The higher-order QED effect of vacuum pair production

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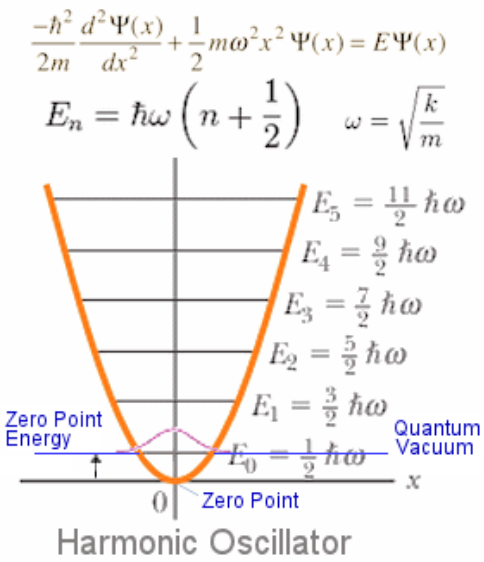
arXiv: 2103.04605

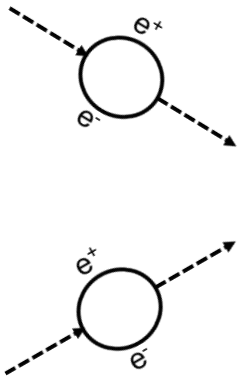
What is the vacuum --- Exactly nothing?

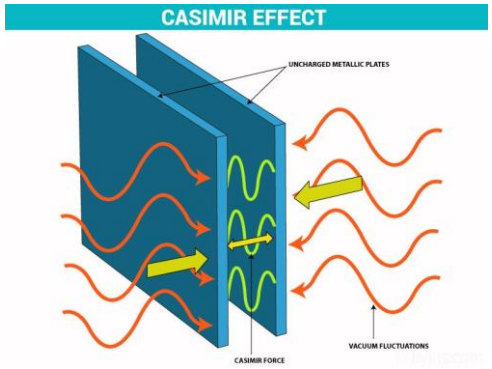
- Occupies all space.
- Has no charge.
- Has no angular momentum (isotropic).
- Has no preferred origin (homogeneous).
- Non-dispersive (c(λ)=constant).
- **Energy density = ?Zero??**



Zero point energy

- ✓ The random quantum fluctuations of virtual pairs.

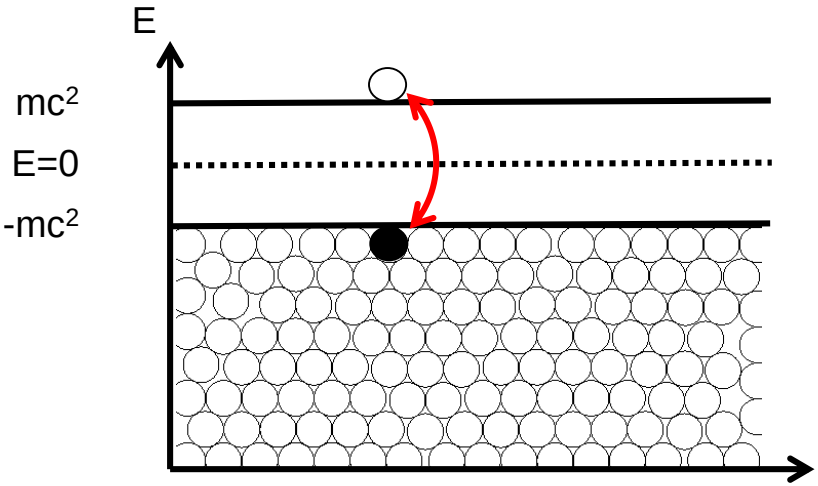




H.B.G. Casimir
Indag.Math. 10 (1948) 261-263

Sparking the vacuum --- The Schwinger Mechanism

Dirac's View of Vacuum



An insulator with a negative energy "sea" filled with electrons.

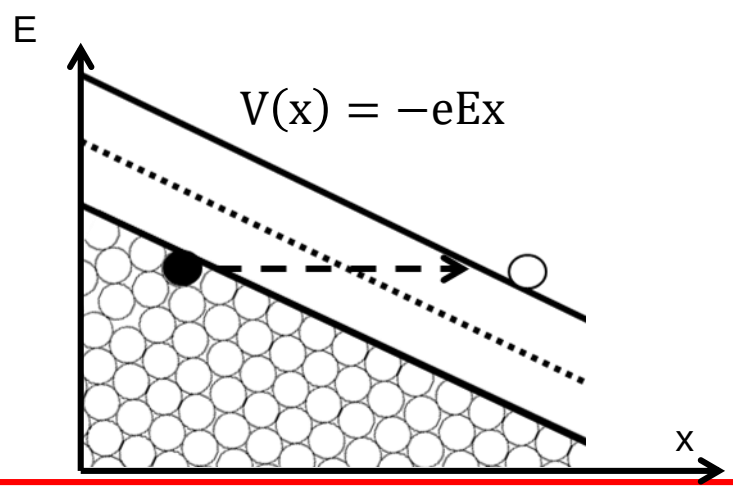
Quantum fluctuations:

Creation and annihilation virtual electron-"hole" pairs.

Virtual pair "lifetime" $\Delta t \approx \hbar/2mc^2$.

Virtual pair "size" $\Delta l \approx 2\hbar/mc$.

With an electric external field



The Schwinger Mechanism

J.S. Schwinger
Phys. Rev. 82 (1951) 664

$$eE_c\Delta l = 2mc^2$$

$$E_c = \frac{m^2c^3}{e\hbar} = 1.3 \times 10^{16}\, V/cm$$

The Schwinger Critical Field

Why should we care?

The production rate of Schwinger Mechanism at a given constant electric field E:

C. Itzykson, J.B. Zuber
Quantum Electrodynamics of Strong Fields

$$\frac{d^4 n_{e^+e^-}}{d^3x dt} \sim \frac{c}{4\pi^3 \lambda_c^4} \exp(-\pi \frac{E_c}{E})$$

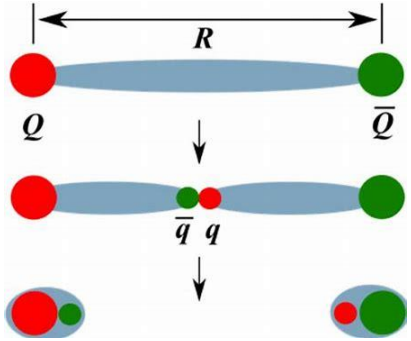
The non-perturbative nature of the production mechanism.

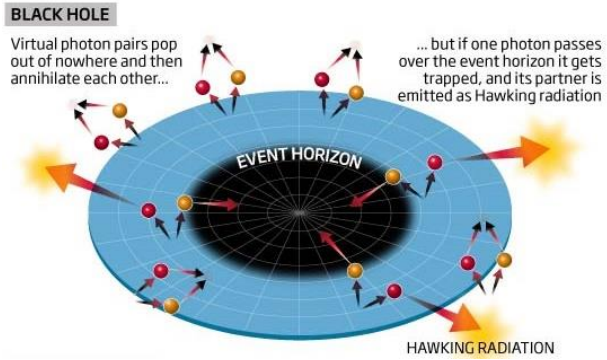
-Related to the "$Z\alpha > 1$" problem.

Motivated analysis in many parts of quantum field theory:

-Models of string breaking in QCD.

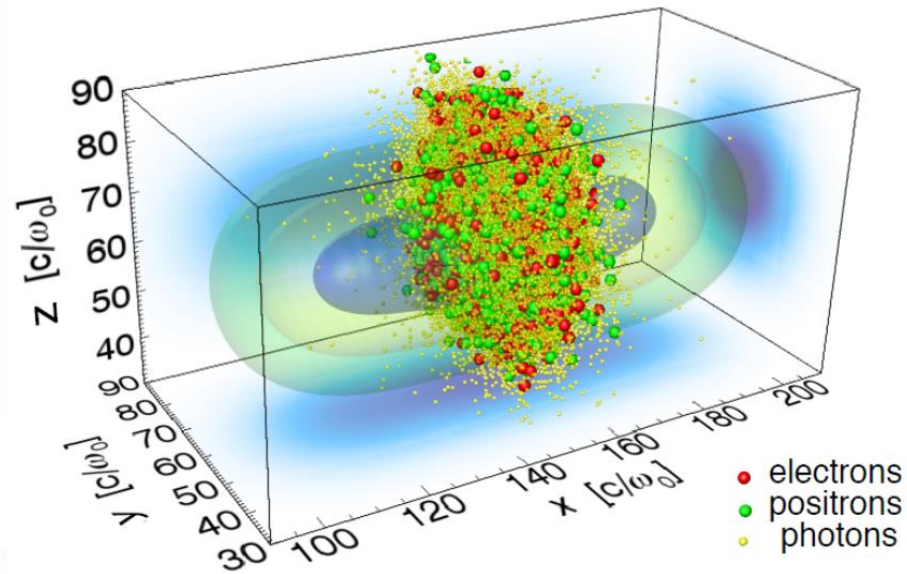
-Insights into Hawking radiation near black holes.

