Un-binned Angular Analysis of $B \to D^* \ell \nu$ and the Right-handed Current

唐儒英
合作者：吕才典研究员，Emi Kou教授，黄卓然博士

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Outline

- Right-handed Current and V_{cb} puzzle
- $<g_i>$ Functions
- χ^2 Fit and Pseudo-Experimental Data
- Sensitivity to the C_{V_R}
- Summary and Conclusions

Right-handed Current and V_{cb} puzzle

- Experiments + Theory

	Inclusive	vs Exclusive
Theory method	Heavy quark effective theory + Operator product expansion + Dispersion relation	Heavy quark effective theory, Lattice QCD, factorization approach, sum rules or … on form factors
Experiments	$B \to X_c \ell \nu$	$B \to D \ell \nu$ $B \to D^* \ell \nu$
Value of $\lvert V_{cb} \rvert$	$(42.2 \pm 0.8) \times 10^{-3}$	$(39.5 \pm 0.9) \times 10^{-3}$

- Marginally consistent with each other

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.

Right-handed Current and V_{cb} puzzle

- Improve the theory calculations and determinations of experiment or New Physics

- Right-handed Vector Current

$$H_{eff} = \frac{4G_F V_{cb}}{\sqrt{2}} \bar{\ell}\gamma^\mu P_L \nu \left[(1 + c_L^{cb}) \bar{q}\gamma_\mu P_L b + c_R^{cb} \bar{q}\gamma_\mu P_R b \right]$$

$$V_{cb} = \frac{V_{cb}^{\mathrm{SM}}}{1 + c_L^{cb} + c_R^{cb}} \quad (B \to D\ell\nu)$$

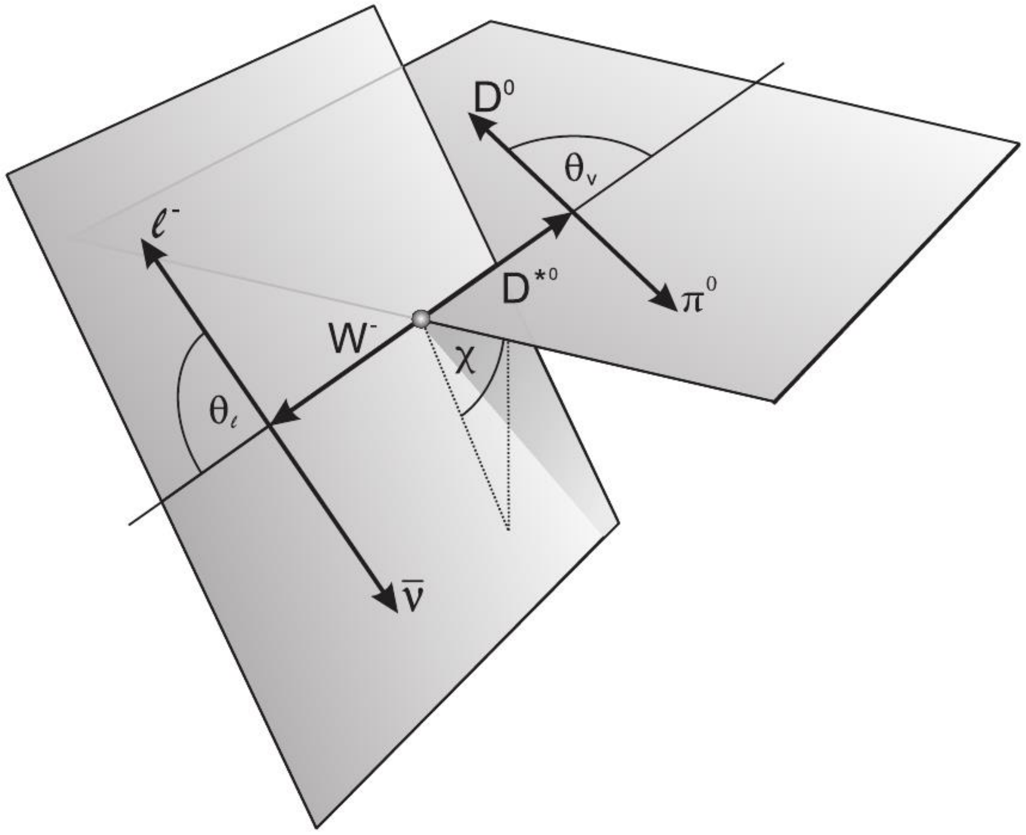
$$V_{cb} = \frac{V_{cb}^{\mathrm{SM}}}{1 + c_L^{cb} - c_R^{cb}} \quad (B \to D^*\ell\nu)$$

$$V_{cb} = \frac{V_{cb}^{\mathrm{SM}}}{1 + c_L^{cb} - 0.34 c_R^{cb}} (Inclusive)$$

