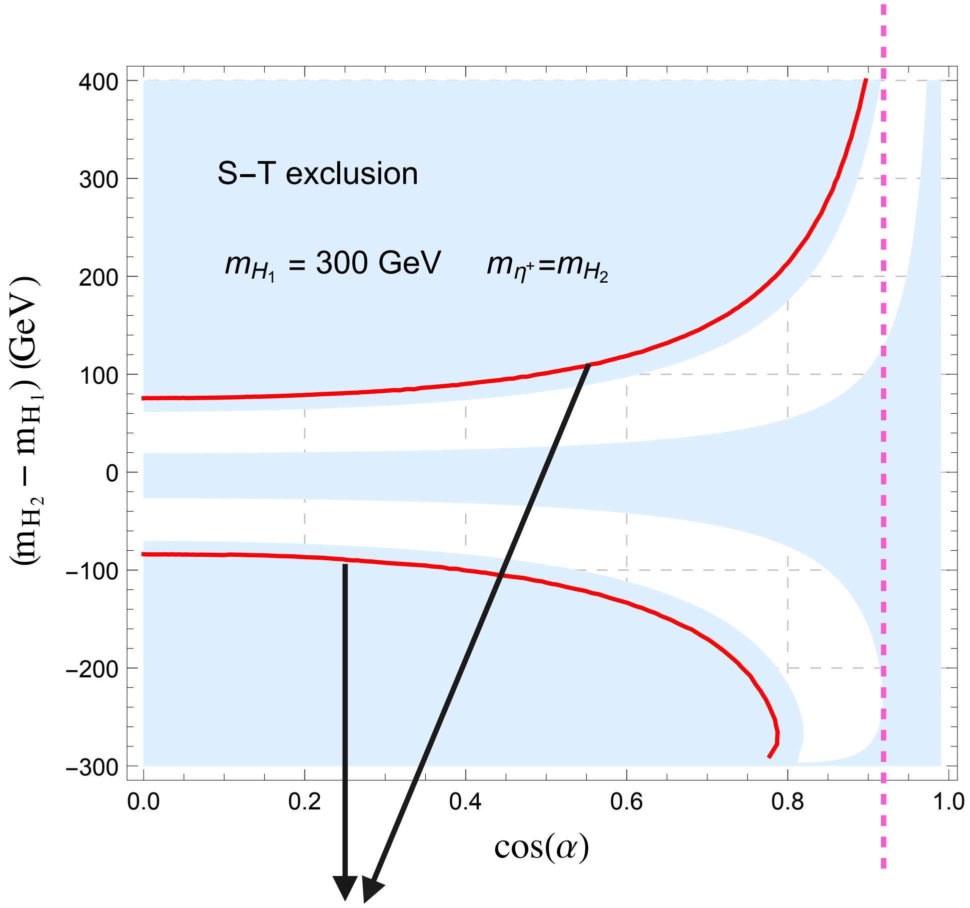
$$(\lambda'_{H\eta}\frac{v_H}{2}\sin\alpha + \lambda_0\frac{v_\varphi}{2}\cos\alpha)^2 = \frac{(m_{H_1}^2 - m_{\eta^+}^2)^2}{v_H^2}\sin^2\alpha$$

$$(\lambda'_{H\eta}\frac{v_H}{2}\cos\alpha - \lambda_0\frac{v_\varphi}{2}\sin\alpha)^2 = \frac{(m_{H_2}^2 - m_{\eta^+}^2)^2}{v_H^2}\cos^2\alpha$$

$$\lambda_0^2 v_\varphi^2 = \frac{4(m_{H_1}^2 - m_{H_2}^2)^2}{v_H^2}\sin\alpha^2\cos^2\alpha$$

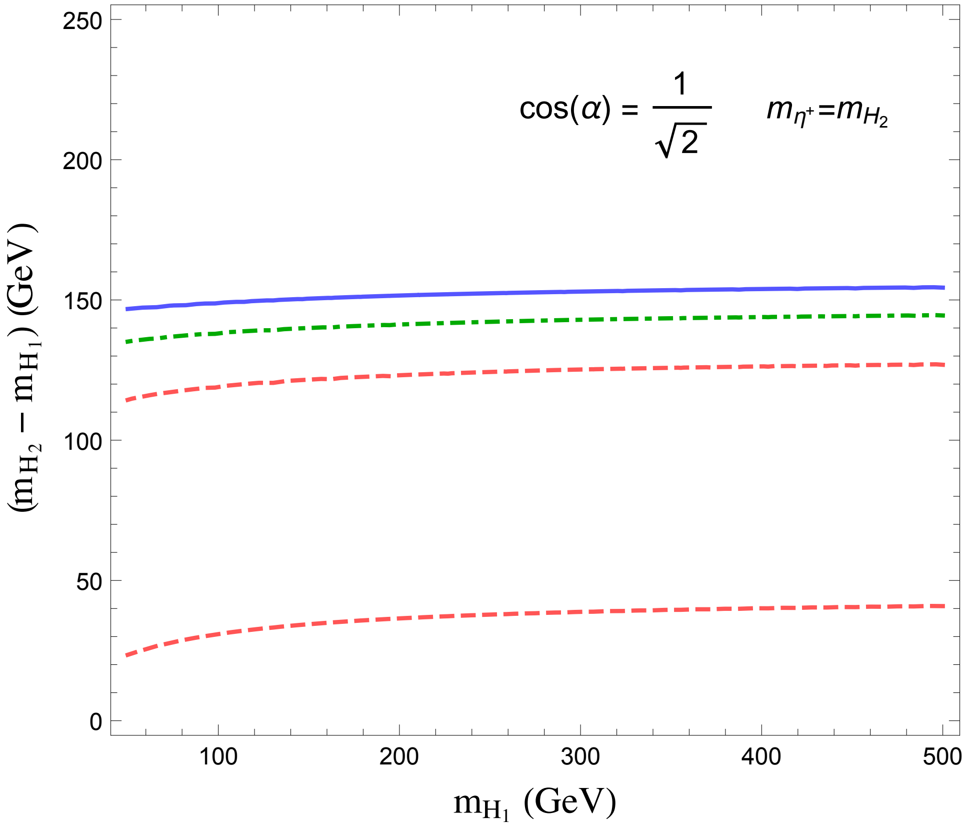
EWPT bounds

1σ fit prefers $m_{H_2} - m_{H_1} > 0$
for $0.92 < \cos\alpha < 1.0$
(two white bands are allowed)