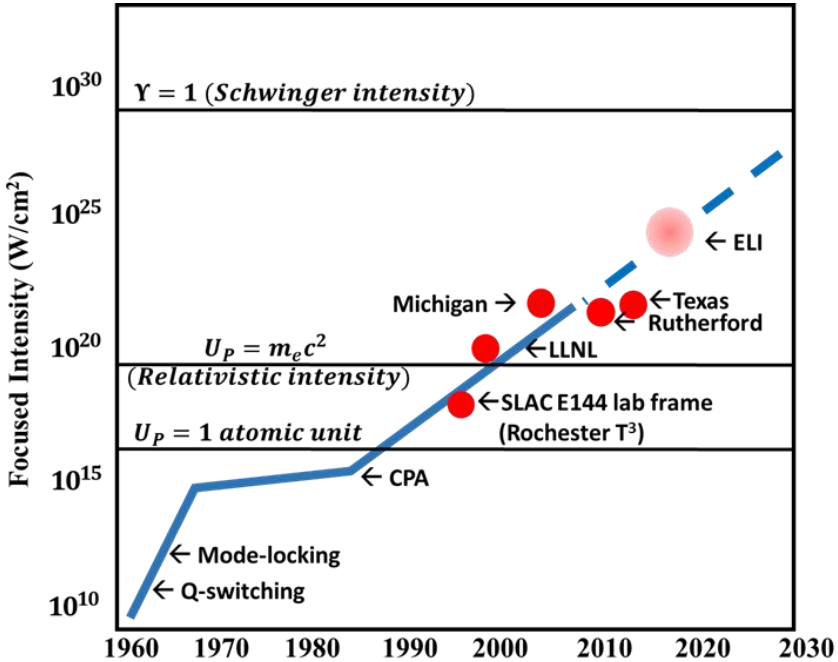




How to achieve in laboratory

Colliding laser beams above the Schwinger threshold



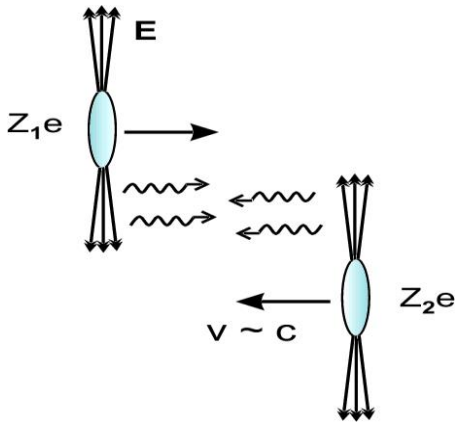


$$E_c = \frac{m^2c^3}{e\hbar} = 1.3 \times 10^{16}\, V/cm$$

$$I_c = \frac{E_c^2}{4\pi} = 2.2 \times 10^{29}\, W/cm^2$$

Still orders of magnitude lower than the Schwinger threshold!

More powerful "laser" facility --- Heavy Ion Collider



Colliding Clouds of Quasi-real photons

At RHIC b = 15 fm:

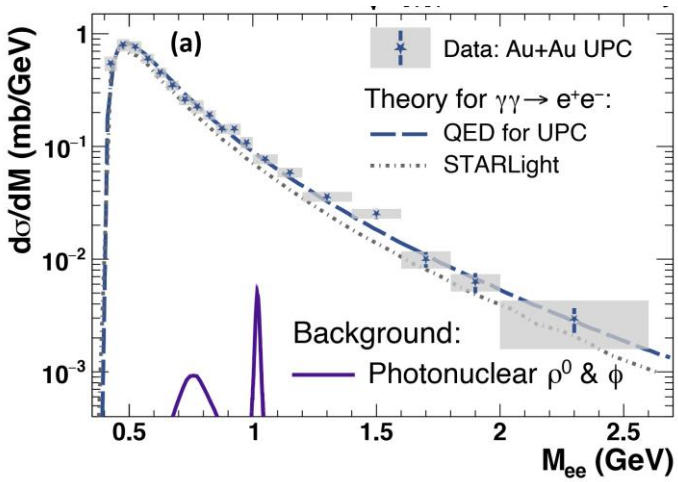
$$E_{Max} = 5.3 \times 10^{16}\ V/cm$$
$$I_{Max} = 9 \times 10^{29}\ W/cm^2$$

At LHC b = 15 fm:

$$E_{Max} = 1.4 \times 10^{18}\ V/cm$$
$$I_{Max} = 2.4 \times 10^{31}\ W/cm^2$$

$$\gg I_c = 2.2 \times 10^{29}\ W/cm^2$$

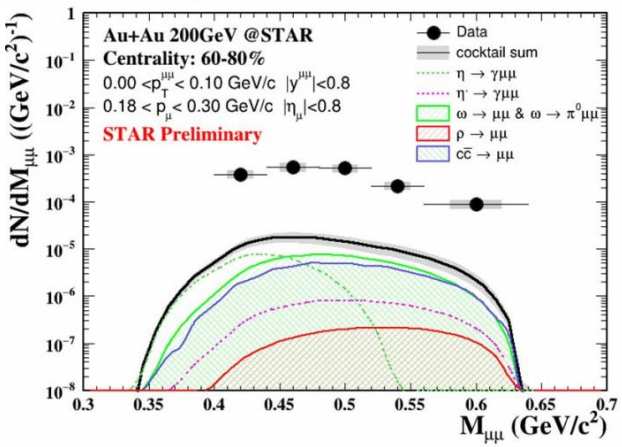
Dielectron pair production from STAR



STAR Rev. Lett. 127 (2021) 052302

Jian Zhou, Next talk

Dimuon pair production from STAR



Still Perturbative, but?

The duration time of the strong field in heavy-ion collisions:

$\Delta t_{HIC} \approx \frac{R_A}{\gamma}$ 0.06 fm/c at RHIC 0.0025fm/c at LHC

The pulse duration of PW laser system: Δt_{Laser} ~ fs ~ 10^8 fm/c

Virtual pair "lifetime" $\Delta t \approx \hbar/2mc^2 \approx 5 \times 10^3$ fm/c

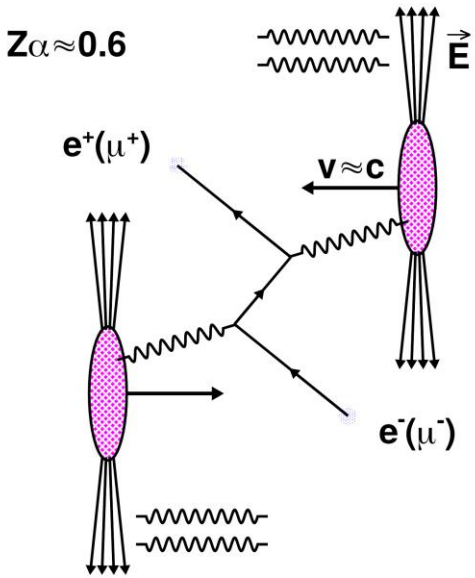
$\Delta t_{Laser} >> \Delta t >> \Delta t_{HIC}$

⇩ ⇩

Non-perturbative Perturbative

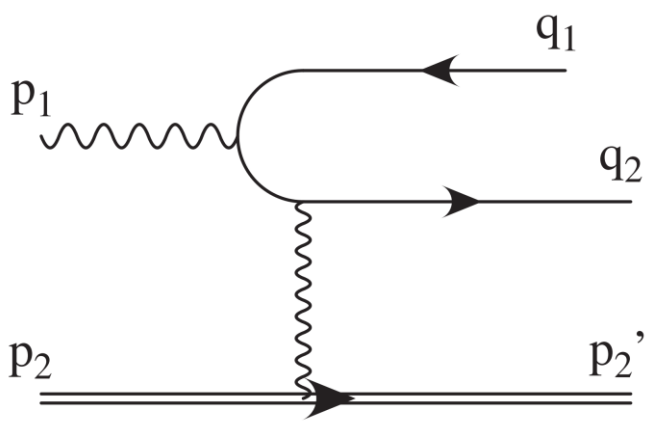
At RHIC and LHC Zα ~ 0.6

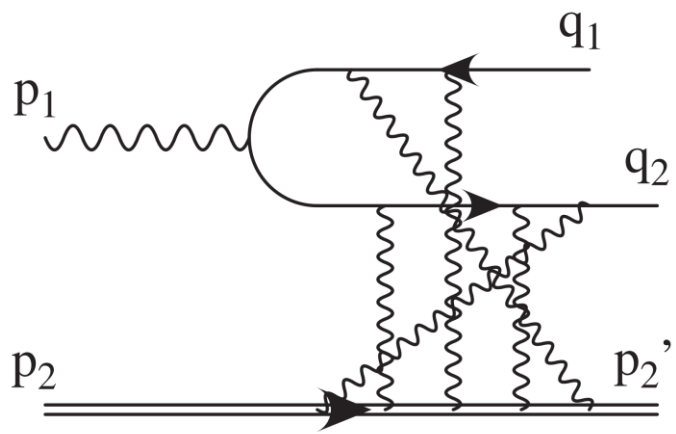
Still Perturbative, but with sizable higher-order effect!



