



X atom

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2021.08.16-20 青岛

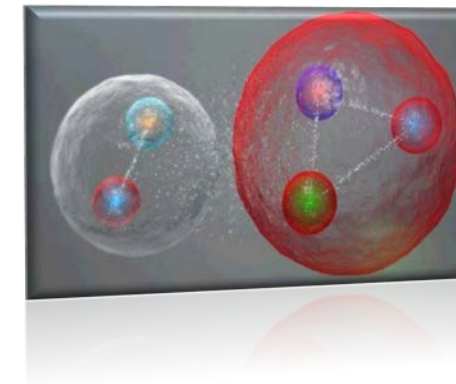
Based on Z.-H. Zhang, F.-K. Guo. Phys. Rev. Lett. 127, 012002 (2021)

X atom: Background



Exotic hadrons are the hadrons **beyond the quark model**.

XYZ states, **Glueballs**, **Pentaquarks**...



$X(3872)$ is one of the most important XYZ states

$X(3872)$ is first discovered in the $J/\psi\pi^+\pi^-$ invariant mass distribution by Belle

Collaboration in 2003, with $I^G J^{PC} = 0^+(1^{++})$, $m_X = (3871.69 \pm 0.17) \text{ MeV}$

Salient features: (a) $\delta = m_{D^0} + m_{D^{*0}} - m_X = (0.00 \pm 0.18) \text{ MeV}$

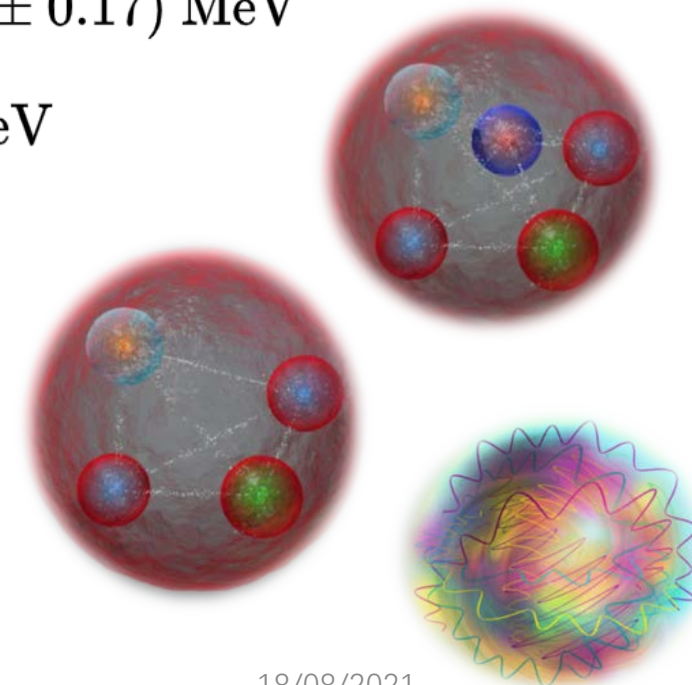
$$(b) \mathcal{B}(X(3872) \rightarrow D^0 \bar{D}^0 \pi^0) > 40\%$$

$$\mathcal{B}(X(3872) \rightarrow D^0 \bar{D}^{*0}) > 30\%$$

At long distance, $D^0 \bar{D}^{*0}$ is dominant in $X(3872)$

$$|X(3872)\rangle = \frac{1}{\sqrt{2}} (|D^0 \bar{D}^{*0}\rangle - |\bar{D}^0 D^{*0}\rangle)$$

$$\begin{pmatrix} \mathcal{C}|D\rangle = |\bar{D}\rangle \\ \mathcal{C}|D^*\rangle = -|\bar{D}^*\rangle \end{pmatrix}$$





X atom: Introduction

Typical size for the $X(3872)$ at long distance: $r_X \simeq \frac{1}{\sqrt{2\mu_c^0\delta}} \gtrsim 10 \text{ fm}$

Typical size (Bohr radius) for the $D^+ D^{*-}$ bound state: $r_B = \frac{1}{\alpha\mu_c} = 27.86 \text{ fm}$

$$\mu_0 = \frac{m_{D^0} m_{D^{*0}}}{\Sigma_0} \quad \mu_c = \frac{m_D m_{D^*}}{\Sigma_c} \quad \Sigma_0 = m_{D^0} + m_{D^{*0}} \quad \Sigma_c = m_D + m_{D^*} = (3879.91 \pm 0.07) \text{ MeV}$$

Coulomb binding energies: $-E_n = -\frac{\alpha^2 \mu_c}{2n^2} = \frac{-E_1}{n^2} = -\frac{25.81 \text{ keV}}{n^2}$

X atom: The ground state $\frac{1}{\sqrt{2}}(|D^+ D^{*-}\rangle - |D^- D^{*+}\rangle)$ atom with $C = +$

Scale separation: $r_B \Lambda_{\text{QCD}} \gg 1$, strong interaction between $D^+ D^{*-}$ is a **correction**

Effects of strong interaction at LO:

(a) **Energy level shift:** $\Delta E_n^{\text{str}} \sim \mathcal{O}(\alpha^3)$ (b) **Decay modes:** $D^0 \bar{D}^{*0}, D^0 \bar{D}^0 \pi^0, J/\psi \pi \pi, \dots$

The strong interaction is non-perturbative due to the existence of the $X(3872)$

Only hadronic atoms with light quarks have been studied

Gasser, Lyubovitskij, Rusetsky, *Phys. Rept.* 456 (2008)

