Systematic study of $\eta_c \to 2\gamma$ from Lattice QCD

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Outline

- Overviews: $\eta_c \to 2\gamma$
  - Experiment & PDG
  - Perturbative and nonperturbative results
- The new method on lattice
- Result and discussion
- Conclusion

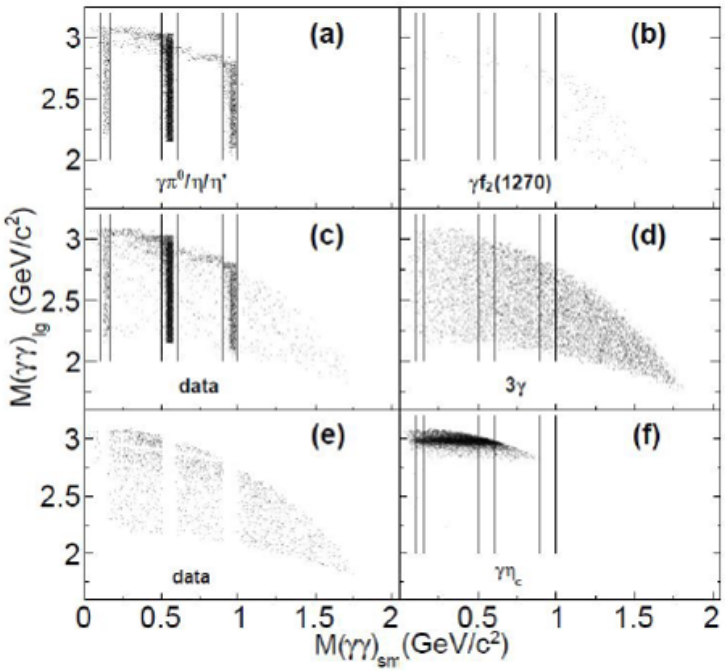
Motivation

- Perturbative and nonperturbative QCD are appliable for $\eta_c \to 2\gamma$
- $\eta_c \to 2\gamma$ provides an ideal ground to test the validity limit of various approximations and techniques e.g.
  - NRQCD
  - DSE
  - Lattice method
  - $\cdots$

$\eta_c \to 2\gamma$: experiments

Exp	$\mathcal{B}r \times 10^4$	Refs
CLEO	$1.4^{+0.7}_{-0.5} \pm 0.3$	PRL 101,101801(2008)
BESIII	$2.7 \pm 0.8 \pm 0.6$	PRD 87,032003(2013)
World average	$1.9^{+0.7}_{-0.6}$	PDG(2021)
Global fit	1.61 ± 0.12	PDG(2021)

- Difficulties: Multi-intermediate state contamination
- Opportunities: As the largest $\tau-$charm factory, BESIII has accumulated 10^{10} J/ψ events.



$J/\psi \to 3\gamma$ from BESIII, $\sim 3 \times 10^7\, J/\psi$

$\eta_c \to 2\gamma$: theories

- Smaller results on lattice

  J. J. Dudek *et al.* Phys. Rev. Lett.**97**,172001(2006)

  T. Chen *et al.* Eur. Phys. J. C **76**, 358 (2016)

  C. Liu *et al.* Phys. Rev. D.**102**,034502(2020)

- Other bigger results:

  NRQCD F. Feng,Y. Jia,W.-L. Sang Phys.Rev.Lett.**119**,252001(2017)

  DSE J. Chen,D.-H. Ding,L. Chang,Y.-X. Liu,Phys.Rev.D.**95**,016010(2017)

We firstly include various systematic errors at the same time

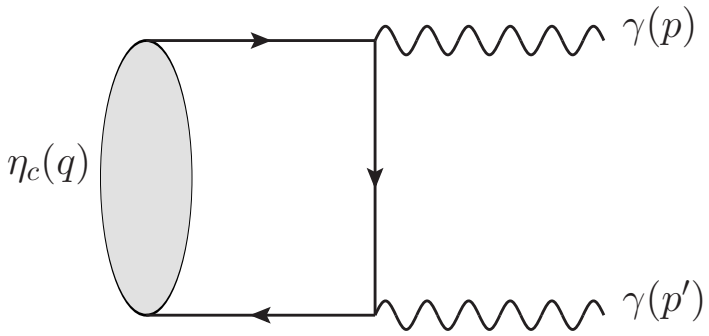
→ lattice discretization effect

→ excited-stated effect

→ mass scaling effect

...

