Remarks on the composite nature of the light scalar mesons $f_0(980)$ and $a_0(980)$

汪泽强

北京师范大学

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指导老师：康现伟

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Contents

Motivation

- The nonperturbative meson-meson interaction and the related scalar meson is a topic of great importance

  The unflavored scalar mesons below 1 GeV

  $\longrightarrow$ like $f_0(500)/\sigma$, $K^*(800)/\kappa$, $f_0(980)$, $a_0(980)$

- The compositeness can give quantitative description of the inner structure of a resonance/molecular

Formulation of the compositeness relation and decay width

Compositeness and elementariness relation [1]

$$X + Z = 1 \tag{1}$$

- X and Z called the compositeness and elementariness.
- Bound state: Z and X are positive real numbers, allowing probabilistic interpretations
- Resonance: Z and X are usually complex numbers.
- For example: The probability of finding the physical deuteron $|d\rangle$ in a bare elementary-particle state $|d_0\rangle$, $Z = |\langle d_0|d\rangle|^2$. If the deuteron is purely elementary, then $Z = 1$. On the contrary, for a purely composite particle made of a proton and a neutron, $Z = 0$. [1,2]

[1] S. Weinberg, Phys. Rev. 137, B672 (1965)

[2] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A.E. Kudryavtsev, Evidence that the a(0)(980) and f(0)(980) are not elementary particles, Phys. Lett. B 586, 53 (2004).

Formulation of the compositeness relation and decay width

Here we will follow Ref. [1], which formulates a probabilistic interpretation of compositeness relation with only positive and real coefficient for resonance. After the proper unitary phase transformation of S-matrix, it offers the partial compositeness coefficient for resonance in the form as

$$X_i = |\gamma_i^2| \left| \frac{\partial G_i(s)}{\partial s} \right|_{s=s_P} \tag{2}$$

compared to the bound state case,

$$X_i = -\gamma_i^2 \frac{\partial G_i(s)}{\partial s} \Big|_{s=s_P}. \tag{3}$$

where $G(s)$ is the one-loop two-point function.

[1] Z.-H. Guo and J. A. Oller, Phys. Rev. D 93, 096001 (2016), 1508.06400.

Formulation of the compositeness relation and decay width

For the different imaginary part of p_1 and p_2, we can define the four different Riemann Sheets as

$$\begin{aligned} \text{Sheet I}: &\quad \mathrm{Im}p_1 > 0,\ \mathrm{Im}p_2 > 0 \\ \text{Sheet II}: &\quad \mathrm{Im}p_1 < 0,\ \mathrm{Im}p_2 > 0 \\ \text{Sheet III}: &\quad \mathrm{Im}p_1 < 0,\ \mathrm{Im}p_2 < 0 \\ \text{Sheet IV}: &\quad \mathrm{Im}p_1 > 0,\ \mathrm{Im}p_2 < 0 \end{aligned} \tag{4}$$

Riemann Sheet II and III are connected to the physical Reimmann Sheet I below and above the $K\overline{K}$ threshold in the real axis, respectively.We have

$$G_i^{II}(s-i\epsilon) = G_i^{I}(s-i\epsilon) - \frac{i}{4\pi}\frac{\sqrt{[s-(m_1+m_2)^2][s-(m_1-m_2)^2]}}{2s} \tag{5}$$

The notion of $s-i\epsilon$ is to distinguish the negative imaginary part. Thus the Sheet I is obtained with $G_1^{I}(s)$, $G_2^{I}(s)$; the Sheet II is obtained with $G_1^{II}(s)$, $G_2^{I}(s)$; the Sheet III is obtained with $G_1^{II}(s)$, $G_2^{II}(s)$; the Sheet IV is obtained with $G_1^{I}(s)$, $G_2^{II}(s)$;

Formulation of the compositeness relation and decay width

The standard formula for the partial decay width:

$$\Gamma_{\pi\pi} = \frac{|\gamma_1|^2 p_1(m_P^2)}{8\pi m_P^2} \tag{6}$$

where p_i is the momentum in the rest frame of resonance. The $K\overline{K}$ threshold is very close to the resonance mass and the effect of the finite width of $f_0(980)$ (around 50 MeV) is not negligible. Or even, the lower limit of $f_0(980)$ mass within the uncertainty region may be smaller than $K\overline{K}$ threshold. We will consider the Lorentzian distribution for the resonance mass, and the partial decay width can be written as

$$\Gamma_{K\overline{K}} = |\gamma_2|^2 \frac{1}{16\pi^2} \int_{m_1+m_2}^{+\infty} dW \frac{p_2(W^2)}{W^2} \frac{\Gamma_P}{(m_P - W)^2 + \Gamma_P^2/4} \tag{7}$$

