

## Momentum dependence of light nuclei production in $pp$ , $p$ -Pb, and Pb-Pb collisions at the LHC

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**References:** PHYSICAL REVIEW C **103**, 064908 (2021),  
Chinese Physics C **43**, 024101 (2019)].

**Collaborators:** Feng-Lan Shao (邵凤兰) & Jun Song (宋军)

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# I. Introduction

## Why light nuclei ?

Probes to explore important issues in relativistic heavy ion collisions

QGP property, QCD phase diagram, system freeze-out information.....

Production mechanism of such composite particles itself is interesting.

directly thermal production or nucleon coalescence production

Methods: Experiments, Phenomenological models

sensitive observables

- yield ratios, e.g.,  $d / p$ ,  ${}^3\text{He} / p$ ,  $pt / d^2$
- the coalescence factor  $B_A$
- correlations and fluctuations
- flows .....

## Statistical models

- A. Mekjian, *Phys. Rev. Lett.* **38**, 640 (1977);  
P. J. Siemens & J. I. Kapusta, *Phys. Rev. Lett.* **43**, 1486 (1979);  
A. Andronic & P. Braun-Munzinger *et al.*, *Phys. Let. B* **697**, 203 (2011),  
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J. Cleymans *et al.*, *Phys. Rev. C* **84**, 054916 (2011);  
Y. Cai *et al.*, *Phys. Rev. C* **100**, 024911 (2019);  
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## Coalescence models

- S. T. Butler & C. A. Pearson, *Phys. Rev.* **129**, 836 (1963);  
A. Schwarzschild & C. Zupancic, *Phys. Rev.* **129**, 854 (1963);  
H. Sato & K. Yazaki, *Phys. Lett. B* **98**, 153 (1981);  
U. Heinz *et al.*, *Phys. Rev. C* **44**, 1636 (1991), *Phys. Rev. C* **59**, 1585 (1999);  
C.M.Ko, L.W.Chen, H.C. Song, W.B. Zhao, & K.J.Sun *et al.*, *Phys. Rev. C* **68**, 017601  
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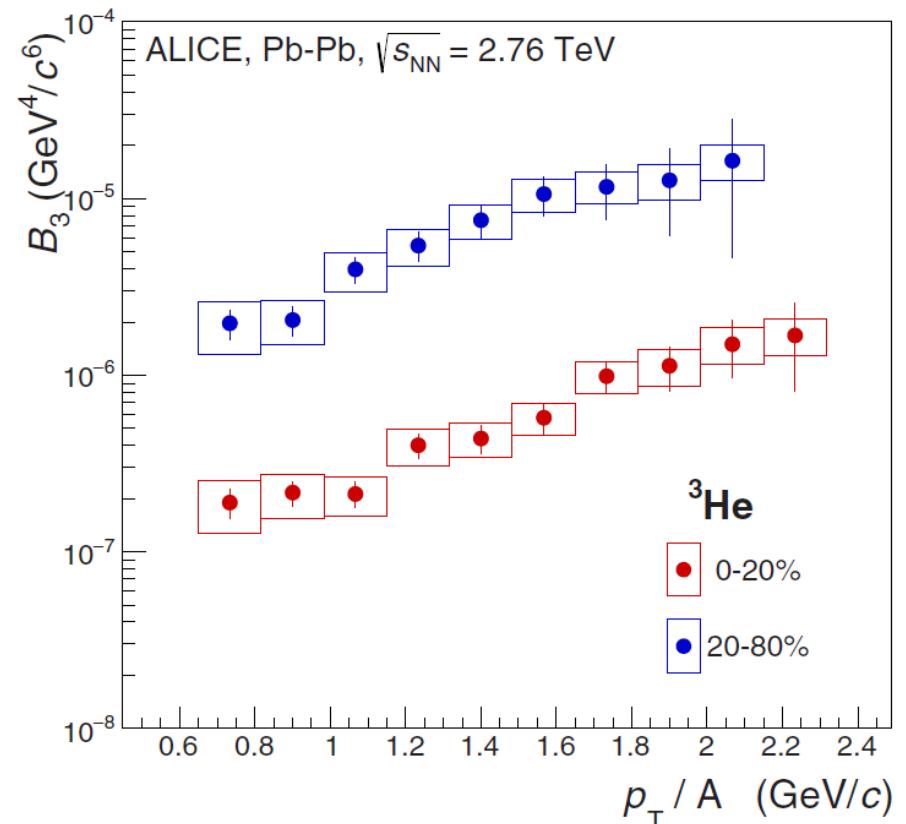
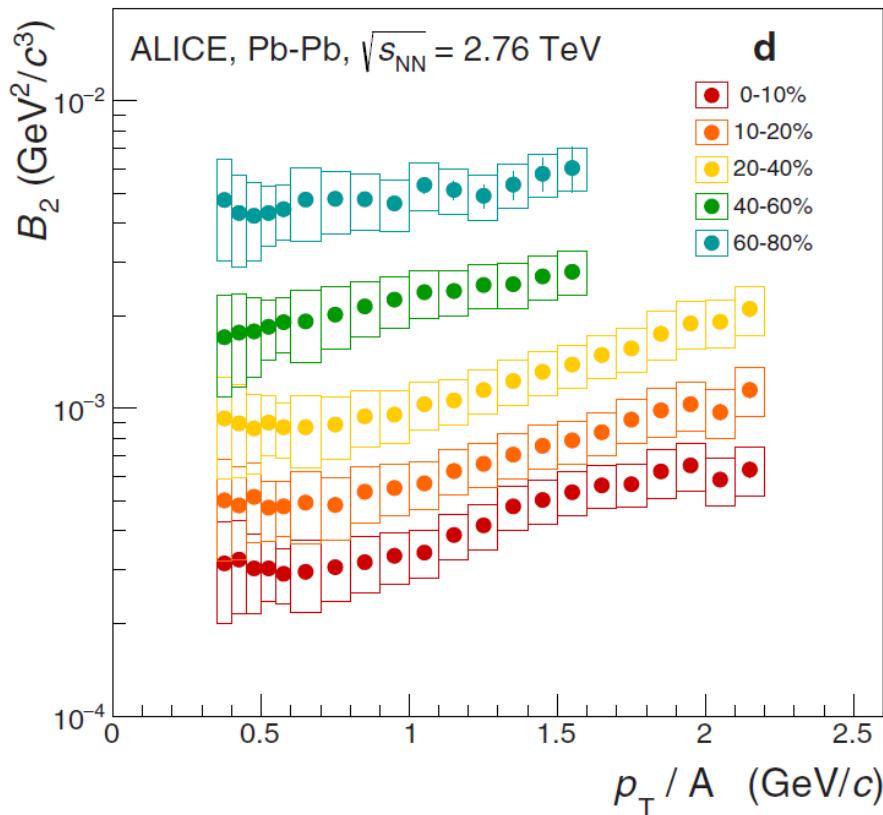
## Transport models

P. Danielewicz & G. F. Bertsch, *Nucl. Phys. A* **533**, 712 (1991);  
S. Sombun, *et al.*, *Phys. Rev. C* **99**, 014901 (2019);  
Y. Oh, Z.-W. Lin, & C. M. Ko, *Phys. Rev. C* **80**, 064902 (2009);  
D. Oliinychenko & L.G.Pang, *et al.*, *Phys. Rev. C* **99**, 044907 (2019), *MDPI Proc.* **10**, 6 (2019);  
.....