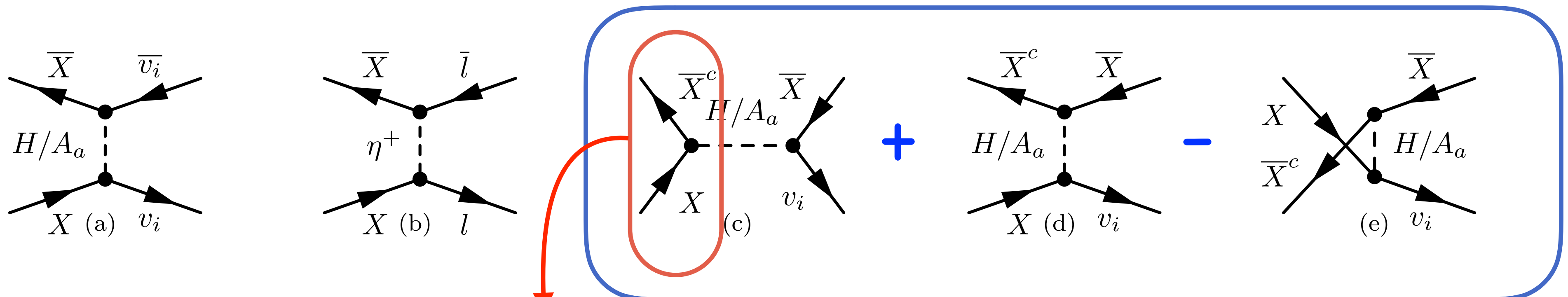
At 3σ C.L. region inside red lines is permitted.

For $\cos\alpha = \frac{1}{\sqrt{2}}$, $m_{\eta^+} = m_{H_2}$
$(m_{H_2} - m_{H_1}) \subset (30, 120)$ GeV
at 68% C.L.



68% (1σ) 95% (2σ) 99% (3σ)

DM (Semi-)annihilations



Symmetric factor 2

Z3 parity

X	ω
$\frac{H_i+iA_i}{\sqrt{2}}$	ω
$\frac{H_i-iA_i}{\sqrt{2}}$	ω^2

$$-\mathcal{L} = \frac{y_{\eta_{i1}}}{\sqrt{2}}\bar{\nu}_i P_R X (s_\alpha H_1 + c_\alpha H_2) - i\frac{y_{\eta_{i1}}}{\sqrt{2}}\bar{\nu}_i P_R X (s_\alpha A_1 + c_\alpha A_2) - y_{\eta_{i1}}\bar{\ell}_i P_R X \eta^- + \frac{y_{\chi_{11}}}{\sqrt{2}}\bar{X}^C X (c_\alpha H_1 - s_\alpha H_2) + i\frac{y_{\chi_{11}}}{\sqrt{2}}\bar{X}^C X (c_\alpha A_1 - s_\alpha A_2) + \text{h.c.}$$

Semi-annihilations modify the density evolution and freeze out temperature

$$\frac{dY_X}{dx} = -\frac{\lambda_{\rm A}}{x^2}[Y_X^2 - Y_X^{\rm eq2}] - \frac{1}{2}\frac{\lambda_{\rm S}}{x^2}[Y_X^2 - Y_X Y_X^{\rm eq}]$$

$$x_f \simeq \ln\left[0.038c(c+2)\langle\sigma v\rangle_{\rm A}\frac{g M_X M_{pl}}{\sqrt{g_* x_f}}\right] + \ln\left[1 + \frac{c+1}{c+2}\,\frac{\langle\sigma v\rangle_{\rm S}}{2\,\langle\sigma v\rangle_{\rm A}}\right]$$

Semi-annihilation

Identical particle effect for S-channel

$$|\bar{\mathcal{M}}_3|^2 = (s_\alpha c_\alpha)^2 \sum_{i=1}^{3} |y_{\chi_{11}} y_{\eta_{i1}}|^2 \Big[8\, |\sum_{a=1}^{2} (-1)^{a+1} S_{inv}^a|^2 (p_1\cdot p_2 - M_X^2)(k_1\cdot k_2)$$
$$+2\,|\sum_{a=1}^{2}(-1)^{a+1} T_{inv}^a|^2 (p_1\cdot k_1 + M_X^2)(p_2\cdot k_2) + 2\,|\sum_{a=1}^{2}(-1)^{a+1} U_{inv}^a|^2 (p_2\cdot k_1 + M_X^2) p_1\cdot k_2$$
$$+2\sum_{a=1}^{2}(-1)^{a+1} S_{inv}^{Re,a} \sum_{a=1}^{2}(-1)^{a+1} T_{inv}^a [(p_1\cdot p_2)(k_1\cdot k_2) - (p_1\cdot k_2)(p_2\cdot k_1) + (p_1\cdot k_1)(p_2\cdot k_2)$$
$$+M_X^2(-p_1\cdot k_2 + p_2\cdot k_2 - k_1\cdot k_2)] + 2\sum_{a=1}^{2}(-1)^{a+1} S_{inv}^{Re,a} \sum_{a=1}^{2}(-1)^{a+1} U_{inv}^a [(p_1\cdot p_2)(k_1\cdot k_2)$$
$$-(p_1\cdot k_1)(p_2\cdot k_2) + (p_1\cdot k_2)(p_2\cdot k_1) + M_X^2(-p_2\cdot k_2 + p_1\cdot k_2 - k_1\cdot k_2)]$$
$$-\sum_{a=1}^{2}(-1)^{a+1} T_{inv}^a \sum_{a=1}^{2}(-1)^{a+1} U_{inv}^a [(p_1\cdot k_1)(p_2\cdot k_2) - (p_1\cdot p_2)(k_1\cdot k_2) + (p_1\cdot k_2)(p_2\cdot k_1)$$
$$+M_X^2(k_1\cdot k_2 + p_1\cdot k_2 + p_2\cdot k_2)]\Big] .$$

