



中国科学技术大学

University of Science and Technology of China

# Master integrals for the mixed QCD-QED corrections to the Drell-Yan process with lepton mass dependence

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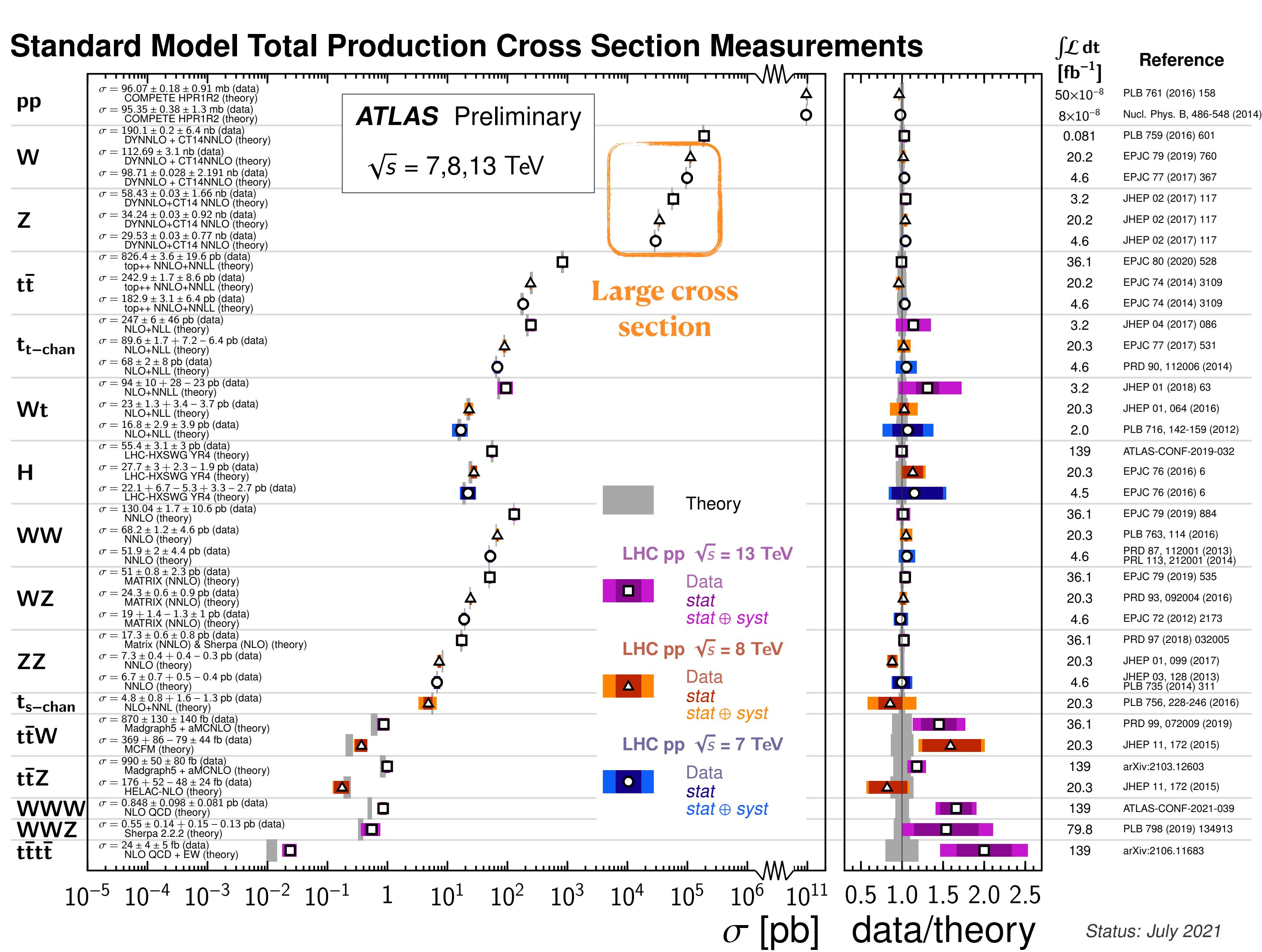
中国物理学会高能物理分会第十三届全国粒子物理学学术会议

# Outline

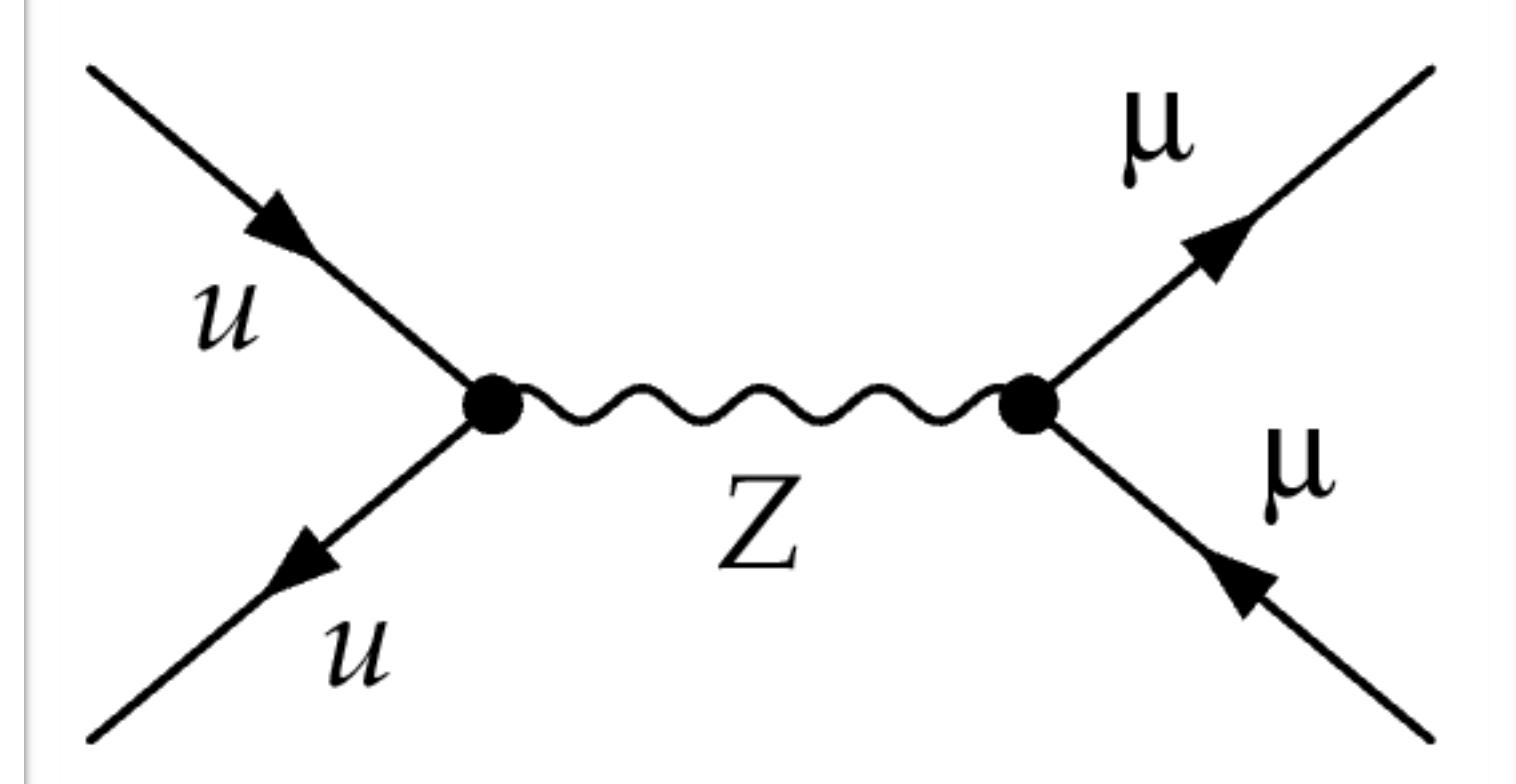
- Motivation
- QCD-QED corrections to  $q\bar{q}' \rightarrow \nu_l l$ 
  - Master integrals with massive lepton
  - Differential equations
  - Solutions
- Conclusions & outlook

