

Trace anomaly contribution to hydrogen atom mass

中国物理学会高能物理分会第十三届全国粒子物理学学术会议 August 17th, 2021

Bao-Dong Sun (孙保东)
Shandong University, Qingdao

In collaboration with Ze-hao Sun, Jian Zhou, Ref: [2012.09443](#)

Proton mass 938 MeV: Higgs mechanism: ~3 MeV per valence quark
quantum effect (**Trace anomaly**)

Ji's decomposition: Ji, PRL.74.1071(1995)

$$H_{\text{QCD}} = H_m + H_q + H_g + H_a = \int d^3x T^{00}(0, \mathbf{x})$$

$$H_m = \int d^3\vec{x} \bar{\psi} m \psi, \quad \longleftarrow \quad \text{Quark mass}$$

$$H_q = \int d^3\vec{x} \bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\alpha}) \psi, \quad \longleftarrow \quad \text{Quark energy}$$

$$H_g = \int d^3\vec{x} \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2), \quad \longleftarrow \quad \text{Gluon energy}$$

$$H_a = \int d^3\vec{x} \frac{9\alpha_s}{16\pi} (\mathbf{E}^2 - \mathbf{B}^2). \quad \longleftarrow \quad \text{Quantum Anomalous Energy (QAE)}$$

Proton mass 938 MeV: Higgs mechanism: ~3 MeV per valence quark
quantum effect (Trace anomaly)

Ji's decomposition: Ji, PRL.74.1071(1995)

$$H_{\text{QCD}} = H_m + H_q + H_g + H_a = \int d^3x T^{00}(0, \mathbf{x})$$

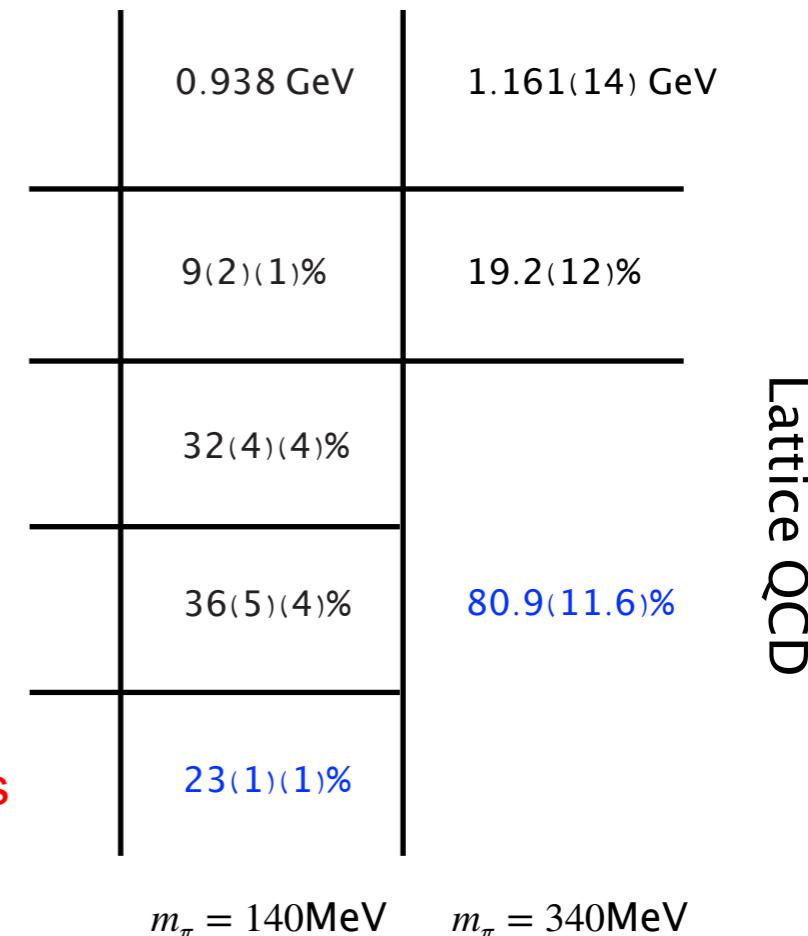
$$H_m = \int d^3\vec{x} \bar{\psi} m \psi, \quad \longleftarrow \quad \text{Quark mass}$$

