Non-Abelian Covariant Chiral Kinetic Equation

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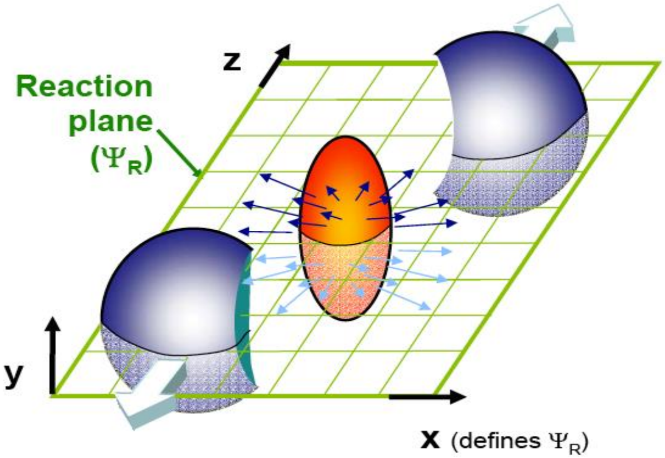
Introduction



- Abelian CKT

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A. Huang, S. Shi, Y. Jiang, J. Liao and P. Zhuang, Phys. Rev. D 98 (2018) no.3, 036010
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S. Lin and L. Yang, Phys. Rev. D 101 (2020) no.3, 034006
……

- non-Abelian CKT

  - QGP in HIC is mainly controlled by QCD— SU(3) non-Abelian gauge field.
  - The decoherence from CGC to QGP is still an open question.
  - Only very restricted work had discussed the CKT in non-Abelian gauge field.

M. Stone and V. Dwivedi, Phys. Rev. D 88 (2013) no.4, 045012;
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T. Hayata and Y. Hidaka, PTEP 2017 (2017) no.7, 073I01;
N. Mueller and R. Venugopalan, Phys. Rev. D 99 (2019) no.5, 056003;
……

The non-Abelian CKT deserves further study

Introduction

➢ Wigner function in Abelian case :

D. Vasak, M. Gyulassy, and H. T. Elze, Annals Phys. 173, 462 (1987)

$$W_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \left\langle \bar{\psi}_\beta(x_2) U(x_2, x_1) \psi_\alpha(x_1) \right\rangle$$

(4×4)spinor

- decompose the Wigner function: $W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^\mu\gamma^5\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$
- define chiral Wigner function: $\mathscr{J}_s^\mu = \frac{1}{2}(\mathscr{V}^\mu + s\mathscr{A}^\mu)\,.$ s = +1/ − 1 ⇨ right/left hand

➢ Wigner function in non-Abelian case:

H. T. Elze, M. Gyulassy and D. Vasak, Nucl. Phys. B 276, 706 (1986)

S. Ochs and U. W. Heinz, Annals Phys. 266, 351 (1998)

$$W^{ab}_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \left\langle \bar{\psi}^c_\beta(x_2) U^{cb}(x_2, x) U^{ad}(x, x_1) \psi^d_\alpha(x_1) \right\rangle$$

(N×N)color ⊗(4×4)spinor

Introduction

- chiral Wigner equation:

S. Ochs and U. W. Heinz, Annals Phys. 266, 351 (1998)

$$
\begin{aligned}
0 &= \{\Pi_\mu, \mathscr{J}_s^\mu\}\,, \\
0 &= [G_\mu, \mathscr{J}_s^\mu]\,, \\
0 &= \{\Pi^\mu, \mathscr{J}_s^\nu\} - \{\Pi^\nu, \mathscr{J}_s^\mu\} - s\hbar\epsilon^{\mu\nu\alpha\beta}\left[G_\alpha, \mathscr{J}_{s\beta}\right],
\end{aligned}
$$