Link the crossover from perturbative to non-perturbative region!

Higher-order correction to Bethe-Heitler process





The lowest order results:

$$\sigma_{\mathrm{BH}} = \frac{28}{9}\frac{\alpha^3 Z^2}{m^2}\left(\log\frac{2\omega}{m} - \frac{109}{42}\right)$$

H. A. Bethe, W. Heitler
Proc. Roy. Soc. Lond. A 146 (1934) 83

H.A. Bethe, L.C. Maximon
Phys. Rev. 93 (1954) 768

Sizable negative correction!

Should hold in heavy-ion collisions.

With higher-order correction:

$$\sigma = \frac{28}{9}\frac{\alpha^3 Z^2}{m^2}\left(\log\frac{2\omega}{m} - \frac{109}{42} - f(Z\alpha)\right)$$

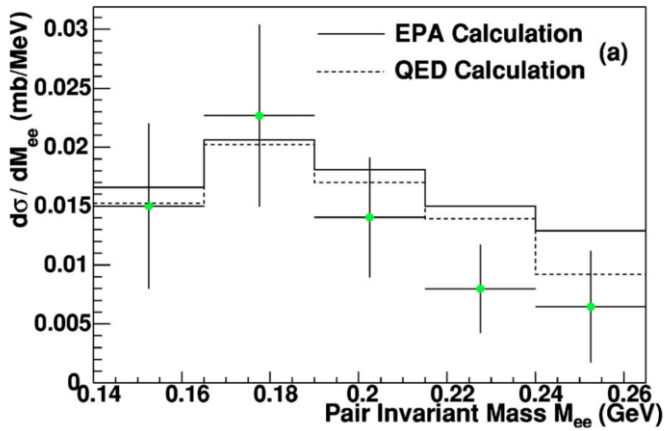
$$f(Z\alpha) = \gamma_{\mathrm{E}} + Re\,\Psi(1+\mathrm{i}Z\alpha) = (Z\alpha)^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+(Z\alpha)^2)}$$

Sommerfeld-Maue type approach

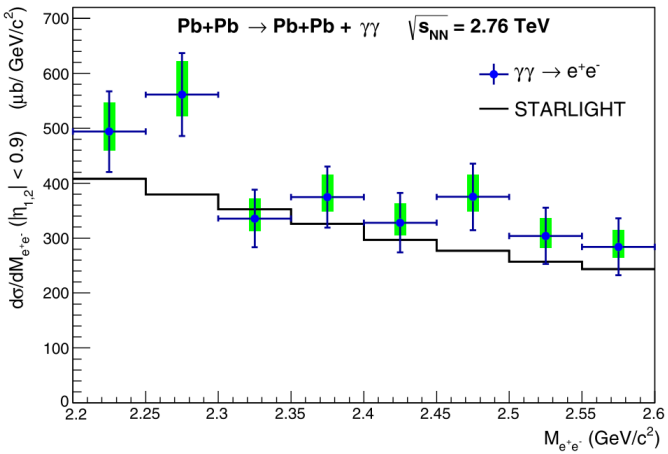
-Equivalent to the standard Feynman diagram approach.

The measurements of vacuum pair production in HIC

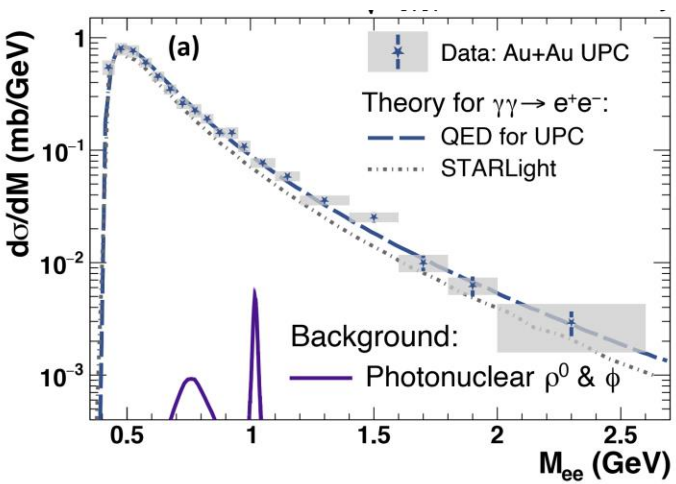
STAR, Phys. Rev. C70 (2004) 031902(R)

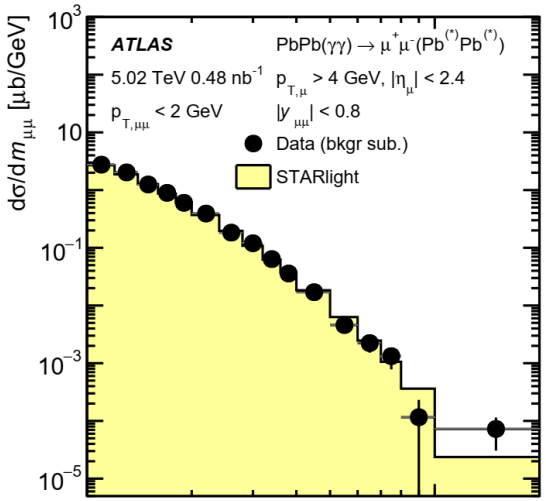


ALICE, Eur. Phys. J. C (2013) 73:2617



STAR, Phys. Rev. Lett. 127 (2021) 052302





ATLAS arXiv (2020) [2011.12211]

Consistent with the lowest-order QED results!

Higher-order Correction?

No room for Higher-order effect!

In April 1990 a workshop took place in Brookhaven with the title 'Can RHIC be used to test QED?' [98]. We think that after about 17 years the answer to this question is 'no'. However, many theorists were motivated to deal with this

G. Baur, K. Hencken and D. Trautmann
Phys. Rep. ***453****, 1 (2007)*

M. Fatyga, M. Rhoades-Brown, and M. Tannenbaum, Can RHIC be used to test QED: Workshop summary, Workshop "Can RHIC be used to test QED?", Upton, N.Y., Apr 20-21, 1990, BNL 52247 Formal Report.

Bethe and Maximon were wrong?

How could we expect the non-perturbative production from Schwinger mechanism?

Theoretical setup: EPA method

$$\sigma(A+A\to A+A+l^+l^-) = \int d\omega_1 d\omega_2 n(\omega_1)n(\omega_2)\sigma(\gamma\gamma\to l^+l^-)$$

Semiclassical part

$$n(\omega, r_\perp) = \frac{4Z^2\alpha}{\omega}\left|\int \frac{\vec{q}_\perp}{(2\pi)^2}\vec{q}_\perp \frac{f(\vec{q})}{q^2} e^{i\vec{q}_\perp\cdot\vec{r}_\perp}\right|^2$$

F. Krauss et al
Prog.Part. Nucl. Phys. 39 (1997) 503

Quantum part

$$\sigma(\gamma\gamma\to l^+l^-) = \frac{4\pi\alpha^2}{W^2}[(2+\frac{8m^2}{W^2}-\frac{16m^4}{W^4})\ln(\frac{W+\sqrt{W^2-4m^2}}{2m}) - \sqrt{1-\frac{4m^2}{W^2}}(1+\frac{4m^2}{W^2})].$$

G. Breit and J.A. Wheeler
Phys. Rev. 46 (1934) 1087

The transverse momentum of vacuum pair is the vector sum of the two photon.

$$\frac{dN}{dk_\perp} = \frac{2Z^2\alpha F^2(k_\perp^2+k^2/\gamma^2)k_\perp^3}{\pi[k_\perp^2+k^2/\gamma^2]^2}$$

Generalized equivalent photon approximation (gEPA)….

