Electroweak Phase Transition in Composite Higgs Model

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In collaboration with Ligong Bian and Ke-Pan Xie, 1909.02014, 1912.xxxxx

Outline

Next to Minimal Composite Higgs Model (NMCHM)

One Higgs doublet and One Scalar Singlet

- Gauge Sector
- Fermion Sector

- Electroweak Phase Transition (EWPT)
  - 1st Order Phase Transition

Scalar Potential

$$V(h,\eta) = \frac{\mu_h^2}{2} h^2 + \frac{\lambda_h}{4} h^4 + \frac{\mu_\eta^2}{2} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

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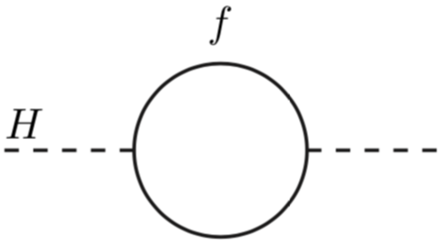
Introduction

Puzzles in SM:

- Hierarchy Problem:

Mass of the elementary scalar particle
Quadratically sensitive to the Cutoff Scale



$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\text{UV}}^2$$

- Baryon Asymmetry of the Universe:

Dynamically Generated:

Sakharov Conditions:

A.D. Sakharov; Pisma Zh.Eksp. Teor.Fiz 5 (1967) 32

1. Baryon number violation
2. C/CP Violation
3. Departure from the Thermal Equilibrium

Electroweak Baryogenesis:
1st Order Phase Transition

Composite Higgs

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Next to Minimal Composite Higgs Model

- Minimal Composite Higgs Model:

$$SO(5)/SO(4)$$

  - Only One Higgs doublet **SM**
- $SO(6)/SO(5)$ Coset
  - One Higgs doublet (h) and One Scalar Singlet (η)

SM

SO(5)/SO(4)

$$\begin{tikzcd} SO(6) \arrow[d, "f"'] \\ SO(5) \end{tikzcd}$$

$$SO(6)\!:\{\underbrace{\overbrace{T_L^a, \quad T_R^a}^{SO(4)\cong SU(2)_L\times SU(2)_R}, \quad T_1^i}_{SO(5)}, \quad \hat{T}_2^r\}$$

SO(6)/SO(5) Coset

$$V(h,\eta) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

- Thermal Correction:

$$\Delta V(h,\eta,T) = \frac{c_h T^2}{2}h^2 + \frac{c_\eta T^2}{2}\eta^2$$

  - $c_h = \frac{3g^2+g'^2}{16} + \frac{y_t^2}{4} + \frac{\lambda_h}{2} + \frac{\lambda_{h\eta}}{12}$
  - $c_\eta = \frac{\lambda_\eta}{4} + \frac{\lambda_{h\eta}}{3}$

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EWPT in NMCHM

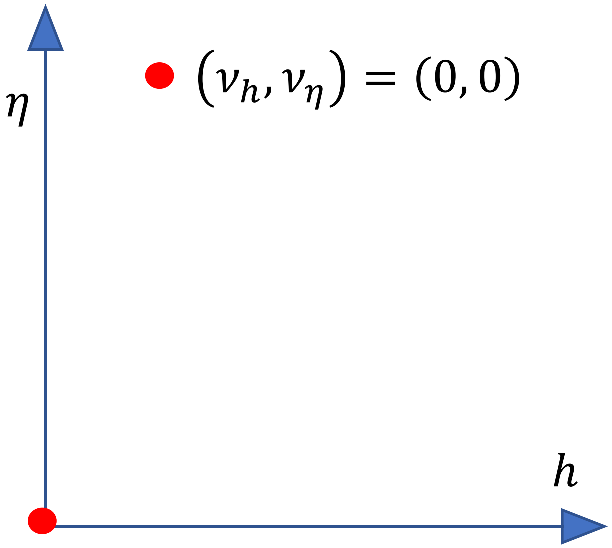
- Scalar Potential:

$$V(h,\eta,T) = \underbrace{\frac{\mu_h^2 + c_h T^2}{2}}_{>\mathbf{0}} h^2 + \frac{\lambda_h}{4} h^4 + \underbrace{\frac{\mu_\eta^2 + c_\eta T^2}{2}}_{>\mathbf{0}} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

EWPT Process

High Temperature

$\boldsymbol{T} \gg$ **EW Scale**



● $(v_h, v_\eta) = (0,0)$

EWPT in NMCHM

- Scalar Potential: $\mu_h^2 < 0 \;\&\; \mu_\eta^2 < 0$

$$V(h,\eta,T) = \underset{\color{red}{>0}}{\boxed{\frac{\mu_h^2 + c_h T^2}{2}}} h^2 + \frac{\lambda_h}{4} h^4 + \underset{\color{red}{<0}}{\boxed{\frac{\mu_\eta^2 + c_\eta T^2}{2}}} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

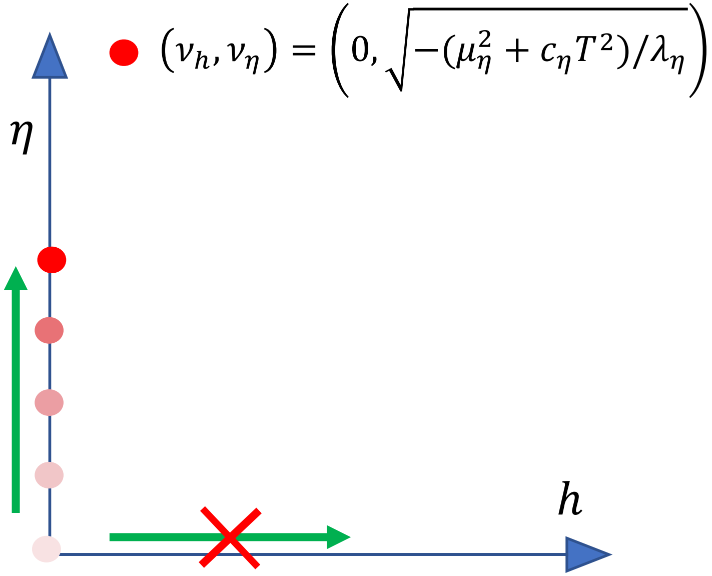
EWPT Process

High Temperature $T \gg$ EW Scale

Minimum @ (0,0)

Temperature Drops $\boldsymbol{c_h > 0}$, $\boldsymbol{c_\eta > 0}$

$$\sqrt{-\mu_h^2/c_h} < T < \sqrt{-\mu_\eta^2/c_\eta}$$



● $(v_h, v_\eta) = \left(0, \sqrt{-(\mu_\eta^2 + c_\eta T^2)/\lambda_\eta}\right)$

EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta,T)=\underset{<\mathbf{0}}{\frac{\mu_h^2+c_hT^2}{2}}h^2+\frac{\lambda_h}{4}h^4+\underset{<\mathbf{0}}{\frac{\mu_\eta^2+c_\eta T^2}{2}}\eta^2+\frac{\lambda_\eta}{4}\eta^4+\frac{\lambda_{h\eta}}{2}h^2\eta^2$$