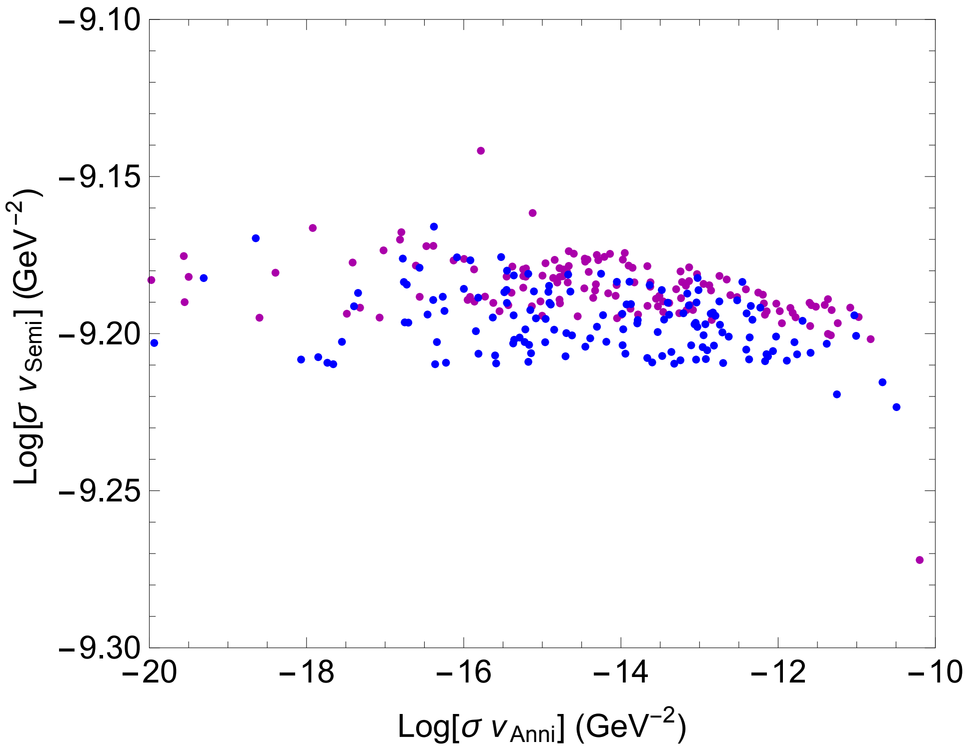
S-T-U interference

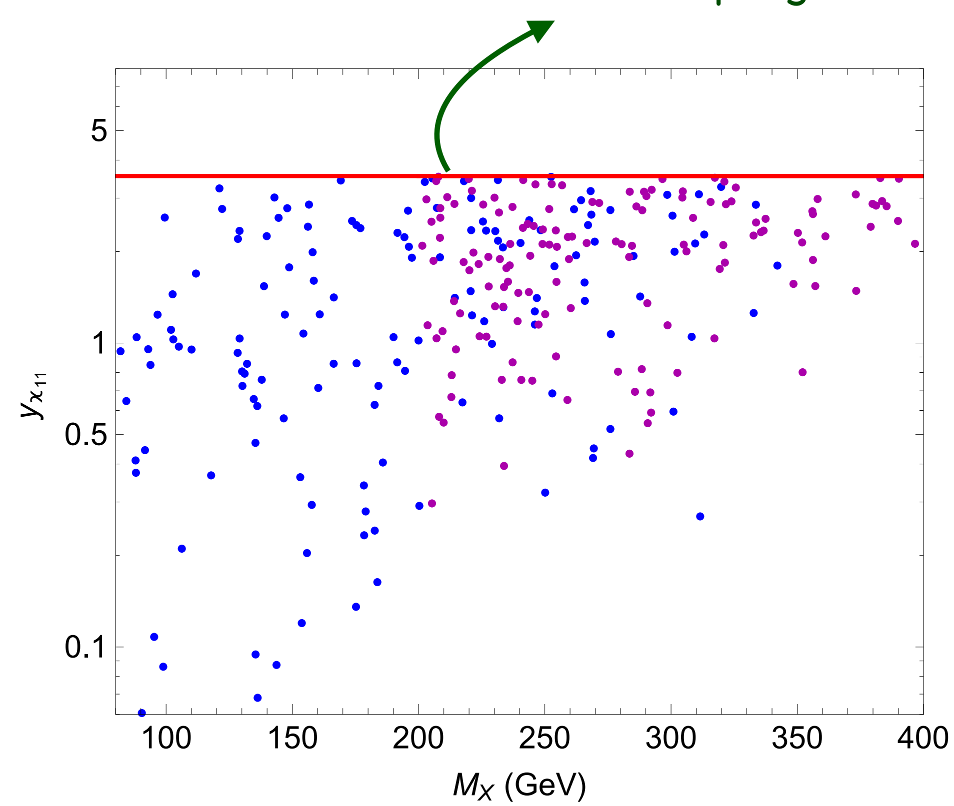
$(-1)^{a+1}$: Phase of two deconstructive complex scalar resonances

$$S_{inv}^a = 1/(s - m_a^2 + i m_a \Gamma_a),\ T_{inv}^a = 1/(2M_X^2 - m_a^2 - 2p_1\cdot k_1),\ \ U_{inv}^a = 1/(M_X^2 - m_a^2 - 2p_1\cdot k_2).$$

