

A simple prescription for Covariant chiral effective theory

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OUTLINE

- Covariance and chiral power counting: a simple strategy
- One-loop examples in SU(2) examples (nucleons)
- One-loop examples in SU(3) situation (octet baryons)
- Summary and prospects

Covariance and chiral power counting: a simple strategy

- Chiral power counting (χ PC) obeyed by the pseudo-scalar meson (Goldstone) sector:

$$Q \sim m_\pi, \Lambda_\chi \gg m_\pi (\epsilon \ll 1) : \text{loop diagrams} = \mathcal{O}\left(\frac{Q^n}{\Lambda_\chi^n}\right) = \mathcal{O}(\epsilon^n)$$

- χ PC violated by the baryonic sector:

$$Q \sim M_B \sim \Lambda_\chi : \text{loop diagrams} = \mathcal{O}\left(\frac{Q^n}{\Lambda_\chi^n}\right) = \mathcal{O}(1)!$$

- Observation: the χ PC-violating pieces are actually LOCAL(!)
- Strategy: simply remove these local χ PC-violating pieces via counter-terms (in any conventional scheme, say MS), χ PC is restored in the terms leftover

$$\mathcal{O}\left(\frac{\tilde{Q}^n}{\Lambda_\chi^n}\right), \tilde{Q} \sim (|Q| - M_B) \sim \mathcal{O}(m_\pi)$$

Lagrangian

- SU(2):

$$\mathcal{L}(\Psi) = \bar{\Psi} \left[i\gamma^\mu \partial_\mu - M_N + \frac{g_A}{2f_\pi} \gamma^5 \gamma^\mu \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi} - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \times \gamma^\mu \partial_\mu \boldsymbol{\pi} + o(\boldsymbol{\pi}^3) \right] \Psi$$

$$\mathcal{L}_{HB,int} = \bar{\Psi}_v \left[-\frac{g_A}{2f_\pi} S^\mu \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi} - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \times v^\mu \partial_\mu \boldsymbol{\pi} + o(\boldsymbol{\pi}^3) \right] \Psi_v$$

$$S^\mu \equiv \frac{i}{2} \gamma^5 \sigma^{\mu\nu} v_\nu = -\frac{1}{2} \gamma^5 (\gamma^\mu \gamma^\alpha v_\alpha - v^\mu), \sigma^{\mu\nu} \equiv \frac{i}{4} [\gamma^\mu, \gamma^\nu], v^2 = 1, S^\mu v_\mu = 0, \Psi_v \equiv e^{iM_N v \cdot x \frac{1}{2}} (1 + \gamma^\alpha v_\alpha) \Psi$$

- SU(3):

$$\mathcal{L}(B, T) = i\text{Tr}[\bar{B}\gamma \cdot DB] - M_B \text{Tr}[\bar{B}B] + D\text{Tr}[\bar{B}\gamma^\mu \gamma^5 \{A_\mu, B\}] + F\text{Tr}[\bar{B}\gamma^\mu \gamma^5 [A_\mu, B]]$$

$$+ i\bar{T}_\mu \gamma^{\mu\nu\alpha} \mathcal{D}_\alpha T_\nu - m_T \bar{T}_\mu \gamma^{\mu\nu} T_\nu + \mathcal{H} \bar{T}^\mu \gamma \cdot A \gamma_5 T_\mu + \mathcal{C} (\bar{T}_\mu \Theta^{\mu\nu} A_\nu B + h.c.)$$

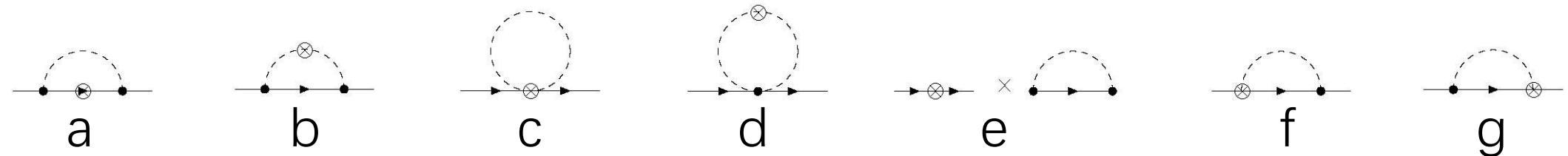
$$+ b_1 \text{Tr}[\bar{B}(\xi^\dagger m \xi^\dagger + \xi m \xi) B] + b_2 \text{Tr}[\bar{B}B(\xi^\dagger m \xi^\dagger + \xi m \xi)]$$

$$-c \bar{T}_\mu \gamma^{\mu\nu} (\xi^\dagger m \xi^\dagger + \xi m \xi) T_\nu + \sigma \text{Tr}[mU + mU^\dagger] \text{Tr}[\bar{B}B] - \bar{\sigma} \text{Tr}[mU + mU^\dagger] \bar{T}^\mu T_\mu$$