$$
\begin{aligned}
\Pi_\mu &= p_\mu - \hbar\frac{ig}{2}\sum_{k=0}^{\infty}\left(-\frac{i\hbar}{2}\right)^k \frac{k+1}{(k+2)!}\left[(\partial_p\cdot\mathscr{D})^k F_{\nu\mu}\right]\partial_p^\nu, \\
G_\mu &= D_\mu - \frac{g}{2}\sum_{k=0}^{\infty}\left(-\frac{i\hbar}{2}\right)^k \frac{1}{(k+1)!}\left[(\partial_p\cdot\mathscr{D})^k F_{\nu\mu}\right]\partial_p^\nu
\end{aligned}
$$

$$
F_{\mu\nu} = -\frac{\hbar}{ig}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{ig}{\hbar}[A_\mu, A_\nu]
$$

$$
\mathscr{D}_\mu(x)\mathcal{T}(x) \equiv [D_\mu(x), \mathcal{T}(x)]\,,
$$

- covariant derivative: $D_\mu(x) = \partial_\mu - \frac{ig}{\hbar}A_\mu(x),$ $\quad F_{\mu\nu}\partial_p^\nu$

- trace expansion in powers of $\hbar$:

$$
\Pi_\mu = \sum_{k=0}^{\infty}\hbar^k\Pi_\mu^{(k)}, \qquad G_\mu = \sum_{k=0}^{\infty}\hbar^k G_\mu^{(k)} \qquad W(x,p) = \sum_{k=0}^{\infty}\hbar^k W^{(k)}(x,p),
$$

Covariant gradient expansion

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- expansion of operator:

$$\Pi_\mu^{(0)} = p_\mu, \quad \Pi_\mu^{(1)} = \frac{ig}{4} F_{\mu\nu}\partial_p^\nu, \quad \Pi_\mu^{(2)} = \frac{g}{12}\left[(\partial_p\cdot\mathscr{D})F_{\mu\nu}\right]\partial_p^\nu, \quad G_\mu^{(0)} = D_\mu + \frac{g}{2}F_{\mu\nu}\partial_p^\nu, \quad G_\mu^{(1)} = -\frac{ig}{8}\left[(\partial_p\cdot\mathscr{D})F_{\mu\nu}\right]\partial_p^\nu$$

- auxiliary vector: $n^\mu, \quad n^2 = 1, \quad X^\mu = X_n n^\mu + \bar{X}^\mu$

- Wigner equation up to $\mathcal{O}(\hbar^0)$

$$0 = \bar{p}^\mu \mathscr{J}_n^{(0)} - p_n \bar{\mathscr{J}}^{(0)\mu}$$

$$\bar{\mathscr{J}}^{(0)\mu} = \bar{p}^\mu \frac{\mathscr{J}_n^{(0)}}{p_n},$$

$$0 = p_n \mathscr{J}_n^{(0)} + \bar{p}_\mu \bar{\mathscr{J}}^{(0)\mu},$$

$$0 = \bar{p}^\mu \bar{\mathscr{J}}^{(0)\nu} - \bar{p}^\nu \bar{\mathscr{J}}^{(0)\mu}$$

$p^2 \frac{\mathscr{J}_n^{(0)}}{p_n} = 0$, time-like component: $\frac{\mathscr{J}_n^{(0)}}{p_n} = f^{(0)}\delta(p^2)$

automatically satisfied

kinetic equation: $0 = \left[G_n^{(0)}, \mathscr{J}_n^{(0)}\right] + \left[\bar{G}_\mu^{(0)}, \bar{\mathscr{J}}^{(0)\mu}\right]$ where $G_n^{(0)} = \partial_t - \frac{ig}{\hbar}A_n + \frac{g}{2}F_{n\nu}\partial_p^\nu$

