



Nuclear Science
Computing Center at CCNU



Correlated Dirac eigenvalues & Axial anomaly In Chiral symmetric QCD

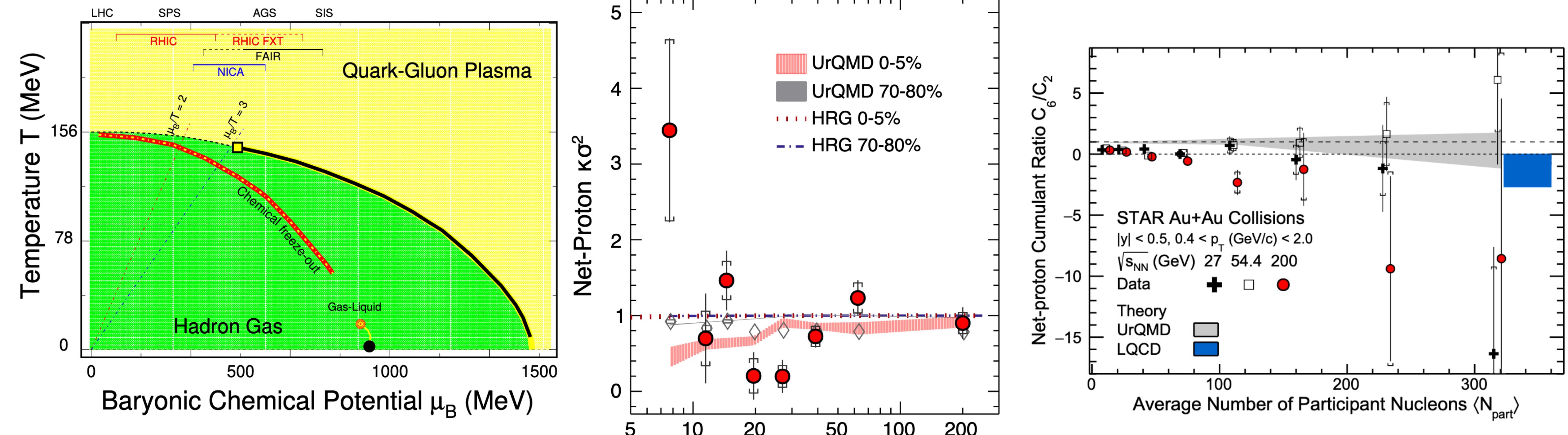
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Central China Normal University (华中师范大学)

based on PRL 126 (2021) 082001 & in collaboration with
Sheng-Tai Li(李胜泰), Akio Tomiya, Swagato Mukherjee, Xiao-Dan Wang(汪晓丹), Yu Zhang(张瑜)

中国物理学会高能物理分会第十三届全国粒子物理学学术会议
2021年8月16-19@山东大学线上

Search for critical end point & criticalities



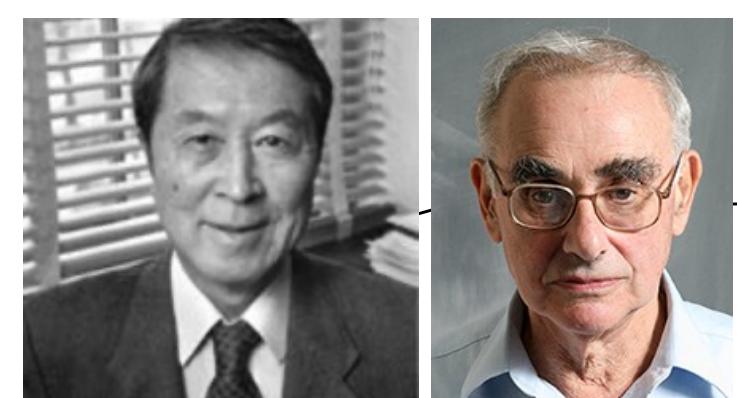
STAR, arXiv: 2001.02852, 2105.14698

Symmetries of QCD in vacuum

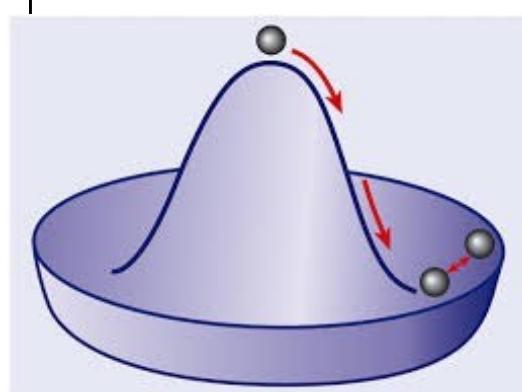
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^\mu(\partial_\mu - igA_\mu) - m_q\right]q$$

Classical QCD symmetry ($m_q=0$)

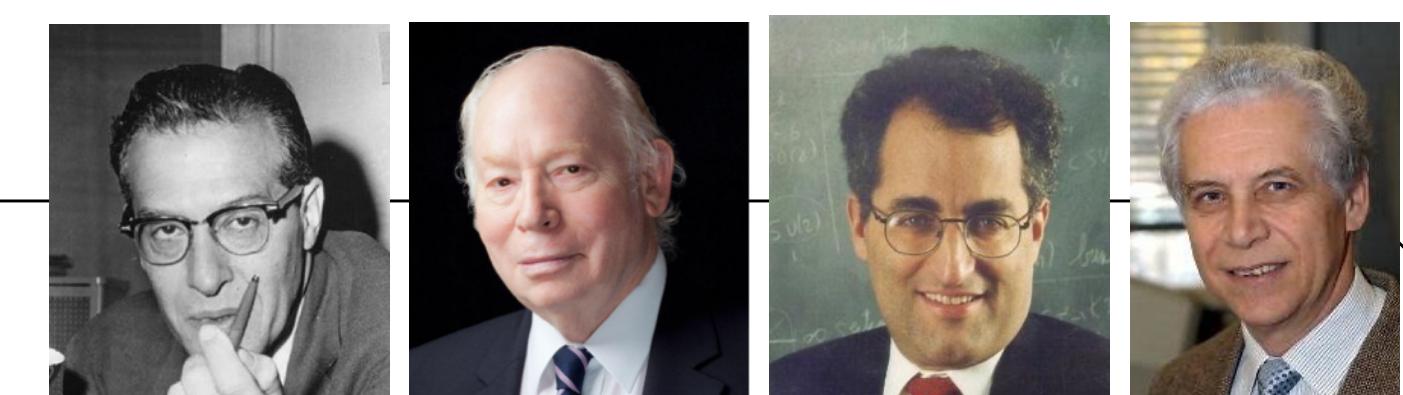
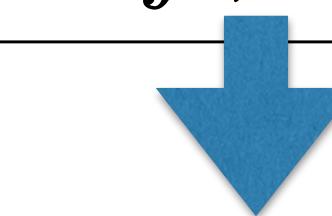
$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$



Chiral condensate:
spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

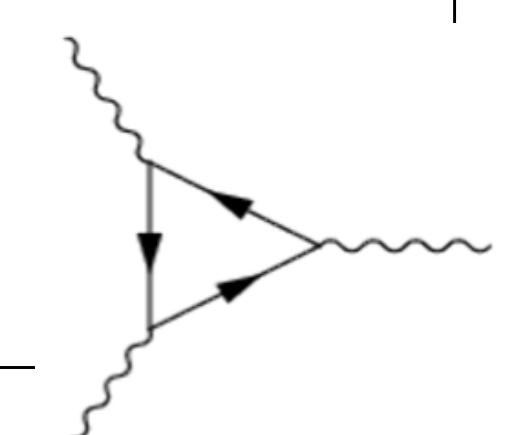


Axial anomaly:
quantum violation of $U(1)_A$

ABJ...

$U(1)$ problem

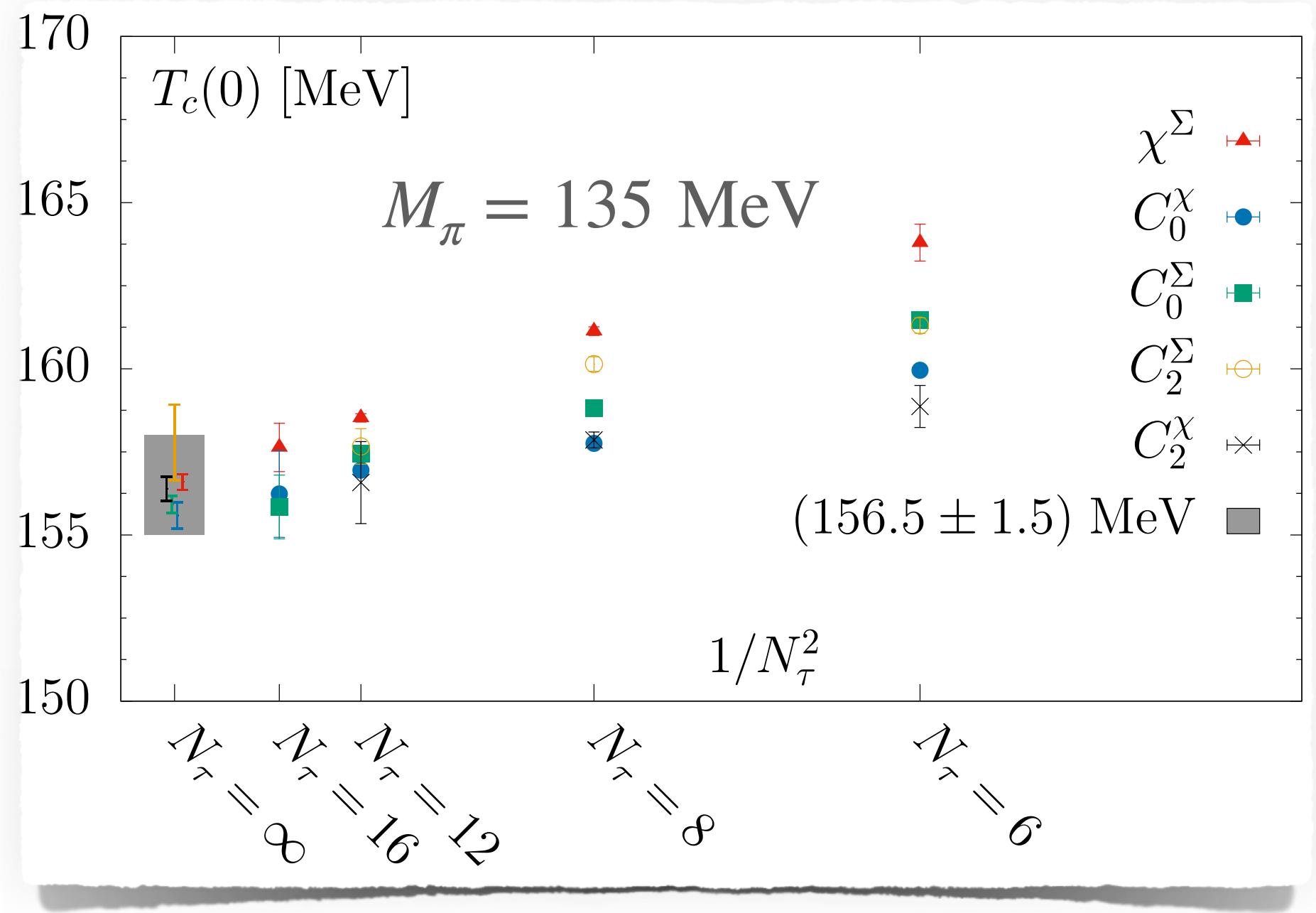
$$\partial_\mu j_5^\mu = \frac{g^2 N_f}{16\pi^2} \text{tr}(\tilde{F}_{\mu\nu} F^{\mu\nu})$$



$$SU(N_f)_V \times U(1)_V$$

Chiral crossover/phase transition temperature at physical point and $m_q \rightarrow 0$