Theoretical setup: QED approach

Straight line approximation

$$A_\mu^{(1,2)}(q) = -2\pi Z e \mu_\mu^{(1,2)} \delta(q\mu^{(1,2)}) \frac{f(q^2)}{q^2} \exp(\pm \mathrm{iqb}/2)$$

The matrix element of lepton pair production from Feynman diagrams:

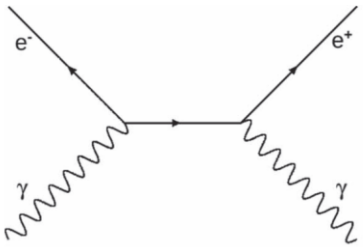
$$\hat{M} = -ie^2 \int \frac{d^4 q_1}{(2\pi)^4} \not{A}^{(1)}(q_1) \frac{\not{p}_- - \not{q}_1 + m}{(p_- - q_1)^2 - m^2} \not{A}^{(2)}(p_+ + p_- - q_1)$$

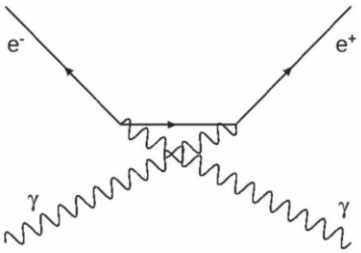
$$- ie^2 \int \frac{d^4 q_1}{(2\pi)^4} \not{A}^{(2)}(p_+ + p_- - q_1) \frac{\not{q}_1 - \not{p}_+ + m}{(q_1 - p_+)^2 - m^2} \not{A}^{(1)}(q_1)$$

$$= -i(\frac{Ze^2}{2\pi})^2 \frac{1}{2\beta} \int d^2 q_{1\perp} \frac{1}{q_1^2} \frac{1}{(p_+ + p_- - q_1)^2} \exp(\mathrm{iq}_{1\perp}\mathrm{b})$$

$$\{\frac{\not{\psi}^{(1)}(\not{p}_- - \not{q}_1 + m)\not{\psi}^{(2)}}{[(p_- - q_1)^2 - m^2]} + \frac{\not{\psi}^{(2)}(\not{q}_1 - \not{p}_+ + m)\not{\psi}^{(1)}}{[(q_1 - p_+)^2 - m^2]}\},$$

K. Hencken, D. Trautmann and G. Baur
Phys. Rev. A ***51*** *(1995) 1874*





Theoretical setup: Sommerfeld-Maue type approach

The δ-function potential from colliding nuclei:

$$V(\boldsymbol{\rho}, z, t) = \delta(z-t)(1-\alpha_z)\Lambda^-(\boldsymbol{\rho}) + \delta(z+t)(1+\alpha_z)\Lambda^+(\boldsymbol{\rho})$$

The exact semiclassical amplitude for lepton pair production:

$$M(p,q) = \int \frac{d^2k}{(2\pi)^2} \exp[i\mathbf{k}\cdot\mathbf{b}]\mathcal{M}(\mathbf{k})F_B(\mathbf{k}) \times F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k}),$$

R. Lee, A. Milstein and V. Strakhovenko Phys. Rev. A ***69*** *(2004) 022708.*

$$\mathcal{M}(\mathbf{k}) = \bar{u}(p)\frac{\alpha\cdot(\mathbf{k}-\mathbf{p}_\perp)+\gamma_0 m}{-p_+q_- - (\mathbf{k}-\mathbf{p}_\perp)^2 - m^2 + i\epsilon}\gamma_- u(-q) + \bar{u}(p)\frac{-\alpha\cdot(\mathbf{k}-\mathbf{q}_\perp)+\gamma_0 m}{-p_-q_+ - (\mathbf{k}-\mathbf{q}_\perp)^2 - m^2 + i\epsilon}\gamma_+ u(-q)$$

$$F(\mathbf{k}) = \int d^2\rho \exp[-i\mathbf{k}\cdot\boldsymbol{\rho}]\{\exp[-iZ\alpha\ln\rho] - 1\}$$

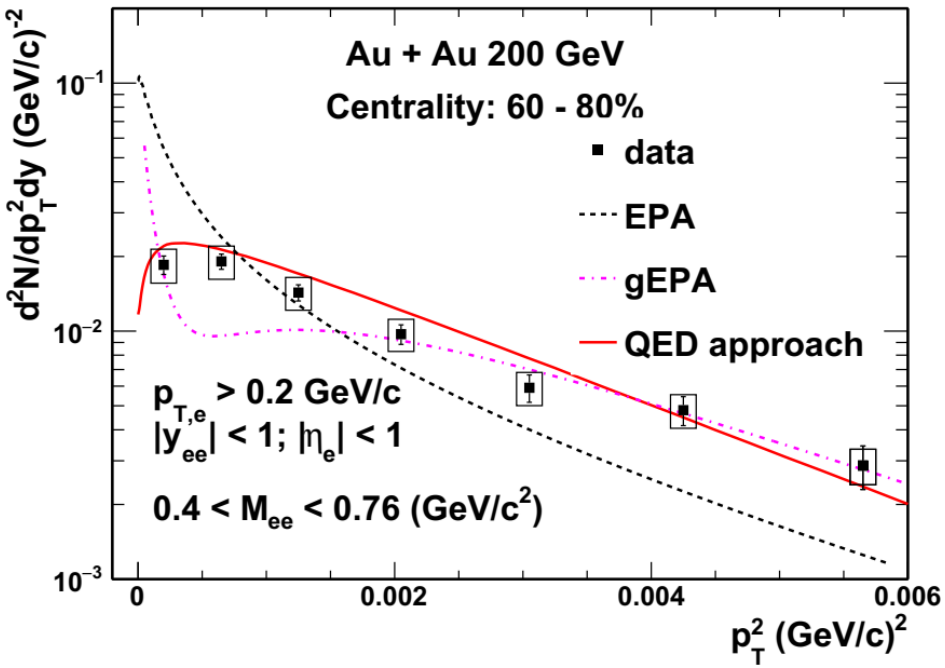
F(k) has to be regularized!

Perturbative limit: $F^0_{A,B}(\mathbf{k}) = \dfrac{4\pi i Z_{A,B}\alpha}{k^2 + \omega^2/\gamma^2}$

Theoretical setup: different approaches

All approaches give identical results on cross section estimation.

- No impact parameter dependence in standard EPA.
- Recovered in gEPA.
- Failed to reproduced the transverse momenta of pairs.
- QED approach describe the data very well!

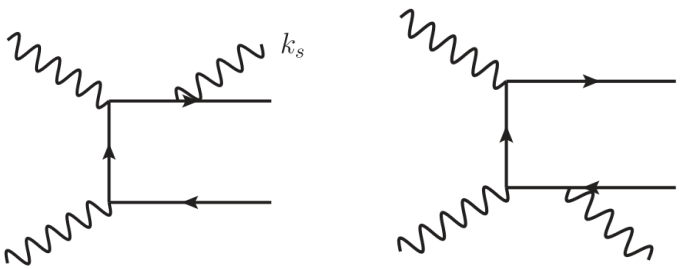


W. Zha etal.
Phys. Lett. B800 (2020) 135089

Employ the QED approach.

Theoretical setup: QED shower

Internal photon radiation

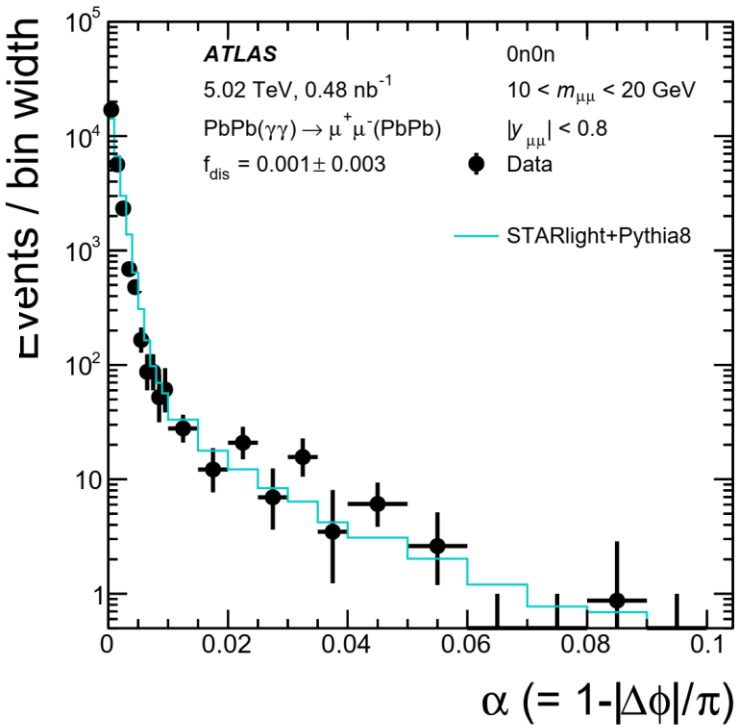


$$\int \frac{d^2 r_\perp}{(2\pi)^2} e^{i r_\perp \cdot q_\perp} e^{-S(Q, r_\perp)} \int d^2 q'_\perp e^{i r_\perp \cdot q'_\perp} d\sigma_0(q'_\perp, \ldots)$$

$$S(Q, r_\perp) = \begin{cases} \frac{\alpha_e}{2\pi} \ln^2 \frac{Q^2}{\mu_r^2}, & \mu_r > m_\mu \\ \frac{\alpha_e}{2\pi} \ln \frac{Q^2}{m_\mu^2} \left[\ln \frac{Q^2}{\mu_r^2} + \ln \frac{m_\mu^2}{\mu_r^2} \right], & \mu_r < m_\mu \end{cases}$$

Bowen Xiao etal., Phys. Rev. Lett. 122 (2019) 132301

ATLAS, arXiv (2020) [2011.12211]



Produce a tail at large transverse momentum.

