Counting Topological Windings of Gauge Fields with Chiral Magnetic Effect

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Thursday, August 16, 2021

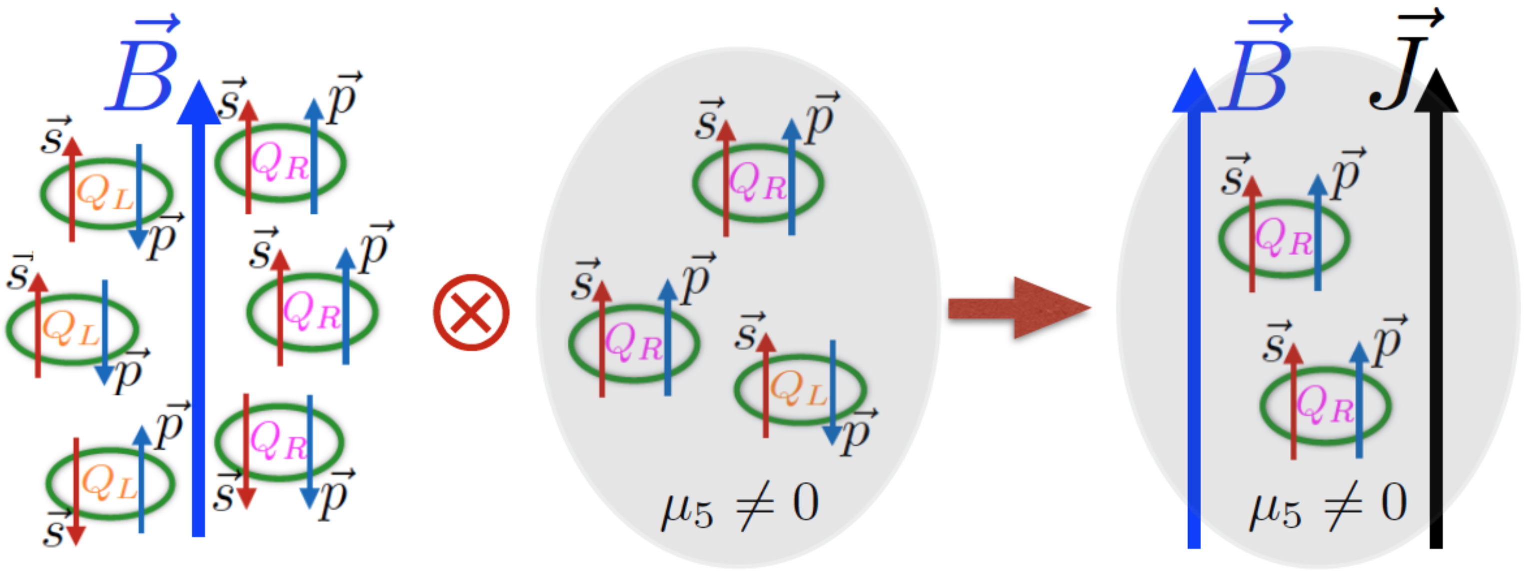
arXiv:2106.10847

Outline

- Introduction
- Motivation
- Framework
- Numerical Results
- Summary

Introduction

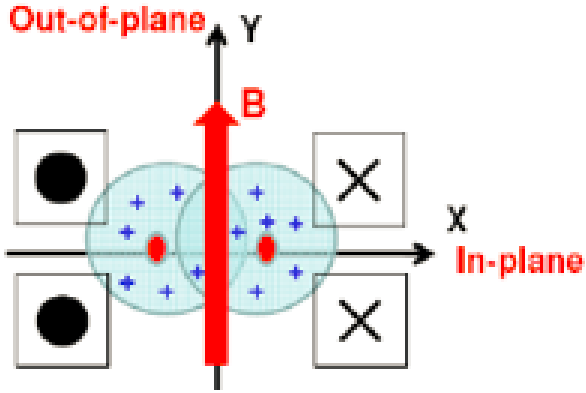
Chiral Magnetic Effect (CME)