X atom: Introduction



The X atom is related to the $X(3872)$ (as a hadronic molecule) by **isospin symmetry**

D^+D^{*-} threshold: $\Sigma_c = m_D + m_{D^*} = (3879.91 \pm 0.07) \text{ MeV}$, **no signal** near the threshold

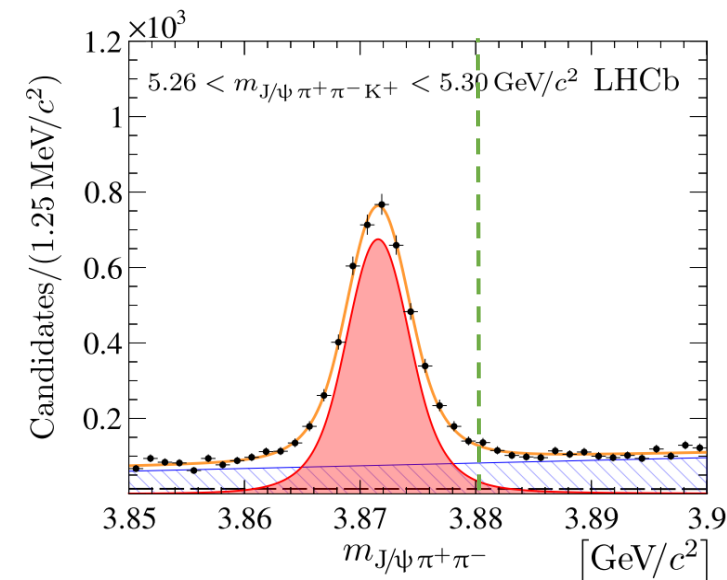
Make use of the zero signal to:

- Put a lower bound on the $X(3872)$ binding energy
- Give a criterion on the $X(3872)$ nature

Scale separation: $r_B \Lambda_{\text{QCD}} \gg 1$; Nonrelativistic effective field theory (**NREFT**) applicable

Approximation: Isospin-1 strong interaction neglected

- No isovector state was found
- Isospin breaking in the couplings is small $\frac{g_{X\rho}}{g_{X\omega}} = 0.26^{+0.08}_{-0.05}$



LHCb, *J. High Energy Phys.* 08 (2020) 123

Hanhart et al., *Phys. Rev. D* 85 (2012) 011501



X atom: NREFT

Coupled channel: CH 1 : $D^+ D^{*-} \rightarrow D^+ D^{*-}$ CH 2 : $D^0 \bar{D}^{*0} \rightarrow D^0 \bar{D}^{*0}$

Non-relativistic effective Lagrangian: Galilean, Gauge invariant; C, P, T

Around threshold, LO Lagrangian: constant contact terms for strong interactions

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\phi=D^\pm, D^0, \bar{D}^0} \phi^\dagger \left(iD_t - m_\phi + \frac{\nabla^2}{2m_\phi} \right) \phi + \sum_{\phi=D^{*\pm}, D^{*0}, \bar{D}^{*0}} \phi^\dagger \left(iD_t - m_\phi + i\frac{\Gamma_\phi}{2} + \frac{\nabla^2}{2m_\phi} \right) \phi \\ & -\frac{C_0}{2} (D^+ D^{*-} - D^- D^{*+})^\dagger (D^+ D^{*-} - D^- D^{*+}) - \frac{C_0}{2} \left[(D^+ D^{*-} - D^- D^{*+})^\dagger (D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0}) + \text{h. c.} \right] \\ & -\frac{C_0}{2} (D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0})^\dagger (D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0}) + \dots \end{aligned}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad D_t \phi = \partial_t \phi \mp iQ A_0 \phi$$

Constant width approximation for D^*

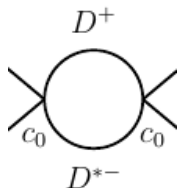
Hanhart, Kalashnikova, Nefediev, *Phys. Rev. D* 81 (2010) 094028

X atom: NREFT

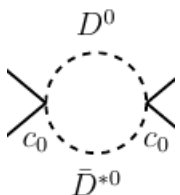


S-wave T -matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel: $T(E) = V[1 - G(E)V]^{-1}$

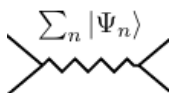
Strong contact term: $V = C_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ **Green's function:** $G(E) = \begin{pmatrix} J_c(E) + J_{|\Psi\rangle}(E) & 0 \\ 0 & J_0(E) \end{pmatrix}$



$$J_c(E) = \frac{\mu_c}{2\pi} \left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_c(E + i\Gamma_c/2)} \right) \quad E = \sqrt{s} - \Sigma_c$$



$$J_0(E) = \frac{\mu_0}{2\pi} \left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_0(E + \Delta + i\Gamma_0/2)} \right) \quad \Delta = \Sigma_c - \Sigma_0$$