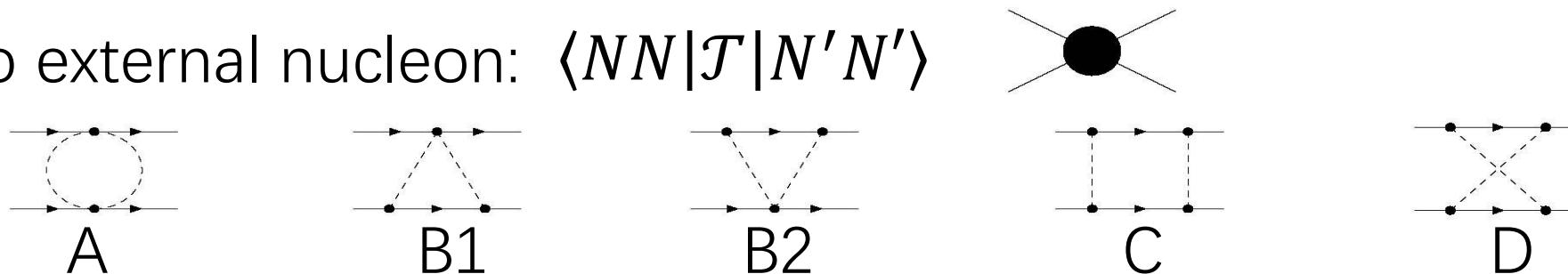
$$\gamma^{\mu\nu\alpha} \equiv \frac{1}{2} \{\gamma^{\mu\nu}, \gamma^\alpha\}, \gamma^{\mu\nu} \equiv \frac{1}{2} [\gamma^\mu, \gamma^\nu], \Theta^{\mu\nu} \equiv g^{\mu\nu} - \gamma^\mu \gamma^\nu; \gamma^\mu T_\mu = 0, \partial_\mu T_\mu = 0; U = \xi^2$$

- One-loop examples in SU(2) situation (Z. Liu, L.-H. Wen, JFY, NuclPhysB963(21)115288)

➤ One external nucleon: $\langle N | \mathcal{O} \dots | N \rangle$



➤ Two external nucleon: $\langle NN | \mathcal{T} | N'N' \rangle$



$\langle N | \mathcal{O} \dots | N \rangle$, primary

- **Fully covariant formalism:** $\Gamma^{tr} \equiv A^{(1)} \bar{u} \tau^a \gamma_\mu u; \ell_\pi \equiv \ln \frac{4\pi\mu^2}{m_\pi^2}, \ell_N \equiv \ln \frac{4\pi\mu^2}{M_N^2}, \rho \equiv \frac{M_N^2}{m_\pi^2}; \mathcal{R}^{\dots} = o\left(\rho^{-\frac{1}{2}}\right)$

$$\text{Fig a} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [-\Gamma(\epsilon) - \ell_\pi - 2 \ln \rho + 3 + \mathcal{R}_{1a}^c] \right\}$$

$$\text{Fig b} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [2\rho(\Gamma(\epsilon) + \ell_N + 1) + \Gamma(\epsilon) + \ell_N - 3 \ln \rho + 7 + \mathcal{R}_{1b}^c] \right\}$$

Fig c = *chiral*

Fig d = *chiral*

$$\text{Fig e} = \Gamma^{tr} \left\{ -\frac{3g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [-\Gamma(\epsilon) - \ell_\pi - 2 \ln \rho + 3 + \mathcal{R}_{1a}^c] \right\}$$

$$\text{Fig f} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [\rho(\Gamma(\epsilon) + \ell_N + 1) + \Gamma(\epsilon) + \ell_N + 2 + \mathcal{R}_{1f}^c] \right\}$$

$$\text{Fig g} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [\rho(\Gamma(\epsilon) + \ell_N + 1) + \Gamma(\epsilon) + \ell_N + 2 + \mathcal{R}_{1f}^c] \right\}$$

$\langle N | \mathcal{O} \dots | N \rangle$, decomposed

- **Fully covariant formalism:** $\langle \ln \rho = \Gamma(\epsilon) + \ell_\pi - (\Gamma(\epsilon) + \ell_N) \rangle$

$$\text{Fig a} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [2(\Gamma(\epsilon) + \ell_N + 3/2) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{1a}^c] \right\}$$

$$\text{Fig b} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [2\rho(\Gamma(\epsilon) + \ell_N + 1) + 4(\Gamma(\epsilon) + \ell_N + 7/4) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{1b}^c] \right\}$$

Fig c = *chiral*

Fig d = *chiral*

$$\text{Fig e} = \Gamma^{tr} \left\{ -\frac{3g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [2\rho(\Gamma(\epsilon) + \ell_N + 3/2) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{1a}^c] \right\}$$

$$\text{Fig f} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [\rho(\Gamma(\epsilon) + \ell_N + 1) + \Gamma(\epsilon) + \ell_N + 2 + \mathcal{R}_{1f}^c] \right\}$$

$$\text{Fig g} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [\rho(\Gamma(\epsilon) + \ell_N + 1) + \Gamma(\epsilon) + \ell_N + 2 + \mathcal{R}_{1f}^c] \right\}$$

$\langle N | \mathcal{O} \dots | N \rangle$, subtracted

- **Fully covariant formalism:**

$$\text{Fig a} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [-3\ell_\pi + o(\rho^{-1/2})] \right\}$$

$$\text{Fig b} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [-3\ell_\pi + o(\rho^{-1/2})] \right\}$$

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$$\text{Fig e} = \Gamma^{tr} \left\{ -\frac{3g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [-3\ell_\pi + o(\rho^{-1/2})] \right\}$$

$$\text{Fig f} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [0 + o(\rho^{-1/2})] \right\}$$

$$\text{Fig g} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} [0 + o(\rho^{-1/2})] \right\}$$