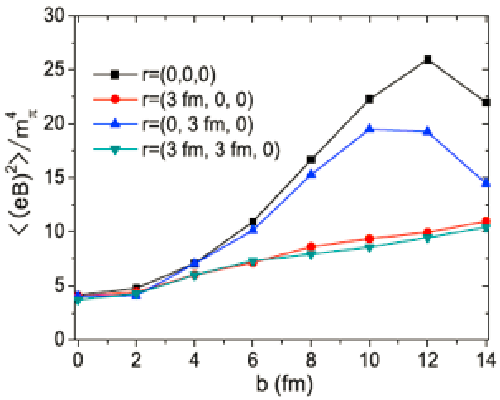


D.E. Kharzeev, J. Liao

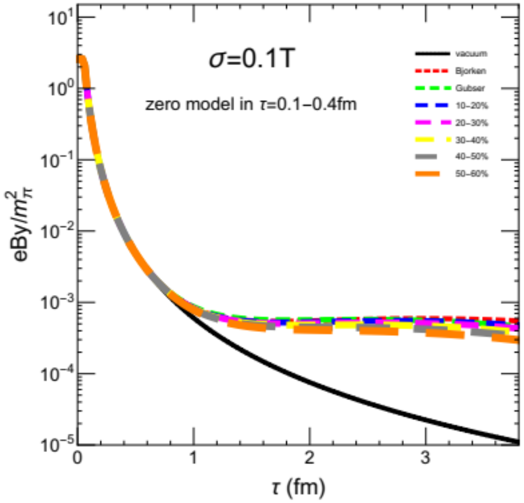
Introduction

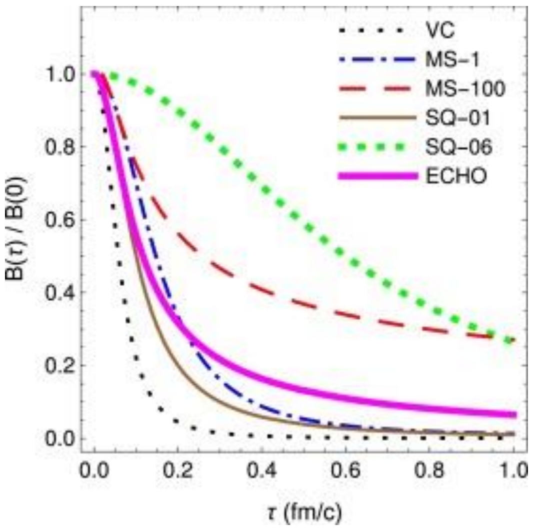
Magnetic field in HIC





Xuguang





This work use this model

$$\boldsymbol{B} = \frac{B_0}{1+\tau^2/\tau_B^2}\hat{\boldsymbol{y}}$$

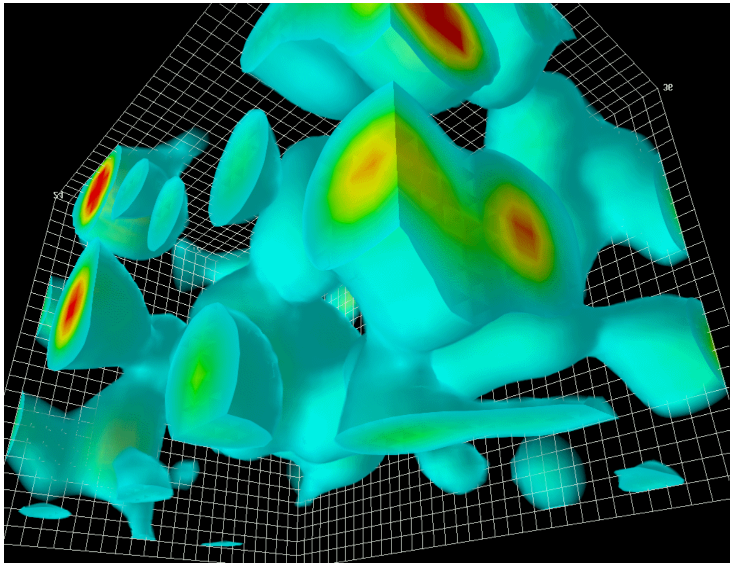
$\boldsymbol{\tau_B} = \mathbf{0.4}$ **or 0.6fm**

Introduction

Winding number in HIC

$$N_R^f - N_L^f = N_5 = 2Q_w, \qquad Q_w = \int d^4x \, -\frac{g^2 \epsilon^{\mu\nu\rho\sigma}}{32\pi^2} Tr\{G_{\mu\nu} G_{\rho\sigma}\}$$

$$\partial_\mu J^\mu_{5,f} = 2q = -\frac{g^2 \epsilon^{\mu\nu\rho\sigma}}{16\pi^2} Tr\{G_{\mu\nu} G_{\rho\sigma}\}$$

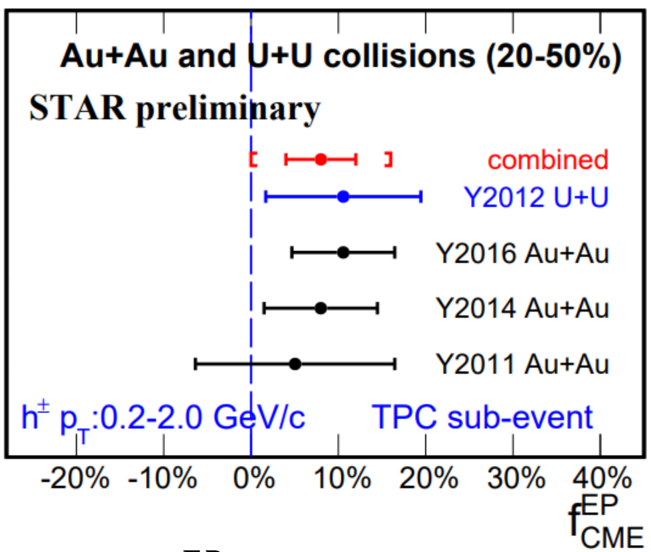


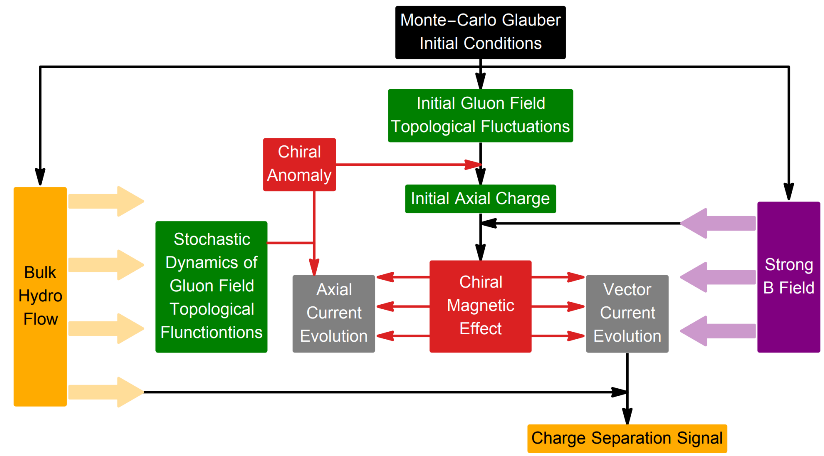
Derek Leinweber

Bubbles with nonzero chirality can be created via topological fluctuations and triangle anomaly --- locally P- & CP-odd environment !