- up to $\mathcal{O}(\hbar^1)$

$$\bar{\mathscr{J}}^{(1)\mu} = \bar{p}^\mu \frac{\mathscr{J}_n^{(1)}}{p_n} + \frac{s}{2p_n}\bar{\epsilon}^{\mu\alpha\beta}\left[G_\alpha^{(0)}, \bar{p}_\beta \frac{\mathscr{J}_n^{(0)}}{p_n}\right] + \frac{1}{2p_n}\left(\left\{\bar{\Pi}^{(1)\mu}, p_n \frac{\mathscr{J}_n^{(0)}}{p_n}\right\} - \left\{\Pi_n^{(1)}, \bar{p}^\mu \frac{\mathscr{J}_n^{(0)}}{p_n}\right\}\right)$$

time-like components are independent

Non-Abelian Wigner functions and kinetic equations

- 0^{th} order

Wigner function: $$\mathscr{J}^{(0)\mu} = p^\mu f^{(0)} \delta(p^2).$$

kinetic equation: $$0 = \left[G_\mu^{(0)}, p^\mu f^{(0)} \delta(p^2)\right]$$

- 1^{st} order

Wigner function: $$\mathscr{J}^{(1)\mu} = p^\mu \left[f^{(1)}\delta(p^2) - \frac{s}{2p_n}\bar{\epsilon}^{\nu\alpha\beta} p_\nu \left\{\frac{g}{2}F_{\alpha\beta}, f^{(0)}\right\}\delta'(p^2) + \left\{\Pi_\nu^{(1)}, p^\nu f^{(0)}\right\}\delta'(p^2)\right] + \frac{s}{2p_n}\bar{\epsilon}^{\mu\alpha\beta}\left[G_\alpha^{(0)}, \bar{p}_\beta \frac{\mathscr{J}_n^{(0)}}{p_n}\right] + \frac{1}{2p_n}\left(\left\{\bar{\Pi}^{(1)\mu}, p_n \frac{\mathscr{J}_n^{(0)}}{p_n}\right\} - \left\{\Pi_n^{(1)}, \bar{p}^\mu \frac{\mathscr{J}_n^{(0)}}{p_n}\right\}\right).$$

kinetic equation: $$\left[G_\mu^{(0)}, p^\mu \frac{\mathscr{J}_n^{(1)}}{p_n}\right] = -\frac{s}{2}\bar{\epsilon}^{\mu\alpha\beta}\left[\bar{G}_\mu^{(0)}, \frac{1}{p_n}\left[G_\alpha^{(0)}, \bar{p}_\beta \frac{\mathscr{J}_n^{(0)}}{p_n}\right]\right] - \left[G_\mu^{(1)}, p^\mu \frac{\mathscr{J}_n^{(0)}}{p_n}\right] - \frac{1}{2}\left[\bar{G}_\mu^{(0)}, \frac{1}{p_n}\left(\left\{\bar{\Pi}^{(1)\mu}, p_n \frac{\mathscr{J}_n^{(0)}}{p_n}\right\} - \left\{\Pi_n^{(1)}, \bar{p}^\mu \frac{\mathscr{J}_n^{(0)}}{p_n}\right\}\right)\right]$$

Constraint equations

- 1^{st} order constraint equation:

$$\left[\tilde{F}^{\alpha\beta}, \mathscr{J}_{\alpha}^{(0)}\right] = 0$$

- 2^{nd} order constraint equation:

$$\left[\tilde{F}_{\alpha\beta}, \mathscr{J}^{(1)\alpha}\right] = -\frac{3}{32}\left[\left[\tilde{F}_{\nu\alpha}\partial^p_\beta - \tilde{F}_{\nu\beta}\partial^p_\alpha, F^{\nu\kappa}\partial^p_\kappa\right], \mathscr{J}^{(0)\alpha}\right]$$

- these constraints are unique for non-Abelian gauge field
- the "covariant gradient expansion" is not completely identical to $\hbar$ expansion for non-Abelian gauge field
- $[D_\mu, D_\nu] = igF_{\mu\nu}/\hbar$ will contribute to the lower power order through iterative process
- we expect that the constraint equation should be satisfied automatically while including all the contributions