$\langle N | \mathcal{O} \dots | N \rangle$, primary

- Mixed formalism=covariant propagators + HB vertices:

$$\text{Fig a} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} \left[\frac{5}{2} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{3}{5} \right) + \frac{3}{4} \left(\Gamma(\epsilon) + \ell_N + \frac{1}{3} \right) - 3 \ln \rho + 6 + \mathcal{R}_{1a}^{HBv} \right] \right\}$$

$$\text{Fig b} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} \left[\frac{5}{8} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{11}{10} \right) - \frac{1}{2} \left(\Gamma(\epsilon) + \ell_\pi + 1 \right) - \frac{5}{2} \ln \rho + 6 + \mathcal{R}_{1b}^{HBv} \right] \right\}$$

Fig c = *chiral*

Fig d = *chiral*

$$\text{Fig e} = \Gamma^{tr} \left\{ -\frac{3g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} \left[\frac{5}{8} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{11}{10} \right) - \frac{1}{2} \left(\Gamma(\epsilon) + \ell_\pi + 1 \right) - \frac{5}{2} \ln \rho + 6 + \mathcal{R}_{1b}^{HBv} \right] \right\}$$

Fig f = 0

Fig g = 0

$\langle N | \mathcal{O} \dots | N \rangle$, decomposed

- Mixed formalism=covariant propagators + HB vertices:

$$\text{Fig a} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} \left[\frac{5}{2} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{3}{5} \right) + \frac{15}{4} \left(\Gamma(\epsilon) + \ell_N + \frac{5}{3} \right) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{1a}^{HBv} \right] \right\}$$

$$\text{Fig b} = \Gamma^{tr} \left\{ \frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2} \left[\frac{5}{8} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{11}{10} \right) + \frac{5}{2} \left(\Gamma(\epsilon) + \ell_N + \frac{11}{5} \right) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{1b}^{HBv} \right] \right\}$$

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$$\text{Fig e} = \Gamma^{tr} \left\{ -\frac{3g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} \left[\frac{5}{8} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{11}{10} \right) + \frac{5}{2} \left(\Gamma(\epsilon) + \ell_N + \frac{11}{5} \right) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{1b}^{HBv} \right] \right\}$$

Fig f = 0

Fig g = 0

$\langle N | \mathcal{O} \dots | N \rangle$, subtracted

- Mixed formalism=covariant propagators + HB vertices:

$$\text{Fig a} = \Gamma^{tr} \left\{ -\frac{g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [-3\ell_\pi + o(\rho^{-1/2})] \right\}$$

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$$\text{Fig e} = \Gamma^{tr} \left\{ -\frac{3g_A^2 m_\pi^2}{4(4\pi f_\pi)^2} [-3\ell_\pi + o(\rho^{-1/2})] \right\}$$

Fig f = 0

Fig g = 0

$\langle NN | \mathcal{T} | N'N' \rangle$, primary

- **Fully covariant formalism:** $(3 \pm 2\langle \tau_1 \cdot \tau_2 \rangle) \equiv 3(\bar{u}_1 u_1)(\bar{u}_2 u_2) \pm 2(\bar{u}_1 \tau_1 u_1) \cdot (\bar{u}_2 \tau_2 u_2)$

Fig A = *chiral*

$$\text{Fig B 1} = \langle \tau_1 \cdot \tau_2 \rangle \left\{ \frac{g_A^2 m_\pi^2}{8(4\pi)^2 f_\pi^4} [2\rho(\Gamma(\epsilon) + \ell_N + 1) + \Gamma(\epsilon) + \ell_N - 3 \ln \rho + 7 + \mathcal{R}_{B1}^c] \right\} = \text{Fig B2}$$

$$\text{Fig C} = -(3 - 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\rho(\Gamma(\epsilon) + \ell_N + 3) - \frac{1}{4}(\Gamma(\epsilon) + \ell_\pi + 1) + 4 \ln \rho - 3 - \frac{12\rho \tan^{-1} \sqrt{4\rho-1}}{\sqrt{4\rho-1}} + \mathcal{R}_C^c \right] \right\}$$

$$\text{Fig D} = -(3 + 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[3\rho \left(\Gamma(\epsilon) + \ell_N + \frac{1}{3} \right) + \frac{1}{4}(\Gamma(\epsilon) + \ell_\pi + 1) - 4 \ln \rho + 4 + \mathcal{R}_D^c \right] \right\}$$

$\langle NN | \mathcal{T} | N'N' \rangle$, decomposed

- Fully covariant formalism:

Fig A = *chiral*

$$\text{Fig B 1} = \langle \tau_1 \cdot \tau_2 \rangle \left\{ \frac{g_A^2 m_\pi^2}{8(4\pi)^2 f_\pi^4} [2\rho(\Gamma(\epsilon) + \ell_N + 1) + 4(\Gamma(\epsilon) + \ell_N + 7/4) - 3(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_{B1}^c] \right\} = \text{Fig B2}$$

$$\text{Fig C} = -(3 - 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\rho(\Gamma(\epsilon) + \ell_N + 3) - 4(\Gamma(\epsilon) + \ell_N + 13/16) - \frac{15}{4}(\Gamma(\epsilon) + \ell_\pi) - \frac{12\rho \tan^{-1} \sqrt{4\rho-1}}{\sqrt{4\rho-1}} + \mathcal{R}_C^c \right] \right\}$$

$$\text{Fig D} = -(3 + 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} [3\rho(\Gamma(\epsilon) + \ell_N + \frac{1}{3}) + 4(\Gamma(\epsilon) + \ell_N + 17/16) - \frac{15}{4}(\Gamma(\epsilon) + \ell_\pi) + \mathcal{R}_D^c] \right\}$$

