

Parton Distribution Functions Matching in LaMET

Ruilin Zhu(朱瑞林)

南京师范大学

中国物理学会高能物理分会第十三届全国粒子物理学学术会议
(2021)

2021年8月16日-20日 山东大学青岛校区 (by Zoom)

Based on recent works

- Next-to-next-to-leading order corrections to non-singlet quark Quasi distribution functions

L.-B. Chen (陈龙斌), *W. Wang* (王伟), *R. Zhu* (朱瑞林),
Phys. Rev. Lett. **126**, 072002 (2021).

- Master Integrals for two-loop QCD corrections to Quasi PDFs

L.-B. Chen, W. Wang, R. Zhu, JHEP10, 079 (2020).

- Quasi parton distribution functions at NNLO: flavor non-diagonal quark contributions

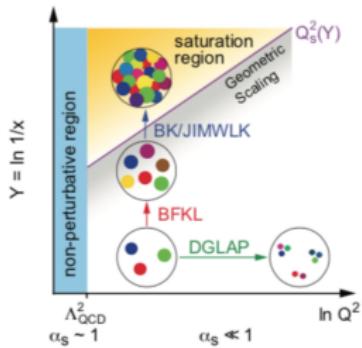
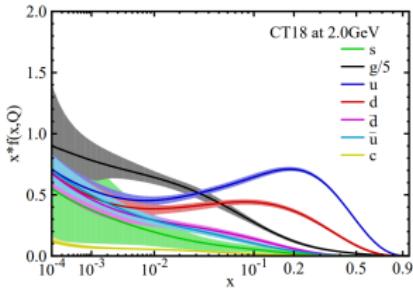
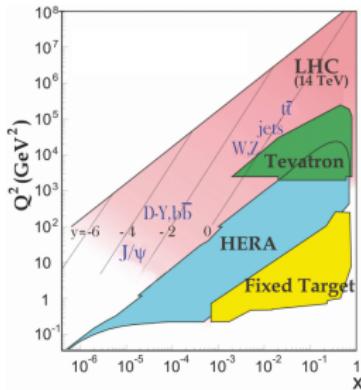
L.-B. Chen, W. Wang, R. Zhu, Phys. Rev. D102, 011503 (2020).

Outline

- ① PDFs and LaMET introduction
- ② Two Loop Calculation of Quark Quasi PDF
- ③ NNLO Results
 - $q \rightarrow q'$ case
 - $q \rightarrow q$ case
- ④ Summary

Available PDFs

PDG2018 and 1912.10053



- Parton distribution functions (PDFs) are fundamental inputs for hadron colliding processes; QCD factorization allows us to extract the partonic structure information of hadrons
Current available PDFs from the global fit
- We know **some** (more on perturbative aspects) of the PDFs at many different facilities over 50 years effort, however, we understand **less** from first principle of QCD due to its light-cone correlation

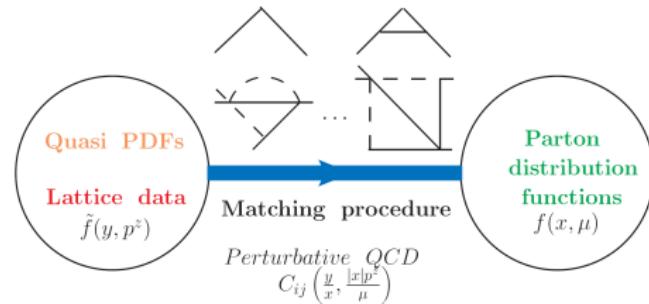
Large Momentum Effective Theory (LaMET)

- LaMET factorization formula

$$\tilde{f}_{i/H}(y, p^z) = \int_{-1}^1 \frac{dx}{|x|} \left[C_{ij} \left(\frac{y}{x}, \frac{|x| p^z}{\mu} \right) f_{j/H}(x, \mu) \right] + \mathcal{O} \left(\frac{m_h^2}{p^{z2}}, \frac{\Lambda_{\text{QCD}}^2}{p^{z2}} \right)$$