Color decomposition

- decompose Wigner function: $\mathscr{J}_\mu(x,p) = \mathscr{J}_\mu^I(x,p)\mathbf{1} + \mathscr{J}_\mu^a(x,p)t^a$
  singlet multiplet
- kinetic equations:

$$
\begin{aligned}
\partial_\mu^x \mathscr{J}^{(0)I\mu} &= -\frac{1}{N} G_\mu^{(0)a} \mathscr{J}^{(0)a\mu}, \\
\mathscr{D}_\mu^{ac} \mathscr{J}^{(0)c\mu} &= -2G_\mu^{(0)a} \mathscr{J}^{(0)I\mu} - d^{bca} G_\mu^{(0)b} \mathscr{J}^{(0)c\mu}, \\
\partial_\mu^x \mathscr{J}^{(1)I\mu} &= -\frac{1}{N} G_\mu^{(0)a} \mathscr{J}^{(1)a\mu}, \\
\mathscr{D}_\mu^{ac} \mathscr{J}^{(1)c\mu} &= -2G_\mu^{(0)a} \mathscr{J}^{(1)I\mu} - d^{bca} G_\mu^{(0)b} \mathscr{J}^{(1)c\mu} - i\hbar f^{bca} G_\mu^{(1)b} \mathscr{J}^{(0)c\mu}.
\end{aligned}
$$

- Wigner functions:

$$
\begin{aligned}
\mathscr{J}^{(0)I\mu} &= p^\mu f^{(0)I} \delta(p^2), \\
\mathscr{J}^{(0)a\mu} &= p^\mu f^{(0)a} \delta(p^2), \\
\mathscr{J}^{(1)I\mu} &= p^\mu f^{(1)I} \delta(p^2) - \frac{s}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \frac{g}{2N} F_{\alpha\beta}^a f^{(0)a} \delta'(p^2) + \frac{s}{2p_n} \bar{\epsilon}^{\mu\alpha\beta} p_\beta \left(\partial_\alpha^x f^{(0)I} + \frac{1}{N} G_\alpha^{(0)a} f^{(0)a} \right) \delta(p^2), \\
\mathscr{J}^{(1)a\mu} &= p^\mu f^{(1)a} \delta(p^2) - s\epsilon^{\mu\nu\alpha\beta} p_\nu \left(\frac{g}{2} F_{\alpha\beta}^a f^{(0)I} + \frac{1}{2} d^{bca} \frac{g}{2} F_{\alpha\beta}^b f^{(0)c} \right) \delta'(p^2) \\
&\quad + \frac{s}{2p_n} \bar{\epsilon}^{\mu\alpha\beta} p_\beta \left(\mathscr{D}_\alpha^{ac} f^{(0)c} + 2G_\alpha^{(0)a} f^{(0)I} + \hbar d^{bca} G_\alpha^{(0)b} f^{(0)c} \right) \delta(p^2) \\
&\quad + \frac{1}{2p_n} i\hbar f^{bca} \left(\bar{\Pi}^{(1)b\mu} \left[p_n f^{(0)c} \delta(p^2) \right] - \Pi_n^{(1)b} \left[\bar{p}^\mu f^{(0)c} \delta(p^2) \right] \right) + i\hbar f^{bca} p^\mu \left[\Pi_\nu^{(1)b} \left(p^\nu f^{(0)c} \right) \right] \delta'(p^2)
\end{aligned}
$$

- the singlet and multiplet distributions are totally coupled with each other

Choose the reference frame with velocity n, n′

- choose a frame with n

$$\mathscr{J}^{(0)\mu} = p^{\mu}\frac{\mathscr{J}_{n}^{(0)}}{p_{n}},$$
$$\mathscr{J}^{(1)\mu} = p^{\mu}\frac{\mathscr{J}_{n}^{(1)}}{p_{n}} - \frac{s}{2p_{n}}\epsilon^{\mu\nu\alpha\beta}n_{\nu}\left[G_{\alpha}^{(0)}, \mathscr{J}_{\beta}^{(0)}\right] + \frac{1}{2p_{n}}\left(\left\{\Pi^{(1)\mu}, \mathscr{J}_{n}^{(0)}\right\} - \left\{\Pi_{n}^{(1)}, \mathscr{J}^{(0)\mu}\right\}\right).$$