**Different phenomenological models have been successfully used to describe different production characteristics of light nuclei.**

# Experiment observe fascinating features of $B_A$

$$E_A \frac{d^3 N_A}{dp_A^3} = B_A (E_p \frac{d^3 N_p}{dp_p^3})^Z (E_n \frac{d^3 N_n}{dp_n^3})^{A-Z}$$

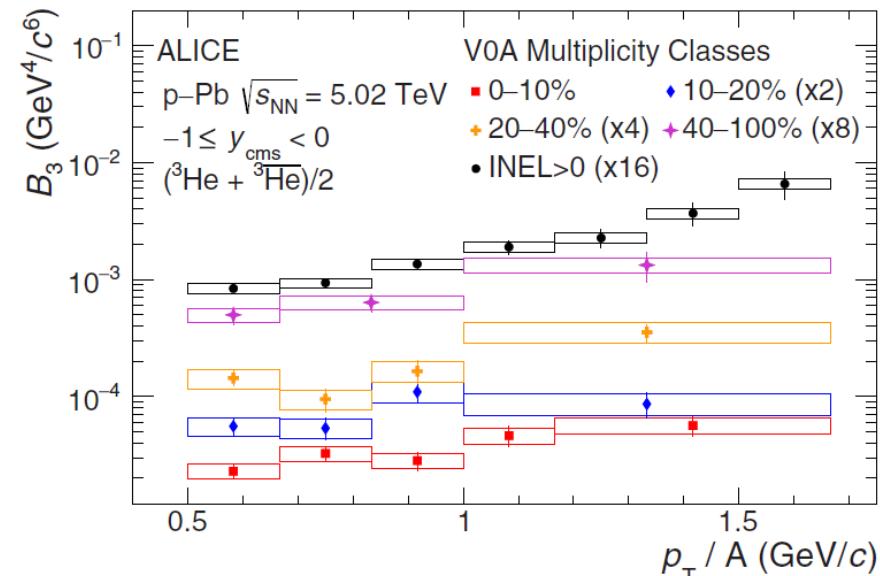
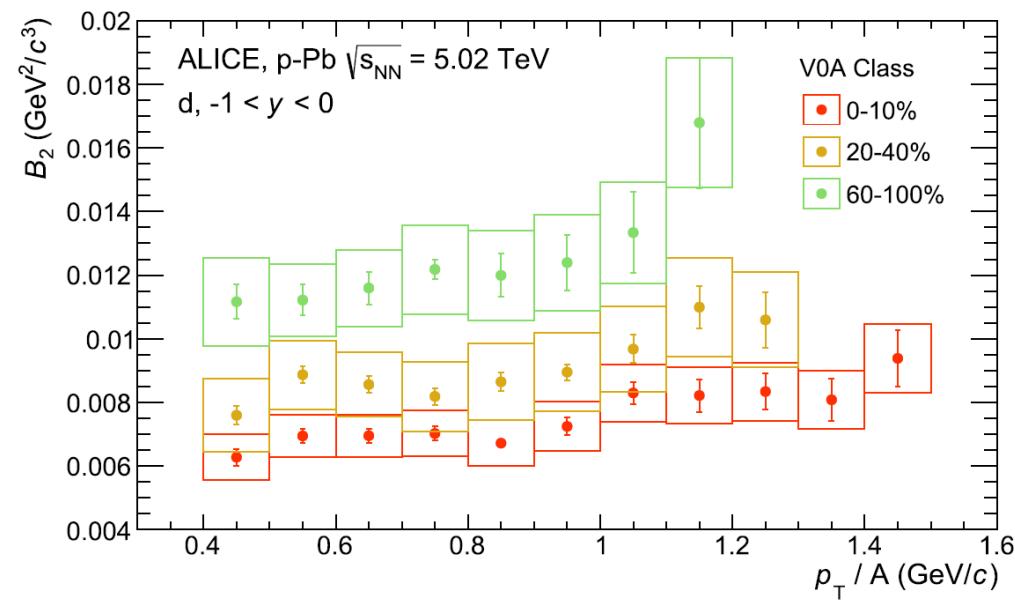