$$H_q = \int d^3\vec{x} \bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\alpha}) \psi, \quad \longleftarrow \quad \text{Quark energy}$$

$$H_g = \int d^3\vec{x} \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2), \quad \longleftarrow \quad \text{Gluon energy}$$

$$H_a = \int d^3\vec{x} \frac{9\alpha_s}{16\pi} (\mathbf{E}^2 - \mathbf{B}^2). \quad \longleftarrow \quad \text{Quantum Anomalous Energy (QAE)}$$

$$T_\mu^\mu = H_m + (\gamma_m H_m + \frac{\beta}{2g} F^2)$$



QCD EMT: non-perturbative; Lattice, Models, Large N_c

[**On the hadron mass decomposition**](#) C. Lorcé, Eur.Phys.J.C 78 (2018) 2, 120

[**Quark and gluon contributions to the QCD trace anomaly**](#) Y. Hatta, A. Rajan, K. Tanaka, JHEP 12 (2018), 008

[**Gauge-Invariant Decomposition of Nucleon Spin**](#) X.-D. Ji, Phys.Rev.Lett. 78 (1997), 610-613

[**Gravitational form factors of a spin one particle**](#) M. V. Polyakov, **B.-D. Sun**, Phys.Rev.D 100 (2019) 3, 036003

[**Covariant multipole expansion of local currents for massive states of any spin**](#) S. Cotogno, et. al., Phys.Rev.D 101 (2020) 5, 056016

[**Gravitational form factors of a baryon with spin-3/2**](#) J.-Y. Kim, **B.-D. Sun**, Eur.Phys.J.C 81 (2021) 1, 85

[**Proton Mass Decomposition from the QCD Energy Momentum Tensor**](#) Y.-B. Yang, et al.: Phys.Rev.Lett. 121 (2018) 21, 212001

[**A Demonstration of Hadron Mass Origin from QCD Trace Anomaly**](#), F. He, P. Sun, Y.-B. Yang e-Print: 2101.04942 [hep-lat]

[**Gluons in charmoniumlike states**](#) xQCD Collaboration • Wei Sun et al. Phys.Rev.D 103 (2021) 9, 094503

[**Gluon gravitational structure of hadrons of different spin**](#) D. A. Pefkou, D. C. Hackett, P. E. Shanahan e-Print: 2107.10368 [hep-lat]

[**Near threshold J/ψ and Υ photoproduction at JLab and RHIC**](#) Y. Hatta, A. Rajan, D.-L. Yang, Phys.Rev.D 100 (2019) 1, 014032

[**Determination of the Proton Trace Anomaly Energy from the Near-Threshold Vector Meson Photoproduction Data**](#)

W. Kou , R. Wang, X. Chen, e-Print: 2103.10017 [hep-ph]

[**Gluon Gravitational Form Factors at Large Momentum Transfer**](#) X.-B. Tong, J.-P. Ma, F. Yuan, e-Print: 2101.02395 [hep-ph]

[**Gravitational form factors of \\$\\rho\\$ meson with a light-cone constituent quark model**](#) **B.-D. Sun**, Y.-B. Dong, Phys.Rev.D 101 (2020) 9, 096008

[**Quantum anomalous energy effects on the nucleon mass**](#) X. Ji, Y. Liu, Sci.China Phys.Mech.Astron. 64 (2021) 8, 281012

QCD EMT: non-perturbative; Lattice, Models, Large $N_c \dots \dots$

QED EMT: perturbative; completely solvable, eg: [Hydrogen atom](#)

Assumption: proton serves as an infinitely heavy charge source, decoupled in our treatment

i.e.: [Omitting proton mass / Coulomb potential](#)

QED EMT in hydrogen atom:
[\(Poincare invariance\)](#)