$x \in [-1, 1], y \in [-\infty, \infty]$

X. Ji, PRL110, 262002 (2013), ... See previous talk by Jian-Hui Zhang



- other approaches such as pseudo-PDFs, Good lattice cross-section,

Radyushkin, 1705.01488; Ma-Qiu, 1709.03018

Perturbative calculation of $C_{ij}^{(0)}, C_{ij}^{(1)}, C_{ij}^{(2)}, \dots$

- $C_{ij} = C_{ij}^{(0)} + \frac{\alpha_s}{2\pi} C_{ij}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_{ij}^{(2)} + \dots$
- Leading order(LO): $C_{ij}^{(0)}(y) = \delta(1-y)$
- Higher-order matching, renormalization scheme dependent.
- Next-to-leading order (NLO) $C_{ij}^{(1)}(y, \frac{p^z}{\mu})$

MS: Izubuchi, Ji, Jin, Stewart, Zhao, 1801.03917;

MMS: Alexandrou, Cichy, Constantinou, Jansen, Scapellato, Steffens, 1803.02685;

RI/MOM: Stewart, Zhao, 1709.04933; Wang, Zhang, Zhao, Zhu, 1904.00978;

Others: Ji, Xiong, Zhang, Zhao, 1310.7471; Ma, Qiu, 1404.6860, Wang, Zhao, Zhu, 1708.02458, ...

3 regions for y: $[-\infty, 0], [0, 1], [1, +\infty]$, 1 color factor: C_F

- Next-to-next-to-leading order(NNLO) $C_{ij}^{(2)}$ (done only for quark case)
 - higher-order corrections are important in QCD
 - $\mu = 2\text{GeV}$, $\alpha_s(\mu = 2\text{GeV}) \sim 0.3$, α_s^2 -correction is needed for a precision prediction
 - factorization proof at NNLO is nontrivial
 - Li, Ma, Qiu 2020; Chen, Wang, Zhu 2020

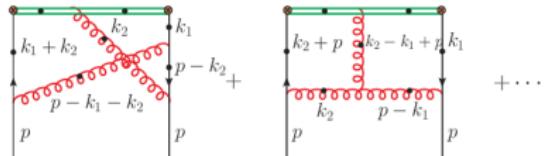
Calculation procedures

- FeynRules and FeynArts to auto produce two-loop Feynman diagrams and amplitudes

Christensen et al, 1310.1921, T. Hahn, 0012260

$$f_{q/H}(x, \mu) = \int \frac{d\xi^-}{4\pi} e^{-ixp^+ \xi^-} \langle p | \bar{q}(\xi^-) \gamma^+ W(\xi^-, 0) q(0) | p \rangle,$$

$$\tilde{f}_{q/H}(y, p^z) = \frac{p^z}{p^0} \int \frac{dz}{4\pi} e^{izyp^z} \langle p | \bar{q}(z) \gamma^0 W(z, 0) q(0) | p \rangle,$$



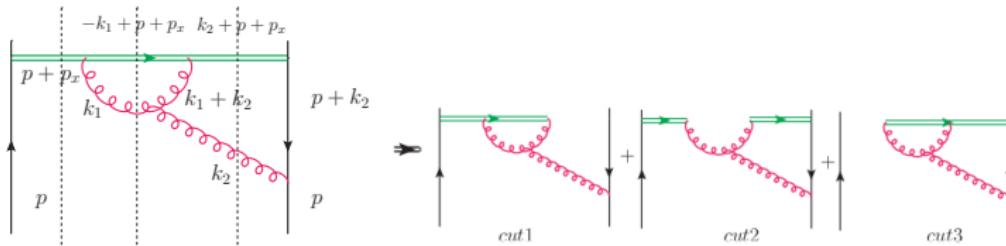
- Cutkosky rules, *J.Math.Phys.1,429(1960)*

$$\delta(k_z - xp_z) = \frac{1}{2\pi i} \left(\frac{1}{k_z - xp_z - i0} - \frac{1}{k_z - xp_z + i0} \right).$$

- Solve Master Integrals(MIs)