Two main equation

- We suppose that the total compositeness coefficient of $f_0(980)$ can be expressed as the sum of those for two channels $\pi\pi$ and $K\bar{K}$:

$$X = X_{\pi\pi} + X_{K\bar{K}} = |\gamma_1|^2 \left| \frac{\partial G_1(s)}{\partial s} \right|_{s=s_P} + |\gamma_2|^2 \left| \frac{\partial G_2(s)}{\partial s} \right|_{s=s_P} \tag{8}$$

- The total decay width of $f_0(980)$ is then

$$\Gamma_P = |\gamma_1|^2 \frac{p_1(m_P^2)}{8\pi m_P^2} + |\gamma_2|^2 \frac{1}{16\pi^2} \int_{m_1+m_2}^{m_P+2\Gamma_P} dW \frac{p(W^2)}{W^2} \frac{\Gamma_P}{(m_P - W)^2 + \Gamma_P^2/4} \tag{9}$$

- Our input parameter of $f_0(980)$ is from the dispersive analysis in a model-independent way, which provide a reliable and accurate determination. [1]

$$m_P = 996 \pm 7 \text{ MeV}, \;\; \Gamma_P = 50^{+20}_{-12} \text{ MeV} \tag{10}$$

[1] R. Garcia-Martin, R. Kaminski, J. R. Pelaez, and J. Ruiz de Elvira, Phys. Rev. Lett. 107, 072001 (2011), 1107.1635.

Results

- The compositeness coefficient $K\overline{K}$ is always much larger than $\pi\pi$. Moreover, $f_0(980)$ couples much more strongly to $K\overline{K}$ channel than to $\pi\pi$
- The partial decay width of $\pi\pi$ channel should be larger than the one for $K\overline{K}$. In other words, smaller $X(X \sim 0.4)$ is preferred. Other components contribute to the elementariness, like $q\bar{q}$, compact $qq\bar{q}\bar{q}$, or their superposition.

For X =0.8, 0.6, 0.4 in Riemann Sheet II and III, we calculate the couplings(columns 3,4), corresponding partial decay width Γ_i (columns 5,6) and individual compositeness coefficients X_i(columns 7,8) for each channel.

X	Riemann Sheet	$\lvert\gamma_{\pi\pi}\rvert$(GeV)	$\lvert\gamma_{K\overline{K}}\rvert$(GeV)	$\Gamma_{\pi\pi}$(MeV)	$\Gamma_{K\overline{K}}$(MeV)	$X_{\pi\pi}$	$X_{K\overline{K}}$
0.8	II	–	–	–	–	–	–
	III	$1.05^{+0.41}_{-0.64}$	$3.86^{+0.36}_{-0.30}$	$21.06^{+19.74}_{-17.90}$	$28.94^{+15.73}_{-12.13}$	$0.0082^{+0.0078}_{-0.0070}$	$0.7918^{+0.0070}_{-0.0078}$
0.6	II	$0.99^{+0.34}_{-0.52}$	$4.00^{+0.45}_{-0.31}$	$18.90^{+15.57}_{-14.66}$	$31.10^{+18.52}_{-12.87}$	$0.0073^{+0.0062}_{-0.0057}$	$0.5927^{+0.0057}_{-0.0062}$
	III	$1.21^{+0.36}_{-0.42}$	$3.33^{+0.31}_{-0.25}$	$28.48^{+19.89}_{-16.45}$	$21.52^{+11.59}_{-9.00}$	$0.0110^{+0.0079}_{-0.0064}$	$0.5890^{+0.0064}_{-0.0079}$
0.4	II	$1.22^{+0.32}_{-0.33}$	$3.24^{+0.35}_{-0.25}$	$29.61^{+17.27}_{-13.84}$	$20.39^{+11.92}_{-8.39}$	$0.0115^{+0.0069}_{-0.0054}$	$0.3885^{+0.0054}_{-0.0069}$
	III	$1.37^{+0.33}_{-0.32}$	$2.70^{+0.24}_{-0.20}$	$35.89^{+20.03}_{-14.99}$	$14.11^{+7.45}_{-5.86}$	$0.0139^{+0.0080}_{-0.0059}$	$0.3861^{+0.0059}_{-0.0080}$