Normalization:

$$\langle P' | P \rangle = 2P^0(2\pi)^3\delta^3(P' - P)$$

hydrogen atom mass:

$$\langle P | T_\mu^\mu | P \rangle = 2M^2$$

M : hydrogen atom mass

m_0 : electron bare mass

Trace of QED EMT:

$$T_\mu^\mu = (1 + \gamma_m) m_0 \bar{\Psi} \Psi + \frac{\beta(e)}{2e} [F^{\mu\nu} F_{\mu\nu}]_R$$



[Trace anomaly](#)

Hydrogen mass at LO (omitting proton mass):

$$M_{H,0} = \frac{\langle H | \int d^3x m_0 \bar{\Psi}(x) \Psi(x) | H \rangle}{\langle H | H \rangle} = m \int d^3x \varphi_0^\dagger(x) \gamma^0 \varphi_0(x)$$

m: electron physical mass

Dirac equation predicts:

$$M_{H,0} = m \sqrt{1 - \alpha^2}$$

φ_0 : ground state wave function

ground state energy of hydrogen: $M_{H,0} - m = m \sqrt{1 - \alpha^2} - m \approx -13.6\text{eV}$

NLO corrections: $\rightarrow m_0 \bar{\Psi}(x) \Psi(x)$ and $F^{\mu\nu}(x) F_{\mu\nu}(x)$ mix \rightarrow Trace anomaly contribution is scheme dependent



Difference between free & bound matters (energy shift)

$$\langle e | [F^{\mu\nu}(x) F_{\mu\nu}(x)]_R | e \rangle = 0$$

$$\langle \gamma | [F^{\mu\nu}(x) F_{\mu\nu}(x)]_R | \gamma \rangle = \langle \gamma | Z_3^{-1} F^{\mu\nu}(x) F_{\mu\nu}(x) | \gamma \rangle_{\text{Tree}}$$

Need to choose a subtraction scheme (most natural one)

(Adler et al, 1977; Rodini et al 2020; Metz et al, 2020)

NLO: group $F^{\mu\nu}(x)F_{\mu\nu}(x)$ and $m_0\bar{\Psi}(x)\Psi(x)$

$$T_\mu^\mu = (1 + \gamma_m) m_0 \bar{\Psi} \Psi + \frac{\beta(e)}{2e} [F^{\mu\nu} F_{\mu\nu}]_R$$



BDS, Sun, Zhou, 2012.09443

Ji, Liu, SCPMA.64.8 (2021)

Note: -2 factor? different methods

$$\text{Fig. 1(a)} = \frac{\left\langle H \left| \int d^3x \frac{\beta}{2e} [F^{\mu\nu}(x)F_{\mu\nu}(x)]_R \mathcal{T} e^{-i \int d^4y H_I(y)} \right| H \right\rangle}{\langle H | H \rangle}$$

$$2 \times \text{Fig. 1(b)} = 8\alpha^2 \int d^3y \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{y}}}{\vec{q}^2 + i\epsilon} \int_0^1 da \frac{a(1-a)m^2}{m^2 + a(1-a)\vec{q}^2} [\bar{\varphi}_0(y)\gamma^0\varphi_0(y)]$$

$$\text{FIG. 1(a)} + 2 \times \text{FIG. 1(b)} \approx \frac{-4\alpha^2}{15m^2} \varphi_0^\dagger(0)\varphi_0(0) \sim O(\alpha^5) \quad \text{NR limit: } |\vec{q}| \ll m$$

Uehling effect: part of Lamb shift, due to vacuum polarization!

