Intermittency analysis in relativistic heavy-ion collisions

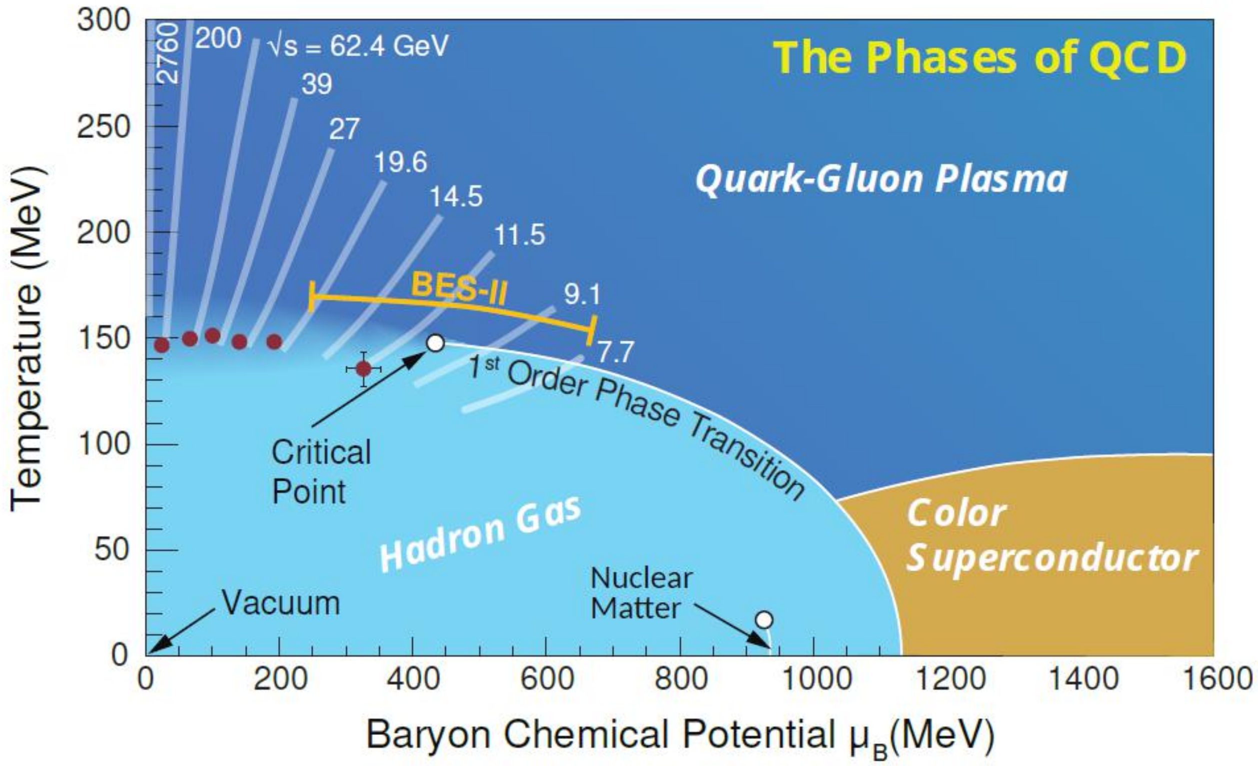
Zhiming Li （李治明）
Central China Normal University (华中师范大学)

Collaborators: Yuanfang Wu (吴元芳), Xiaofeng Luo (罗晓峰), Jin Wu(吴锦) and Yufu Lin(林裕富)

- Introduction
- Two important issues of measuring intermittency in heavy-ion collisions
- Results from the UrQMD model
- Summary

arXiv: 2104.11524 (2021)
Phys. Lett. B, 801, 135186 (2020)

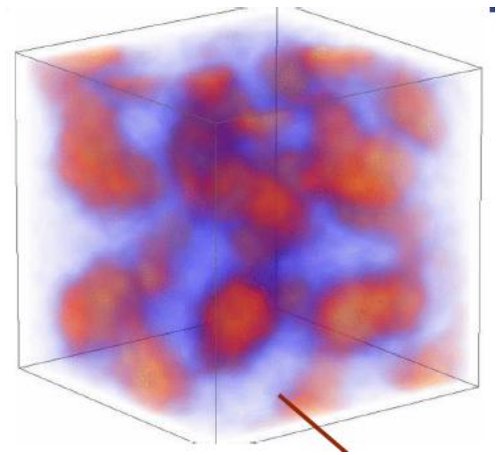
中国物理学会高能物理分会第十三届全国粒子物理学术会议



A. Bzdak, et al., Phys. Rep., 853, 1 (2020)

Experimental observables:

- Event-by-event (global) fluctuations: Variance, Skewness, Kurtosis.
- Light nuclei production.
- Local density fluctuation (cluster): Intermittency (self-similar behavior)
- ...



N. Antoniou, et al., Phy. Rev. Lett. 97,032002 (2006)
J. Wu, Z. Li, et al., Phys. Lett. B 801, 135186 (2020)

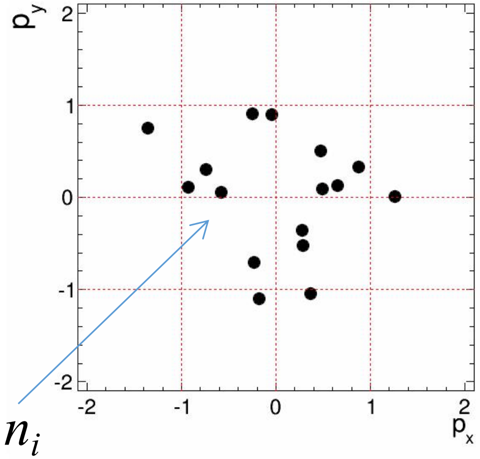
Goal: exploring the QCD phase boundary and Critical Point.

Scaled factorial moment (SFM) and intermittency

- The scaled factorial moments $F_q(M)$,

$$\boxed{F_q(M) = \frac{<\frac{1}{M^D}\sum_{i=1}^{M^D} n_i(n_i-1)\ldots(n_i-q+1)>}{<\frac{1}{M^D}\sum_{i=1}^{M^D} n_i>^q}}$$

Transverse momentum space is partitioned into M^2 cells, q is tl order of moments, <···> denote averaging over events.



Particle multiplicity in the *i-th* cell n_i

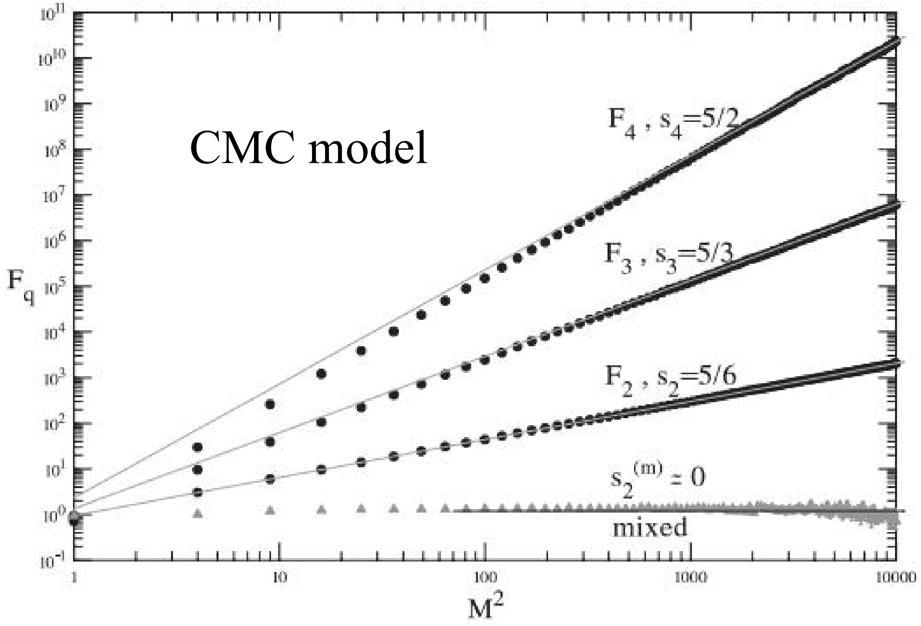
- Intermittency refers to the power law behavior:

$$\boxed{F_q(M) \propto (M^2)^{\phi_q}, M \to \infty}$$

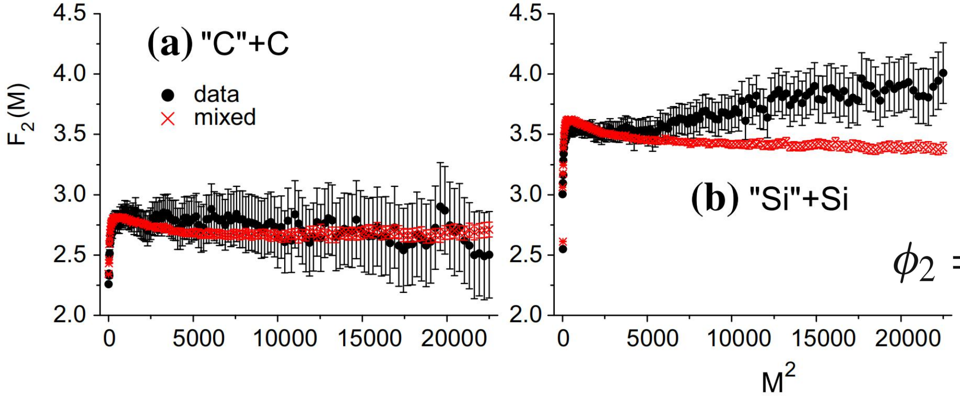
- For a critical system of the 3D Ising universality class of baryon numbers:

$$\phi_2^{critical} = 5/6$$

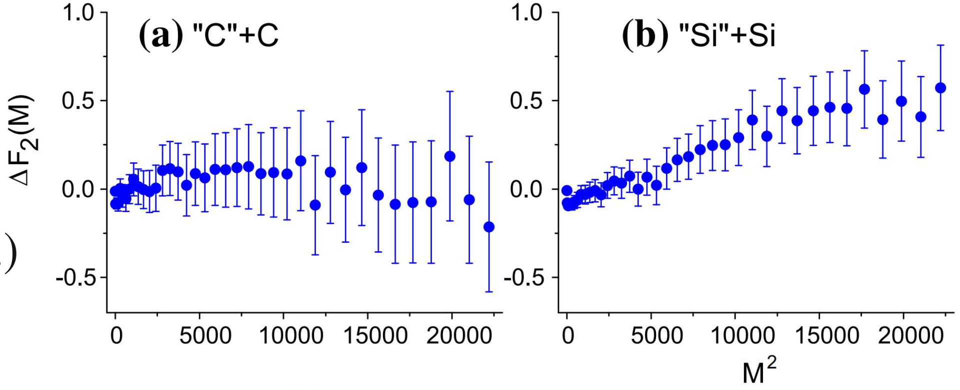
N. Antoniou, et al., Phy. Rev. Lett. 97,032002 (2006)
T. Anticic, et al., Eur. Phys. J. C 75, 587 (2015)

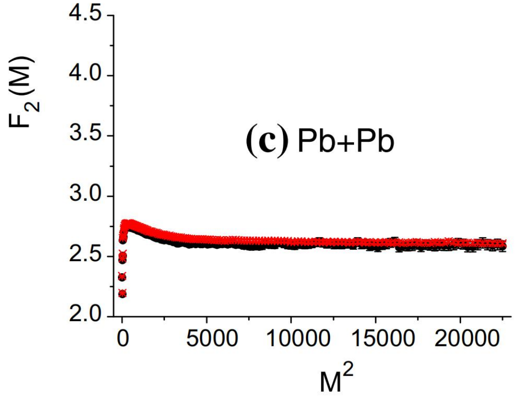


Intermittency in NA49 experiment



$$\phi_2 = 0.96^{+0.38}_{-0.25}\,(\text{stat.}) \pm 0.16\,(\text{syst.})$$

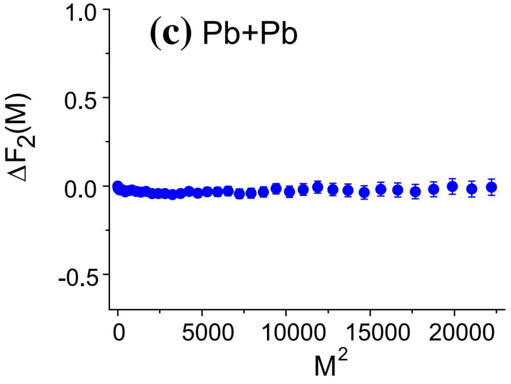




$$\Delta F_2^{(e)}(M) = F_2^{(d)}(M) - F_2^{(m)}(M)$$

N. Davis, et al. (NA49 Coll.), QM2018

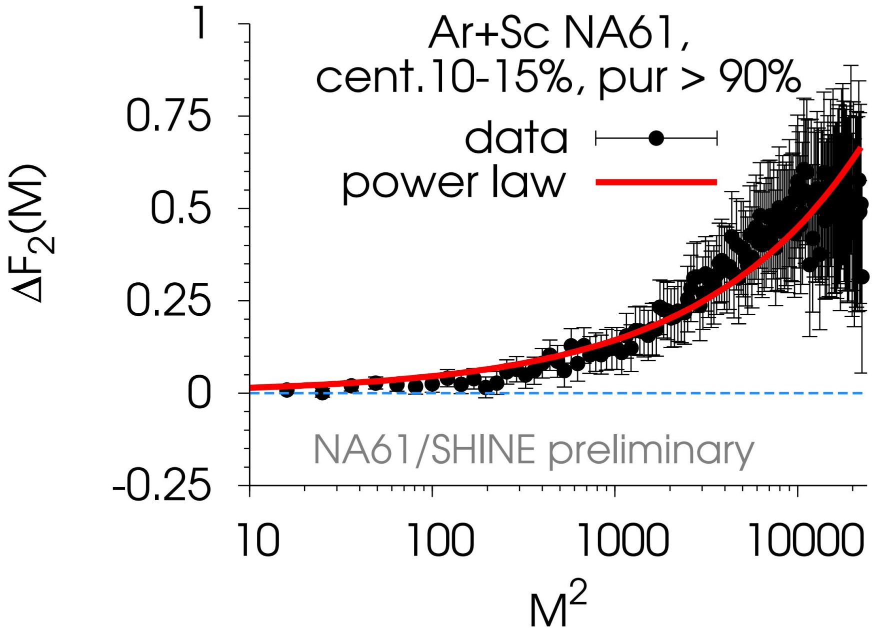
T. Anticic, et al., Eur. Phys. J. C 75, 587 (2015)



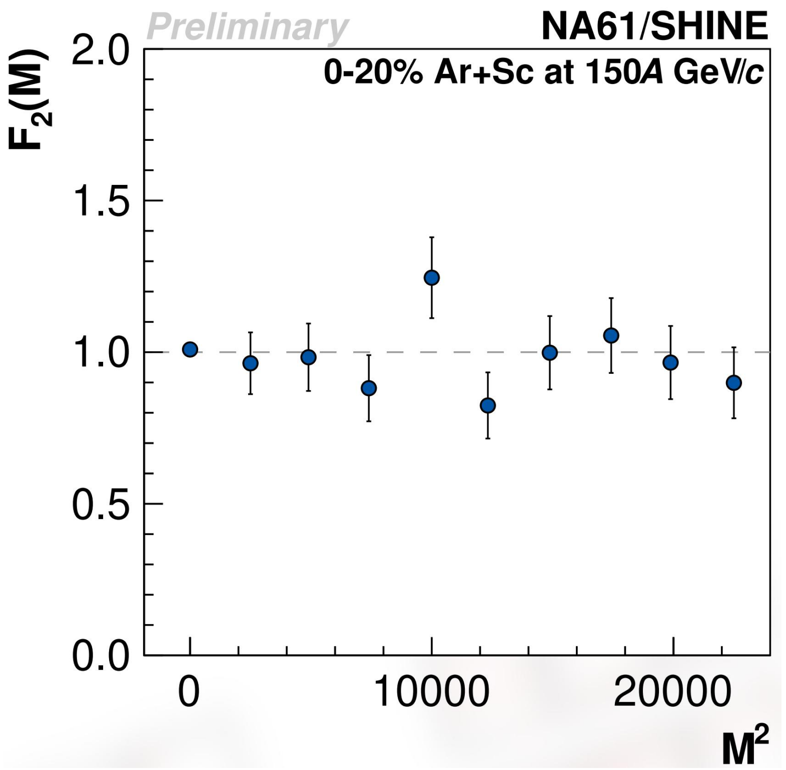
- Intermittency of NA49 experiment reveals significant power-law fluctuations of proton density in **Si + Si** collisions at $\sqrt{s_{NN}}$ = 17.3 GeV. But no intermittent behavior is visible in C+C or Pb + Pb collisions.

- NA49 uses the mixed event method to estimate and subtract background by assuming that the particle multiplicity in each cell can be simply divided into background and critical contributions.