- choose another frame with n'

$$\mathscr{J}^{(0)\mu} = p^{\mu}\frac{\mathscr{J}_{n'}^{(0)}}{p_{n'}},$$
$$\mathscr{J}^{(1)\mu} = p^{\mu}\frac{\mathscr{J}_{n'}^{(1)}}{p_{n'}} - \frac{s}{2p_{n'}}\epsilon^{\mu\nu\alpha\beta}n'_{\nu}\left[G_{\alpha}^{(0)}, \mathscr{J}_{\beta}^{(0)}\right] + \frac{1}{2p_{n'}}\left(\left\{\Pi^{(1)\mu}, \mathscr{J}_{n'}^{(0)}\right\} - \left\{\Pi_{n'}^{(1)}, \mathscr{J}^{(0)\mu}\right\}\right).$$

Wigner functions $\mathscr{J}^{(0)\mu}$ and $\mathscr{J}^{(1)\mu}$ should not depend on the auxiliary vector

- the modified Lorentz transformation

$$\delta\left(\frac{\mathscr{J}_{n}^{(0)}}{p_{n}}\right) = \frac{\mathscr{J}_{n'}^{(0)}}{p_{n'}} - \frac{\mathscr{J}_{n}^{(0)}}{p_{n}} = 0,$$
$$\delta\left(\frac{\mathscr{J}_{n}^{(1)}}{p_{n}}\right) = \frac{\mathscr{J}_{n'}^{(1)}}{p_{n'}} - \frac{\mathscr{J}_{n}^{(1)}}{p_{n}}$$
$$= \frac{s\epsilon^{\mu\nu\alpha\beta}n_{\mu}n'_{\nu}}{2p_{n}p_{n'}}\left[G_{\alpha}^{(0)}, \mathscr{J}_{\beta}^{(0)}\right] - \frac{\left(n_{\mu}n'_{\nu} - n_{\nu}n'_{\mu}\right)}{2p_{n}p_{n'}}\left\{\Pi^{(1)\mu}, \mathscr{J}^{(0)\nu}\right\}.$$

Frame dependence of distribution function

- the transformation after color decomposition

0$^{\text{th}}$ order:

$$\begin{aligned} \delta(p^2)\delta f^{(0)I} &= 0, \\ \delta(p^2)\delta f^{(0)a} &= 0, \end{aligned}$$

Lorentz scalar

1$^{\text{st}}$ order:

$$\begin{aligned} \delta(p^2)\delta f^{(1)I} &= -\delta(p^2)\frac{s\epsilon^{\mu\nu\alpha\beta}n_\mu n'_\nu p_\beta}{2p_n p_{n'}}\left[\partial^x_\alpha f^{(0)I} + \frac{\hbar^2}{N}G^{(0)a}_\alpha f^{(0)a}\right], \\ \delta(p^2)\delta f^{(1)a} &= -\delta(p^2)\frac{s\epsilon^{\mu\nu\alpha\beta}n_\mu n'_\nu p_\beta}{2p_n p_{n'}}\left[\mathscr{D}^{ac}_\alpha f^{(0)c} + 2G^{(0)a}_\alpha f^{(0)I} + \hbar d^{bca}G^{(0)b}_\alpha f^{(0)c}\right] \\ &\quad \boxed{-\frac{n_\mu n'_\nu - n_\nu n'_\mu}{2p_n p_{n'}} i\hbar f^{bca}\Pi^{(1)b\mu}\left(p^\nu f^{(0)c}\delta(p^2)\right).} \end{aligned}$$

unique for non-Abelian gauge field

- 1$^{\text{st}}$ order distribution functions depend on the 0$^{\text{th}}$ order distribution functions .
- these non-trivial transformation dependence are important for choosing some specific solutions.