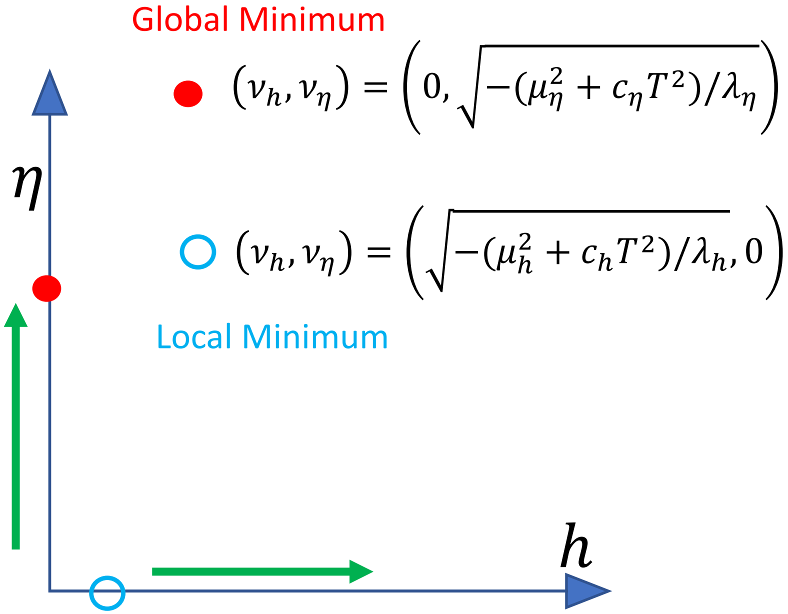
EWPT Process

High Temperature $T \gg$ **EW Scale**

Minimum @ (0,0)

Temperature Drops $\boldsymbol{c_h > 0}$, $\boldsymbol{c_\eta > 0}$

$$T<\sqrt{-\mu_h^2/c_h}$$



Global Minimum

● $(v_h, v_\eta)=\left(0,\sqrt{-(\mu_\eta^2+c_\eta T^2)/\lambda_\eta}\right)$

○ $(v_h, v_\eta)=\left(\sqrt{-(\mu_h^2+c_h T^2)/\lambda_h},0\right)$

Local Minimum

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EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta,T)=\underset{<\mathbf{0}}{\boxed{\frac{\mu_h^2+c_hT^2}{2}}}h^2+\frac{\lambda_h}{4}h^4+\underset{<\mathbf{0}}{\boxed{\frac{\mu_\eta^2+c_\eta T^2}{2}}}\eta^2+\frac{\lambda_\eta}{4}\eta^4+\frac{\lambda_{h\eta}}{2}h^2\eta^2$$

EWPT Process

High Temperature $T \gg$ **EW Scale**

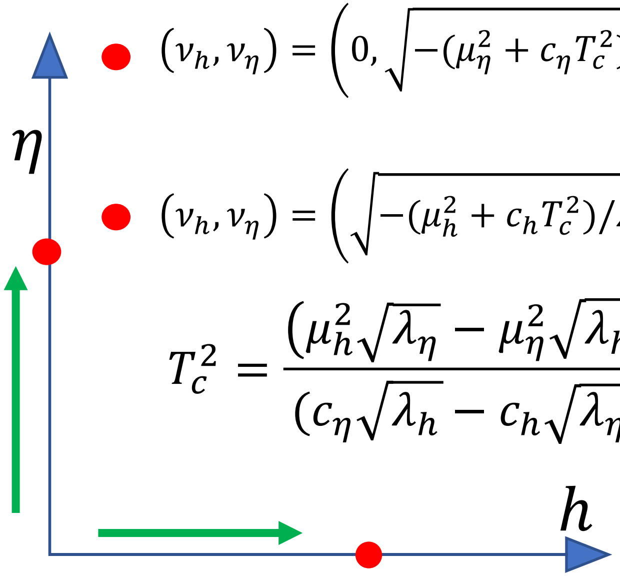
Minimum @ (0,0)

Temperature Drops $c_h > 0, \quad c_\eta > 0$

Critical Temperature $\boldsymbol{T = T_c}$

Two degenerate Minima

$$V_h=-\frac{\left(\mu_h^2+c_hT_c^2\right)^2}{4\lambda_h} = V_\eta=-\frac{\left(\mu_\eta^2+c_\eta T_c^2\right)^2}{4\lambda_\eta}$$



● $(v_h, v_\eta)=\left(0,\sqrt{-(\mu_\eta^2+c_\eta T_c^2)/\lambda_\eta}\right)$

● $(v_h, v_\eta)=\left(\sqrt{-(\mu_h^2+c_h T_c^2)/\lambda_h},0\right)$

$$T_c^2=\frac{\left(\mu_h^2\sqrt{\lambda_\eta}-\mu_\eta^2\sqrt{\lambda_h}\right)}{\left(c_\eta\sqrt{\lambda_h}-c_h\sqrt{\lambda_\eta}\right)}$$

EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta,T)=\underset{<\mathbf{0}}{\frac{\mu_h^2+c_hT^2}{2}}h^2+\frac{\lambda_h}{4}h^4+\underset{<\mathbf{0}}{\frac{\mu_\eta^2+c_\eta T^2}{2}}\eta^2+\frac{\lambda_\eta}{4}\eta^4+\frac{\lambda_{h\eta}}{2}h^2\eta^2$$

EWPT Process

High Temperature $T \gg$ **EW Scale**

Minimum @ (0,0)

Temperature Drops $c_h > 0$, $c_\eta > 0$

Critical Temperature $T = T_c$

Two degenerate Minima

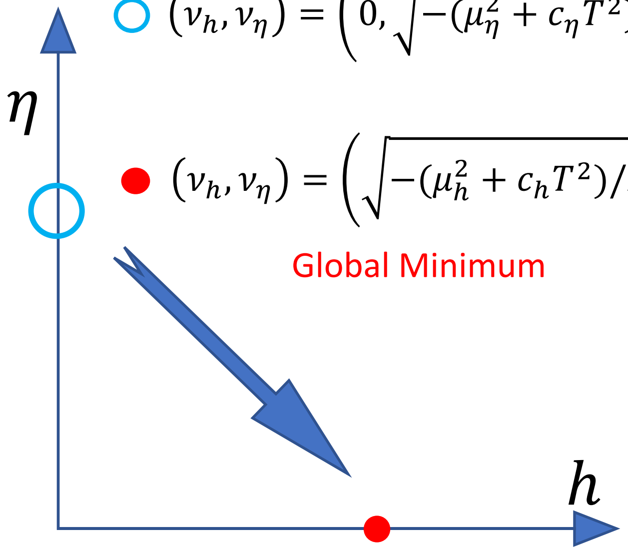
Temperature Drops $\boldsymbol{T \sim T_n \lesssim T_c}$

