Multi-sites Reconstruction for Kinetic Energy Reconstruction

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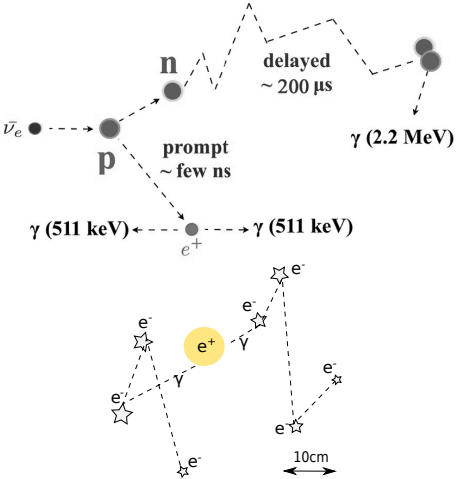
Kinetic Positron Isolation

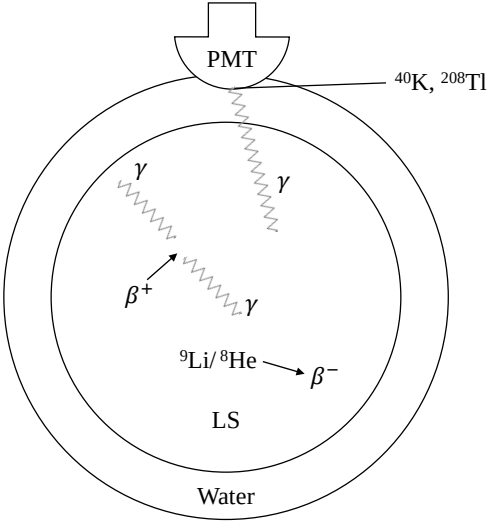
- for reactor $\bar{\nu}_e$
  Imagine a detector only sensitive to β^+ and blind to γ, the equivalent visible energy resolution can be improved.
- Mean free path of γ is about $\sim$ 10 cm in LS, while the resolution can be $\sim$ cm.



Applications

The multi-site technology is potentially applicable to the key issues of neutrino physics.

1. $\beta^+/\beta^-/\gamma$ discrimination, reject ^{9}Li/^{8}He background.
2. probabilistic dark noise model.



Detector response for single-site events

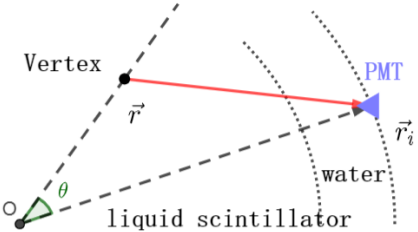
The key point is to obtain a response $R_j(t)$ for a point source $\delta(\mathbf{r}, t_0)$ on j-th PMT, considering the spherical symmetry.

$$\delta(\mathbf{r}, t_0) \rightarrow R_j^{\delta}(t; r, \theta_j, E) \tag{1}$$

while R_j^{δ} is an inhomogeneous Poisson process, called Probe function.

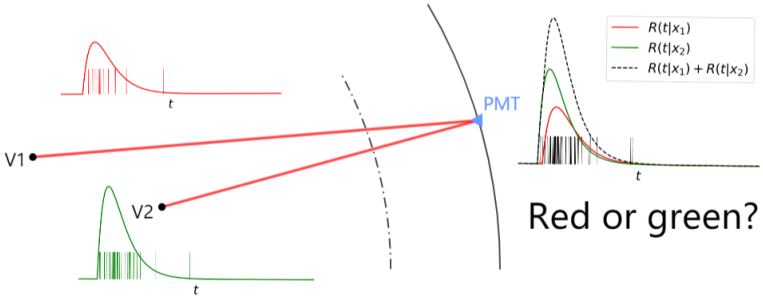
R_j can be obtained by MC/calibration data.

- Photon propagation
- Regression (see Yuyi Wang's talk)
- Neural networks, and others...



Extend 1 to K

- Parameters: $E_1, E_2, \cdots, E_K$, $\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_K}$.
- Observation: hits and its time on each PMT.
- Hidden variables: which vertex each hit belongs to.



Likelihood for 1 class

- For only 1 class, with $\lambda_j = \int R_j(t|\mathbf{x})\mathrm{d}t$, $p_j(t_i) = \tilde{R}_j(t_i|\mathbf{x})$, the likelihood function is

$$\mathcal{L}(t_0, \mathbf{x}, E|t_i) \propto \overbrace{\prod_j e^{-\lambda_j}}^{\text{nonhit}} \cdot \overbrace{\prod_i R_{j_i}(t_i|\mathbf{x})}^{\text{hit}} \tag{2}$$

  - j is PMT index, i is the hit index.
  - $R_j(t|\mathbf{x})$ is the point response on the j-th PMT;
  - t_i, j_i are the timing and PMT index of the i-th PE.