Differential equation, A.V.Kotikov, PLB254, 158(1991); PLB267, 123(1991)

An example in Feynman gauge



- From auxiliary field back to Wilson line, we need to do the cuts. For cut1, we have $p_x = -p - k_2$

$$\mathcal{M}|_{\text{cut1}} = \mu^{4\epsilon} \int \int \frac{d^{4-2\epsilon} k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon} k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut1} \times \delta(k_2^z + p^z - yp^z),$$

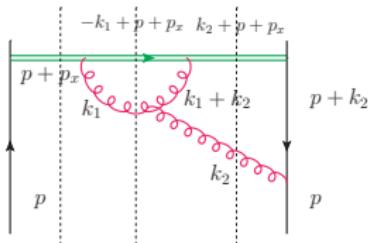
- For cut2, we have $p_x = -p + k_1$; both them give real contribution

$$\mathcal{M}|_{\text{cut2}} = \mu^{4\epsilon} \int \int \frac{d^{4-2\epsilon} k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon} k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut2} \times \delta(-k_1^z + p^z - yp^z),$$

- For cut3, we have $p_x = -p$ and it gives virtual contribution

$$\mathcal{M}|_{\text{cut3}} = \mu^{4\epsilon} \int \int \frac{d^{4-2\epsilon} k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon} k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut3} \times \delta(1 - y),$$

An example in Feynman gauge



- Use the identity

$$\frac{1}{k_1 \cdot n k_2 \cdot n} = \frac{1}{(k_1 \cdot n + k_2 \cdot n) k_2 \cdot n} + \frac{1}{k_1 \cdot n (k_1 \cdot n + k_2 \cdot n)},$$

- do the momentum transformation, then

$$\begin{aligned} \mathcal{M}|_{cut1+cut2+cut3} &= \left[\mu^{4\epsilon} \int \int \frac{d^{3-2\epsilon} k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon} k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut1}'|_{k_1^z=yp_z} \right]_+ \\ &+ \left[\mu^{4\epsilon} \int \int \frac{d^{3-2\epsilon} k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon} k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut2}'|_{k_1^z=yp_z} \right]_+ \end{aligned}$$

- It includes both the virtual and real contributions

Master Integrals Calculation:Differential Equations

- To calculate MIs f_i , we can set up a differential equation with respect to Lorentz invariant kinematics z , for example $z = \frac{p^0}{p^z}$ (or p^2)
- If the number of MIs is larger than 1, A is $n \times n$ coefficient matrix and depends on both z and ϵ

$$\frac{d}{dz} \begin{pmatrix} f_1(z, \epsilon) \\ \vdots \\ f_n(z, \epsilon) \end{pmatrix} = \begin{pmatrix} A_{11}(z, \epsilon) & \dots & A_{1n}(z, \epsilon) \\ \vdots & & \vdots \\ A_{n1}(z, \epsilon) & \dots & A_{nn}(z, \epsilon) \end{pmatrix} \begin{pmatrix} f_1(z, \epsilon) \\ \vdots \\ f_n(z, \epsilon) \end{pmatrix}$$

A.V.Kotikov, PLB254, 158(1991); PLB267, 123(1991)

- It is not easy to determine all the boundary condition for MIs f_i

A suitable choice of basis: Canonical basis

$$\frac{d}{dz} \begin{pmatrix} g_1(z; \epsilon) \\ \vdots \\ g_n(z; \epsilon) \end{pmatrix} = \epsilon \begin{pmatrix} B_{11}(z) & \dots & B_{1n}(z) \\ \vdots & & \vdots \\ B_{n1}(z) & \dots & B_{nn}(z) \end{pmatrix} \begin{pmatrix} g_1(z; \epsilon) \\ \vdots \\ g_n(z; \epsilon) \end{pmatrix}$$

where

$$\vec{f} = T \vec{g}$$

$$B = T^{-1} A T - T^{-1} \partial_z T$$

- New strategy in dimensional regularization with $D = 4 - 2\epsilon$
- A linear transformation of MIs to the canonical basis
- The coefficient matrix B only depends on z