PbPb 2.76 TeV

ALICE, *Phys.Rev.C93, 024917, 2016*

***p*-Pb 5.02 TeV**

**less  $p_T$  dependent, compared to Pb-Pb**

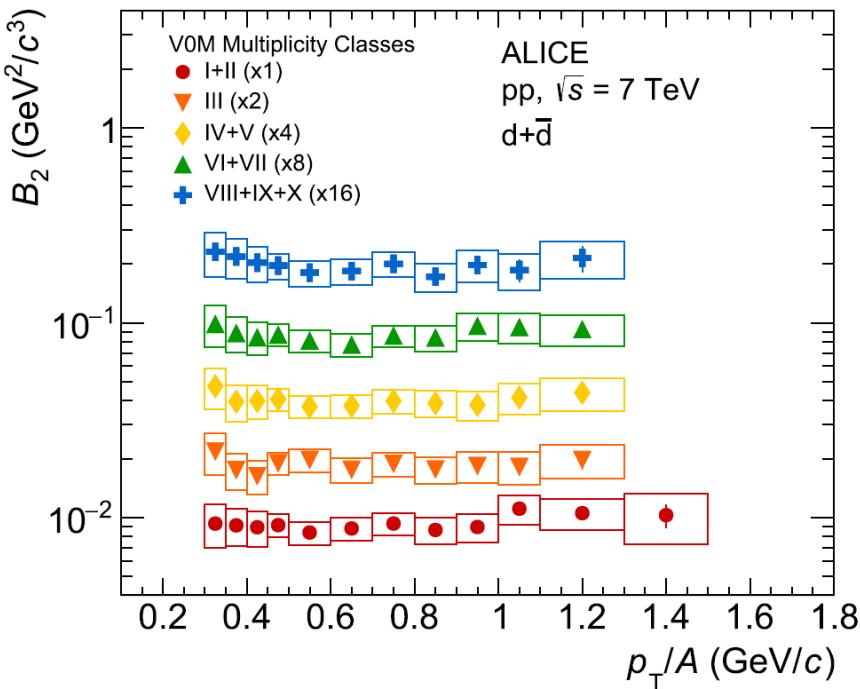


ALICE, *Phys.Lett.B800, 135043, 2020*

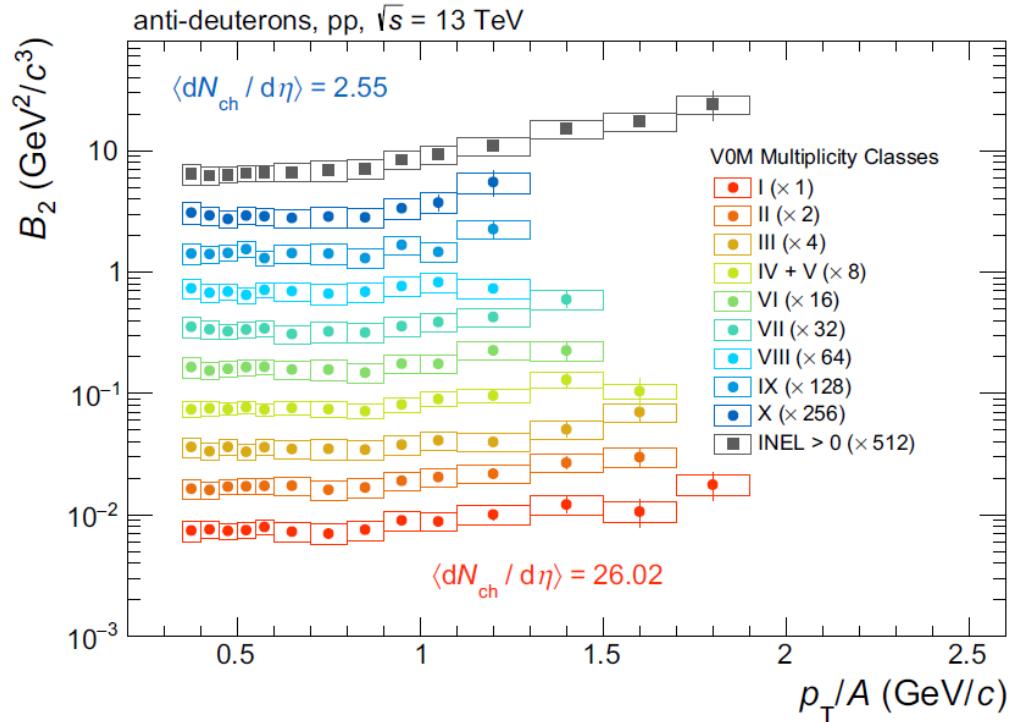
ALICE, *Phys.Rev.C101, 044906, 2020*

*pp 7 & 13 TeV*

**nearly constant with increasing  $p_T$**

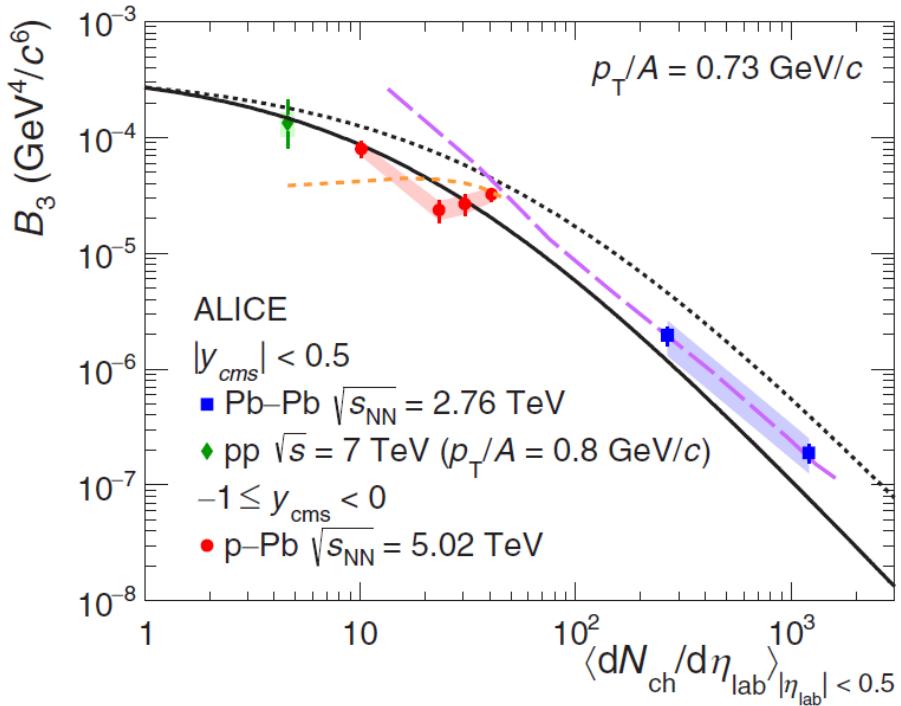
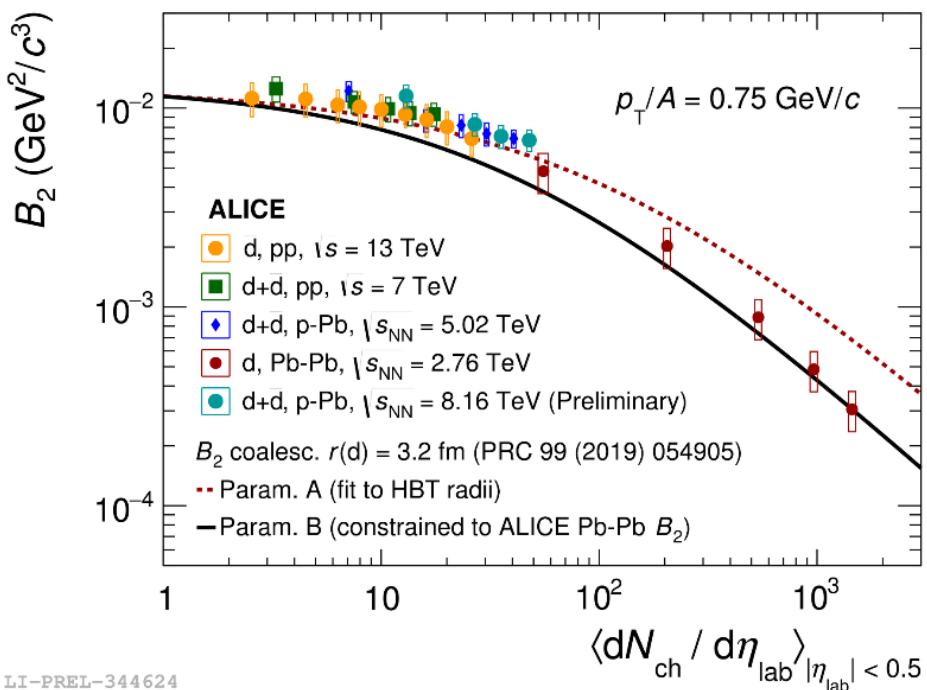


ALICE, *Phys.Lett. B 794, 50, 2019*



ALICE, *Eur. Phys. J. C 80, 889, 2020*

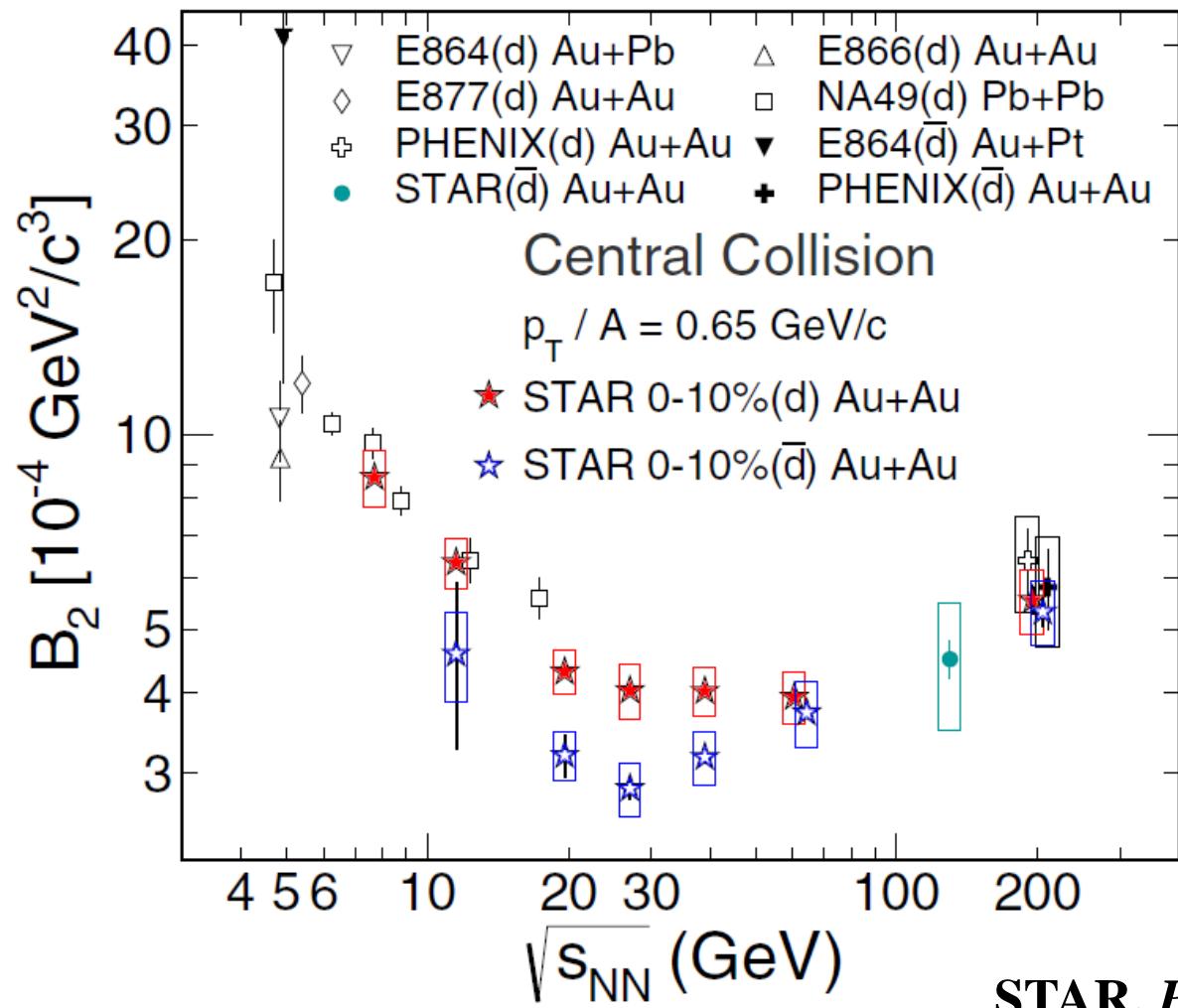
# significant collision system size dependent behaviors of $B_2$ and $B_3$