# Motivation

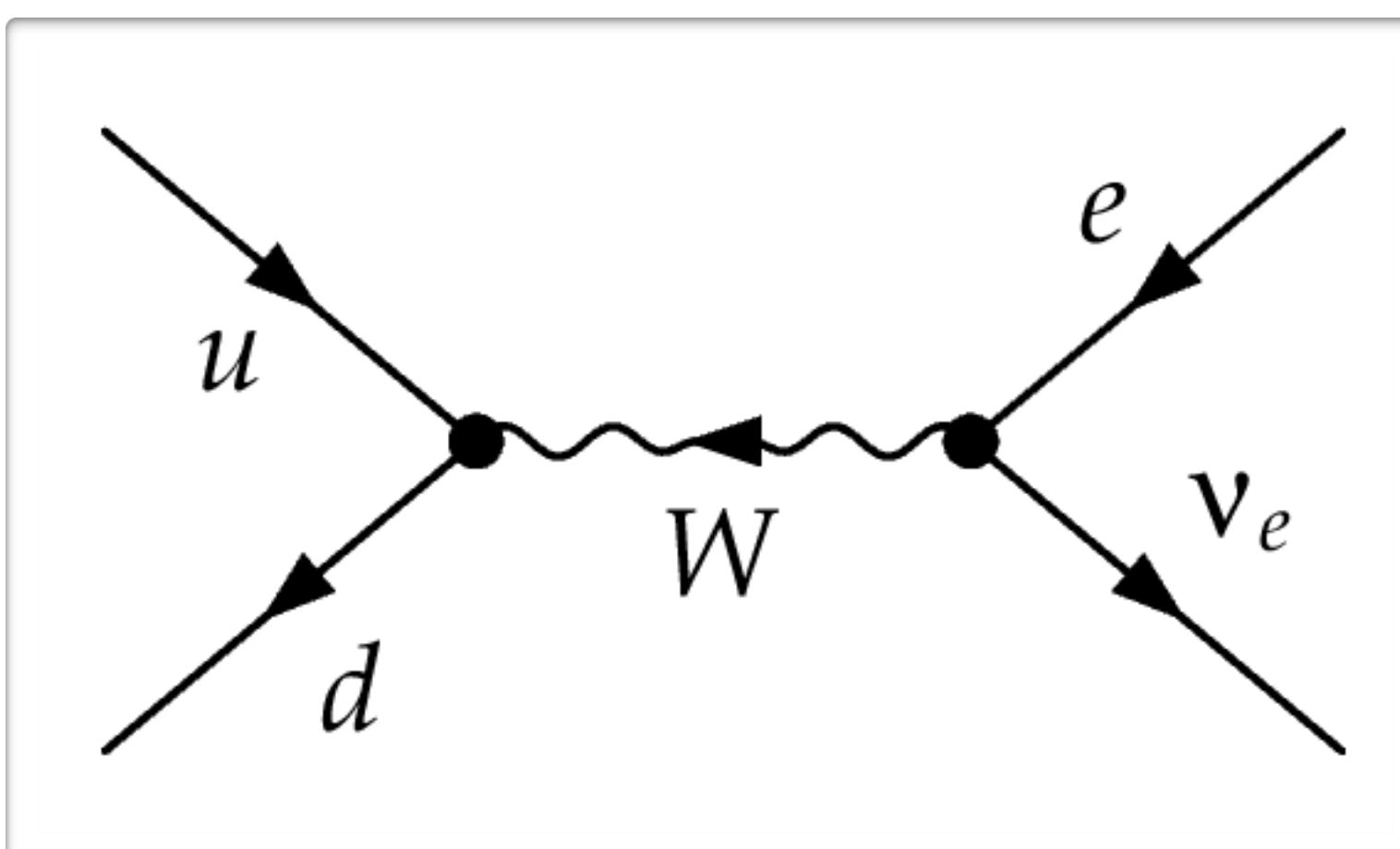
# Drell-Yan processes @ LHC



Neutral-current

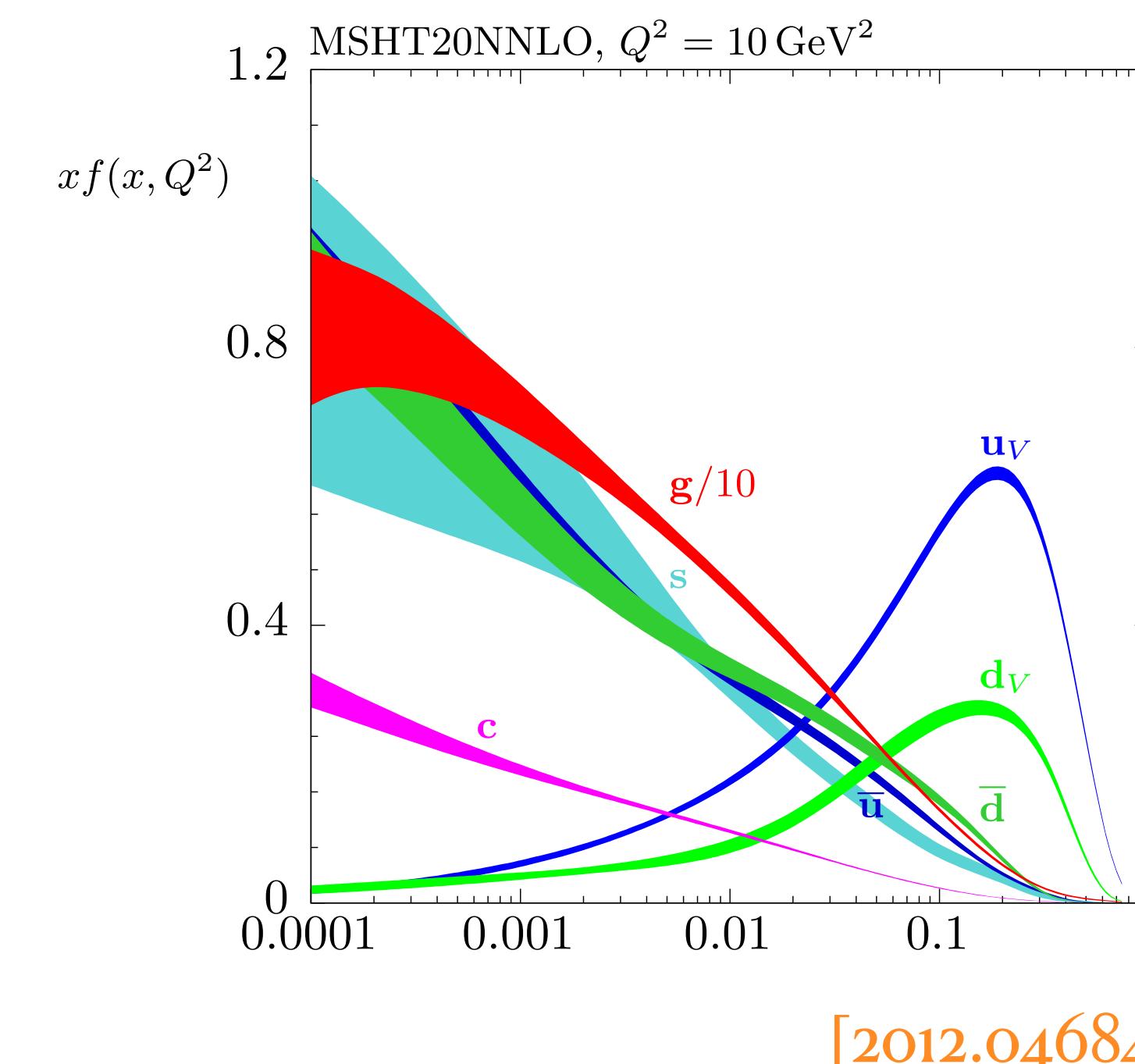


Charged-current

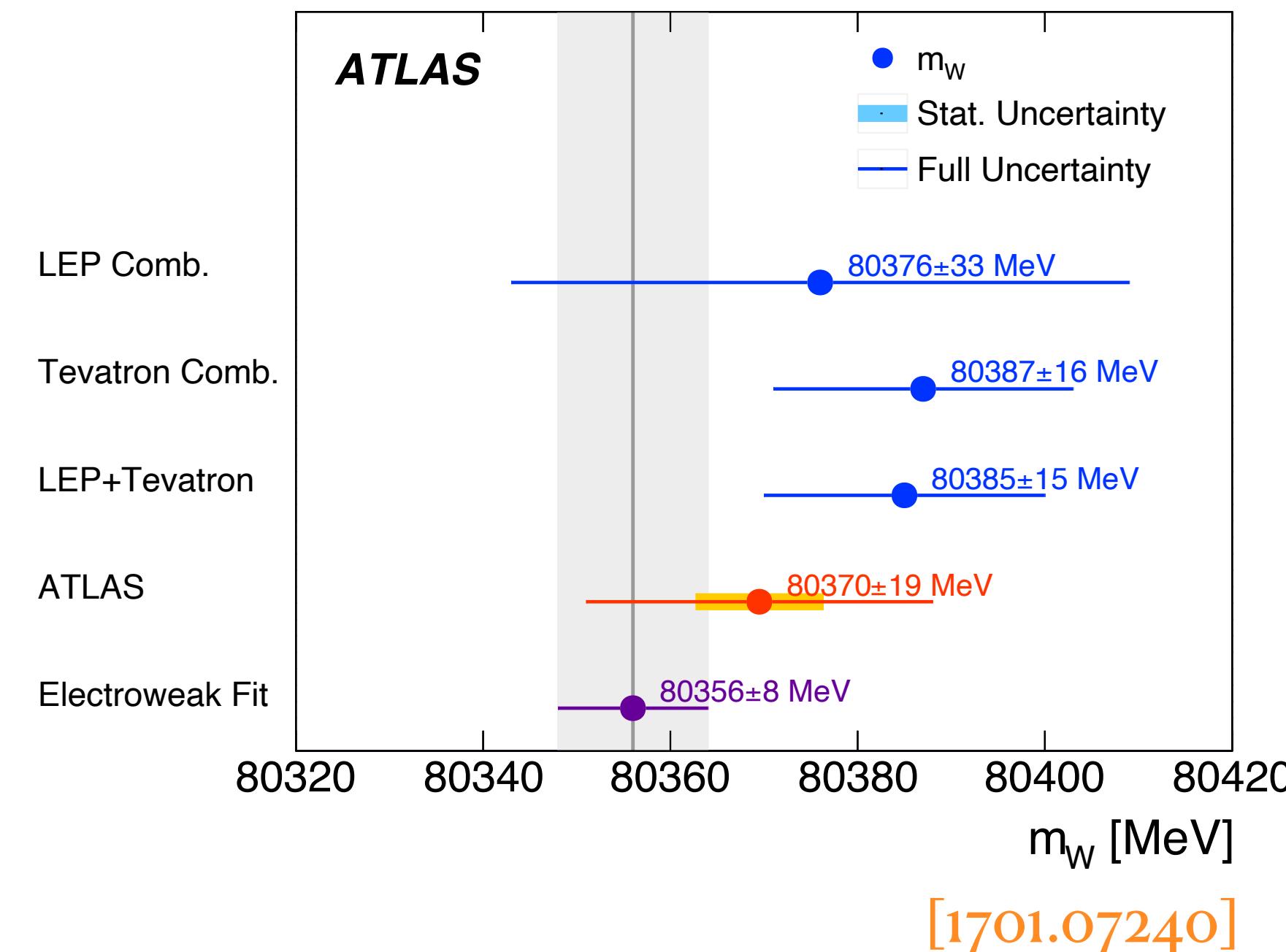


# Drell-Yan processes @ LHC

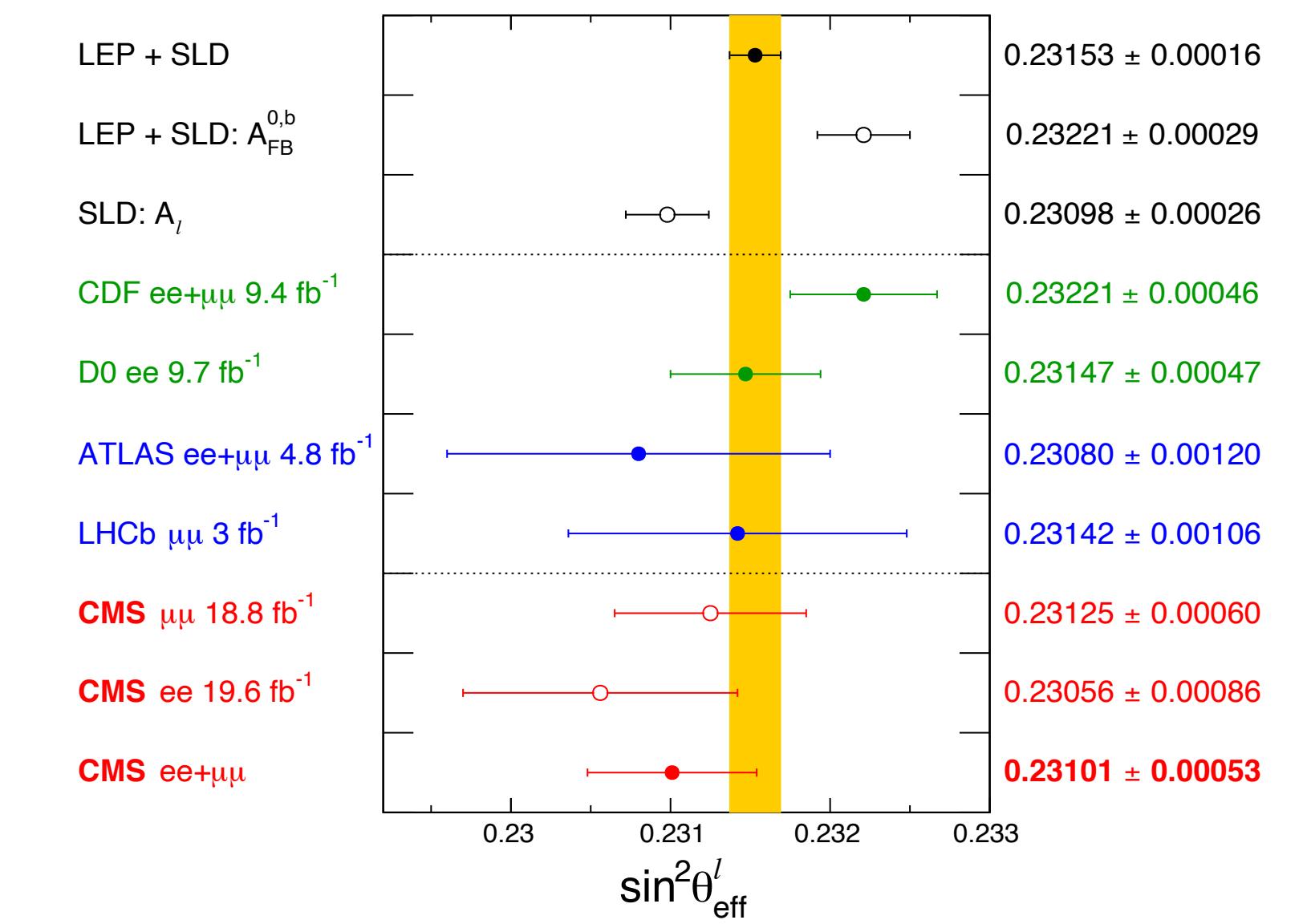
● PDF fit



● W mass measurement



● Weak mixing angle measurement



# Drell-Yan processes: Fixed-order calculations



## ● NLO

- ▶ QCD corrections. [Politzer '77, Altarelli et al '78/79]
- ▶ EW corrections. [Baur et al '98/01/04, Dittmaier et al '02/10, Zykunov '06/07, Carloni Calame et al '06/07]