- Un-binned Angular Analysis

A. Crivellin and S. Pokorski, Phys. Rev. Lett. 114, no.1, 011802 (2015)

<g_i> Functions

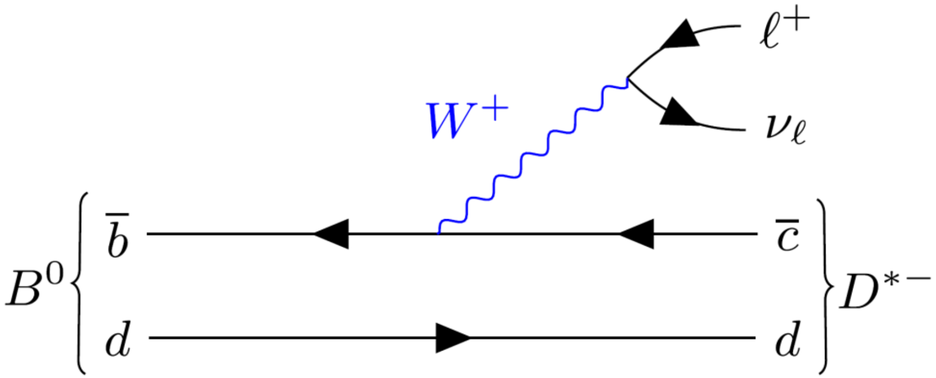


- Weak effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[C_{V_L} \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R} \right] + \text{h.c.}$$

where

$$\mathcal{O}_{V_L} = (\bar{c}_L \gamma^\mu b_L)(\bar{\ell}_L \gamma_\mu \nu_L)\,, \quad \mathcal{O}_{V_R} = (\bar{c}_R \gamma^\mu b_R)(\bar{\ell}_L \gamma_\mu \nu_L)$$



E. Waheed et al. [Belle], Phys. Rev. D 100, no.5, 052007 (2019)

<g_i> Functions

$$
\begin{aligned}
&\frac{\mathrm{d}\Gamma(\overline{B} \to D^*(\to D\pi)\,\ell^-\,\bar{\nu}_\ell)}{\mathrm{d}w\,\mathrm{d}\cos\theta_V\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\chi} \\
= \quad & \frac{6 m_B m_{D^*}^2}{8(4\pi)^4}\sqrt{w^2-1}(1-2\,w\,r+r^2)\,G_F^2\,|V_{cb}|^2\,\mathcal{B}(D^*\to D\pi) \\
\times \quad & \Big\{ J_{1s}\sin^2\theta_V + J_{1c}\cos^2\theta_V + (J_{2s}\sin^2\theta_V + J_{2c}\cos^2\theta_V)\cos 2\theta_\ell + J_3\sin^2\theta_V\sin^2\theta_\ell\cos 2\chi \\
+ \quad & J_4\sin 2\theta_V\sin 2\theta_\ell\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\cos\chi + (J_{6s}\sin^2\theta_V + J_{6c}\cos^2\theta_V)\cos\theta_\ell \\
+ \quad & J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\sin\chi + J_9\sin^2\theta_V\sin^2\theta_\ell\sin 2\chi \Big\}
\end{aligned}
$$

$$
\begin{gathered}
J_{1s} = \tfrac{3}{2}(H_+^2 + H_-^2)(|C_{V_L}|^2 + |C_{V_R}|^2) - 6H_+H_-\mathrm{Re}[C_{V_L}C_{V_R}^*]\,, \quad J_{1c} = 2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_{2s} = \tfrac{1}{2}(H_+^2 + H_-^2)(|C_{V_L}|^2 + |C_{V_R}|^2) - 2H_+H_-\mathrm{Re}[C_{V_L}C_{V_R}^*]\,, \quad J_{2c} = -2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_3 = -2H_+H_-(|C_{V_L}|^2 + |C_{V_R}|^2) + 2(H_+^2 + H_-^2)\mathrm{Re}[C_{V_L}C_{V_R}^*]\,, \quad J_4 = (H_+H_0 + H_-H_0)(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\mathrm{Re}[C_{V_L}C_{V_R}^*]) \\
J_5 = -2(H_+H_0 - H_-H_0)(|C_{V_L}|^2 - |C_{V_R}|^2)\,, \quad J_{6s} = -2(H_+^2 - H_-^2)(|C_{V_L}|^2 - |C_{V_R}|^2)\,, \quad J_{6c} = 0\,, \quad J_7 = 0 \\
J_8 = 2(H_+H_0 - H_-H_0)\mathrm{Im}[C_{V_L}C_{V_R}^*]\,, \quad J_9 = -2(H_+^2 - H_-^2)\mathrm{Im}[C_{V_L}C_{V_R}^*]
\end{gathered}
$$

