

Recent progresses on perturbative QCD at high-energy frontier

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QCD factorization

long-distance and short-distance contributions due to asymptotic freedom



• QCD factorization theorem is essential for theoretical predictions in high energy scatterings; separation of

$$) = \sum_{i=q,\bar{q},g} \int_0^1 d\xi \, C_2^i(x/\xi, Q^2/\mu_r^2, \mu_f^2/\mu_r^2, \alpha_s(\mu_r^2)) \times f_{i/h}(\xi, \mu_f)$$

QCD at precision frontier

SM electroweak sector and searches of new physics beyond SM, recap of LEP legacy

effective NC couplings of charged leptons [LEP&SLC, hep-ex/0509008]



Improvements on QCD predictions will be a prerequisite for precision study at the next generations of e+e- colliders

Improvements on QCD predictions not only benefit study of QCD itself but also are crucial for test of the

FB asymmetry of bottom quarks, 2.8σ



QCD at precision frontier





+ Improvements on QCD predictions not only benefit study of QCD itself but also are crucial for test of the SM electroweak sector and searches of new physics beyond SM, W-mass at LHC as an example

- * W mass determined from charged-lepton pT(mT) spectrum in relative to those spectrum from Z boson
- LHC aims at a target precision of ~10 MeV
- * QCD modeling (PDFs and QCD corrections) affect the ratio of W/Z distributions and dominate in the total uncertainties

Domestic efforts



[https://indico.ihep.ac.cn/event/13260/timetable/#20210514]

- * many interesting works are presented in PQFT workshop (all slides are on indico) based on which my talk is prepared
- It the second series of PQFT workshop will be held in 2022 at Hang Zhou

• Domestically we have a long tradition on perturbative calculations of quantum field theory, and in the younger generation, a relative small theory community working on diverse topics of QCD at high energies

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Outline

◆ 1. Phenomenology at LHC, SMEFT, Higgs boson, Drell-Yan, top quark

◆ 2. Deep Inelastic Scattering, parton distributions, heavy-quark scheme

♦ 3. Phenomenology at Lepton Colliders, jet substructures, event shapes

◆ 4. Methodology, multi-loop Feynman integrals

QCD improved SMEFT



 $\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}$



* QCD corrections of EFT contributions can be important due to possible operator mixings and complicated interference with the SM

张岑 1984-2021

Automated one-loop computations in the SMEFT

Céline Degrande,^{1,*} Gauthier Durieux,^{2,†} Fabio Maltoni,^{1,3,‡} Ken Mimasu,^{1,§} Eleni Vryonidou,^{4,¶} and Cen Zhang^{5,6,**}

Interpretation of searches of new physics with SM effective field theory can be improved with higher-order QCD corrections; dedicated tools have been developed (MG5 SMEFT@NLO) [Cen Zhang +, 2021]

SMEFT
$$\mathcal{L}_{\rm SM} + \sum_{i} \frac{c_i^{(6)} O_i^{(6)}}{\Lambda^2}$$

measurements

$$p_n^{\text{EXP}} - o_n^{\text{SM}} = \sum_i \frac{a_{n,i}^{(6)}(\mu) c_i^{(6)}(\mu)}{\Lambda^2}$$

SMEFT operator in four top quark production

${\cal O}(\Lambda^{-2}$	²)		$\mathcal{O}(\Lambda^{-4})$			
LO	NLO	K	LO	NLO	K	
$081^{+55\%}_{-33\%}$ [-0.358]	$0.090^{+4\%}_{-11\%}$	1.1	$0.115^{+46\%}_{-29\%}$	$0.158^{+4\%}_{-11\%}$	1.37	
$274^{+54\%}_{-33\%}$ [-0.639]	$0.311^{+5\%}_{-10\%}$	1.14	$0.342^{+46\%}_{-29\%}$	$0.378^{+4\%}_{-13\%}$	1.10	
$.242^{+55\%}_{-33\%}$ [-1.07]	$0.24(3)^{+3\%}_{-18\%}$	0.99	$1.039^{+47\%}_{-29\%}$	$1.41^{+4\%}_{-11\%}$	1.36	
$098(10)^{+38\%}_{-33\%} \ [0.862]$	$-0.019(9)^{+63\%}_{-27\%}$	1.9	$1.406^{+46\%}_{-30\%}$	$1.86^{+4\%}_{-10\%}$	1.32	
$483^{+55\%}_{-33\%}$ [-1.86]	$0.53(8)^{+3\%}_{-10\%}$	1.10	$4.154^{+47\%}_{-29\%}$	$5.61^{+4\%}_{-11\%}$	1.35	

