Accurate and Robust PMT Waveform Analysis

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Motivation

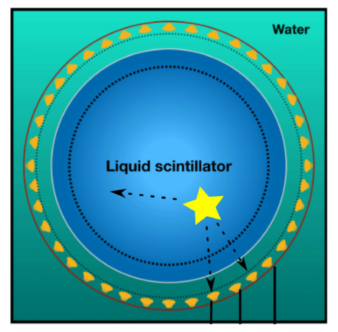

Figure 1. An Event in Detector

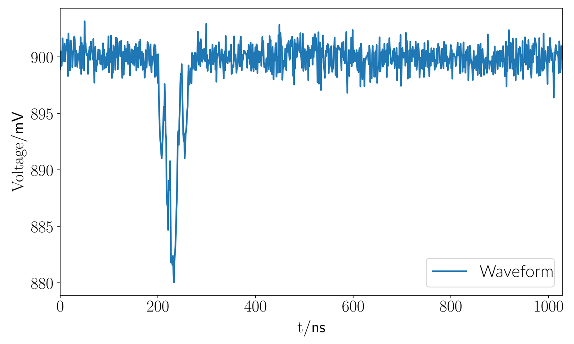

Figure 2. A PMT Waveform

Waveform analysis, which means extracting time and charge information from PMT waveforms, is the bedrock of subsequent analysis such as event reconstruction.

Simulation Setup

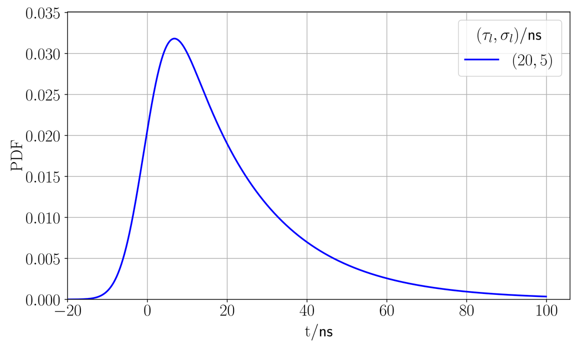

Figure 3. Time Profile $\phi(t)$ of Events

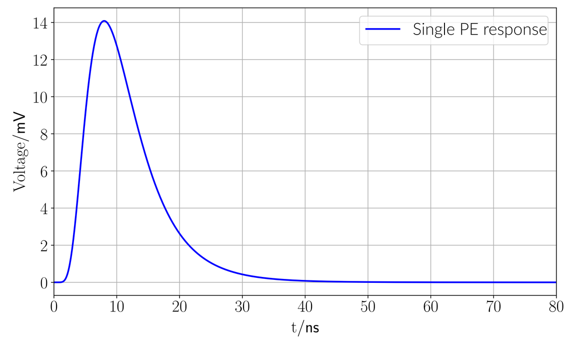

Figure 4. Single PE response $V_\mathrm{PE}(t)$[1]

$$\phi(t) = \mathcal{N}(t|\sigma_l^2) \otimes \mathrm{Exp}(t|\tau_l)$$
$$= \frac{1}{2\tau_l}\exp\left(\frac{\sigma_l^2}{2\tau_l^2} - \frac{t}{\tau_l}\right)\left[1 - \mathrm{erf}\left(\frac{\sigma_l}{\sqrt{2}\tau_l} - \frac{t}{\sqrt{2}\sigma_l}\right)\right]$$

$$V_\mathrm{PE}(t) = V_0 \exp\left[-\frac{1}{2}\left(\frac{\log(t/\tau_\mathrm{PE})}{\sigma_\mathrm{PE}}\right)^2\right]$$

Data Input & Output

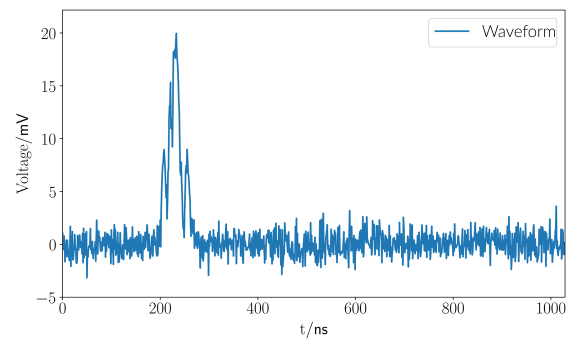

Figure 5. Input Waveform (Pedestal free)