$\langle NN | \mathcal{T} | N'N' \rangle$, subtracted

- Fully covariant formalism:

Fig A = *chiral*

$$\text{Fig B 1} = \langle \tau_1 \cdot \tau_2 \rangle \left\{ \frac{g_A^2 m_\pi^2}{8(4\pi)^2 f_\pi^4} [-3\ell_\pi + o(\rho^{-1/2})] \right\} = \text{Fig B2}$$

$$\text{Fig C} = -(3 - 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{15}{4} \ell_\pi - \frac{12\rho \tan^{-1} \sqrt{4\rho-1}}{\sqrt{4\rho-1}} + o(\rho^{-1/2}) \right] \right\}$$

$$\text{Fig D} = -(3 + 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[-\frac{15}{4} \ell_\pi + o(\rho^{-1/2}) \right] \right\}$$

$\langle NN | \mathcal{T} | N'N' \rangle$, primary

- Mixed formalism:

Fig A = chiral

$$\text{Fig B 1} = \langle \tau_1 \cdot \tau_2 \rangle \left\{ \frac{g_A^2 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{5}{8} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{11}{10} \right) - \frac{1}{2} (\Gamma(\epsilon) + \ell_\pi + 1) - \frac{5}{2} \ln \rho + 6 + \mathcal{R}_{B1}^{HBv} \right] \right\} = \text{Fig B2}$$

$$\text{Fig C} = -(3 - 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{45}{64} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{881}{270} \right) - \frac{133}{64} (\Gamma(\epsilon) + \ell_N) + \frac{15}{4} \ln \rho - \frac{3745}{384} - \frac{12\rho \tan^{-1} \sqrt{4\rho-1}}{\sqrt{4\rho-1}} + \mathcal{R}_C^{HBv} \right] \right\}$$

$$\text{Fig D} = -(3 + 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{417}{64} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{587}{834} \right) + \frac{197}{64} (\Gamma(\epsilon) + \ell_N) - \frac{15}{4} \ln \rho + \frac{1399}{128} + \mathcal{R}_D^{HBv} \right] \right\}$$

$\langle NN | \mathcal{T} | N'N' \rangle$, decomposed

- Mixed formalism:

Fig A = chiral

$$\text{Fig B 1} = \langle \tau_1 \cdot \tau_2 \rangle \left\{ \frac{g_A^2 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{5}{8} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{11}{10} \right) + \frac{5}{2} \left(\Gamma(\epsilon) + \ell_N + \frac{11}{5} \right) - 3 \left(\Gamma(\epsilon) + \ell_\pi \right) + \mathcal{R}_{B1}^{HBv} \right] \right\} = \text{Fig B2}$$

$$\text{Fig C} = -(3 - 2 \langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{45}{64} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{881}{270} \right) - 5 \frac{53}{64} \left(\Gamma(\epsilon) + \ell_N + \frac{3745}{2388} \right) + \frac{15}{4} \left(\Gamma(\epsilon) + \ell_\pi \right) - \frac{12\rho \tan^{-1} \sqrt{4\rho-1}}{\sqrt{4\rho-1}} + \mathcal{R}_C^{HBv} \right] \right\}$$

$$\text{Fig D} = -(3 + 2 \langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{417}{64} \rho \left(\Gamma(\epsilon) + \ell_N + \frac{587}{834} \right) + 6 \frac{53}{64} \left(\Gamma(\epsilon) + \ell_N + \frac{1399}{874} \right) - \frac{15}{4} \left(\Gamma(\epsilon) + \ell_\pi \right) + \mathcal{R}_D^{HBv} \right] \right\}$$

$\langle NN | \mathcal{T} | N'N' \rangle$, subtracted

- Mixed formalism:

Fig A = *chiral*

$$\text{Fig B 1} = \langle \tau_1 \cdot \tau_2 \rangle \left\{ \frac{g_A^2 m_\pi^2}{8(4\pi)^2 f_\pi^4} [-3\ell_\pi + o(\rho^{-1/2})] \right\} = \text{Fig B2}$$

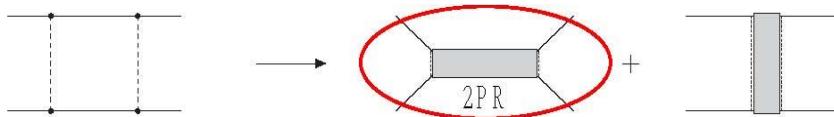
$$\text{Fig C} = -(3 - 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{15}{4} \ell_\pi - \frac{12\rho \tan^{-1} \sqrt{4\rho-1}}{\sqrt{4\rho-1}} + o(\rho^{-1/2}) \right] \right\}$$

$$\text{Fig D} = -(3 + 2\langle \tau_1 \cdot \tau_2 \rangle) \left\{ \frac{g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[-\frac{15}{4} \ell_\pi + o(\rho^{-1/2}) \right] \right\}$$

NN scattering: pionful to pionless mapping

- Dominant contribution to $C_0(\bar{\psi}\psi)^2$ from one-loop diagram ([JFY, ModPhysLettA29\(13\)1450043](#)):