$\eta_c \to 2\gamma$



- Amplitude:

$$\begin{aligned} \mathcal{M} &= e^2 \epsilon_\mu(p)\epsilon_\nu(p') H_{\mu\nu}(p,q) \\ H_{\mu\nu}(p,q) &= \int d^4x e^{-ipx} \mathcal{H}_{\mu\nu}(x,q), \quad \mathcal{H}_{\mu\nu} = \langle 0|\mathrm{Tr}[J_\mu(x)J_\nu(0)]|\eta_c(q)\rangle \end{aligned}$$

- Form factor: $$H_{\mu\nu}(p,q) = \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta F(p^2)$$

- Decay width:

$$\Gamma_{\eta_c\gamma\gamma} = \alpha_{\mathrm{em}}^2 \frac{\pi}{4} m_{\eta_c}^3 |F_{\eta_c\gamma\gamma}|^2, \quad F_{\eta_c\gamma\gamma} = F(0)$$

Traditional method for on-shell form factor

- Off-shell form factors by projecting discrete momentum $\vec{p} = \frac{2\pi\vec{n}}{L}$

$$F(p^2) \xrightarrow{\text{Cont.Limit}} F(0)$$

  - Leading to additional computation cost and systematic source.

J. J. Dudek *et al.* Phys.Rev.Lett.**97**,172001(2006)

Chuan Liu *et al.* Phys.Rev.D.**102**,034502(2020)

- Without considering various systematic effects
  - Lattice discretization effect: only one or two lattice spacings used
  - Excited-state contamination: without considering the excited-state effect on the lattice

Direct approach to on-shell form factor

- Construct a scalar function[infinite volume]

$$\begin{aligned}\mathcal{I} \equiv & \quad \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta H_{\mu\nu}(p,q) \\ = & \quad m_{\eta_c} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{\boldsymbol{x}} e^{-i\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{x}}} \epsilon_{\mu\nu\alpha 0} \frac{\partial \mathcal{H}_{\mu\nu}(x,q)}{\partial x_\alpha}\end{aligned}$$

- Averaging the $\vec{p}$ directions and projecting $|\vec{p}| = m_{\eta_c}/2$[infinite volume]

$$\begin{aligned}F_{\eta_c\gamma\gamma} \quad \equiv & \quad \frac{\mathcal{I}}{[\epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta][\epsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma]} \\ = & \quad -\frac{1}{2m_{\eta_c}} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{\boldsymbol{x}} \frac{j_1(|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|)}{|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x,q)\end{aligned}$$