Motivation

Counting the topological winding numbers of gauge field with measurements of the CME.





$$f_{CME}^{EP} = 8 \pm 4\%$$

AVFD

(Anomalous-Viscous Fluid Dynamics)

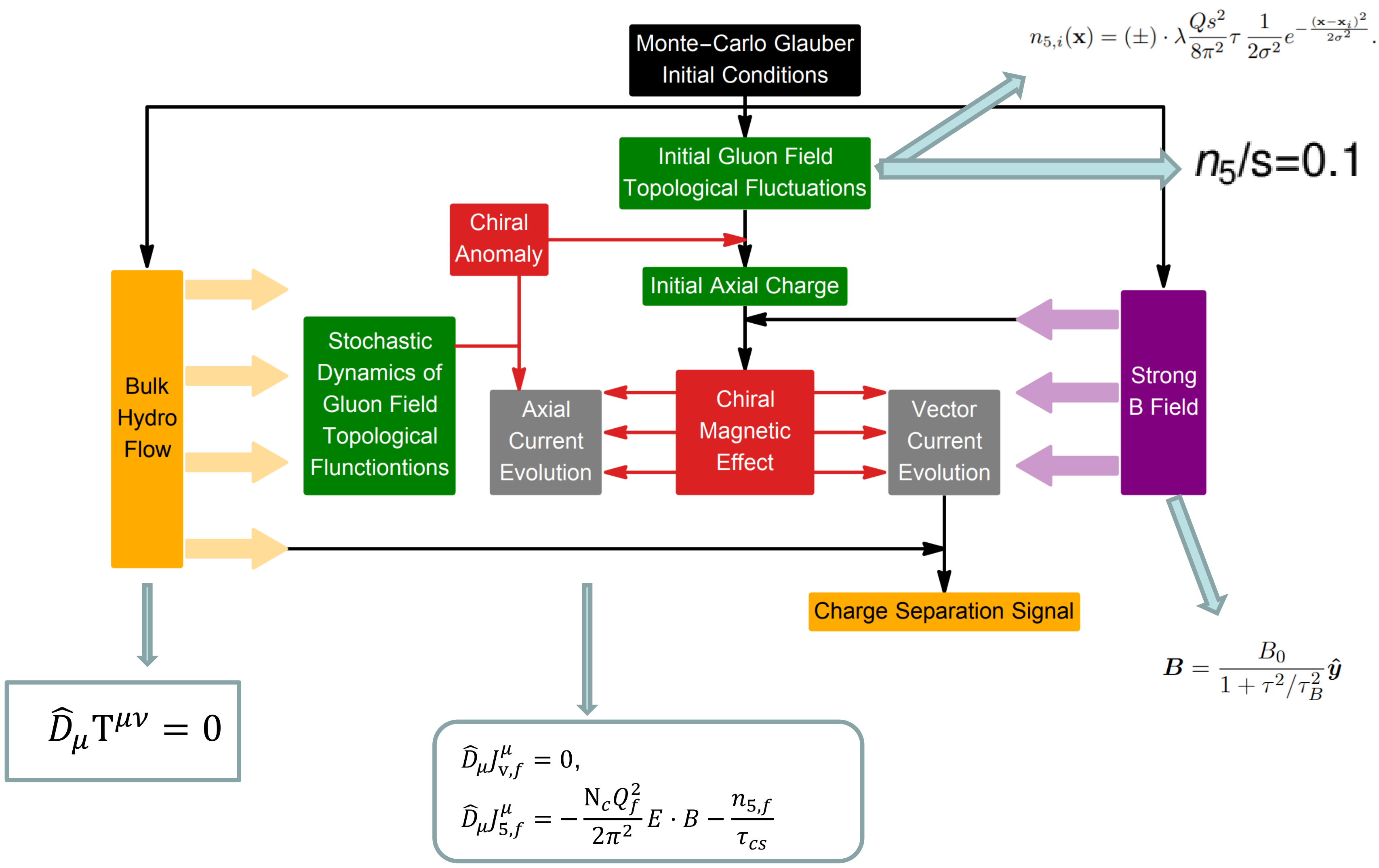
$$\triangle\gamma_{\rm CME} = f_{CME}^{EP} * \triangle\gamma_{\rm inclusive} = 1.46 * 10^{-5} \pm 7.28 * 10^{-6}$$

$$\Delta\gamma_{CME} = \gamma^{OS} - \gamma^{SS} = 2(a_1^{+})^2$$

$$N_R^f - N_L^f = N_5 = 2Q_w, \qquad Q_w = \int d^4x \, \frac{g^2 \epsilon^{\mu\nu\rho\sigma}}{32\pi^2} Tr\{G_{\mu\nu} G_{\rho\sigma}\}$$

Framework

The flow chart of AVFD



Framework

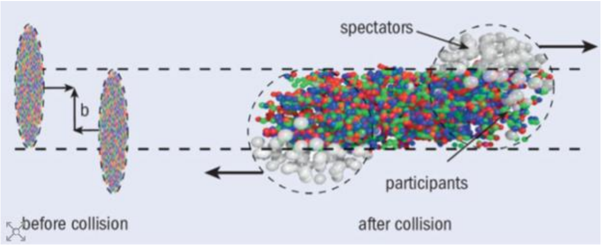
Initial condition

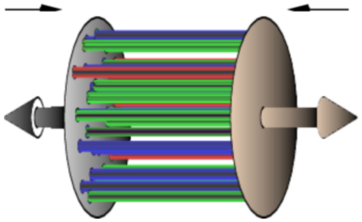
1. MC method

$$n_{5,i}(\tau,x,y,\eta) = (\pm)\cdot\lambda\frac{Qs^2}{8\pi^2}\tau\,\frac{1}{2\sigma^2}e^{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}}$$

$$n_5(\mathbf{x}) = \sum_i n_{5,i}(\mathbf{x}),$$

$$\partial_\mu J^\mu_{5,f} = 2q = -\frac{g^2\epsilon^{\mu\nu\rho\sigma}}{16\pi^2}Tr\{G_{\mu\nu}G_{\rho\sigma}\} = \frac{g^2N_f}{4\pi^2}\,\vec{E}^a\cdot\vec{B}_a$$