The **anomalous** piece comes from the two-particle reducible (2PR) component in Fig C



$$\left\{ \frac{(3 - 2\langle \tau_1 \cdot \tau_2 \rangle)g_A^4 m_\pi^2}{8(4\pi)^2 f_\pi^4} \left[\frac{12\rho \tan^{-1} \sqrt{4\rho - 1}}{\sqrt{4\rho - 1}} \right] \approx \frac{3(3 - 2\langle \tau_1 \cdot \tau_2 \rangle)g_A^4 M_N m_\pi}{128\pi f_\pi^4} \right\}$$

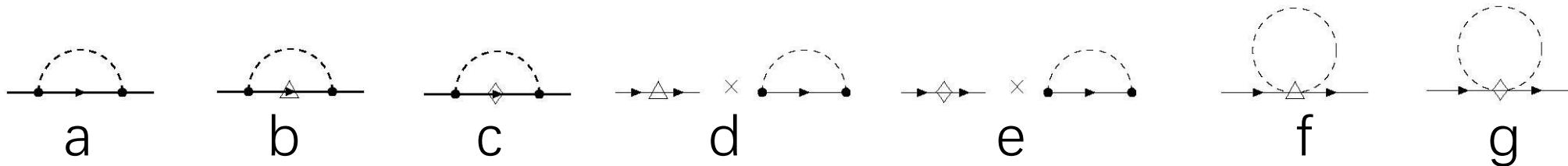
With parametrization

$$C_0 = \frac{4\pi}{M_N \Lambda} \Rightarrow \Lambda(\text{Fig C}) \approx \frac{512\pi^2 f_\pi^4}{9g_A^4 M_N^2 m_\pi} \approx \alpha m_\pi, \quad \alpha = \begin{cases} 0.97, g_A = 1.26 \\ 0.88, g_A = 1.29 \\ 0.80, g_A = 1.32 \end{cases}$$

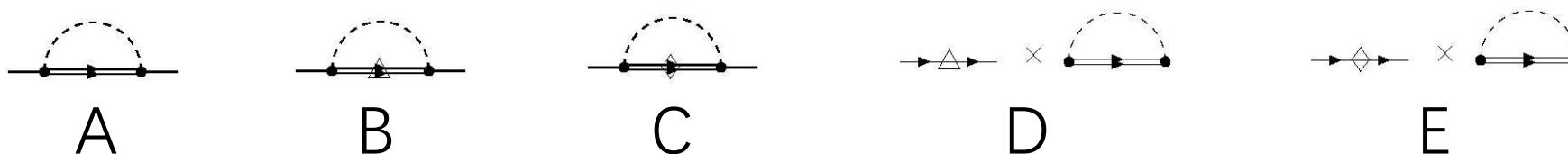
➤ Higher order such ‘anomalous’ contributions need to be computed and summed up

- One-loop examples in SU(3) situation (H.-F. Zhou, Z. Liu, J.-F. Yang):
 $\langle B | \mathcal{M}_B | B \rangle$

➤ Octet to octet



➤ Decuplet to octet



$\langle B | \mathcal{M}_B | B \rangle$, decomposed

- **O to O:** $\ell_\phi \equiv \ln \frac{4\pi\mu^2}{m_\phi^2}$, $\ell_B \equiv \ln \frac{4\pi\mu^2}{M_B^2}$, $\rho_B \equiv \frac{M_B^2}{m_\phi^2}$; $\mathcal{R}^{\dots} = o\left(\rho_B^{-1/2}\right)$

$$\text{Fig a} = \lambda_a \frac{1}{(4\pi F)^2} \left\{ 2M_B^3 [\Gamma(\epsilon) + \ell_B + 1] + 2M_B m_\phi^2 [\Gamma(\epsilon) + \ell_B + 2] + \mathcal{R}_a^c \right\}$$

$$\text{Fig b} = \lambda_b \frac{m_s m_\phi^2}{(4\pi F)^2} \left\{ 6\rho_B \left[\Gamma(\epsilon) + \ell_B + \frac{1}{3} \right] + 4 \left[\Gamma(\epsilon) + \ell_B + \frac{3}{4} \right] - 3(\Gamma(\epsilon) + \ell_\phi) + \mathcal{R}_b^c \right\}$$

$$\text{Fig c} = -\lambda_c \frac{m_s m_\phi^2}{(4\pi F)^2} \left\{ 6\rho_B \left[\Gamma(\epsilon) + \ell_B + \frac{1}{3} \right] + 4 \left[\Gamma(\epsilon) + \ell_B + \frac{3}{4} \right] - 3(\Gamma(\epsilon) + \ell_\phi) + \mathcal{R}_b^c \right\}$$

$$\text{Fig d} = \lambda_d \frac{m_s m_\phi^2}{(4\pi F)^2} \left\{ -2 \left[\Gamma(\epsilon) + \ell_B + \frac{3}{2} \right] + 3(\Gamma(\epsilon) + \ell_\phi) + \mathcal{R}_d^c \right\}$$

$$\text{Fig e} = \lambda_e \frac{m_s m_\phi^2}{(4\pi F)^2} \left\{ -2 \left[\Gamma(\epsilon) + \ell_B + \frac{3}{2} \right] + 3(\Gamma(\epsilon) + \ell_\phi) + \mathcal{R}_d^c \right\}$$