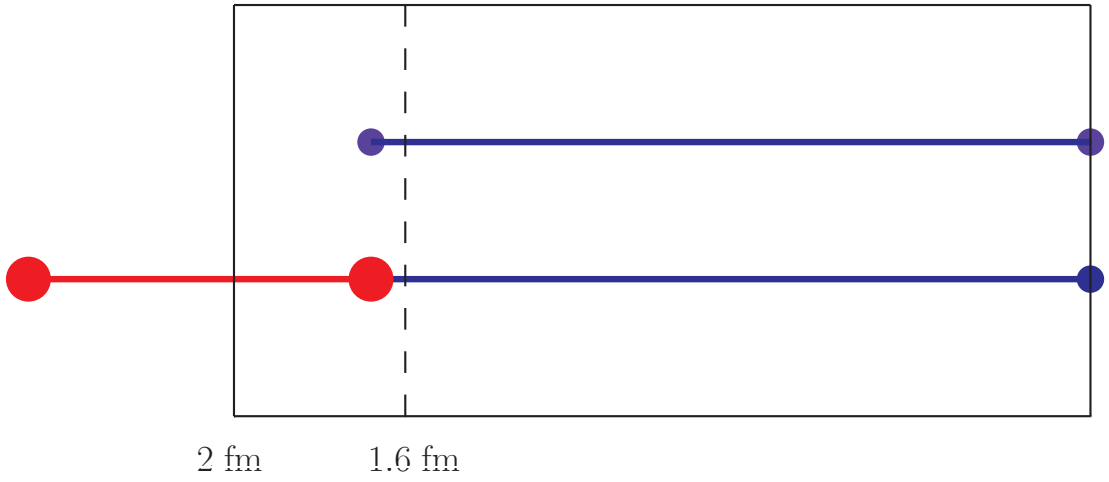
- Replaced by finite-volume version

$$\epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x,q) \to \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^L_{\mu\nu}(x,q)$$

  - Exponentially supressed with the distance

Excited-state effect

- Two-point function: $\langle \mathcal{O}_{\eta_c}(t)\mathcal{O}^\dagger_{\eta_c}(0)\rangle \xRightarrow{t\gtrsim 1.6\mathrm{fm}}$ ground state η_c
- Three-point function: $\langle J_\mu(t)J_\nu(0)\mathcal{O}^\dagger_{\eta_c}(-\Delta t)\rangle \xRightarrow[\Delta t>1.6\mathrm{fm}]{t\gtrsim 0.8\mathrm{fm}} \eta_c \to 2\gamma$
  - Large Δt, t is not enough to obtain the signal
  - Small Δt, excited-state contamination



Extraction of η_c ground-state contribution

- Ground and first excited states dominate

$$\langle J_\mu(\vec{x},t)J_\nu(0)\mathcal{O}^\dagger_{\eta_c}(-\Delta t)\rangle = \frac{Z_0}{2m}\mathcal{H}^{(0)}_{\mu\nu}e^{-m\Delta t} + \frac{Z_1}{2m_1}\mathcal{H}^{(1)}_{\mu\nu}e^{-m_1\Delta t}$$

$$\mathcal{H}_{\mu\nu} = \mathcal{H}^{(0)}_{\mu\nu} + \frac{Z_1 m}{Z_0 m_1}\mathcal{H}^{(1)}_{\mu\nu} \times e^{-(m_1-m)\Delta t}$$

- Extract the ground-state form factor

$$F_{\eta_c\gamma\gamma} = F^{(0)}_{\eta_c\gamma\gamma} + C_F \times e^{-(m_1-m)\Delta t}$$

- Extract mass of ground and first excited state

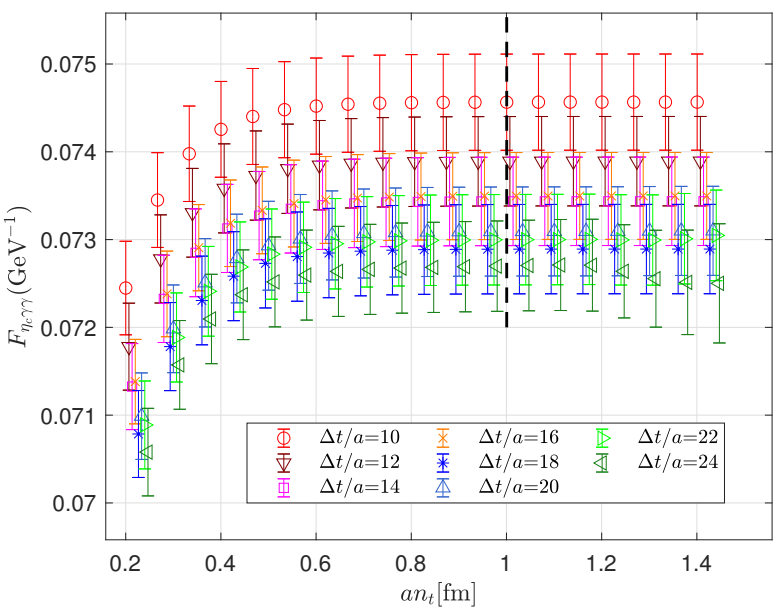
$$\langle \mathcal{O}_{\eta_c}(t)\mathcal{O}^\dagger_{\eta_c}(0)\rangle = A_1\left(e^{-mt} + e^{-m(T-t)}\right) + A_2\left(e^{-m_1 t} + e^{-m_1(T-t)}\right), m_1 \geq m$$