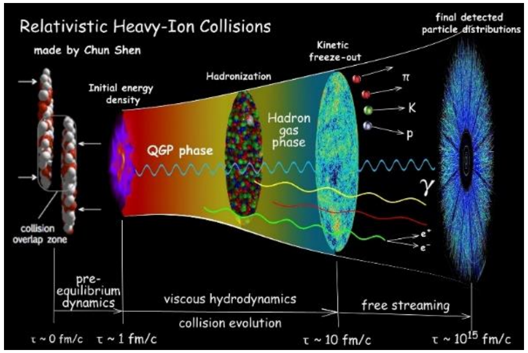




arXiv:1205.0579

2. Given value

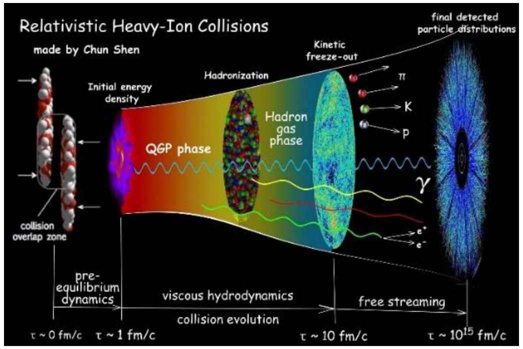
n_5/s=0.1



Framework

stochastic dynamics of gluon field topological fluctuations

During the hydrodynamic evolution, there exist random topological fluctuations of the gluon fields that would necessarily in fluence the axial current evolution. These fluctuations eventually amount to a relaxation effect toward equilibrium with vanishing topological charge on long time scale.

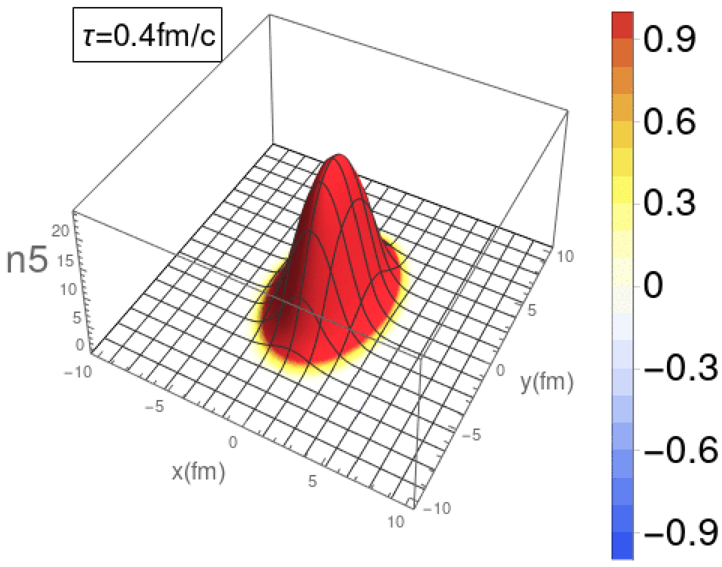


In Milne space $$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f}}{\tau_{cs}}$$

Shu Lin et al $$\tau_{cs} = \chi * \frac{T}{2\Gamma_{cs}}, \Gamma_{cs} = 30(\alpha_s T)^4$$