- Given the branching ratio value $\Gamma_{\pi\pi}/(\Gamma_{\pi\pi}+\Gamma_{K\overline{K}})$. Let $\Gamma_{\pi\pi}/(\Gamma_{\pi\pi}+\Gamma_{K\overline{K}})=b$ and $\Gamma_{\pi\pi}+\Gamma_{K\overline{K}}=\Gamma_P$, the value of compositeness $X_{\pi\pi}$ and $X_{K\overline{K}}$ will be given by

$$X_{\pi\pi} = \frac{8\pi b\Gamma_P m_P^2}{p_1(m_P^2)}\left|\frac{\partial G_1(s)}{\partial s}\right|_{s=s_P} \tag{11}$$

$$X_{K\overline{K}} = \frac{16\pi^2(1-b)}{\int_{m_1+m_2}^{m_P+2\Gamma_P} dW\,\frac{p(W^2)}{W^2}\frac{1}{(m_P-W)^2+\Gamma_P^2/4}}\left|\frac{\partial G_2(s)}{\partial s}\right|_{s=s_P} \tag{12}$$

Results

- In the Sheet II, the range of the total compositeness varies from 0.2 to 0.45, while in the Sheet III the range becomes 0.3 to 0.65. From the "molecular" point of view, it is also more suitable for interpreting $f_0(980)$ as a resonance locating at the Sheet III.

Combining the ratio of $\Gamma_{\pi\pi}/(\Gamma_{\pi\pi}+\Gamma_{K\overline{K}})$, as well as $\Gamma_{\pi\pi}+\Gamma_{K\overline{K}}=\Gamma_P$, the corresponding values are uniquely predicted in the II and III Riemann Sheet.

$\frac{\Gamma_{\pi\pi}}{\Gamma_{\pi\pi}+\Gamma_{K\overline{K}}}$	Riemann Sheet	$\lvert\gamma_{\pi\pi}\rvert$(GeV)	$\lvert\gamma_{K\overline{K}}\rvert$(GeV)	$\Gamma_{\pi\pi}$(MeV)	$\Gamma_{K\overline{K}}$(MeV)	$X_{\pi\pi}$	$X_{K\overline{K}}$
0.52[1]	II	$1.16^{+0.21}_{-0.15}$	$3.52^{+0.79}_{-0.71}$	$26.00^{+10.40}_{-6.24}$	$24.00^{+9.60}_{-5.76}$	$0.0101^{+0.0042}_{-0.0025}$	$0.4574^{+0.1351}_{-0.1341}$
	III	$1.16^{+0.21}_{-0.15}$	$3.52^{+0.79}_{-0.71}$	$26.00^{+10.40}_{-6.24}$	$24.00^{+9.60}_{-5.76}$	$0.0101^{+0.0042}_{-0.0025}$	$0.6567^{+0.2456}_{-0.2385}$
0.68[2]	II	$1.33^{+0.25}_{-0.17}$	$2.87^{+0.64}_{-0.58}$	$34.00^{+13.60}_{-8.16}$	$16.00^{+6.40}_{-3.84}$	$0.0132^{+0.0055}_{-0.0033}$	$0.3049^{+0.0901}_{-0.0894}$
	III	$1.33^{+0.25}_{-0.17}$	$2.87^{+0.64}_{-0.58}$	$34.00^{+13.60}_{-8.16}$	$16.00^{+6.40}_{-3.84}$	$0.0132^{+0.0055}_{-0.0033}$	$0.4378^{+0.1637}_{-0.1590}$
0.78[3]	II	$1.43^{+0.27}_{-0.19}$	$2.38^{+0.53}_{-0.48}$	$39.00^{+15.6}_{-9.36}$	$11.00^{+4.40}_{-2.64}$	$0.0151^{+0.0063}_{-0.0037}$	$0.2096^{+0.0619}_{-0.0615}$
	III	$1.43^{+0.27}_{-0.19}$	$2.38^{+0.53}_{-0.48}$	$39.00^{+15.6}_{-9.36}$	$11.00^{+4.40}_{-2.64}$	$0.0151^{+0.0063}_{-0.0037}$	$0.3010^{+0.1125}_{-0.1093}$

[1] B. Aubert et al. (BaBar), Phys. Rev. D 74, 032003 (2006), hep-ex/0605003.