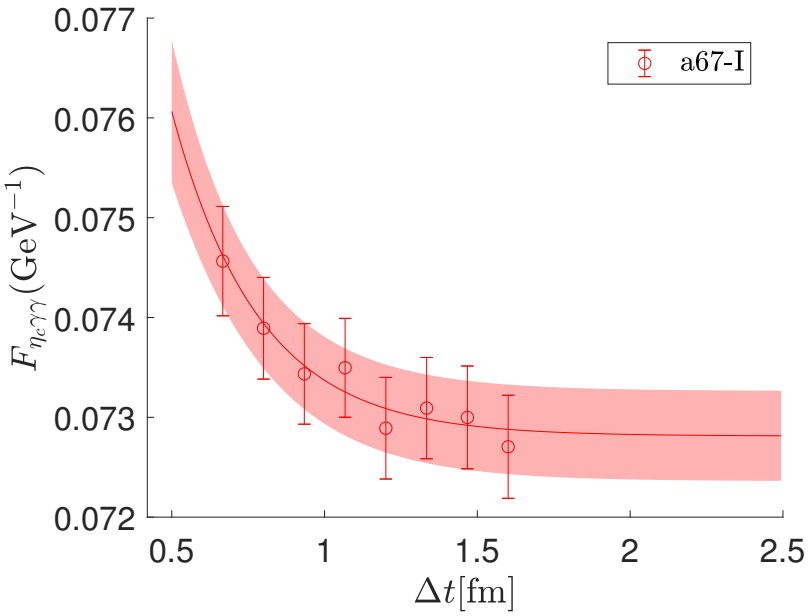
$N_f = 2$ twisted-mass

Ens	a(fm)	V/a^4	$a\mu_{sea}$	$N_{\rm conf} \times T_s$	$\Delta t/a$
a98	0.098	$24^3 \times 48$	0.0060	235×48	7:1:17
a85	0.085	$24^3 \times 48$	0.0040	197×48	8:1:18
a67	0.0667	$32^3 \times 64$	0.0030	197×64	10:2:24

- All ensembles have similar physical spatial volume: $2.04 \sim 2.35$ fm.
- All ensembles have similar pion mass: $300 \sim 360$ MeV.
- A series of Δt to extract the ground-state contribution for $\eta_c \to 2\gamma$.
- Charm quark mass is tuned by physical η_c and J/ψ mass, respectively, and take the difference as our systematic error.