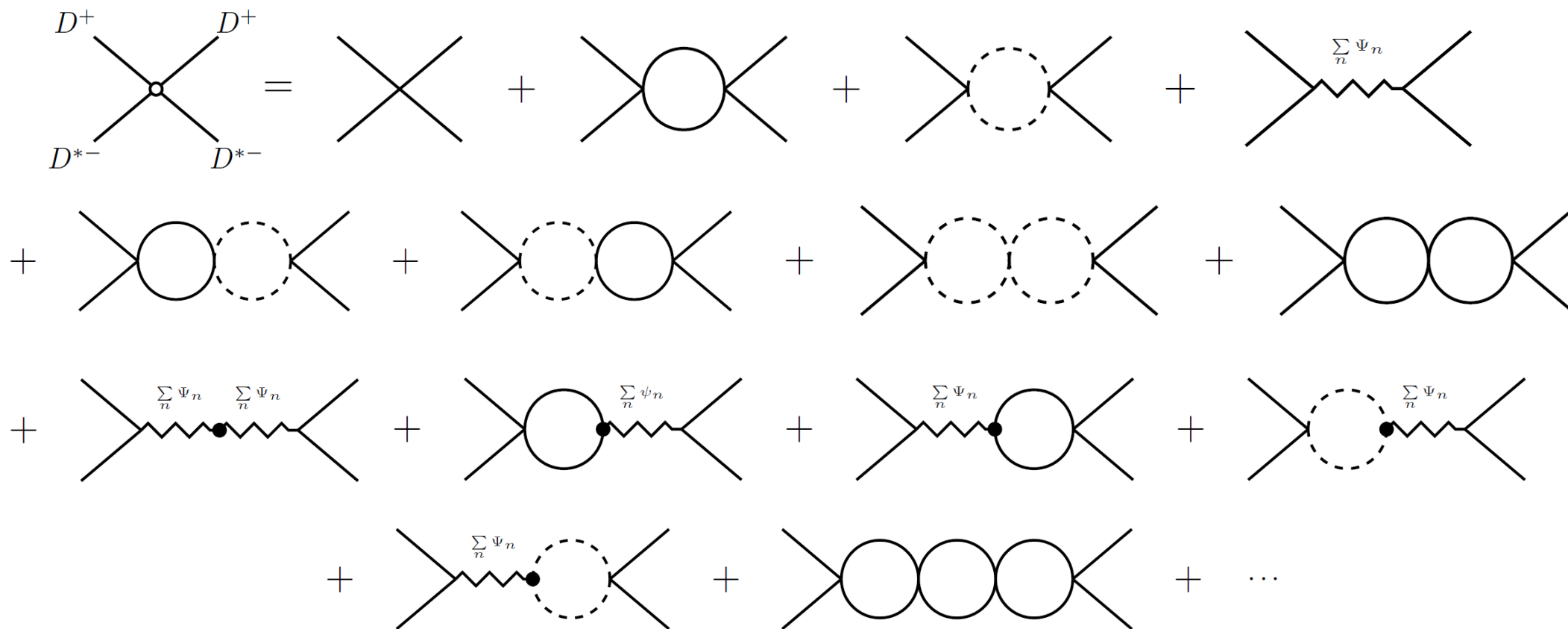
$$J_{|\Psi\rangle}(E) = \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{E + E_n + i\Gamma_c/2} \quad \Gamma_c \equiv \Gamma_{D^*}, \quad \Gamma_0 \equiv \Gamma_{D^{*0}}$$

X atom: NREFT



***S*-wave *T*-matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel:**

$$T(E) = \frac{1}{C_0^{-1} - [J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$





X atom: Strong Energy Level Shift

S-wave T -matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel:

$$T(E) = \frac{1}{C_0^{-1} - [J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Renormalization: $C_{0R}^{-1} = C_0^{-1} + \Lambda(\mu_0 + \mu_c)/\pi^2$

The $X(3872)$ and hadronic atoms appear as **poles of the T -matrix**

$X(3872)$ pole: $E = -\Delta - \delta - i\frac{\Gamma_0}{2}$ $\delta\Gamma = \Gamma_c - \Gamma_0$

$$C_{0R}^{-1} = \frac{\mu_0}{2\pi} \sqrt{2\mu_0\delta} + \frac{\mu_c}{2\pi} \sqrt{2\mu_c \left(\Delta + \delta - i\frac{\delta\Gamma}{2} \right)} - \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{\Delta + \delta - E_n - i\delta\Gamma/2} = \frac{\mu_c}{2\pi} \sqrt{2\mu_c\Delta} \left[1 + \mathcal{O}\left(\frac{\delta}{\Delta}, \frac{\delta\Gamma}{\Delta}, \frac{\alpha^3 \mu_c^{3/2}}{\Delta^{3/2}} \right) \right]$$

S-wave hadronic atom poles: $E = -E_{An} - i\frac{\Gamma_c}{2}$

$$0 = C_{0R}^{-1} + i\frac{\mu_0}{2\pi} \sqrt{2\mu_0 \left(\Delta - E_{An} - i\frac{\delta\Gamma}{2} \right)} - \frac{\mu_c}{2\pi} \sqrt{2\mu_c E_{An}} - \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{-E_{An} + E_n}$$



X atom: Strong Energy Level Shift

Strong energy level shift: $\Delta E_n = E_{An} - E_n$

$$\Delta E_n = \frac{2\alpha^3 \mu_c^2}{n^3 \sqrt{2\mu_c \Delta}} \left[-1 - i + \mathcal{O}\left(\alpha \sqrt{\frac{\mu_c}{\Delta}}\right) \right]^{-1}$$

S-wave hadronic atom poles: $E = -E_{An} - i\frac{\Gamma_c}{2} = -E_n - \Delta E_n - i\frac{\Gamma_c}{2}$

Ground state: $n = 1$

Binding energy: $\text{Re } E_{A1} = E_1 - \frac{\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} \simeq 22.92 \text{ keV} \quad M_{A1} = (3879.89 \pm 0.07) \text{ MeV}$