Local Minimum

○ $(v_h, v_\eta) = \left(0, \sqrt{-(\mu_\eta^2 + c_\eta T^2)/\lambda_\eta}\right)$

● $(v_h, v_\eta) = \left(\sqrt{-(\mu_h^2 + c_h T^2)/\lambda_h}, 0\right)$

Global Minimum



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EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta,T) = \underbrace{\frac{\mu_h^2 + c_h T^2}{2}}_{<\mathbf{0}} h^2 + \frac{\lambda_h}{4} h^4 + \underbrace{\frac{\mu_\eta^2 + c_\eta T^2}{2}}_{<\mathbf{0}} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

EWPT Process

High Temperature $T \gg$ **EW Scale**

Minimum @ (0,0)

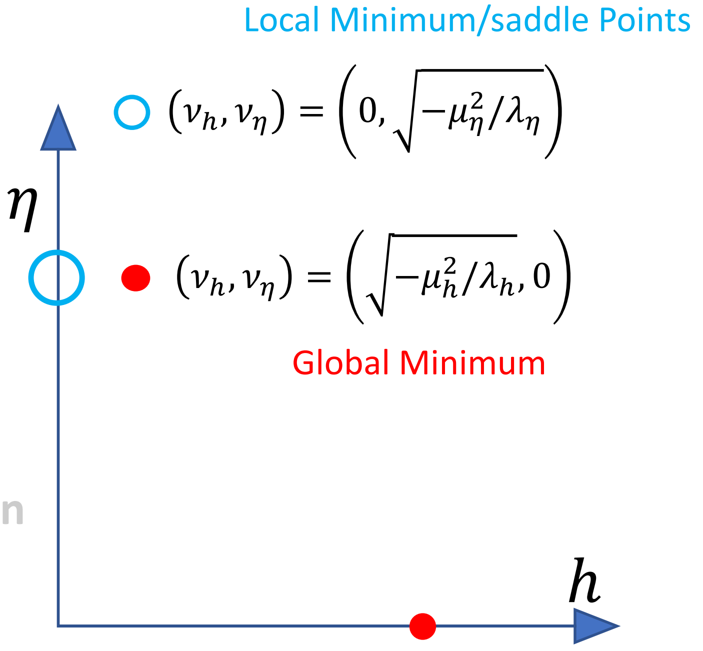
Temperature Drops $c_h > 0, \quad c_\eta > 0$

Critical Temperature $T = T_c$

Two degenerate Minima

Temperature Drops $T \sim T_n \lesssim T_c$ **Transition**

Zero Temperature m_h, m_W, m_Z, m_t



Local Minimum/saddle Points

○ $(v_h, v_\eta) = \left(0, \sqrt{-\mu_\eta^2/\lambda_\eta}\right)$

● $(v_h, v_\eta) = \left(\sqrt{-\mu_h^2/\lambda_h}, 0\right)$

Global Minimum

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EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta,T)=\frac{\mu_h^2+c_hT^2}{2}h^2+\frac{\lambda_h}{4}h^4+\frac{\mu_\eta^2+c_\eta T^2}{2}\eta^2+\frac{\lambda_\eta}{4}\eta^4+\frac{\lambda_{h\eta}}{2}h^2\eta^2$$

EWPT Process

$T \gg$ EW Scale

Conditions for Successful 2-step EWPT:

$$\boldsymbol{\frac{c_\eta}{c_h}<\frac{\mu_\eta^2}{\mu_h^2}<\frac{\sqrt{\lambda_\eta}}{\sqrt{\lambda_h}}<\frac{\lambda_{h\eta}}{\lambda_h}}$$

degenerate Mini

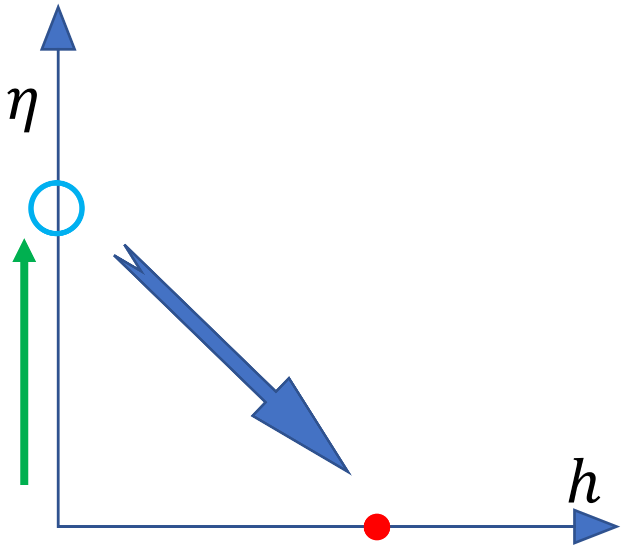
vev first develop along η

Local Minima
Not Saddle Points

EW vev
the global one

Zero Temp

m_h, m_W, m_Z, m_t



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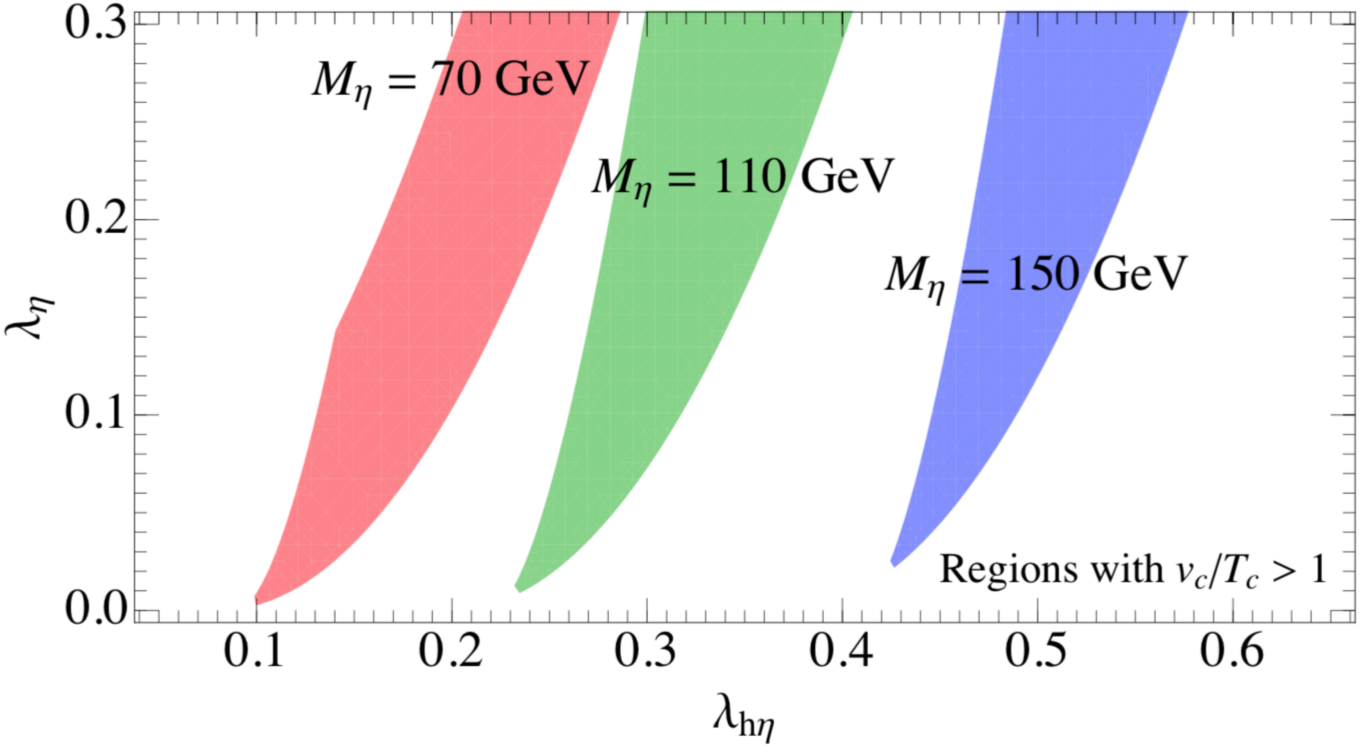
EWPT in NMCHM