-Affect the acceptance for experimental measurements.

QED + Pythia8.3 with QED shower

The UPC trigger probability

The UPC trigger: no hadronic interactions $b > R_A + R_B$

In p + p collisions: $P_{\mathrm{H}} = |1 - \exp(-\mathrm{b}^2/(2\mathrm{B}))|^2$ no inelastic

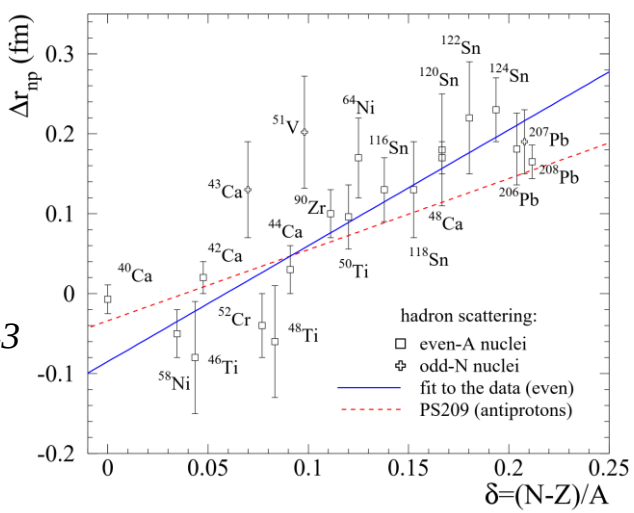
In heavy-ion collisions: optical Glauber model

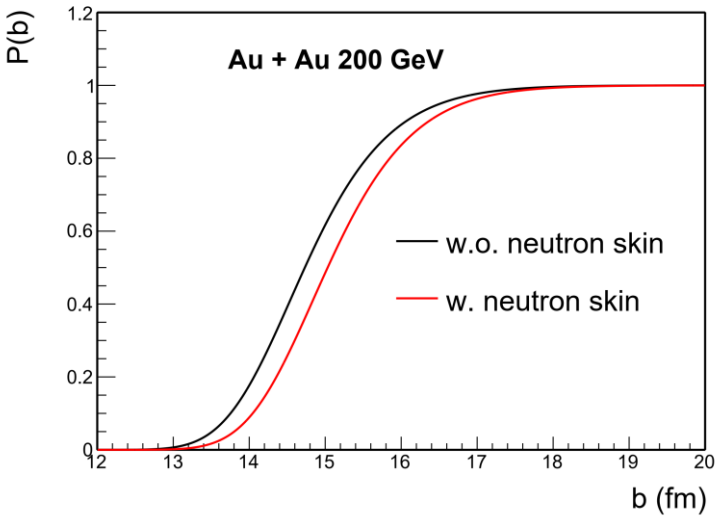
$$m_H(b) = \int d^2 r_\perp T_A(r_\perp - b)\{1 - \exp[-\sigma_{\mathrm{NN}} \mathrm{T_A}(\mathrm{r}_\perp)]\}$$

$$\rho_N(r) = \frac{Z}{A}\rho_p(r) + \frac{N}{A}\rho_n(r) \qquad P_H(b) = \exp[-\mathrm{m_H(b)}]$$

Neutron Skin effects!

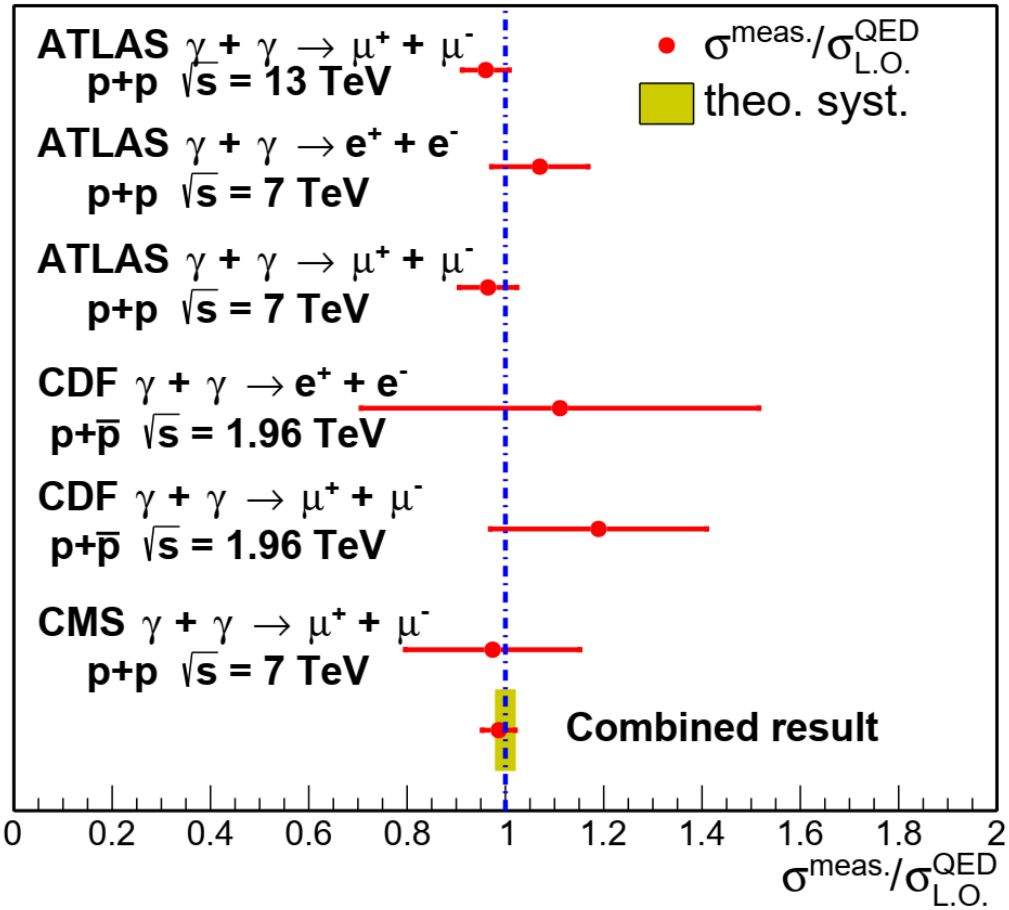
J. Jastrze etal. Inter. Jour. of Mod. Phys. E 13 (2004) 343





The lowest-order baseline

The perturbative limit: Z = 1 p + p collisions

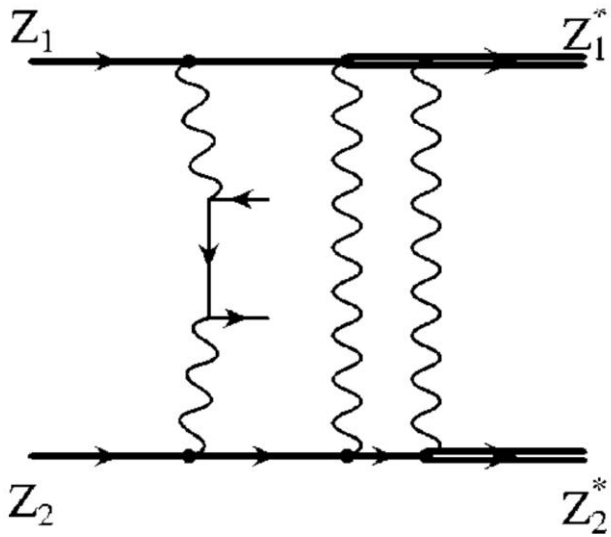


CDF collaboration
Phys. Rev. Lett. ***102*** *(2009) 242001.*
CDF collaboration
Phys. Rev. Lett. ***98*** *(2007) 112001.*
ATLAS collaboration
Phys. Lett. B ***749*** *(2015) 242*
ATLAS collaboration
Phys. Lett. B 777 (2018) 303
CMS collaboration
JHEP ***11*** *(2012) 080*

- ✓ The electric proton size
- ✓ The UPC trigger probability

Consistent with the lowest order QED calculations!