* now fully automated with NLO QCD corrections matched with parton shower and hadronizations

QCD improved SMEFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{2\pi}{\Lambda^2} \sum_{n \in \{1,3,5\}} c_n O_n$$
 quark compositer (contact interaction)

$$O_1 = \delta_{ij} \delta_{kl} \left(\sum_{c=1}^3 \overline{q}_{Lci} \gamma_\mu q_{Lcj} \sum_{d=1}^3 \overline{q}_{Ldk} \gamma^\mu q_{Ldl} \right)$$

_	-	•
Data sets		Partial χ^2 / N_{dp}
HERA I+II neutral current	e^+ p, $E_p = 920 \text{GeV}$	402/332
HERA I+II neutral current	$e^+ p, E_p = 820 \text{GeV}$	60/63
HERA I+II neutral current	$e^+ p, E_p = 575 \text{GeV}$	198/234
HERA I+II neutral current	$e^+ p, E_p = 460 \text{GeV}$	208/187
HERA I+II neutral current	e ⁻ p	223/159
HERA I+II charged current	$e^+ p$	46/39
HERA I+II charged current	e ⁻ p	55/42
CMS 13 TeV tī 3D		23/23
CMS inclusive jets 13 TeV	0.0 < y < 0.5	13/22
	0.5 < y < 1.0	28/21
	1.0 < y < 1.5	13/19
	1.5 < y < 2.0	33/16
Correlated χ^2		121
Log penalty χ^2		-12.39
Global χ^2 / N_{dof}		1411/1143

a fit at NLO for both SM and CIs

◆ Interpretation of searches of new physics with SM effective field theory requires a spontaneous fit incorporating both new physics and non-perturbative QCD inputs (e.g. parton distributions) [CMS+ JG, 2021]

- ness ions)
- Cls constrained by jets production at the LHC, resulting in different energy scaling behavior and angular distributions
- Input parton distributions are also constrained by the same jet data, thus requiring a spontaneous fit





Higgs associated production at two-loops

predictions of cross sections; two-loop calculations involves multi-mass scales are challenging



- small-mass expansion proved to be fast, smooth, and sufficient accurate through entire phase space

+ HL-LHC aims at measurements of Higgs couplings at a few percent level, requiring similar precision on

performance comparison to direct

[Wang, Xu, Xu, Yang, 2021]

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Higgs associated production at two-loops

predictions of cross sections; two-loop calculations involves multi-mass scales are challenging

1.2 dơ/dM_{ZH}(fb/GeV) NLO 0.9 LHC 13 TeV, $\mu_{def} = M_{ZH}$ 0.6 NNPDF31_nnlo_as_0118 0.3 0.0 Ratio 200 400 600 800 M_{7н}(GeV)

ZH invariant mass distribution

- ◆ gluon fusion contributes to 20% of the total ZH production at LHC 13 TeV
- ✤ A first phenomenological study on gg->ZH at two-loops (NLO) in QCD 1.2 GeV)

+ HL-LHC aims at measurements of Higgs couplings at a few percent level, requiring similar precision on

cross sections at LHC 13 TeV

$\mu_r = \mu_f \Big $	$\sigma_{ m LO}^{gg}$	$\sigma^{gg}_{ m NLO}$	$\sigma_{pp \to ZH}^{\text{no } gg}$	$\sigma_{pp o ZH}$	$\left\ \sigma_{\mathrm{NLO}}^{gg,m_t \to \infty}\right\ $	$\sigma^{m_t \to \infty}_{pp \to ZH}$
$m_{ZH}/3$	310.5(3)	551(1)	2106(2)	2658(2)	568(3)	2675(3)
m_{ZH}	233.7(2)	451(1)	2104(2)	2555(2)	$ \ 476(2) $	2579(3)
$3m_{ZH}$	179.0(2)	373(1)	2113(2)	2485(2)	$ \ 396(2) $	2509(3)

cross sections at HE-LHC 27 TeV

$\mu_r = \mu_f \Big $	$\sigma^{gg}_{ m LO}$	$\sigma^{gg}_{ m NLO}$	$\sigma_{pp \to ZH}^{\text{no } gg}$	$\sigma_{pp \to ZH}$	$\sigma_{\rm NLO}^{gg,m_t \to \infty}$	$\sigma_{pp\to ZH}^{m_t\to\infty}$
$M_{ZH}/3$	73.56(7)	129.4(3)	784.0(7)	913.4(7)	133.6(6)	917.6(9
M_{ZH}	51.03(5)	101.7(2)	781.1(7)	882.9(7)	106.0(4)	887.2(8
$3M_{ZH}$	36.62(4)	80.4(2)	780.7(8)	861.1(8)	84.0(3)	864.8(9