## ● NNLO

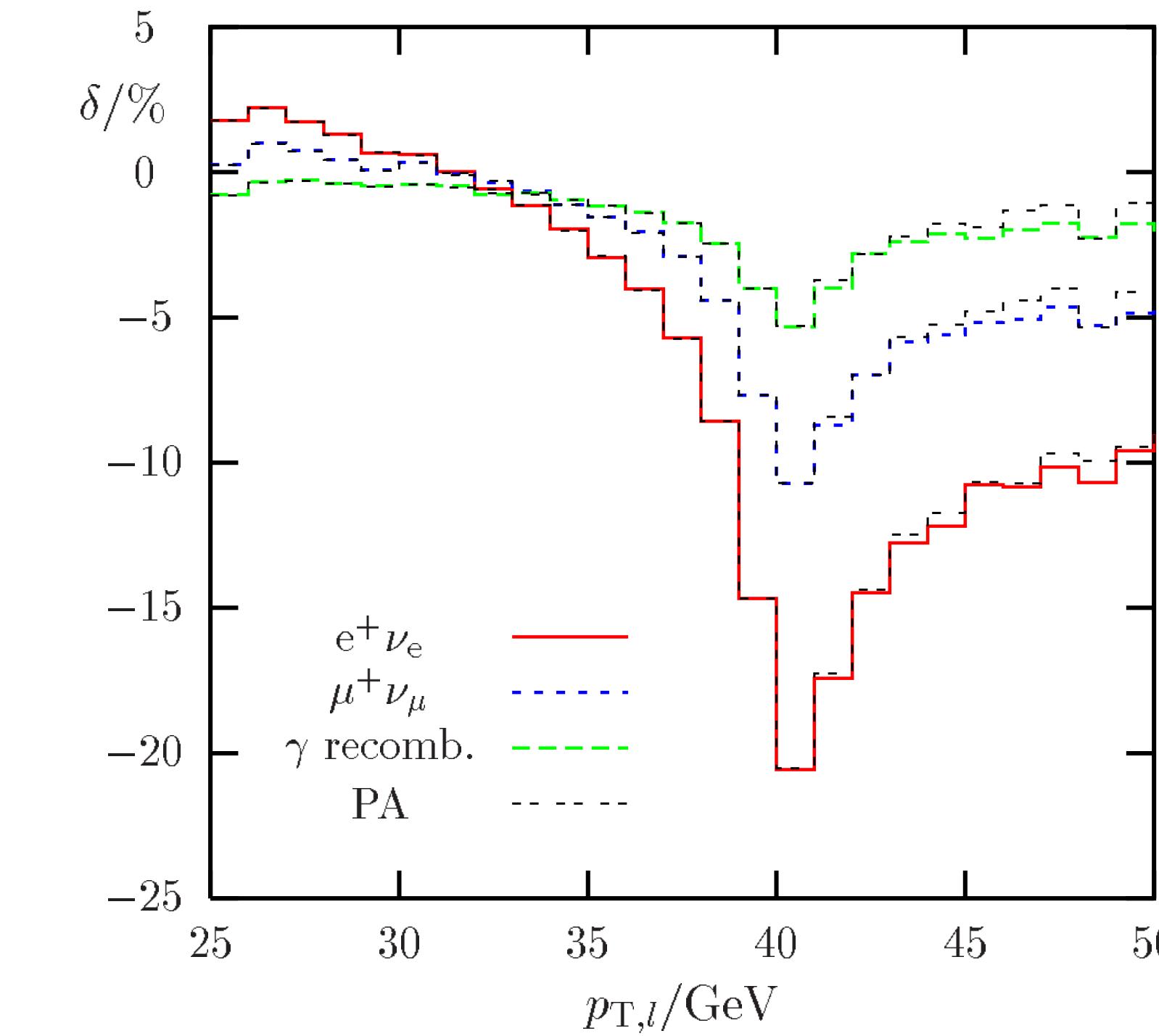
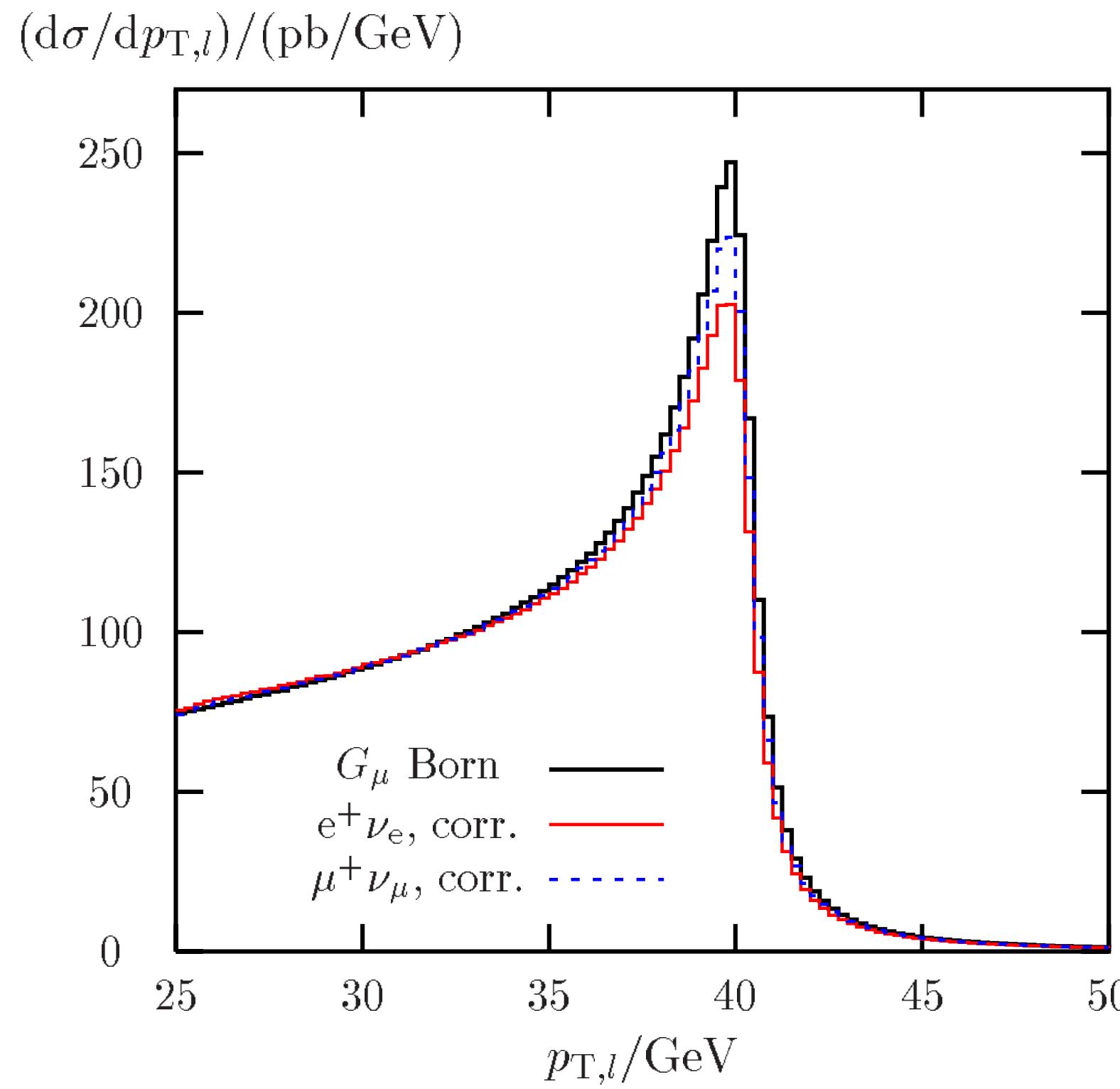
- ▶ QCD corrections. [Hamberg et al '91, Harlander et al '02, Anastasiou et al '04, Melnikov et al '06, Catani et al '09/10]
- ▶ Factorizable categories
  - QCD-EW corrections to hadronic decays of W&Z. [Czarnecki et al '96, Kara et al '13]
  - QCD-EW corrections to Z production form factors. [Kotikov et al '08]
  - QCD-QED corrections to Z production, on-shell & off-shell. [Florian et al '18/20, Delta et al '20, Cieri et al '20]
  - QCD-EW corrections to Z&W production. [Bonciani et al '20, Behring et al '20, Dittmaier et al '20]
- ▶ Non-factorizable categories
  - QCD-QED corrections to lepton pairs. [Kilgore et al '12]
  - QCD-EW corrections to DY processes in the resonance region. [Dittmaier et al '14/16]
  - QCD-EW corrections to  $pp \rightarrow l\nu_l + X$ . [Buonocore et al '21]
  - First complete QCD-EW corrections to lepton pairs production. [Bonciani et al '21]
  - Master integral and amplitude calculations. [Bonciani et al '16, Manteuffel et al '17, Heller et al '20/21, Hasan et al '20]

## ● N<sub>3</sub>LO QCD

- ▶ Inclusive Z&W production. [Duhr et al '20]
- ▶ First differential distribution study. [X. Chen et al '21] (see talk of HuaXing Zhu)

# Massless or massive lepton

- NLO EW, distributions of lepton's  $p_T$  [0109062, Dittmaier]



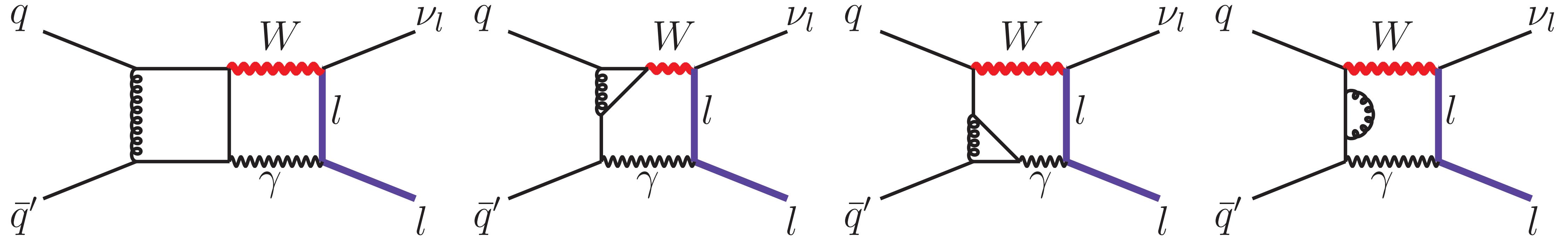
- Lepton's mass might affect observables in presence of cuts for identifying lepton which gives rise to logarithm in the form of  $\log(m_l^2/Q)$ , where  $Q$  is the characteristic scale of the process, such as collision energy or vector boson mass.
- We can focus on the QED part of the QCD-EW correction calculations, since the logarithm originates from photon emissions and exchanges.
- Need to keep the mass of lepton in the calculations of scattering amplitudes which requires 4-point master integrals with massive lepton.

# **QCD-QED corrections to**

$$q\bar{q}' \rightarrow \nu_l l$$

# Charged-current DY with massive lepton: Conventions

## ○ Feynman diagrams



## ○ Notations

$$q(p_2) + \bar{q}'(p_1) \rightarrow \nu_l(p_3) + l(p_4), \quad l = e, \mu,$$

where  $p_1^2 = p_3^2 = p_4^2 = 0$ ,  $p_2^2 = m_l^2$  and

$$s = (p_1 + p_2)^2, \quad t = (p_1 + p_4)^2, \quad u = (p_2 + p_4)^2, \quad \text{with } s + t + u = m_l^2.$$

The integral family is defined by 9 inverse propagators,

$$D_1 = l_1^2,$$

$$D_2 = (l_1 + p_1)^2,$$

$$D_3 = (l_1 + p_1 + p_2)^2,$$

$$D_4 = (l_2 + p_1 + p_2)^2 - m_W^2,$$

$$D_5 = (l_2 - p_4)^2 - m_l^2,$$

$$D_6 = l_2^2,$$

$$D_7 = (l_1 - l_2)^2,$$

$$D_8 = (l_1 - p_4)^2,$$

$$D_9 = (l_2 + p_1)^2.$$

# Charged-current DY with massive lepton: DEs

- Forty-six master integrals after IBP reduction

$$f_i = \int \mathcal{D}^d l_1 \mathcal{D}^d l_2 \frac{1}{D_1^{\alpha_1} \dots D_9^{\alpha_9}}, \quad \alpha_i \in \mathbb{Z}.$$