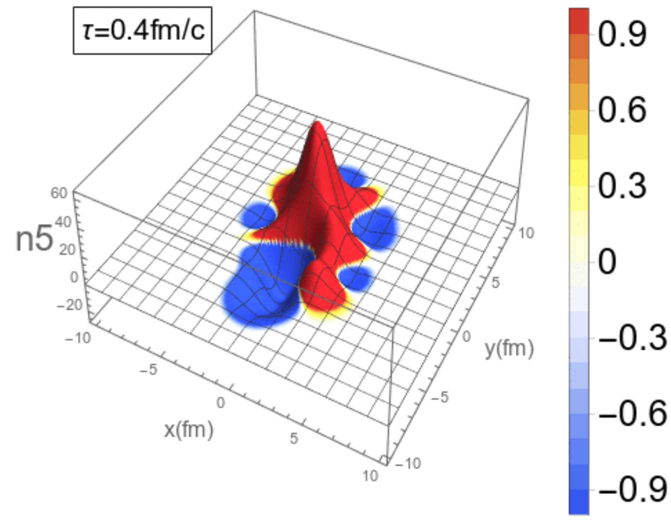
Results

1. axial charge density changed with time



Given a value of the initial n_5 by

$n_5/s=0.1$



Random sample the initial n_5 by

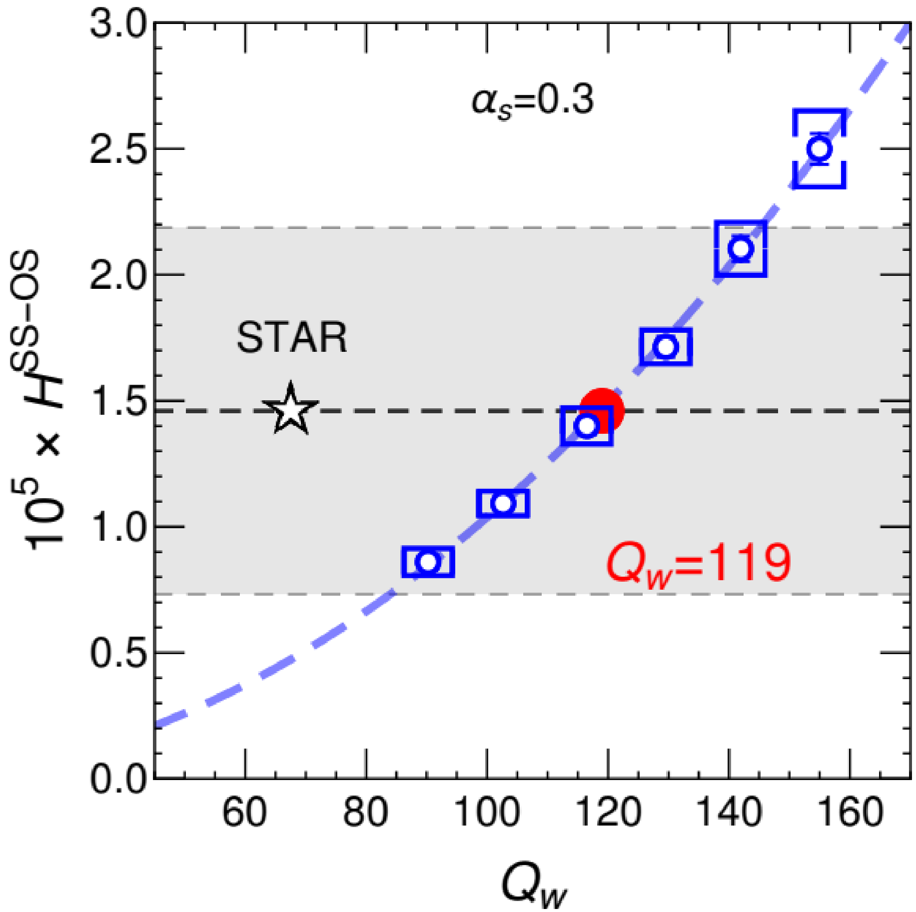
$$n_{5,i}(\mathbf{x}) = (\pm) \cdot \lambda \frac{Qs^2}{8\pi^2} \tau \frac{1}{2\sigma^2} e^{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}}.$$

$$n_5(\mathbf{x}) = \sum_i n_{5,i}(\mathbf{x}).$$

Results

2.1 Counting Topological Winding number of gluone Fields

$$H^{SS-OS} = 1.038 \times 10^{-9}\, Q_w^2$$



$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E\cdot B - \frac{n_{5,f}}{\tau_{cs}}$$

$$\tau_{cs} = \chi * \frac{T}{2\Gamma_{cs}},\ \Gamma_{cs} = 30(\alpha_s T)^4$$

$\boldsymbol{\tau_B = 0.6}$ **fm**

$$n_{5,i}(\mathbf{x}) = (\pm)\cdot\lambda\frac{Qs^2}{8\pi^2}\tau\,\frac{1}{2\sigma^2}e^{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}}.$$

$$n_5(\mathbf{x}) = \sum_i n_{5,i}(\mathbf{x}).$$

$$N_5 \equiv N_R - N_L = 2Q_w$$

$N_5 = \sqrt{< N_5^2 >_{event}}$ or $Q_w = \sqrt{< Q_w^2 >_{event}}$

Results

2.2 Counting Topological Winding number of gluone Fields

$$\widehat{D}_\mu J^\mu_{v,f} = 0,$$
$$\widehat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f}}{\tau_{cs}}$$