Relic density analysis

$\langle \sigma v_{\rm Anni} \rangle$ v.s. $\langle \sigma v_{\rm Semi} \rangle$ at x_f





$100 < m_{H_2} - m_{H_1} < 150$ GeV

$115 < m_{H_2} - m_{H_1} < 125$ GeV

Case I) $m_{H_1} \simeq 2M_X \Rightarrow y_{\chi 11} \gtrsim 0.1$

Case II) $m_{H_2} \simeq 2M_X \Rightarrow y_{\chi 11} \gtrsim 0.5$

An inverse seesaw model

$$-\mathcal{L}_Y = y_{N_{aa}} \bar{L}_{L_a} \tilde{H}' N_{R_a} + y_{N\varphi_{aa}} \bar{N}_{L_a} N_{R_a} \varphi + y_{N\varphi'_{ab}} \bar{N}^C_{L_a} N_{L_b} \varphi' + y_{U\varphi_{aa}} \bar{U}_{R_a} U_{L_a} \varphi^* + y_{D\varphi_{aa}} \bar{D}_{R_a} D_{L_a} \varphi + y_{E\varphi_{aa}} \bar{E}_{R_a} E_{L_a} \varphi + (y_{u\chi})_{ia} \bar{u}_{R_i} U_{L_a} \chi^* + (y_{d\chi})_{ia} \bar{d}_{R_i} D_{L_a} \chi + (y_{e\chi})_{ia} \bar{e}_{R_i} E_{L_a} \chi + \text{h.c.},$$

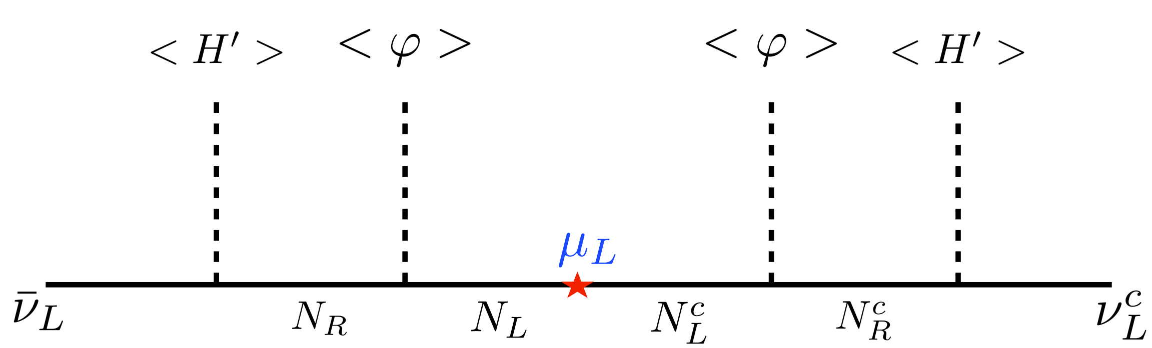
The neutrino mass matrix (9×9) in the basis of $\left(\nu_L^i, N_R^{C,a}, N_L^a\right)$, is given by M_N, with $m_D \equiv y_N v_{H'}/\sqrt{2}$, $M \equiv y_{N\varphi} v_\varphi/\sqrt{2}$, and $\mu_L \equiv y_{N\varphi'} v_{\varphi'}/\sqrt{2}$.

$$\mu_L \lesssim m_D \ll M \quad \Longrightarrow \quad m_\nu \approx m_D^* M^{-1} \mu_L (M^T)^{-1} m_D^\dagger$$

$$m_{N_{1,2}} \approx M \pm \mu_L/2$$

Imposing Casas-Ibarra Parametrisation $\Longrightarrow$ $m_D = U_{MNS} \sqrt{D_\nu} O_{mix} R_N^{-1}$

$$M_N = \begin{bmatrix} 0 & m_D^* & 0 \\ m_D^\dagger & 0 & M^T \\ 0 & M & \mu_L \end{bmatrix}$$