Rigorous definition from $O(4)$ universality class

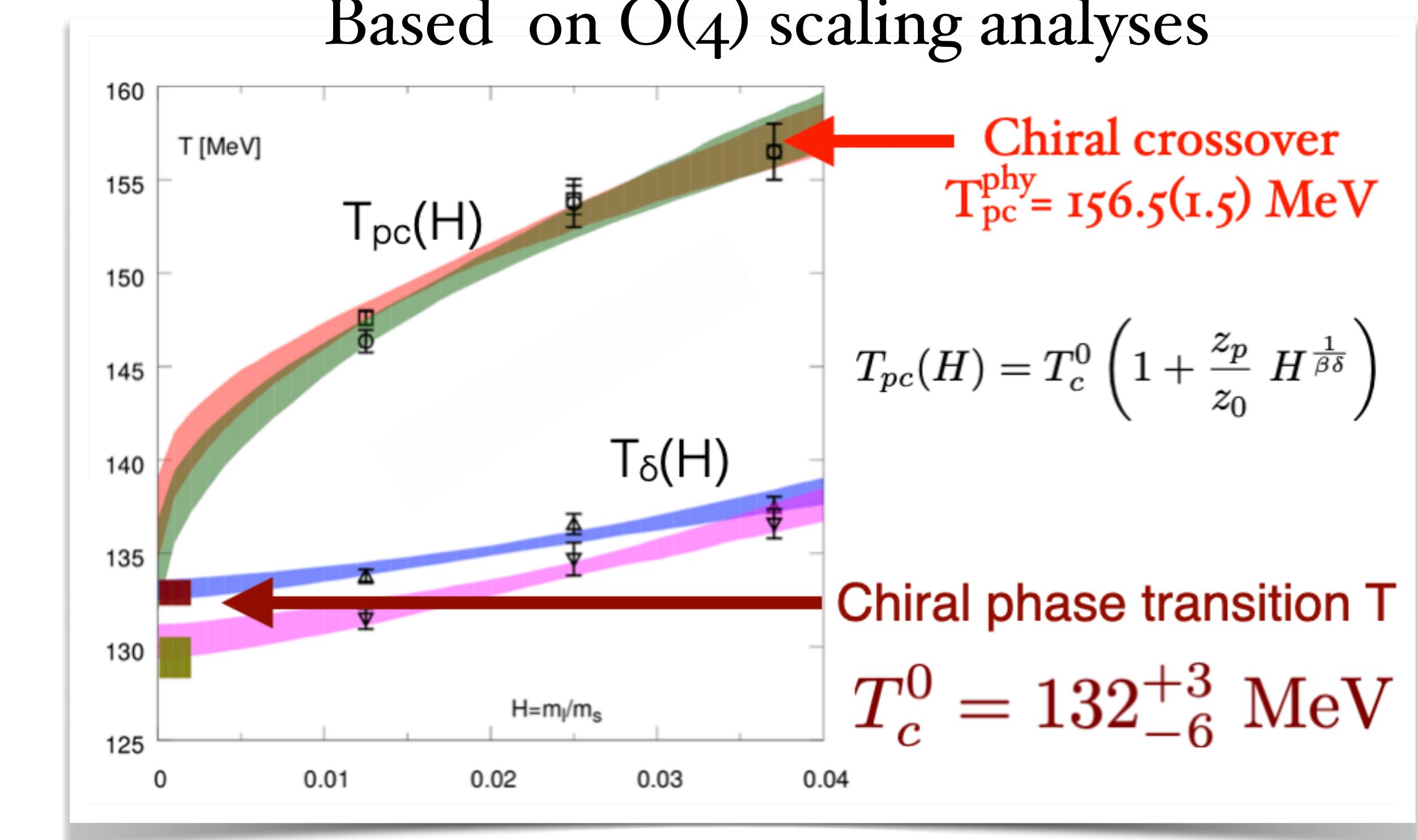


A. Bazavov, HTD, P. Hegde et al. [HotQCD],
Phys. Lett. B795 (2019) 15

Chiral crossover transition
 $T=156.5(1.5)$ MeV

See also Wuppertal-Budapest, PRD125 (2020) 052001

Based on $O(4)$ scaling analyses

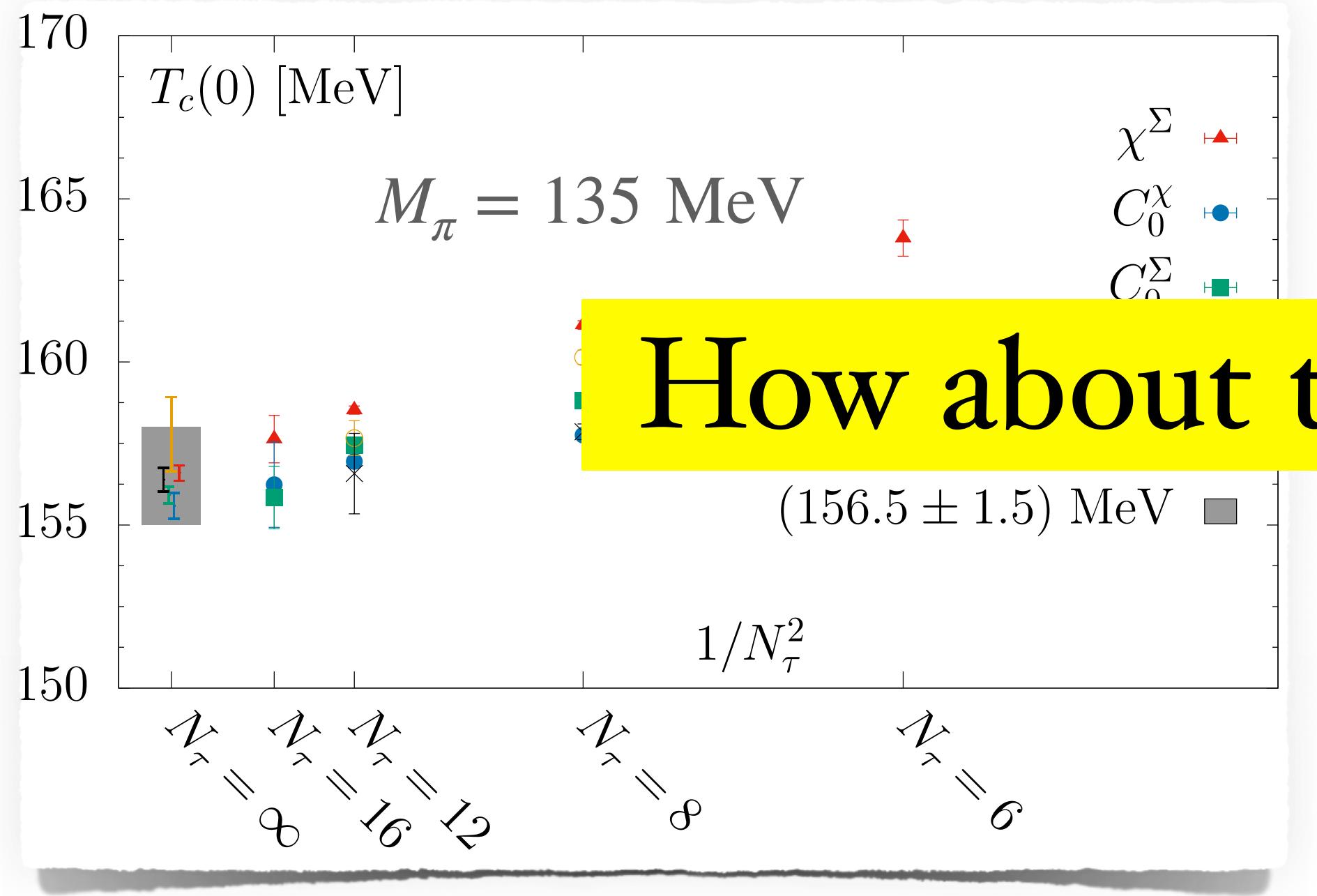


HTD, P. Hegde, O. Kaczmarek et al. [HotQCD],
Phys. Rev. Lett. 123 (2019) 062002

Chiral phase transition T : possible upper bound of T_c
at critical end point in the T - μ phase diagram

Chiral crossover/phase transition temperature at physical point and $m_q \rightarrow 0$

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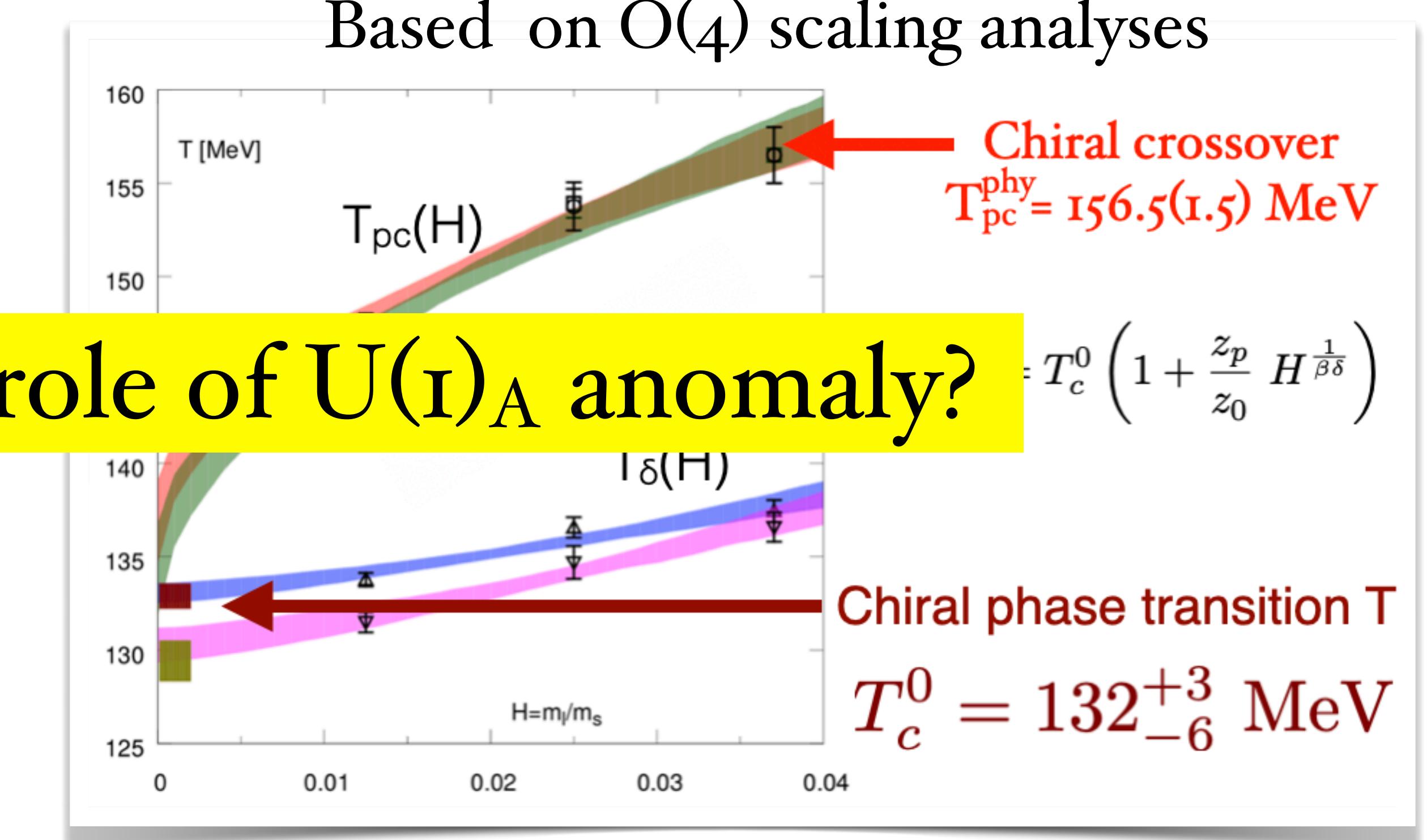


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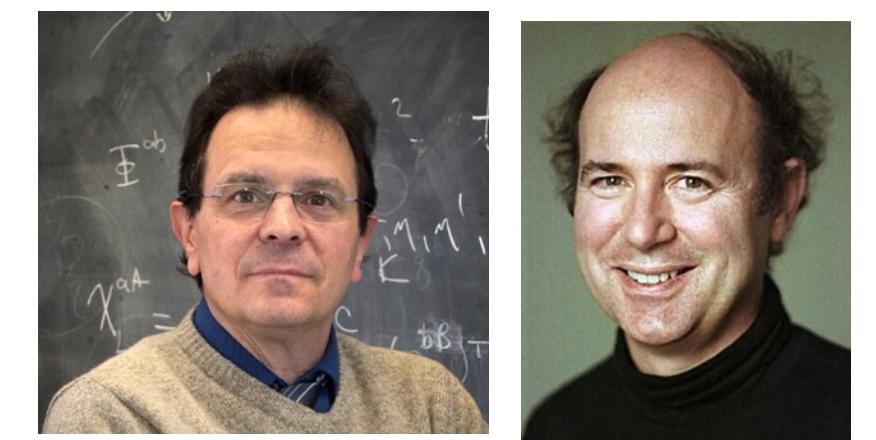


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Chiral phase transition T : possible upper bound of T_c
at critical end point in the T -mu phase diagram

$$\partial_\mu j_5^\mu = \frac{g^2 N_f}{16\pi^2} \text{tr}(\tilde{F}_{\mu\nu} F^{\mu\nu})$$

Landau functional of QCD



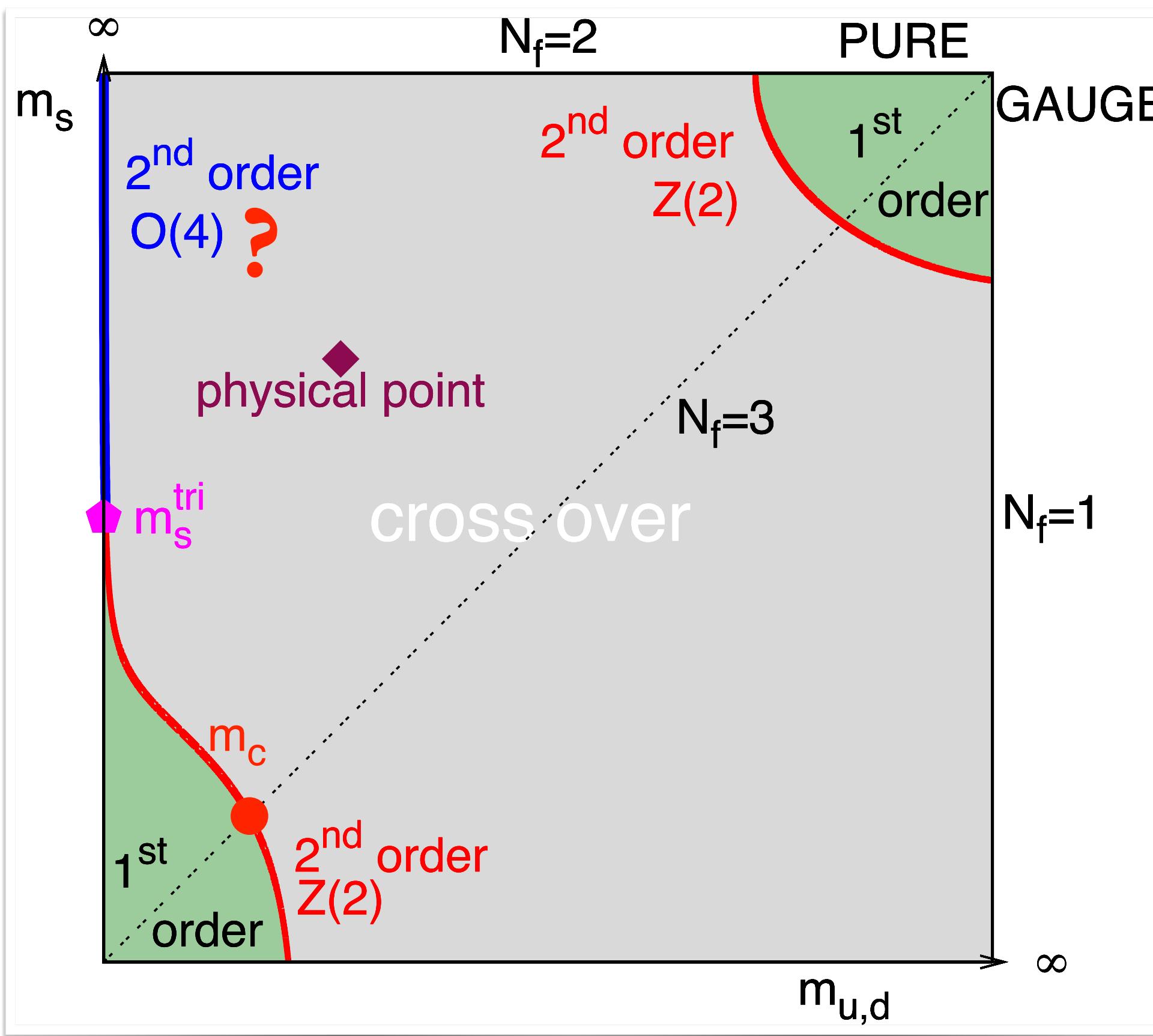
Symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Pisarski & Wilczek,
PRD 84'