✤ in total cross sections, residual scale variations are ~3%, dominated by gluon fusion; top-quark mass effects are relevant, at the level of ~1%





Higgs pair production at N3LO



+ Higgs boson pair production probe the Higgs trilinear coupling and dynamics of EW phase transition; and theoretically has been calculated fully differentially to N3LO in QCD in the heavy top-quark limit



Drell-Yan production at N3LO

QCD; and theoretically has been calculated fully differentially to N3LO in QCD



- underestimate the true perturbative uncertainty

• Drell-Yan lepton pair production via virtual photon is the standard candle process at the LHC for testing of

[Chen, Gehrmann, Glover, Huss, Yang, Zhu, 2021]







predictions at top pair threshold vs. CMS



cross section close to threshold



• Being abundantly produced at LHC via strong interactions, top-quark pair is the key process for searches of new physics, test of SM; QCD Coulomb corrections at threshold calculated to next-to-leading power

top quark mass dependence

[Ju, Wang, Wand, Xu, Xu, Yang, 2020]

Single top production at NNLO



cross sections vs. QCD scale

- μ/m_t
- * scale variations reduced to ~1%; demonstrate the better stability of 5FS comparing to 4FS

• Single top-quark production probes directly charged-current coupling of top quark, setting constraints on various new physics extensions; production with subsequent top decays calculated to full NNLO



averaged lepton pT vs. top mass

[Gao, Yuan, JG, Berger, 2020]

- single top production can provide independent and complementary determination on top quark mass
- NNLO calculation gives precise modeling on decayed leptonic kinematics, thus a top mass precision of ~1GeV 14

Progress on global analysis of PDFs^{1.0}

LHC data now plays important role, especially precision $\frac{1}{2}$ /Z, jet, and top-quark pair data

[CT18 PDFs, Hou, JG+, 2019]



◆ 11 LHC data sets out of 39 total, ~800 data points out of ~3700 total, for CT18 PDF fits



- PDF turns to be the dominant source of theoretical uncertainty in many cases at LHC
- smaller uncertainty from a single group may not be the truth; a benchmarking effort is ongoing towards a PDF4LHC21 combination similar to previous PDF4LHC15 PDF sets 15

PDFs from Large momentum effective theory

Lattice QCD; exciting progresses towards new theory inputs for hadron colliders





[**Ji**, 2013]

PDF: light-cone separation; cannot be calculated on the lattice

$$\tilde{f}_{i/H}(y, p^z) = \int_{-1}^{1} \frac{dx}{|x|} \left[C_{ij} \left(\frac{y}{x}, \frac{|x|p^z}{\mu} \right) f_{j/H}(x) \right]$$

* prove of factorization ensure a perturbative matching of the PDF and quasi-PDF due to same IR behavior

• Development of LaMET leads to realistic simulation of parton distributions at large Bjorken-x region from

[Chen, Wang, Zhu, 2020]

- (x, μ)
- * the matching kernels have recently been calculated to two-loops in QCD
- extracted PDFs from lattice simulation show promising trends comparing to those from fit to data



Novel observables for jet substr

er results analytically at two-loops, and resummed QCD predictions



$$\mathsf{Cuctures}_{\mathrm{EEC}}(\{\chi\}) = \frac{1}{\sigma_{\mathrm{tot}}} \sum_{i_1, i_2, \dots, i_N} \int d\sigma_{e^+e^- \to i_1, \dots, i_N + X} \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{E_{\mathrm{tot}}^N} \delta(\chi_{12} - \chi_{i_1 i_2}) \delta(\chi_{12$$

- energy-energy correlations (EEC) and its extensions in study of jet substructures; working



- reformulation on QCD factorization of EEC and generalized EEC observables
- * apply operator product expansion (OPE) of light-ray operators in QCD, leading to improved understanding of all-order structures/resummations for EEC-like observables

[Chen, Luo, Moult, Zhu, 2020]

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Novel observables for jet substructures

Renovation of energy-energy correlations (EEC) and its extensions in study of jet substructures; open new directions with rich phenomenologies, taking three-point EECs as an example

squeeze limit of 3-point EEC



- * unique feature not yet been observed experimentally, not even produced by event generator until recently