- Normalize  $f_i$  to be dimensionless

$$f_i \rightarrow (m_W^2)^{\sum_j^9 \alpha_j - d}$$

- Introduce 3 dimensionless ratios

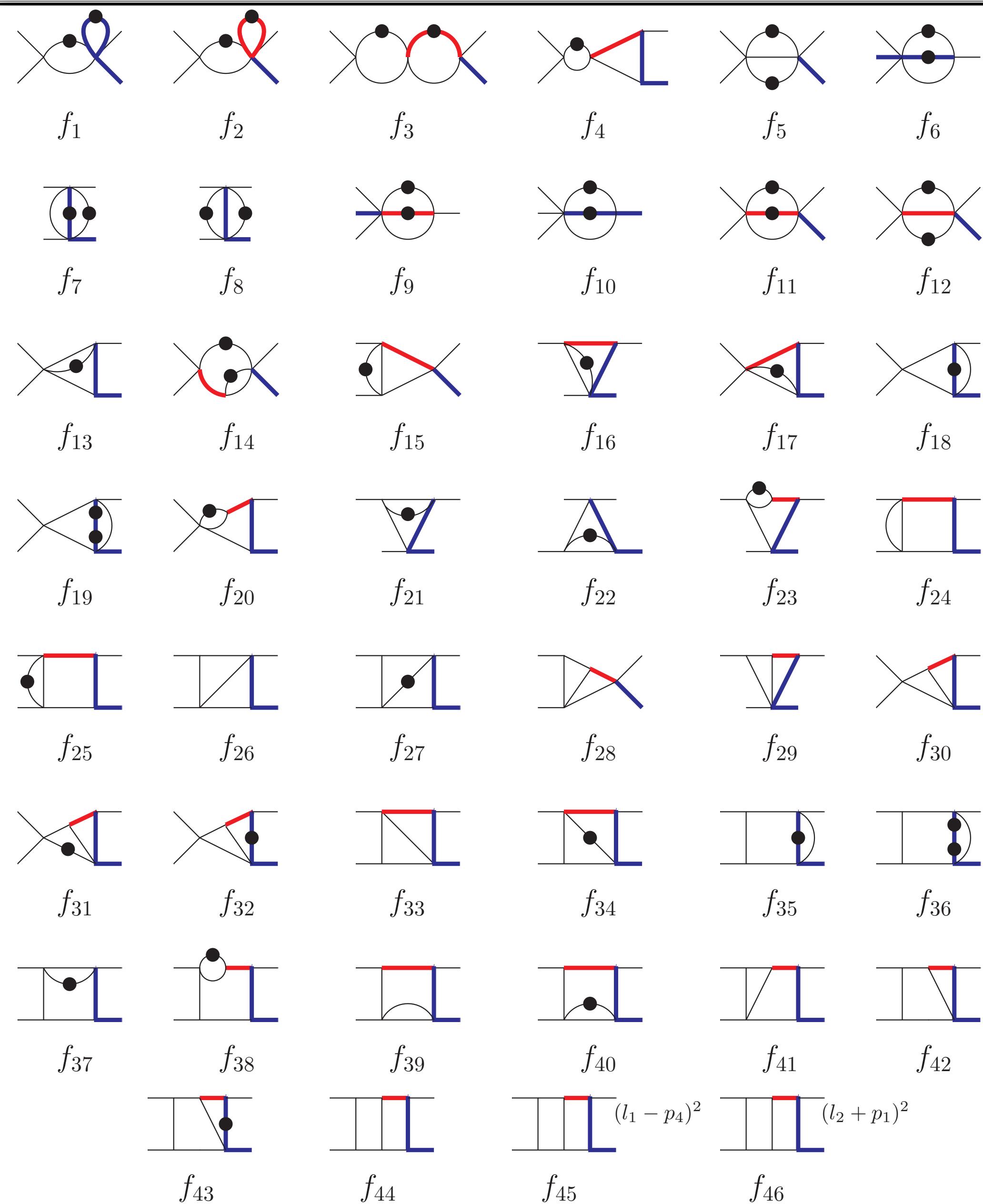
$$x = -\frac{s}{m_W^2}, \quad y = -\frac{t}{m_W^2}, \quad z = \frac{m_l^2}{m_W^2}.$$

- Differential equation system

$$\frac{\partial \vec{f}}{\partial x} = B_x(x, y, z; \epsilon) \vec{f}, \quad \frac{\partial \vec{f}}{\partial y} = B_y(x, y, z; \epsilon) \vec{f}, \quad \frac{\partial \vec{f}}{\partial z} = B_z(x, y, z; \epsilon) \vec{f}.$$

- Canonical basis  $\vec{g} = T \vec{f}$

$$\frac{\partial \vec{g}}{\partial x} = \epsilon \mathbb{A}_x(x, y, z) \vec{g}, \quad \frac{\partial \vec{g}}{\partial y} = \epsilon \mathbb{A}_y(x, y, z) \vec{g}, \quad \frac{\partial \vec{g}}{\partial z} = \epsilon \mathbb{A}_z(x, y, z) \vec{g}.$$



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Rationalize the root?

$$\frac{\partial \vec{g}}{\partial x} = \epsilon \mathbb{A}_x(x, y, z) \vec{g}, \quad \frac{\partial \vec{g}}{\partial y} = \epsilon \mathbb{A}_y(x, y, z) \vec{g}, \quad \frac{\partial \vec{g}}{\partial z} = \epsilon \mathbb{A}_z(x, y, z) \vec{g}.$$

$g_{27} = \epsilon^3 f_{27} x (y + z),$	$g_{28} = \epsilon^4 f_{28} x,$
$g_{29} = \epsilon^4 f_{29} y,$	$g_{30} = \epsilon^4 f_{30} (x + z),$
$g_{31} = \epsilon^3 f_{31} x (x + z),$	$g_{32} = \epsilon^3 f_{32} (1 - z) (x + z),$
$g_{33} = \epsilon^4 f_{33} (x + y + z),$	$g_{34} = \epsilon^3 f_{34} (x + 1) (y + z),$
$g_{35} = \epsilon^3 f_{35} x y,$	$g_{36} = \epsilon^2 x z ((y + z) f_{36} + \epsilon f_{35}),$
$g_{37} = \epsilon^3 f_{37} x (y + z),$	$g_{38} = \epsilon^3 (y + z) ((x + 1) f_{38} + f_{37}),$
$g_{39} = \epsilon^3 (1 - 2\epsilon) f_{39} y,$	$g_{40} = \epsilon^3 f_{40} (x + 1) (y + z),$
$g_{41} = \epsilon^4 f_{41} \lambda,$	$g_{42} = \epsilon^4 f_{42} x y,$

where only  $g_{41}$  needs a square root  $\lambda$ ,

$$\lambda = \sqrt{x^2 (y^2 + 2y(z+1) + (z-1)^2) + 2xy(y+z+1) + y^2}.$$

# Charged-current DY with massive lepton: Solutions

- Canonical differential system

$$\frac{\partial \vec{g}}{\partial x} = \epsilon \mathbb{A}_x(x, y, z) \vec{g}, \quad \frac{\partial \vec{g}}{\partial y} = \epsilon \mathbb{A}_y(x, y, z) \vec{g}, \quad \frac{\partial \vec{g}}{\partial z} = \epsilon \mathbb{A}_z(x, y, z) \vec{g}.$$

- Expand the DE system around  $z=0$  due to large mass gap between lepton and W boson. No roots in expanded system.

$$\mathbb{A}_x = \mathbb{A}_{x,0} + z \mathbb{A}_{x,1} + \mathcal{O}(z^2), \quad \mathbb{A}_y = \mathbb{A}_{y,0} + z \mathbb{A}_{y,1} + \mathcal{O}(z^2), \quad \mathbb{A}_z = \frac{\mathbb{A}_{z,-1}}{z} + \mathbb{A}_{z,0} + \mathcal{O}(z^1).$$

- Solutions evaluate to Goncharov polylogarithms (GPLs)

$$G(w_n, \dots, w_1; a) = \int_0^a \frac{1}{t - w_n} G(w_{n-1}, \dots, w_1; t) dt, \quad G(\underbrace{0, \dots, 0}_{n \text{ times}}; t) = \frac{\log^n(t)}{n!}.$$

❖ Taylor series

$$\vec{g} = \sum_{i=0}^{\infty} \vec{g}^{(i)} \epsilon^i,$$

$$\vec{g}^{(n)} = \sum_{i=0}^n \mathbb{M}^{(n-i)} \vec{c}^{(i)},$$

❖ Iteration

$$\begin{aligned} \mathbb{M}_x^{(n)} &= \int \mathbb{A}_x \mathbb{M}^{(n-1)} dx, \\ \mathbb{M}_y^{(n)} &= \int \left( \mathbb{A}_y \mathbb{M}^{(n-1)} - \partial_y \mathbb{M}_x^{(n)} \right) dy, \\ \mathbb{M}_z^{(n)} &= \int \left( \mathbb{A}_z \mathbb{M}^{(n-1)} - \partial_z \mathbb{M}_x^{(n)} - \partial_z \mathbb{M}_y^{(n)} \right) dz, \\ \mathbb{M}^{(n)} &= \mathbb{M}_x^{(n)} + \mathbb{M}_y^{(n)} + \mathbb{M}_z^{(n)} \end{aligned}$$