J.M. Henn, PRL 110, 251601 (2013)

Outline

1 PDFs and LaMET introduction

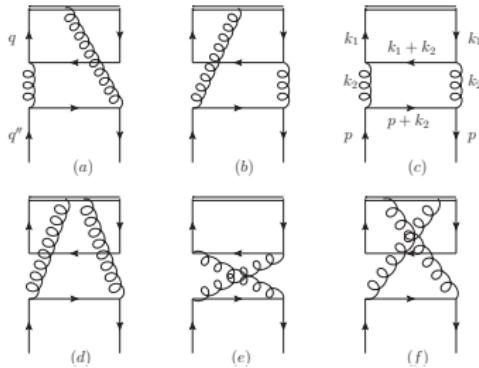
2 Two Loop Calculation of Quark Quasi PDF

3 NNLO Results

- $q \rightarrow q'$ case
- $q \rightarrow q$ case

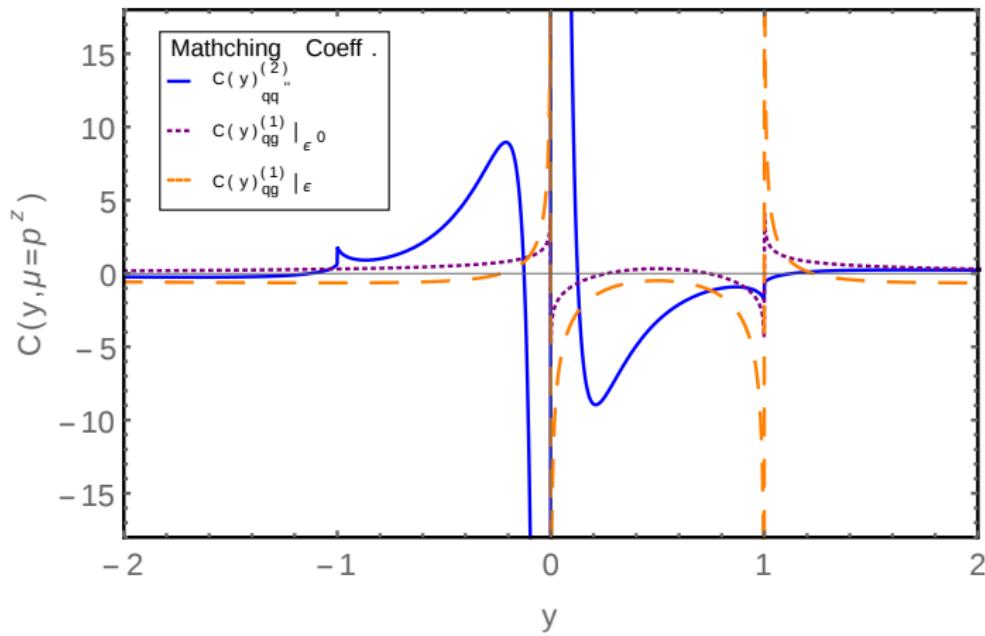
4 Summary

$q \rightarrow q'$ case



$q \rightarrow q'$ Feynman diagrams

$$\begin{aligned}
 \tilde{f}_{q/q'}^{(2)}(y, \frac{p^z}{\mu}, \epsilon_{\text{IR}}) &= C_{qq''}^{(2)} \left(\frac{y}{x}, \frac{|x| p^z}{\mu} \right) \otimes f_{q''/q'}^{(0)}(x, \epsilon_{\text{IR}}) \\
 &\quad + C_{qq''}^{(1)} \left(\frac{y}{x}, \frac{|x| p^z}{\mu} \right) \otimes f_{q''/q'}^{(1)}(x, \epsilon_{\text{IR}}) \\
 &\quad + C_{qq''}^{(0)} \left(\frac{y}{x}, \frac{|x| p^z}{\mu} \right) \otimes f_{q''/q'}^{(2)}(x, \epsilon_{\text{IR}}).
 \end{aligned}$$