- Scalar Potential:

$$V(h,\eta,T) = \frac{\mu_h^2 + c_h T^2}{2} h^2 + \frac{\lambda_h}{4} h^4 + \frac{\mu_\eta^2 + c_\eta T^2}{2} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

Providing observed $m_h, v_h\ (m_W/m_Z)$ → Fixing μ_h^2, λ_h



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Potential in NMCHM

- Explicitly Breaking of Global Symmetry
  - Gauge Sector
  - Fermion Sector

- Minimal Higgs Potential (MHP)

D. Marzocca, M. Serone, J. Shu, JHEP08(2012)013

  - Calculable part dominant
- Higher Order Operators
  - Naïve Dimensional Analysis (NDA)

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Potential in NMCHM MHP

- The Gauge Sector

Vector Resonance: ρ, a $\mathbf{10}, \mathbf{5}$ of $SO(5)$

$$\mathcal{L}_g = -\frac{1}{4} Tr[\rho_{\mu\nu}\rho^{\mu\nu}] + \frac{M_\rho^2}{2g_\rho^2} Tr\left[\left(g_\rho \rho_\mu - e_\mu\right)^2\right] - \frac{1}{4} Tr[a_{\mu\nu}a^{\mu\nu}] + \frac{M_a^2}{2} Tr[a_\mu a^\mu]$$

$$V_g^{\mathrm{IR}}(h) \approx \frac{6}{2}\int \frac{d^4Q}{(2\pi)^4} \ln\left(1 + \frac{\Pi_1}{4\Pi_W}\frac{h^2}{f^2}\right) + \frac{3}{2}\int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + \left(\frac{g_0'^2}{g_0^2}\frac{\Pi_1}{4\Pi_B} + \frac{\Pi_1}{4\Pi_W}\right)\frac{h^2}{f^2}\right]$$

$$\Pi_W = Q^2 + \Pi_0 \qquad \Pi_B = Q^2 + (g_0'^2/g_0^2)\Pi_0$$

$$\Pi_0 = \sum_{n=1}^{N_\rho} g_0^2 \frac{Q^2 f_{\rho(n)}^2}{Q^2 + M_{\rho(n)}^2} \qquad \Pi_1 = g_0^2 f^2 + 2g_0^2 \left(\sum_{n=1}^{N_a} \frac{Q^2 f_{a(n)}^2}{Q^2 + M_{a(n)}^2} - \sum_{n=1}^{N_\rho} \frac{Q^2 f_{\rho(n)}^2}{Q^2 + M_{\rho(n)}^2}\right)$$

Potential in NMCHM MHP

- The Fermion Sector: Partial Compositeness
  - Fermion Resonances: **Representation of $\boldsymbol{SO(5)}$** Extra $U(1)_X$
    - $\boldsymbol{\psi_1: 1_{2/3}}$
    - $\boldsymbol{\psi_5: 5_{2/3} \to 2_{7/6} \oplus 2_{1/6} \oplus 1_{2/3}}$
    - $\boldsymbol{\psi_{10}: 10_{2/3} \to 3_{2/3} \oplus 1_{5/3} \oplus 1_{2/3} \oplus 1_{-1/3} \oplus 2_{7/6} \oplus 2_{1/6}}$
  - Fermions in SM: **Embedded in incomplete representation of $\boldsymbol{SO(6)}$**
    - $\boldsymbol{1_{2/3}}$
    - $\boldsymbol{6_{2/3} \to 2_{7/6} \oplus 2_{1/6} \oplus 1_{2/3} \oplus 1_{2/3}}$
    - $\boldsymbol{15_{2/3} \to 3_{2/3} \oplus 1_{5/3} \oplus 1_{2/3} \oplus 1_{-1/3} \oplus 2_{7/6} \oplus 2_{1/6} \oplus}$ $\boldsymbol{2_{7/6} \oplus 2_{1/6} \oplus 1_{2/3}}$

Embedding of q_L	**Embedding of t_R**
$\boldsymbol{15_{2/3}}$	$\boldsymbol{15_{2/3}}$
$\boldsymbol{6_{2/3}}$	$\boldsymbol{6_{2/3}}$
	$\boldsymbol{1_{2/3}}$

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Potential in NMCHM

- The Fermion Sector: Partial Compositeness
  - Fermion Resonances: **Representation of $SO(5)$** Extra $U(1)_X$
    - $\boldsymbol{\psi_1, \psi_5, \psi_{10}}$
  - Fermions in SM: **Embedded in incomplete representation of $SO(6)$**
    - $\mathbf{1_{2/3}, 6_{2/3}, 15_{2/3}}$

(q_L, t_R)

Example Potential from (6, 6)

$$V_f^{\rm IR}(h,\eta) \approx -2N_c \int \frac{d^4Q}{(2\pi)^4}\Bigg\{ \ln\left(1+\frac{\Pi_1^q}{2\Pi_0^q}\frac{h^2}{f^2}\right) + \ln\left[1+\frac{\Pi_1^t}{\Pi_0^t}\left(s_\theta^2\left(1-\frac{h^2}{f^2}\right)+c_{2\theta}\frac{\eta^2}{f^2}\right)\right] + \ln\left[1+\frac{1}{Q^2}\frac{|M_1^t|^2}{2\Pi_0^q\Pi_0^t}\frac{h^2}{f^2}\left(s_\theta^2\left(1-\frac{h^2}{f^2}\right)+c_{2\theta}\frac{\eta^2}{f^2}\right)\right]\Bigg\},$$