[2] J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997), [Erratum: Nucl.Phys.A 652, 407 – 409 (1999)], hep-ph/9702314.

[3] W. Wetzel, K. Freudenreich, F. X. Gentit, P. Muhlemann, W. Beusch, A. Birman, D. Websdale, P. Astbury, A. Harckham, and M. Letheren, Nucl. Phys. B 115, 208 (1976).

Results of $a_0(980)$

A recent coupled-channel analysis of antiproton-proton annihilation data is presented in Ref. [1], where the pole parameters and partial decay width of $a_0(980)$ are discussed; In Sheet II,

$$m_P = 1004.1 \pm 6.67 \text{ MeV}, \Gamma_P = 97.2 \pm 6.01 \text{ MeV}, \Gamma_{K\overline{K}}/\Gamma_{\pi\eta} = 13.8 \pm 3.5 \ \% \tag{13}$$

and in the Sheet III,

$$m_P = 1002.4 \pm 6.55 \text{ MeV}, \Gamma_P = 127.0 \pm 7.08 \text{ MeV}, \Gamma_{K\overline{K}}/\Gamma_{\pi\eta} = 14.9 \pm 3.9 \ \% \tag{14}$$

TABLE V: Given the total compositeness X varying from 0.2 to 0.6 in different Riemann Sheets for $a_0(980)$, we predict the couplings, the compositeness coefficients and the partial decay widths. The label 1 denotes the channel $\pi\eta$ and 2 the channel $K\overline{K}$.

X	Riemann Sheet	$\lvert\gamma_{\pi\eta}\rvert$(GeV)	$\lvert\gamma_{K\overline{K}}\rvert$(GeV)	$\Gamma_{\pi\eta}$(MeV)	$\Gamma_{K\overline{K}}$(MeV)	$X_{\pi\eta}$	$X_{K\overline{K}}$
0.6	II	$1.61^{+0.23}_{-0.31}$	$4.82^{+0.14}_{-0.13}$	$34.24^{+10.59}_{-11.89}$	$62.96^{+11.23}_{-10.30}$	$0.0305^{+0.0104}_{-0.0111}$	$0.5695^{+0.0111}_{-0.0104}$
	III	$2.56^{+0.16}_{-0.18}$	$3.78^{+0.11}_{-0.10}$	$87.04^{+11.06}_{-11.86}$	$39.96^{+6.10}_{-5.74}$	$0.0790^{+0.0122}_{-0.0123}$	$0.5210^{+0.0123}_{-0.0122}$
0.4	II	$2.11^{+0.16}_{-0.19}$	$3.77^{+0.10}_{-0.09}$	$58.77^{+9.14}_{-9.97}$	$38.43^{+6.64}_{-6.14}$	$0.0524^{+0.0096}_{-0.0099}$	$0.3476^{+0.0099}_{-0.0096}$
	III	$2.80^{+0.13}_{-0.14}$	$2.90^{+0.08}_{-0.07}$	$103.52^{+9.71}_{-10.22}$	$23.48^{+3.42}_{-3.24}$	$0.0939^{+0.0113}_{-0.0112}$	$0.3061^{+0.0112}_{-0.0113}$
0.2	II	$2.51^{+0.11}_{-0.12}$	$2.27^{+0.05}_{-0.04}$	$83.30^{+7.70}_{-8.05}$	$13.90^{+2.08}_{-1.97}$	$0.0743^{+0.0088}_{-0.0087}$	$0.1257^{+0.0087}_{-0.0088}$
	III	$3.01^{+0.10}_{-0.11}$	$1.58^{+0.10}_{-0.10}$	$120.01^{+8.36}_{-8.58}$	$6.99^{+1.50}_{-1.28}$	$0.1089^{+0.0104}_{-0.0101}$	$0.0911^{+0.0101}_{-0.0104}$

[1] M. Albrecht et al. (Crystal Barrel), Eur. Phys. J. C 80, 453 (2020), 1909.07091.