Intermittency in NA61/SHINE experiment



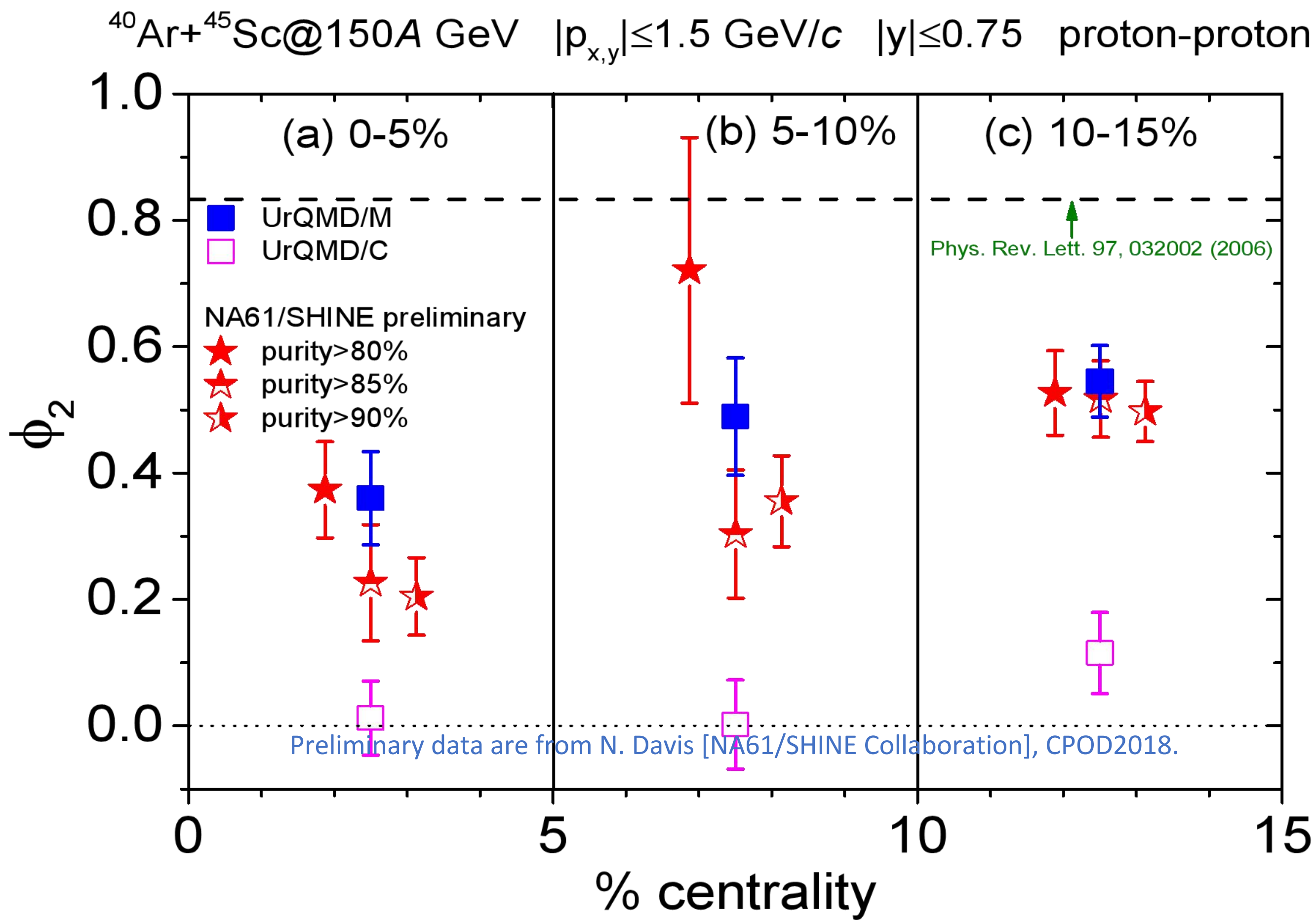
N. Davis, et al. (NA61/SHINE Coll.), CPOD2018
D. Prokhorova and N. Davis, Universe 5, 103 (2019)



T. Czopowicz (NA61/SHINE Coll.), CPOD2021.

- Preliminary NA61/SHINE result at CPOD2018 with mixed-event method exhibits a power-law scaling of $\Delta F_2(M)$ of proton density for **Ar + Sc** collisions at 150A GeV/c.

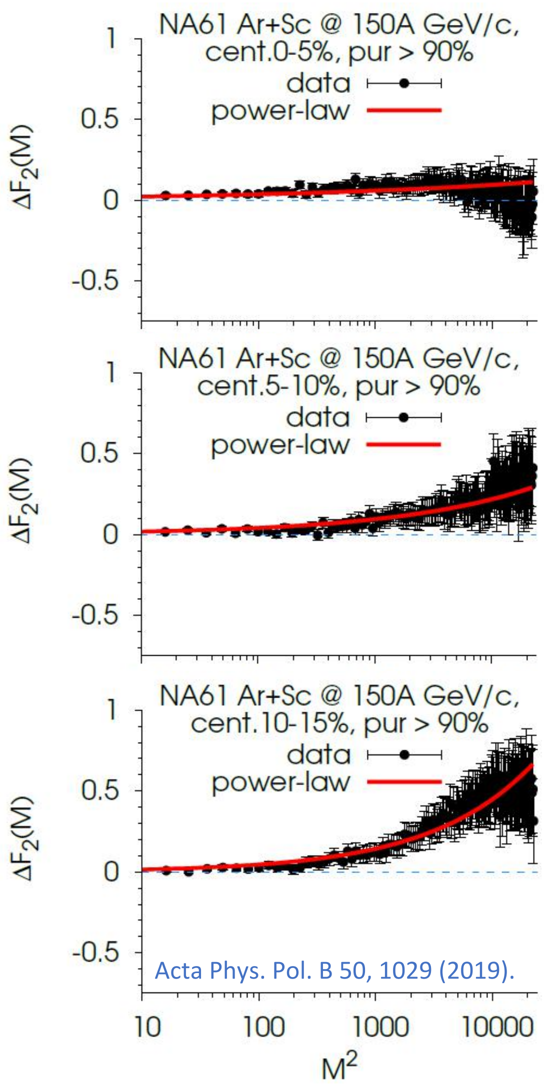
- New NA61/SHINE result at CPOD2021, which is with statistically independent points and cumulative variables, shows no indication of a power-law increase.



P. Li, Q. Li, et al., Phys. Lett. B 818, 136393 (2021)

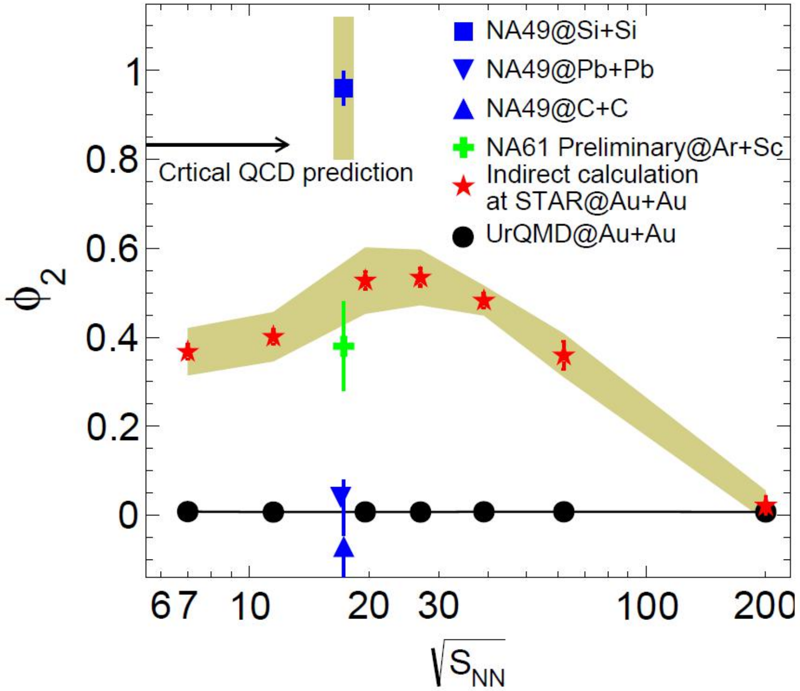
UrQMD/M (mean-field mode): with the hadronic potentials

UrQMD/C (cascade mode): without the hadronic potentials



- The intermittency index, ϕ_2, calculated by the UrQMD/C model is around zero.
- With the inclusion of hadronic potentials, it is found that ϕ_2 from the UrQMD/M model is comparable to the NA61/SHINE data in different centrality bins.