- $\Pi_{0,1}^{t,q}, M_1^t$: Form Factor from top partner

Potential in NMCHM

- Confronting the EWPT:
  - Can’t provide 1st order EWPT

Embeddings (q_L, t_R)	Reason
$(\mathbf{6}, \mathbf{1})$	**No η Potential**
$(\mathbf{6}, \mathbf{6})$	**Local minimum $\eta > f$**
$(\mathbf{6}, \mathbf{15})$	**No η Potential**
$(\mathbf{15}, \mathbf{1})$	**No top mass term**
$(\mathbf{15}, \mathbf{6})$	**No two local minima at $T = T_c$**
$(\mathbf{15}, \mathbf{15})$	**Local minimum $\eta > f$**

- $(\mathbf{6}, \mathbf{6}), (\mathbf{15}, \mathbf{6}), (\mathbf{15}, \mathbf{15})$:
  - Can be improved by Adding higher order corrections to potential

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Beyond Minimal Potential

Spurion Method

Example for (6, 6)

$$c_f^L \left|y_L\right|^2 f^4 \Sigma^\dagger \mathcal{Q}^{\mathbf{6}} \mathcal{Q}^{\mathbf{6}\dagger} \Sigma, \quad c_f^R \left|y_R\right|^2 f^4 \Sigma^\dagger \mathcal{T}^{\mathbf{6}} \mathcal{T}^{\mathbf{6}\dagger} \Sigma,$$

$$\frac{d_f^L}{16\pi^2} \left|y_L\right|^4 f^4 \left(\Sigma^\dagger \mathcal{Q}^{\mathbf{6}} \mathcal{Q}^{\mathbf{6}\dagger} \Sigma\right)^2, \quad \frac{d_f^R}{16\pi^2} \left|y_R\right|^4 f^4 \left(\Sigma^\dagger \mathcal{T}^{\mathbf{6}} \mathcal{T}^{\mathbf{6}\dagger} \Sigma\right)^2,$$

$$V(h,\eta) = \frac{\mu_h^2}{2} h^2 + \frac{\lambda_h}{4} h^4 + \frac{\mu_\eta^2}{2} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

$$\begin{aligned}
(\mu_f^2)^{\mathrm{UV}} &= c_f^L \left|y_L\right|^2 f^2 - 2c_f^R \left|y_R\right|^2 f^2 s_\theta^2 - \frac{d_f^R}{4\pi^2} \left|y_R\right|^4 f^2 s_\theta^4, \\
(\mu_\eta^2)^{\mathrm{UV}} &= 2c_f^R \left|y_R\right|^2 f^2 c_{2\theta} + \frac{d_f^R}{4\pi^2} \left|y_R\right|^4 f^2 s_\theta^2 c_{2\theta}, \\
(\lambda_f)^{\mathrm{UV}} &= \frac{d_f^L}{16\pi^2} \left|y_L\right|^4 + \frac{d_f^R}{4\pi^2} \left|y_R\right|^4 s_\theta^4, \\
(\lambda_\eta)^{\mathrm{UV}} &= \frac{d_f^R}{4\pi^2} \left|y_R\right|^4 c_{2\theta}^2, \quad (\lambda_{h\eta})^{\mathrm{UV}} = -\frac{d_f^R}{4\pi^2} \left|y_R\right|^4 s_\theta^2 c_{2\theta}.
\end{aligned}$$

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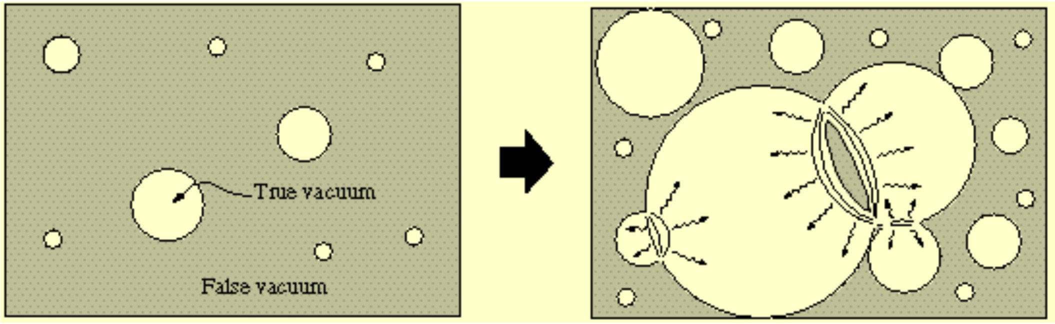
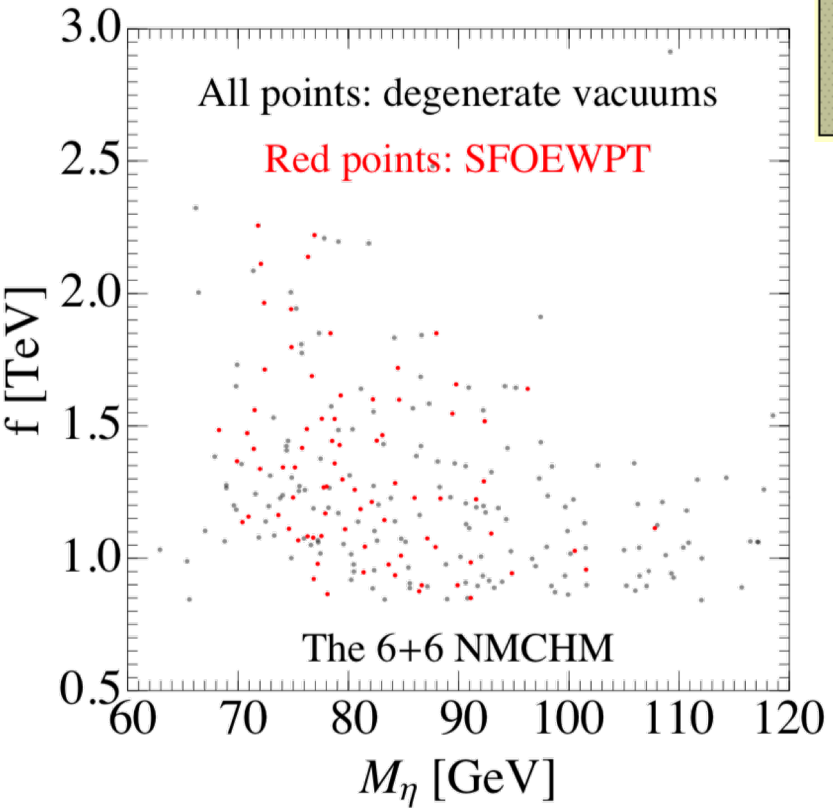
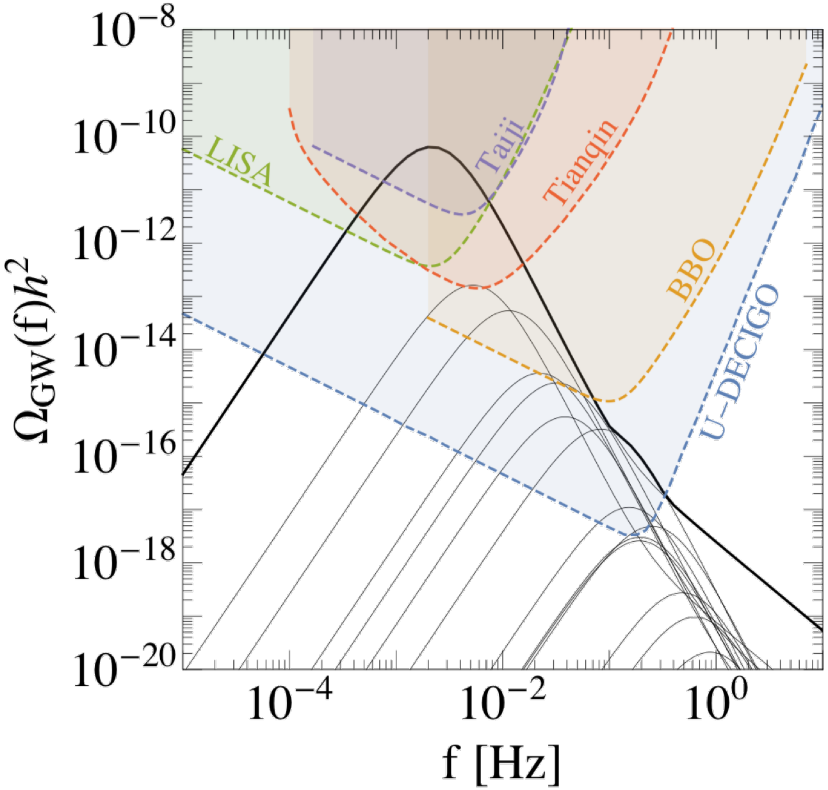
EWPT in NMCHM