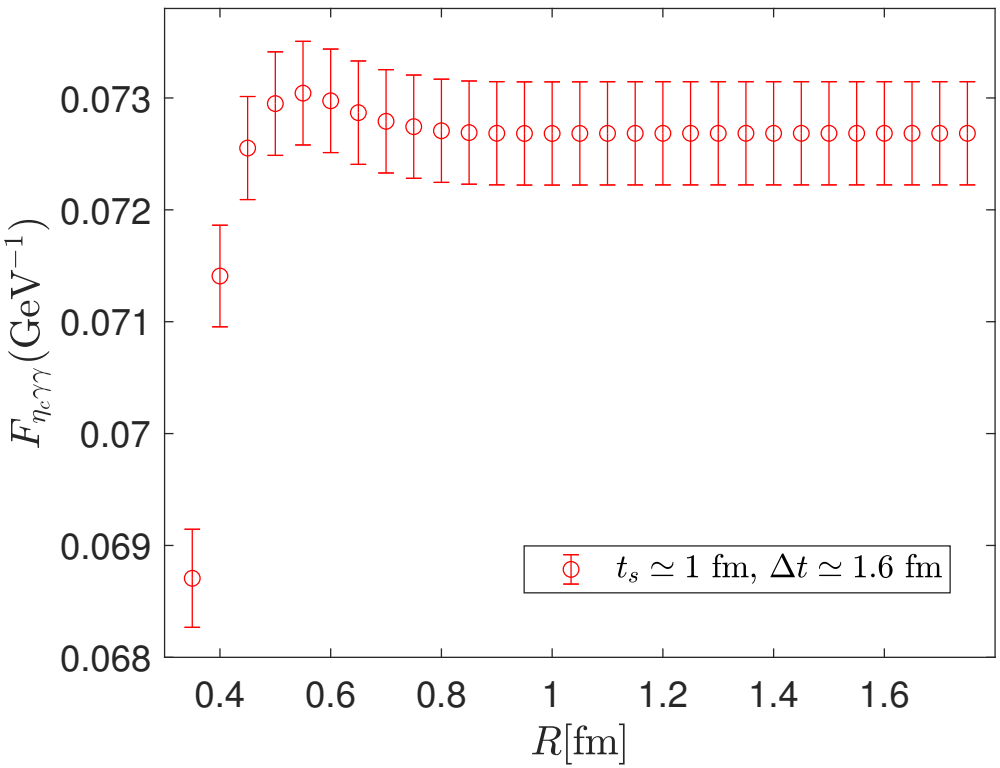
Form factor on lattice





- An obvious dependene on excited-state of η_c.
- Dashed black line: a suitable time truncation $t_{\mathrm{cut}} \sim 1$ fm.
- The right: an extrapolation for the form factors at t_{cut}.