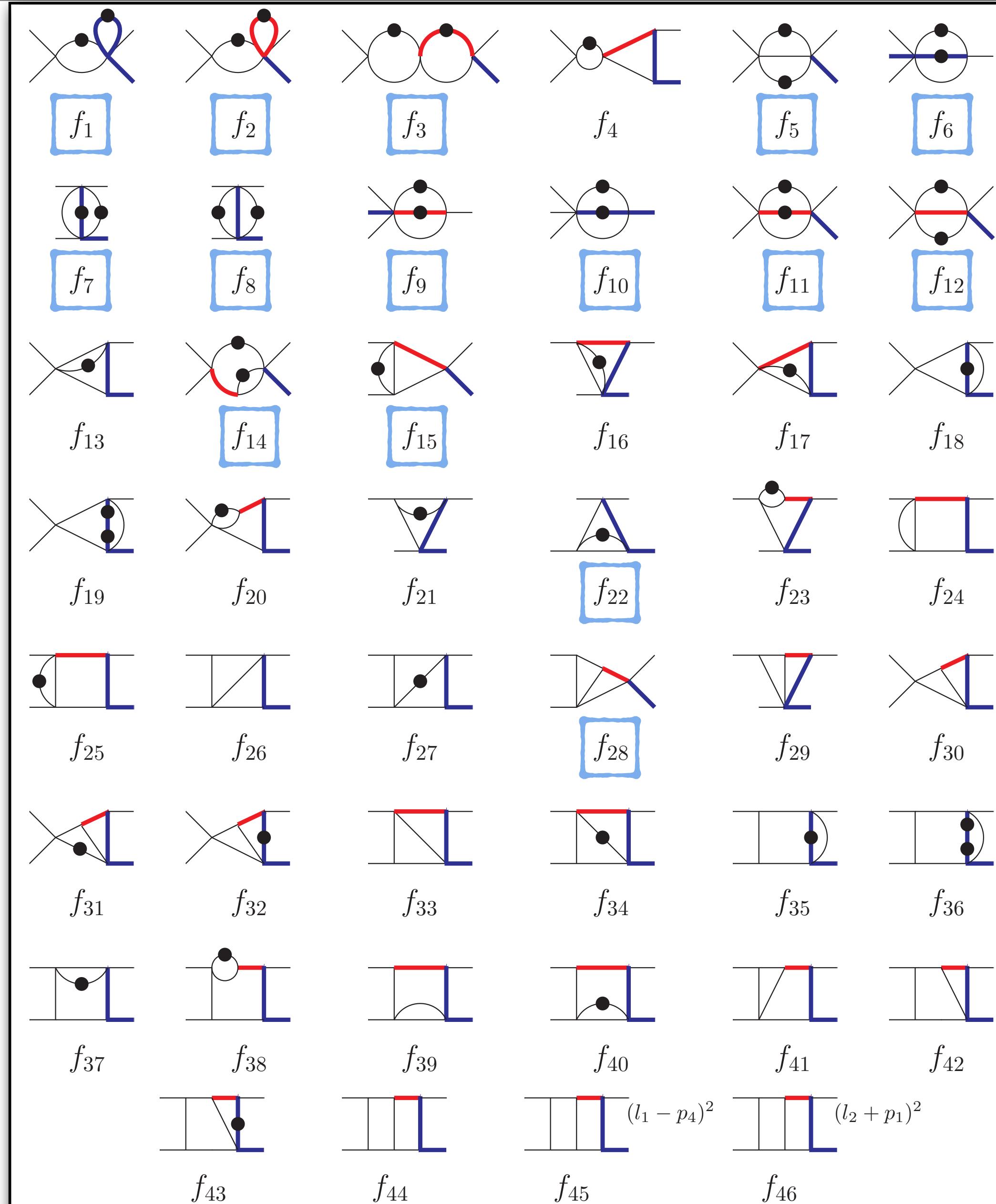
❖ Alphabet

$$\begin{array}{ll} \alpha_1 = x, & \alpha_2 = 1 + x, \\ \alpha_3 = x + y, & \alpha_4 = x + y + xy, \\ \alpha_5 = y, & \alpha_6 = 1 - y, \\ \alpha_7 = z. & \end{array}$$

# Charged-current DY with massive lepton: Solutions

Boundary conditions

1.  $g_{1,2,3,5,\dots,12,14,15,22,28}$  as input integrals.



$$(l_1 - p_4)^2$$

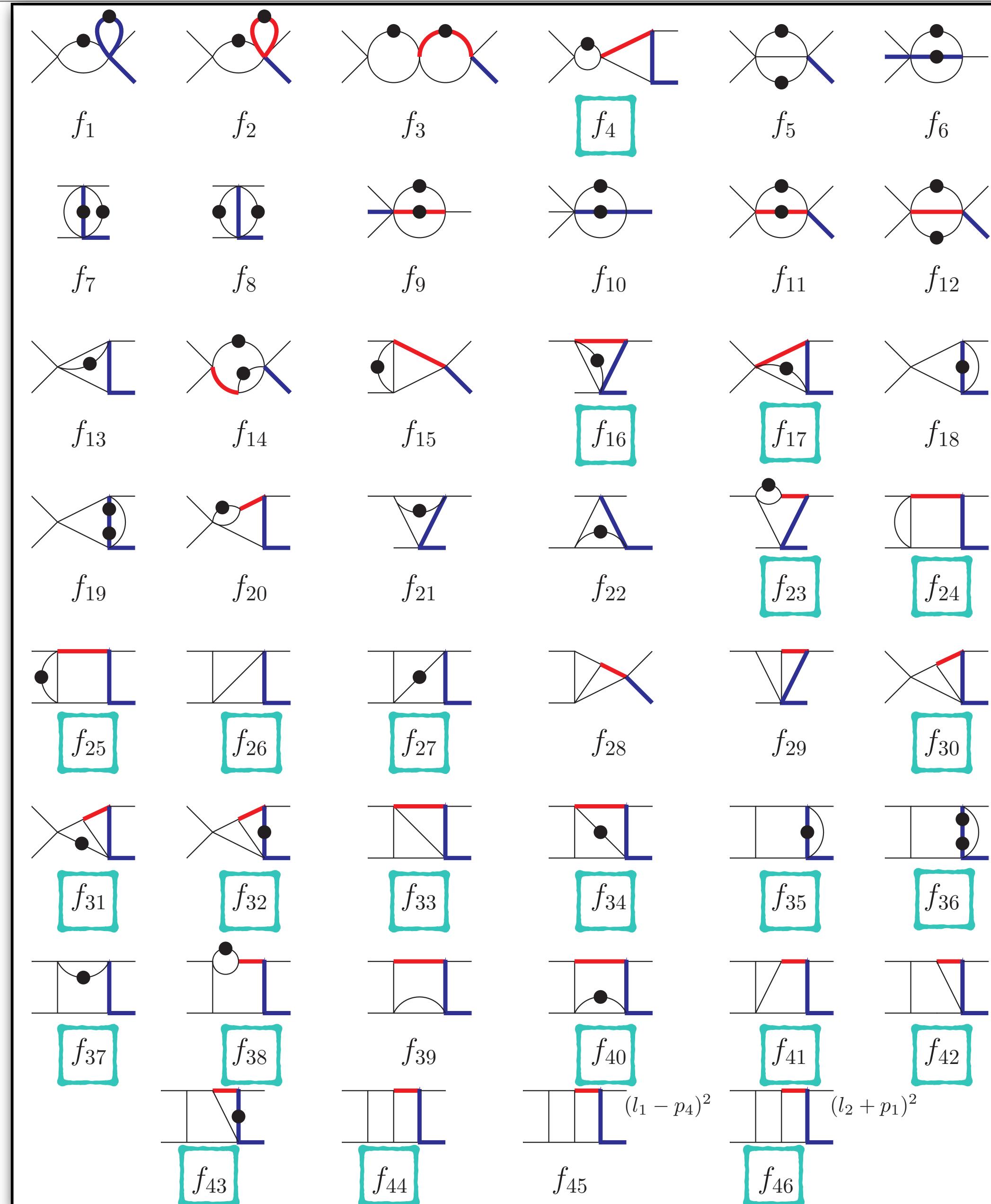
$$(l_2 + p_1)^2$$

# Charged-current DY with massive lepton: Solutions

## Boundary conditions

1.  $g_{1,2,3,5,\dots,12,14,15,22,28}$  as input integrals.
2. Remaining ones except  $g_{13,18,21,30,45}$  are fixed by imposing the regularity conditions below

	limit	regular MI
	0	$g_{4,17,24,25,30,31,34,40,46}$
$s \rightarrow$	$-t$	$g_{26,27,33,35,36,37,38,40,41,42,43,44}$
	$m_W^2$	$g_{32}$
$t \rightarrow$	$-m_W^2$	$g_{16,23}$
	$-\frac{m_W^2 s}{m_W^2 - s}$	$g_{41,44}$