- Bubble Nucleation







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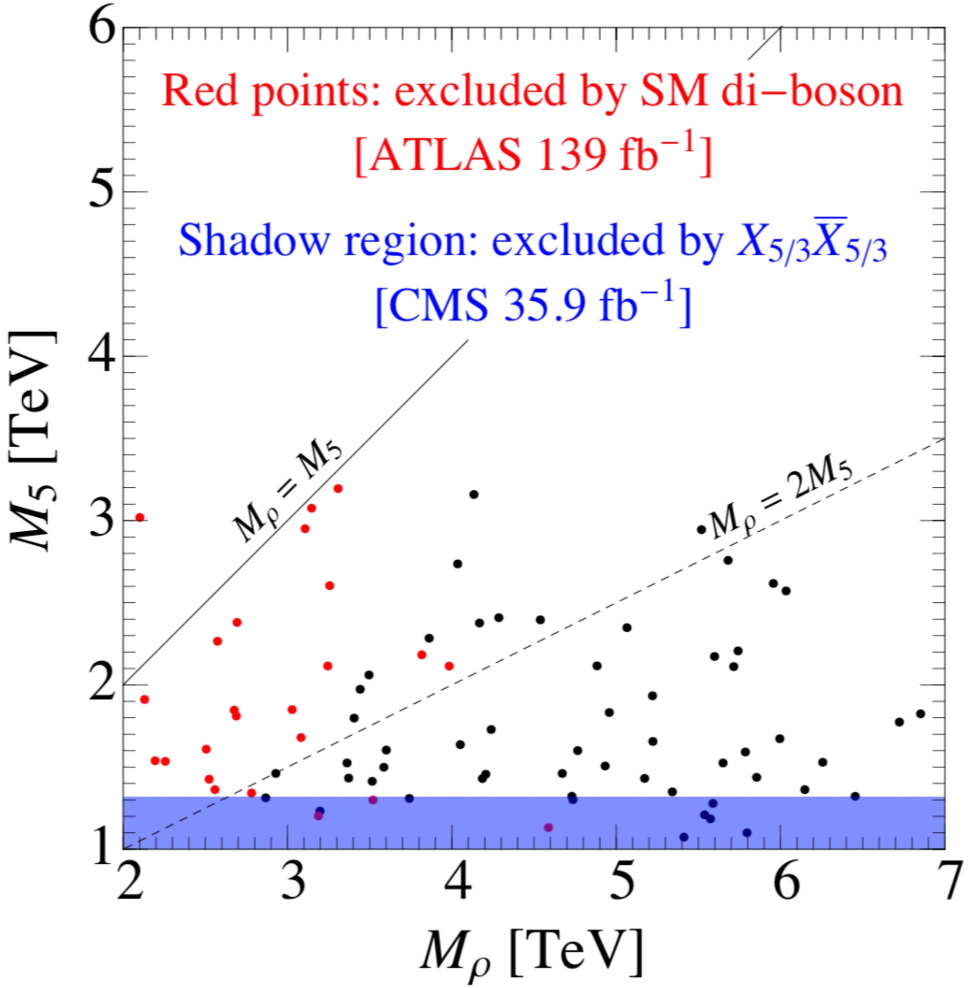
Collider Searches



$\boldsymbol{p,p \to \rho^{\pm,0} \to W^+W^-, W^\pm Z, \ldots\ldots}$

$\boldsymbol{p,p \to X_{5/3}\bar{X}_{5/3} \to t\,\bar{t}\,W^+W^-}$

$p,p \ \to \rho^{\pm,0} \to X_{5/3}\bar{X}_{2/3} + X_{5/3}\bar{X}_{2/3} + etc.$

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Summary

Next to Minimal Composite Higgs Model (NMCHM)

- Gauge Sector
- Fermion Sector

- Electroweak Phase Transition (EWPT)
  - 1st Order Phase Transition

Scalar Potential

$$V(h,\eta) = \frac{\mu_h^2}{2} h^2 + \frac{\lambda_h}{4} h^4 + \frac{\mu_\eta^2}{2} \eta^2 + \frac{\lambda_\eta}{4} \eta^4 + \frac{\lambda_{h\eta}}{2} h^2 \eta^2$$