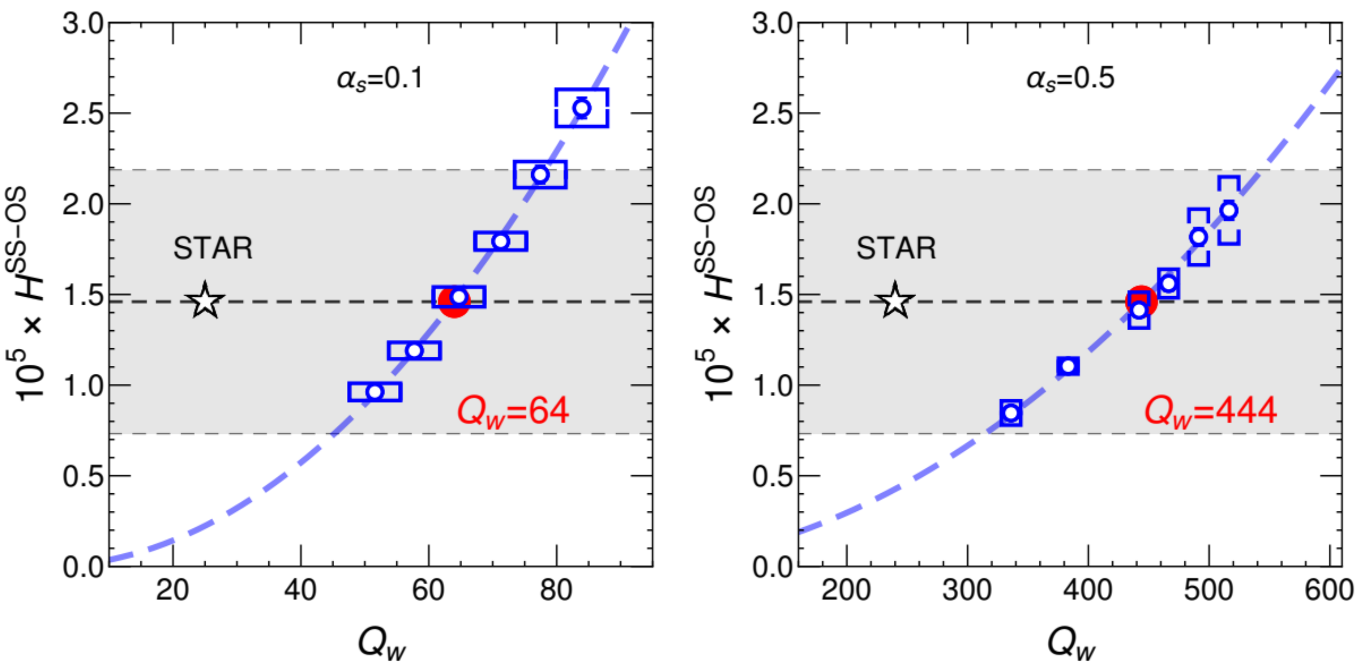
$\boldsymbol{\tau_B = 0.6}$ **fm**

$$n_{5,i}(\mathbf{x}) = (\pm) \cdot \lambda \frac{Qs^2}{8\pi^2} \tau \frac{1}{2\sigma^2} e^{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}}. \qquad n_5(\mathbf{x}) = \sum_i n_{5,i}(\mathbf{x})$$