Decay width: $\Gamma_c + 2 \text{Im } E_{A1} = \Gamma_c + \frac{2\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} = (89.2 \pm 1.8) \text{ keV}$

$$D^* \rightarrow D\pi, D\gamma, \dots \quad \Gamma_c = (83.4 \pm 1.8) \text{ keV}$$

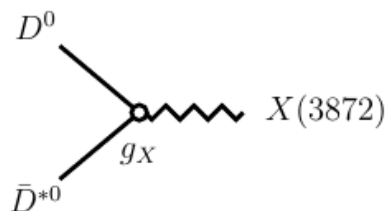
$$A (\text{X atom}) \rightarrow D^0 \bar{D}^{*0} (\bar{D}^0 D^{*0}) \quad \Gamma_s = 2\text{Im}E_{A1} = 5.8 \text{ keV}$$

X atom: Effective Coupling



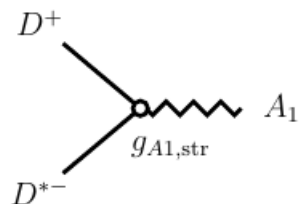
The effective coupling squared is the residue of the T -matrix at the pole

$$D^0 \bar{D}^{*0} \rightarrow X(3872) \quad X(3872) \text{ pole: } E = -\Delta - \delta - i\frac{\Gamma_0}{2}$$



$$g_X^2 = \lim_{E \rightarrow -\Delta - \delta - i\frac{\Gamma_0}{2}} \left(E + \Delta + \delta + i\frac{\Gamma_0}{2} \right) T_{22}(E) = \frac{2\pi}{\mu_0^2} \sqrt{2\mu_0 \delta} \left[1 + \mathcal{O}\left(\frac{\delta^{1/2}}{\Delta^{1/2}} \right) \right]^{-1}$$

$$D^+ D^{*-} \rightarrow A_1 \quad \text{Hadronic atom poles: } E = -E_{A_1} - i\frac{\Gamma_c}{2}$$



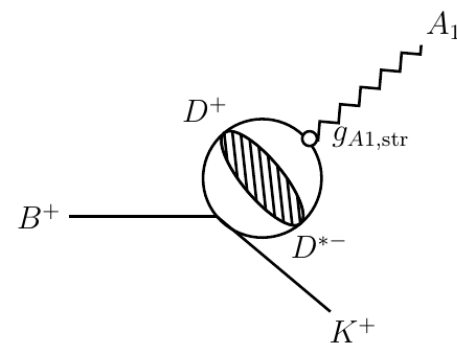
$$g_{A1,\text{str}}^2 = \lim_{E \rightarrow -E_{A1} - i\frac{\Gamma_c}{2}} \left(E + E_{A1} + i\frac{\Gamma_c}{2} \right) T_{11}(E) = -i\frac{\pi\alpha^3}{\Delta} \left[1 + \mathcal{O}\left(\frac{\alpha^2\mu_c}{\Delta} \right) \right]^{-1}$$

X atom: Production

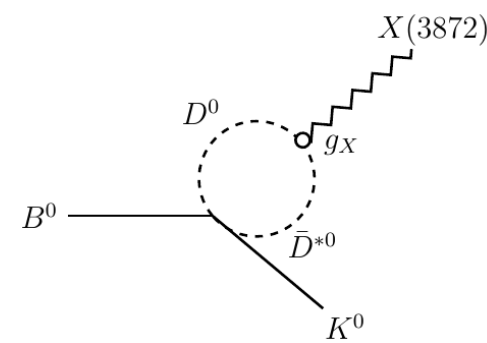


Production in exclusive B decays:

$$\begin{aligned} \text{Diagram with shaded blob} &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \\ &= \text{Diagram 4} + \text{Diagram 5} \end{aligned}$$



$$B^+ \rightarrow (DD^*)_+ K^+ \rightarrow A_1 K^+$$



$$B^0 \rightarrow (DD^*)_+^0 K^0 \rightarrow X K^0$$

$$\mathcal{A}_{B^+ \rightarrow A_1 K^+} = \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{(\Lambda)} G_C(\Lambda, E) g_{A_1, \text{str}}$$

$$\mathcal{A}_{B^0 \rightarrow X K^0} = \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{(\Lambda)} G_0(\Lambda, E) g_X$$

$$G_C(\Lambda, E) = -\frac{\mu_c \Lambda}{\pi^2} - \frac{\alpha \mu_c^2}{\pi} \left[\ln \frac{\Lambda}{\alpha \mu_c} + \ln(x) + \frac{1}{2x} - \psi(-x) - \gamma_E \right]$$

$$G_0(\Lambda, E) = -\frac{\mu_c^0 \Lambda}{\pi^2} + \frac{\mu_c^0}{2\pi} \left(\sqrt{-2\mu_c^0 E - i\epsilon} \right)$$

$$\begin{aligned} |(DD^*)_+^0\rangle &= \frac{1}{\sqrt{2}} (|D^0 \bar{D}^{*0}\rangle - |\bar{D}^0 D^{*0}\rangle) \\ |(DD^*)_+\rangle &= \frac{1}{\sqrt{2}} (|D^+ D^{*-}\rangle - |D^- D^{*+}\rangle) \end{aligned}$$

$$x = \frac{\alpha \mu_c}{\sqrt{-2\mu_c(E + i\frac{\Gamma_c}{2})}} \quad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

Kong, Ravndal, *Nucl. Phys. A* 665 (2000)