$q \rightarrow q'$ case $q \rightarrow q'$ two loop matching coefficients

Outline

1 PDFs and LaMET introduction

2 Two Loop Calculation of Quark Quasi PDF

3 NNLO Results

- $q \rightarrow q'$ case
- $q \rightarrow q$ case

4 Summary

Renormalization

- Renormalization procedure

$$\tilde{f}(y, \frac{p^z}{\mu}, \epsilon_{\text{IR}}) = \int \frac{dy_1}{|y_1|} \left[Z_q \tilde{Z} \left(\frac{y}{y_1} \right) \right] \left[Z_q^{-1} \tilde{f} \left(y_1, \frac{p^z}{\mu}, \epsilon \right) \right].$$

Z_q is quark renormalization constant, \tilde{Z} is quasi distribution renormalization factor

$$\tilde{Z}(\xi) = \delta(1 - \xi) \left(1 + \frac{\alpha_s}{2\pi} \frac{\tilde{Z}^{(1)}}{\epsilon_{UV}} + \left(\frac{\alpha_s}{2\pi} \right)^2 \frac{\tilde{Z}^{(2)}}{\epsilon_{UV}^2} \right),$$

$$\tilde{Z}^{(1)} = -\frac{3C_F S_\epsilon}{2}, \quad \tilde{Z}^{(2)} = S_\epsilon^2 \left(\frac{a + 9C_F^2}{4} + \frac{b}{4}\epsilon \right)$$

X. Ji and J.H. Zhang, 1505.07699;

Braun, Chetyrkin and Kniehl, 2004.01043

IR behavior in Quasi PDF

- Soft divergences are cancelled
- Reducible collinear divergences

$$\tilde{f}_{q/q}^{(2)}\left(y, \frac{p^z}{\mu}, \epsilon_{\text{IR}}\right)|_{\text{div.part.1}} = C_{qq}^{(1)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes \left[-\frac{(1+x^2)}{(1-x)}\right]_+ \frac{1}{\epsilon_{\text{IR}}}.$$

- “Irreducible” collinear divergences

the same as light cone PDFs, including both $\frac{1}{\epsilon_{\text{IR}}}$ and $\left(\frac{1}{\epsilon_{\text{IR}}}\right)^2$

$$\tilde{f}_{i/j}^{(2)}\left(y, \frac{p^z}{\mu}, \epsilon_{\text{IR}}\right)|_{\text{div.part.2}} = f_{i/j}^{(2)}(x, \epsilon_{\text{IR}}).$$

$$f_{i/j}^{(2)}(x) = \frac{1}{2\epsilon_{\text{IR}}^2} \left[\sum_k P_{ik}^{(0)}(z) \otimes P_{kj}^{(0)}(x) + \beta_0 P_{ij}^{(0)}(z) \right] - \frac{P_{ij}^{(1)}(x)}{\epsilon_{\text{IR}}}$$

Factorization formula at NNLO

- Matching procedure between renormalized quasi and light-cone PDFs:

$$\tilde{f}_{i/k}^{(0)}\left(y, \frac{p^z}{\mu}\right) = C_{ij}^{(0)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f_{j/k}^{(0)}(x),$$

$$\begin{aligned} \tilde{f}_{i/k}^{(1)}\left(y, \frac{p^z}{\mu}, \epsilon_{\text{IR}}\right) &= C_{ij}^{(1)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f_{j/k}^{(0)}(x) \\ &\quad + C_{ij}^{(0)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f_{j/k}^{(1)}(x, \epsilon_{\text{IR}}), \end{aligned}$$