NLO: $m_0 \bar{\Psi}(x) \Psi(x)$ in **free** state, self-energy corrections

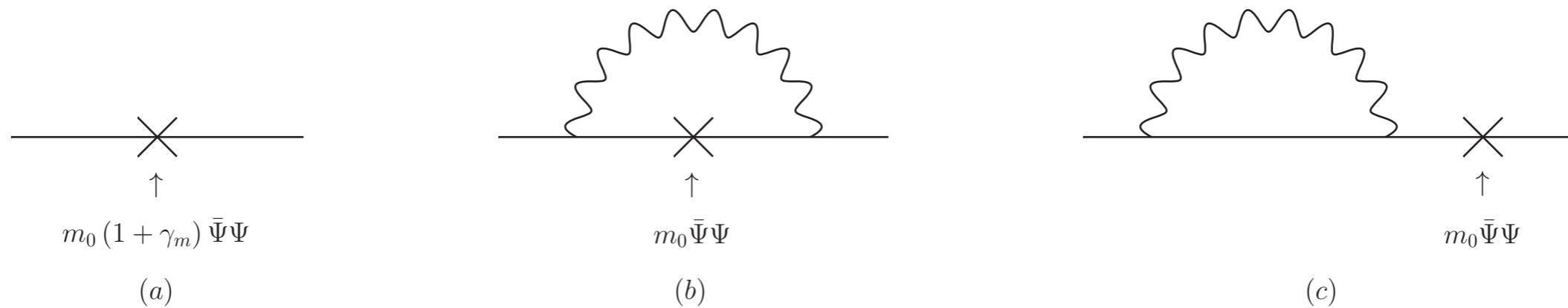


FIG. 2: NLO corrections to the electron mass term.

$$\text{Fig.2(a)} = \gamma_m m_0 + m - \delta m = \gamma_m m_0 + m - \frac{\alpha_{em}}{2\pi} m_0 \int_0^1 da (2-a) \ln \frac{a\Lambda^2}{(1-a)^2 m_0^2}$$

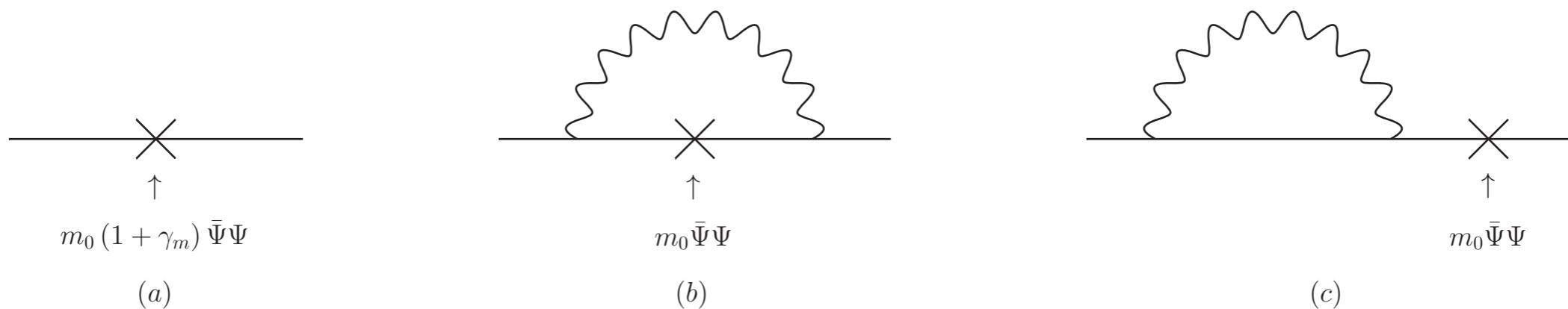
$$\text{Fig.2(b)} = \frac{\alpha_{em}}{2\pi} m_0 \int_0^1 da \left\{ 2 \ln \frac{a\Lambda^2}{(1-a)^2 m_0^2} - \frac{2(2-a)}{(1-a)} \right\}$$

$$2 \times \text{Fig.2(c)} = \frac{\alpha_{em}}{2\pi} m_0 \int_0^1 da \left\{ -a \ln \frac{a\Lambda^2}{(1-a)^2 m_0^2} + \frac{2a(2-a)}{(1-a)} \right\}$$

$$\rightarrow \frac{\langle e | (1 + \gamma_m) m_0 \int d^3x \bar{\Psi}(x) \Psi(x) | e \rangle}{\langle e | e \rangle} = m \quad \text{physical mass of a free electron}$$