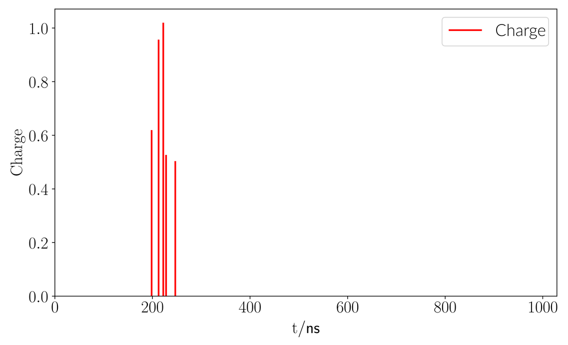

Figure 6. Output Time and Charge $\hat{\phi}(t)$

$$w(t) = \tilde{\phi}(t) \otimes V_\mathrm{PE}(t) + \epsilon(t) = \sum_{i=1}^{N_\mathrm{PE}} q_i V_\mathrm{PE}(t - t_i) + \epsilon(t) \qquad \tilde{\phi}(t) = \sum_{i=1}^{N_\mathrm{PE}} q_i \delta(t - t_i),\ N_\mathrm{PE} \sim \mathrm{Poisson}(\mu)$$

Wasserstein Distance[2] as Evaluation Criteria

$\tilde{\phi}(t)$ (simulation truth) is an approximation of $\phi(t)$ (time profile).

$\hat{\phi}(t)$ (reconstruction result) should be consistent with $\tilde{\phi}(t)$.

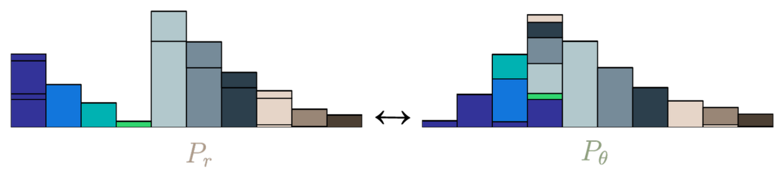

Figure 7. Wasserstein Distance D_w when $p = 1$: Earth Mover Distance

$$D_w\left[\hat{\phi}_*, \tilde{\phi}_*\right] = \inf_{\gamma \in \Gamma}\left[\int |t_1 - t_2|^p \gamma(t_1, t_2)\mathrm{d}t_1\mathrm{d}t_2\right]^{\frac{1}{p}}$$

$$\Gamma = \left\{\gamma(t_1, t_2) \middle| \int \gamma(t_1, t_2)\mathrm{d}t_1 = \hat{\phi}_*(t_2), \int \gamma(t_1, t_2)\mathrm{d}t_2 = \tilde{\phi}_*(t_1)\right\}$$

when $p = 1$, Cumulative distribution function (CDF) of $\phi(t)$ is $\Phi(t)$, D_w is a ℓ_1-distance:

$$D_w\left[\hat{\phi}_*, \tilde{\phi}_*\right] = \int \left|\hat{\Phi}(t) - \tilde{\Phi}(t)\right| \mathrm{d}t$$