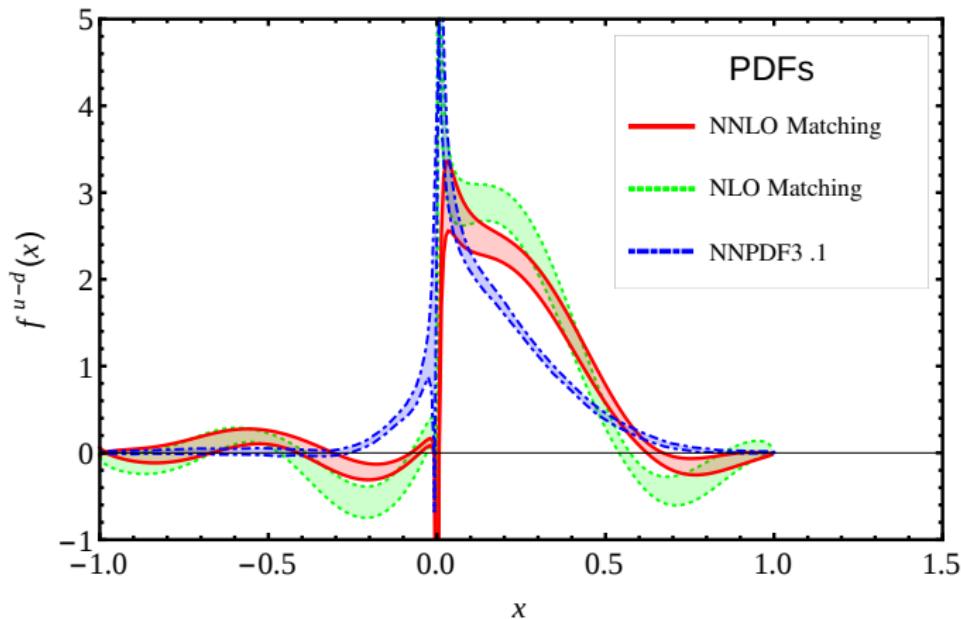
$$\begin{aligned} \tilde{f}_{i/k}^{(2)}\left(y, \frac{p^z}{\mu}, \epsilon_{\text{IR}}\right) &= C_{ij}^{(2)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f_{j/k}^{(0)}(x) \\ &\quad + C_{ij}^{(1)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f_{j/k}^{(1)}(x, \epsilon_{\text{IR}}) \\ &\quad + C_{ij}^{(0)} \left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f_{j/k}^{(2)}(x, \epsilon_{\text{IR}}). \end{aligned}$$

NNLO matching coefficients $C_{qq}^{(2)}$

- consistent results in $\overline{\text{MS}}$ scheme by Li-Ma-Qiu [*Phys.Rev.Lett.* 126, 072001 \(2021\)](#)
- We also obtained $C_{qq}^{(2)}(y, \frac{p^z}{\mu})$ in both RI/MOM and $\overline{\text{MMS}}$ scheme
- 4 regions for y and 3 color structures ($C_F, C_A, nf T_F$) C_F
- the final asymptotic behavior: $C_{qq}^{(2), \overline{\text{MMS}}}|_{y \rightarrow \infty} \propto \frac{1}{y^2}$

$$\begin{aligned}
 & C_{qq}^{(2), \overline{\text{MMS}}}(y, \frac{p^z}{\mu}) \\
 &= [C_{qq}^{(2), \overline{\text{MMS}}}(y, \frac{p^z}{\mu})|_{y>1}]_+ + [C_{qq}^{(2), \overline{\text{MMS}}}(y, \frac{p^z}{\mu})|_{0 < y < 1}]_+ \\
 &+ [C_{qq}^{(2), \overline{\text{MMS}}}(y, \frac{p^z}{\mu})|_{-1 < y < 0}]_+ + [C_{qq}^{(2), \overline{\text{MMS}}}(y, \frac{p^z}{\mu})|_{y < -1}]_+
 \end{aligned}$$

PDFs from NNLO Matching



using ETMC data(PRD99,114504) with $z_{cut} = 10a$, $p^z = 2.3\text{GeV}$, $\mu = 2\text{GeV}$ and in modified $\overline{\text{MS}}$ scheme; uncertainty is from lattice data

Summary

- NNLO correction is important
- NNLO matching coefficients of quark PDF are obtained
- Complete cancellation of IR divergence is confirmed, which nontrivially validates the LaMET factorization at NNLO
- Outlook
 - *Gluon quasi distribution functions at NNLO*
 - *Pion quasi distribution amplitudes at NNLO*
 - *A new stage of lattice calculation of PDFs with NNLO matching*

Thank you a lot!