<g_i> Functions

$$\frac{d\Gamma}{dw} = \frac{6 m_B m_{D^*}^2}{8(4\pi)^4} G_F^2\, \eta_{\mathrm{EW}}^2\, |V_{cb}|^2\, \mathcal{B}(D^* \to D\pi) \frac{8\pi}{9} \left\{ 6J'_{1s} + 3J'_{1c} - 2J'_{2s} - J'_{2c} \right\}$$

where $J'_i \equiv J_i \sqrt{w^2 - 1}(1 - 2wr + r^2)$

- Separating full w range into 10 bins
- Integration by different w interval
- Normalization factor

$$\begin{aligned} \langle\Gamma\rangle_{w-\mathrm{bin}} &= \frac{6 m_B m_{D^*}^2}{8(4\pi)^4} G_F^2\, \eta_{\mathrm{EW}}^2\, |V_{cb}|^2 \times \mathcal{B}(D^* \to D\pi) \\ &\times \frac{8\pi}{9} \left\{ 6\langle J'_{1s}\rangle_{w-\mathrm{bin}} + 3\langle J'_{1c}\rangle_{w-\mathrm{bin}} - 2\langle J'_{2s}\rangle_{w-\mathrm{bin}} - \langle J'_{2c}\rangle_{w-\mathrm{bin}} \right\} \end{aligned}$$

$<g_i>$ Functions

$$
\begin{aligned}
&\hat{f}_{\langle\vec{g}\rangle}(\cos\theta_V, \cos\theta_\ell, \chi) \\
= \quad & \frac{9}{8\pi}\Big\{\frac{1}{6}(1 - 3\langle g_{1c}\rangle + 2\langle g_{2s}\rangle + \langle g_{2c}\rangle)\sin^2\theta_V + \langle g_{1c}\rangle\cos^2\theta_V \\
+ \quad & (\langle g_{2s}\rangle\sin^2\theta_V + \langle g_{2c}\rangle\cos^2\theta_V)\cos 2\theta_\ell + \langle g_3\rangle\sin^2\theta_V\sin^2\theta_\ell\cos 2\chi \\
+ \quad & \langle g_4\rangle\sin 2\theta_V\sin 2\theta_\ell\cos\chi + \langle g_5\rangle\sin 2\theta_V\sin\theta_\ell\cos\chi + (\langle g_{6s}\rangle\sin^2\theta_V + \langle g_{6c}\rangle\cos^2\theta_V)\cos\theta_\ell \\
+ \quad & \langle g_7\rangle\sin 2\theta_V\sin\theta_\ell\sin\chi + \langle g_8\rangle\sin 2\theta_V\sin 2\theta_\ell\sin\chi + \langle g_9\rangle\sin^2\theta_V\sin^2\theta_\ell\sin 2\chi\Big\}
\end{aligned}
$$

where $\langle g_i\rangle \equiv \dfrac{\langle J'_i\rangle}{6\langle J'_{1s}\rangle + 3\langle J'_{1c}\rangle - 2\langle J'_{2s}\rangle - \langle J'_{2c}\rangle}$

- 11 normalized functions, in which the overall factor is canceled such like $|V_{cb}|$.
- Can be determined in future experiments.