Factorized amplitudes:

$$\mathcal{A}_{B^+ \rightarrow A_1 K^+} = \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{\text{s.d.}} g_{A_1, \text{str}}$$

$$\mathcal{A}_{B^0 \rightarrow X K^0} = \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} g_X$$

Braaten, Kusunoki, *Phys. Rev. D* 72 (2005) 014012

Isospin symmetry:

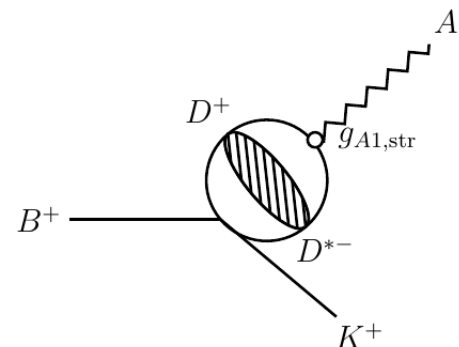
$$\left| \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{\text{s.d.}} \right| = \left| \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} \right|$$

X atom: Production

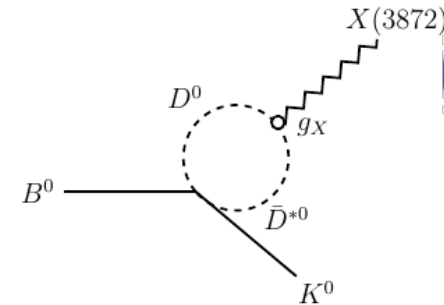
Production in exclusive B decays:

$$|(DD^*)_+^0\rangle = \frac{1}{\sqrt{2}} (|D^0\bar{D}^{*0}\rangle - |\bar{D}^0D^{*0}\rangle)$$

$$|(DD^*)_+\rangle = \frac{1}{\sqrt{2}} (|D^+D^{*-}\rangle - |D^-D^{*+}\rangle)$$



$$B^+ \rightarrow (DD^*)_+ K^+ \rightarrow A_1 K^+$$



$$B^0 \rightarrow (DD^*)_+^0 K^0 \rightarrow X K^0$$



Factorized amplitudes:

$$\mathcal{A}_{B^+ \rightarrow A_1 K^+} = \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{\text{s.d.}} g_{A1,\text{str}} \quad \mathcal{A}_{B^0 \rightarrow X K^0} = \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} g_X$$

Isospin symmetry:

$$|\mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{\text{s.d.}}| = |\mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}}|$$

Lower bound on the $X(3872)$ binding energy:

$$R_\Gamma \equiv \frac{\Gamma_{B^+ \rightarrow A_1 K^+}}{\Gamma_{B^0 \rightarrow X K^0}} = \frac{|g_{A1,\text{str}}|^2}{|g_X|^2} \quad \delta \simeq \frac{0.25 \text{ eV}}{R_\Gamma^2}$$

Production in inclusive pp collisions:

$$R_\sigma \equiv \frac{d\sigma_{pp \rightarrow A_1 + y}}{d\sigma_{pp \rightarrow X + y}} = \frac{|g_{A1,\text{str}}|^2}{|g_X|^2} \quad \delta \simeq \frac{0.25 \text{ eV}}{R_\sigma^2} \quad R_\Gamma \simeq R_\sigma \gtrsim 1 \times 10^{-3}$$



X atom: Decay

Constituent D^* decay: $D^* \rightarrow D\pi, D\gamma, \dots$ $\Gamma_c = (83.4 \pm 1.8) \text{ keV}$

Decay into neutral pair: $A \text{ (X atom)} \rightarrow D^0 \bar{D}^{*0} (\bar{D}^0 D^{*0})$ $\Gamma_s = 2\text{Im}E_{A1} = 5.8 \text{ keV}$

Decay into $J/\psi\pi\pi$ & $J/\psi\pi^+\pi^-\pi^0$ (like the $X(3872)$) $A \rightarrow J/\psi\pi\pi, J/\psi\pi^+\pi^-\pi^0$

Ratio of branchings for the $X(3872)$: $\frac{\text{Br}_{[X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0]}^{\text{exp}}}{\text{Br}_{[X(3872) \rightarrow J/\psi\pi^+\pi^-]}^{\text{exp}}} = 1.1 \pm 0.4$

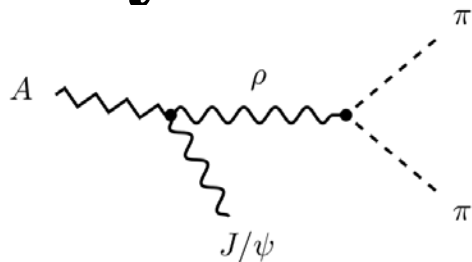
Isospin breaking: $R_X = \frac{g_{[X(3872) \rightarrow J/\psi\rho]}}{g_{[X(3872) \rightarrow J/\psi\omega]}} = 0.26$ **C. Hanhart et al., *Phys. Rev. D* 85 (2012) 011501**

$D^+ D^{*-}$ atom (A): $m_A = 3879.89 \pm 0.07 \text{ MeV}$

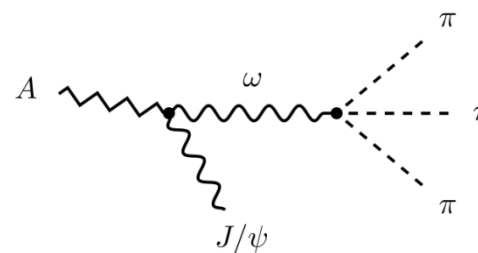
Isospin breaking negligible: $|D^+ D^{*-}\rangle = \frac{1}{\sqrt{2}}(|I=1\rangle + |I=0\rangle)$ $R_A = \frac{g_{[A \rightarrow J/\psi\rho]}}{g_{[A \rightarrow J/\psi\omega]}} = 1$