Principle of Maximal Conformality

conventional pQCD calculation

$$\rho(Q) = \sum_{n \ge 1} r_{n,0} \alpha(\mu)^{n+p-1} + \sum_{n \ge 1} \left[(n+p-1)\alpha(\mu)^{n+p-2} \beta \right] \sum_{j \ge 1} (-1)^j \Delta_n^{(j-1)} r_{j,0}$$

the PMC method (single scale version)

$$\rho(Q) = \sum_{n \ge 1} \hat{r}_{n,0} \alpha(Q_{\star})^{n+p-1}$$
$$\ln \frac{Q_{\star}^2}{Q^2} = T_0 + T_1 \alpha(Q) + T_2 \alpha^2(Q) + \cdots$$

- * conventional pQCD calculations have been criticized by the "arbitrary" scale choice and its variation ranges
- In PMC the non-conformal terms are absorbed into an effective running scale of the QCD coupling
- * PMC scale and the remaining conformal coefficients can be set by matching to conventional pQCD results

Principle of Maximal Conformality provides a rigorous method for eliminating renormalization schemeand-scale ambiguities in perturbative QCD calculations; unique view on various QCD predictions

extraction of QCD coupling constant from event shape distribution





Efficient multi-loop integrations

two-loop or even three-loop Feynman integrals

method of auxiliary mass flow

$$I_{\text{mod}}(\vec{\nu};\eta) = \int \prod_{i=1}^{L} \frac{\mathrm{d}^{D}\ell_{i}}{\mathrm{i}\pi^{D/2}} \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}}\cdots\mathcal{D}_{K+1}}{(\mathcal{D}_{1}+\mathrm{i}\eta)^{\nu_{1}}\cdots(\mathcal{D}_{K+1})}$$

boundary conditions fixed at $\eta = \infty$



• Recent developments on auxiliary mass flow method provide solutions to calculations of cutting-edge

physical integrals



solving differential equations numerically





Recent developments on auxiliary mass flow method provide solutions to calculations of cutting-edge

time consumed in CPU core hours for a single phase-space point

Family	dp	(a)	(b)	(c)	(d)	(e)	(f)
$T_{ m setup}$	6	20	18	8	1	25	30
$T_{\rm solve}$	7	11	15	6	3	15	42
P_1	95%	99%	96%	99%	98%	94%	93%
$T_{\vec{s}}$	2	916	64	1305	30	1801	63

- * master integrals considered relevant for, two-loop EW correction to HZ production at CEPC, two-loop QCD correction fo H/W/Z+2j, ttbar+H, 4j, three-loop QCD correction for ttbar at LHC, mostly not known previously
- * AMF method provides accurate numerical results with a reasonable time cost; further developments towards realistic pheno applications are ongoing

[Liu, Ma, Wang, 2017, 2021]

Charged-current DIS with full mass at NNLO

neutrino charged-current cross sections vs. energy



* CC DIS is the dominant process to detect high-energy neutrinos from DUNE all the way to IceCube; also can be measured in electron scattering at EIC, EICC

+ Heavy-quark (HQ) contributions, especially charm quarks, is vital to achieving high-precision in theoretical calculations of DIS cross sections; now been calculated to NNLO in QCD for CC DIS

power term of mc

logarithms of mc



[JG, Hobbs, Nadolsky, Sun, Yuan, 2021]

- * factorization on heavy-quark contribution to DIS was proved by Collins and result in ACOT-like HQ schemes to account for full mass dependence across wide energies
- It the coefficient functions for CC DIS in simplified ACOT-**x** scheme have now been calculated to NNLO with full mass and with additional massless N3LO terms



Charged-current DIS with full mass at NNLO

e-p collision with EIC setup



* At high Q² seeing a very good convergence of the GM predictions and sequential reduction of scale variations, <1% for GM N3LO'

+ Heavy-quark (HQ) contributions, especially charm quarks, is vital to achieving high-precision in theoretical calculations of DIS cross sections; significant implications of the new GM predictions

neutrino scattering on iso-scalar target



Inite mass corrections are about 2% at low Ev comparing GM and ZM N2LO; scale variations are 1~3% for GM N2LO, and <1% for N3LO' at large Ev



总结

◆ 高能量前沿的精确计算和蒙卡模拟对于进一步检验标准模型以及寻找新物理至关重要

◆ 高能微扰QCD方向国内的各位理论同行作出了众多有特色和影响力的工作

◆期待未来理论与实验学家更加密切的合作,理论研究结合学会的战略规划,例如CEPC实验

总结

◆ 高能量前沿的精确计算和蒙卡模拟对于进一步检验标准模型以及寻找新物理至关重要

◆ 高能微扰QCD方向国内的各位理论同行作出了众多有特色和影响力的工作



◆期待未来理论与实验学家更加密切的合作,理论研究结合学会的战略规划,例如CEPC实验