χ^2 Fit and Pseudo-Experimental Data

$$\chi^2(\vec{v}) = \chi^2_{\text{angle}}(\vec{v}) + \chi^2_{\text{lattice}}(\vec{v}) + \chi^2_{w-\text{bin}}(\vec{v})$$

$$\chi^2_{\text{angle}}(\vec{v}) = \sum_{w-\text{bin}=1}^{10} \left[\sum_{ij} N_{\text{event}} (\langle g_i \rangle^{\text{exp}} - \langle g_i^{\text{th}}(\vec{v}) \rangle) \hat{V}_{ij}^{-1} (\langle g_j \rangle^{\text{exp}} - \langle g_j^{\text{th}}(\vec{v}) \rangle) \right]_{w-\text{bin}}$$

$$\chi^2_{\text{lattice}}(v_i) = \left(\frac{v_i^{\text{lattice}} - v_i}{\sigma_{v_i}^{\text{lattice}}} \right)^2$$

$$\chi^2_{w-\text{bin}}(\vec{v}) = \sum_{w-\text{bin}=1}^{10} \frac{([N]_{w-\text{bin}} - \alpha \langle \Gamma \rangle_{w-\text{bin}})^2}{[N]_{w-\text{bin}}}$$

where $\vec{v} = (h_{A_1}(1), \rho^2_{D^*}, R_1(1), R_2(1), C_{V_R}, \ldots)$ for CLN parameterization

or $\vec{v} = (a^g_{0,1,\ldots}, a^f_{0,1,\ldots}, a^{\mathcal{F}_1}_{1,2,\ldots}, C_{V_R}, \ldots)$ for BGL parameterization

χ^2 Fit and Pseudo-Experimental Data

- CLN:

$$
\begin{aligned}
H_\pm(w) &= m_B\sqrt{r}(w+1)h_{A_1}(w)\left[1\mp\sqrt{\frac{w-1}{w+1}}R_1(w)\right]\\
H_0(w) &= m_B^2\sqrt{r}(w+1)\frac{1-r}{\sqrt{q^2}}h_{A_1}(w)\left[1+\frac{w-1}{1-r}(1-R_2(w))\right]\\
&\quad \text{where } r = m_{D^*}/m_B
\end{aligned}
$$

$$
\begin{aligned}
h_{A_1}(w) &= h_{A_1}(1)(1-8\rho_{D^*}^2 z+(53\rho_{D^*}^2-15)z^2-(231\rho_{D^*}^2-91)z^3)\\
R_1(w) &= R_1(1)-0.12(w-1)+0.05(w-1)^2\\
R_2(w) &= R_2(1)+0.11(w-1)-0.06(w-1)^2\\
&\quad \text{where } z = (\sqrt{w+1}-\sqrt{2})/(\sqrt{w+1}+\sqrt{2})
\end{aligned}
$$

BGL:

$$
\begin{aligned}
H_\pm(w) &= f(w)\mp m_B|\mathbf{p}_{D^*}|g(w)\\
H_0(w) &= \frac{\mathcal{F}_1(w)}{\sqrt{q^2}}\\
g(z) &= \frac{1}{P_g(z)\phi_g(z)}\sum_{n=0}^{N} a_n^g z^n\\
f(z) &= \frac{1}{P_f(z)\phi_f(z)}\sum_{n=0}^{N} a_n^f z^n\\
\mathcal{F}_1(z) &= \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)}\sum_{n=0}^{N} a_n^{\mathcal{F}_1} z^n
\end{aligned}
$$

- Values of CLN and BGL parameters come from Belle' 18.
- Total number of events: ~ 95000

Sanity Check

- CLN:

	Un-binned Fit	Belle'18
$\rho^2_{D^*}$	1.106(19)	1.106(31)(7)
$R_1(1)$	1.229(11)	1.229(28)(9)
$R_2(1)$	0.852(11)	0.852(21)(6)
$\lvert V_{cb}\rvert$	0.0387(6)	0.0384(2)(6)(6)

BGL:

	Un-binned Fit	Belle'18
a_0^f	0.0132(2)	0.0131(1)(2)
a_1^f	0.0169(28)	0.0169(44)(23)
$a_1^{\mathcal{F}_1}$	0.0070(11)	0.0070(17)(6)
$a_2^{\mathcal{F}_1}$	-0.0853(199)	-0.0848(324)(117)
a_0^g	0.0242(4)	0.0241(5)(3)
$\lvert V_{cb}\rvert$	0.0384(6)	0.0383(3)(7)(6)

- Errors of form factors ~50% smaller partially due to the un-binned analysis.