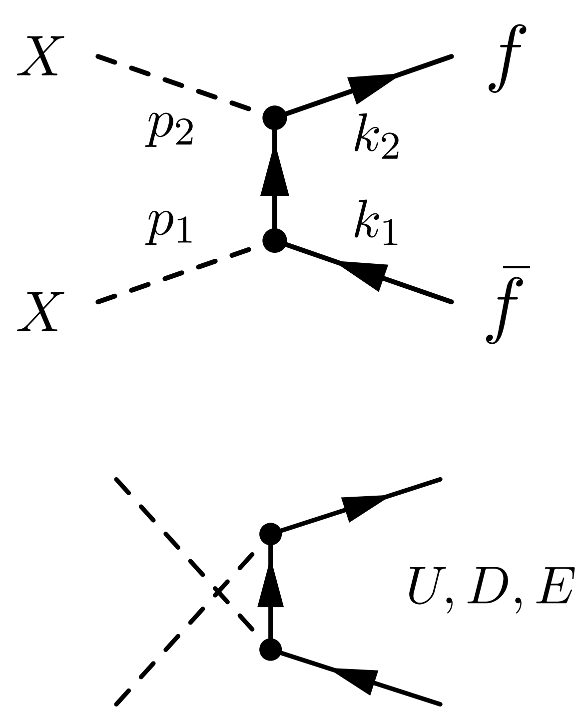
Bound on DM mass

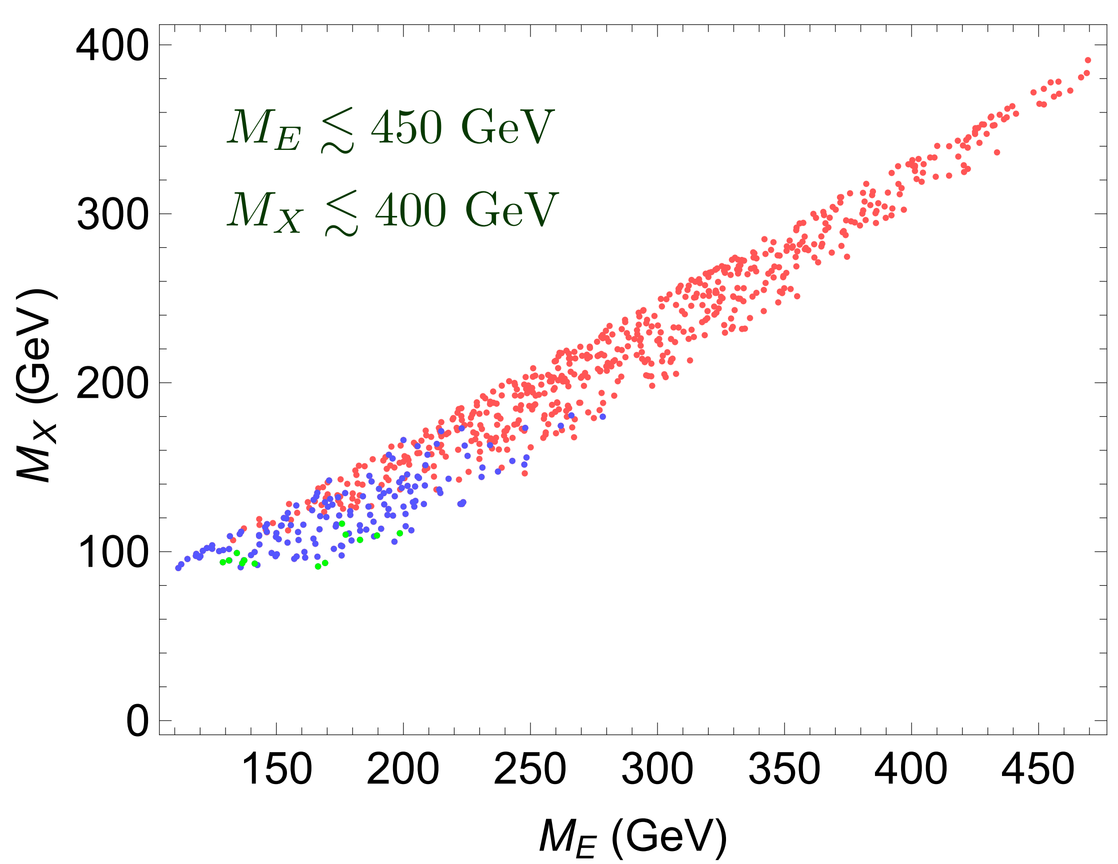
We can fix dark matter to be $X = \chi_R$, and the constraints are from relic density, muon g-2, flavour violation processes from lepton and Z decay.

$$m^2_{\chi_R} = \mu^2_\chi + \frac{1}{2}(\lambda_{H\chi}v^2_H + \lambda_{H'\chi}v^2_{H'} + \lambda_{\varphi\chi}v^2_\varphi + \lambda_{\varphi'\chi}v^2_{\varphi'}) - \sqrt{2}\mu v_{\varphi'}$$

$$m^2_{\chi_I} = \mu^2_\chi + \frac{1}{2}(\lambda_{H\chi}v^2_H + \lambda_{H'\chi}v^2_{H'} + \lambda_{\varphi\chi}v^2_\varphi + \lambda_{\varphi'\chi}v^2_{\varphi'}) + \sqrt{2}\mu v_{\varphi'}$$

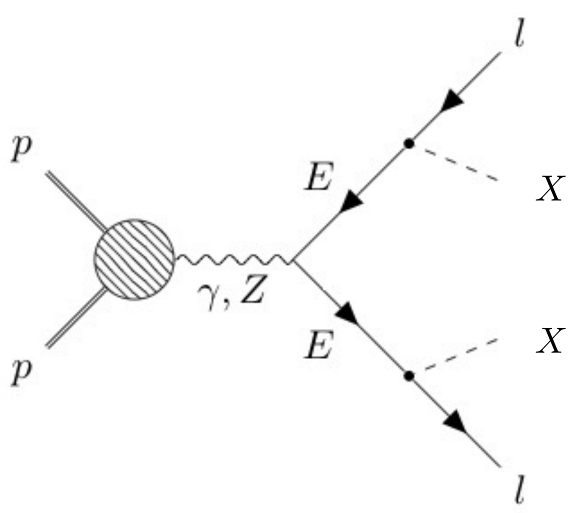
Mass splitting
If assume mu >0,
real part is DM