Likelihood for K classes

- For K classes, non-hit part

$$\exp(-\lambda_j) \to \exp\left[-\sum_{k=1}^{K} E_k \int R_j(t|\mathbf{x}_k)\mathrm{d}t\right] \tag{3}$$

- hit part (**mixture model**)

$$R_{j_i}(t_i;\Theta) \to \sum_k E_k R_{j_i}(t_i|\mathbf{x}_k) \tag{4}$$

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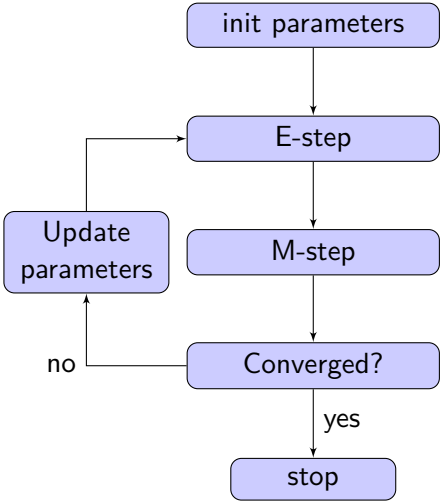
Difficulties

Mixture model is hard to optimize. Expectation Maximization (EM) is an effective method.

EM flow chart



- **Key point**: Add hidden variables $Q_k(t_i)$ – the probability of which class each hit belongs to, write down the new likelihood $F(Q, \Theta)$.
- **Init**: Give an initial guess of all parameter $\Theta^{[0]} : E_k^{[0]}, \mathbf{x}_k^{[0]}$.
- **E-step**: $Q_k^{[\text{new}]} \leftarrow \arg\max_Q F(Q, \Theta^{[\text{old}]})$
  Calculate the **posterior** of each hit.

$$Q_k(t_i) = \frac{E_k R_{j_i}(t_i|\mathbf{x}_k)}{\sum_{k=0}^{K} E_k R_{j_i}(t_i|\mathbf{x}_k)} \tag{5}$$

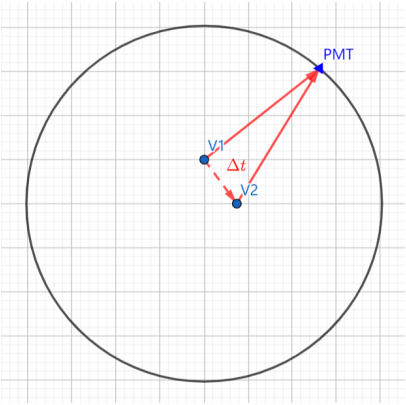
- **M-step**: $\Theta^{[\text{new}]} \leftarrow \arg\max_\Theta F(Q^{[\text{new}]}, \Theta)$
  This process is **similar** single site reconstruction.
- Iterate E-step and M-step until convergence.

Try on simulation data

A TAO-like detector (arXiv:2005.08745) is a paradise for reconstruction!

- No total reflection
- radius: $\sim 0.9\,\mathrm{m}$
- SiPM: ~ 4000
- QE: ~ 0.5
- Light yield: ~ 12000 photons/MeV

Simply, we simulate 2 point-like events in the detector and manually add the propagation time of the γ.



Get Probe

The expected PE λ on each PMT is

$$\lambda(r, \theta_j, E) = \bar{E} Y \frac{\Omega}{4\pi} \exp\left(-\frac{d}{L}\right) \eta \tag{6}$$

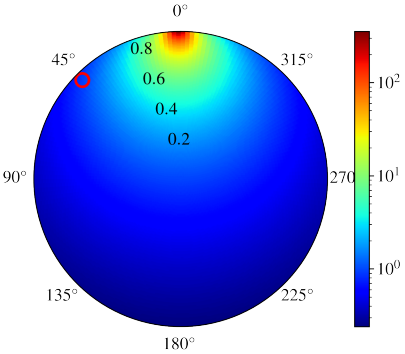
- $\bar{E}$ - a fit parameter, relative to energy
- Y - light yield
- Ω - solid angle
- L - attenuation length
- η - quantum efficiency