Chiral field: $\Phi_{ij} \sim \frac{1}{2} \bar{q}^j (1 - \gamma_5) q^i = \bar{q}_R^j q_L^i$ Chiral transformation: $\Phi \rightarrow e^{-2i\alpha_A} V_L \Phi V_R^\dagger$

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{1}{2} \text{tr} \partial\Phi^\dagger \partial\Phi + \frac{a}{2} \text{tr} \Phi^\dagger \Phi \\ & + \frac{b_1}{4!} (\text{tr} \Phi^\dagger \Phi)^2 + \frac{b_2}{4!} \text{tr} (\Phi^\dagger \Phi)^2 \xrightarrow{\quad \quad \quad} \text{SU}(N_f)_L \times \text{SU}(N_f)_R \times U(1)_A \\ & - \frac{c}{2} (\det\Phi + \det\Phi^\dagger) \xrightarrow{\quad \quad \quad} \text{SU}(N_f)_L \times \text{SU}(N_f)_R \\ & - \frac{d}{2} \text{tr} h (\Phi + \Phi^\dagger) . \xrightarrow{\quad \quad \quad} \text{Quark mass term} \end{aligned}$$

Role of axial anomaly in the nature of chiral phase transition



$$\partial_\mu j_5^\mu = \frac{g^2 N_f}{16\pi^2} \text{tr}(\tilde{F}_{\mu\nu} F^{\mu\nu})$$

- The strength of axial U(1) anomaly could get reduced at high temperature

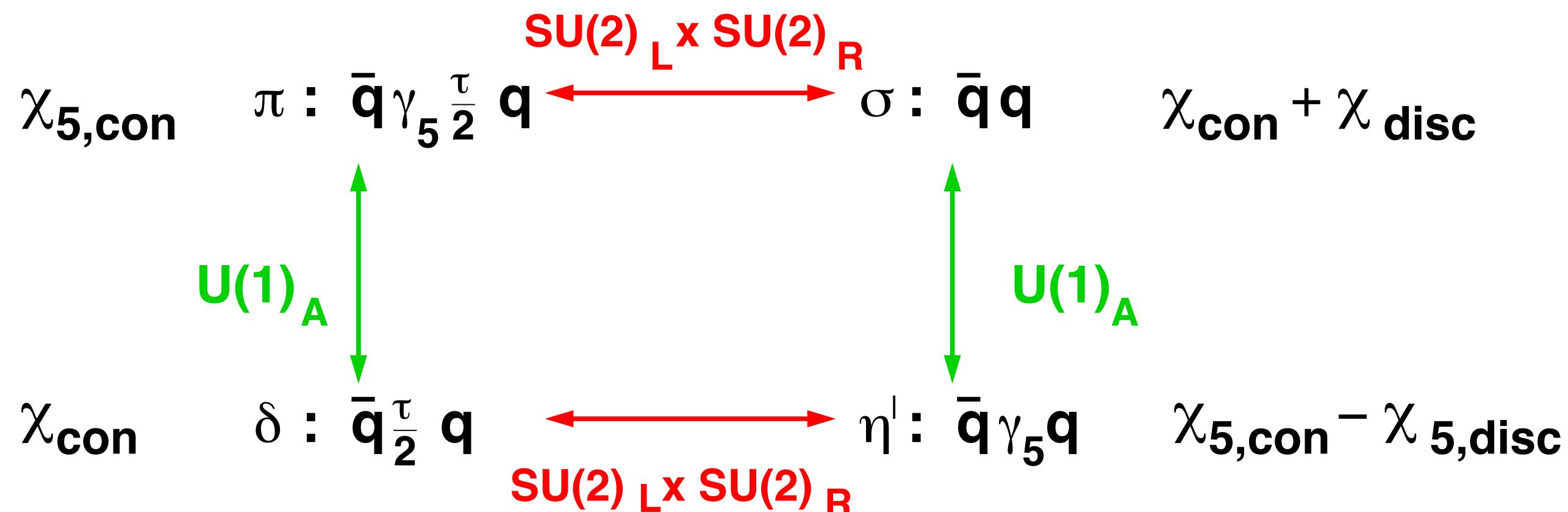
Pisarski and Wilczek, PRD 29 (1984) 338
Butti, Pelissetto and Vicari, JHEP 08 (2003) 029
Pelissetto & Vicari, PRD 88 (2013) 105018
Grahl, PRD 90 (2014) 117904

- “Effectively” restored: 1st or 2nd order ($U(2)_L \otimes U(2)_R / U(2)_V$)
- Broken: 2nd order (O(4)) phase transition

Signatures of symmetry restorations ?

- Susceptibilities defined as integrated two point correlation functions of the local operators, e.g. $\chi_\pi = \int d^4x \langle \pi^i(x) \pi^i(0) \rangle$ with $\pi^i(x) = i\bar{\psi}_l(x)\gamma_5\tau^i\psi_l(x)$

E. Shuryak, Comments Nucl.Part.Phys. 21 (1994) 4, 235,
[hep-ph/9310253](https://arxiv.org/abs/hep-ph/9310253)



Restoration of $SU(2)_L \times SU(2)_R$:

$$\begin{aligned} \chi_\pi - \chi_\sigma &= 0 \\ \chi_\delta - \chi_\eta &= 0 \end{aligned} \rightarrow \chi_\pi - \chi_\delta = \chi_{\text{disc}} = \chi_{5,\text{disc}}$$

Effective restoration of $U(1)_A$:

$$\begin{aligned} \chi_\pi - \chi_\delta &= 0 \\ \chi_\sigma - \chi_\eta &= 0 \end{aligned} \rightarrow \chi_\pi - \chi_\delta = \chi_{\text{disc}} = \chi_{5,\text{disc}} = 0$$

$$\chi_{\text{disc}} = \frac{T}{V} \int d^4x \left\langle [\bar{\psi}(x)\psi(x) - \langle \bar{\psi}(x)\psi(x) \rangle]^2 \right\rangle$$

Dirac Eigenvalue spectrum ρ

$$\langle \bar{\psi} \psi \rangle = \int_0^\infty \frac{4m_l \rho}{\lambda^2 + m_l^2} d\lambda \quad \chi_\pi - \chi_\delta = \int_0^\infty d\lambda \frac{8m_l^2 \rho}{(\lambda^2 + m_l^2)^2} \quad \chi_{\text{disc}} = \int_0^\infty d\lambda \frac{4m_l \partial \rho / \partial m_l}{\lambda^2 + m_l^2}$$

$$\rho(\lambda, m) = c_o + c_I \lambda + c_2 m^2 \delta(\lambda) + c_3 m + c_4 m^2 + O(\lambda, m)$$

$$\langle \bar{\psi} \psi \rangle = 2c_0\pi - 4c_1 m \ln(m) + 2c_2 m + 2\pi c_3 m + 2\pi c_4 m^2$$

$$\chi_\pi - \chi_\delta = 2c_0\pi/m + 4c_1 + 4c_2 + 2\pi c_3 + 2\pi c_4 m$$

Ansatz	$\langle \bar{\psi} \psi \rangle$	χ_π	χ_δ	$\chi_\pi - \chi_\delta$	χ_{disc}
c	$2c\pi$	$2c\pi/m$	0	$2c\pi/m$	0
λ	$-4m \ln(m)$	$-4 \ln(m)$	$-4 \ln(m)$	4	0
$m^2 \delta(\lambda)$	$2m$	2	-2	4	4
m	$2\pi m$	2π	0	2π	2π
m^2	$2\pi m^2$	$2\pi m$	0	$2\pi m$	$2\pi m$
m^3	$2\pi m^3$	$2\pi m^2$	0	$2\pi m^2$	$2\pi m^2$

c_o & c_I terms: break both symmetries

c_2 : near zero mode contribution

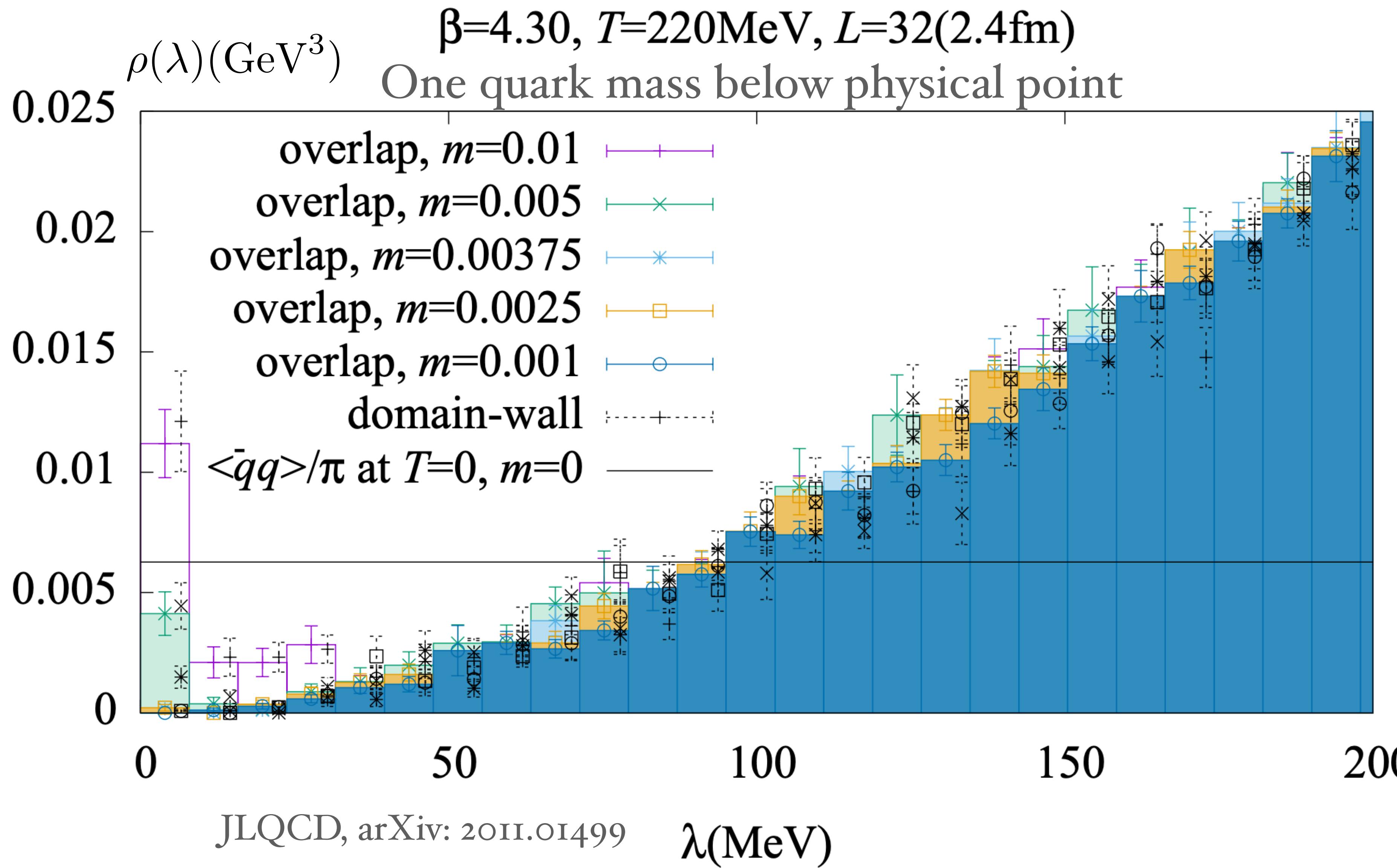
c_3 : another $U(I)_A$ breaking term

c_4 : Not manifested in 2-pt correlators

Smilga & Stern, PLB 93,
Gross, Yaffe & Pisarski,
RMP 81'

Aoki, Fukaya & Taniguchi,
PRD 12'