Fig f= *chiral*

Fig g= *chiral*

$\langle B | \mathcal{M}_B | B \rangle$, subtracted

- **O to O:** $\mathcal{R}_{\dots} = o\left(\rho_B^{-1/2}\right)$ (useful for extrapolation in meson masses!)

$$\text{Fig a} = \lambda_a \frac{1}{(4\pi F)^2} \{0 + \mathcal{R}_c^c\}$$

$$\text{Fig b} = \lambda_b \frac{m_s m_\phi^2}{(4\pi F)^2} \{-3\ell_\phi + \mathcal{R}_a^c\}$$

$$\text{Fig c} = -\lambda_c \frac{m_s m_\phi^2}{(4\pi F)^2} \{-3\ell_\phi + \mathcal{R}_a^c\}$$

$$\text{Fig d} = \lambda_d \frac{m_s m_\phi^2}{(4\pi F)^2} \{3\ell_\phi + \mathcal{R}_d^c\}$$

$$\text{Fig e} = \lambda_e \frac{m_s m_\phi^2}{(4\pi F)^2} \{3\ell_\phi + \mathcal{R}_d^c\}$$

Fig f= *chiral*

Fig g= *chiral*

$\langle B | \mathcal{M}_B | B \rangle$, decomposed

- D to O:** $\eta_B \equiv \frac{M_B}{M_T}$, $\Omega \equiv \frac{M_B^2 - M_T^2 - m_\phi^2}{2m_\phi M_B} = \frac{\rho_B(1 - \eta_B^{-1}) - 1}{2\sqrt{\rho_B}}$, $\mathcal{R}_{\dots} = o(\rho_B^{-1/2})$

$$\text{Fig A} = \tilde{\lambda}_A \frac{1}{24(4\pi F)^2} \left\{ M_T^3 [(-\eta_B^{-3} - 2\eta_B^{-2} + 2\eta_B^{-1} + 6 + 6\eta_B^2 + 2\eta_B^3 - 2\eta_B^4 - \eta_B^5)(\Gamma(\epsilon) + \ell_B) + (-2\eta_B^{-1} + 44 + 51\eta_B - 4\eta_B^2 - 10\eta_B^3 - 4\eta_B^5)] + 2M_T m_\phi^2 [(3\eta_B^2 + 2\eta_B^3 + 2\eta_B^{-3} + 3\eta_B^{-2})(\Gamma(\epsilon) + \ell_B) + (3\eta_B^{-1} + 28 + 14\eta_B - 2\eta_B^2 + 7\eta_B^3)] + [M_T^3 (\eta_B^{-3} + 2\eta_B^{-2} - 2\eta_B^{-1} - 6 + 6\eta_B^2 + 2\eta_B^3 - 2\eta_B^4 - \eta_B^5) + 2M_T m_\phi^2 (-2\eta_B^{-3} - 3\eta_B^{-2} + 3\eta_B^2 + 2\eta_B^3)] (\Gamma(\epsilon) + \ell_\phi) + 4[M_T^2 m_\phi (\eta_B^{-3} + 2\eta_B^{-2} - \eta_B^{-1} - 4 - \eta_B + 2\eta_B^2 + \eta_B^3) - m_\phi^3 (3\eta_B^{-3} + 4\eta_B^{-2} + 2\eta_B^{-1} + 4 + 3\eta_B)] \sqrt{1 - \Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_A^c \right\}$$

$$\text{Fig B} = -\tilde{\lambda}_B \frac{m_s}{36(4\pi F)^2} \left\{ M_T^2 [3(3\eta_B^{-3} + 5\eta_B^{-2} - 4\eta_B^{-1} - 9 - 3\eta_B^2 - \eta_B^4 - \eta_B^5)(\Gamma(\epsilon) + \ell_B) + (-3\eta_B^{-1} + 269 + 441\eta_B - 26\eta_B^2 - 174\eta_B^3 - 12\eta_B^5)] + m_\phi^2 [3(-8\eta_B^2 - 9\eta_B^3 + 3\eta_B^{-3} + 4\eta_B^{-2})(\Gamma(\epsilon) + \ell_B) + 6(3\eta_B^{-1} + 50 + 50\eta_B - \eta_B^2 + 2\eta_B^3)] + 3[M_T^2 (-3\eta_B^{-3} - 5\eta_B^{-2} + 4\eta_B^{-1} + 9 - 3\eta_B^2 - \eta_B^4 - \eta_B^5) + m_\phi^2 (8\eta_B^{-3} + 9\eta_B^{-2} + 3\eta_B^2 + 4\eta_B^3)] (\Gamma(\epsilon) + \ell_\phi) + 12[M_T m_\phi (-3\eta_B^{-3} - 5\eta_B^{-2} + \eta_B^{-1} + 4 + \eta_B + \eta_B^2 + \eta_B^3)] \sqrt{1 - \Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_B^c \right\}$$

