Hamiltonian Effective Field Theory (HEFT) in Elongated or Moving Finite Volume

Speaker: Yan Li

Collaborators: Jia-Jun Wu, Derek B. Leinweber, Anthony W. Thomas

PRD 103, 094518 (2021)

Experiment vs Lattice QCD

Two particle system

- Experiment: Infinite volume
  - Two particle scattering
  - Scattering phase shifts $\delta_l(E)$

- Lattice QCD: Finite periodic box (with length L)
  - Two particle bound states
  - Finite-volume energy levels $E_n(L)$

$$\langle\Omega|\,\chi(t)\chi^{\dagger}(0)\,|\Omega\rangle = \sum_n C_n e^{-E_n t}$$

What is HEFT

- Not itself an EFT, but an extension of Chiral EFT

$$\begin{tikzcd} \text{Chiral EFT} \arrow[r, Rightarrow] & H \arrow[r, Leftrightarrow] \arrow[d, Rightarrow] & \text{Experiment} \\ & H_L \arrow[r, Leftrightarrow] & \text{Lattice} \end{tikzcd}$$

H: Infinite-volume Hamiltonian

Experiment: $\delta_l(E)$

H_L: Finite-volume Hamiltonian

Lattice: $E_n(L)$

Why HEFT

M. Lüscher:
Commun. Math. Phys., 105:153-188, 1986.
Commun. Math. Phys., 104:177, 1986.
Nucl. Phys., B354:531-578, 1991.

Lüscher's Method: model-independent

still needs parametrization because of limited data

HEFT:

- Parametrization respecting Chiral EFT, incorporating quark-mass dependence.
- Hamiltonian has a clear physical picture
- Composition of finite-volume eigenstates

$$\langle\Omega|\,\chi(t)\chi^{\dagger}(0)\,|\Omega\rangle = \sum_n C_n e^{-E_n t}$$

qualitative information from Lattice simulation

$$\Lambda(1405):\ uds, \pi\Sigma, \bar{K}N$$

PRL 114, 132002 (2015)

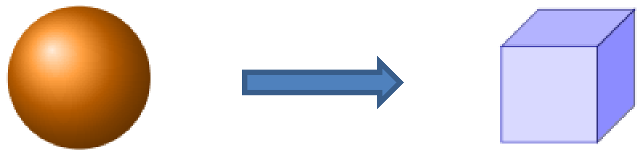
周一下午的报告 by 杨智

HEFT 应用到$c\bar{s}$体系

Previous Work

PRD 101, 114501 (2020)

Partial-wave mixing in rest-frame cube



E.g.: s- and g-waves are mixed in finite volume

Infinite volume:

$$\int d\Omega_{\hat{\mathbf{k}}} Y^*_{l'm'}(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{k}}) = \delta_{l',l}\delta_{m',m}$$

Finite volume:

$$4\pi \sum_{|\mathbf{n}|^2=N} Y^*_{l'm'}(\hat{\mathbf{n}}) Y_{lm}(\hat{\mathbf{n}})$$

Reduction of matrix dimension: ~60000 -> ~1000

Current Work

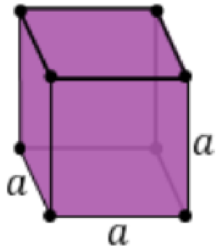
- From H in rest-frame infinite-volume, how to get H_L in moving-frame elongated cube

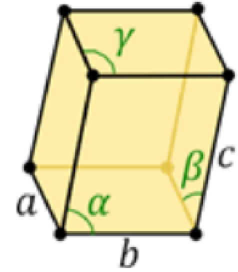
- Partial-wave mixing in elongated or moving finite volume

Previous and current works can be used not only in HEFT, but also in any Hamiltonian formalism

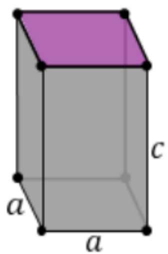
Parallelepiped and elongated cube

Cube

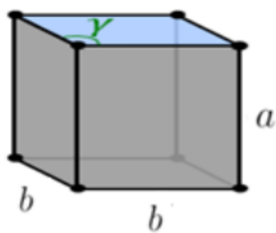




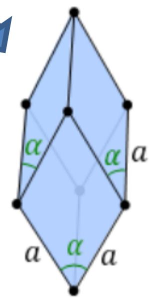
Parallelepiped



(0,0,1)



(0,1,1)



(1,1,1)

Elongated cube

$$M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$k_i^* = \sum_j M_{ij}^{-1} \frac{2\pi}{L} n_j$$

Moving frame

$$\int \frac{d^3\mathbf{k}_\mathrm{r}}{(2\pi)^3} \to L^{-3}\sum_{\mathbf{k}_\mathrm{r}}$$

H $\longrightarrow$ H_L in rest frame

?