Collider signal: 2 tau + MET

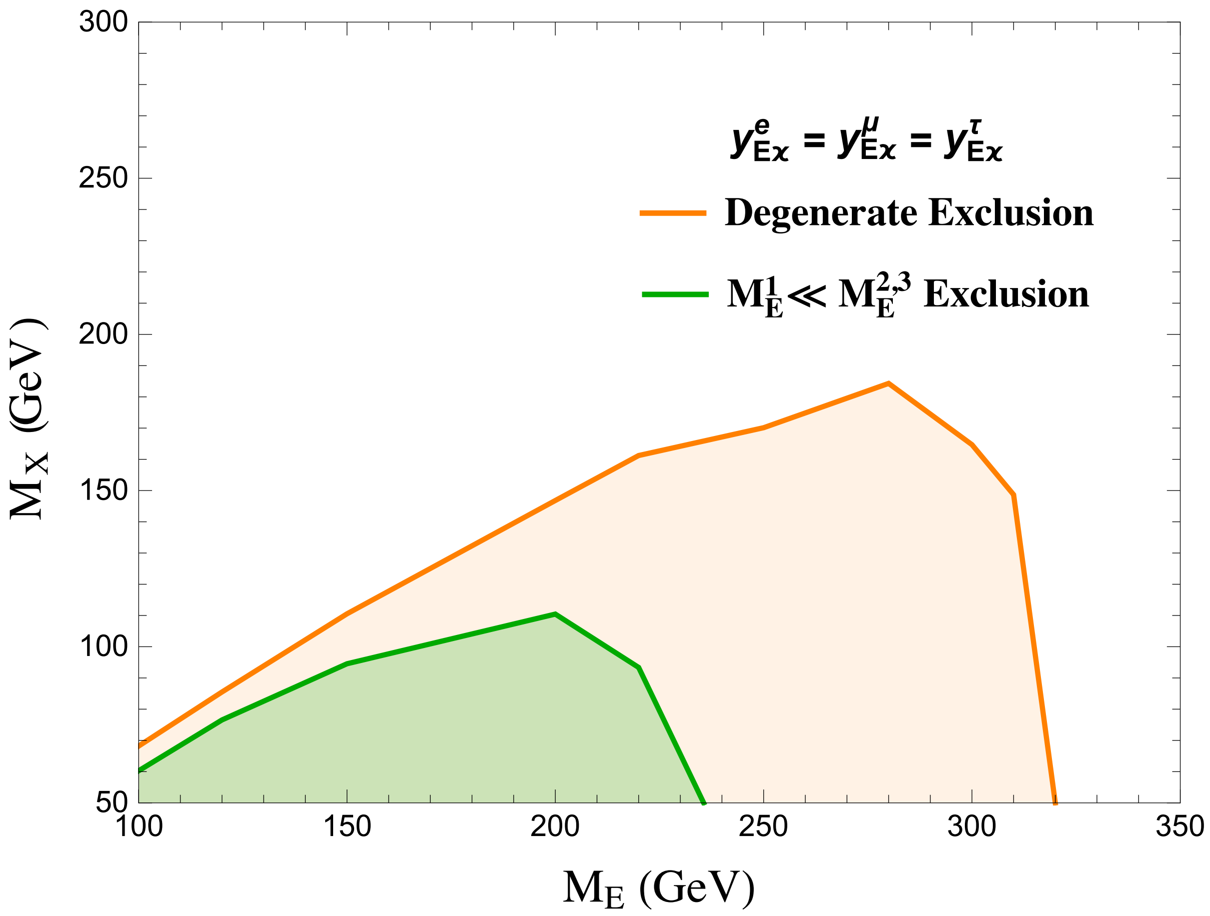
Drell-Yan pair production of VLL, and decay into 2 tau + MET:



$$M_T(q, \vec{p}_T^{\,miss}) \equiv \sqrt{2E_{T,q}E_T^{miss}(1-\cos\Delta\phi)},$$

$$M_{T2} = \min_{\vec{p}_T^{\,X(1)}+\vec{p}_T^{\,X(2)}=\vec{p}_T^{\,miss}} \left[\max\left(M_T^{(1)}, M_T^{(2)}\right)\right]$$

LHC bound is very loose due to low sensitivity to hadronic taus



No constraint for M_E > 320 GeV

More model detail in the backup slides

Conclusion

- Neutrino as the most abundant fermion in universe is special. For model building, seesaw mechanism is implemented to generate < 0.1 eV order mass (Dirac or Majorana), $\lesssim 10^{-6} \times m_E (= 0.511 \mathrm{MeV})$.
- Many neutrino models postulate extra neutral fermions and scalars, with discrete symmetry (Abelian or non-Abelian), giving rise to DM candidate.
- New signals inherent normally are: Lepton flavour violation processes, collider signatures at LHC.

Back up Slides
For Inverse Seesaw Model

The model

Charge assignments of our fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_H \times Z_2$, where all the SM fields are zero charges under the $U(1)_H$ symmetry and even under the Z_2.

	$U_R(U_L)$	$D_R(D_L)$	$E_R(E_L)$	$N_R(N_L)$	H'	φ	φ'	χ
$SU(3)_C$	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_L$	**1**	**1**	**1**	**1**	**2**	**1**	**1**	**1**
$U(1)_Y$	$\frac{2}{3}$	$-\frac{1}{3}$	-1	0	$\frac{1}{2}$	0	0	0
$U(1)_H$	$4(1)$	$-4(-1)$	$-4(-1)$	$4(1)$	4	-3	-2	1
Z_2	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$-$

Non-trivial anomaly free conditions involve with products of $U(1)$s.