The pdf of timing is fit by Poisson regression using Legendre polynomials, then

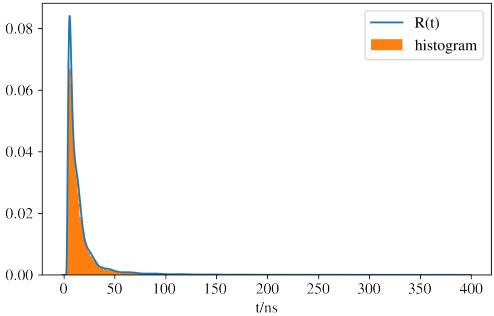
$$R_j(t, r, \theta_j, E) = \lambda(r, \theta_j, E) \exp(\sum_m a_m P_m(t)) \tag{7}$$

where $P_m(t)$ is the m-th Legendre polynomial.

Graphs of Probe



(a) Fix SiPM at $\theta = 0$, the heatmap of expected PE contribution at each vertex.



(b) The fit light curve vs simulation when restricted to red circle of Fig. (a)

Merge events

- The Probe can be used in vertex reconstruction directly!
- However, we need multi-sites events.

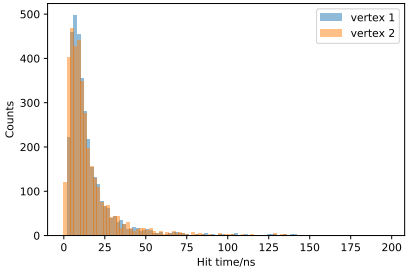


图: Timing of 2 vertices

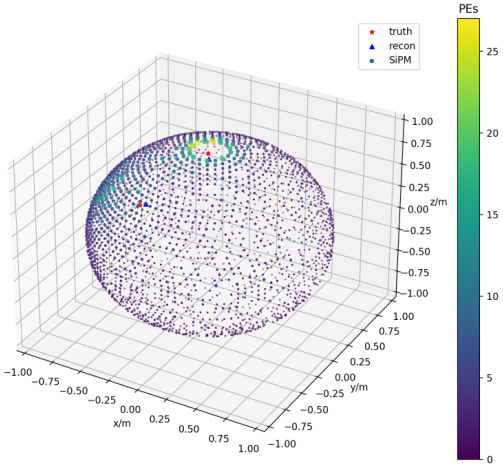


图: PE of 2 vertices

Estimate the reconstruction performance I

The more similar of the 2 vertices, the harder to discriminate.

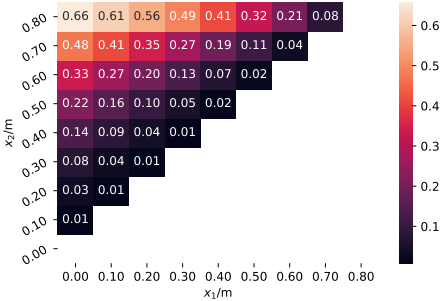
- Estimate the similarity between different points, define coherence Φ

$$\Phi = \frac{\sum_j \int R_j(t|\mathbf{x}_1) R_j(t|\mathbf{x}_2) \mathrm{d}t}{\sqrt{\sum_j \int R_j^2(t|\mathbf{x}_1)\mathrm{d}t}\sqrt{\sum_j \int R_j^2(t|\mathbf{x}_2)\mathrm{d}t}} \tag{8}$$

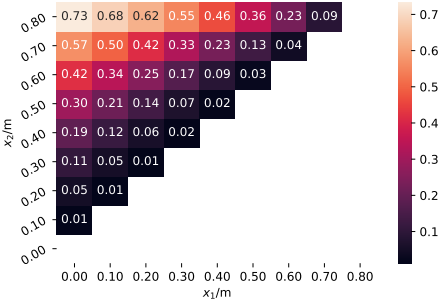
  j is the index of PMT.
- Φ is similar to intersection angle between 2 vectors on j, t space.
- Define $1 - \Phi$ as cosine distance, the closer to 0 indicate more similar!

Estimate the reconstruction performance II

Assuming 2 points on x axis.



(a) Simultaneously.

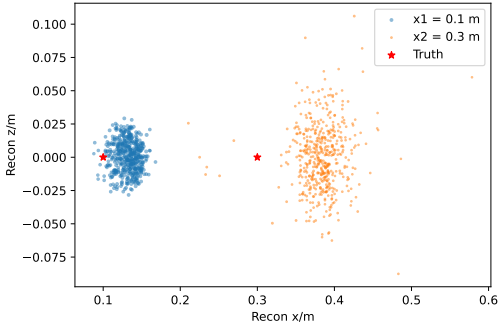


(b) Add the propagation time.