Go beyond Minimal Higgs Potential Hypothesis

At least for 1, 6, 15 representations for fermion embedding

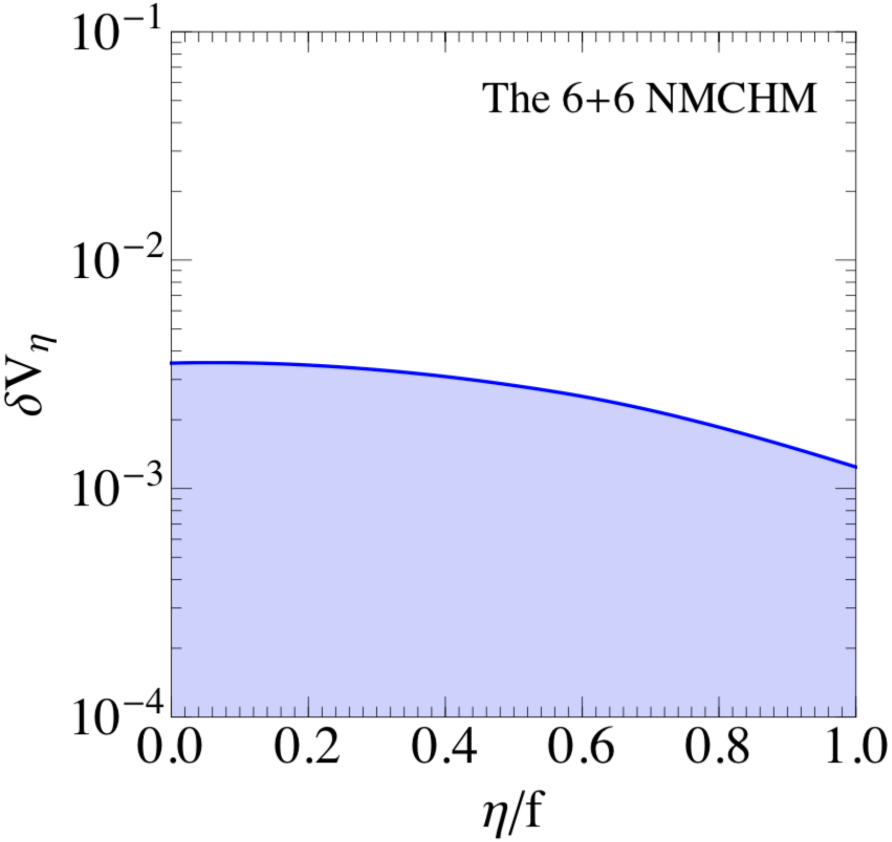
Trigger Strong 1st order EWPT

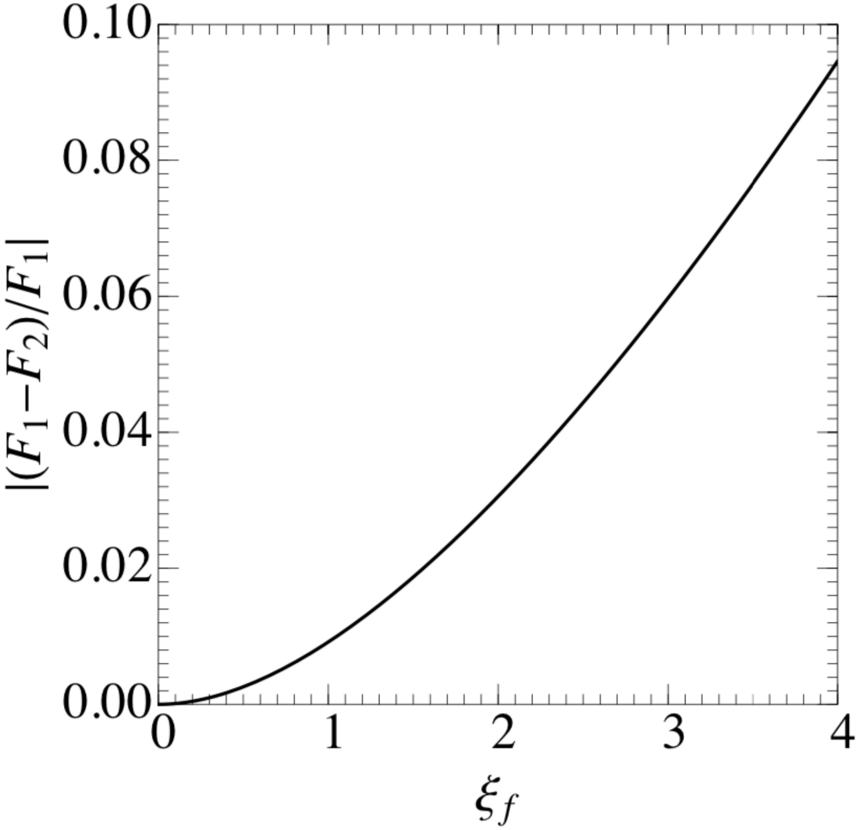
Observable GW signals

Thanks!

Backups

Validation of Polynomial Expansion





2019-11-22

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Details about the Fermion sector induced Potential

The $6+1$ model:

$$\mathcal{L}_{\mathbf{6+1}} \supset y_L^{\mathbf{5}} f (\bar{q}_L^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_L^{\mathbf{1}} f (\bar{q}_L^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}} + \text{h.c.}\ ,$$

The $6+6$ model:

$$\mathcal{L}_{\mathbf{6+6}} \supset y_L^{\mathbf{5}} f (\bar{q}_L^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_L^{\mathbf{1}} f (\bar{q}_L^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + y_R^{\mathbf{5}} f (\bar{t}_R^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_R^{\mathbf{1}} f (\bar{t}_R^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + \text{h.c.}\ .$$

The $6+15$ model:

$$\begin{aligned}\mathcal{L}_{\mathbf{6+15}} \supset\ & y_L^{\mathbf{5}} f (\bar{q}_L^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_L^{\mathbf{1}} f (\bar{q}_L^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} \\ & + y_R^{\mathbf{10}} f (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_R^{\mathbf{5}} f \Sigma_I^\dagger (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r + \text{h.c.}\ .\end{aligned}$$

The $15+1$ model:

$$\mathcal{L}_{\mathbf{15+1}} \supset y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r + y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}} + \text{h.c.}\ ,$$

The $15+6$ model:

$$\begin{aligned}\mathcal{L}_{\mathbf{15+6}} \supset\ & y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r \\ & + y_R^{\mathbf{5}} f (\bar{t}_R^{\mathbf{6}})_I U_{Ir} \Psi_{\mathbf{5}}^r + y_R^{\mathbf{1}} f (\bar{t}_R^{\mathbf{6}})_I U_{I6} \Psi_{\mathbf{1}} + \text{h.c.}\ .\end{aligned}$$

The $15+15$ model:

$$\begin{aligned}\mathcal{L}_{\mathbf{15+15}} \supset\ & y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r \\ & + y_R^{\mathbf{10}} f (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_R^{\mathbf{5}} f \Sigma_I^\dagger (\bar{t}_R^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r + \text{h.c.}\ .\end{aligned}$$

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Details about the Fermion sector induced Potential

(15,1)

The $\mathbf{15+1}$ model:

$$\mathcal{L}_{\mathbf{15+1}} \supset \underline{y_L^{\mathbf{10}} f (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{10}}^{rs} [U^\dagger]_{sI} + y_L^{\mathbf{5}} f \Sigma_I^\dagger (\bar{q}_L^{\mathbf{15}})_{IJ} U_{Jr} \Psi_{\mathbf{5}}^r} + \underline{y_R^{\mathbf{1}} f \bar{t}_R^{\mathbf{1}} \Psi_{\mathbf{1}}} + \text{h.c.} \ ,$$