**ALICE, Phys.Rev.C101, 044906, 2020**

2010.02632, Chiara Pinto  
on behalf of the ALICE

## nontrivial collision energy dependent behavior of $B_2$



STAR, *Phys.Rev.C99, 064905, 2019*

**How to understand such nontrivial behaviors of  $B_A$  depending on collision energy, system size, and the momentum?**

### **Universal Nucleon Coalescence Mechanism**

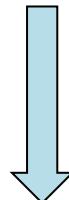
**light nuclei size**

**hadronic system size**

**instantaneous coalescence in the nucleon rest frame**

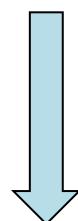
# our method

**Basic ideas of the nucleon coalescence mechanism**



**Coordinate and momentum factorization**

**Momentum-dependent production formulas of  
light nuclei (analytical expressions of  $B_A$ )**

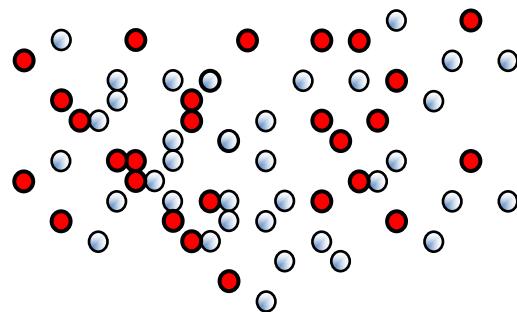


**Comparison to data in high energy collisions at RHIC & LHC**

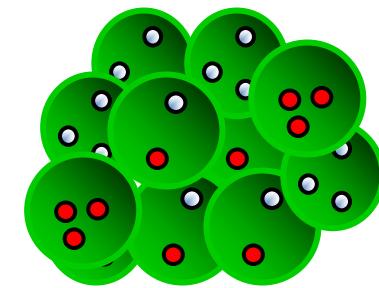
**Obtain: production characteristics, freeze-out properties**

## II. Light nuclei production in Coalescence Mechanism

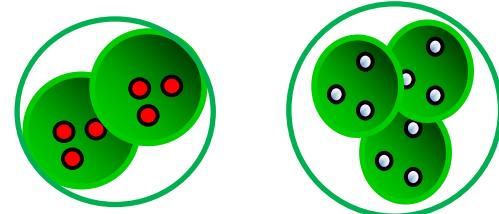
quark-antiquark system



hadronic system



combination



coalescence

light nuclei such as deuterons,  
tritons, helions .....

## General formalism

$$f_d(\vec{p}) = \int d\vec{x}_1 d\vec{x}_2 d\vec{p}_1 d\vec{p}_2 \textcolor{blue}{N}_{pn} f_{pn}^{(n)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) \textcolor{magenta}{R}_d(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2, \vec{p})$$

$N_{pn} = N_p N_n$  number of all the possible  $pn$ -pairs

$f_{pn}^{(n)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2)$  normalized two-nucleon joint  
coordinate-momentum distribution

kernel function

$$R_d(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2, \vec{p}) = g_d \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}) R_d^{(x,p)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2)$$

spin degeneracy factor

momentum conservation

the coordinate and  
momentum dependent part

## With coordinate and momentum factorization assumption

$$f_{pn}^{(n)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) = f_{pn}^{(n)}(\vec{x}_1, \vec{x}_2) f_{pn}^{(n)}(\vec{p}_1, \vec{p}_2)$$

$$R_d^{(x,p)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) = R_d^{(x)}(\vec{x}_1, \vec{x}_2) R_d^{(p)}(\vec{p}_1, \vec{p}_2)$$

we have

$$f_d(\vec{p}) = g_d N_{pn} A_d M_d(\vec{p})$$

$$A_d = \int d\vec{x}_1 d\vec{x}_2 f_{pn}^{(n)}(\vec{x}_1, \vec{x}_2) R_d^{(x)}(\vec{x}_1, \vec{x}_2)$$

$$M_d(\vec{p}) = \int d\vec{p}_1 d\vec{p}_2 f_{pn}^{(n)}(\vec{p}_1, \vec{p}_2) R_d^{(p)}(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_1 + \vec{p}_2 - \vec{p})$$

## Compute $A_d$

$$A_d = \int d\vec{x}_1 d\vec{x}_2 f_{pn}^{(n)}(\vec{x}_1, \vec{x}_2) R_d^{(x)}(\vec{x}_1, \vec{x}_2)$$

$$\begin{cases} \vec{R} = \frac{\vec{x}_1 + \vec{x}_2}{2} \\ \vec{r} = \vec{x}_1 - \vec{x}_2 \end{cases} \quad f_{pn}^{(n)}(\vec{r}) = \frac{1}{(\pi C R_f^2)^{3/2}} e^{-\frac{\vec{r}^2}{C R_f^2}} \quad R_d^{(x)}(\vec{x}_1, \vec{x}_2) = 8e^{-\frac{(\vec{x}_1' - \vec{x}_2')^2}{\sigma_d^2}}$$

$$A_d = \frac{8\sigma_d^3}{(C R_f^2 + \sigma_d^2) \sqrt{C(R_f/\gamma)^2 + \sigma_d^2}}$$

## Compute $M_d$

$$M_d(\vec{p}) = \int d\vec{p}_1 d\vec{p}_2 f_{pn}^{(n)}(\vec{p}_1, \vec{p}_2) R_d^{(p)}(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_1 + \vec{p}_2 - \vec{p})$$

$$R_d^{(p)}(\vec{p}_1, \vec{p}_2) = e^{-\frac{\sigma_d^2 (\vec{p}_1' - \vec{p}_2')^2}{4\hbar^2 c^2}} \Rightarrow \delta \text{ function approximation}$$

$$M_d(\vec{p}) = \left(\frac{\hbar c}{\sigma_d} \sqrt{\pi}\right)^3 \gamma f_{pn}^{(n)}\left(\frac{\vec{p}}{2}, \frac{\vec{p}}{2}\right)$$

$$f_d(\vec{p}) = \frac{8(\sqrt{\pi} \hbar c)^3 g_d \gamma}{(CR_f^2 + \sigma_d^2) \sqrt{C(R_f / \gamma)^2 + \sigma_d^2}} f_p\left(\frac{\vec{p}}{2}\right) f_n\left(\frac{\vec{p}}{2}\right)$$

## Coalescence factor

$$\begin{aligned} B_2 &= (E_d \frac{d^3 N_d}{d\vec{p}_d^3}) / \left[ (E_p \frac{d^3 N_p}{d\vec{p}_p^3})(E_n \frac{d^3 N_n}{d\vec{p}_n^3}) \right] \\ &= \frac{32 g_d (\sqrt{\pi} \hbar c)^3}{m_d (CR_f^2 + \sigma_d^2) \sqrt{C(R_f / \gamma)^2 + \sigma_d^2}} \end{aligned}$$