An all order proof: Callan-Symanzik equation
(Adler, Collins, Duncan, 1977):

NLO: $m_0 \bar{\Psi}(x) \Psi(x)$ in **bound state**, self-energy corrections: (BDS, Sun, Zhou, 2020)



NRQED, Coulomb gauge:

$$\mathcal{L} = \psi^\dagger \left(i\partial^0 - eA^0 - \frac{\vec{p}^2}{2m_0} + \frac{e}{2m_0}(\vec{p}' + \vec{p}) \cdot \vec{A} - \frac{e^2}{2m_0} \vec{A}^2 - (1 + O(\alpha_{em})) \frac{ie}{2m_0} \sigma \cdot [(\vec{p} - \vec{p}') \times \vec{A}] \right) \psi + \dots$$

$$\text{FIG. 2(b)} + 2 \times \text{FIG. 2(c)} = \frac{\left\langle e \left| m_0 \int d^3x \left[\bar{\Psi}_R(x) \Psi_R(x) - \Psi_R^\dagger(x) \Psi_R(x) \right] \right| e \right\rangle}{\langle e | e \rangle}$$

(2)

$$\approx \frac{\left\langle e \left| \int d^3x \left\{ \psi^\dagger \left[\frac{e}{2m_0} (\vec{p}' + \vec{p}) \cdot \vec{A} - \frac{\vec{p}^2}{2m_0} - \frac{e^2}{2m_0} \vec{A}^2 - \frac{ie}{2m_0} \sigma \cdot [(\vec{p} - \vec{p}') \times \vec{A}] \right] \psi \right\} \right| e \right\rangle}{\langle e | e \rangle}$$

spinor in nonrelativistic limit:

A trick: $Z_2 = Z_3$
(vector current conservation)

dipole vertex

same as 10

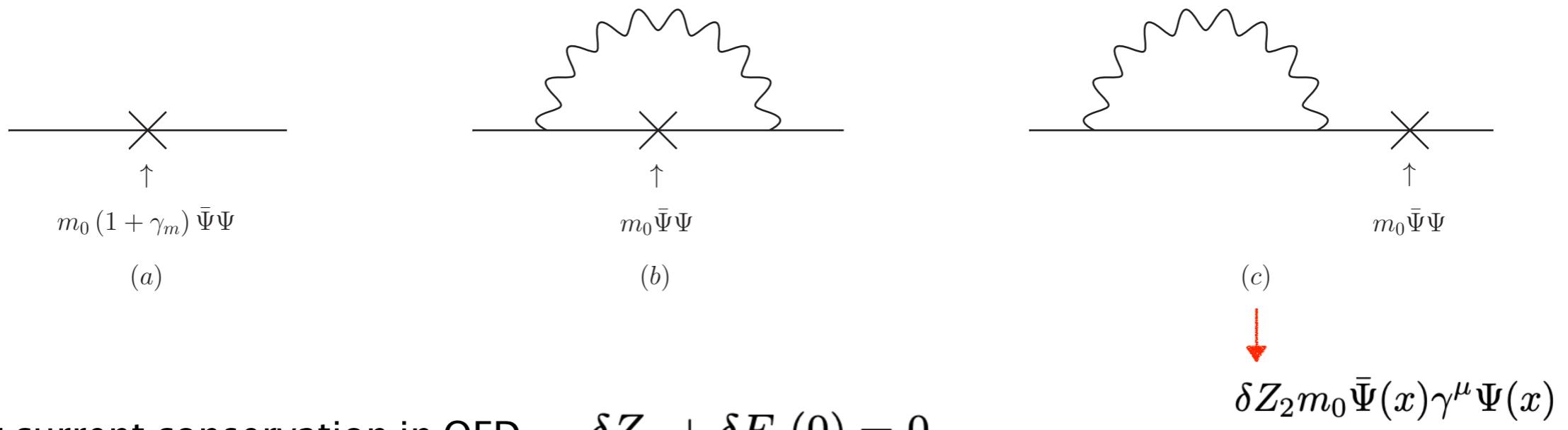
spin-orbital term $\rightarrow 0$



$$\vec{p} \approx \vec{p}'$$



NLO: $m_0 \bar{\Psi}(x) \Psi(x)$ in bound state, self-energy corrections: (BDS, Sun, Zhou, 2020)