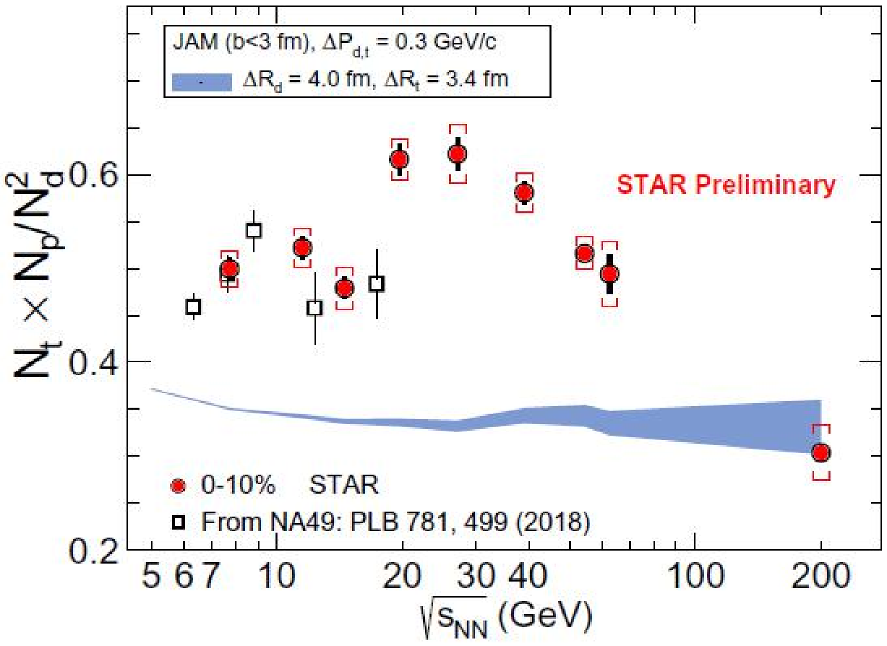
Indirect mapping of intermittency via densitity fluctuations at RHIC/STAR



J. Wu, Z. Li, et al., Phys. Lett. B 801, 135186 (2020)

$$(N_t \times N_p)/N_d^2 = g(1+\Delta n)$$

$$\Delta n = \frac{\langle(\delta n)^2\rangle}{\langle n\rangle^2} = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle^2}$$

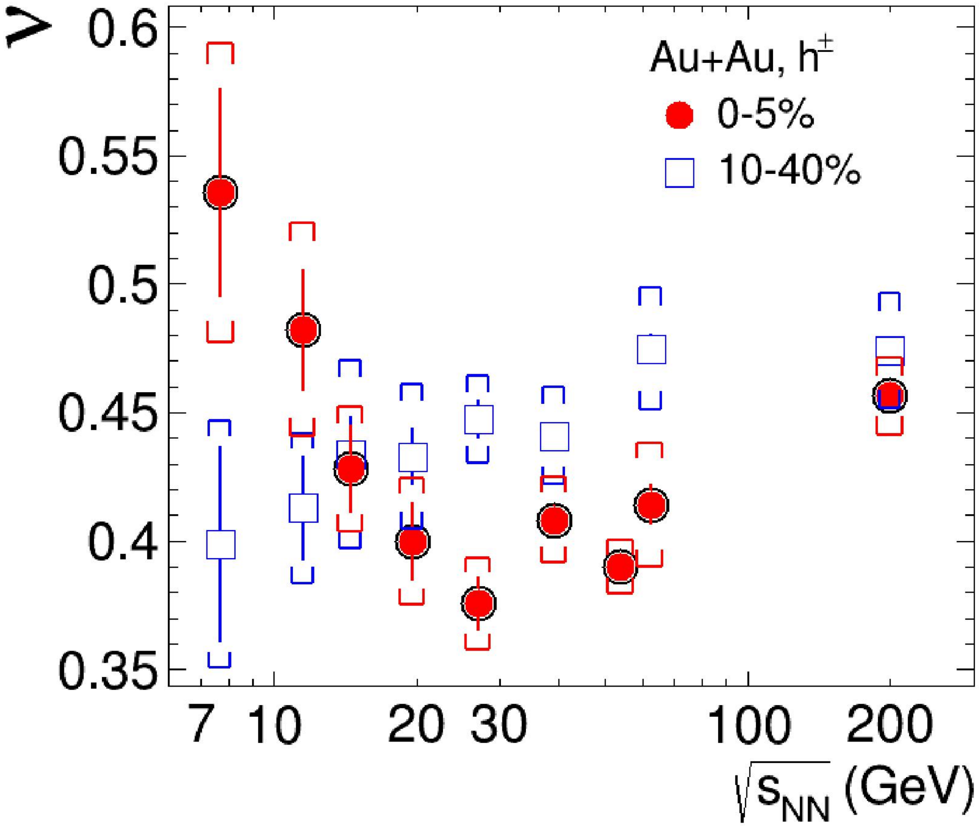


D. Zhang (for STAR Coll.), Nucl. Phys. A1005,12185 (2021)

- The second-order intermittency index is gained indirectly by mapping the relative density fluctuations into the relation between Δn and ϕ_2.

- The energy dependence of ϕ_2 displays a non-monotonic behavior with a peak at energy around 20-30GeV, indicating that the strength of intermittency becomes the largest in this region.

Intermittency in RHIC/STAR experiment



$$F_q(M) = \frac{< \frac{1}{M^D} \sum_{i=1}^{M^D} n_i(n_i-1)\ldots(n_i-q+1) >}{< \frac{1}{M^D} \sum_{i=1}^{M^D} n_i >^q}$$

$$\Delta F_q(M) \propto \Delta F_2(M)^{\beta_q}$$

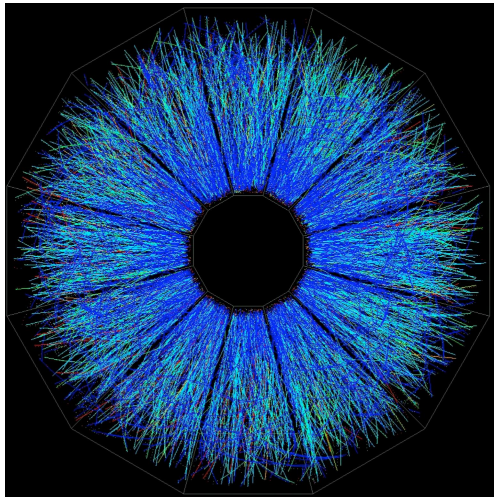
$$\boxed{\beta_q \propto (q-1)^{\nu}}$$

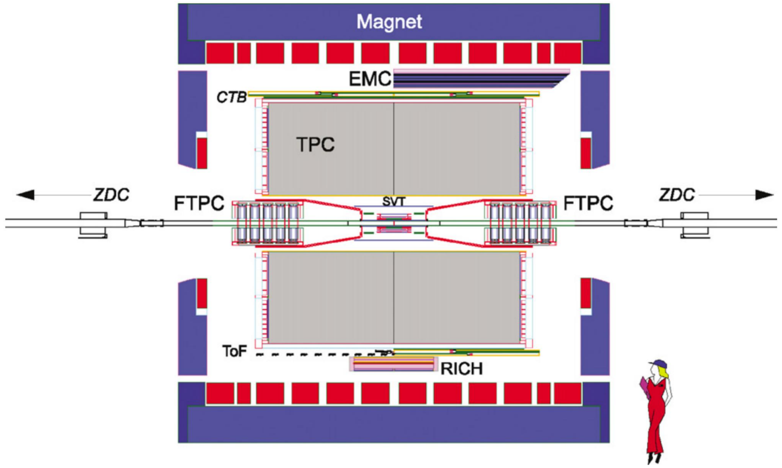
R. Hua and M. Nazirov, Phy. Rev. Lett. 69, 741 (1992)
R. Hua and C. Yang, Phy. Rev. C 85, 044914 (2012)

Poster section: Jin Wu at 14:24 on 17th

- In the most central collisions, the scaling exponent ν exhibits a non-monotonic behavior on collision energy and seems to reach a minimum around $\sqrt{s_{NN}}$ = 20-30 GeV.
- In 10-40% central collisions, ν monotonically increases with increasing collision energy at $\sqrt{s_{NN}}$ =7.7 – 200 GeV.