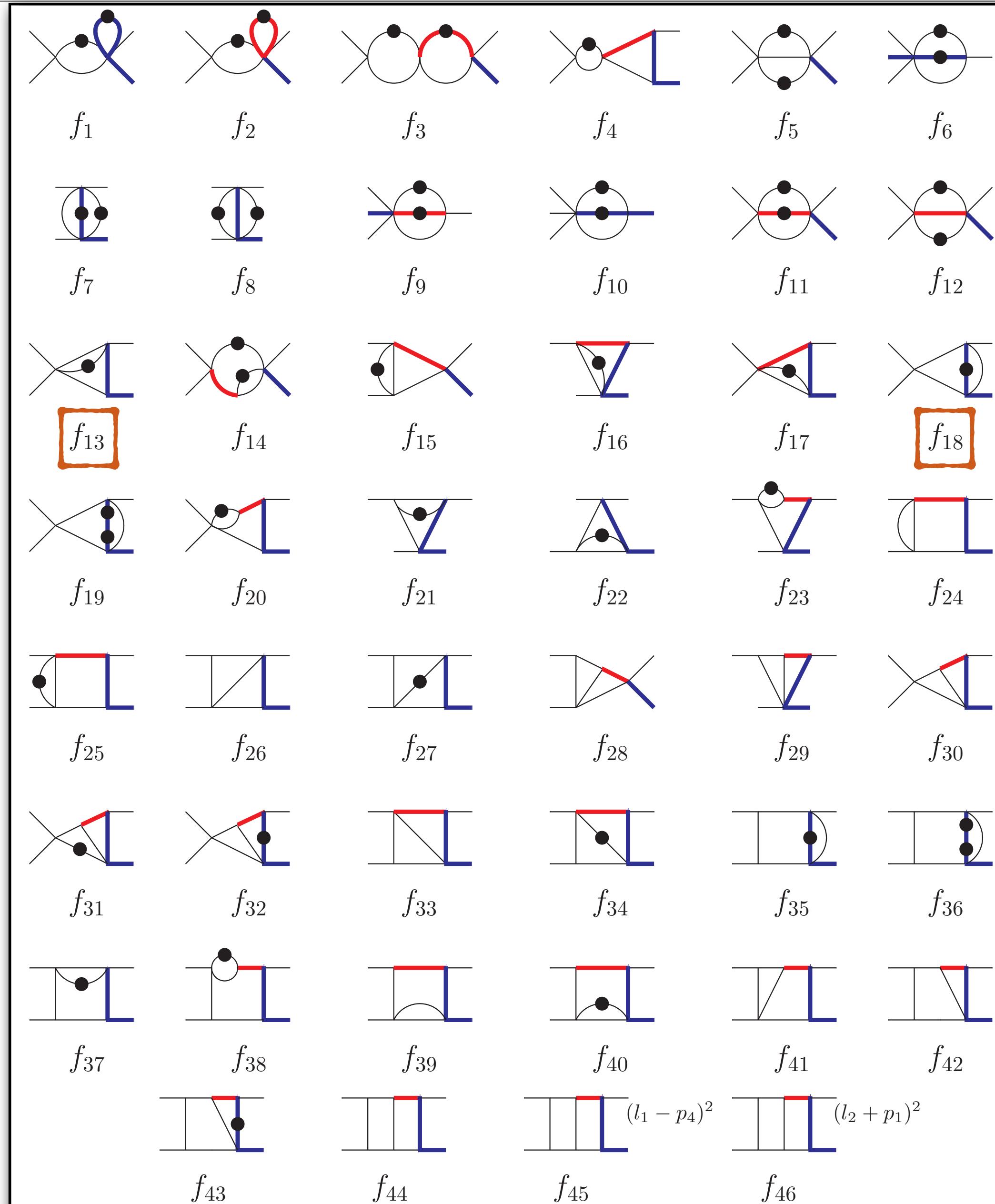
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3.  $g_{13,18}$  are matched against the counterparts with full lepton mass dependence.



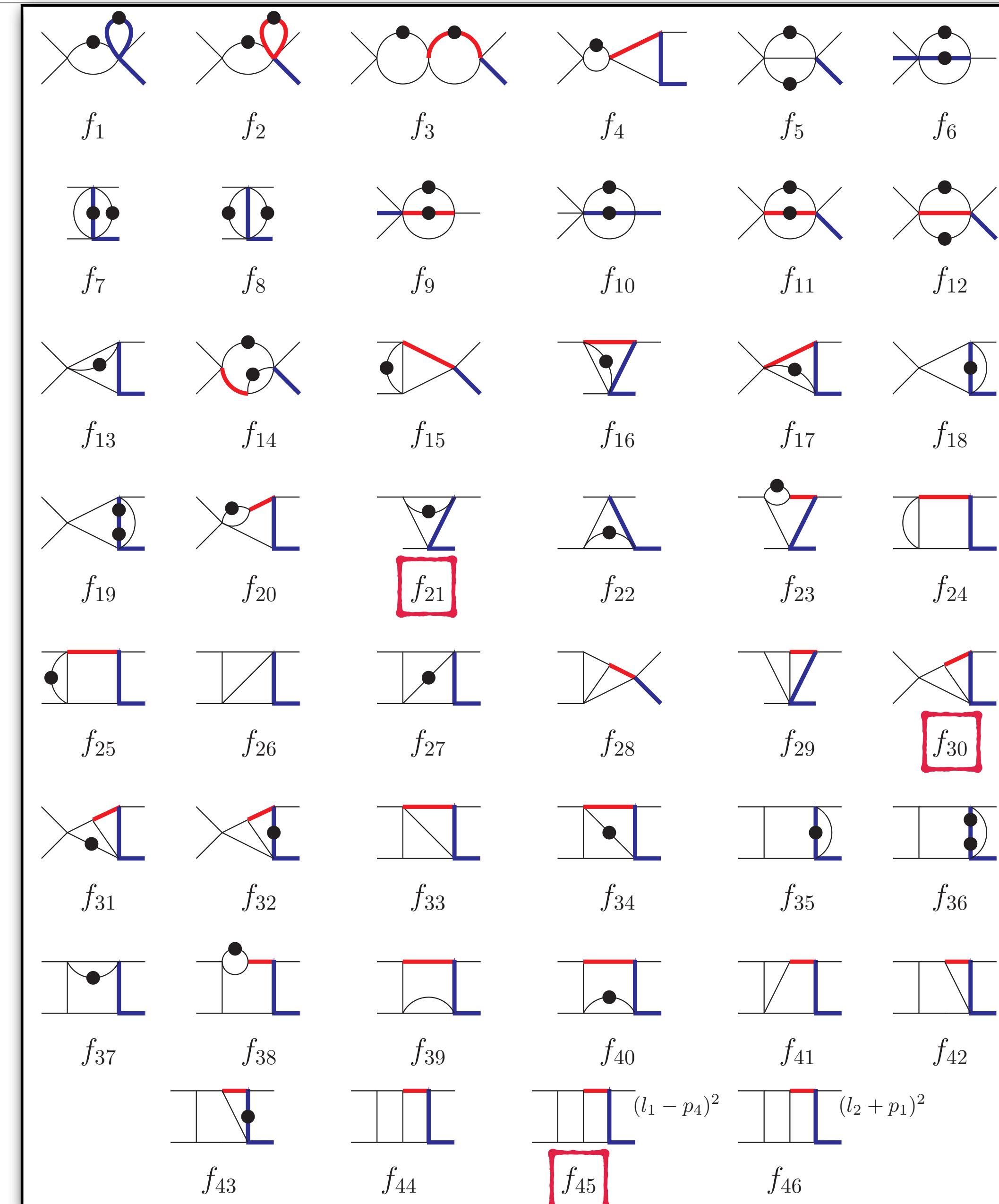
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3.  $g_{13,18}$  are matched against the counterparts with full lepton mass dependence.
4.  $g_{21,30,45}$  take use of the asymptotic behavior in the limit  $z \rightarrow 0$ .