Vector current conservation in QED: $\delta Z_2 + \delta F_1(0) = 0$

and for $\delta F_1(0)$:

$$\left\langle H \left| m_0 \int d^3x \bar{\Psi}_R(x) \gamma^\mu \Psi_R(x) \right| H \right\rangle_{\text{NLO}} = \delta F_1(0) \left\langle H \left| \int d^3x \bar{\Psi}_R(x) \gamma^\mu \Psi_R(x) \right| H \right\rangle_{\text{Tree}}$$

Choose $\mu = 0$:

$$2 \times \text{Fig.2}(c) = - \frac{\left\langle H \left| m_0 \int d^3x \bar{\Psi}_R(x) \gamma_0 \Psi_R(x) \right| H \right\rangle_{\text{NLO}}}{\left\langle H \left| \int d^3x \Psi_R^\dagger(x) \Psi_R(x) \right| H \right\rangle_{\text{Tree}}} \\ = - \frac{\left\langle H \left| m_0 \int d^3x \Psi_R^\dagger(x) \Psi_R(x) \right| H \right\rangle_{\text{NLO}}}{\langle H | H \rangle}$$

A trick: $Z_2 = Z_3$
(vector current conservation)

NLO: $m_0 \bar{\Psi}(x) \Psi(x)$ in **bound** state, self-energy corrections: (BDS, Sun, Zhou, 2020)

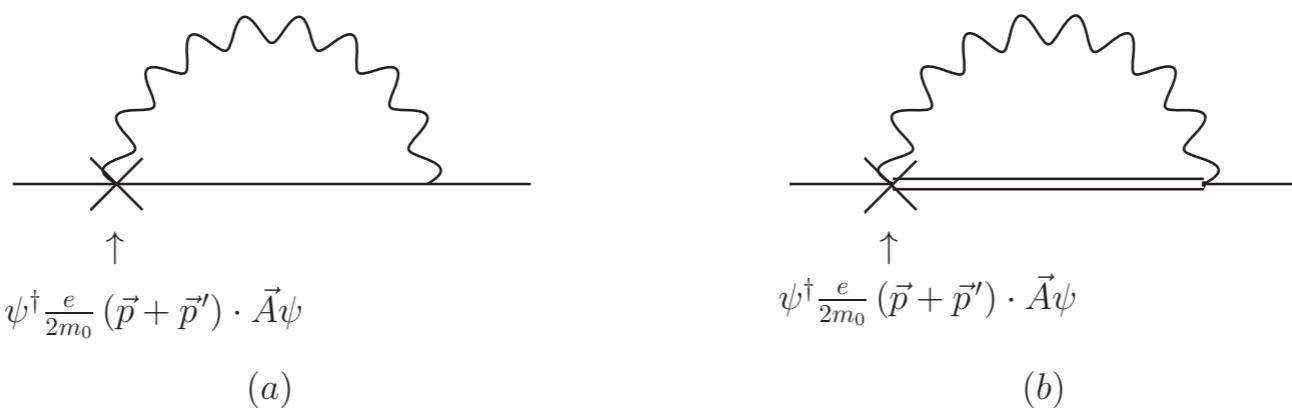


FIG. 3: Part of the NLO correction to the electron mass term in NRQED in vacuum(diagram a), and that in the bound state(diagram b).

- electron propagator: $\sum_M \frac{\varphi_M(x)\varphi_M^\dagger(y)}{\Delta E_M + i\epsilon}$

- photon propagator: $\frac{1}{k^2 + i\epsilon} \rightarrow \frac{1}{k^2 + i\epsilon} - \frac{1}{k^2 - \mu^2 + i\epsilon}$
(scale cut off $\mu < m$)

- energy shift for ground state:

$$Fig.3(b) - Fig.3(a) \approx \frac{4\alpha_{em}^2}{3m^2} \varphi_0^\dagger(0) \varphi_0(0) \left[\ln \frac{\mu}{2\Delta E} + \frac{5}{6} \right]$$

consist with Bethe 1947's result (with $\mu = m$)

(Lamb, et al, 1947)

Note: We did not compute the self energy contribution directly, instead evaluated the expectation value of the mass operator at the NLO.