Non-Abelian chiral anomaly

- distribution functions in free Dirac field:

$n_s^i/\bar{n}_s^i$:quark/antiquark number density, i, j: color index

$$f_s^{(0)I} = \frac{1}{4\pi^3 N}\sum_i \left[\theta(p_0)n_s^i + \theta(-p_0)\bar{n}_s^i\right] - \frac{1}{4\pi^3}\theta(-p_0), \quad f_s^{(0)a} = \frac{1}{2\pi^3}\sum_i t_{ii}^a \left[\theta(p_0)n_s^i + \theta(-p_0)\bar{n}_s^i\right].$$

vacuum contribution $f_{sv}^{(0)}$

- axial Wigner function: $\mathscr{A}^\mu = \sum_{s=\pm 1} s\mathscr{J}^\mu,$
- axial currents: $j_5^\mu = \int d^4p \mathscr{A}^\mu,$

- chiral anomaly:

0th order:

$$\begin{aligned} \partial_\mu^x j_5^{(0)I\mu} &= -\frac{1}{N}\int d^4p G_\mu^{(0)a}\mathscr{A}^{(0)a\mu}, \\ \mathscr{D}_\mu^{ac} j_5^{(0)c,\mu} &= -2\int d^4p \left(G_\mu^{(0)a}\mathscr{A}^{(0)I\mu} - d^{bca}G_\mu^{(0)b}\mathscr{A}^{(0)c\mu}\right) \end{aligned};$$

1st order:

$$\begin{aligned} \partial_\mu^x j_5^{(1)I\mu} &= -\frac{1}{N}\int d^4p G_\mu^{(0)a}\mathscr{A}^{(1)a\mu}, \\ \mathscr{D}_\mu^{ac} j_5^{(1)c,\mu} &= -2\int d^4p \left(G_\mu^{(0)a}\mathscr{A}^{(1)I\mu} - d^{bca}G_\mu^{(0)b}\mathscr{A}^{(1)c\mu} - if^{bca}G_\mu^{(1)b}\mathscr{A}^{(0)c\mu}\right) \end{aligned}$$

where $G_\mu^{(0)a} = \frac{g}{2}F_{\mu\nu}^a\partial_p^\nu, \quad G_\mu^{(1)b} = -\frac{ig}{8}\left[(\partial_p\cdot\mathscr{D})F_{\mu\nu}\right]^b\partial_p^\nu$

the only possible nonvanishing contribution is from the singular vacuum term

Non-Abelian chiral anomaly

- vacuum contribution: $f_{sv}^{(0)} = -\frac{1}{4\pi^3}\theta(-p_0)$

- 0^{th} order : $\partial_\mu^x j_5^{(0)I\mu} = 0, \quad \mathscr{D}_\mu^{ac} j_5^{(0)c,\mu} = 0$ conservation of chiral current

- 1^{st} order : $\partial_\mu^x j_5^{(1)I\mu} = \frac{g^2}{2N} E^a \cdot B^a C_v, \quad \mathscr{D}_\mu^{ac} j_5^{(1)c,\mu} = \frac{g^2}{2} d^{bca} E^b \cdot B^c C_v.$

where $C_v = -\frac{1}{2\pi^2}\int \frac{d^3\mathbf{p}}{2\pi}\partial_{\mathbf{p}} \cdot \left(\frac{\hat{\mathbf{p}}}{2\mathbf{p}^2}\right) = -\frac{1}{2\pi^2}$

Berry curvature

- non-Abelian chiral anomaly

$$\partial_\mu^x j_5^{(1)I\mu} = -\frac{g^2}{4\pi^2 N} E^a \cdot B^a, \quad \mathscr{D}_\mu^{ac} j_5^{(1)c\mu} = -\frac{g^2}{4\pi^2} d^{bca} E^b \cdot B^c.$$

the non-Abelian chiral anomaly originates from the Berry curvature of the vacuum contribution