Fast Bayesian Matching Pursuit[3] in waveform analysis

- Fast Bayesian Matching Pursuit (FBMP) is a sparse regression algorithm, which origins from the field of signal processing.
- Time in DAQ window is divided into time bins: $\vec{t}$, whose length is N. Each time bin can have 1 PE. As long as the bin width is small, the timing resolution will be retained.
- Model vector: $\vec{z}$. $z_i = 0 \implies q_i = 0$ and $z_i = 1 \implies q_i \neq 0$. When z_i is 0, the corresponding charge of PE in time bin t_i will be 0, otherwise it may not be zero.
- Linear Model: $\vec{w} = \boldsymbol{V}_\mathrm{PE}\vec{z} + \vec{\epsilon}$. This process is equivalent to $\tilde{\phi}$ convoluting with Single PE, and merely time is digitized.
- $$\begin{bmatrix}\vec{w}\\ \vec{q}\end{bmatrix}\middle|\vec{z} \sim \mathrm{Normal}\left(\begin{bmatrix}\boldsymbol{V}_\mathrm{PE}\vec{z}\\ \vec{z}\end{bmatrix}, \begin{bmatrix}\boldsymbol{\Sigma}_z & \boldsymbol{V}_\mathrm{PE}\boldsymbol{Z}\\ \boldsymbol{Z}\boldsymbol{V}_\mathrm{PE}^\intercal & \boldsymbol{Z}\end{bmatrix}\right)$$
  $$\boldsymbol{\Sigma}_z = \boldsymbol{V}_\mathrm{PE}\boldsymbol{Z}\boldsymbol{V}_\mathrm{PE}^\intercal + \sigma_\epsilon^2\boldsymbol{I}$$
  where $\boldsymbol{Z}$ is the diagonal matrix of vector $\vec{z}$ controlling q_i
- $\mathcal{Z} = \{\vec{z}_j\}$ contains 2^N model vectors

FBMP Evaluation

- Calculation of 2^N model vectors is impossible!
- Most of $p(\vec{w}|\vec{z}) \to 0$!

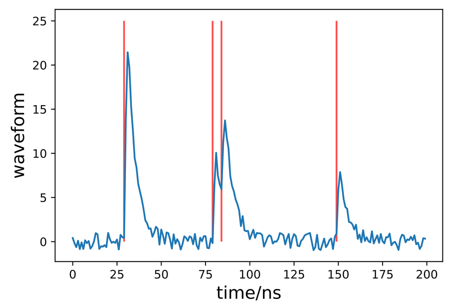

Figure 8. perfect PE matching waveform, $p(\vec{z}|\vec{w})$ hit maximum

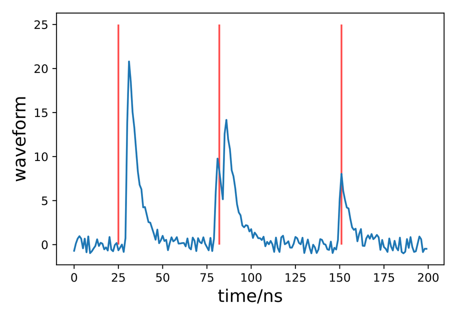

Figure 9. not so perfect, $p(\vec{z}|\vec{w})$ is smaller but still > 0

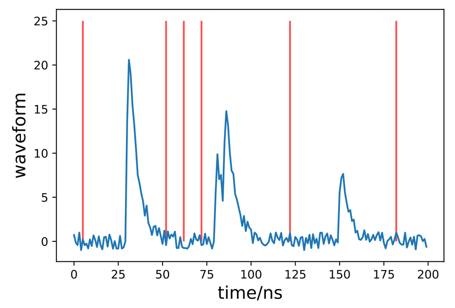

Figure 10. Completely mismatch the waveform, $p(\vec{z}|\vec{w}) \to 0$

- Most z can be ignored because most z does not correspond to the waveform. If we only consider the model vector z with a relatively large posterior probability, the calculation effort will be reduced.
- $$\begin{aligned}\log[p(\vec{w}, \vec{z})] &= \log[p(\vec{w}|\vec{z})p(\vec{z})]\\ &= -\frac{1}{2}(\vec{w} - \boldsymbol{V}_\mathrm{PE}\vec{z})^\intercal\boldsymbol{\Sigma}_z^{-1}(\vec{w} - \boldsymbol{V}_\mathrm{PE}\vec{z}) - \frac{1}{2}\log\det\boldsymbol{\Sigma}_z\\ &\quad - \frac{N}{2}\log 2\pi - \mu + \sum_{i|z_i=1}\log\frac{\mu\phi(t_i' - t_0)\Delta t'}{1 - \mu\phi(t_i' - t_0)\Delta t'}\end{aligned}$$
- A repeated greedy search(RGS) is performed to construct the target set $\mathcal{Z}'$, which contains only the $\vec{z}$ giving large $p(\vec{w}|\vec{z})$.