Difficult to access m dep. of ρ directly from LQCD



No clear gap

At $m < 0.01$ and $\lambda > 0$,
m dependence can be
hardly seen

Novel relation: Light quark mass derivative of ρ and C_n

$$\rho(\lambda, m_l) = \frac{T}{VZ[\mathcal{U}]} \int \mathcal{D}[\mathcal{U}] e^{-S_G[\mathcal{U}]} \det [\not{D}[\mathcal{U}] + m_s] (\det [\not{D}[\mathcal{U}] + m_l])^2 \rho_U(\lambda)$$

Partition function $Z[\mathcal{U}] = \int \mathcal{D}[\mathcal{U}] e^{-S_G[\mathcal{U}]} \det [\not{D}[\mathcal{U}] + m_s] (\det [\not{D}[\mathcal{U}] + m_l])^2$

Eigenvalue spectrum per ensemble $\rho_U(\lambda) = \sum_j \delta(\lambda - \lambda_j)$

Quark mass dependence of ρ is enclosed in

$$\det [\not{D}[\mathcal{U}] + m_l] = \prod_j (+i\lambda_j + m_l)(-i\lambda_j + m_l) = \exp \left(\int_0^\infty d\lambda \rho_U(\lambda) \ln [\lambda^2 + m_l^2] \right)$$



$$\frac{V}{T} \frac{\partial \rho}{\partial m_l} = \int_0^\infty d\lambda_2 \frac{4m_l C_2}{\lambda_2^2 + m_l^2}$$

$$C_2(\lambda, \lambda_2) = \langle \rho_U(\lambda) \rho_U(\lambda_2) \rangle - \langle \rho_U(\lambda) \rangle \langle \rho_U(\lambda_2) \rangle$$

Relation between ρ derivatives and C_{n+1}

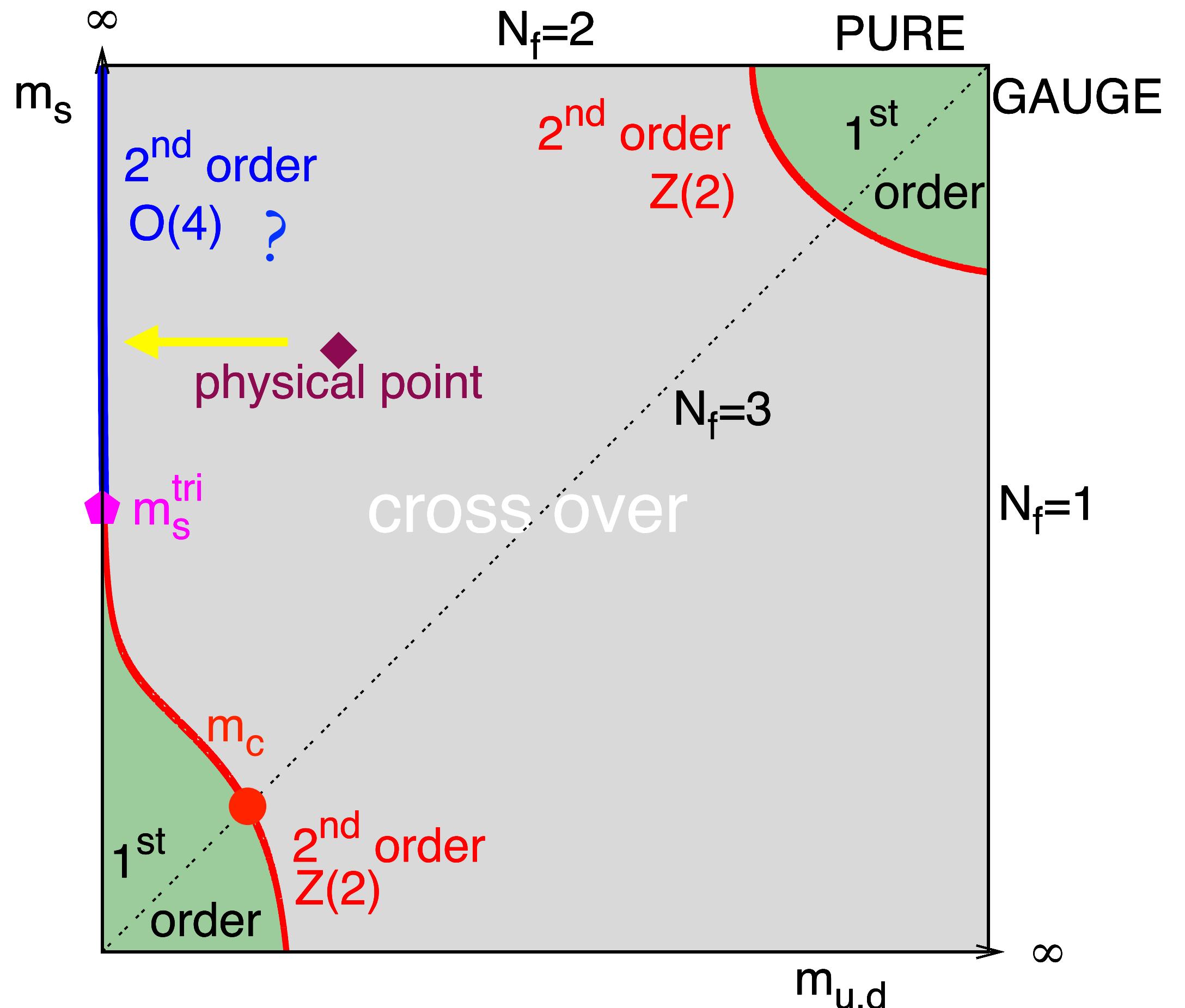
$$\frac{V}{T} \frac{\partial^2 \rho}{\partial m_l^2} = \int_0^\infty d\lambda_2 \frac{4(\lambda_2^2 - m_l^2) C_2}{(\lambda_2^2 + m_l^2)^2} + \int_0^\infty d\lambda_2 d\lambda_3 \frac{(4m_l)^2 C_3}{(\lambda_2^2 + m_l^2)(\lambda_3^2 + m_l^2)}$$

... ...

... ...

$$C_n(\lambda_1, \dots, \lambda_n; m_l) = \left\langle \prod_{i=1}^n [\rho_U(\lambda_i) - \langle \rho_U(\lambda_i) \rangle] \right\rangle$$

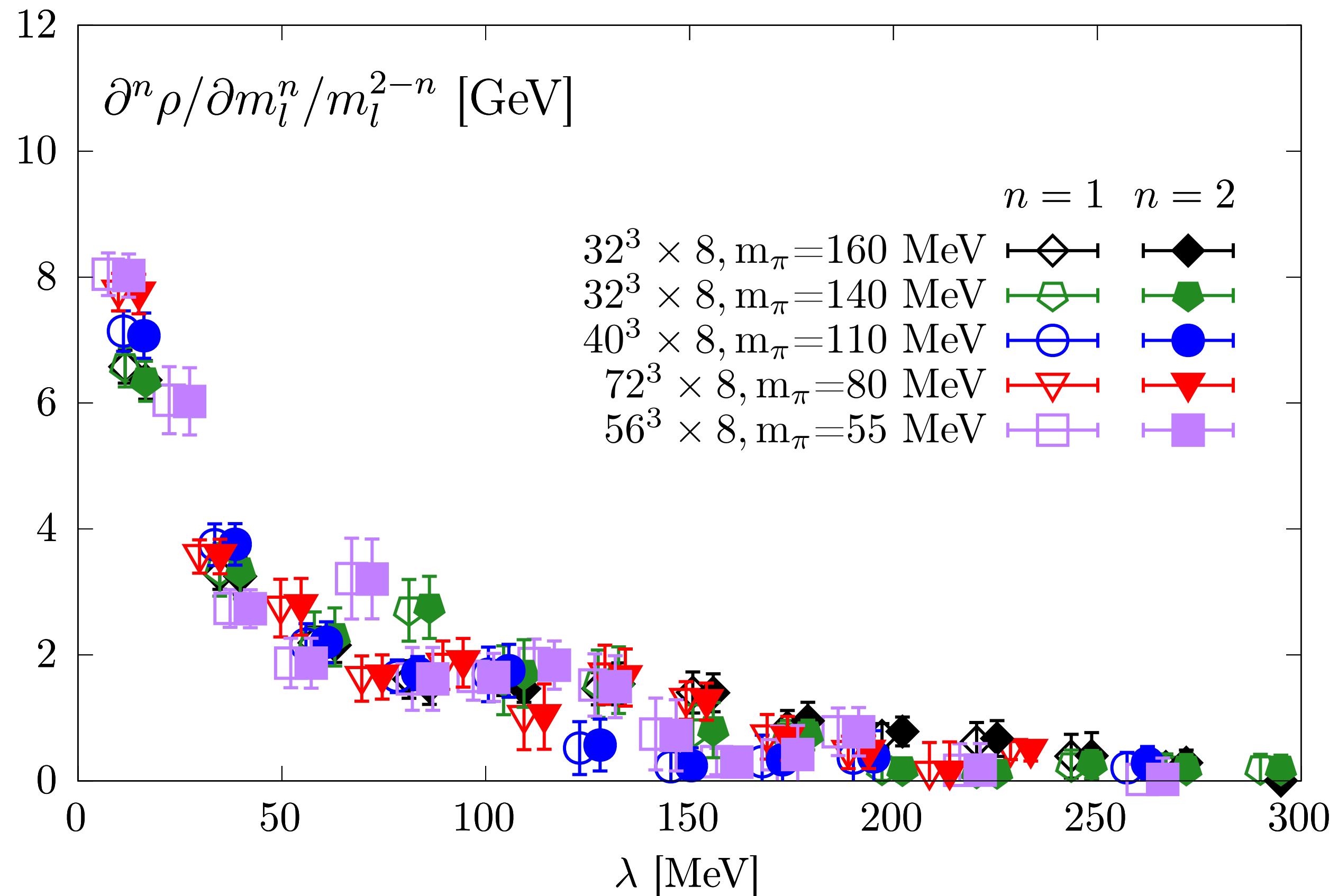
Lattice setup



- At a single $T \sim 205$ MeV
- HISQ/tree action
- $N_f=2+1$:
 - $N_t=8, 12, 16$ ($a=0.12, 0.08, 0.06$ fm)
 - $m_s^{\text{phy}}/\mathbf{m}_I = 20, 27, 40, 80, 160$
 - $m_\pi \approx 160, 140, 110, 80, 55$ MeV
 - $9 \geq N_s/N_t \geq 4$
- Chebyshev filtering technique to compute ρ_u

HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang
 Phys. Rev. Lett. 126 (2021) 082001

1st & 2nd mass derivative of ρ on $N_\tau=8$ lattices



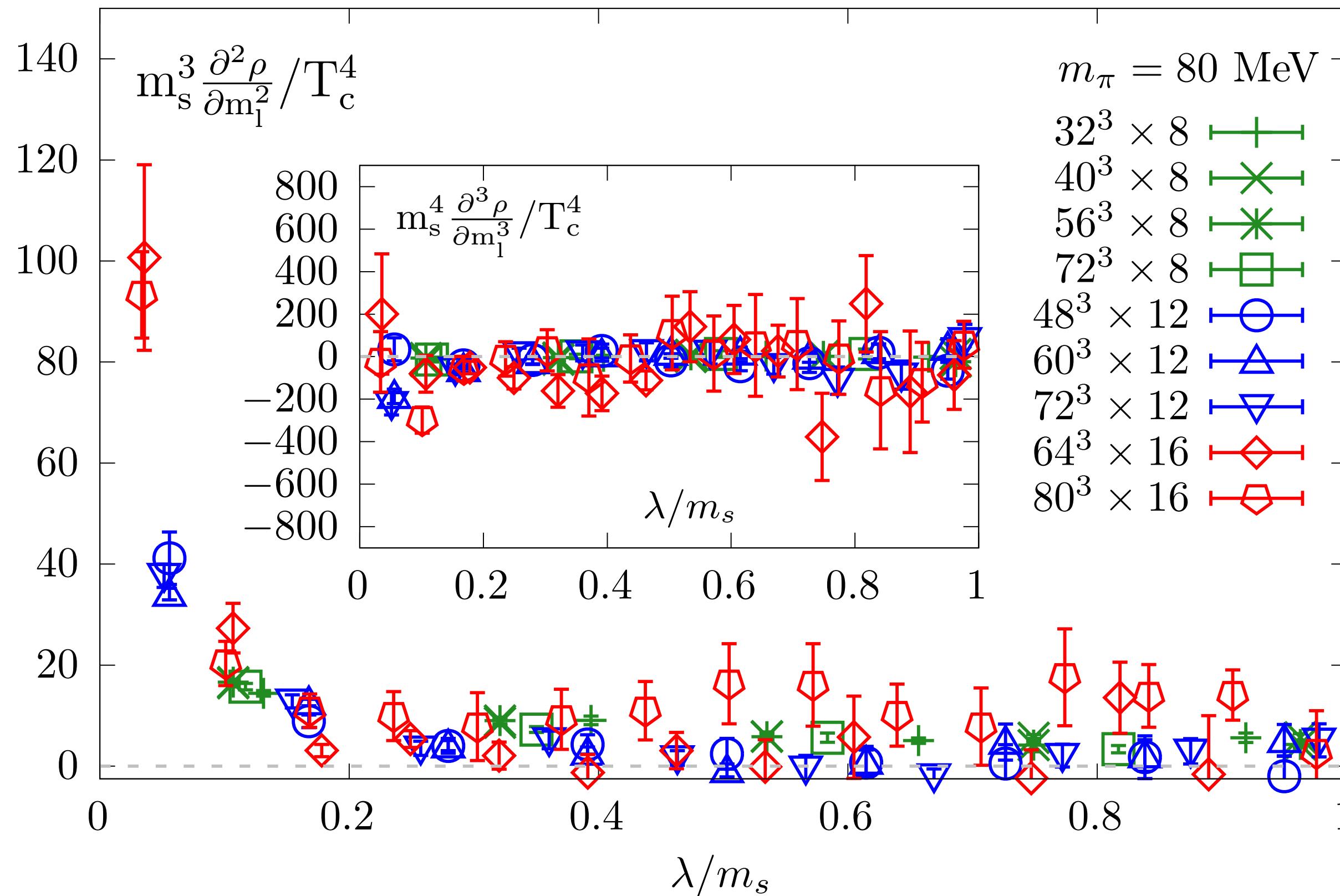
$$m_l^{-1} \partial \rho / \partial m_l \approx \partial^2 \rho / \partial m_l^2$$