Mutual Coulomb Dissociation



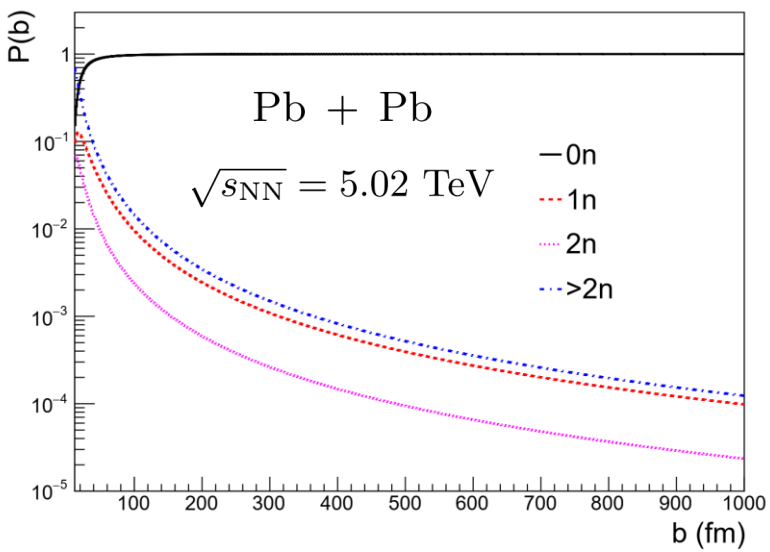
$$m_{Xn}(b) = \int dkn(b,E)\sigma_{\gamma A \to A^*}(E)$$

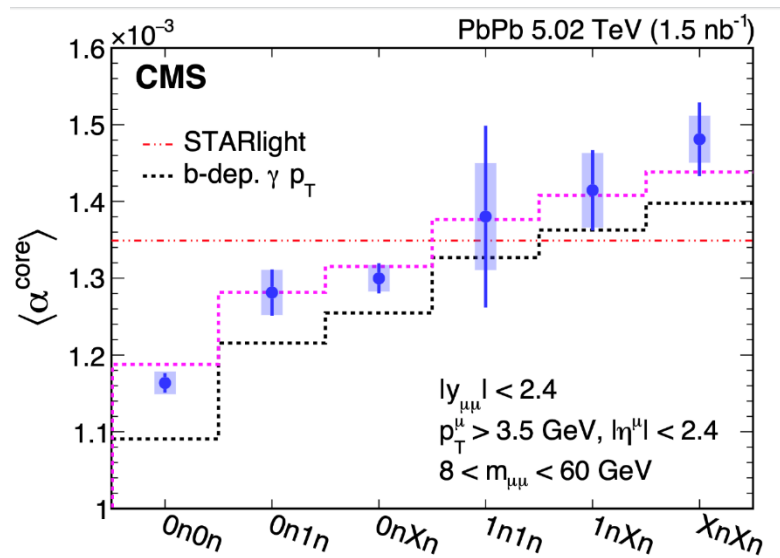
$$P_{0n}(b) = e^{-m_{Xn}(b)}$$

$$P_{XnXn}(b) = (1.0 - e^{-m_{Xn}(b)})^2,$$

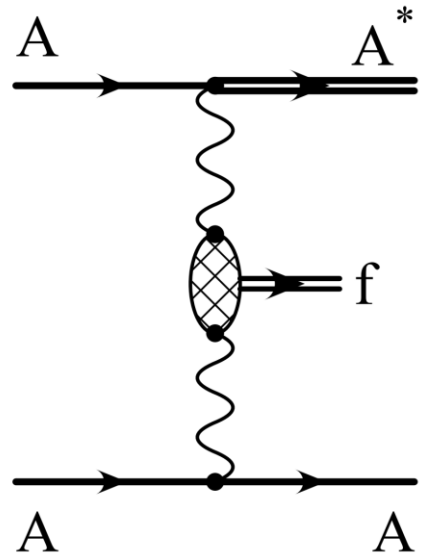
$$P_{0nXn}(b) = 2(1.0 - e^{-m_{Xn}(b)})e^{-m_{Xn}(b)},$$

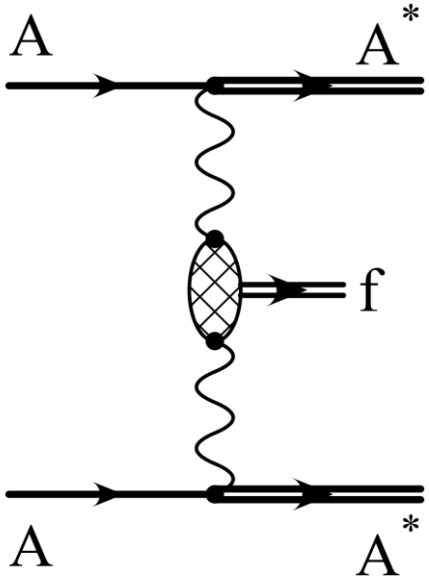
CMS[PAS-HIN-19-014]





Incoherent photon emission

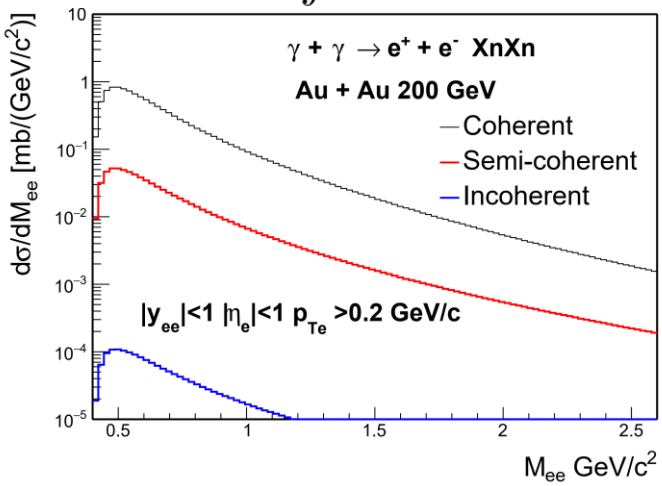




Excited and dissociated by photon emission:

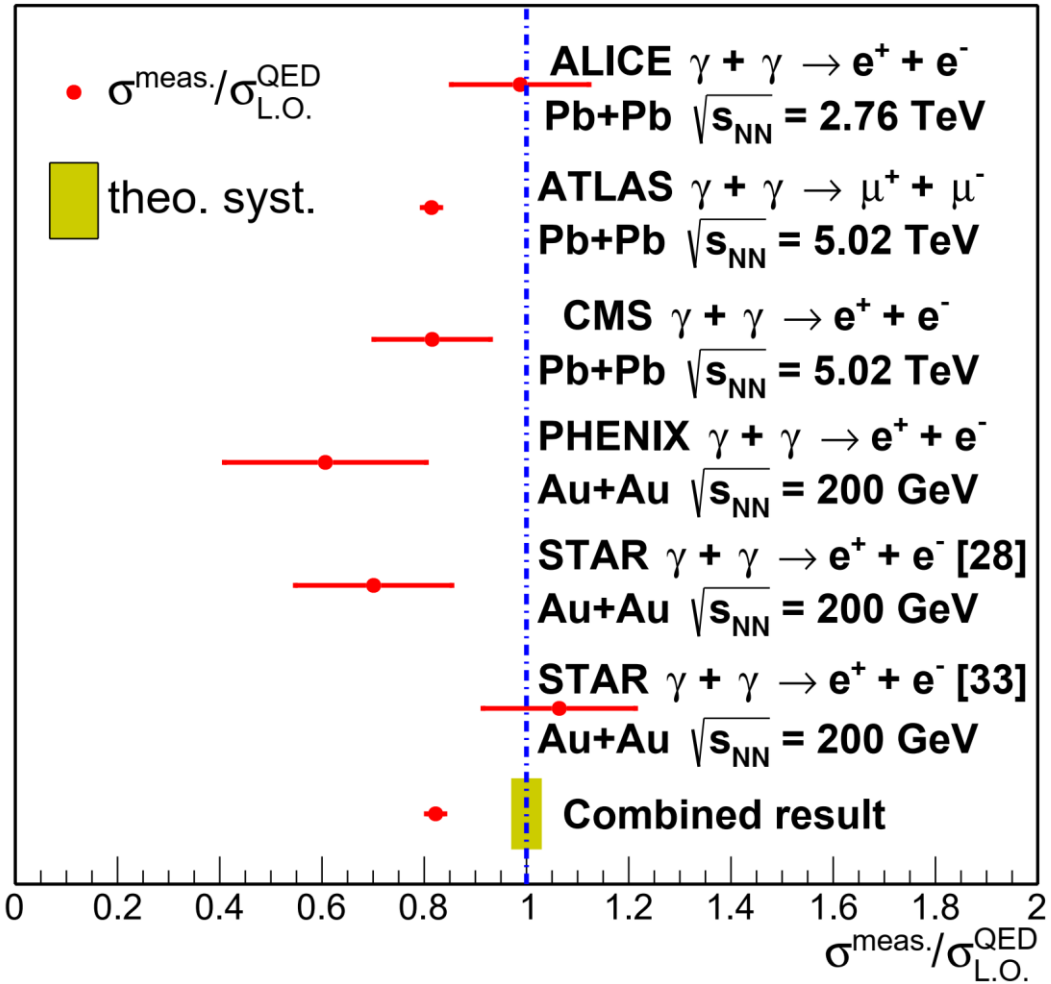
- ✓ MCD (XnXn) + Coherent photon-photon interaction.
- ✓ MCD (Xn*n) + Semi-coherent photon-photon interaction (one incoherent photon).
- ✓ Incohrent photon-photon interaction (two incoherent photons).