For $[U(1)_H]^2[U(1)_Y]$:

$$n_f\left[3\cdot\tfrac{2}{3}(4^2-1)-3\cdot\tfrac{1}{3}(4^2-1)-(4^2-1)\right]=0$$

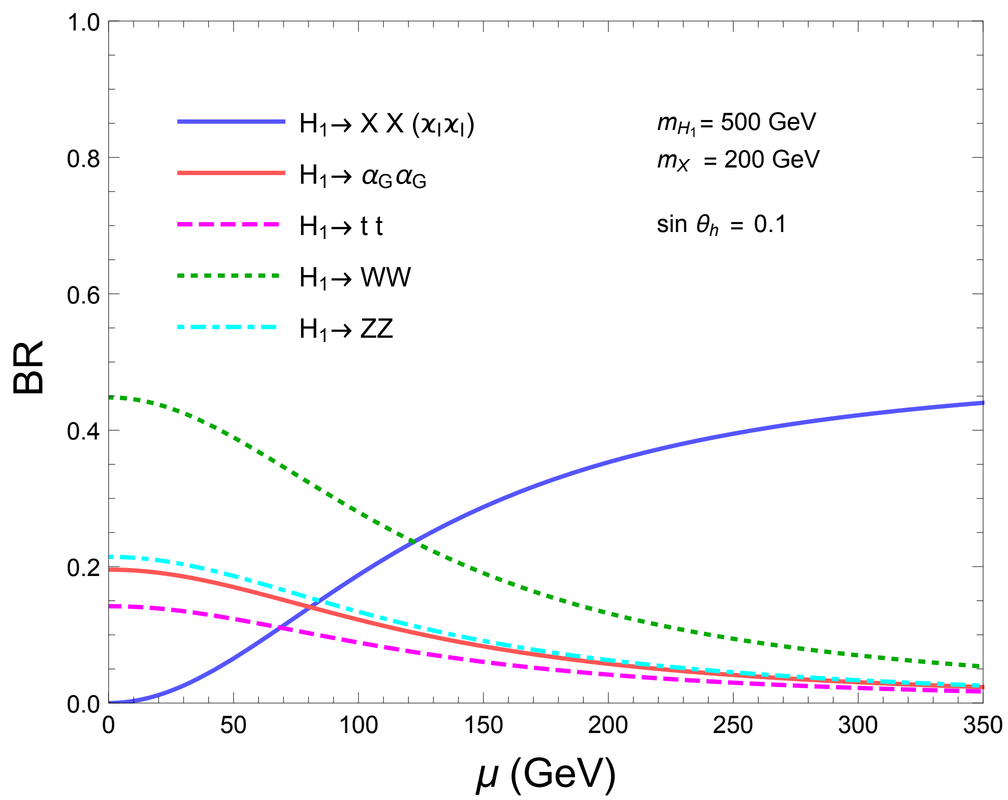
For $[U(1)_H][U(1)_Y]^2$:

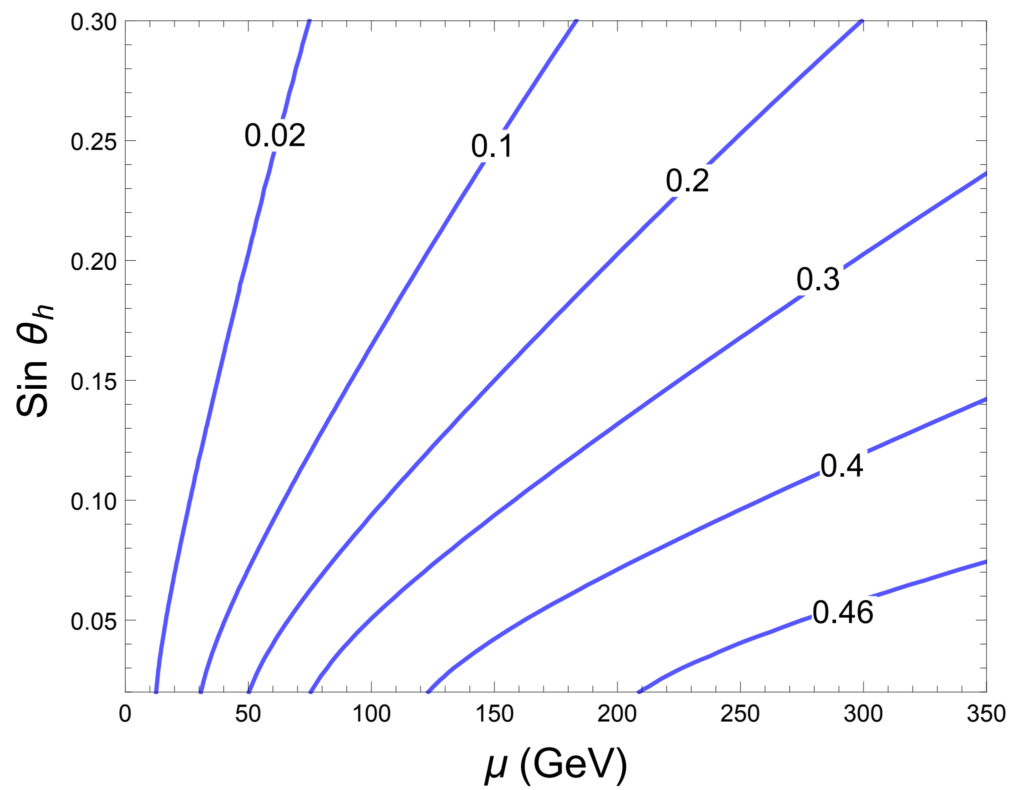
$$n_f\left[3\cdot\left(\tfrac{2}{3}\right)^2(4-1)+3\cdot\left(-\tfrac{1}{3}\right)(4-1)+(-4+1)\right]=0$$

The scalar sector

$$V = \sum_{\phi}^{H,H',\varphi,\varphi',\chi} \left[\mu_\phi^2 \phi^\dagger\phi + \lambda_\phi |\phi^\dagger\phi|^2\right] + \frac{1}{2}\sum_{\phi\neq\phi'}^{H,H',\varphi,\varphi',\chi} \lambda_{\phi\phi'}|\phi|^2|\phi'|^2 + \lambda'_{HH'}(H^\dagger H')(H'^\dagger H)$$

$$+ \left[\lambda_0 (H^\dagger H')\varphi'^2 - \mu\chi\chi\varphi' + \text{h.c.}\right],$$