The phase space of the $D^+ D^{*-}$ atom is larger than the phase space of the $X(3872)$

X atom: Decay



$$A \rightarrow J/\psi \pi \pi$$



$$A \rightarrow J/\psi \pi^+ \pi^- \pi^0$$

C. Hanhart et al. , *Phys. Rev. D* **85** (2012) 011501

O. Kaymakcalan, S. Rajeev, and J. Schechter, *Phys. Rev. D* **30**, 594 (1984)

E. A. Kuraev and Z. K. Silagadze, *Phys. At. Nucl.* **58**, 1589 (1995)

$$\frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]}}{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^-]}} = 1.09$$

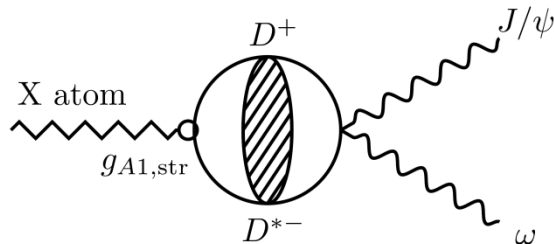
$$\frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]}^{\text{exp}}}{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^-]}^{\text{exp}}} = 1.1 \pm 0.4$$

Effective couplings: $R_A = \frac{g_{[A \rightarrow J/\psi \rho]}}{g_{[A \rightarrow J/\psi \omega]}} = 1$ $R_X = \frac{g_{[X(3872) \rightarrow J/\psi \rho]}}{g_{[X(3872) \rightarrow J/\psi \omega]}} = 0.26$

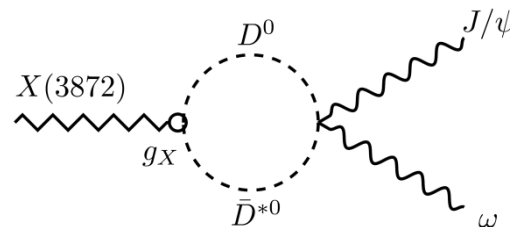
Ratio of branchings: $\frac{\text{Br}_{[A \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}} = 3.34$ $\frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}} = 0.91$

$$\frac{\text{Br}_{[A \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}} \simeq 3.65 \frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}$$

X atom: Decay



$$A \rightarrow J/\psi \omega$$



$$X(3872) \rightarrow J/\psi \omega$$

Factorized amplitudes : $\mathcal{A}_{[A \rightarrow J/\psi \omega]} = g_{A1, \text{str}} \mathcal{A}_{[(DD^*)_+ \rightarrow J/\psi \omega]}^{\text{s.d.}}$ $\mathcal{A}_{[X(3872) \rightarrow J/\psi \omega]} = g_X \mathcal{A}_{[(DD^*)^0_+ \rightarrow J/\psi \omega]}^{\text{s.d.}}$

Ratio of phase spaces : $\frac{\Phi_{[A \rightarrow J/\psi \pi^+ \pi^- \pi^0]}}{\Phi_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]}} = 3.76$

Ratio of decay widths : $\frac{\Gamma_{[A \rightarrow J/\psi \pi^+ \pi^- \pi^0]}}{\Gamma_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]}} = \frac{|g_{A1, \text{str}}|^2}{|g_X|^2} \frac{\Phi_{[A \rightarrow J/\psi \pi^+ \pi^- \pi^0]}}{\Phi_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]}} \gtrsim 3.76 \times 10^{-3}$

$$\frac{\text{Br}_{[A \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}} \simeq 3.65 \frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}$$

$$\frac{\Gamma_{[A \rightarrow J/\psi \pi \pi]}}{\Gamma_{[X(3872) \rightarrow J/\psi \pi \pi]}} \gtrsim 1.37 \times 10^{-2}$$

X atom: Results

(a) Binding Energy and Decay Width for the X Atom

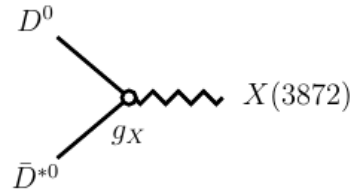
$$\text{Re } E_{A1} = E_1 - \frac{\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} \simeq 22.92 \text{ keV}$$

$$M_{A1} = (3879.89 \pm 0.07) \text{ MeV}$$

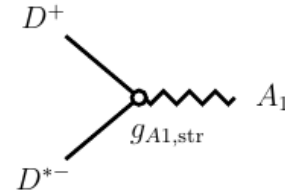
$$\Gamma_c + 2 \text{Im } E_{A1} = \Gamma_c + \frac{2\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} = (89.2 \pm 1.8) \text{ keV}$$

$$\Gamma_c = (83.4 \pm 1.8) \text{ keV}$$

(b) LO Effective Couplings



$$g_X^2 = \frac{2\pi}{\mu_0^2} \sqrt{2\mu_0 \delta}$$



$$|g_{A1,\text{str}}|^2 = \frac{\pi \alpha^3}{\Delta}$$

(c) Lower bound on the $X(3872)$ binding energy

$$\delta \simeq \frac{0.25 \text{ eV}}{R_{\Gamma(\sigma)}^2}$$

$$R_{\Gamma} \equiv \frac{\Gamma_{B^+ \rightarrow A_1 K^+}}{\Gamma_{B^0 \rightarrow X K^0}}$$