H_L in moving frame

$$\int \frac{d^3\mathbf{k}_\mathrm{r}}{(2\pi)^3} \to \int \frac{d^3\mathbf{k}_\mathrm{m}}{(2\pi)^3}\left|\frac{\partial \mathbf{k}_\mathrm{r}}{\partial \mathbf{k}_\mathrm{m}}\right|$$

$$\int \frac{d^3\mathbf{k}_\mathrm{m}}{(2\pi)^3} \to L^{-3}\sum_{\mathbf{k}_\mathrm{m}}$$

Elongated moving system

Case	$\mathbf{d}_\eta$	η	$\mathbf{d}_\gamma$	$m_1 = m_2$?	Degenerate shell
A	any	$=1$	0	Any	$\mathbf{n}^2$
B	$\mathbf{d} \neq \mathbf{0}$	Any	$\mathbf{d} \neq \mathbf{0}$	No	$(\mathbf{n}^2, (\mathbf{d}-\mathbf{n})^2)$ or $(\mathbf{n}^2, \mathbf{n}\cdot\mathbf{d})$
C1	$\mathbf{d} \neq \mathbf{0}$	$\neq 1$	0	Any	$(\mathbf{n}^2, \lvert\mathbf{n}\cdot\mathbf{d}\rvert)$
C2	$\mathbf{d} \neq \mathbf{0}$	Any	$\mathbf{d} \neq \mathbf{0}$	Yes	$\{\mathbf{n}^2, (\mathbf{d}-\mathbf{n})^2\}$

$\boldsymbol{d}_\eta$: elongated direction η: elongated magnitude
$\boldsymbol{d}_\gamma$: moving direction

Degenerate shell: degenerate eigenstates of H_0

Example: Isospin-2 ππ system

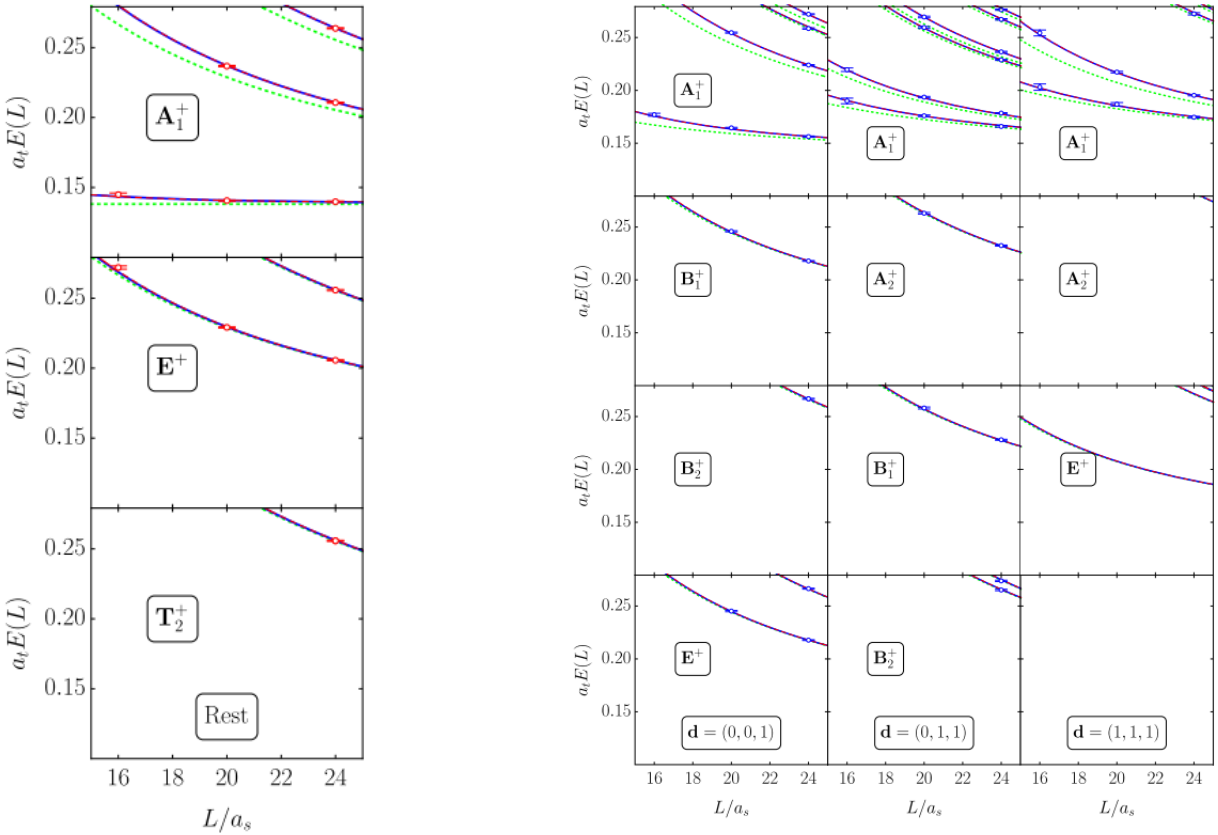
- Partial-wave cut: s-, d- and g-waves
- Parametrization: separable potentials

Dimension of Hamiltonian matrix

Case	$N_{cut} = 100$ **~ 4GeV**	$N_{cut} = 600$ **~ 10GeV**
Before reduction	~4,000	~60,000
Rest cube	~100	~1,000
Elongated or moving	~500	~5,000