Table 7. Individual contributions to the Lamb shift E_L , in MHz.

one-loop self-energy
 Uehling one-loop
 vacuum polarization
 QED trace anomaly

$$E_{2S_{1/2}} - E_{2P_{1/2}} \approx 1057.83 \text{ MHz}$$

Yerokhin, Pachucki, Patkos,
 Annalen Phys. 531.5 (2019)

	1S	2S	2P _{1/2}
$Z = 1, {}^1\text{H}, R_C = 0.840\,87(39) \text{ fm}, M/m = 1836.152\,673\,346(81)$			
SE	8 396.453 556 (1)	<u>1 072.958 455</u>	-12.858 661 (1)
Ue	-215.170 186	<u>-26.897 303</u>	-0.000 347
WK	0.002 415	0.000 302	0
Ue(μ had)	-0.008 48 (8)	-0.001 06 (1)	0
SESE	2.335 0 (13)	0.292 48 (16)	0.027 253 (4)
SEVP	0.288 39 (16)	0.036 015 (20)	-0.001 241
VPVP	-1.895 224	-0.236 911	-0.000 003
QED(ho)	0.001 83 (96)	0.000 23 (12)	-0.000 216
RRM	-12.765 917	-1.633 931	0.011 741
REC	2.402 830	0.340 469	-0.016 656
REC(ho)	0.013 16 (74)	-0.003 227 (92)	-0.001 335 (4)
FNS	1.107 6 (10)	0.138 45 (13)	0
NUCL5	-0.000 109 (1)	-0.000 014	0
NUCL6	0.001 07 (39)	0.000 140 (49)	0.000 001
FNS(rad)	-0.000 135 (1)	-0.000 017	0
NSE	0.004 63 (16)	0.000 585 (20)	0.000 001 (20)
Total	8 172.770 4 (18)(10)	<u>1 044.994 66 (23)(13)</u>	<u>-12.839 463 (21)(0)</u>
	

SE, one-loop self-energy; Ue, Uehling one-loop vacuum polarization; WK, Wichmann–Kroll one-loop vacuum-polarization; Ue(μ had), Uehling muon and hadronic vacuum polarization; SESE, two-loop self-energy; SEVP, electron self-energy with vacuum-polarization insertions; VPVP, two-loop vacuum-polarization; QED(ho), three-loop QED correction; RRM, relativistic reduced mass correction (see text); REC, recoil correction E_{REC} ; REC(ho), sum of higher-order recoil corrections $E_{\text{REC},2}$, $E_{\text{REC,hfs}}$, and E_{RREC} ; FNS, leading-order fns correction $E_{\text{nuc}}^{(4)}$; NUCL5, $(Z\alpha)^5$ nuclear correction $E_{\text{nuc}}^{(5)}$; NUCL6, $(Z\alpha)^6$ nuclear correction $E_{\text{nuc}}^{(6)}$; FNS(rad), radiative fns correction $E_{\text{fns,rad}}$; NSE, nuclear self-energy correction E_{NSE} .

Summary

- QED EMT Trace Anomaly contribution to Hydrogen atom mass, which turns out to be part of Lamb shift.
- The combination of the trace anomaly (F^2) contribution and the NLO correction to the $m_0 \bar{\Psi} \Psi$ from the vacuum polarization diagram is related to the Uehling effect which is the small part of the Lamb shift.
- It is in sharp contrast to proton mass decomposition case where trace anomaly part is considered to be a new additional contribution to the bound state mass other than parton kinematic energy and potential energy.
- The method developed in this note can potentially be applied to analysis the mechanical properties of hydrogen system.