$$R_{\sigma} \equiv \frac{d\sigma_{pp \rightarrow A_1 + y}}{d\sigma_{pp \rightarrow X + y}}$$

$$\delta = m_{D^0} + m_{D^{*0}} - m_X$$

$$R_{\Gamma} \simeq R_{\sigma} \gtrsim 1 \times 10^{-3}$$

(d) Ratio of decay widths

$$\frac{\text{Br}[A \rightarrow J/\psi \pi \pi]}{\text{Br}[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]} \simeq 3.65 \frac{\text{Br}[X(3872) \rightarrow J/\psi \pi \pi]}{\text{Br}[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}$$

$$\frac{\Gamma[A \rightarrow J/\psi \pi^+ \pi^- \pi^0]}{\Gamma[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]} \gtrsim 3.76 \times 10^{-3}$$

$$\frac{\Gamma[A \rightarrow J/\psi \pi \pi]}{\Gamma[X(3872) \rightarrow J/\psi \pi \pi]} \gtrsim 1.37 \times 10^{-2}$$



X atom: Summary

- We show that a null signal of the X atom can be used to put a lower limit on the binding energy of the $X(3872)$.
- If the binding energy of the $X(3872)$ is measured, the lower limit could give a criterion on the $X(3872)$ nature.
- From more and more events collected at the PANDA and LHCb experiments for the $X(3872)$, we can except the signal from the X atom.

Thank you for your attention!

Back Up

Line Shape of the X atom

Binding energy of the X atom: $E_{XA} \sim \text{Re } E_{A1} = E_1 - \frac{\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} \simeq 22.92 \text{ keV}$

Decay width of the X atom: $\Gamma_{XA} \sim \Gamma_c + 2 \text{Im } E_{A1} = \Gamma_c + \frac{2\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} = (89.2 \pm 1.8) \text{ keV} \gg E_{XA}$

The line shape of the X atom is more like the line shape of the Toponium.

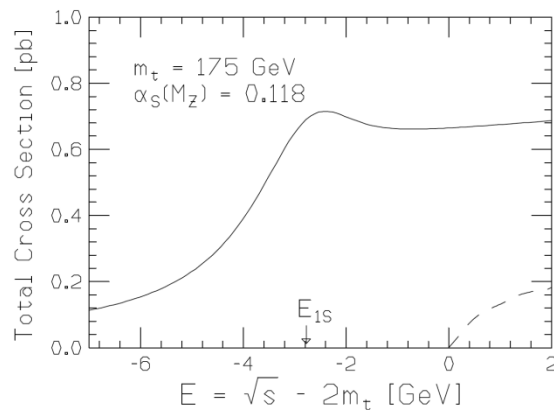


Fig. 4. The total cross section vs. energy, $E = \sqrt{s} - 2m_t$. The solid curve is calculated from the Green function. The dashed curve shows the tree-level total cross section for a stable top quark.

Total cross section of the Toponium.

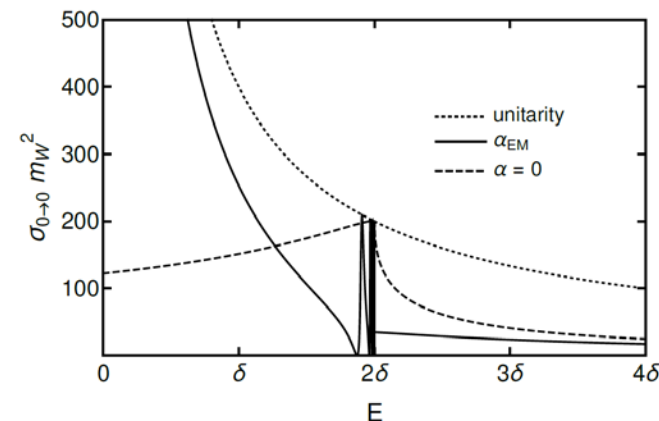


Figure 4. Neutral-wino elastic cross section $\sigma_{0 \rightarrow 0}$ as a function of the energy E . The cross section for $M_* = 2.39 \text{ TeV}$ is shown for $\alpha = 1/137$ (solid curve) and for $\alpha = 0$ (dashed curve). The S-wave unitarity bound is shown as a dotted curve.

Total cross section of the Neutral-wino.

Y. Sumino, *Adv. Ser. Direct. High Energy Phys.* 19, 135(2005) E. Braaten, E. Johnson and H. Zhang, *J. High Energy Phys.* 02(2018) 150