## Check the validity of the delta function approximation

$$M_d(\vec{p}) = \int d\vec{p}_1 d\vec{p}_2 f_{pn}^{(n)}(\vec{p}_1, \vec{p}_2) R_d^{(p)}(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_1 + \vec{p}_2 - \vec{p})$$

$$R_d^{(p)}(\vec{p}_1, \vec{p}_2) = e^{-\frac{\sigma_d^2 (\vec{p}_1' - \vec{p}_2')^2}{4\hbar^2 c^2}} \quad f_{p,n}^{(n)}(\vec{p}) = \frac{1}{(2\pi mT)^{3/2}} e^{-\frac{\vec{p}^2}{2mT}}$$

$$f_d(\vec{p}) = \frac{8g_d \sigma_d^3 (\pi mT)^{3/2}}{(CR_f^2 + \sigma_d^2) \sqrt{C(R_f/\gamma)^2 + \sigma_d^2} \left(1 + \frac{mT\sigma_d^2}{\hbar^2 c^2}\right) \sqrt{1 + \frac{mT\sigma_d^2}{\gamma^2 \hbar^2 c^2}}} f_p\left(\frac{\vec{p}}{2}\right) f_n\left(\frac{\vec{p}}{2}\right)$$

$$\because mT\sigma_d^2 \gg \gamma^2 \hbar^2 c^2,$$

$$\therefore B_2 = \frac{32g_d (\sqrt{\pi} \hbar c)^3}{m_d (CR_f^2 + \sigma_d^2) \sqrt{C(R_f/\gamma)^2 + \sigma_d^2}}$$

## Similarly for ${}^3\text{He}$

$$f_{{}^3\text{He}}(\vec{p}) = \frac{8^2 (\sqrt{\pi} \hbar c)^6 g_{{}^3\text{He}} \gamma^2}{3\sqrt{3} \left( \frac{C}{2} R_f^2 + \sigma_{{}^3\text{He}}^2 \right) \sqrt{\frac{C}{2} (R_f / \gamma)^2 + \sigma_{{}^3\text{He}}^2}}$$

$$\times \frac{1}{\left( \frac{2C}{3} R_f^2 + \sigma_{{}^3\text{He}}^2 \right) \sqrt{\frac{2C}{3} (R_f / \gamma)^2 + \sigma_{{}^3\text{He}}^2}} f_p\left(\frac{\vec{p}}{3}\right) f_p\left(\frac{\vec{p}}{3}\right) f_n\left(\frac{\vec{p}}{3}\right)$$

$$B_3 = \frac{192\sqrt{3}(\pi \hbar^2 c^2)^3 g_{{}^3\text{He}}}{m_{{}^3\text{He}}^2 \left( \frac{C}{2} R_f^2 + \sigma_{{}^3\text{He}}^2 \right) \sqrt{\frac{C}{2} (R_f / \gamma)^2 + \sigma_{{}^3\text{He}}^2}}$$

$$\times \frac{1}{\left( \frac{2C}{3} R_f^2 + \sigma_{{}^3\text{He}}^2 \right) \sqrt{\frac{2C}{3} (R_f / \gamma)^2 + \sigma_{{}^3\text{He}}^2}}$$

### III. Applications in $pp$ , $p\text{-Pb}$ and $\text{Pb-Pb}$ collisions at LHC

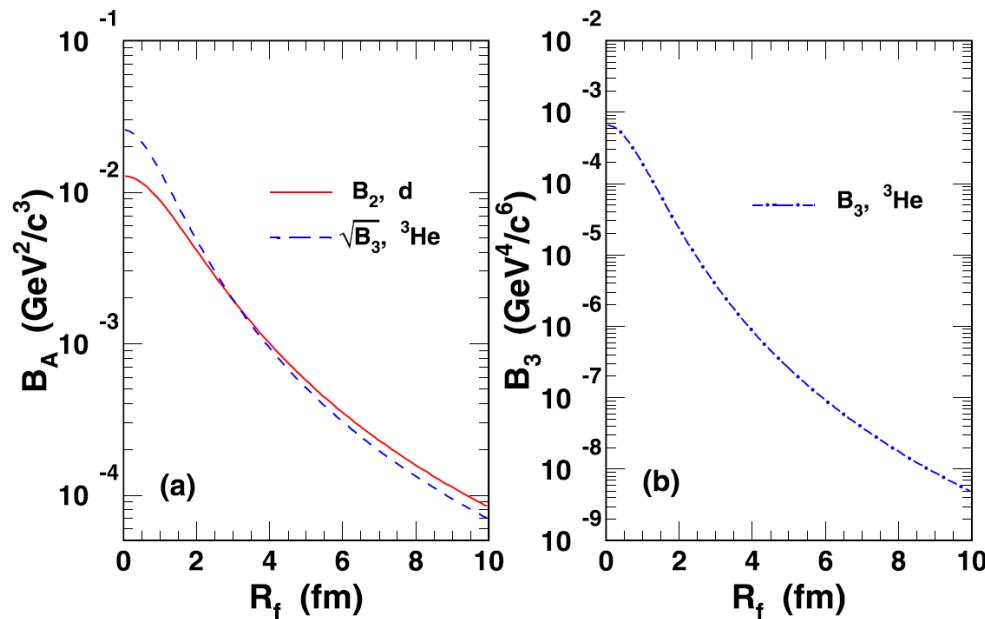
$$B_3 = \frac{192\sqrt{3}(\pi\hbar^2c^2)^3 g_{^3\text{He}}}{m_{^3\text{He}}^2 \left( \frac{C}{2} R_f^2 + \sigma_{^3\text{He}}^2 \right) \sqrt{\frac{C}{2} (R_f / \gamma)^2 + \sigma_{^3\text{He}}^2} \left( \frac{2C}{3} R_f^2 + \sigma_{^3\text{He}}^2 \right) \sqrt{\frac{2C}{3} (R_f / \gamma)^2 + \sigma_{^3\text{He}}^2}}$$

$$B_2 = \frac{32(\sqrt{\pi}\hbar c)^3 g_d}{m_d (C R_f^2 + \sigma_d^2) \sqrt{C (R_f / \gamma)^2 + \sigma_d^2}}$$

For large  $R_f$

$$B_2 \propto R_f^{-3} \propto V_f^{-1}$$

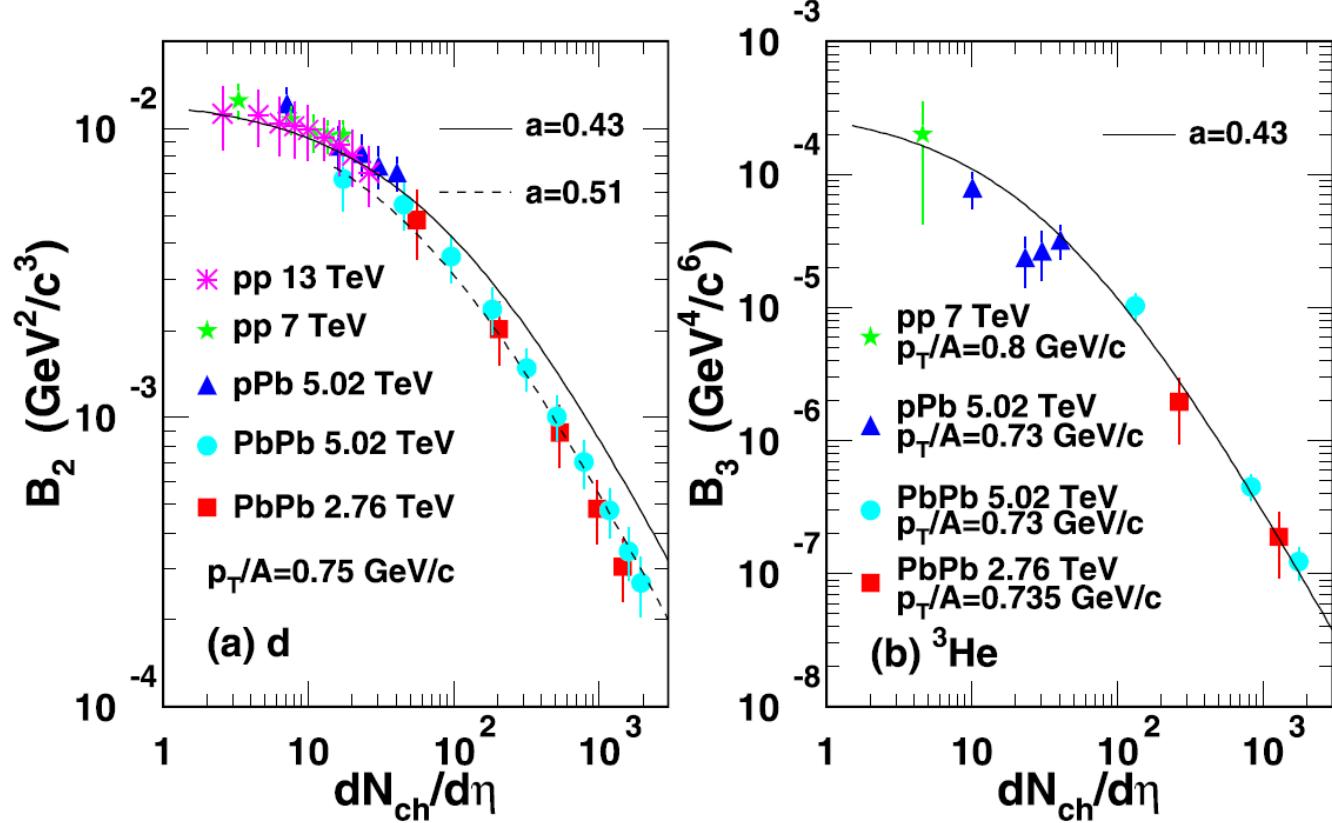
$$B_3 \propto R_f^{-6} \propto V_f^{-2}$$