No top mass

(6,1) and (6,15) no η potential

$$\delta q_L^{\mathbf{6}} = 0, \quad \delta t_R^{\mathbf{1}} = 0, \quad \delta t_R^{\mathbf{15}} = 0.$$

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Details about the Fermion sector induced Potential

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2,$$

For (6, 6):

$$\alpha_{q,t} = \frac{N_c}{f^2}\int \frac{d^4Q}{(2\pi)^4}\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}, \quad \beta_{q,t} = \frac{N_c}{f^4}\int \frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2, \quad \epsilon = \frac{N_c}{f^4}\int \frac{d^4Q}{(2\pi)^4}\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t},$$

$$\begin{aligned}
(\mu_f^2)^{\mathrm{IR}} &= -2\alpha_q + 4s_\theta^2\alpha_t - 4s_\theta^4 f^2\beta_t - 2s_\theta^2 f^2\epsilon, \\
(\lambda_f)^{\mathrm{IR}} &= \beta_q + 4s_\theta^4\beta_t + 4s_\theta^2\epsilon, \\
(\lambda_{h\eta})^{\mathrm{IR}} &= -4c_{2\theta}s_\theta^2\beta_t - 2c_{2\theta}\epsilon.
\end{aligned}$$

$$\boxed{\begin{aligned}
(\mu_\eta^2)^{\mathrm{IR}} &= -4c_{2\theta}\alpha_t + 4c_{2\theta}s_\theta^2 f^2\beta_t, \\
(\lambda_\eta)^{\mathrm{IR}} &= 4c_{2\theta}^2\beta_t,
\end{aligned}}$$

$$\langle\eta\rangle \gg f$$

Details about the Fermion sector induced Potential

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2,$$

For (15, 15):

$$\alpha_{q,t} = \frac{N_c}{f^2}\int \frac{d^4Q}{(2\pi)^4}\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}, \quad \beta_{q,t} = \frac{N_c}{f^4}\int \frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2, \quad \epsilon = \frac{N_c}{f^4}\int \frac{d^4Q}{(2\pi)^4}\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t},$$

$$\begin{aligned}(\mu_f^2)^{\mathrm{IR}} &= -\alpha_q - \alpha_t(1-3s_\theta^2) + \frac{\beta_t f^2}{2}(1-3s_\theta^2)s_\theta^2 - \frac{\epsilon f^2}{2}s_\theta^2,\\ (\lambda_f)^{\mathrm{IR}} &= \frac{\beta_q}{4} + \frac{\beta_t}{4}(1-3s_\theta^2)^2 + \epsilon s_\theta^2,\\ (\lambda_{h\eta})^{\mathrm{IR}} &= \frac{\beta_q}{2} - \frac{1-3s_\theta^2}{4}\epsilon.\end{aligned} \quad \boxed{\begin{aligned}(\mu_\eta^2)^{\mathrm{IR}} &= -4\alpha_q,\\ (\lambda_\eta)^{\mathrm{IR}} &= 2\beta_q,\end{aligned}}$$

$\langle\eta\rangle \gg f$

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Details about the Fermion sector induced Potential

$$V_f(h,\eta) = \frac{\mu_f^2}{2}h^2 + \frac{\lambda_f}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2,$$

For (15, 15):

$$\alpha_{q,t} = \frac{N_c}{f^2}\int\frac{d^4Q}{(2\pi)^4}\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}, \quad \beta_{q,t} = \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\left(\frac{\Pi_1^{q,t}}{\Pi_0^{q,t}}\right)^2, \quad \epsilon = \frac{N_c}{f^4}\int\frac{d^4Q}{(2\pi)^4}\frac{|M_1^t|^2}{Q^2\Pi_0^q\Pi_0^t},$$

$$(\mu_f^2)^{\rm IR} = -\alpha_q + 4\alpha_t s_\theta^2 - 4s_\theta^4 f^2\beta_t - c_\theta^2 f^2\epsilon, \quad (\mu_\eta^2)^{\rm IR} = -4\alpha_q - 4\alpha_t c_{2\theta} + 4c_{2\theta}s_\theta^2\beta_t f^2,$$

$$(\lambda_f)^{\rm IR} = \frac{\beta_q}{4} + 4s_\theta^4\beta_t, \quad (\lambda_\eta)^{\rm IR} = 2\beta_q + 4c_{2\theta}^2\beta_t,$$

$$(\lambda_{h\eta})^{\rm IR} = \frac{\beta_q}{2} - 4c_{2\theta}s_\theta^2\beta_t.$$

$$\begin{aligned}(\lambda_{h\eta}^2)^{\rm IR} - (\lambda_h)^{\rm IR}(\lambda_\eta)^{\rm IR} &\approx (\lambda_{h\eta}^2)^{\rm IR} - (\lambda_f)^{\rm IR}(\lambda_\eta)^{\rm IR} \\ &= -\frac{1}{4}\beta_q\left[\beta_q + 8(1-c_{2\theta})\beta_t + 2(1+c_{4\theta})\beta_t\right] < 0,\end{aligned}$$

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Gravitational Wave:

$$\alpha = \frac{\epsilon}{\rho_{\rm rad}}, \quad \epsilon = -\Delta V_T + T_n \Delta \frac{\partial V_T}{\partial T}\Big|_{T_n}, \quad \rho_{\rm rad} = \frac{\pi^2}{30} g_* T_n^4,$$

$$\beta = \frac{d}{dt}\left(\frac{S_3}{T}\right)\Big|_{t=t_n}, \quad \frac{\beta}{H_n} = T_n \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T=T_n},$$

Bubble Collision

$$v_b \simeq \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1+\alpha}, \qquad \kappa \simeq \frac{0.715\alpha + \frac{4}{27}\sqrt{3\alpha/2}}{1+0.715\alpha}$$

$$\Omega_{\rm col} h^2 = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{0.11 v_b^3}{0.42 + v_b^2}\right) \frac{3.8(f/f_{\rm env})^{2.8}}{1 + 2.8(f/f_{\rm env})^{3.8}}$$

$$f_{\rm env} = 16.5 \times 10^{-6} \left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

Sound Wave

$$\Omega_{\rm sw} h^2 = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} v_w \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4 + 3(f/f_{\rm sw})^2}\right)^{7/2},$$

Turbulence

$$\Omega_{\rm turb} h^2 = 3.35 \times 10^{-4} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\rm turb} \alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_*}\right)^{1/3} v_w \frac{(f/f_{\rm turb})^3}{[1 + (f/f_{\rm turb})]^{11/3}(1 + 8\pi f/h_*)},$$

$\kappa_v \approx \alpha(0.73 + 0.083\sqrt{\alpha} + \alpha)^{-1}$ and $\kappa_{\rm turb} \approx 0.1\kappa_v$

$$f_{\rm sw} = 1.9 \times 10^{-5} \frac{1}{v_b} \left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz},$$

$$f_{\rm turb} = 2.7 \times 10^{-5} \frac{1}{v_b} \left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$