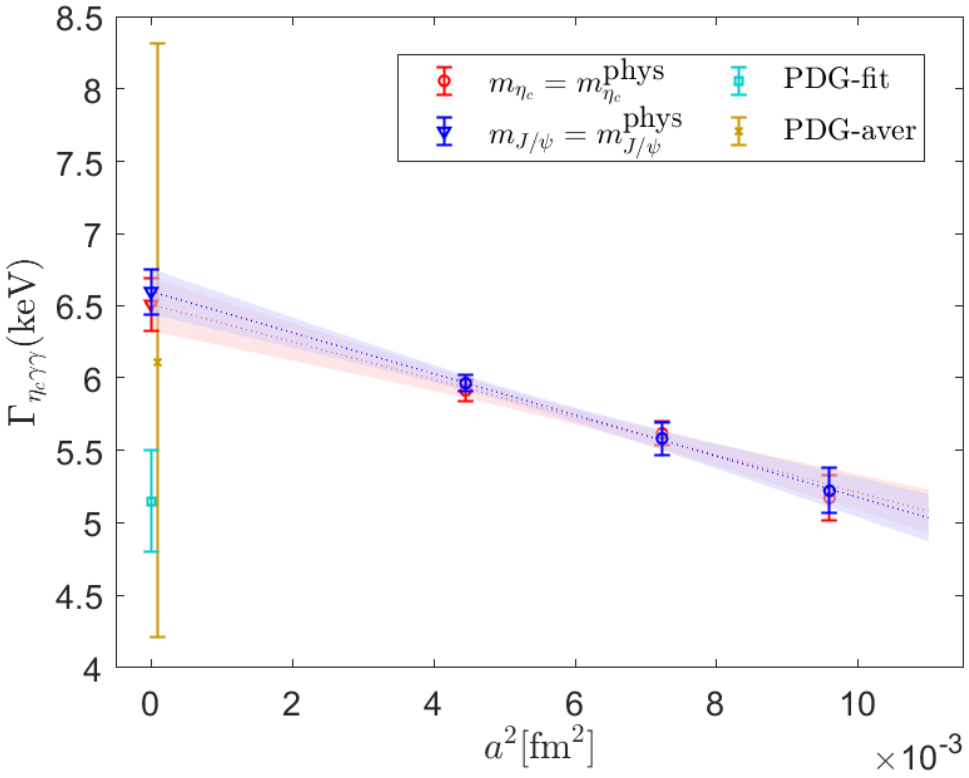
Finite volume effect



- a67: R-dependence of form factor on spatial summation with $|\vec{x}| \leq R$ for $t_{\mathrm{cut}} \simeq 1$ fm and $\Delta t \simeq 1.6$ fm, take a67-l as an example.

Continuous limit

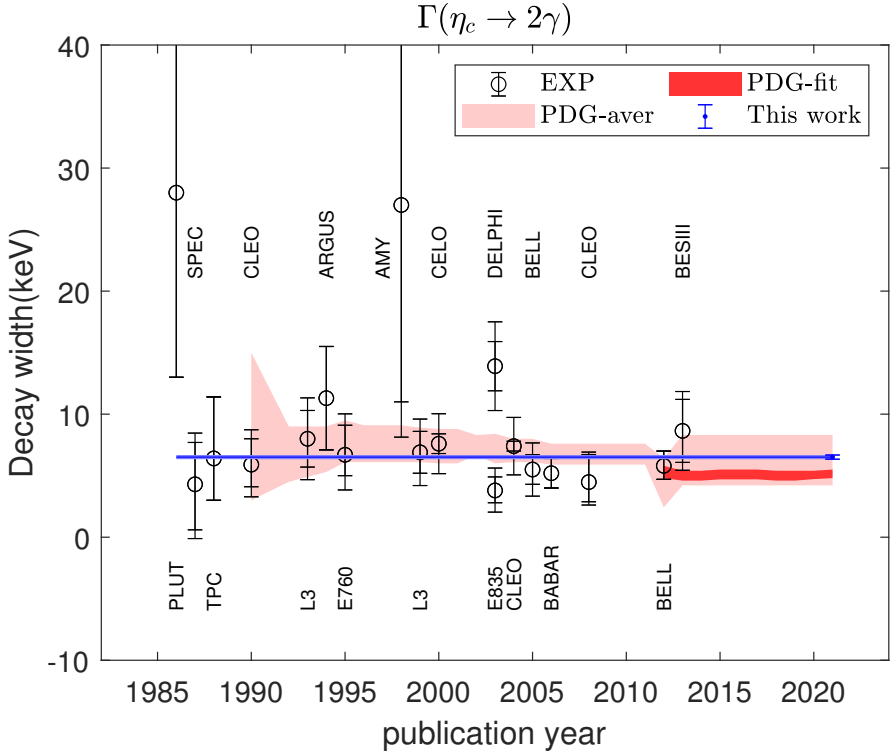
- Automatic $O(a)$ improved;
- PDG-fit: global fit of PDG;
- PDG-aver: world average of PDG;
- DSE ~ 6.4 PRD**95**,016010(2017)
  NRQCD ~ 10 PRL**119**,252002(2017)



$$\Gamma(\eta_c \to 2\gamma) = \begin{cases} 6.51(18)\,\text{keV} & m_{\eta_c} = m_{\eta_c}^{\rm phys} \\ 6.60(16)\,\text{keV} & m_{J/\psi} = m_{J/\psi}^{\rm phys} \\ 5.15(35)\,\text{keV} & \text{PDG-fit} \\ 6.11^{+2.2}_{-1.9}\,\text{keV} & \text{PDG-aver} \end{cases}$$