Two important issues of measuring intermittency

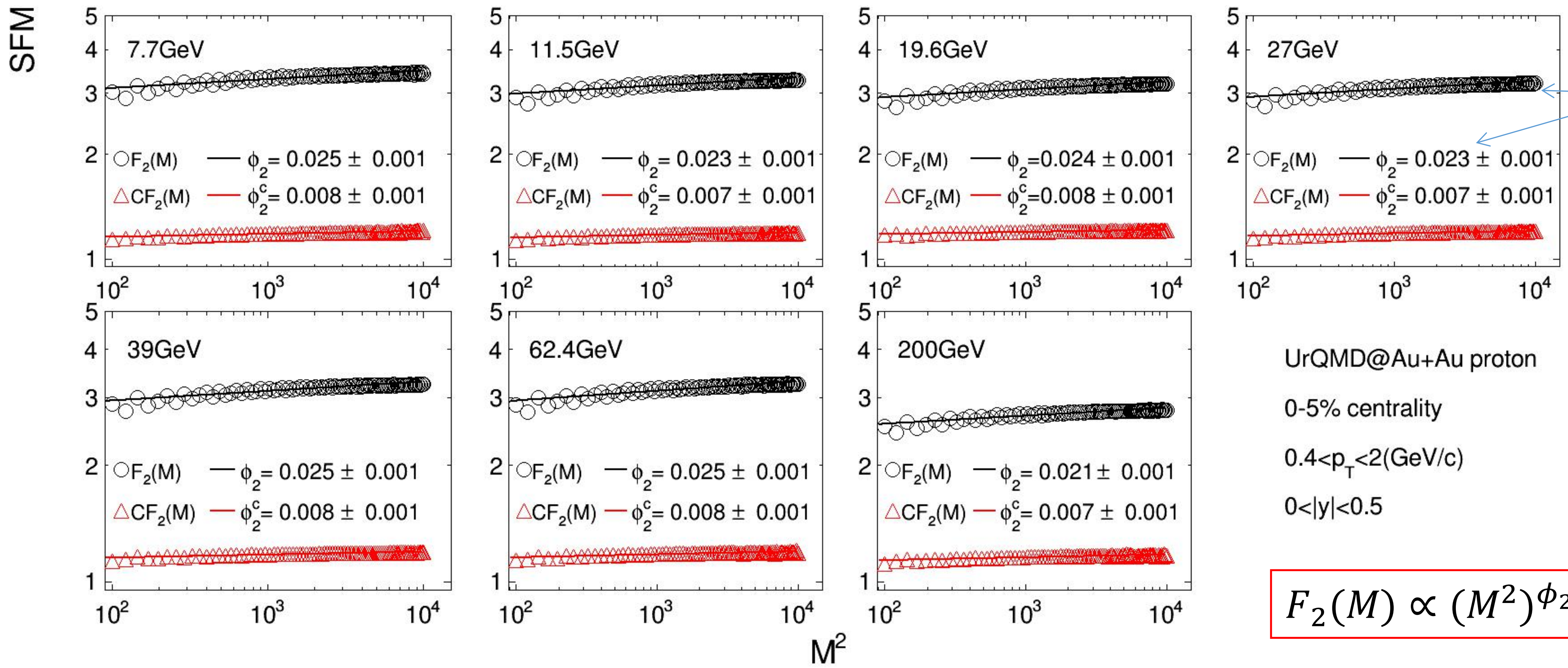




- **Background subtraction:** the non-critical background will change the inclusive single-particle multiplicity distributions in the measured finite momentum space
- **Experimental detector efficiency correction:** not all particles are detected, some leave the detector without any trace, some escape through not sensitive detector areas (holes, cracks for e.g. water cooling and gas pipes, electronics, mechanics)

J. Wu, Z. Li, et al., arXiv: 2104.11524 (2021)

Energy dependence of SFM from UrQMD model

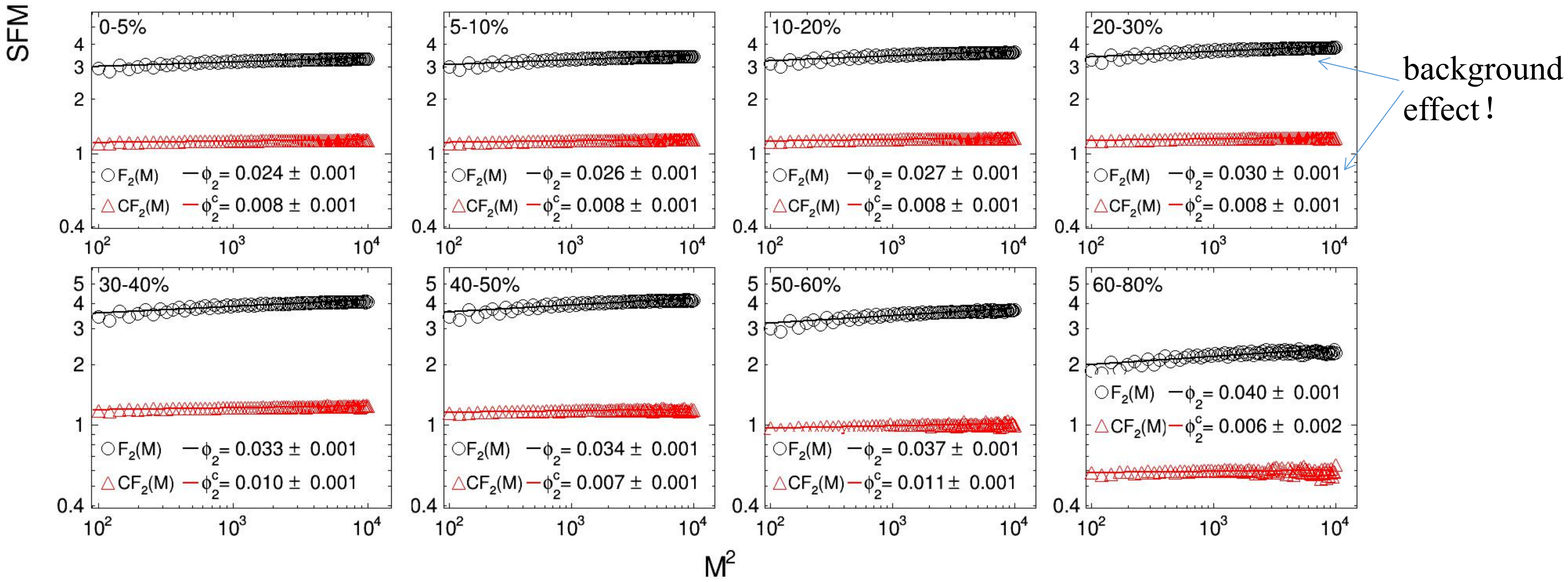


background effect!

$$\boxed{F_2(M) \propto (M^2)^{\phi_2}}$$

- SFM increases slowly with increasing number of dividing bins
- The slopes of the fitting, *i.e.* the intermittency indices ϕ_2, are found to be small but non-zero at all energies.

Centrality dependence of SFM from UrQMD model



- The directly calculated SFMs can be fitted with a small but non-zero intermittency index.

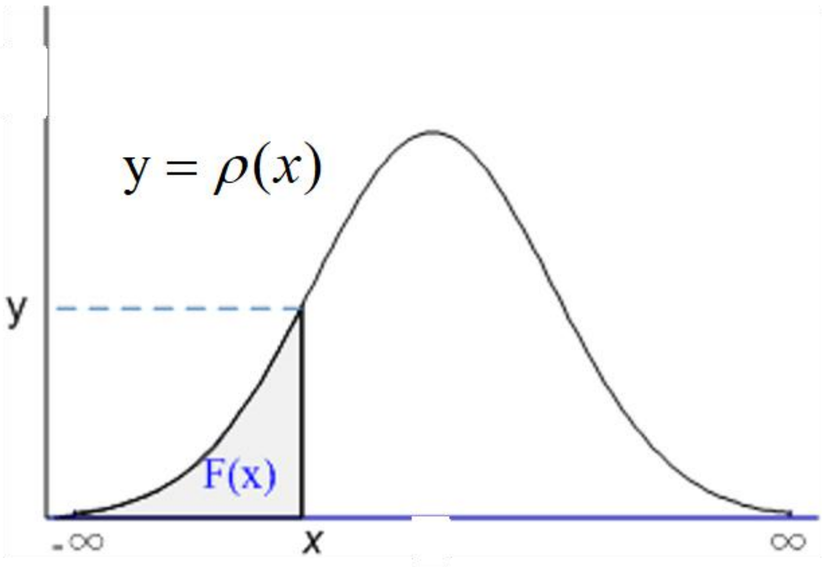
- The values of ϕ_2 increase slightly from the most central (0−5%) to the most peripheral (60 − 80%) collisions.