Results of $a_0(980)$

TABLE VI: Using $\Gamma_{K\overline{K}}/\Gamma_{\eta\pi}$ values from PDG and the condition of $\Gamma_{K\overline{K}}+\Gamma_{\eta\pi}=\Gamma_P$, we predict the compositeness coefficient X_i.

$\frac{\Gamma_{K\overline{K}}}{\Gamma_{\eta\pi}}$	Riemann Sheet	$\lvert\gamma_{\pi\eta}\rvert$(GeV)	$\lvert\gamma_{K\overline{K}}\rvert$(GeV)	$\Gamma_{\pi\eta}$(MeV)	$\Gamma_{K\overline{K}}$(MeV)	$X_{\pi\eta}$	$X_{K\overline{K}}$
0.138[1]	II	$2.54^{+0.08}_{-0.08}$	$2.09^{+0.16}_{-0.16}$	$85.41^{+5.28}_{-5.28}$	$11.79^{+0.73}_{-0.73}$	$0.0761^{+0.0067}_{-0.0064}$	$0.1066^{+0.0132}_{-0.0134}$
	III	$2.90^{+0.08}_{-0.08}$	$2.35^{+0.15}_{-0.14}$	$111.60^{+6.22}_{-6.22}$	$15.40^{+0.86}_{-0.86}$	$0.1013^{+0.0082}_{-0.0079}$	$0.2008^{+0.0291}_{-0.0274}$
0.177[2]	II	$2.50^{+0.08}_{-0.08}$	$2.32^{+0.18}_{-0.17}$	$82.58^{+5.11}_{-5.11}$	$14.62^{+0.90}_{-0.90}$	$0.0736^{+0.0064}_{-0.0061}$	$0.1322^{+0.0164}_{-0.0166}$
	III	$2.85^{+0.08}_{-0.08}$	$2.61^{+0.16}_{-0.16}$	$107.90^{+6.02}_{-6.02}$	$19.10^{+1.06}_{-1.06}$	$0.0979^{+0.0079}_{-0.0076}$	$0.2490^{+0.0361}_{-0.0340}$
0.230[3]	II	$2.44^{+0.08}_{-0.08}$	$2.59^{+0.20}_{-0.19}$	$79.02^{+4.89}_{-4.89}$	$18.18^{+1.12}_{-1.12}$	$0.0704^{+0.0062}_{-0.0059}$	$0.1644^{+0.0204}_{-0.0206}$
	III	$2.79^{+0.08}_{-0.08}$	$2.92^{+0.18}_{-0.18}$	$103.25^{+5.76}_{-5.76}$	$23.75^{+1.32}_{-1.32}$	$0.0937^{+0.0076}_{-0.0073}$	$0.3096^{+0.0449}_{-0.0423}$

[1] M. Albrecht et al. (Crystal Barrel), Eur. Phys. J. C 80, 453 (2020), 1909.07091.

[2] P. A. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020).

[3] A. Abele et al., Phys. Rev. D 57, 3860 (1998).

Flatté parameterization

- We consider the propagator of a resonance following a Flatté parameterization [1]

$$D(E) = E - E_f + i\frac{\Gamma_1}{2} + \frac{i}{2} g_2 \sqrt{m_K E} \tag{15}$$

We look for the zero of $D(E_R) = 0$ to determine the resonance pole position E_R. We have the equation:

$$E_R - E_f + i\frac{\Gamma_1}{2} = -\frac{i}{2} g_2 \sqrt{m_K E} \tag{16}$$

We have to distinguish two cases according to the sign of $m_K g_2^2/16 - E_f$,

i) $\frac{m_K g_2^2}{16} - E_f > 0$

$$E_R = E_f - \frac{i}{2}\Gamma_1 - \frac{1}{8} m_K g_2^2 - \sqrt{\frac{m_K g_2^2}{4}} \left(\left(\frac{m_K g_2^2}{16} - E_f \right)^2 + \frac{\Gamma_1^2}{4} \right)^{\frac{1}{4}} \exp\left(\frac{i}{2} \arctan \frac{\Gamma_1/2}{m_K g_2^2/16 - E_f} \right) \tag{17}$$

ii) $\frac{m_K g_2^2}{16} - E_f < 0$

$$E_R = E_f - \frac{i}{2}\Gamma_1 - \frac{1}{8} m_K g_2^2 - \sqrt{\frac{m_K g_2^2}{4}} \left(\left(\frac{E_f - m_K g_2^2}{16} \right)^2 + \frac{\Gamma_1^2}{4} \right)^{\frac{1}{4}} \exp \frac{i}{2} \left(\pi - \arctan \frac{\Gamma_1/2}{E_f - m_K g_2^2/1} \tag{18}$$