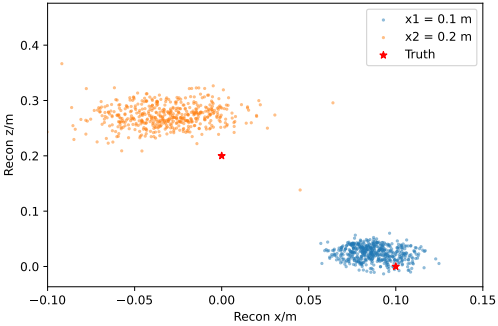
- Propagation time of particle helps to separation.
- Outer region is more likely to discriminate.

Vertex reconstruction

On xOz plane:



(a) truth: (0.1,0,0) m and (0.3,0,0) m

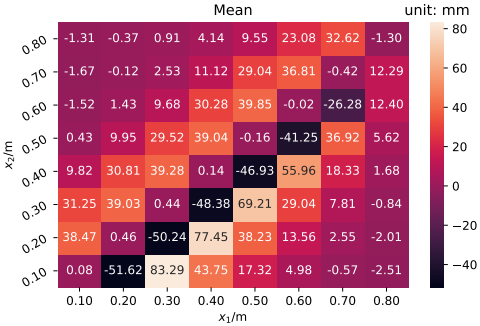


(b) truth: (0.1,0,0) m and (0,0,0.2) m

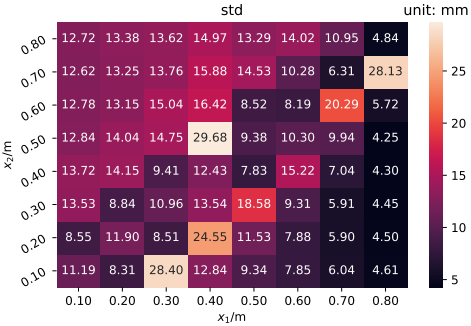
Pile-up events occurring simultaneously can be separated!

Bias and resolution of vertex reconstruction

Fix 2 points at x axis



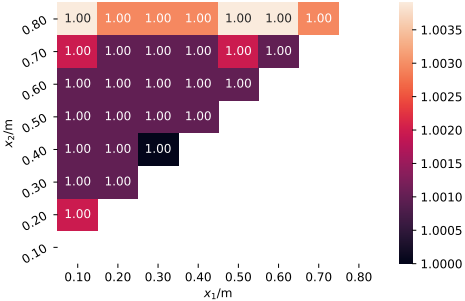
(a) bias of x_1



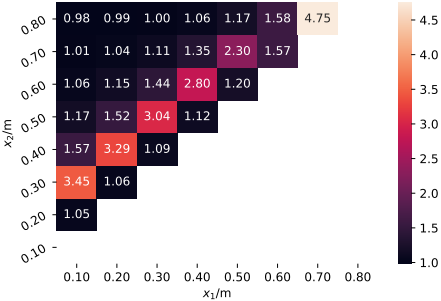
(b) standard deviation of x_1

Distance $\geq$ 30 cm can be reconstructed well.

Energy reconstruction



(a) $E_1 + E_2$: recon/truth, expected 1



(b) single energy ratio: E_1/E_2, expected 1

Total energy is accurate, while single energy is biased.

Summary

- Multi-sites reconstruction is based on single-site, and EM is preliminary implemented and proves effective.

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2. Give an accurate analysis when vertices are in high correlation.

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Thank you!

Backup

- Difficulties: log sum is hard to optimize directly.
- Hint by Expectation Maximization, the optimize from log sum to sum log:

$$\begin{aligned}\sum_i \log \sum_{k=0}^{K} E_k R_{j_i}(t_i|\mathbf{x}_k) &= \sum_i \log \sum_{k=0}^{K} \frac{E_k R_{j_i}(t_i|\mathbf{x}_k) Q_k(t_i)}{Q_k(t_i)} \\ &\geq \sum_{k=0}^{K} \sum_i Q_k(t_i) \log \frac{E_k R_{j_i}(t_i|\mathbf{x}_k)}{Q_k(t_i)}\end{aligned} \tag{9}$$

- Get equality only when

$$Q_k(t_i) = \frac{E_k R_{j_i}(t_i|\mathbf{x}_k)}{\sum_{k=0}^{K} E_k R_{j_i}(t_i|\mathbf{x}_k)} \tag{10}$$

- Meaning of $Q_k(t_i)$: the posterior of i-th hit belongs to k-th class.