Background subtraction: cumulative variable method

- Following Ochs and Bialas, the cumulative variable $X(x)$ is related to the single-particle density distribution $\rho(x)$ through:

$$X(x) = \frac{\int_{x_{min}}^{x} \rho(x)dx}{\int_{x_{min}}^{x_{max}} \rho(x)dx}$$

Phys. Lett. B 214, 617 (1988);
Phys. Lett. B 247, 101 (1990);
Z. Phys. C 50, 339 (1991);
Phys. Lett. B 252,483 (1990)



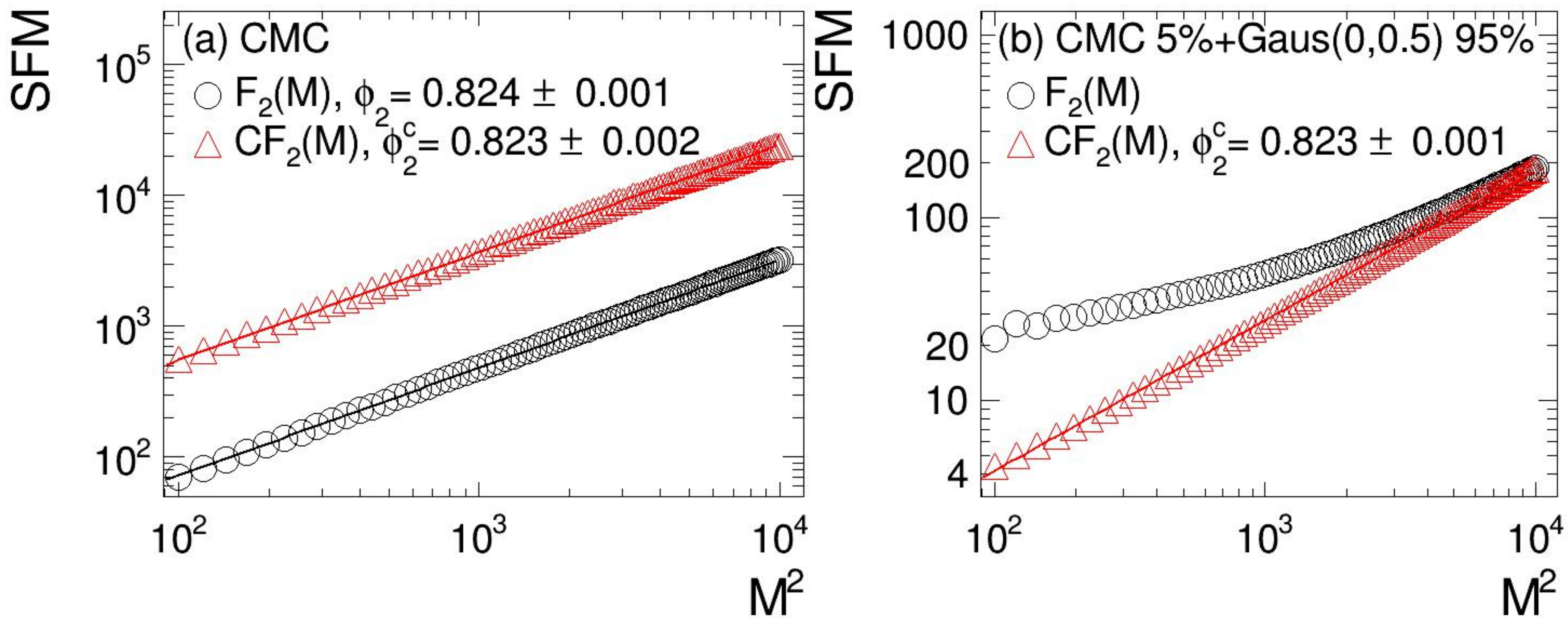
Instead of using p_x and p_y, one can use cumulative variables P_X and P_Y

$$P_X = \frac{\int_{p_{x,min}}^{p_x} \rho(p_x)dp_x}{\int_{p_{x,min}}^{p_{x,max}} \rho(p_x)dp_x} \qquad P_Y = \frac{\int_{p_{y,min}}^{p_y} \rho(p_y)dp_y}{\int_{p_{y,min}}^{p_{y,max}} \rho(p_y)dp_y}$$

Advantages of cumulative variable:

a) It does not depend on the choice of the original variable x, but is uniquely determined by the shape of density distribution $\rho(x)$.
b) The density distribution of cumulative variable, $X(x)$, is uniform in the interval from 0 to 1.

Testing the validity of the cumulative variable method

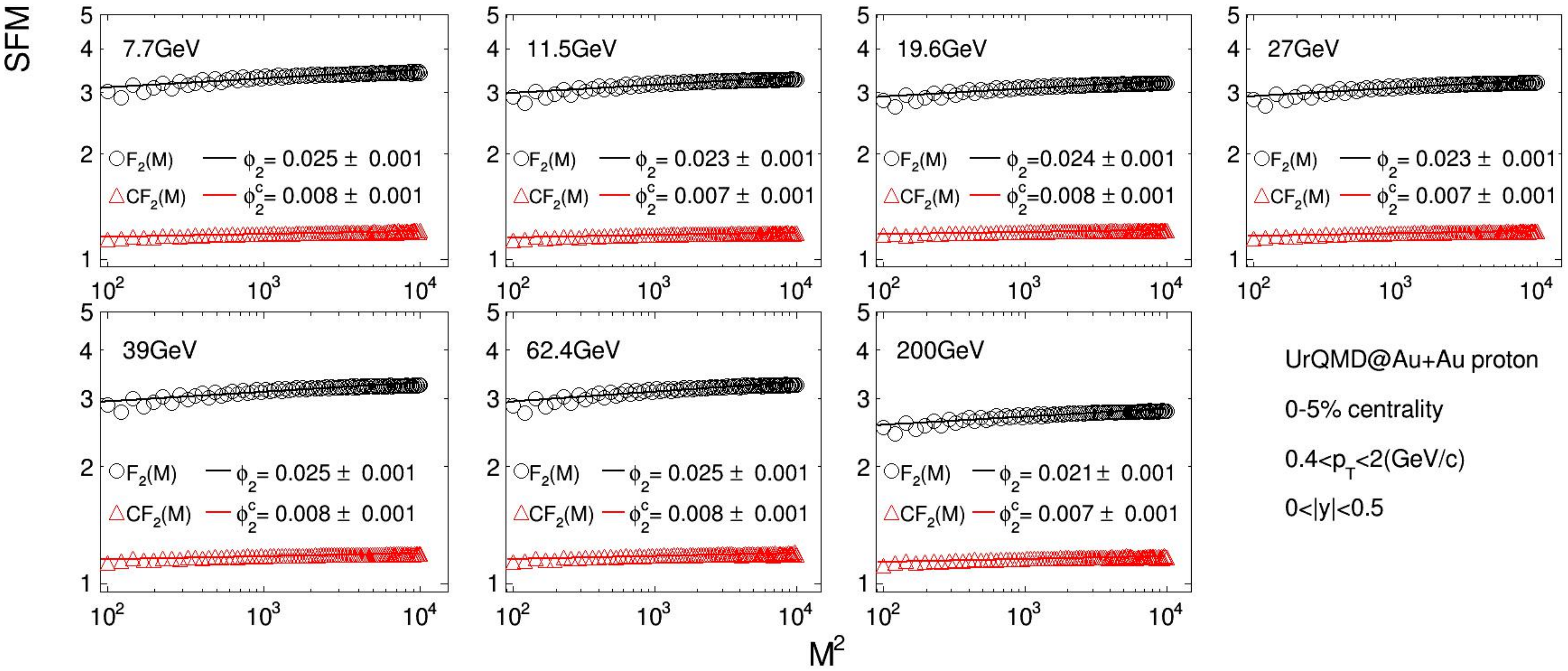


$$F_2(M) \propto (M^2)^{\phi_2}$$

- $CF_2(M)$ follows a good power-law behavior as $F_2(M)$. Cumulative variable method does not change the intermittency behavior for a pure critical signal event sample.

- The directly calculated $F_2(M)$ deviates substantially from the linear dependence, i.e. violation of the scaling law because of the Gaussian background contribution. However, ϕ_2 calculated from $CF_2(M)$ keeps unchanged when comparing to the one in original CMC sample.

Energy dependence of $CF_2(M)$ from UrQMD model



- $CF_2(M)$ is found to be nearly flat with increasing number of cells. The intermittency index, ϕ_2, with the value nearly around 0, is much smaller than the one directly calculated from $F_2(M)$.
- It indicates that the background of non-critical effect can be efficiently removed by the cumulative variable method in the calculation of SFMs in UrQMD model.

- The detector response is assumed to follow a binomial probability distribution function.

$$B(n,N;\varepsilon) = \frac{N!}{n!\,(N-n)!}\epsilon^{n}(1-\varepsilon)^{(N-n)}$$

- The true factorial moment is recovered by dividing the measured factorial moment with appropriate power of the detection efficiency.

$$f_q^{corrected} = \frac{f_q^{measured}}{\varepsilon^q} = \frac{< n(n-1)...(n-q+1) >}{\varepsilon^q}$$

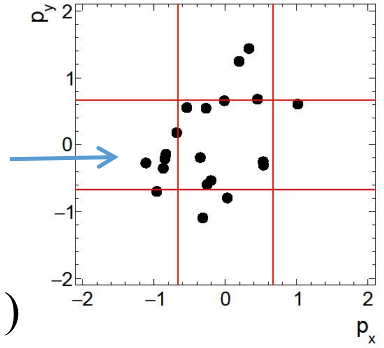
- Definition of $F_q(M)$:

$$F_q(M) = \frac{< \frac{1}{M^D}\sum_{i=1}^{M^D} n_i(n_i-1)\ldots(n_i-q+1) >}{< \frac{1}{M^D}\sum_{i=1}^{M^D} n_i >^q}$$

- Since the available region of phase space is partitioned into a lattice of M^2 equal-size cells, every element of measured $F_q(M)$ should be corrected one by one.