3-body treatment for the $X(3872)$

V. Baru et al. , *Phys. Rev. D* **84** (2011) 074029

V. BARU *et al.*

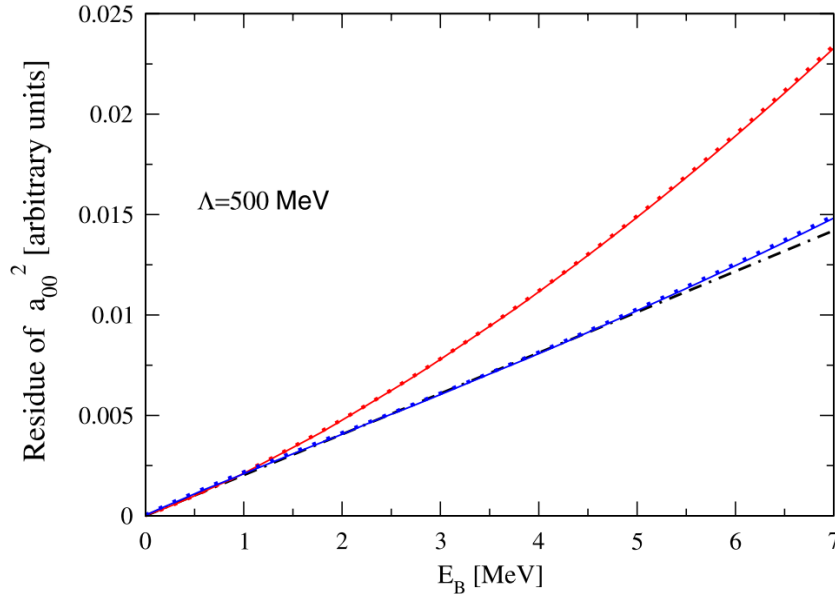


FIG. 3 (color online). Residue of the $D^0 \bar{D}^{*0}$ scattering amplitude squared versus the binding energy in the $D^0 \bar{D}^{*0}$ system. The upper, red (lower, blue) dotted curve corresponds to the solution of the single(two)-channel $D^0 \bar{D}^{*0}$ problem with the contact $D\bar{D}^*$ interaction. Solutions of the full three-body equation with dynamical pions are given by the solid lines: upper, red line—for the single-channel case and lower, blue line—for the two-channel case. The straight dot-dashed line (black) is shown to guide the eye.

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Three-body $D\bar{D}\pi$ dynamics for the $X(3872)$

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Single Channel $D^0 \bar{D}^{*0}$

Coupled Channel $D^0 \bar{D}^{*0} \quad D^+ D^{*-}$

In addition, we found that the residue for $X \rightarrow D\bar{D}^*$ is weakly dependent on the kind of pion dynamics included. Especially, the dependence of the residue on the X binding energy is very close for a fully dynamical calculation and for a calculation with a contact-type interaction only. A deviation between the coupled-channel and the single-channel treatment is clearly observed but with the larger effect for binding energies beyond 1 MeV.

3-body treatment for the $X(3872)$

V. Baru et al. , *Phys. Rev. D* 84 (2011) 074029

THREE-BODY $D\bar{D}\pi$ DYNAMICS FOR THE $X(3872)$

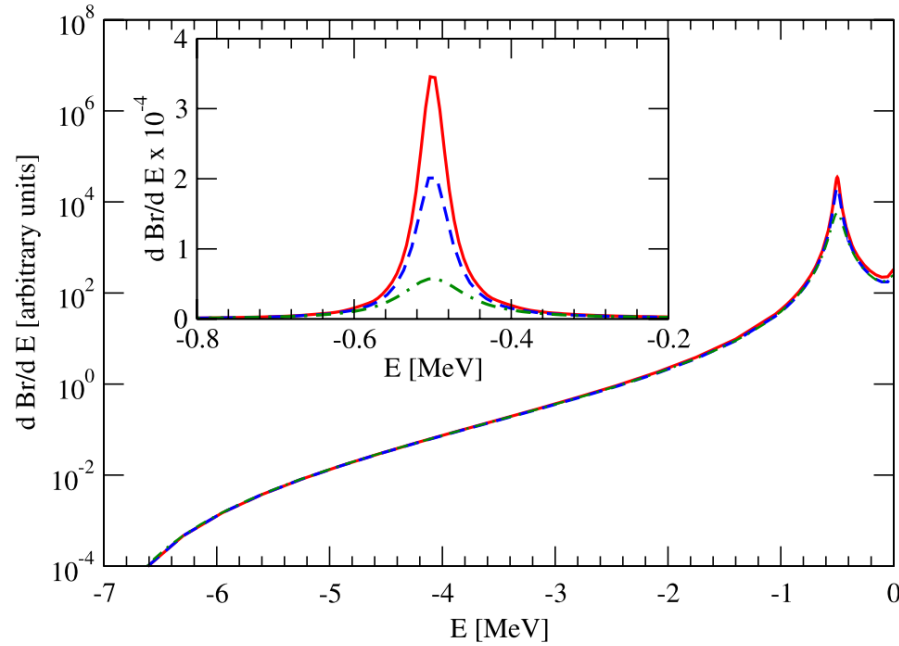


FIG. 5 (color online). Production rate (in logarithmic scale) for the three calculations as described in the text: (i) solution of the single-channel problem in the static limit—(green) dot-dashed line; (ii) solution of the single-channel dynamical calculation—(blue) dashed line; (iii) solution of the full two-channel dynamical problem—(red) solid line. All curves are normalized near the $D^0\bar{D}^0\pi^0$ threshold, located at $E = -7$ MeV. The inlay shows a zoom into the peak region in linear scale.

The most striking effect of dynamical pions is observed in their impact on the X line shapes: in the fully dynamical calculation the width from the $D\bar{D}\pi$ intermediate states appears to be reduced by about a factor of 2, from 102 keV down to 44 keV, assuming that the $X(3872)$ corresponds to a resonance state with a peak at 0.5 MeV below the $D\bar{D}^*$ threshold. Stated differently, by using the naive static approximation for the $D\bar{D}\pi$ intermediate states one overestimates substantially their effect on the X width.

On the contrary, the effect of the coupled-channel dynamics on the X width turned out to be rather moderate, which can be attributed to the fact that both the real part of the resonance pole E_B and the X width Γ_X are small as compared to the separation ΔM between the neutral and the charged thresholds.