$B_A$  as the function of the system size denoted by  $R_f$

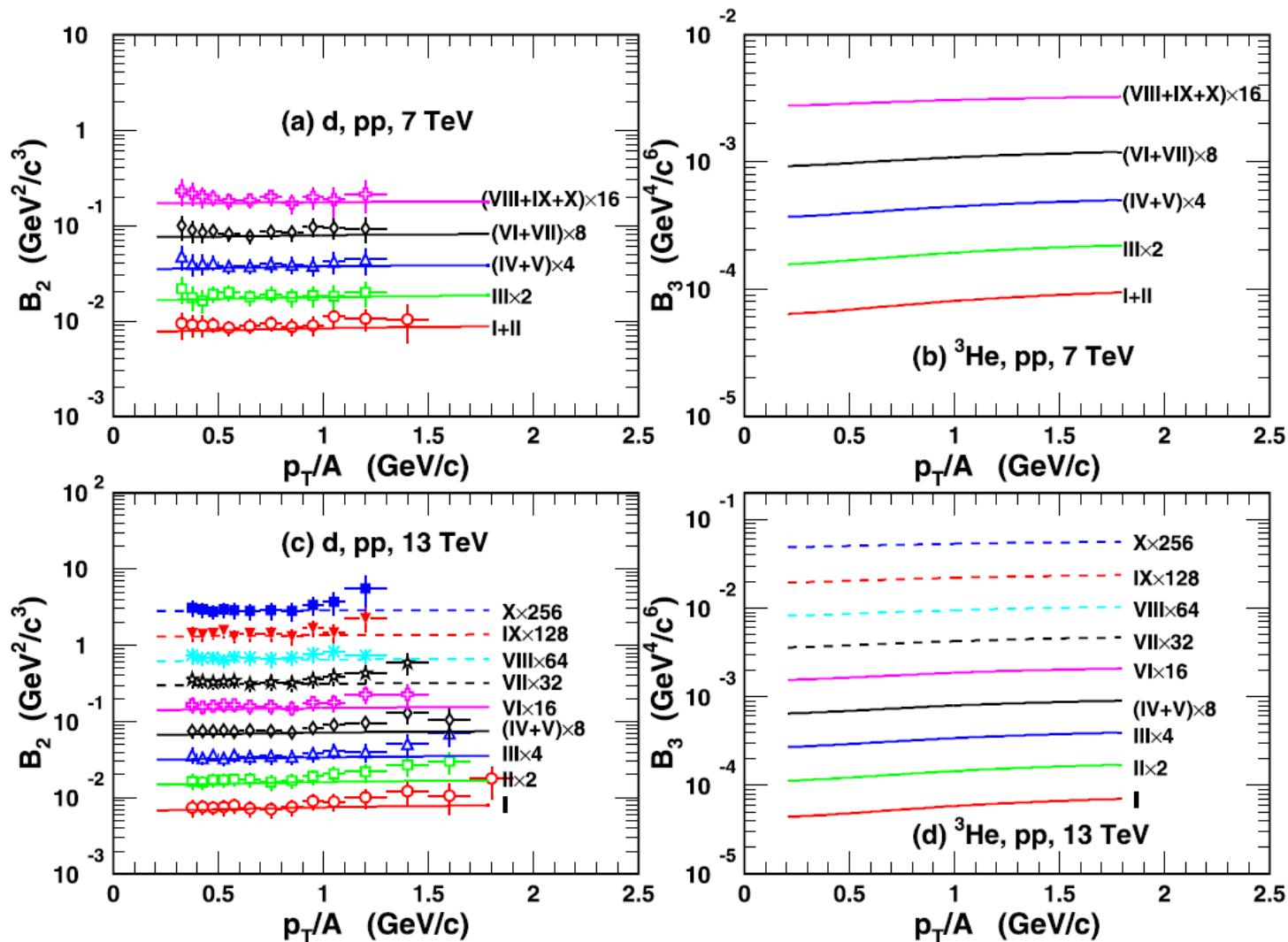
# Results as the function of the system size denoted by $dN_{ch}/d\eta$

$$R_f = a \left( \frac{dN_{ch}}{d\eta} \right)^{1/3}$$

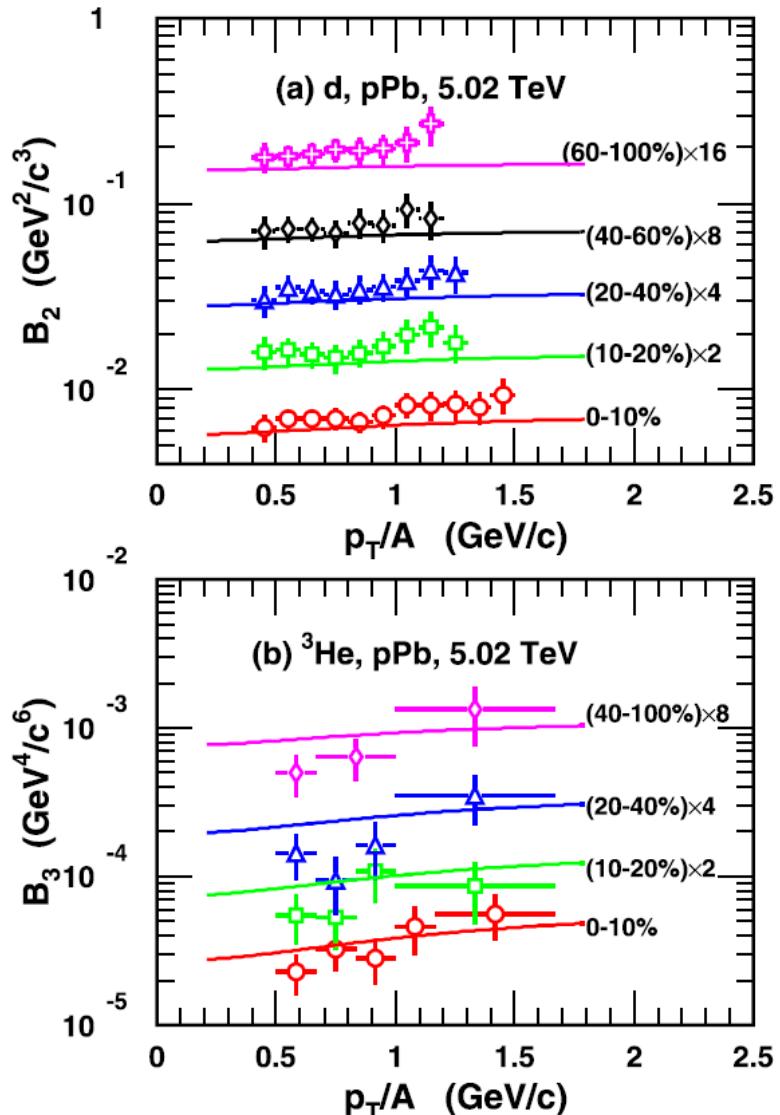


Our results reproduce the system size dependent behaviors of  $B_A$  in pp, p-Pb, and Pb-Pb collisions at the LHC. Larger value of  $a$  parameter for deuteron in Pb-Pb collisions might indicate a later freeze-out for deuteron compared with  ${}^3\text{He}$  in Pb-Pb collisions.