Thanks for your attention!

Hydrogen energy levels

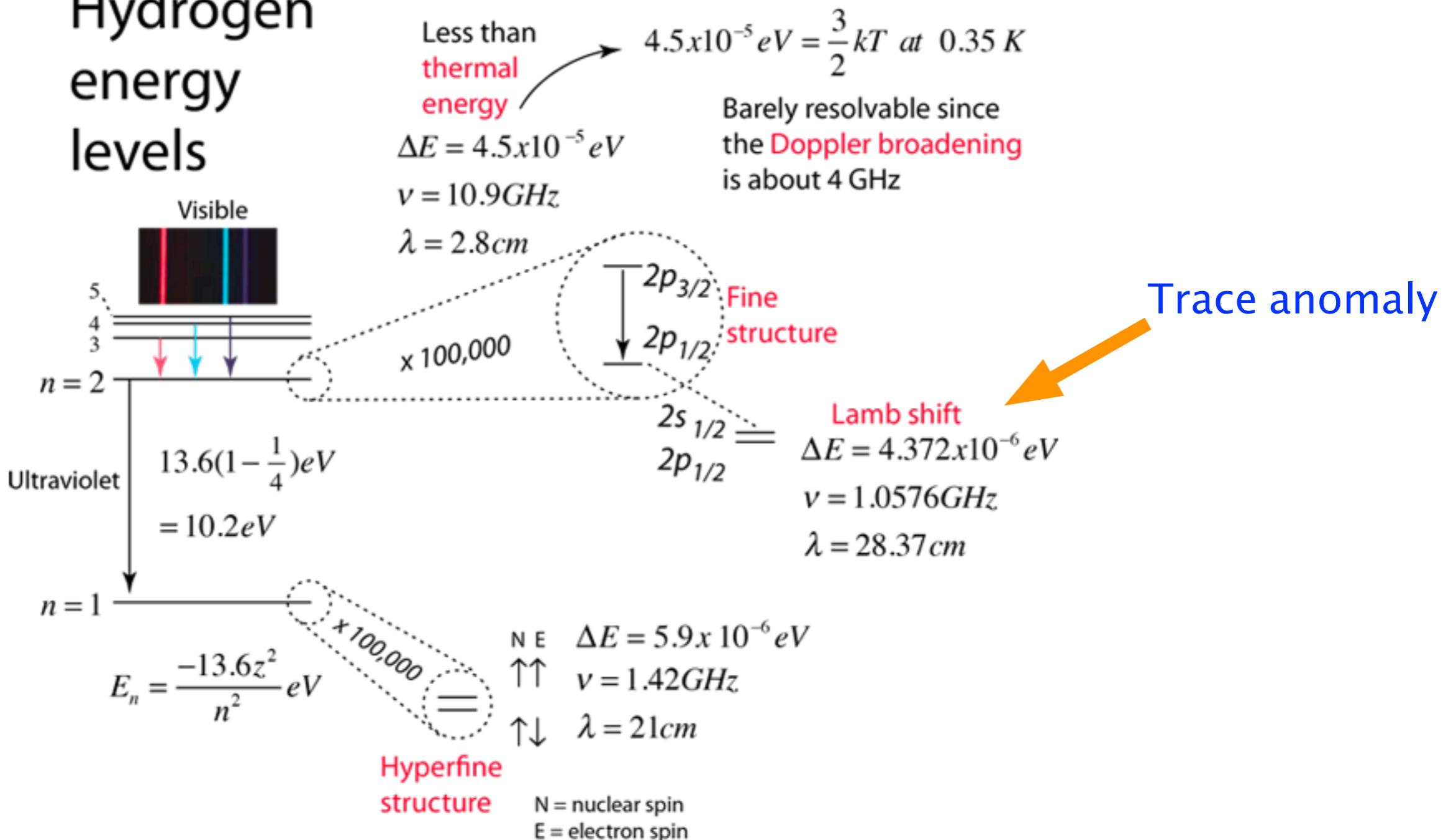


Fig ref: <http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/lamb.html>