[1] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A.E. Kudryavtsev, Evidence that the a(0)(980)

Flatté parameterization

We introduce the auxiliary angle ϕ defined by

$$\phi = \arctan \frac{\Gamma_1/2}{E_f - m_K g_2^2/16} \tag{19}$$

For the discussions we write down explicitly the expressions for the cases i) and ii) for the mass and width of the resonance as obtained from the pole position

Case i)

$$\begin{aligned} M_R &= -\frac{m_K g_2^2}{16} - \frac{1}{2}\Gamma_R \cot\frac{\phi}{2} - \frac{\Gamma_1}{4}\left(\tan\frac{\phi}{2} + \cot\frac{\phi}{2}\right) \\ \Gamma_R &= \Gamma_1 + \frac{1}{2}\sqrt{m_K g_2^2 \Gamma_1 |\tan\frac{\phi}{2}|} \end{aligned} \tag{20}$$

Case ii)

$$\begin{aligned} M_R &= -\frac{m_K g_2^2}{16} - \frac{1}{2}\Gamma_R \tan\frac{\phi}{2} + \frac{\Gamma_1}{4}\left(\tan\frac{\phi}{2} + \cot\frac{\phi}{2}\right) \\ \Gamma_R &= \Gamma_1 + \frac{1}{2}\sqrt{m_K g_2^2 \Gamma_1 \cot\frac{\phi}{2}} \end{aligned} \tag{21}$$

Flatté parameterization

We need to fix fix Γ_1, ϕ and g_2

Case i)

$$M_R = -\frac{m_K g_2^2}{16} - \frac{1}{2}\Gamma_R \cot\frac{\phi}{2} - \frac{\Gamma_1}{4}\left(\tan\frac{\phi}{2} + \cot\frac{\phi}{2}\right) \tag{22}$$

$$\Gamma_R = \Gamma_1 + \frac{1}{2}\sqrt{m_K g_2^2 \Gamma_1 |\tan\frac{\phi}{2}|}$$

Finally, if $X = X_1 + X_2$, M_P and Γ_P are the inputs, then we have the extra equation or branching ratio Γ_1/Γ_R

$$X = \frac{8\pi M_R^2 \Gamma_1}{q_1}\left|\frac{\partial G_1(s)}{\partial s}\right|_{s=s_P} + |\gamma_2^2|\left|\frac{\partial G_2(s)}{\partial s}\right|_{s=s_P} \tag{23}$$

Flatté parameterization

We have the $g_2\sqrt{m_K E}$ is the width to the $K\overline{K}$ channel at energy E. Then,

$$\gamma_2^2 \frac{\sqrt{(2m_K + M_R)^2 - 4m_K^2}}{16\pi(M_R + 2m_K)^2} \to g_2\sqrt{m_K M_R} \tag{24}$$

Therefore,

$$\gamma_2^2 = \frac{16\pi(M_R + 2m_K)^2}{\sqrt{4 + M_R/m_K}} \approx 32\pi m_K^2 g_2 \tag{25}$$

where at the end we taken $M_R \ll 2m_K$ (because the resonance mass is near the two-kaon threshold and this is taken as energy reference).