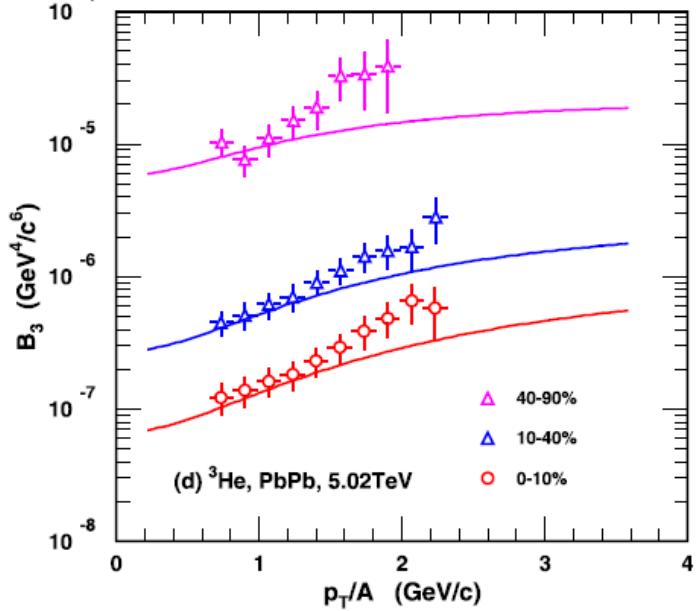
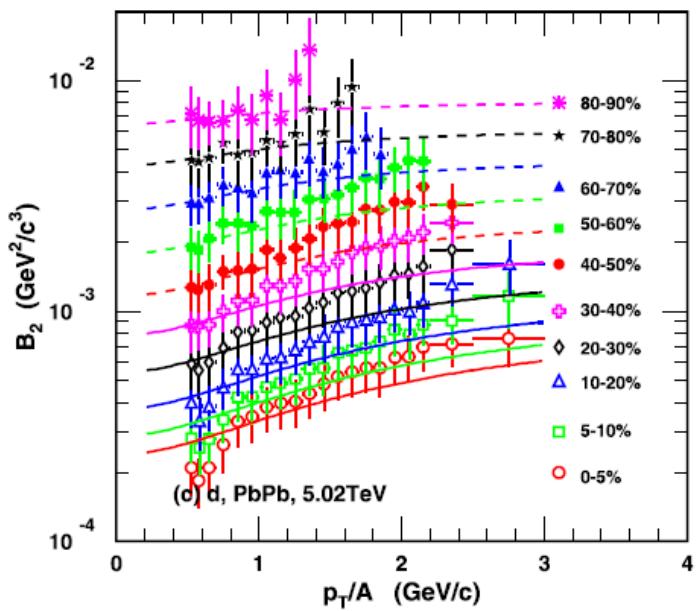
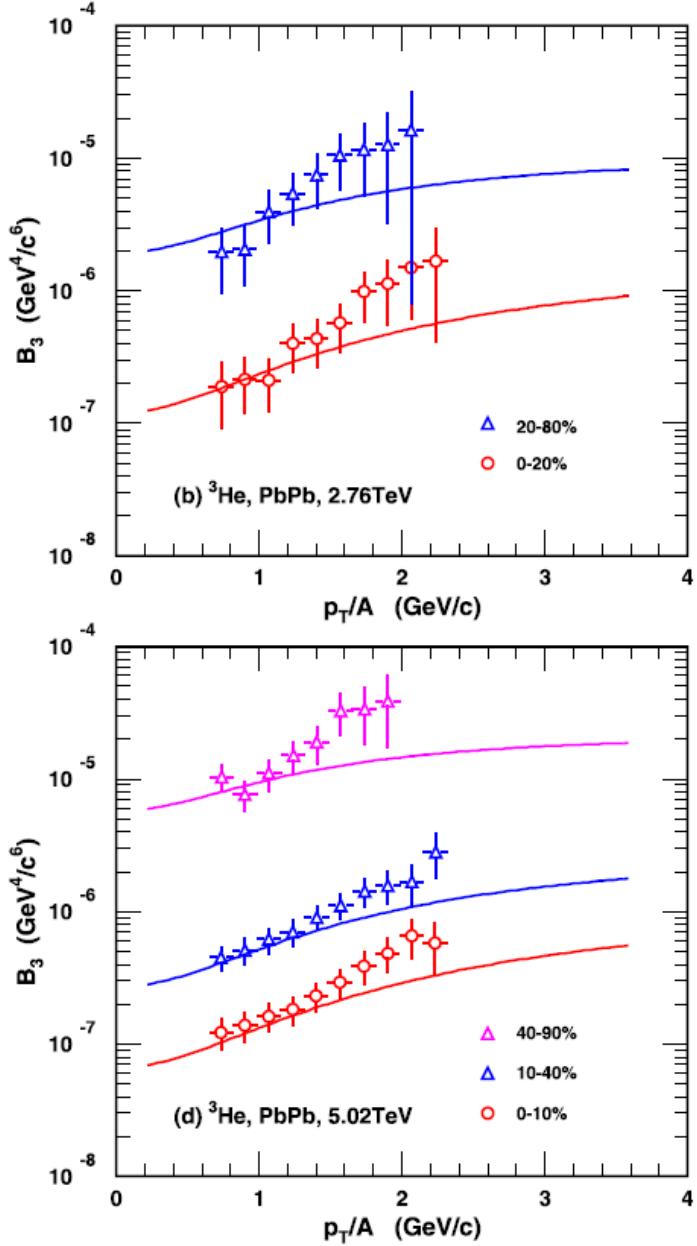
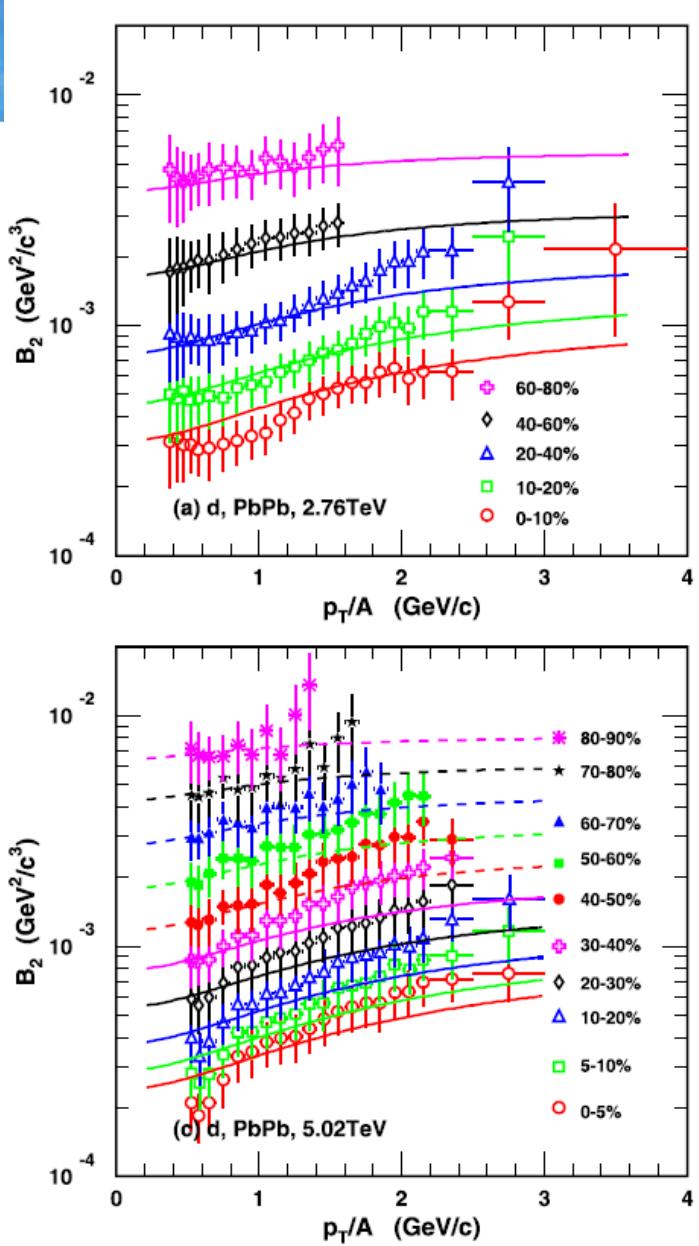
# Results as the function of the momentum per nucleon $p_T/A$