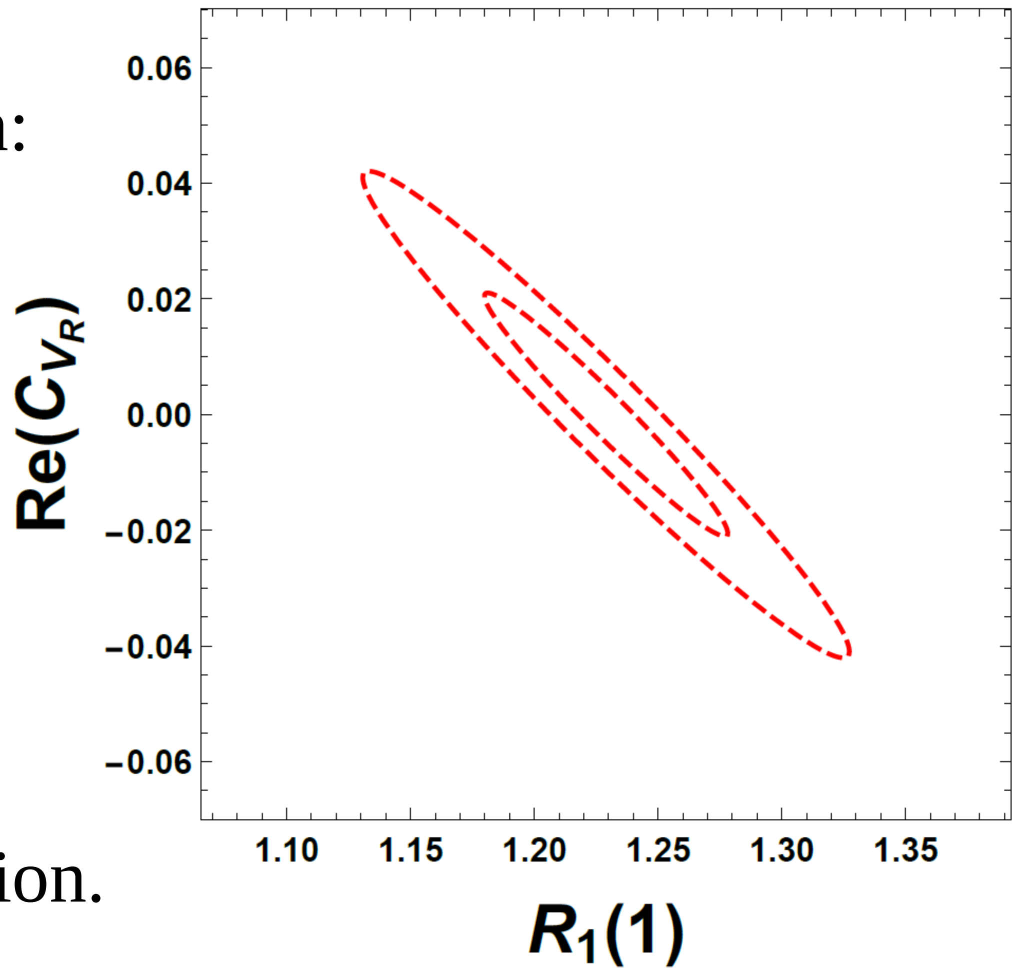
Sensitivity to the C_{V_R}

- Real C_{V_R} with CLN, $R_1(1)$ at a 4% precision:

$$\vec{v} = (\rho^2_{D^*}, R_1(1), R_2(1), C_{V_R}) = (1.106, 1.229, 0.852, 0)$$

$$\sigma_{\vec{v}} = (3.177, 0.049, 0.018, 0.021)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & -0.016 & -0.763 & \boxed{0.095} \\ -0.016 & 1. & 0.006 & \boxed{-0.973} \\ -0.763 & 0.006 & 1. & \boxed{-0.117} \\ 0.095 & -0.973 & -0.117 & 1. \end{pmatrix}$$



- The C_{V_R} can be determined at a 2.1% precision.