Quark mass independent

Peaked structure developed in
the small λ region

Drops rapidly towards zero
for $\lambda/T > 1$

2nd & 3rd mass derivative of ρ : volume and a dependences

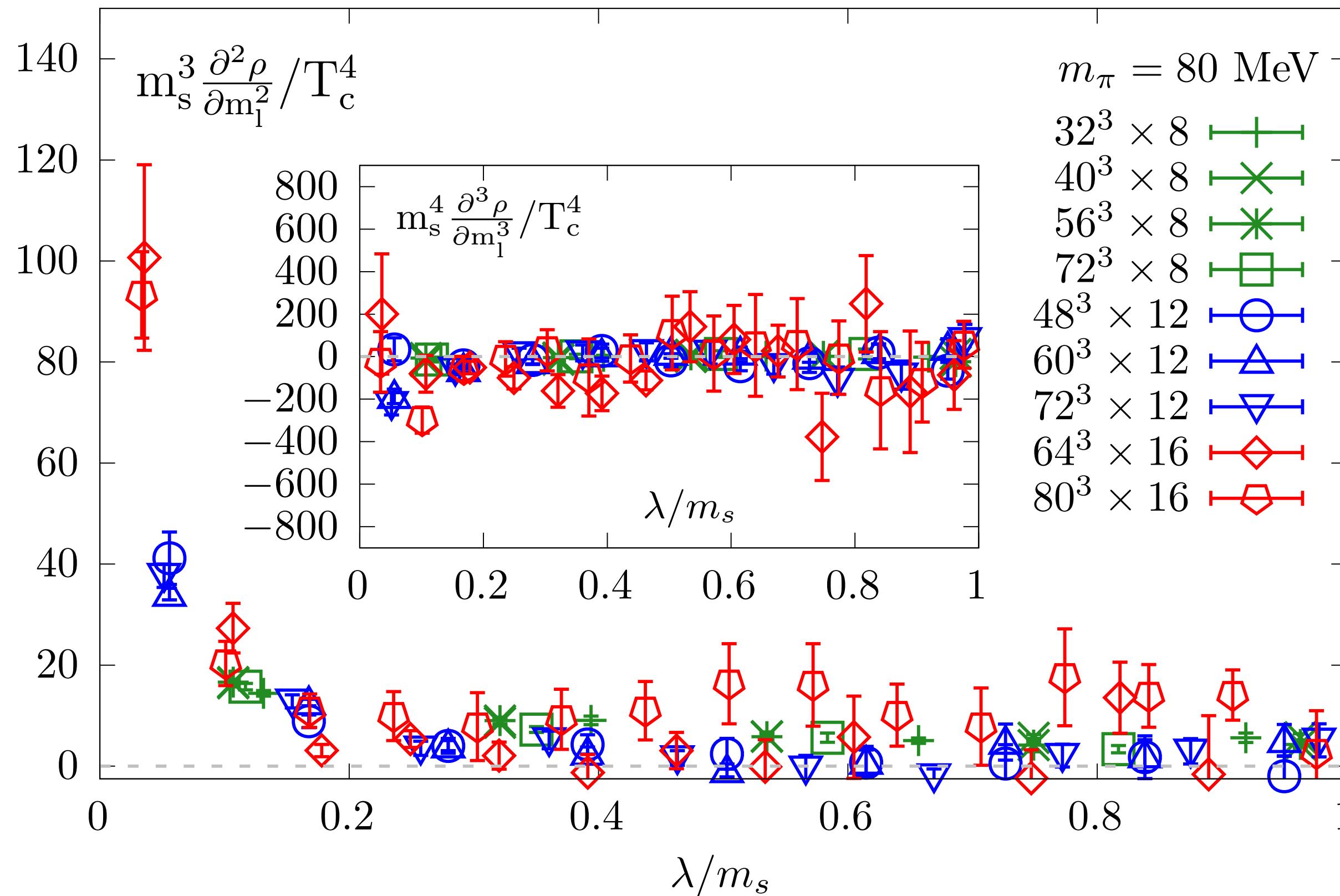


Peaked structure becomes sharper
towards continuum limit

Mild volume dependence

$$\partial^3 \rho / \partial m_l^3 \approx 0$$

2nd & 3rd mass derivative of ρ : volume and a dependences

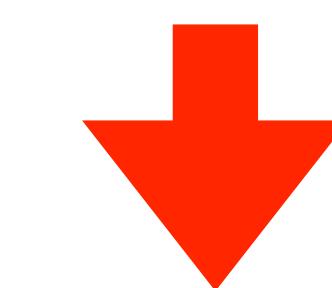


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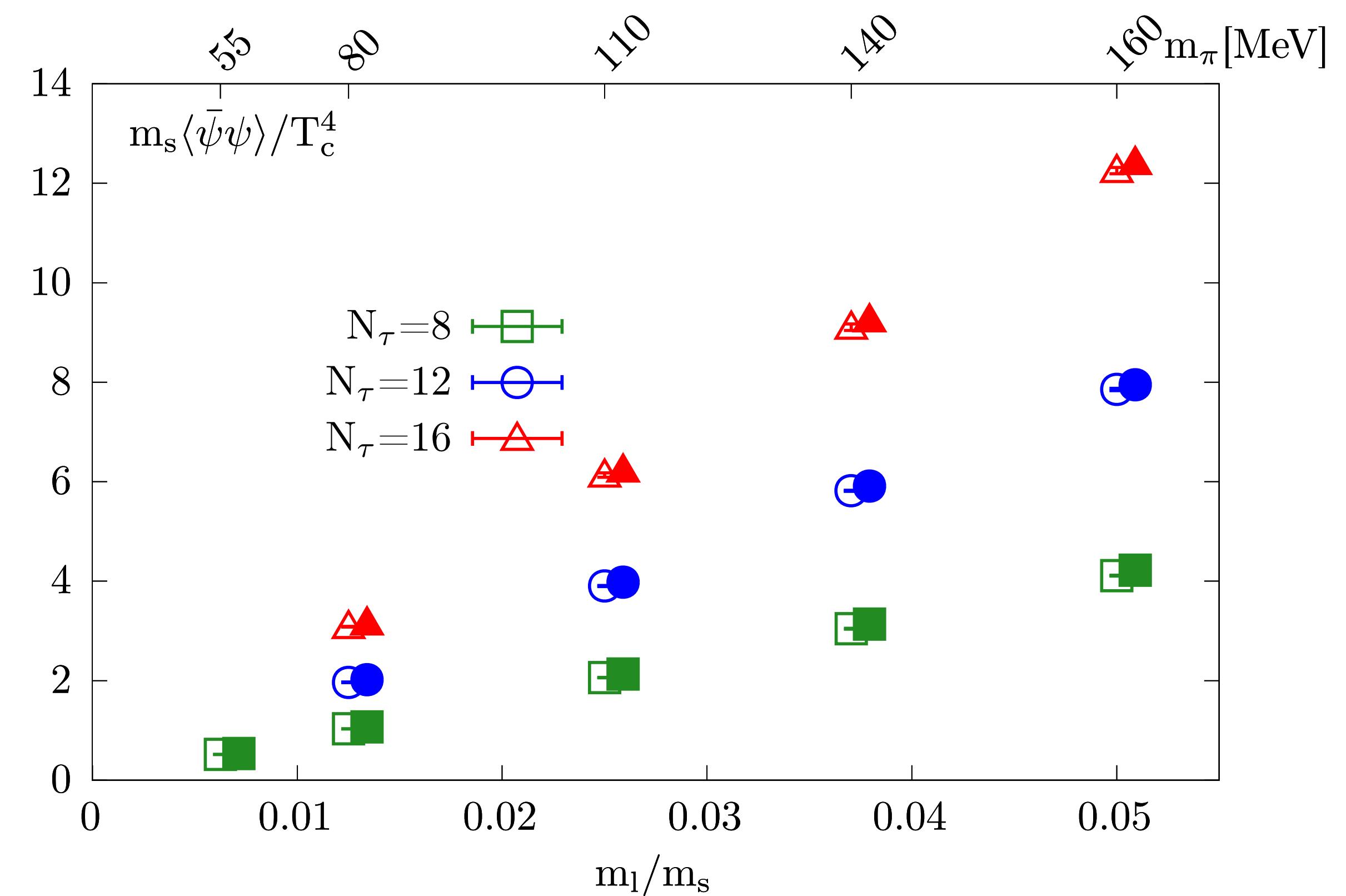
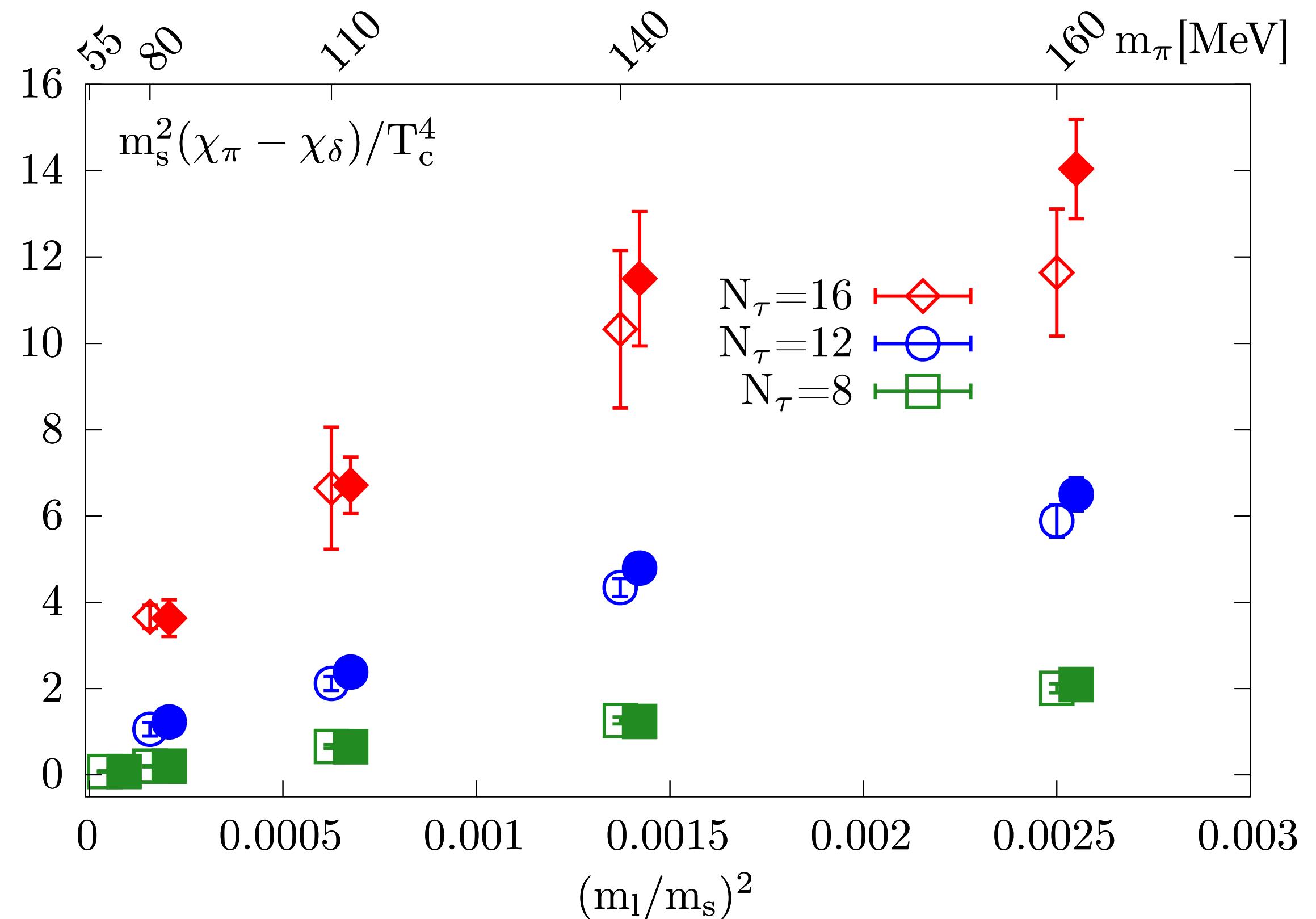
$$m_l^{-1} \partial \rho / \partial m_l \approx \partial^2 \rho / \partial m_l^2$$