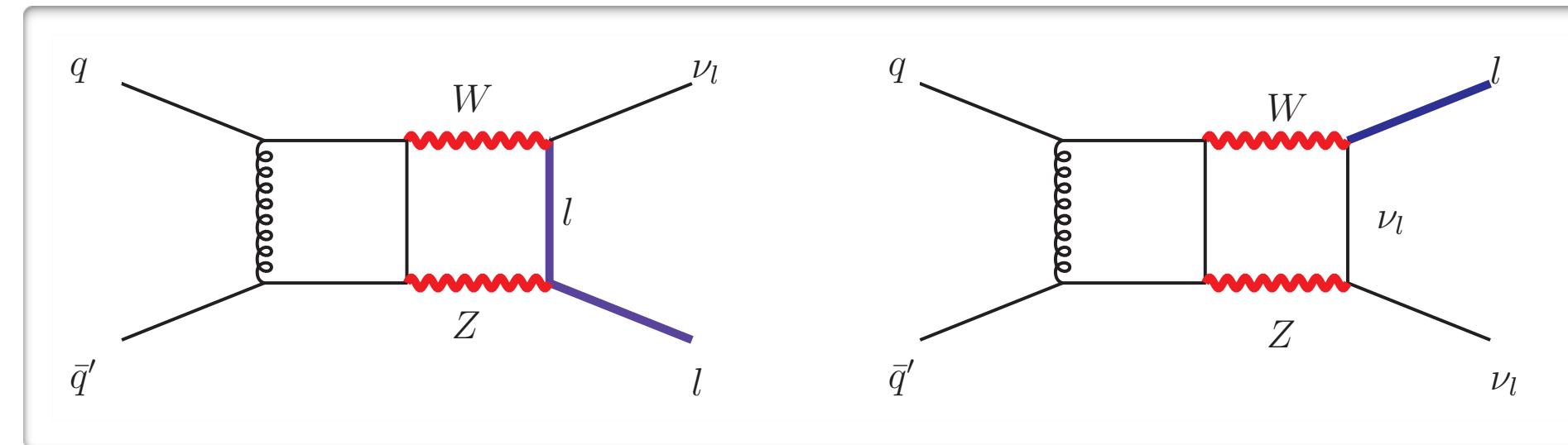


# Conclusions & Outlook

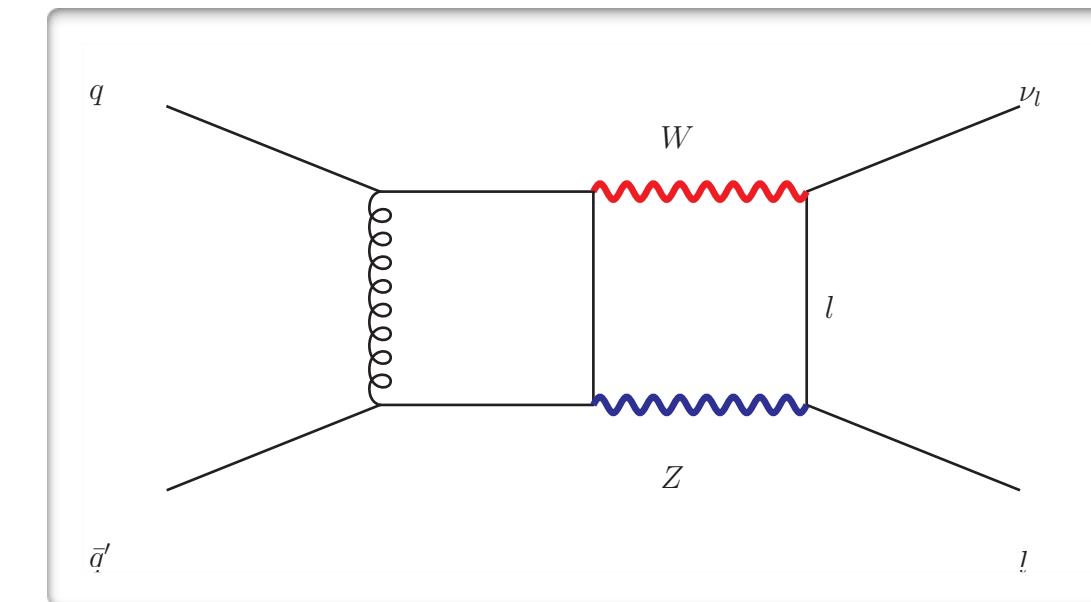
- The Drell-Yan processes provide the stringent precision test of SM.
- More works need to be done for QCD-EW corrections to Drell-Yan processes.
- The two-loop master integrals for QCD-QED corrections to charged-current Drell-Yan process with massive lepton are calculated analytically.
- Our result could help to implement a flexible MC program for real physics analysis when lepton mass is under consideration.

# Conclusions & Outlook

- Amplitude calculations for QCD-EW corrections to charged-current process.
- As an interesting question: what if we want to keep the mass of lepton in the complete QCD-EW calculations?



- What if  $m_Z \neq m_W$  but  $m_l = 0$  when trying to compute master integrals analytically?



- Have found canonical basis for both cases. Too many square roots. Expansion or new method([\[1907.00491\]](#), [\[2105.08046\]](#), [\[2108.03828\]](#))?

**Thank you!**

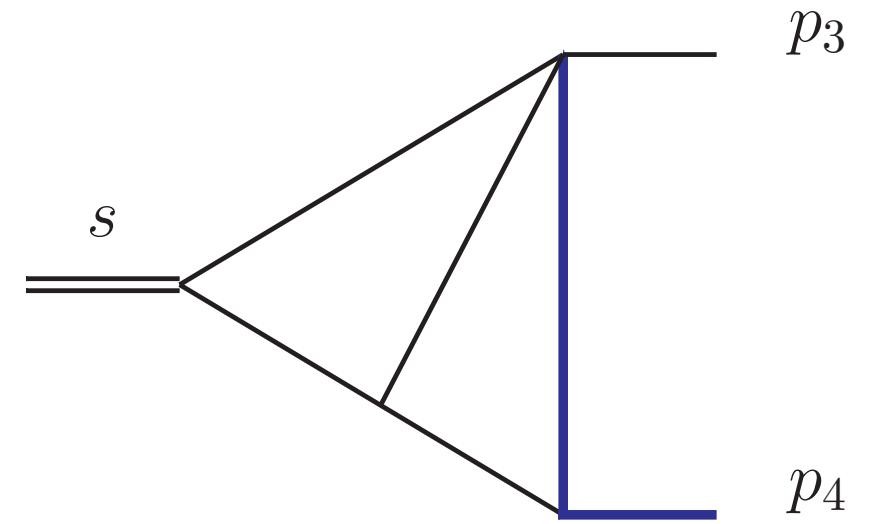
# Back Up

# Boundaries of $g_{13,18}$

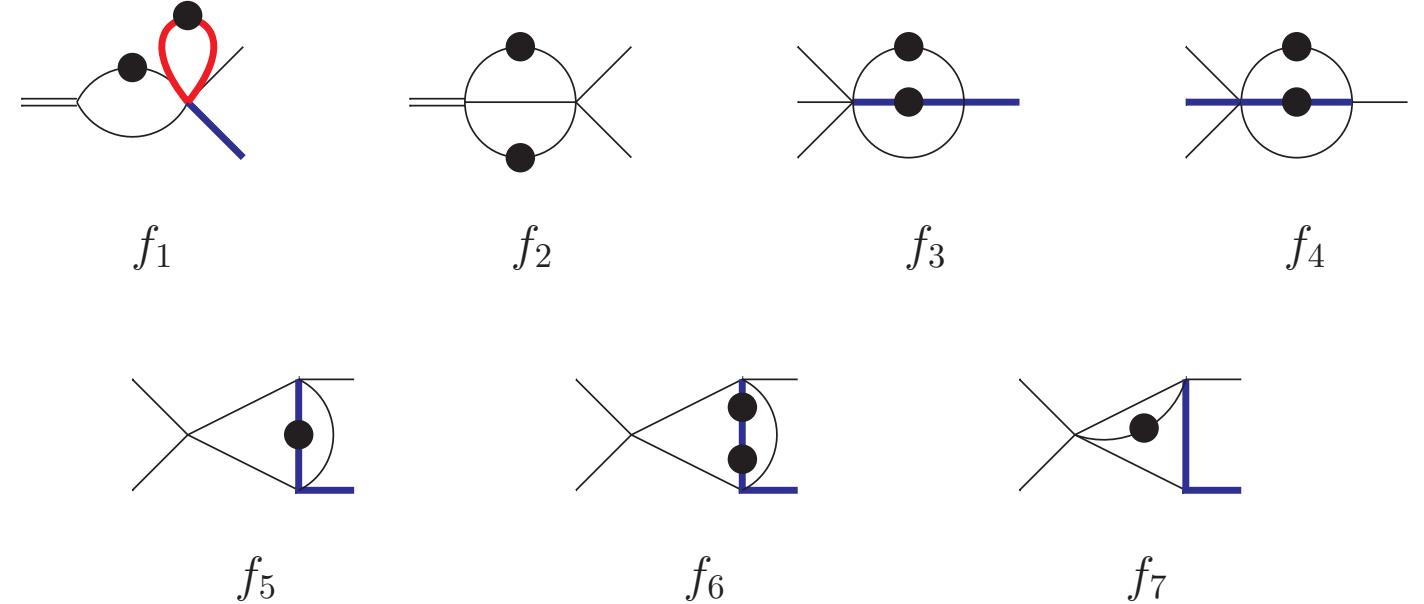
Look at a smaller system defined by

$$\begin{aligned} D_1 &= l_1^2, & D_2 &= l_2^2, & D_3 &= (l_1 - p_3 - p_4)^2, \\ D_4 &= (l_2 - p_4)^2 - m_l^2, & D_5 &= (l_1 - l_2)^2, & D_6 &= (l_1 - p_4)^2, \\ D_7 &= (l_2 - p_3 - p_4)^2, \end{aligned}$$

and consider only the sector (including its subsectors)



Seven MIs in this family



The canonical basis is defined as ( $u = -m_l^2/s$ )

$$\begin{aligned} g_1 &= \epsilon^2 f_1, & g_2 &= \epsilon^2 f_2, \\ g_3 &= \epsilon^2 f_3 u, & g_4 &= (1 - \epsilon) \epsilon f_4 u, \\ g_5 &= \epsilon^3 f_5 (u + 1), & g_6 &= \epsilon^2 f_6 u (u + 1), \\ g_7 &= \epsilon^3 f_7 (u + 1) \end{aligned}$$

Performing an expansion around  $u = 0$  on complete solutions provides the requisite boundaries of  $g_{13,18}$  in our main calculation

$$\begin{aligned} g_5 &= \epsilon^3 \left( \frac{1}{3} \pi^2 G(0,u) + G(0,0,0,u) - \zeta(3) \right) \\ &\quad + \epsilon^4 \left( -\zeta(3)G(0,u) - \frac{7}{6} \pi^2 G(0,0,u) - 5G(0,0,0,0,u) - \frac{7\pi^4}{45} \right) + \mathcal{O}(\epsilon^5), \\ g_7 &= \epsilon^2 \left( G(0,0,u) + \frac{\pi^2}{2} \right) + \epsilon^3 \left( -\frac{5}{6} \pi^2 G(0,u) - 3G(0,0,0,u) + 2\zeta(3) \right) \\ &\quad + \epsilon^4 \left( -6\zeta(3)G(0,u) + \frac{3}{2} \pi^2 G(0,0,u) + 7G(0,0,0,0,u) + \frac{89\pi^4}{360} \right) + \mathcal{O}(\epsilon^5), \end{aligned}$$

# Asymptotic behavior in $z \rightarrow 0$

Perform a Jordan decomposition of  $\mathbb{A}_{z,-1}$

$$\mathbb{A}_{z,-1} = \mathbb{S} \mathbb{J} \mathbb{S}^{-1},$$

Let  $\vec{h} = \mathbb{S}^{-1} \vec{g}$ , then in the limit  $z \rightarrow 0$

$$\partial_z \vec{h} \approx \epsilon \frac{1}{z} \mathbb{J} \vec{h},$$

Easy to identify an  $h$  integral satisfying

$$\partial_z h_i = 0.$$

On the other hand, at integrand level

$$g_i \Big|_{z=0} = \sum_{j=1}^{31} \kappa_j \mathbf{I}_j,$$

where the 31  $\mathbf{I}$  integrals are known [1604.08581]. Following equivalence can provide extra missing boundaries

$$\lim_{z \rightarrow 0} \sum_i \beta_i g_i = \sum_j \gamma_j \mathbf{I}_j.$$