- The discrepancy bewteen PDG-fit and lattice result is beyond 3 σ

Lattice & Experiments



2021 Review of Particle Physics.
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys.

$\eta_c(1S) \to \gamma\gamma$

expand all datablocks

$\Gamma(\eta_c(1S) \to \gamma\gamma)$

VALUE (keV)	EVTS	DOCUMENT ID
5.15 ± 0.35	OUR FIT	

• • • We do not use the following data for averages, fits, limits, etc. • • •

- OUR FIT: a minimum χ^2-fit for the branching ratios from lots of experimental measurements on different decay channels.
- The fitting errors are consistent with world average for all decay channels fitted, but except for $\eta_c \to 2\gamma$.

PDG global fit

CONSTRAINED FIT INFORMATION

show precise values? ☐

An overall fit to total width, 8 combinations of particle width obtained from integrated cross section, 19 branching ratios uses 93 measurements and one constraint to determine 13 parameters. The overall fit has a χ^2 = 117.8 for 81 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $<\delta x_i \sim \delta x_j> / (\delta x_i \cdot \delta x_j)$, in percent, from the fit to parameters p_i, including the branching fractions, $x_i = \Gamma_i / \Gamma_{total}$ The fit constrains the x_i whose labels appear in this array to sum to one.

	x_4	x_7	x_{15}	x_{27}	x_{28}	x_{31}	x_{35}	x_{38}	x_{41}	x_{43}	x_{49}	Γ	x_{999}
x_4	100												
x_7	16	100											
x_{15}	3	5	100										
x_{27}	18	35	6	100									
x_{28}	9	17	3	47	100								
x_{31}	10	18	3	21	10	100							
x_{35}	7	13	2	21	10	8	100						
x_{38}	12	22	4	25	12	14	10	100					
x_{41}	11	20	4	27	13	12	10	15	100				
x_{43}	3	5	1	6	3	3	2	4	23	100			
x_{49}	-27	-51	-9	-59	-28	-32	-23	-38	-38	-9	100		
Γ	-1	-3	0	-3	-1	-2	-1	-2	6	1	-27	100	
x_{999}	0	0	0	0	0	0	0	0	0	0	0	0	100

	Mode	Rate (MeV)	Scale factor
Γ_4	$\eta_c(1S) \to K^*(892)\overline{K}^*(892)$	0.0069 ± 0.0013	
Γ_7	$\eta_c(1S) \to \phi\phi$	0.00174 ± 0.00019	
Γ_{15}	$\eta_c(1S) \to f_2(1270)f_2(1270)$	0.0098 ± 0.0025	
Γ_{27}	$\eta_c(1S) \to K\overline{K}\pi$	0.073 ± 0.004	
Γ_{28}	$\eta_c(1S) \to K\overline{K}\eta$	0.0136 ± 0.0015	
Γ_{31}	$\eta_c(1S) \to K^+K^-\pi^+\pi^-$	0.0066 ± 0.0011	
Γ_{35}	$\eta_c(1S) \to 2(K^+K^-)$	0.00143 ± 0.00030	
Γ_{38}	$\eta_c(1S) \to 2(\pi^+\pi^-)$	0.0091 ± 0.0012	
Γ_{41}	$\eta_c(1S) \to p\overline{p}$	0.00144 ± 0.00014	
Γ_{43}	$\eta_c(1S) \to \Lambda\overline{\Lambda}$	0.00106 ± 0.00023	
Γ_{49}	$\eta_c(1S) \to \gamma\gamma$	$(1.61 \pm 0.12) \times 10^{-4}$	