Non-Abelian distribution function

The specific solution near equilibrium

- 0^{th} order number density:

$$n_s^i = \frac{1}{1+e^{(u\cdot p-\mu_s^i)/T}}, \qquad \bar{n}_s^i = \frac{1}{1+e^{(-u\cdot p+\mu_s^i)/T}}$$

where $\mu_s^i = \mu^i + s\mu_5^i$, i is color index

- constraint equation

$$\partial_\mu^x \frac{u_\nu}{T} + \partial_\nu^x \frac{u_\mu}{T} = 0, \qquad \partial_\mu \frac{\mu_s^i}{T} = g\xi^a t_{ii}^a \frac{E_\mu}{T},$$

this 0^{th} order Wigner function indeed satisfies the 0^{th} order Wigner equation

- modified Lorentz transformation:

$$\delta(p^2)\delta f_s^{(1)I} = -\delta(p^2)\frac{sn'_\nu\tilde{\Omega}^{\nu\sigma}p_\sigma}{2(n'\cdot p)}\frac{df_s^{(0)I}}{dy} + \delta(p^2)\frac{sn_\nu\tilde{\Omega}^{\nu\sigma}p_\sigma}{2(n\cdot p)}\frac{df_s^{(0)I}}{dy},$$

$$\delta(p^2)\delta f_s^{(1)a} = -\delta(p^2)\frac{sn'_\nu\tilde{\Omega}^{\nu\sigma}p_\sigma}{2(n'\cdot p)}\frac{df_s^{(0)a}}{dy} + \delta(p^2)\frac{sn_\nu\tilde{\Omega}^{\nu\sigma}p_\sigma}{2(n\cdot p)}\frac{df_s^{(0)a}}{dy},$$

where $\Omega_{\mu\nu} = \frac{1}{2}\left(\partial_\mu^x \frac{u_\nu}{T} - \partial_\nu^x \frac{u_\mu}{T}\right), \quad \tilde{\Omega}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\Omega^{\alpha\beta}, \quad y = u\cdot p/T.$

- 1^{st} order distribution function :

$$f_s^{(1)I} = -\frac{sn_\nu\tilde{\Omega}^{\nu\sigma}p_\sigma}{2(n\cdot p)}\frac{df_s^{(0)I}}{dy}, \qquad f_s^{(1)a} = -\frac{sn_\nu\tilde{\Omega}^{\nu\sigma}p_\sigma}{2(n\cdot p)}\frac{df_s^{(0)a}}{dy}.$$

Non-Abelian anomalous currents

- 1st order Wigner function:

$$\mathscr{J}_s^{(1)I\mu} = -\frac{s}{2}\tilde{\Omega}^{\mu\nu}p_\nu\frac{df_s^{(0)I}}{dy}\delta(p^2) - \frac{sg}{2N}\tilde{F}^{a,\mu\nu}p_\nu f_s^{(0)a}\delta'(p^2),$$
$$\mathscr{J}_s^{(1)a,\mu} = -\frac{s}{2}\tilde{\Omega}^{\mu\nu}p_\nu\frac{df_s^{(0)a}}{dy}\delta(p^2) - sg\tilde{F}^{b,\mu\nu}p_\nu\left(\delta^{ab}f_s^{(0)I} + \frac{1}{2}d^{bca}f_s^{(0)c}\right)\delta'(p^2)$$

- right/left hand currents:

$$j_s^{(1)I\mu} = \xi_s^I\omega^\mu + \xi_{Bs}^{Ia}B^{a\mu}, \qquad j_s^{(1)a\mu} = \xi_s^a\omega^\mu + \xi_{Bs}^{ab}B^{b\mu}$$

where

$$\xi_s^I = s\left(\frac{T^2}{12} + \frac{1}{4\pi^2 N}\sum_i \mu_s^{i\,2}\right), \qquad \xi_{Bs}^{Ia} = -\frac{sg}{4\pi^2 N}\sum_i t_{ii}^a\mu_s^i,$$
$$\xi_s^a = \frac{s}{2\pi^2}\sum_i t_{ii}^a\mu_s^{i\,2}, \qquad \xi_{Bs}^{ab} = -\frac{sg}{4\pi^2}\left(\frac{\delta^{ab}}{N}\sum_i \mu_s^i + d^{bca}\sum_i t_{ii}^c\mu_s^i\right)$$