$$\text{Fig C} = -\tilde{\lambda}_C \frac{m_s}{36(4\pi F)^2} \left\{ M_T^2 [3(3\eta_B^{-3} + 5\eta_B^{-2} - 4\eta_B^{-1} - 9 - 3\eta_B^2 - x\eta_B^4 - \eta_B^5)(\Gamma(\epsilon) + \ell_B) + (-3\eta_B^{-1} + 269 + 441\eta_B - 26\eta_B^2 - 174\eta_B^3 - 12\eta_B^5)] + m_\phi^2 [3(-8\eta_B^2 - 9\eta_B^3 + 3\eta_B^{-3} + 4\eta_B^{-2})(\Gamma(\epsilon) + \ell_B) + 6(3\eta_B^{-1} + 50 + 50\eta_B - \eta_B^2 + 2\eta_B^3)] + 3[M_T^2 (-3\eta_B^{-3} - 5\eta_B^{-2} + 4\eta_B^{-1} + 9 - 3\eta_B^2 - \eta_B^4 - \eta_B^5) + m_\phi^2 (8\eta_B^{-3} + 9\eta_B^{-2} + 3\eta_B^2 + 4\eta_B^3)] (\Gamma(\epsilon) + \ell_\phi) + 12[M_T m_\phi (-3\eta_B^{-3} - 5\eta_B^{-2} + \eta_B^{-1} + 4 + \eta_B + \eta_B^2 + \eta_B^3)] \sqrt{1 - \Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_B^c \right\}$$

$$\text{Fig D} = \tilde{\lambda}_D \frac{m_s}{36(4\pi F)^2} \left\{ M_T^2 [(3\eta_B^{-4} + 4\eta_B^{-3} - 2\eta_B^{-2} + 12\eta_B + 6\eta_B^2 - 8\eta_B^3 - 5\eta_B^4)(\Gamma(\epsilon) + \ell_B) + (2\eta_B^{-2} + 8\eta_B^{-1} - 5 - 8\eta_B + 2\eta_B^2 + 8\eta_B^3 + 4\eta_B^4)] + 2m_\phi^2 [-9\eta_B^{-2} - 8\eta_B^{-1} - 2 + 12\eta_B + 15\eta_B^2] + [M_T^2 (-3\eta_B^{-4} - 4\eta_B^{-3} + 2\eta_B^{-2} + 12\eta_B + 6\eta_B^2 - 8\eta_B^3 - 5\eta_B^4) + 24m_\phi^2 (\eta_B + \eta_B^2)] (\Gamma(\epsilon) + \ell_\phi) + 4M_T m_\phi (-3\eta_B^{-4} - 4\eta_B^{-3} + \eta_B^{-2} - 1 + 4\eta_B + 3\eta_B^2) \sqrt{1 - \Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_D^c \right\}$$

$$\text{Fig E} = \tilde{\lambda}_D \frac{m_s}{36(4\pi F)^2} \left\{ M_T^2 [(3\eta_B^{-4} + 4\eta_B^{-3} - 2\eta_B^{-2} + 12\eta_B + 6\eta_B^2 - 8\eta_B^3 - 5\eta_B^4)(\Gamma(\epsilon) + \ell_B) + (2\eta_B^{-2} + 8\eta_B^{-1} - 5 - 8\eta_B + 2\eta_B^2 + 8\eta_B^3 + 4\eta_B^4)] + 2m_\phi^2 [-9\eta_B^{-2} - 8\eta_B^{-1} - 2 + 12\eta_B + 15\eta_B^2] + [M_T^2 (-3\eta_B^{-4} - 4\eta_B^{-3} + 2\eta_B^{-2} + 12\eta_B + 6\eta_B^2 - 8\eta_B^3 - 5\eta_B^4) + 24m_\phi^2 (\eta_B + \eta_B^2)] (\Gamma(\epsilon) + \ell_\phi) + 4M_T m_\phi (-3\eta_B^{-4} - 4\eta_B^{-3} + \eta_B^{-2} - 1 + 4\eta_B + 3\eta_B^2) \sqrt{1 - \Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_D^c \right\}$$

$\langle B | \mathcal{M}_B | B \rangle$, subtracted

- **D to O:** $\xi_{BT2}(\eta_B) = o(\rho_B^{-1})$, $\xi_{BT1\phi}(\eta_B) = o\left(\rho_B^{-1/2}\right)$; $\xi_{AT3}(\eta_B) = o\left(\rho_B^{-3/2}\right)$, $\xi_{AT1\phi}(\eta_B) = o\left(\rho_B^{-1/2}\right)$, $\xi_{AT2\phi}(\eta_B) = o(\rho_B^{-1})$; $\xi_{DT2}(\eta_B) = o(\rho_B^{-1})$, $\xi_{DT1\phi}(\eta_B) = o\left(\rho_B^{-1/2}\right)$; $a_{...} = b_{...} = 0$ in HB formulation

$$\begin{aligned} \text{Fig A} &= \tilde{\lambda}_A \frac{1}{24(4\pi\tilde{f})^2} \left\{ [M_T^3 \xi_{AT3}(\eta_B) + 2M_T m_\phi^2 \xi_{AT1\phi}(\eta_B)] \ell_\phi + 4[M_T m_\phi^2 \xi_{AT\phi 2}(\eta_B) - m_\phi^3 (3\eta_B^{-3} + 4\eta_B^{-2} + 2\eta_B^{-1} + 4 + 3\eta_B)] \sqrt{1-\Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_A^c \right\} \\ &= \tilde{\lambda}_A \frac{m_\phi^3}{(4\pi\tilde{f})^2} \left\{ -\frac{4}{3}\pi + a_A \ell_\phi + b_A + \frac{1}{24} \mathcal{R}_A^c \right\} \end{aligned}$$