Results of Flatté parameterization

X	Riemann Sheet	$\lvert\gamma_{\pi\pi}\rvert$(GeV)	$\lvert\gamma_{K\overline{K}}\rvert$(GeV)	$\Gamma_{\pi\pi}$(MeV)	$\Gamma_{K\overline{K}}$(MeV)	$X_{\pi\pi}$	$X_{K\overline{K}}$
0.4	II	$0.81^{+0.26}_{-0.49}$	$3.27^{+0.37}_{-0.25}$	$12.60^{+9.66}_{-10.70}$	$37.40^{+21.51}_{-13.35}$	$0.0049^{+0.0038}_{-0.0042}$	$0.3951^{+0.0042}_{-0.0038}$
	III	$1.12^{+0.34}_{-0.39}$	$2.71^{+0.25}_{-0.20}$	$24.25^{+16.96}_{-13.93}$	$25.75^{+13.22}_{-9.29}$	$0.0094^{+0.0067}_{-0.0054}$	$0.3906^{+0.0054}_{-0.0067}$
0.3	II	$1.08^{+0.27}_{-0.31}$	$2.81^{+0.31}_{-0.21}$	$22.43^{+12.65}_{-11.16}$	$27.57^{+15.54}_{-9.77}$	$0.0087^{+0.0051}_{-0.0044}$	$0.2913^{+0.0044}_{-0.0051}$
	III	$1.27^{+0.32}_{-0.31}$	$2.33^{+0.20}_{-0.17}$	$31.02^{+17.92}_{-13.51}$	$18.98^{+9.53}_{-6.80}$	$0.0120^{+0.0072}_{-0.0053}$	$0.2880^{+0.0053}_{-0.0072}$
0.2	II	$1.30^{+0.28}_{-0.26}$	$2.25^{+0.23}_{-0.16}$	$32.26^{+15.63}_{-11.62}$	$17.75^{+9.58}_{-6.19}$	$0.0125^{+0.0063}_{-0.0046}$	$0.1875^{+0.0046}_{-0.0063}$
	III	$1.40^{+0.31}_{-0.27}$	$1.87^{+0.15}_{-0.13}$	$37.78^{+18.88}_{-13.10}$	$12.22^{+5.86}_{-4.31}$	$0.0147^{+0.0076}_{-0.0052}$	$0.1853^{+0.0052}_{-0.0076}$

$\frac{\Gamma_{\pi\pi}}{\Gamma_{\pi\pi}+\Gamma_{K\overline{K}}}$	Riemann Sheet	$\lvert\gamma_{\pi\pi}\rvert$(GeV)	$\lvert\gamma_{K\overline{K}}\rvert$(GeV)	$\Gamma_{\pi\pi}$(MeV)	$\Gamma_{K\overline{K}}$(MeV)	$X_{\pi\pi}$	$X_{K\overline{K}}$
0.52[1]	II	$1.16^{+0.21}_{-0.15}$	$2.62^{+0.42}_{-0.40}$	$26.00^{+10.40}_{-6.24}$	$24.00^{+9.60}_{-5.76}$	$0.0101^{+0.0042}_{-0.0025}$	$0.2536^{+0.0457}_{-0.0518}$
	III	$1.16^{+0.21}_{-0.15}$	$2.62^{+0.42}_{-0.40}$	$26.00^{+10.40}_{-6.24}$	$24.00^{+9.60}_{-5.76}$	$0.0101^{+0.0042}_{-0.0025}$	$0.3641^{+0.0839}_{-0.1031}$
0.68[2]	II	$1.33^{+0.25}_{-0.17}$	$2.14^{+0.34}_{-0.33}$	$34.00^{+13.60}_{-8.16}$	$16.00^{+6.40}_{-3.84}$	$0.0132^{+0.0055}_{-0.0033}$	$0.1601^{+0.0298}_{-0.0350}$
	III	$1.33^{+0.25}_{-0.17}$	$2.14^{+0.34}_{-0.33}$	$34.00^{+13.60}_{-8.16}$	$16.00^{+6.40}_{-3.84}$	$0.0132^{+0.0055}_{-0.0033}$	$0.2427^{+0.055}_{-0.069}$
0.78[3]	II	$1.43^{+0.27}_{-0.19}$	$1.77^{+0.28}_{-0.27}$	$39.00^{+15.6}_{-9.36}$	$11.00^{+4.40}_{-2.64}$	$0.0151^{+0.0063}_{-0.0037}$	$0.1162^{+0.0209}_{-0.0237}$
	III	$1.43^{+0.27}_{-0.19}$	$1.77^{+0.28}_{-0.27}$	$39.00^{+15.6}_{-9.36}$	$11.00^{+4.40}_{-2.64}$	$0.0151^{+0.0063}_{-0.0037}$	$0.1669^{+0.0385}_{-0.0472}$

Summary

- We predict the couplings, compositeness coefficients and partial widths of the $\pi\pi$ and $K\overline{K}$ channels.
- We also roughly discuss the compositeness of the resonance $a_0(980)$ under the assumption of molecular interpretation.
- We use the Flatté parameterization to discuss the compositeness coefficients.
- The compositeness concept, as a quantitative examination of the inner structure of a resonance/molecular is very crucial to promote a step forward the understanding of hadron structure.

THANK YOU