$B_A$  exhibits nearly constant behavior with the momentum in  $pp$  collisions.



$B_A$  exhibits weak  $p_{\text{T}}$  dependent behavior in  $p\text{-Pb}$  collisions.



$B_A$  exhibits significant  $p_T$  dependent behavior in  $\text{Pb-Pb}$  collisions.

## V. Summary and outlook

- ◆ Momentum-dependent production formulas of light nuclei are derived analytically in the coalescence mechanism.
- ◆ Analytical expressions for the coalescence factors  $B_2$  and  $B_3$  are obtained and are successfully used to describe their interesting behaviors observed in  $pp$ ,  $p\text{-Pb}$ , and  $\text{Pb-Pb}$  collisions at the LHC.
- ◆ We find that instantaneous coalescence occurring in the nucleon rest frame can give natural explanations for the obvious growth of  $B_A$  against  $p_T$  for all centralities in  $\text{Pb-Pb}$  collisions and for the relatively weak  $p_T$  dependence of  $B_A$  in  $pp$  and  $p\text{-Pb}$  collisions at the LHC.
- ◆ More results for  $p_T$  distributions and yield ratios are continued.

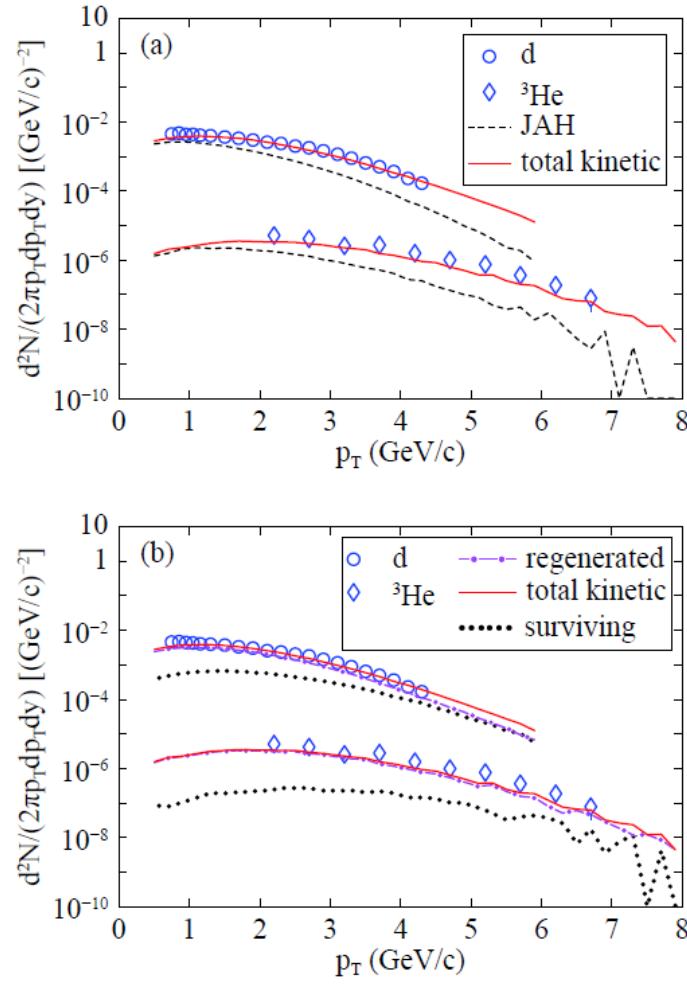
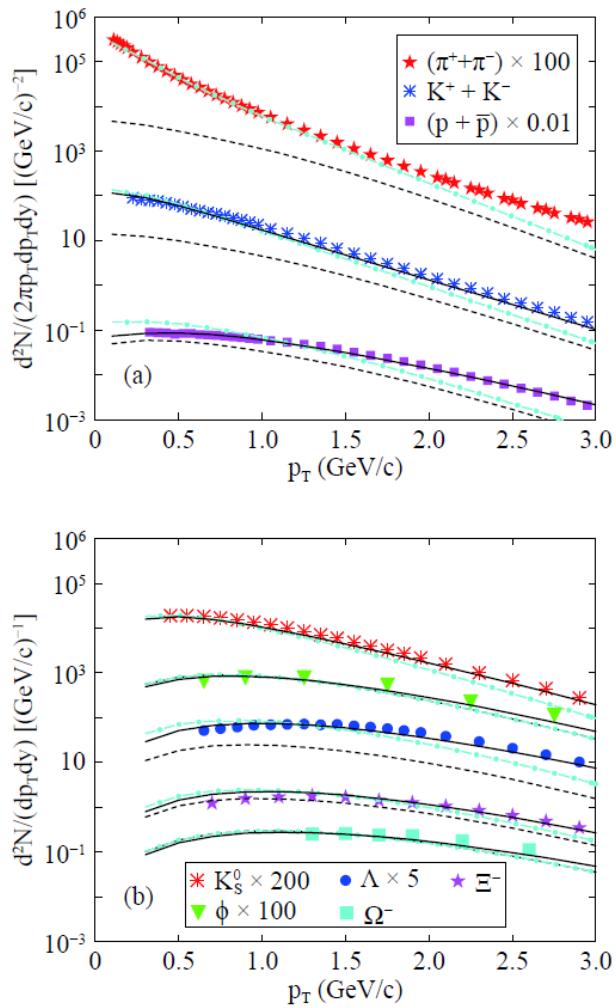
谢谢！

$$O_{p-d-t} \equiv \frac{N_p N_t}{N_d^2}$$

$$= \frac{g_t (C_W R_f^2 + \sigma_d^2)^2 (\frac{C_W R_f^2}{\gamma^2} + \sigma_d^2)}{8 g_d^2 (C_1 R_f^2 + \sigma_t^2) \sqrt{\frac{C_1 R_f^2}{\gamma^2} + \sigma_t^2} (C_2 R_f^2 + \sigma_t^2) \sqrt{\frac{C_2 R_f^2}{\gamma^2} + \sigma_t^2}}$$

$$R_f \gg \sigma_d \sim \sigma_t \quad \quad O_{p-d-t} \approx \frac{3 \sqrt{3} g_t}{8 g_d^2} \approx 0.29$$

$$R_f \ll \sigma_d \sim \sigma_t \quad \quad O_{p-d-t} \approx \frac{g_t \sigma_d^6}{8 g_d^2 \sigma_t^6} \approx 3.44$$



**More results of  $p_T$  distributions are being calculated by X.Y. Zhao & Y.H. Li.**