$$\begin{aligned} \text{Fig B} &= -\tilde{\lambda}_B \frac{m_s}{36(4\pi\tilde{f})^2} \left\{ 3[M_T^2 \xi_{BT2}(\eta_B) + m_\phi^2 (8\eta_B^{-3} + 9\eta_B^{-2} + 3\eta_B^{-2} + 4\eta_B^3)] (\ell_\phi) + 12M_T m_\phi \xi_{BT1\phi}(\eta_B) \sqrt{1-\Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_B^c \right\} \\ &= -\tilde{\lambda}_B \frac{m_s m_\phi^2}{(4\pi\tilde{f})^2} \left\{ (2 + a_B) \ell_\phi + b_B + \frac{1}{36} \mathcal{R}_B^c \right\} \end{aligned}$$

$$\begin{aligned} \text{Fig C} &= -\tilde{\lambda}_C \frac{m_s}{36(4\pi\tilde{f})^2} \left\{ 3[M_T^2 \xi_{BT2}(\eta_B) + m_\phi^2 (8\eta_B^{-3} + 9\eta_B^{-2} + 3\eta_B^{-2} + 4\eta_B^3)] (\ell_\phi) + 12M_T m_\phi \xi_{BT1\phi}(\eta_B) \sqrt{1-\Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_B^c \right\} \\ &= -\tilde{\lambda}_C \frac{m_s m_\phi^2}{(4\pi\tilde{f})^2} \left\{ (2 + a_B) \ell_\phi + b_B + \frac{1}{36} \mathcal{R}_B^c \right\} \end{aligned}$$

$$\text{Fig D} = \tilde{\lambda}_D \frac{m_s}{24(4\pi\tilde{f})^2} \left\{ [M_T^2 \xi_{DT2}(\eta_B) + 24m_\phi^2 (\eta_B + \eta_B^2)] (\ell_\phi) + 4[M_T m_\phi \xi_{DT1\phi}(\eta_B)] \sqrt{1-\Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_D^c \right\} = \tilde{\lambda}_D \frac{m_s m_\phi^2}{(4\pi\tilde{f})^2} \left\{ (2 + a_D) \ell_\phi + b_D + \frac{1}{24} \mathcal{R}_D^c \right\}$$

$$\text{Fig E} = \tilde{\lambda}_E \frac{m_s}{24(4\pi\tilde{f})^2} \left\{ [M_T^2 \xi_{DT2}(\eta_B) + 24m_\phi^2 (\eta_B + \eta_B^2)] (\ell_\phi) + 4[M_T m_\phi \xi_{DT1\phi}(\eta_B)] \sqrt{1-\Omega^2} \cos^{-1}(-\Omega) + \mathcal{R}_D^c \right\} = \tilde{\lambda}_E \frac{m_s m_\phi^2}{(4\pi\tilde{f})^2} \left\{ (2 + a_D) \ell_\phi + b_D + \frac{1}{24} \mathcal{R}_D^c \right\}$$

$\langle B | \mathcal{M}_B | B \rangle$, comparison to HB calculation

- GMO relations:

$$\frac{1}{4}(\mathcal{M}_\Lambda + 3\mathcal{M}_\Sigma - 2\mathcal{M}_N - 2\mathcal{M}_\Xi) = 0$$

➤ Jenkins(NuclPhys**B**368(92)190):

$$\begin{aligned} & \frac{1}{4}(\mathcal{M}_\Lambda + 3\mathcal{M}_\Sigma - 2\mathcal{M}_N - 2\mathcal{M}_\Xi) \\ &= \left(1 - \frac{2}{\sqrt{3}}\right) \left[\frac{2}{3}(D^2 - 3F^2) - \frac{1}{9}\mathcal{C}^2 \right] \frac{m_K^3}{16\pi f^2} - \left[\frac{16}{3}b_D D^2 + \left(\frac{2}{3}b_D + \frac{5}{3}b_F + \frac{5}{9}c \right)\mathcal{C}^2 \right] \frac{m_s m_K^3}{(4\pi f)^2} \ln \frac{m_K^2}{4\pi\mu^2} \end{aligned}$$

➤ Cov EFT: $\tilde{f}^2 = 2f^2$, $b_1 = b_D + b_F$, $b_2 = b_D - b_F$

$$\begin{aligned} & \frac{1}{4}(\mathcal{M}_\Lambda + 3\mathcal{M}_\Sigma - 2\mathcal{M}_N - 2\mathcal{M}_\Xi) \\ &= \left(1 - \frac{2}{\sqrt{3}}\right) \left[\frac{4}{3}(D^2 - 3F^2) - \frac{2}{9}\mathcal{C}^2 \right] \frac{m_K^3}{16\pi \tilde{f}^2} - \left[\frac{32}{3}b_D D^2 + \left(\frac{4}{3}b_D + \frac{10}{3}b_F + \frac{10}{9}c \right)\mathcal{C}^2 \right] \frac{m_s m_K^3}{(4\pi \tilde{f})^2} \ln \frac{m_K^2}{4\pi\mu^2} + \Delta(\rho_B) \\ & \quad \lim_{HB\ limit} \Delta(\rho_B) = 0 \end{aligned}$$

Summary and prospects

Prescription: Covariant Chiral EFT--@each divergent loop with baryonic lines

- χPC is restored after the large (non-chiral) and local pieces are isolated and subtracted, the residuals are characterized by the chiral scale $Q \sim m_\phi$.
 - **But,** there may be nonlocal/definite ‘anomalous’ terms associated with threshold effects/IR enhancement, which must be kept and summed up somehow via relativistic propagators.
- To be examined and applied to more topics in hadronic and nucleonic systems.

谢谢！