Chiral phase transition $T_c = 132$ MeV is used from HTD et al., [HotQCD] PRL 19,

$$\rho(\lambda \rightarrow 0, m_l \rightarrow 0) \propto m_l^2$$

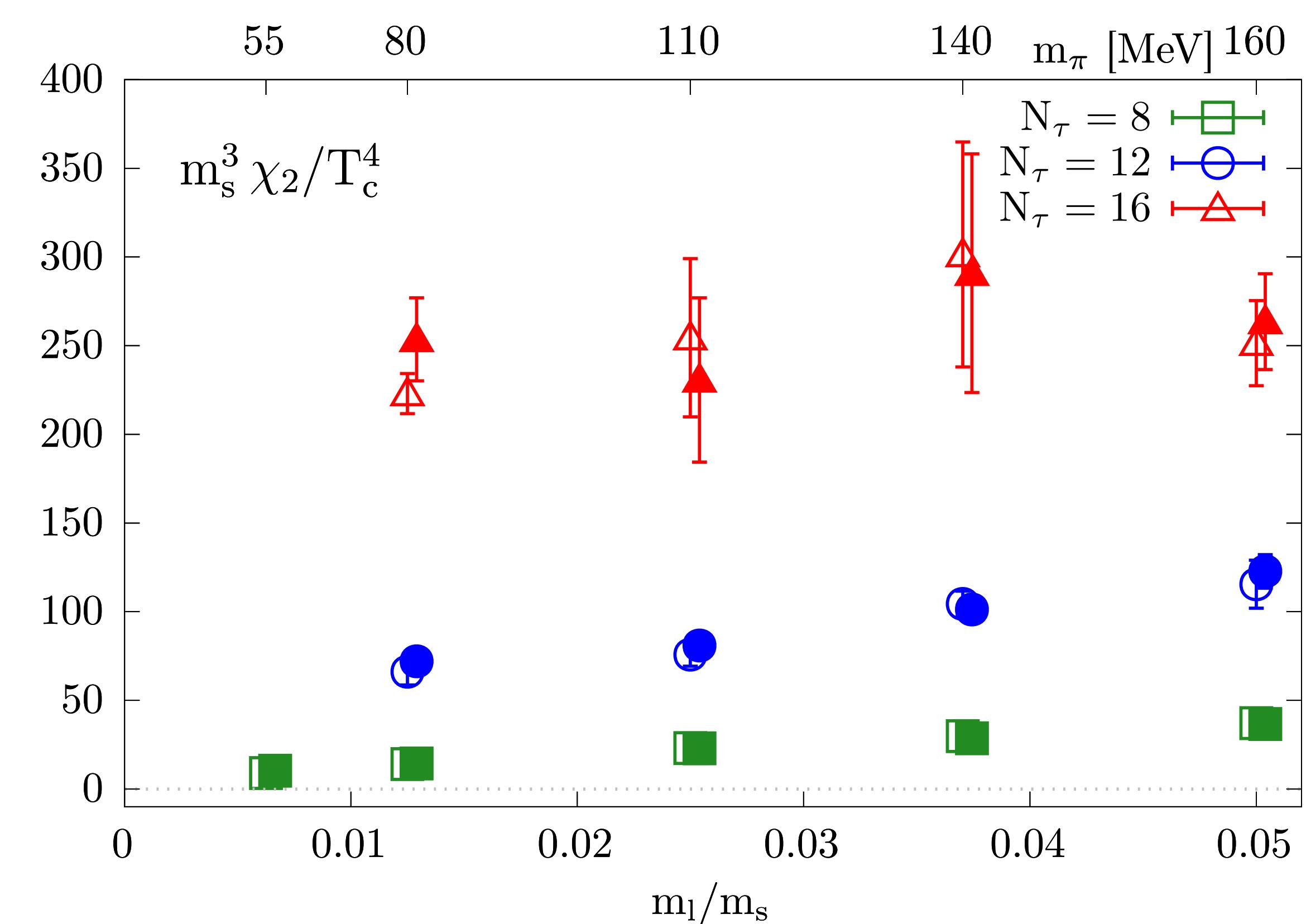
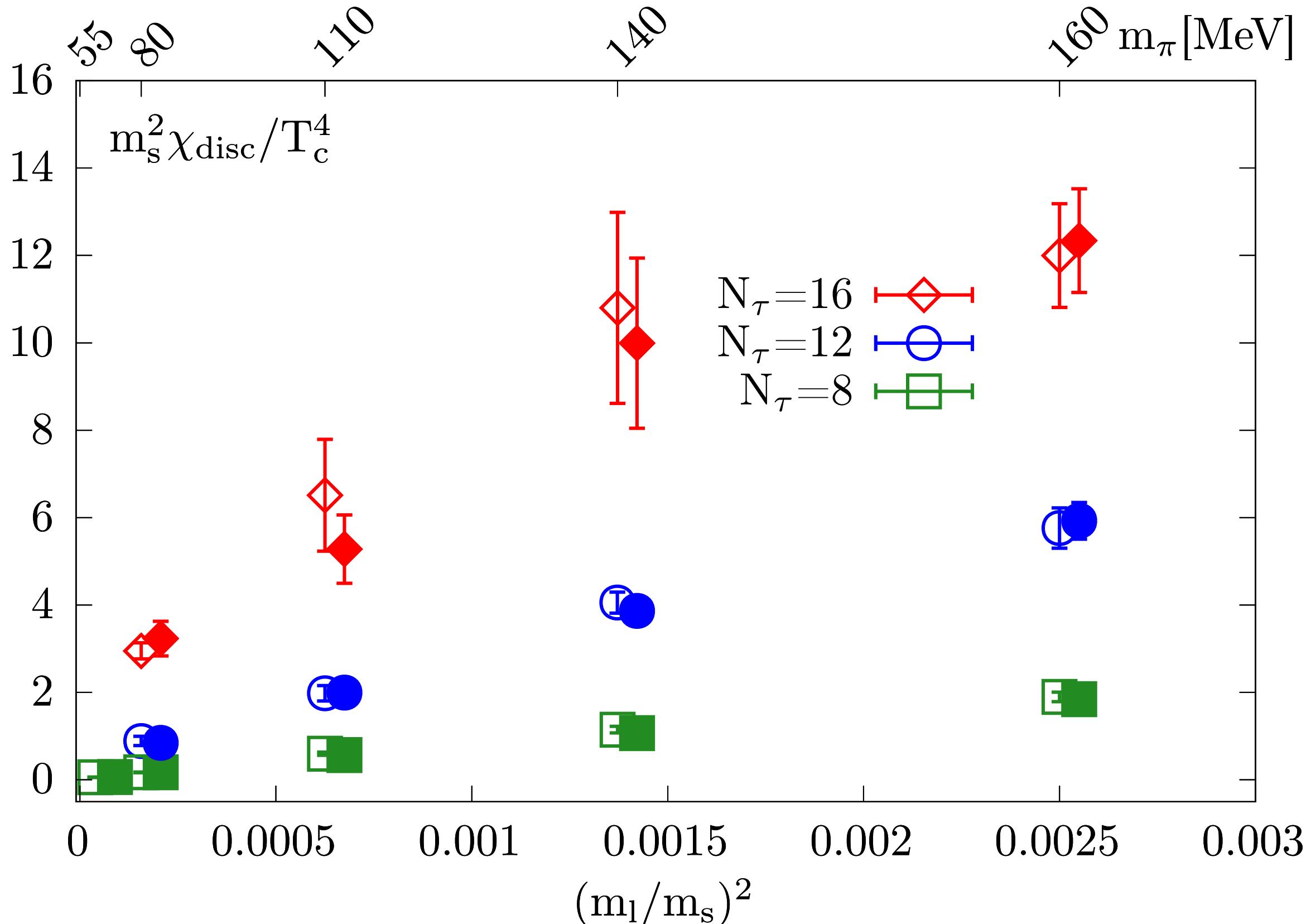
Quantities related to ρ



$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \frac{8m_l^2 \rho}{(\lambda^2 + m_l^2)^2}$$

$$\langle \bar{\psi} \psi \rangle = \int_0^\infty \frac{4m_l \rho}{\lambda^2 + m_l^2} d\lambda$$

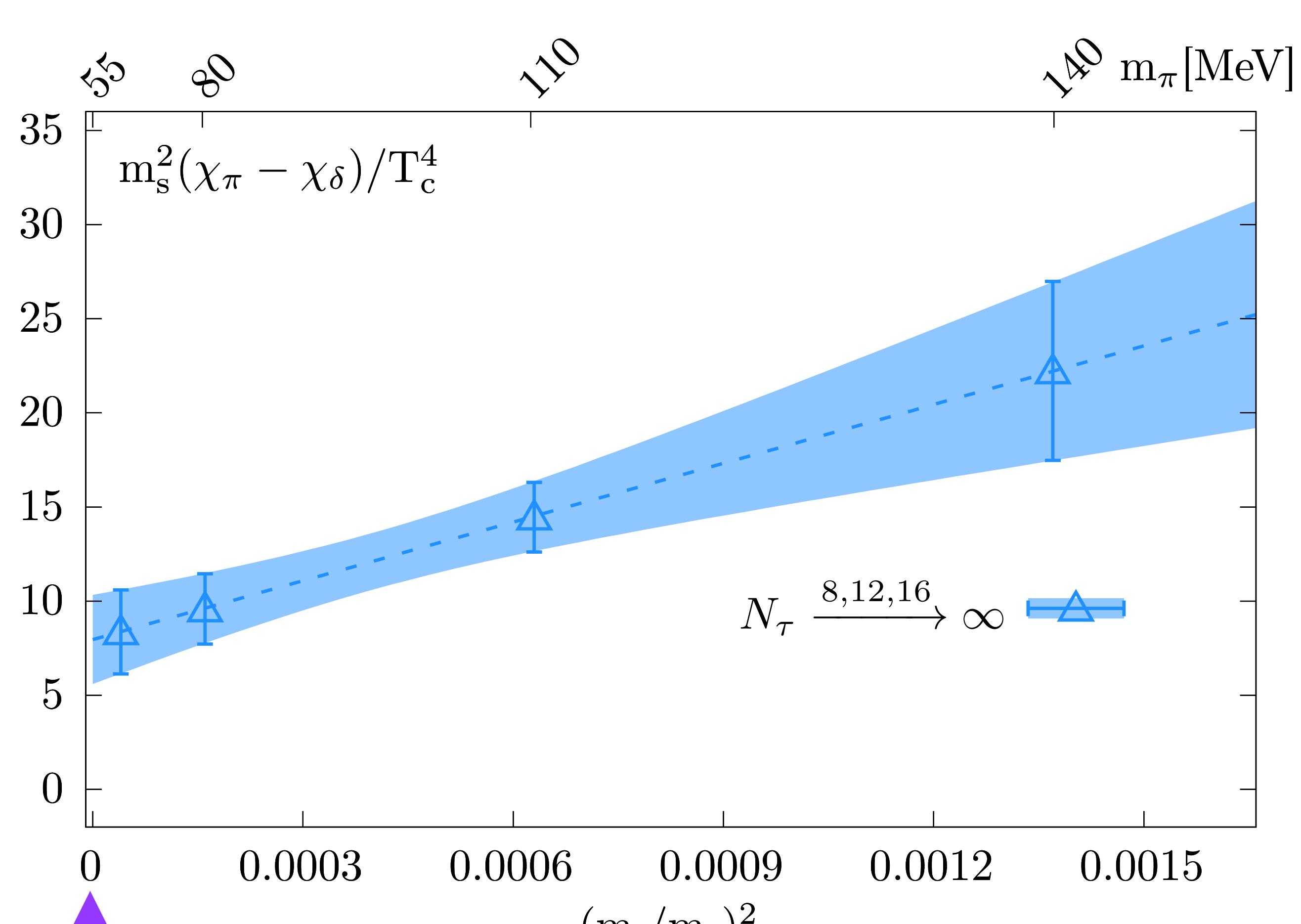
Quantities related to 1st & 2nd derivatives of ρ



$$\chi_{disc} = \int_0^\infty d\lambda \frac{4m_l \partial\rho/\partial m_l}{\lambda^2 + m_l^2}$$

$$\chi_2 = \int_0^\infty d\lambda \frac{4m_l \partial^2\rho/\partial m_l^2}{\lambda^2 + m_l^2}$$