$$\tau_{cs} = \chi * \frac{T}{2\Gamma_{cs}}, \; \Gamma_{cs} = 30(\alpha_s T)^4$$

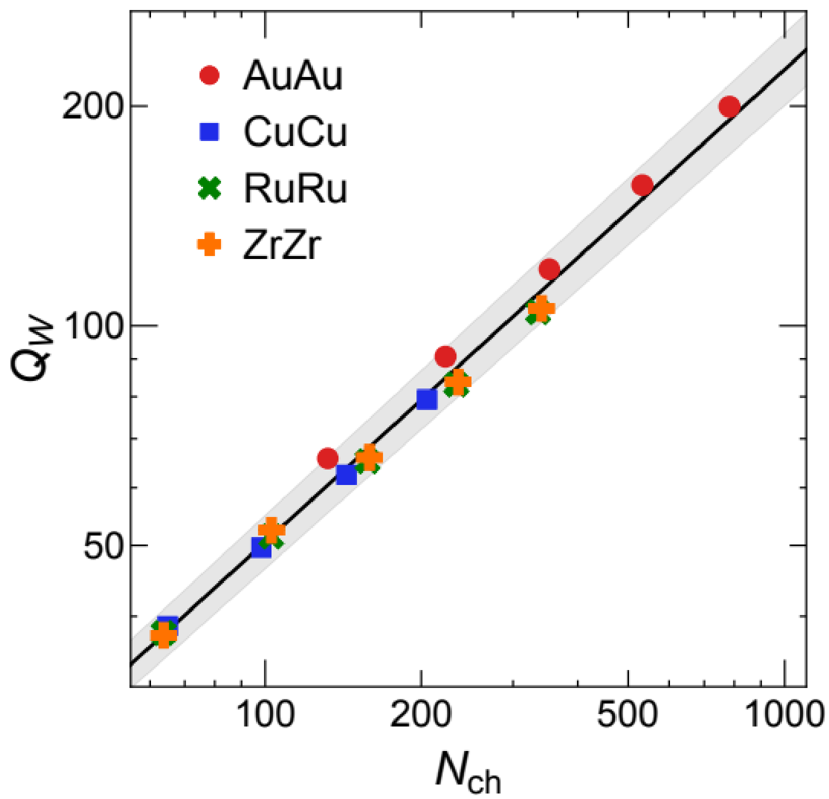
$$N_5 \equiv N_R - N_L = 2Q_w$$

$N_5 = \sqrt{< N_5^2 >_{event}}$ or $Q_w = \sqrt{< Q_w^2 >_{event}}$



Results

3. Winding number versus the charged hadron multiplicity N_{ch}



$$\hat{D}_\mu J^\mu_{v,f} = 0,$$

$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f}}{\tau_{cs}}$$

$\boldsymbol{\tau_B = 0.6}$ **fm**

$$N_5 \equiv N_R - N_L = 2Q_w$$

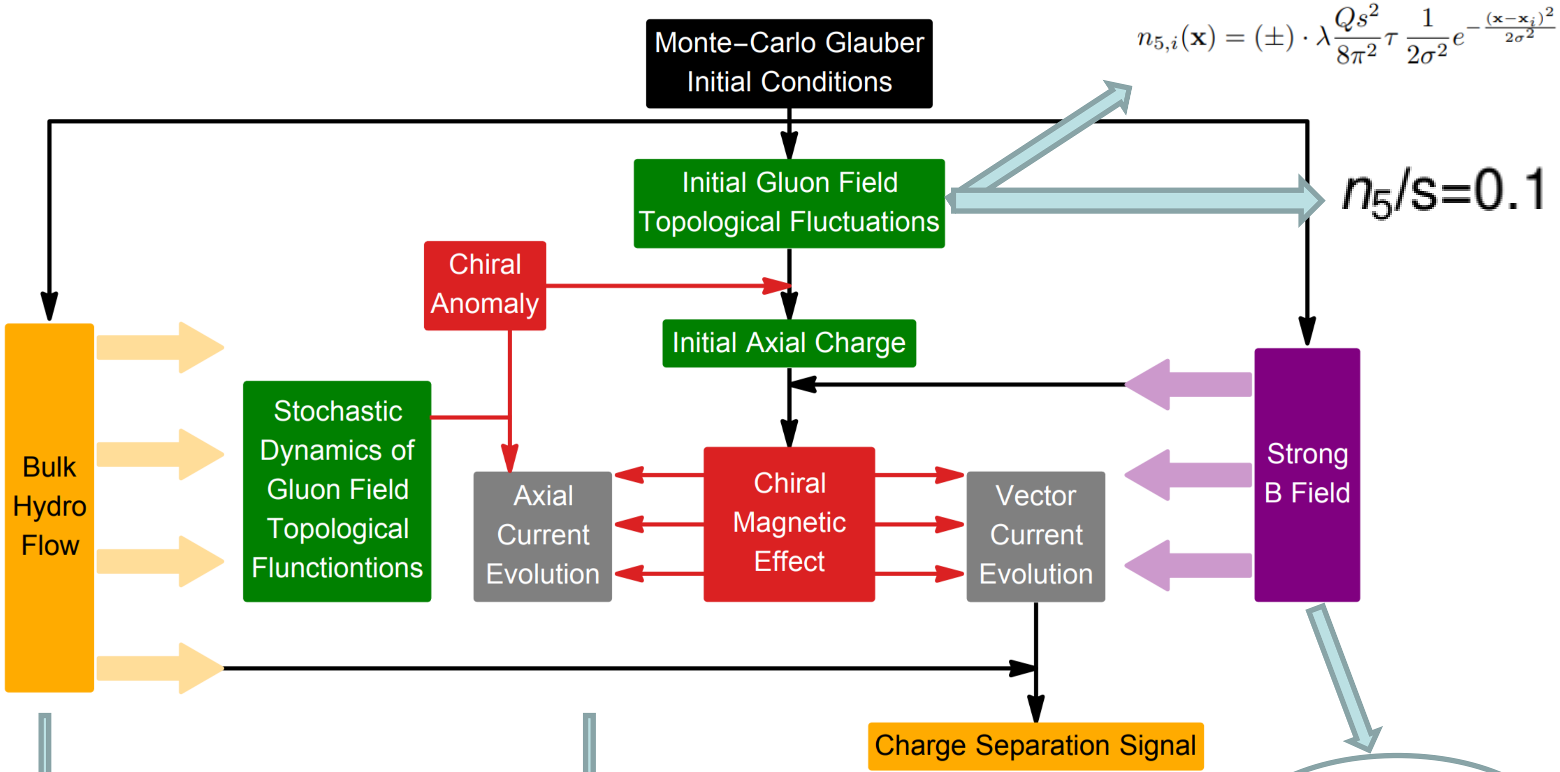
$$Q_w = \alpha\ N_{ch}^{\beta}\ ,$$

$$\alpha = 2.56 \pm 0.17\quad,\quad \beta = 0.648 \pm 0.013$$

Summary and outlook

Combined Maxwell equation and AVFD

$$n_{5,i}(\mathbf{x}) = (\pm) \cdot \lambda \frac{Qs^2}{8\pi^2}\tau \frac{1}{2\sigma^2} e^{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}}.$$



$$\widehat{D}_\mu \mathrm{T}^{\mu\nu} = 0$$

$$\widehat{D}_\mu J^\mu_{v,f} = 0,$$

$$\widehat{D}_\mu J^\mu_{5,f} = -\frac{\mathrm{N}_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f}}{\tau_{total}}$$

$$\partial_\mu F^{\mu\nu} = J^\nu,$$

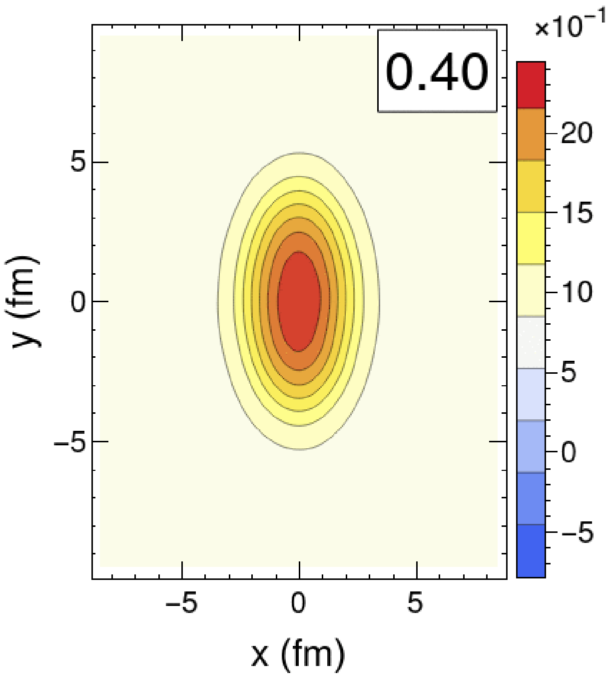
$$\partial_\mu \widetilde{F}^{\mu\nu} = 0.$$

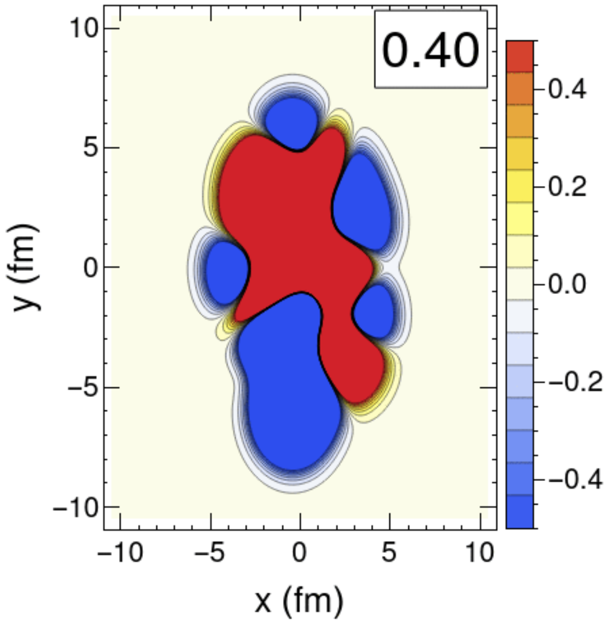
$$\boldsymbol{B} = \frac{B_0}{1+\tau^2/\tau_B^2}\hat{\boldsymbol{y}}$$

Thank You!