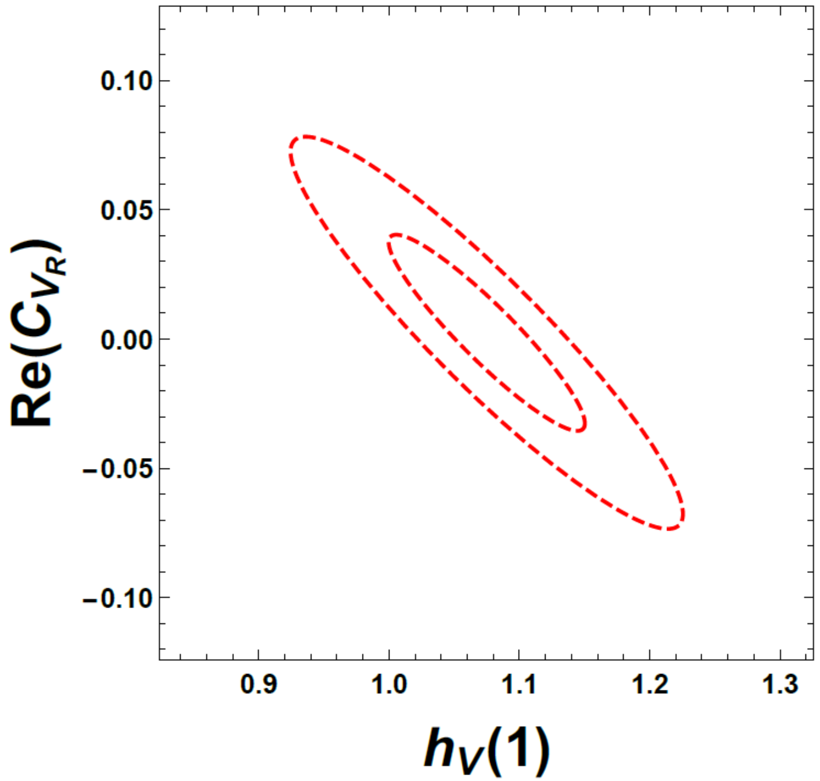
T. Kaneko et al. [JLQCD], PoS LATTICE2019, 139 (2019)

Sensitivity to the C_{V_R}

- Real C_{V_R} with BGL, $h_V(1)$ at a 7% precision :

$$h_V(1) = \frac{m_B\sqrt{r}}{P_g(0)\phi_g(0)} a_0^g$$

$$\vec{v} = (a_0^f, a_1^f, a_1^{\mathcal{F}_1}, a_2^{\mathcal{F}_1}, a_0^g, C_{V_R})$$
$$= (0.0132, 0.0169, 0.0070, -0.0852, 0.0241, 0.0024)\,,$$
$$\sigma_{\vec{v}} = (0.0002, 0.0109, 0.0026, 0.0352, 0.0017, 0.0379)\,,$$
$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.022 & 0.039 & -0.035 & 0.000 & 0.189 \\ 0.022 & 1. & 0.860 & -0.351 & 0.000 & 0.316 \\ 0.039 & 0.860 & 1. & -0.762 & 0.000 & 0.283 \\ -0.035 & -0.351 & -0.762 & 1. & 0.000 & -0.119 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1. & -0.923 \\ 0.189 & 0.316 & 0.283 & -0.119 & -0.923 & 1. \end{pmatrix}$$



- The C_{V_R} can be determined at a 3.79% precision.
- We get the result without knowing the value of V_{cb}.

Sensitivity to the C_{V_R}

- Forward-Backward Asymmetry:

$$\langle \mathcal{A}_{\theta_\ell} \rangle \equiv \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell}{\int_0^1 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell + \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_\ell} \mathrm{d}\cos\theta_\ell} = 3\langle g_{6s} \rangle$$

$$\vec{v} = (\rho^2_{D^*}, R_1(1), R_2(1), C_{V_R}) =$$
$$(1.106, 1.229, 0.852, 0.000)$$
$$\sigma_{\vec{v}} = (2.200, 0.049, 0.031, 0.022)$$
$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.008 & -0.873 & 0.262 \\ 0.008 & 1. & -0.040 & -0.931 \\ -0.873 & -0.040 & 1. & -0.296 \\ 0.262 & -0.931 & -0.296 & 1. \end{pmatrix}$$

- As good as the former case, the BGL's situation is similar.
- Besides, Im(C_{V_R}) can be determined at a 0.7 % precision, both for CLN and BGL.

Summary and Conclusions

- We investigated an application of the un-binned angular analysis of the $B \to D^* \ell \nu$ decay to search for the right-handed contributions.

- We introduced the 11 normalized $\langle g_i \rangle$ angular functions, in which the overall factor including V_{cb} is canceled. Therefore the right-handed contributions can be determined by circumventing the V_{cb} puzzle.

- The vector form factor is necessary for the determination of the C_{V_R}.
- In the un-binned angular analysis, the real part of the C_{V_R} can be determined at a precision of 2-4 %, and the imaginary part of the C_{V_R} can be determined at a ~1 % precision.

- The result of using the forward-back asymmetry A_{FB} only is almost equally good as full analysis of $\langle g_i \rangle$. The future measurement of A_{FB} will be particularly useful for constraining C_{V_R}.

Thank you!