Continuum and chiral extrapolations with $m_\pi \leq 140$ MeV data

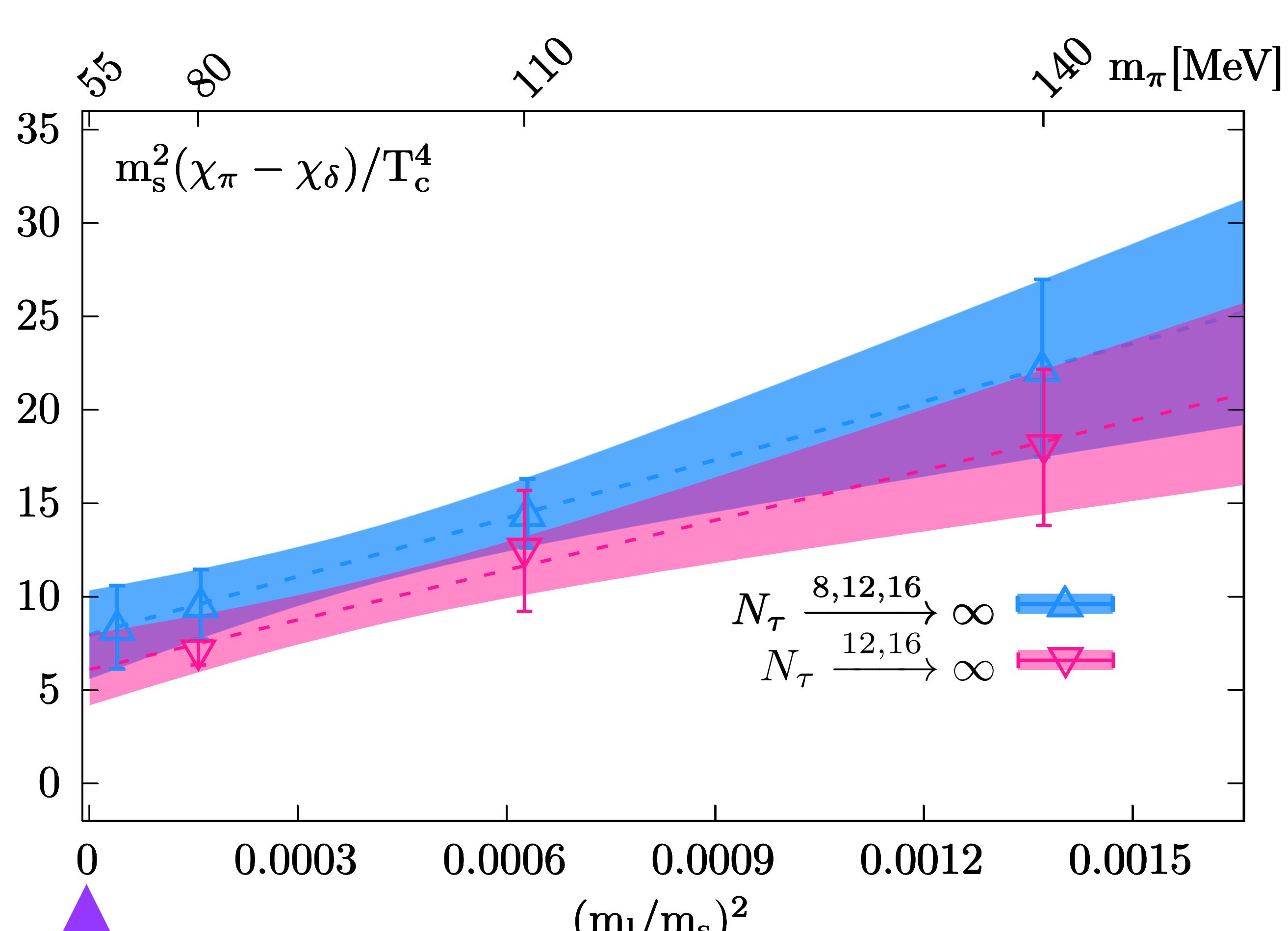


Joint fit: simultaneous fits

Continuum: $c_0 + c_1/N_\tau^2 + c_2/N_\tau^4$
Chiral: quadratic in quark mass

Value at $N_\tau \rightarrow \infty$ and $m \rightarrow 0$:
 8.0 ± 2.4

Continuum and chiral extrapolations with $m_\pi \leq 140$ MeV data



↑
chiral limit

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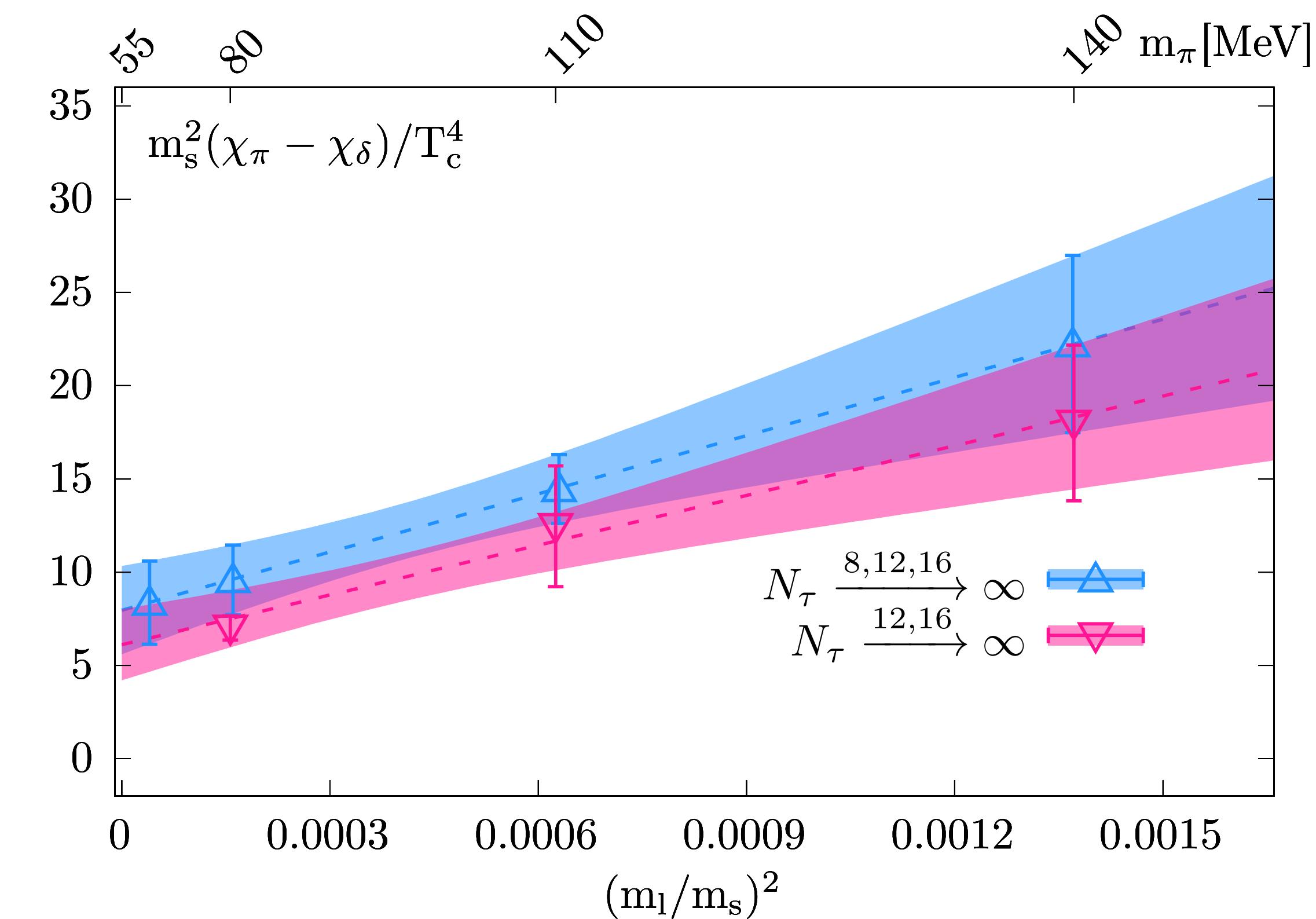
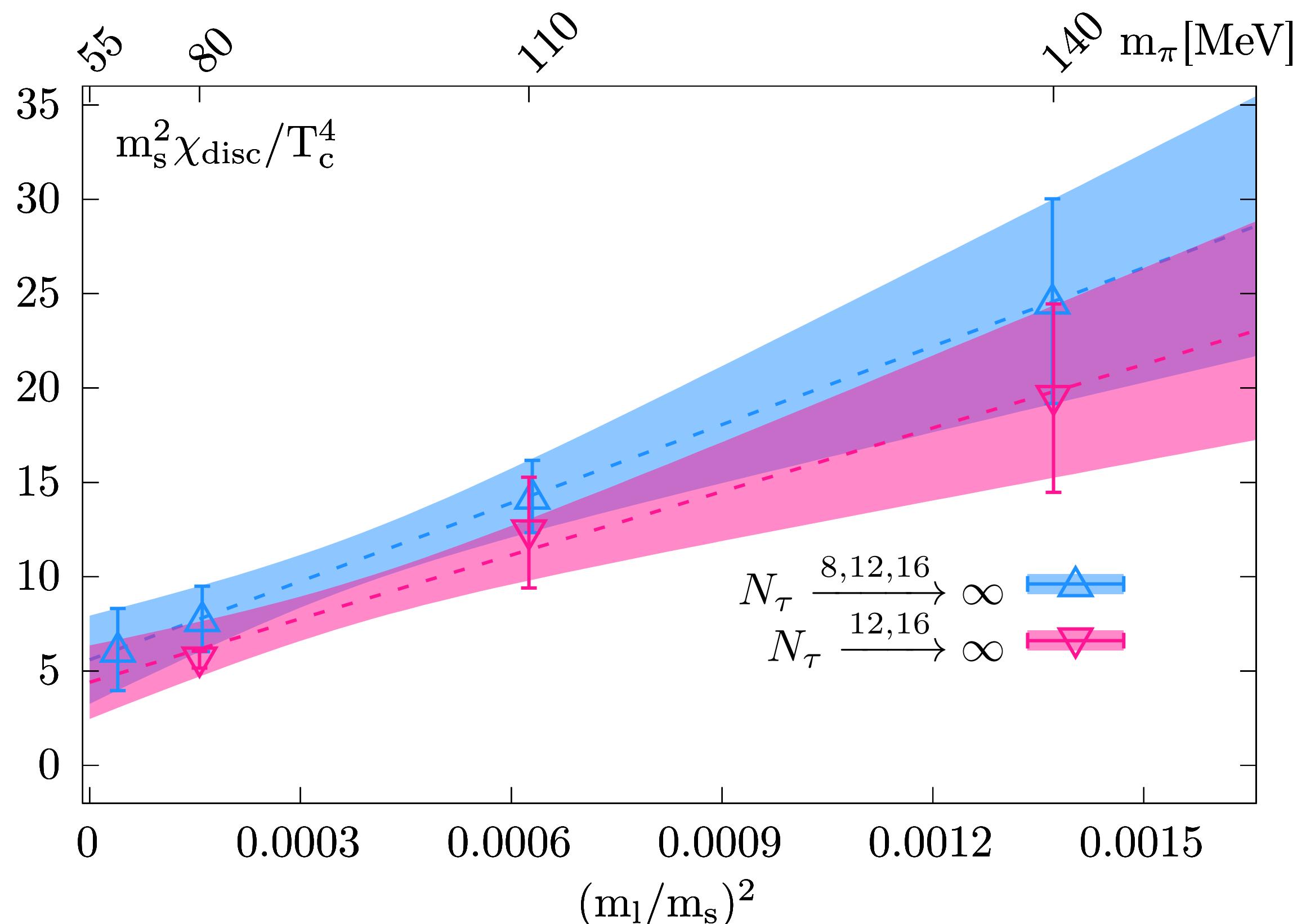
Sequential fit: first continuum and then chiral extrapol.

Continuum: quadratic in $1/N_\tau$ with $N_\tau=12$ & 16 data

Chiral: quadratic in quark mass

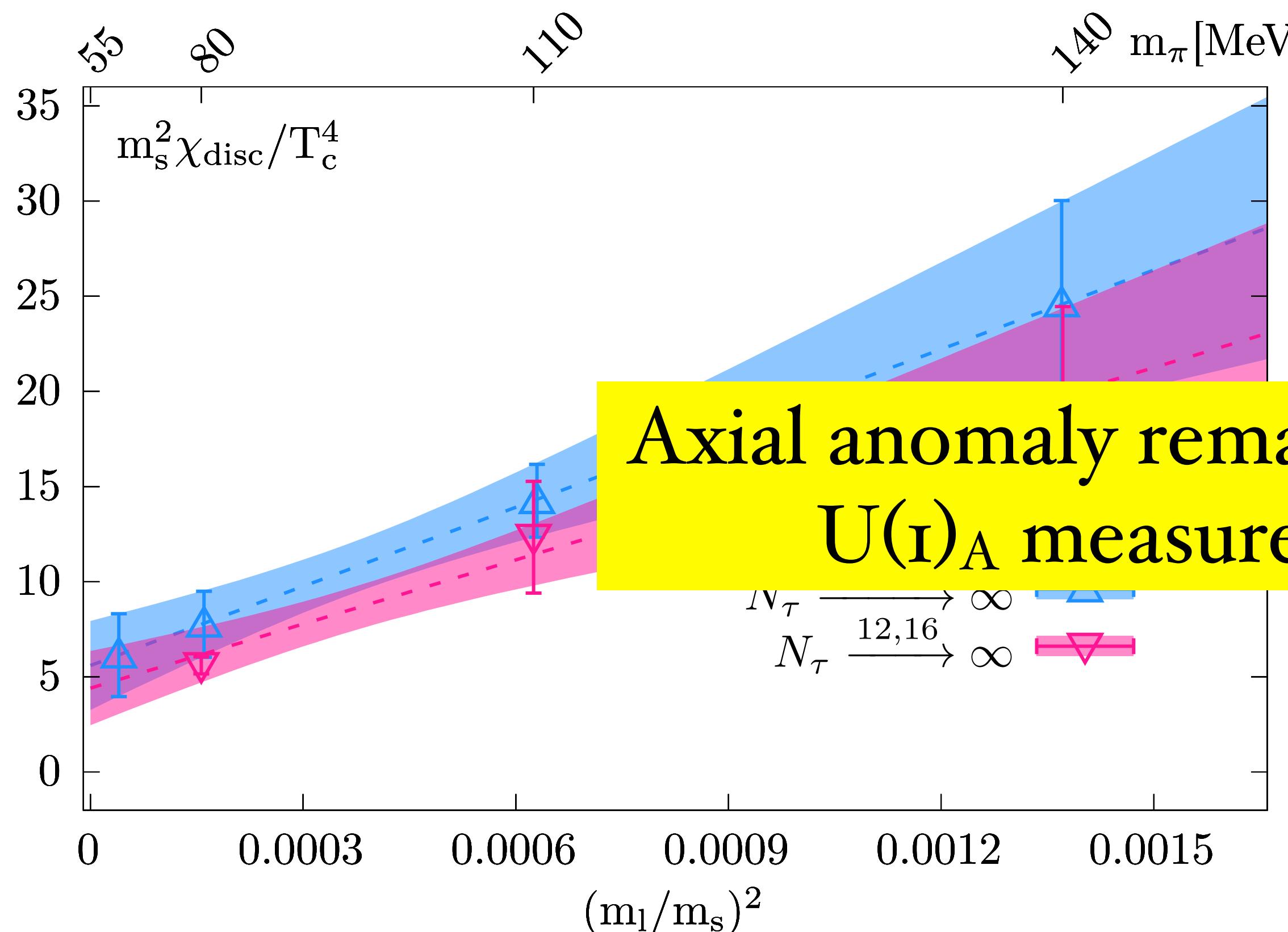
Value at $N_\tau \rightarrow \infty$ and $m \rightarrow 0$:
 6.1 ± 1.9