$$p(\vec{z}|\vec{w}) = \frac{p(\vec{w}|\vec{z})p(\vec{z})}{\sum_{\vec{z}'\in\mathcal{Z}} p(\vec{w}|\vec{z}')p(\vec{z}')} \approx \frac{p(\vec{w}|\vec{z})p(\vec{z})}{\sum_{\vec{z}'\in\mathcal{Z}'} p(\vec{w}|\vec{z}')p(\vec{z}')}$$

FBMP's Bayesian interface

- PE Time: $\vec{t}$
- Models: $\mathcal{Z}' = \{\vec{z}_j\}$
- Charge:

$$\hat{\vec{q}}_z = E(\vec{q}|\vec{w}, \vec{z}) = \vec{z} + \boldsymbol{Z}\boldsymbol{V}_\mathrm{PE}^\intercal\boldsymbol{\Sigma}_z^{-1}(\vec{w} - \boldsymbol{V}_\mathrm{PE}\vec{z})$$

- Model's posterior probability: $p(\vec{z}|\vec{w})$
- The final result of FBMP is the time vector, several models, and their corresponding charge vectors. Additionally, the posterior probability of each model. In commonly used fitting methods such as Maximum likelihood estimation (MLE), the posterior distribution is approximated into delta function or normal distribution.

Provides opportunity for subsequent Bayesian analysis!