$$n^{incoh}(\omega, b) = \int d^2 r_\perp T_Z(r_\perp) n^{proton}(\omega, b + r_\perp) - n(\omega, b)/Z$$



Sizable contribution from incoherent Contribution !

The lowest order results for HIC



*STAR, Phys. Rev. C **70** (2004) 031902.*
STAR, STAR, Phys. Rev. Lett. 127 (2021) 052302
*PHENIX, Phys. Lett. B **679** (2009) 321.*
*ALICE, Eur. Phys. J. C **73** (2013) 2617.*
*CMS, Phys. Lett. B **797** (2019) 134826.*
ATLAS , arXiv (2020) [2011.12211]

Theoretical uncertainties:

- ✓ Nuclear density distribution.
- ✓ UPC trigger probability.
- ✓ MCD probability.
- ✓ Incoherent contribution.

5.2 σ deviation from Lowest order calculations !

Higher order effect

Conveniently considered in Sommerfeld-Maue type approach.

-Presented by the regularization to the photon propagator.

$$F(k) = \int d^2 r_\perp \exp(-\mathrm{ikr}_\perp)\{\exp[-\mathrm{i}\chi(\mathrm{r}_\perp)] - 1\} \qquad \chi(r_\perp) = \int_{-\infty}^{+\infty} dz V(r_\perp, z)$$

$$F(k) = \int d^2 r_\perp \exp(-\mathrm{ikr}_\perp)[\exp(-2\mathrm{iz}\alpha \ln \mathrm{r}_\perp) - 1] \quad V(r_\perp, z) = -Z\alpha/\sqrt{r_\perp^2 + z^2}$$

Regularize at large distance $r_\perp$: $F(k) = \dfrac{4\pi\alpha Z}{k^{2-2i\alpha Z}}$

A low k cutoff at ω/γ: $F(k) = \dfrac{4\pi\alpha Z}{(k^2 + \omega^2/\gamma^2)^{1-i\alpha Z}}$

Higher order effects!

A.J. Baltz and L.D. McLerran, Phys. Rev. C ***58*** *(1998) 1679*

Absent of higher order correction for total cross section!

Revealed in the differential cross section versus impact parameter!

Contradict with the Bethe-Maximon correction!

Higher order effect

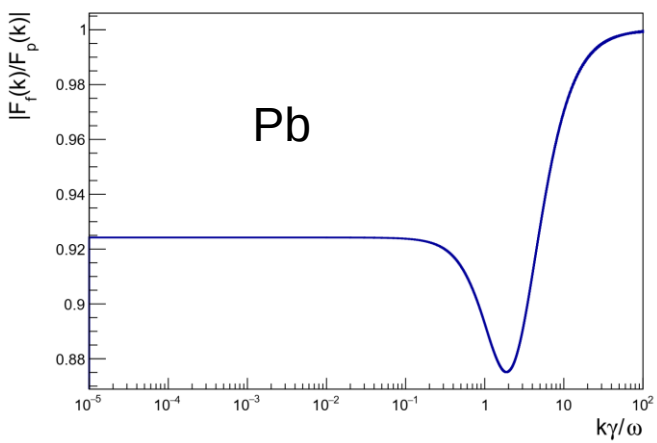
Introduce the screening of the Coulomb potential:

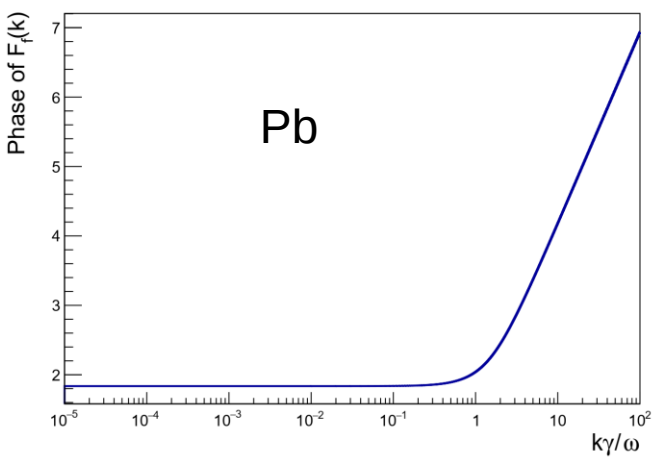
R. Lee, A. Milstein and V. Strakhovenko, Phys. Rev. A ***69*** *(2004) 022708*

$$V(r_\perp, z) = \frac{-Z\alpha \exp(-\sqrt{r_\perp^2 + z^2}\omega/\gamma)}{\sqrt{r_\perp^2 + z^2}} \qquad \chi(r_\perp) = -2Z\alpha K_0(r_\perp \omega/\gamma)$$

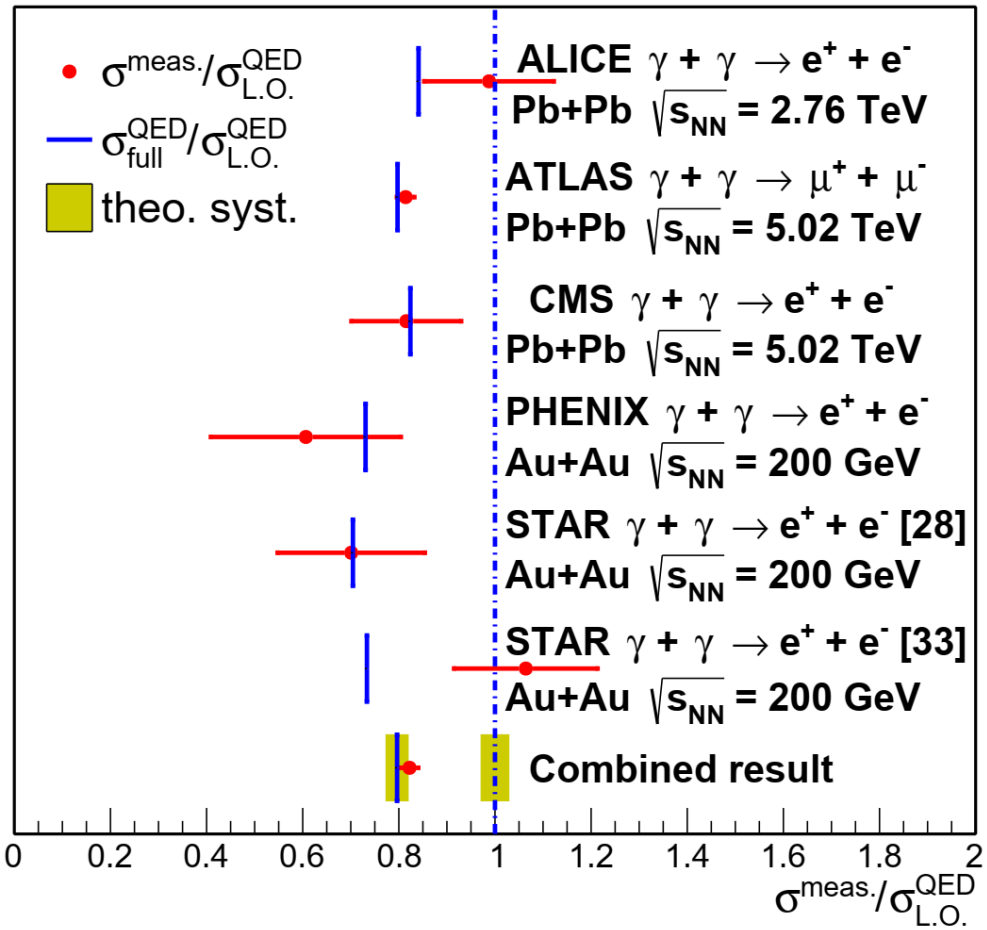
$$F(k) = \frac{1}{2}\int dr_\perp r_\perp J_0(kr_\perp)\{\exp[2iZ\alpha K_0(r_\perp\omega/\gamma)] - 1\}$$