Continuum and chiral extrapolations with $m_\pi \leq 140$ MeV data

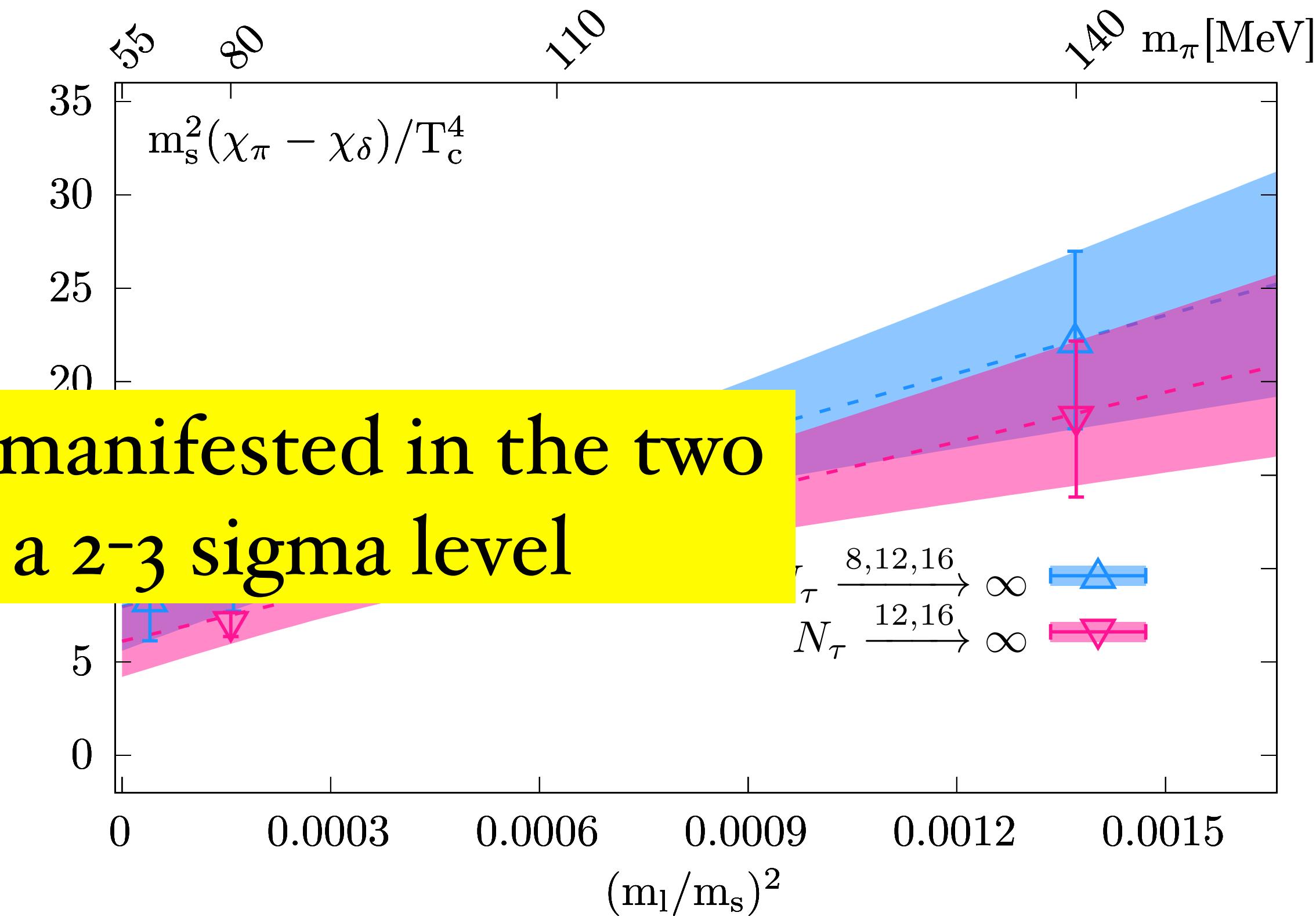


$N_\tau \rightarrow \infty$ and $m \rightarrow 0$	$m_s^2 \chi_{disc} / T_c^4$	$m_s^2 (\chi_\pi - \chi_\delta) / T_c^4$
Joint fit	5.6 ± 2.3	8.0 ± 2.4
Sequential fit	4.4 ± 1.9	6.1 ± 1.9

Continuum and chiral extrapolations with $m_\pi \leq 140$ MeV data



Axial anomaly remains manifested in the two $U(1)_A$ measures at a 2-3 sigma level

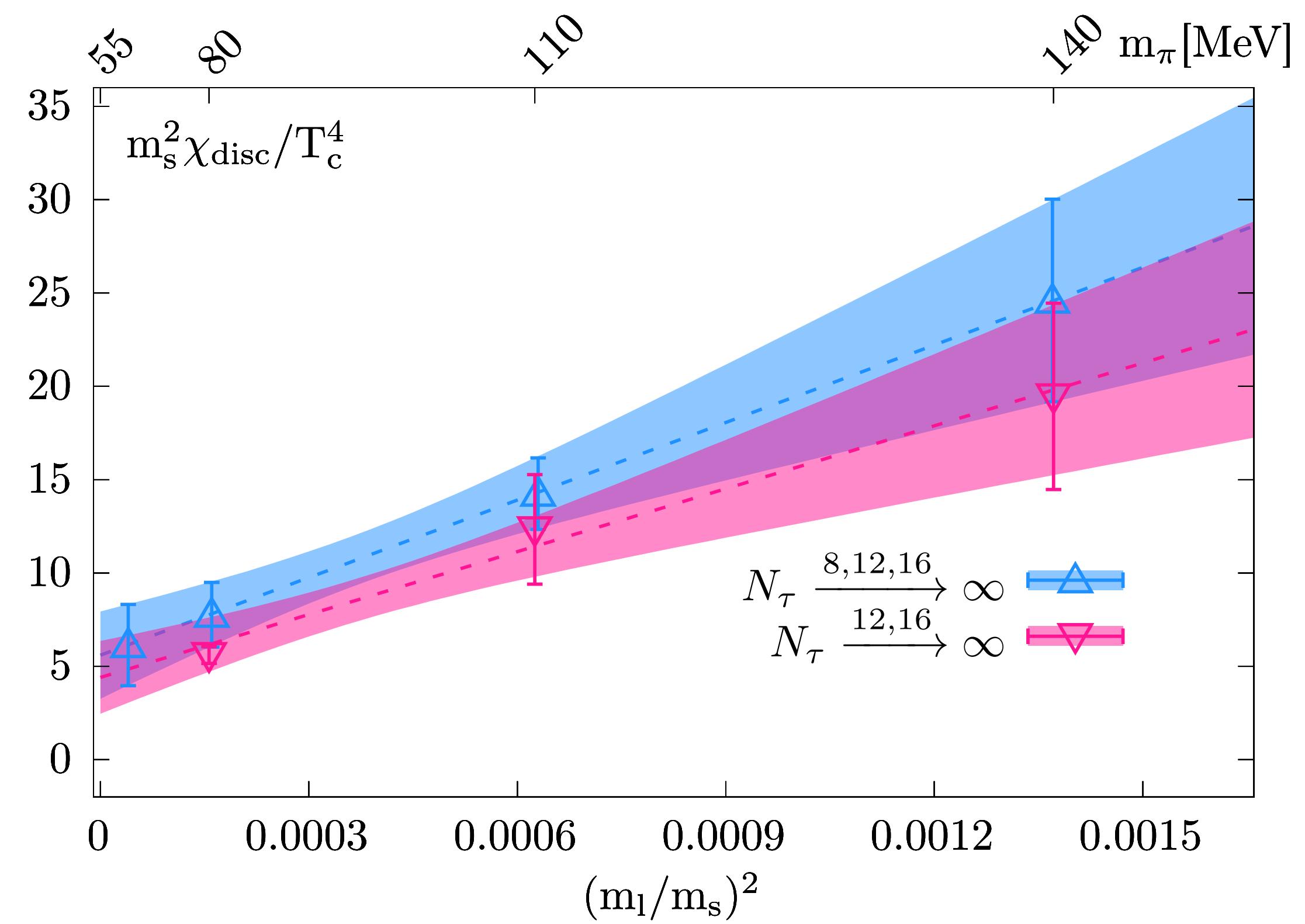
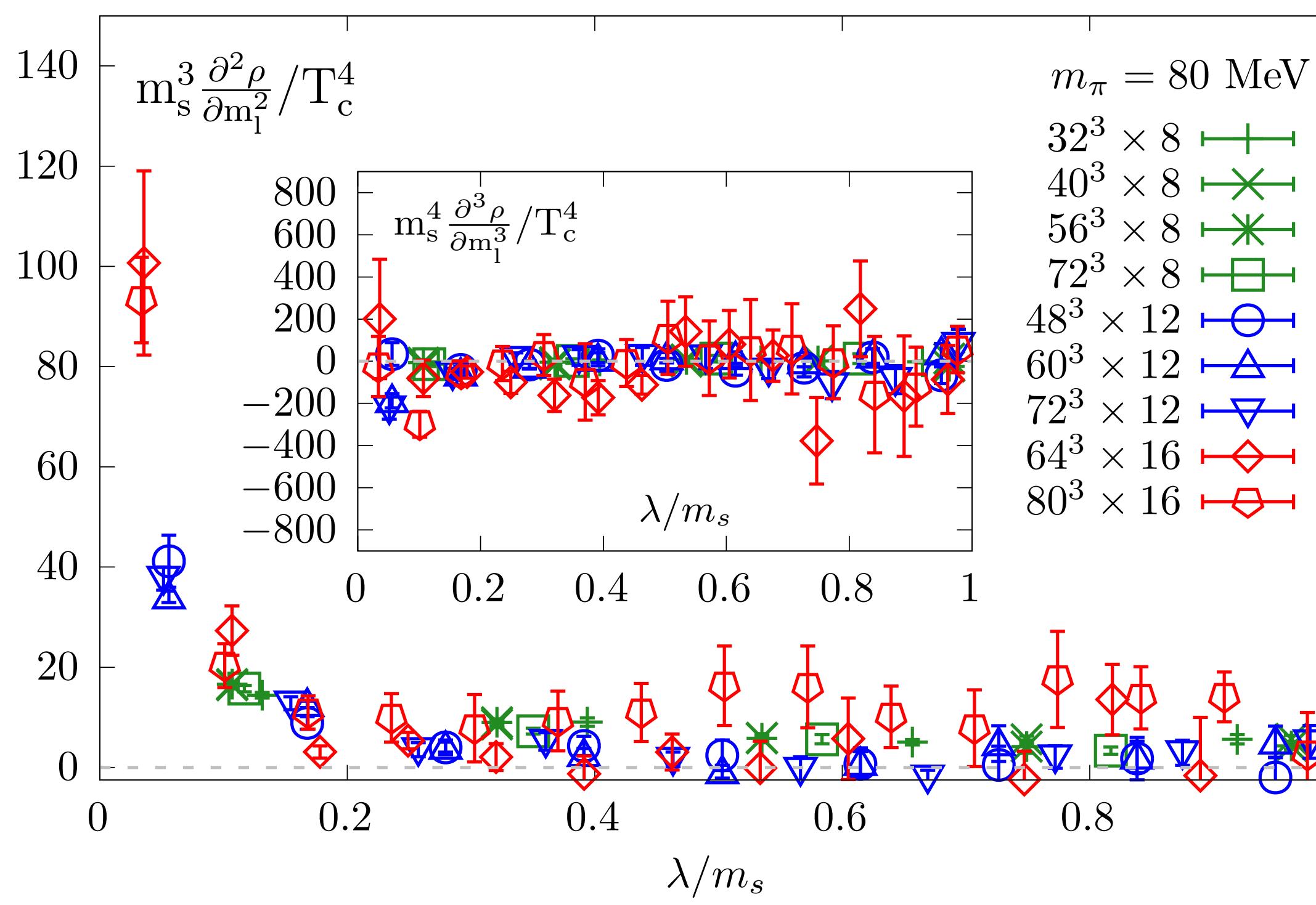


$N_\tau \rightarrow \infty$ and $m \rightarrow 0$	$m_s^2 \chi_{disc}/T_c^4$	$m_s^2 (\chi_\pi - \chi_\delta)/T_c^4$
Joint fit	5.6 ± 2.3	8.0 ± 2.4
Sequential fit	4.4 ± 1.9	6.1 ± 1.9

Summary & Conclusion

- We established novel relations between $\partial^n Q / \partial m^n$ & C_{n+1}

In (2+1)-flavor QCD at $T \approx 1.6 T_c$



Summary & Conclusion

Our study suggests:

- ▶ At $T \gtrsim 1.6 T_c$ the microscopic origin of axial anomaly is driven by the weakly interacting (quasi-) instanton gas motivated $\varrho(\lambda \rightarrow 0, m \rightarrow 0) \propto m^2 \delta(\lambda)$
- ▶ $N_f=2+1$ QCD: 2nd order chiral phase transition belonging to 3-d $O(4)$

Outlook:

- the methodology would be useful for other discretization schemes

Thanks for your attention!