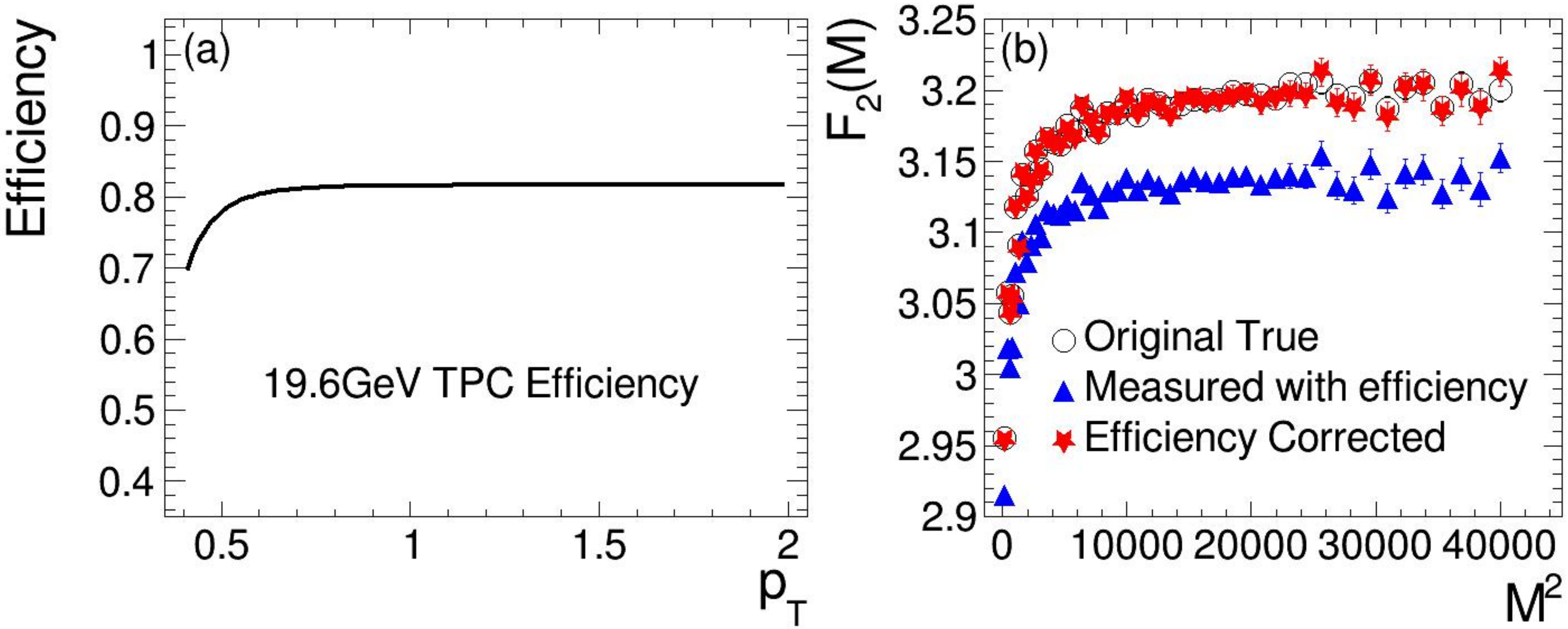
$$F_q^{corrected}(M) = \frac{< \frac{1}{\mathrm{M}^2}\sum_{i=1}^{M^2} \frac{n_i(n_i-1)\ldots(n_i-q+1)}{\bar{\varepsilon}_i^q} >}{< \frac{1}{M^2}\sum_{i=1}^{M^2} \frac{n_i}{\bar{\varepsilon}_i} >^q}$$

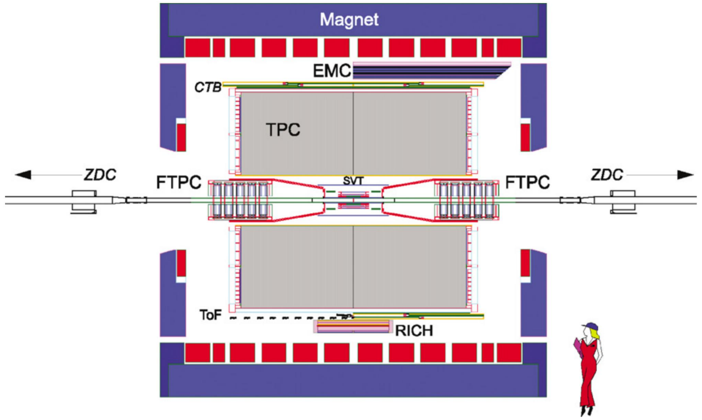
$\bar{\varepsilon}_i = < \frac{1}{n_i}\sum_{j=1}^{n_i} \varepsilon_j >$ in i-th cell



J. Wu, Z. Li, et al., arXiv: 2104.11524 (2021)

Efficiency correction (STAR TPC efficiency)





- The STAR TPC tracking efficiency firstly increases with increasing p_T , and then gets saturated in higher p_T regions.

- The measured SFMs with efficiency are systematically smaller than the true ones, especially in the large number of partitioned cells. However, the efficiency corrected SFMs are found to be well consistent with the original true ones.

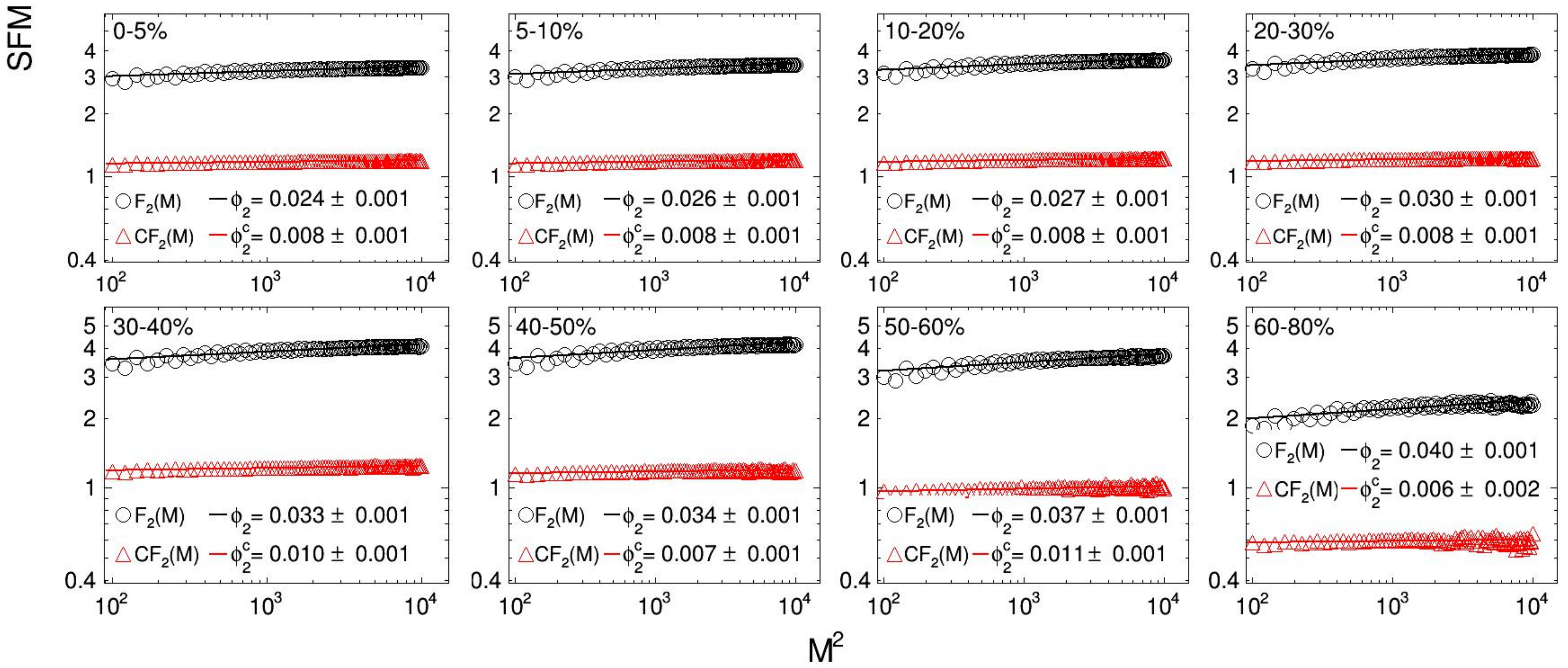
Summary

- The energy and centrality dependence of the SFMs are investigated in Au + Au collisions at $\sqrt{s_{NN}}$ = 7.7 – 200GeV by using the UrQMD model. The second-order intermittency index is found to be small but non-zero in the transport model without implementing any critical related self-similar fluctuations.
- A cumulative variable method can successfully reduce the distortion of a Gaussian background contribution from a pure self-similar event sample generated by the CMC model. After applying the method to the UrQMD event sample, it confirms that the non-critical background effect can be removed and the value of the intermittency index is close to zero.
- We derive a cell-by-cell formula in the calculation of SFMs in heavy-ion collisions. The validity of the method has been checked in the UrQMD event sample which is employed with detector tracking efficiencies used in the RHIC/STAR experiment.
- The cell-by-cell method provides a precise and effective way for the efficiency correction on SFMs. The correction method is universal and can be applied to the ongoing studies of intermittency in heavy-ion experiments.