Results

axial charge density changed with time





Given a value by

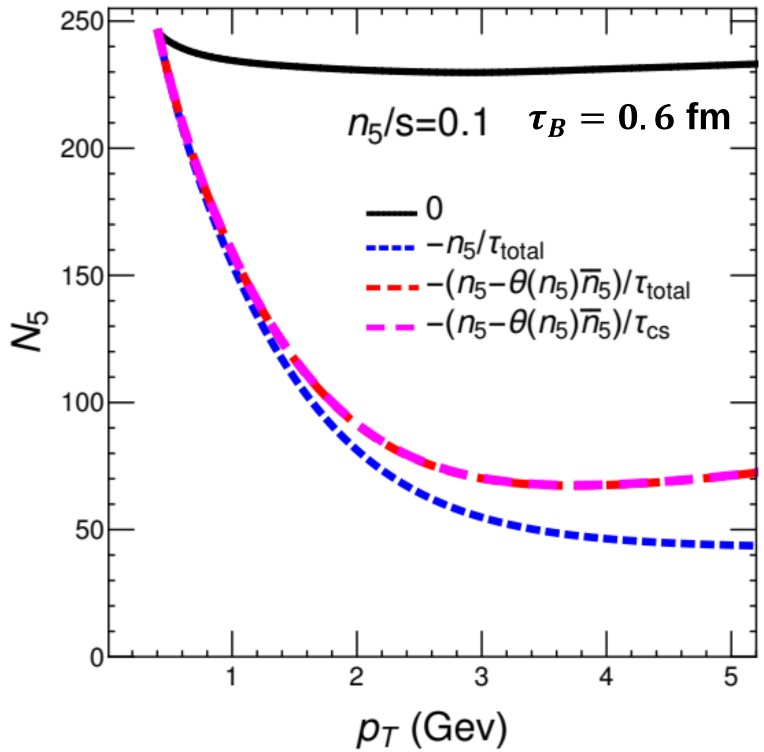
$n_5/s=0.1$

Random sample by

$$n_{5,i}(\mathbf{x}) = (\pm)\cdot\lambda\frac{Qs^2}{8\pi^2}\tau\,\frac{1}{2\sigma^2}e^{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}}.$$

Results

1. Damping effect of total axial charge



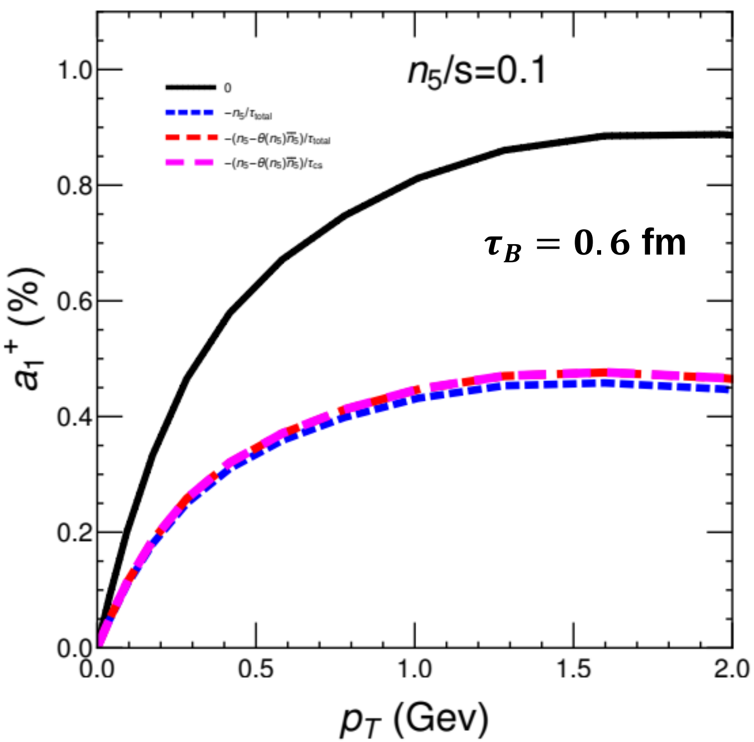
$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B$$

$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f}}{\tau_{cs}}$$

$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f} - \theta(n_{5,f})\,\bar{n}_{5,f}}{\tau_{cs}}$$

Results

4. CME signal for different damping case



$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B$$

$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f}}{\tau_{cs}}$$

$$\hat{D}_\mu J^\mu_{v,f} = 0,$$
$$\hat{D}_\mu J^\mu_{5,f} = -\frac{N_c Q_f^2}{2\pi^2} E \cdot B - \frac{n_{5,f} - \theta(n_{5,f})\,\bar{n}_{5,f}}{\tau_{cs}}$$

$$\frac{dN^\pm}{d\phi} \propto 1 + 2a_1^\pm \sin(\phi - \Psi_{RP}) + 2v_2 \cos(2\phi - 2\Psi_{RP}) + \ldots$$