- vector currents:

$$j^{(1)I\mu} = j_{+1}^{(1)I\mu} + j_{-1}^{(1)I\mu},$$
$$j^{(1)a\mu} = j_{+1}^{(1)a\mu} + j_{-1}^{(1)a\mu},$$

- axial currents:

$$j_5^{(1)I\mu} = j_{+1}^{(1)I\mu} - j_{-1}^{(1)I\mu},$$
$$j_5^{(1)a\mu} = j_{+1}^{(1)a\mu} - j_{-1}^{(1)a\mu}$$

Non-Abelian anomalous currents

- anomalous current:

$$j^{(1)I\mu} = \xi^I \omega^\mu + \xi_B^{Ia} B^{a\mu}, \quad j^{(1)a\mu} = \xi^a \omega^\mu + \xi_B^{ab} B^{b\mu},$$
$$j_5^{(1)I\mu} = \xi_5^I \omega^\mu + \xi_{B5}^{Ia} B^{a\mu}, \quad j_5^{(1)a\mu} = \xi_5^a \omega^\mu + \xi_{B5}^{ab} B^{b\mu}$$

- anomalous transport coefficients:

$$\xi^I = \frac{1}{\pi^2 N}\sum_i \mu^i \mu_5^i, \qquad \xi_B^{Ia} = -\frac{g}{2\pi^2 N}\sum_i t_{ii}^a \mu_5^i,$$
$$\xi^a = \frac{2}{\pi^2}\sum_i t_{ii}^a \mu^i \mu_5^i, \qquad \xi_B^{ab} = -\frac{g}{2\pi^2}\left(\frac{\delta^{ab}}{N}\sum_i \mu_5^i + d^{bca}\sum_i t_{ii}^c \mu_5^i\right)$$

$$\xi_5^I = \frac{T^2}{6N} + \frac{1}{2\pi^2 N}\sum_i (\mu^{i\,2} + \mu_5^{i\,2}), \qquad \xi_{B5}^{Ia} = -\frac{g}{2\pi^2 N}\sum_i t_{ii}^a \mu^i,$$
$$\xi_5^a = \frac{1}{\pi^2}\sum_i t_{ii}^a \left(\mu^{i2} + \mu_5^{i2}\right), \qquad \xi_{B5}^{ab} = -\frac{g}{2\pi^2}\left(\frac{\delta^{ab}}{N}\sum_i \mu^i + \frac{d^{bca}}{\hbar}\sum_i t_{ii}^c \mu^i\right)$$

- these are just the non-Abelian counterparts of the CME, CVE and CSE.
- the singlet current are very similar to the coefficients in the Abelian case
- the coefficients in the last line are unique for the non-Abelian currents

and similar results were also obtained in different approaches.

D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009).
K. Landsteiner, E. Megias and F. Pena-Benitez, Phys. Rev. Lett. 107, 021601 (2011).

Summary

The non-Abelian CKT from Wigner approach by covariant gradient expansion:

- Only the time-like component of the chiral Wigner function is independent while other components can be explicit derivative.
- The kinetic equations of the singlet component and multiplet components are totally coupled with each other.
- We also have constraint equations which are unique for non-Abelian cases.
- Distribution functions have non-trivial dependence in different reference frames.
- The chiral anomaly from non-Abelian gauge field arises naturally from the Berry curvature from the vacuum contribution.
- The anomalous currents as non-Abelian counterparts of chiral magnetic effect and chiral vortical effect have also been derived from the non-Abelian chiral kinetic equation.

Thanks!