Thank you!

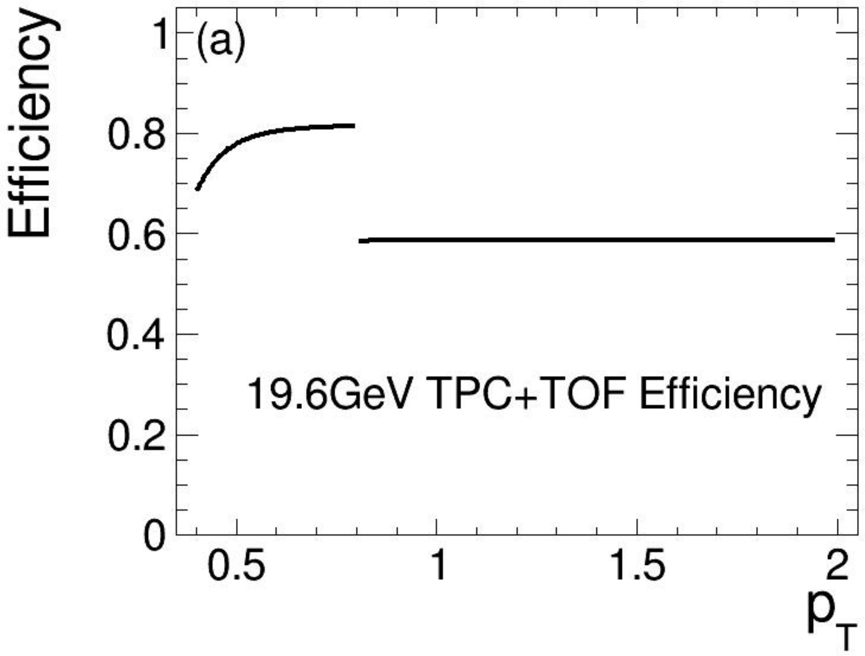
Backup slides

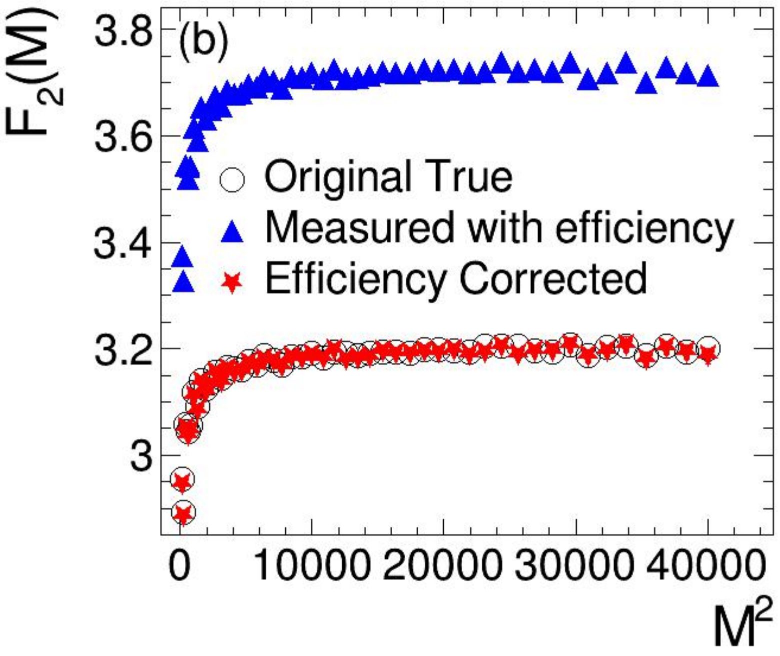
Centrality dependence of $CF_2(M)$ from UrQMD model

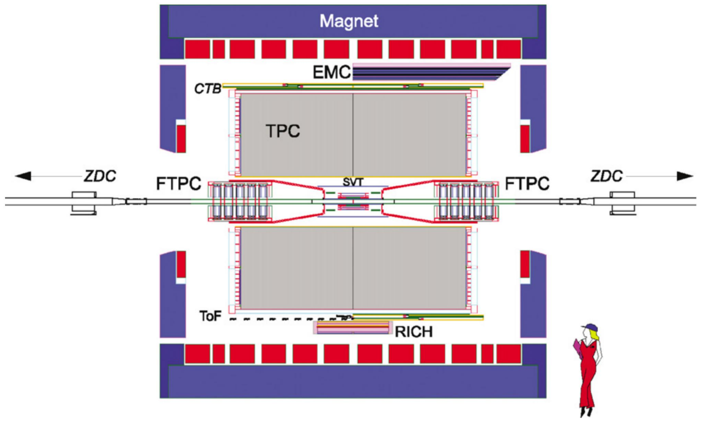


- It confirms that the cumulative variable method can effectively subract the background contribtuion in the intermittency analysis in UrQMD model.

Efficiency correction (TPC+TOF)







- There is a steplike dependence of the efficiencies on p_T since the particle identification method is different between TPC and TOF detectors at STAR experiment.

- The SFMs corrected by the proposed cell-by-cell method are verified to be coincide with the original true ones.

UrQMD : Ultra-relativistic Quantum Molecular Dynamics

- The UrQMD model is a microscopic transport approach to pp, pA and AA interactions at relativistic energies.

Prog. Part. Nucl. Phys. 41, 255 (1998)
J. Phys. G 25, 1859 (1999)

- The UrQMD model has been successfully used to predict and interpret experimental data at various energies and for a multitude of observables and reaction systems.

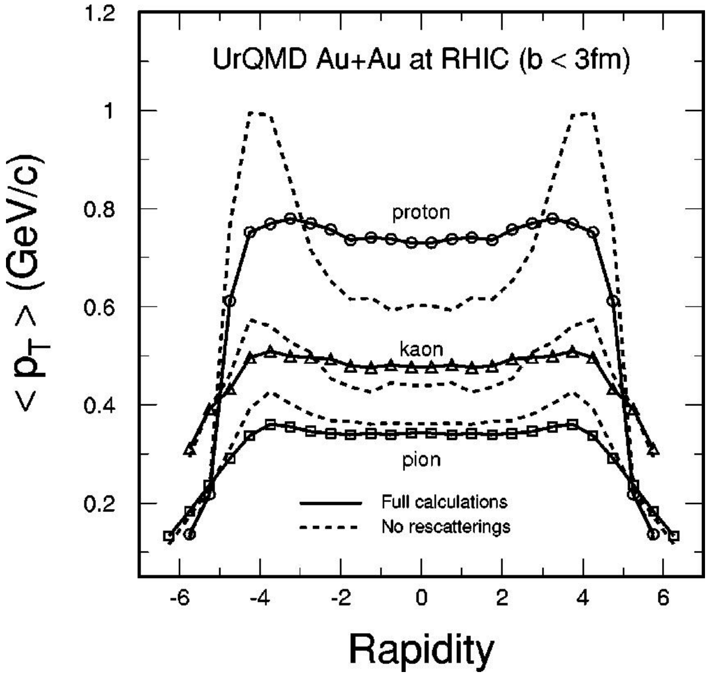
- **Hadron cascade (version 2.3):**
  - Out-of-equilibrium transport model (Boltzmann equation).
  - Particles interact via:

    - Measured and calculated cross sections.

    - String excitation and fragmentation.

    - Formation and decay of resonances.

- Its a suitable simulator to estimate the non-critical contribution and other trivial background effects in the measurement of correlations and fluctuations in heavy-ion collisions.



Phys. Rev. C 62, 024904 (2000)
Phys. Rev. C 69, 054907 (2004)