In the perturbative limit Zα → 0 $F^0(k) = \dfrac{4\pi i Z\alpha}{k^2 + \omega^2/\gamma^2}$





The higher order results for HIC



*STAR, Phys. Rev. C **70** (2004) 031902.*
STAR, STAR, Phys. Rev. Lett. 127 (2021) 052302
*PHENIX, Phys. Lett. B **679** (2009) 321.*
*ALICE, Eur. Phys. J. C **73** (2013) 2617.*
*CMS, Phys. Lett. B **797** (2019) 134826.*
ATLAS , arXiv (2020) [2011.12211]

Theoretical uncertainties:

- ✓ Nuclear density distribution.
- ✓ UPC trigger probability.
- ✓ MCD probability.
- ✓ Incoherent contribution.

Consistent with the higher order results !

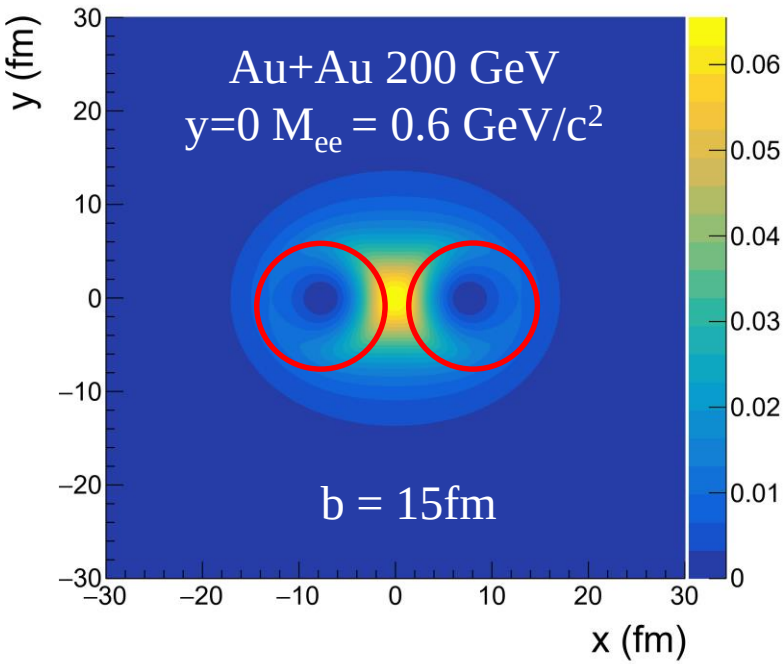
The missing part in STARLight model

S.R. Klein et al., Comput. Phys. Commun. ***212*** *(2017) 258*

STARLight model: Standard equivalent photon approximation method.

The photon flux: $n(\omega, r_\perp) = \frac{Z^2\alpha}{\pi^2\omega r_\perp^2} x^2 K_1^2(x), x = \omega r_\perp/\gamma$ Point-like

Exclude the production within the transverse geometry radius of nuclei.



Pb+Pb 5.02 TeV
UPC ATLAS acceptance

Production within nuclei: 28%
Higher order correction: 20%

These two effects compensate with each other!

Higher order effect?

QED type Wilson line

$$\mathcal{U}_{\mathrm{QED}}(x_\perp) = \exp\left[iZe^2 G(x_\perp)\right] \qquad G(x_\perp) = \frac{1}{2\pi}\mathrm{K}_0(\lambda x_\perp)$$

QED Multiple scattering contribution

Point-like assumption

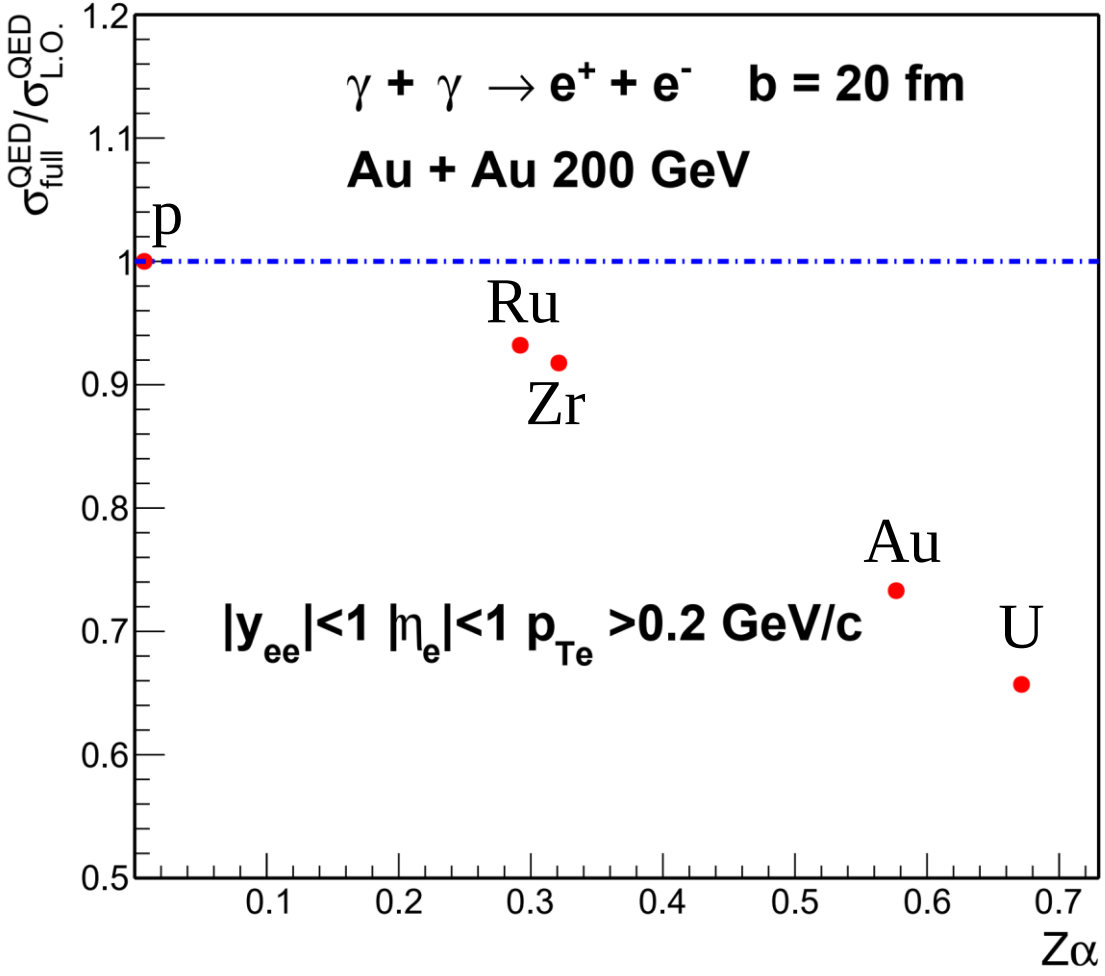
$$\mathcal{U}_{\mathrm{QED}}(b_\perp + \frac{1}{2}r_\perp)\mathcal{U}^\dagger_{\mathrm{QED}}(b_\perp - \frac{1}{2}r_\perp)$$
$$= \exp\left[2iZ\alpha \ln \frac{|b_\perp + \frac{1}{2}r_\perp|}{|b_\perp - \frac{1}{2}r_\perp|}\right].$$

The higher-order effect should be small!

S.R. Klein et al., Phys. Rev. D ***102*** *(2020) 094013*

Z. Sun et al., Phys. Lett. B ***808*** *(2020) 135679*

Vary the coupling constant



Vary the coupling constant by colliding different species.

The dependence of negative correction on colliding species is significant!

Leading order $\propto Z^4$

Higher order ! $\propto Z^4$

Summary

- In p+p collisions ($Z\alpha \rightarrow 0$), the lepton pair production is consistent with the lowest order QED calculation.

- In heavy-ion collisions ($Z\alpha \sim 0.6$), the lepton pair production is about 20% higher than the lowest order result, and consistent with the higher order QED calculation.

- The discovery of higher order QED effect for vacuum pair production**?**
  - ✓ More precise experimental measurements
  - ✓ More theoretical investigations