Example: Isospin-2 ππ system

Data from PRD 83:071504,2011



Data: 11(Rest)+38(Moving)=49

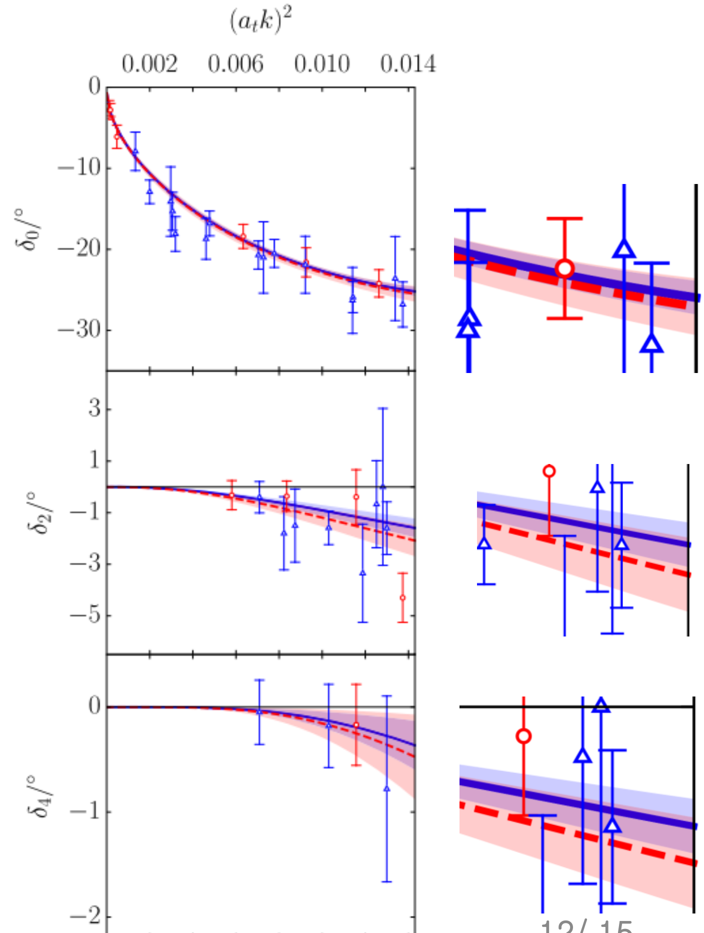
Example: Isospin-2 ππ system

Red: rest-frame only

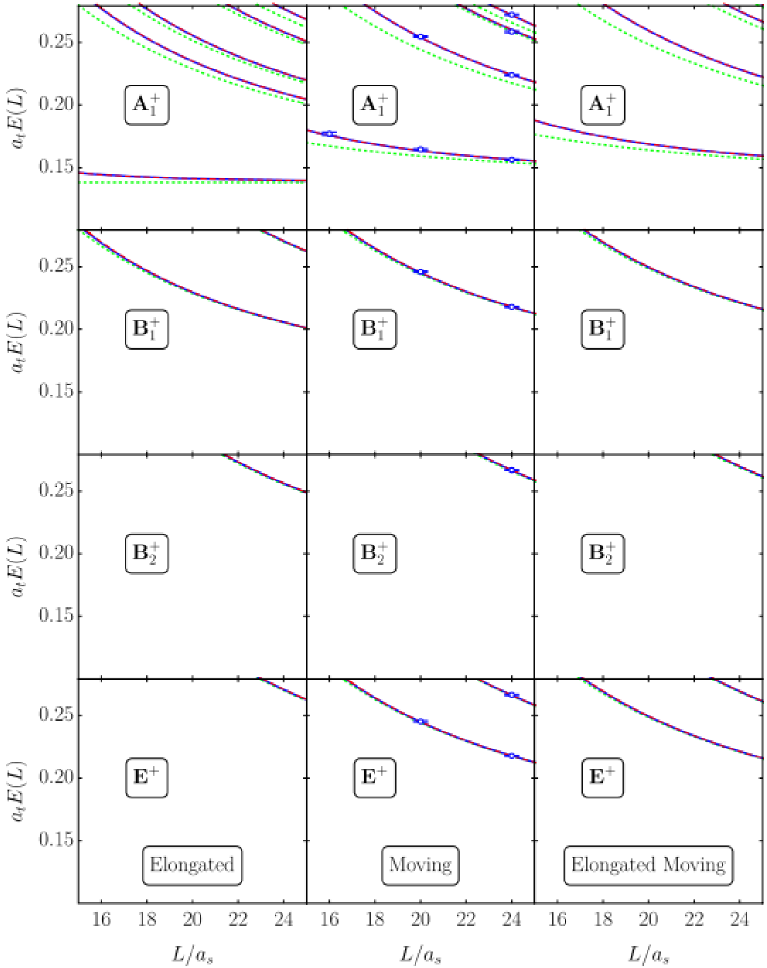
Blue: rest & moving

After including the moving

- Central values shift slightly
- Error bands improved a lot



Example: Isospin-2 ππ system



Summary

- HEFT is now ready for all two-particle system

Outlook

- More applications
- Three-particle system

THANKS

中国科学院大学
University of Chinese Academy of Sciences