Casas-Ibarra Parametrisation

$$U_{\text{MNS}} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \text{diag}(1, \mathrm{e}^{\mathrm{i}\frac{\alpha_{21}}{2}}, \mathrm{e}^{\mathrm{i}\frac{\alpha_{31}}{2}})$$

$$m_\nu \approx m_D^* M^{-1} \mu_L (M^T)^{-1} m_D^\dagger \qquad m_D = U_{MNS} \sqrt{D_\nu} O_{mix} R_N^{-1}$$

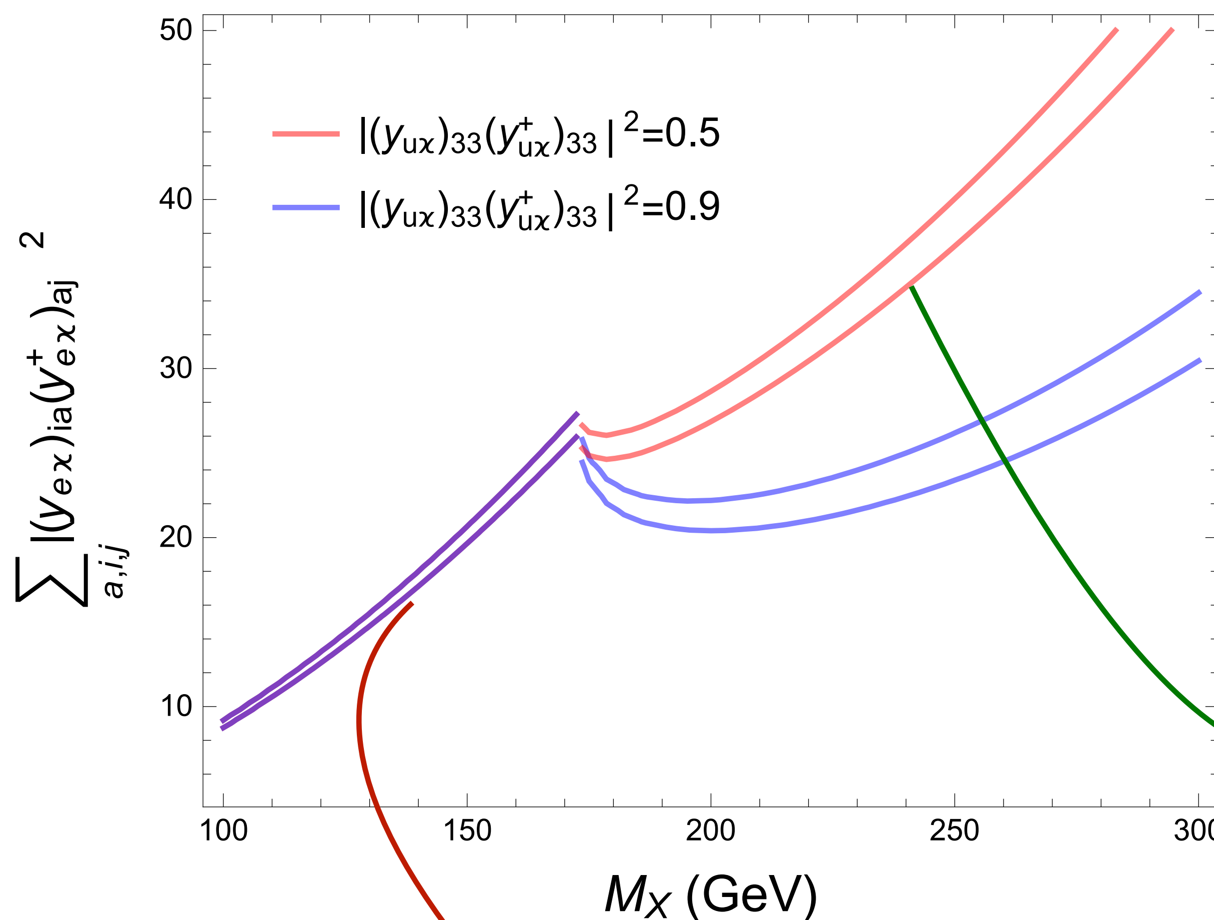
$$R_N^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{d}{ab} & \frac{1}{b} & 0 \\ \frac{-be+df}{abc} & \frac{f}{bc} & \frac{1}{c} \end{bmatrix}, \quad \mu_M = M^{-1} \mu_L (M^T)^{-1}$$

$$a = \sqrt{\mu_{M,11}}, \quad d = \frac{\mu_{M,12}}{a}, \quad b = \sqrt{\mu_{M,22} - d^2}, \quad f = \frac{d\ \mu_{M,13} - a\ \mu_{M,23}}{ab}$$

$$e = \frac{\mu_{M,13}}{a} + 2\frac{d}{b} f, \quad c = \sqrt{\mu_{M,33} - \left(e - 2\frac{d}{b} f\right)^2 - f^2}$$

Top quark threshold

The contours delimit the DM-lepton couplings using the bound of relic density



Adding s- and p-waves from top quark final states

Only d-wave for $m_f \to 0$

Direct Detection Bound

$$\sigma_{\chi_R - n} = \frac{\sin^2\theta_h \cos^2\theta_h}{\pi} \frac{\mu_{nX}^2}{M_X^2} \frac{\mu^2 m_n^2 f_N^2}{v^2 m_h^4}$$

$$\simeq 5.3 \times 10^{-43} \left(\frac{\mu \sin\theta_h \cos\theta_h}{M_X}\right)^2 \quad [\mathrm{cm}^2]$$

The bound of $\sigma_{\chi_R - n}$ fixes the ratio of $\frac{\mu \sin\theta_h \cos\theta_h}{M_X}$