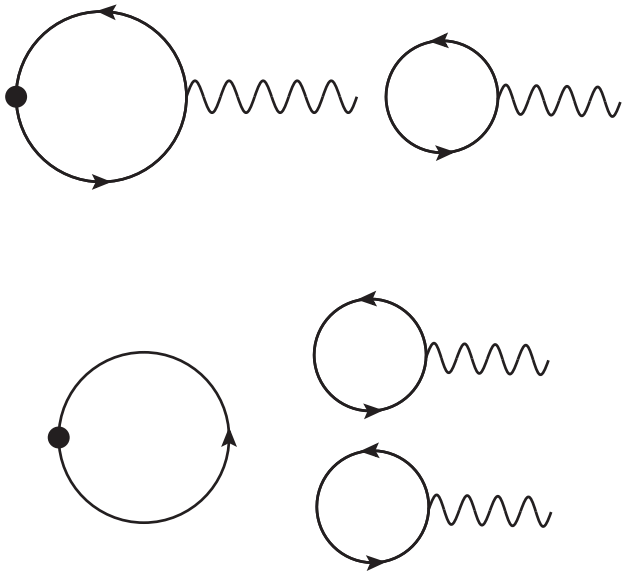
- The smaller errros of fitting may due to: (i)large uncertainties of $\eta_c \to 2\gamma$; (ii)Highly correlated with other decay channels ; (iii)The other channels have high precisions.

What we have considered

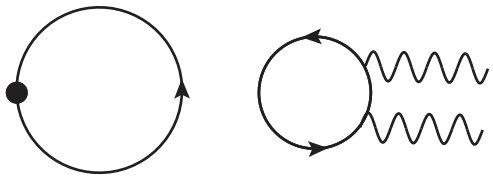
- Excited-state effect $\rightarrow$ A series of Δt
- Lattice discretization effect $\rightarrow$ Three lattice spacings
- Scale setting effect $\rightarrow$ Two mass tunings
- Finite volume effect $\rightarrow$ R-dependence

Systematic effects we have considered

What we have ingored



Type-I



Type-II

- Much smaller contribution due to
  - Type-I: SU(3) asymptotic symmetry with $m_{u,d,s} \ll m_c$
  - Type-II: OZI supressed with $\alpha_s(m_c)$

Conclusion

- Propose a direct method to calculate the on-shell form factor of $\eta_c \to 2\gamma$.
- The method can be applied for processes involving the leptonic or radiative particles in the final states:
  - $\pi^0 \to 2\gamma$
  - $J/\psi \to 3\gamma$
  - $J/\psi \to \gamma\eta_c$
  - $J/\psi \to \gamma\nu\bar{\nu}$
- Various systematic effects are examined carefully.
- In agreement with the world average of PDG, but different from the global fit beyond 3σ.

We expect more precise analysis from BESIII with $10^{10} J/\psi$.

完

Thank you!

Back up

Infinite volume reconstruction [1812.0981]

- Divide the time integral into two parts: short distance calculated on lattice, long distance reconstructed with the short-distance hadroinc function:

$$\mathcal{H}_{\mu\nu}(\vec{x},t)|_{t>t_s} = \int \frac{d^3\vec{p}}{(2\pi)^3} \int d^3\vec{x'} \mathcal{H}_{\mu\nu}(\vec{x'},t_s) e^{-E_{\vec{p}}(t-t_s)+i\vec{p}\cdot(\vec{x}-\vec{x'})}$$

- Long-distance form factor:

$$F^{(\infty)}_{\eta_c\gamma\gamma} = -\frac{1}{m_{\eta_c}} \frac{e^{|\vec{\boldsymbol{p}}|t_s}}{\sqrt{m^2_{J/\psi}+|\vec{\boldsymbol{p}}|^2}-|\vec{\boldsymbol{p}}|} \int d^3\vec{\boldsymbol{x}} \frac{j_1(|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|)}{|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(t_s,\vec{\boldsymbol{x}})$$

- The total:

$$F_{\eta_c\gamma\gamma} = F^{(L)}_{\eta_c\gamma\gamma} + F^{(\infty)}_{\eta_c\gamma\gamma}$$