FBMP Demonstration

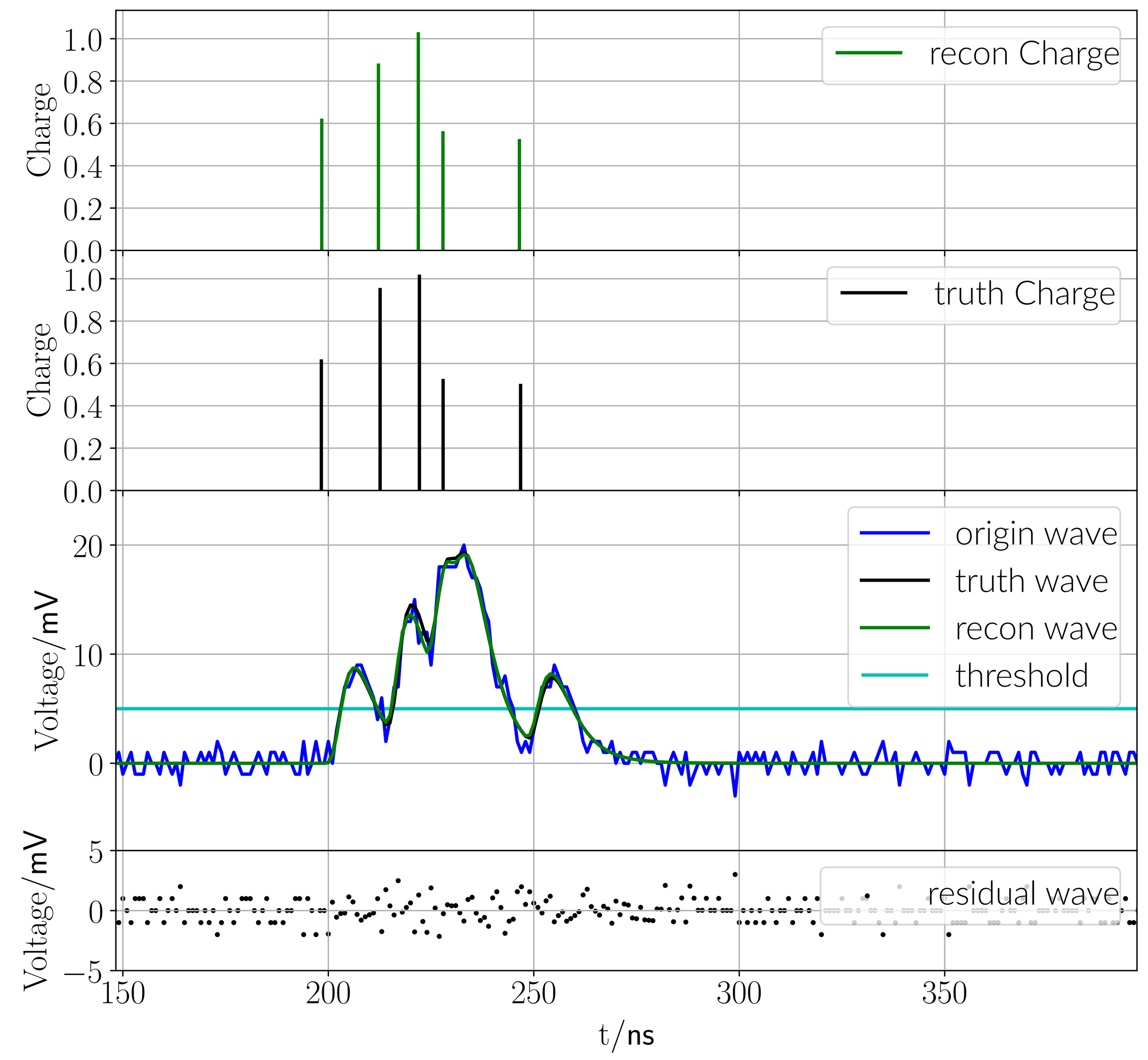

Figure 11. Max posterior probability model in FBMP's result, with $D_w = 0.63$ ns

FBMP's Performance of Evaluation Criteria and Charge Posterior

For dataset (μ, τ, σ)/ns $= (4, 20, 5)$, 10^4 waveforms:

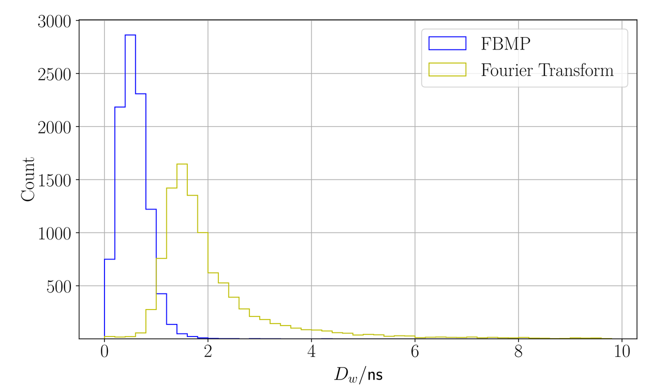

Figure 12. D_w of methods

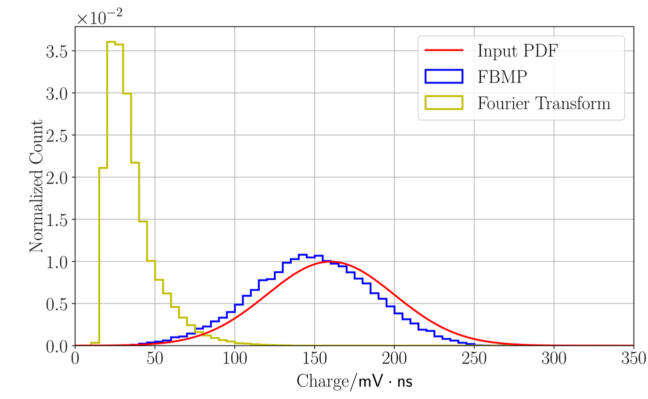

Figure 13. $\hat{q}$ histogram of methods

FBMP performs good on D_w and retains charge distribution of PE. When the truth distribution of charge is normal distribution, only FBMP can retain the distribution, which means every charge in FBMP method can be regarded as one PE, while other methods can not work due to these fragments of charge.

References

[1] Sören Jetter, Dan Dwyer, Wen-Qi Jiang, Dong-Wei Liu, Yi-Fang Wang, Zhi-Min Wang, and Liang-Jian Wen. PMT waveform modeling at the Daya Bay experiment. *Chinese Physics C*, 36(8):733–741, August 2012. Publisher: IOP Publishing.

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