



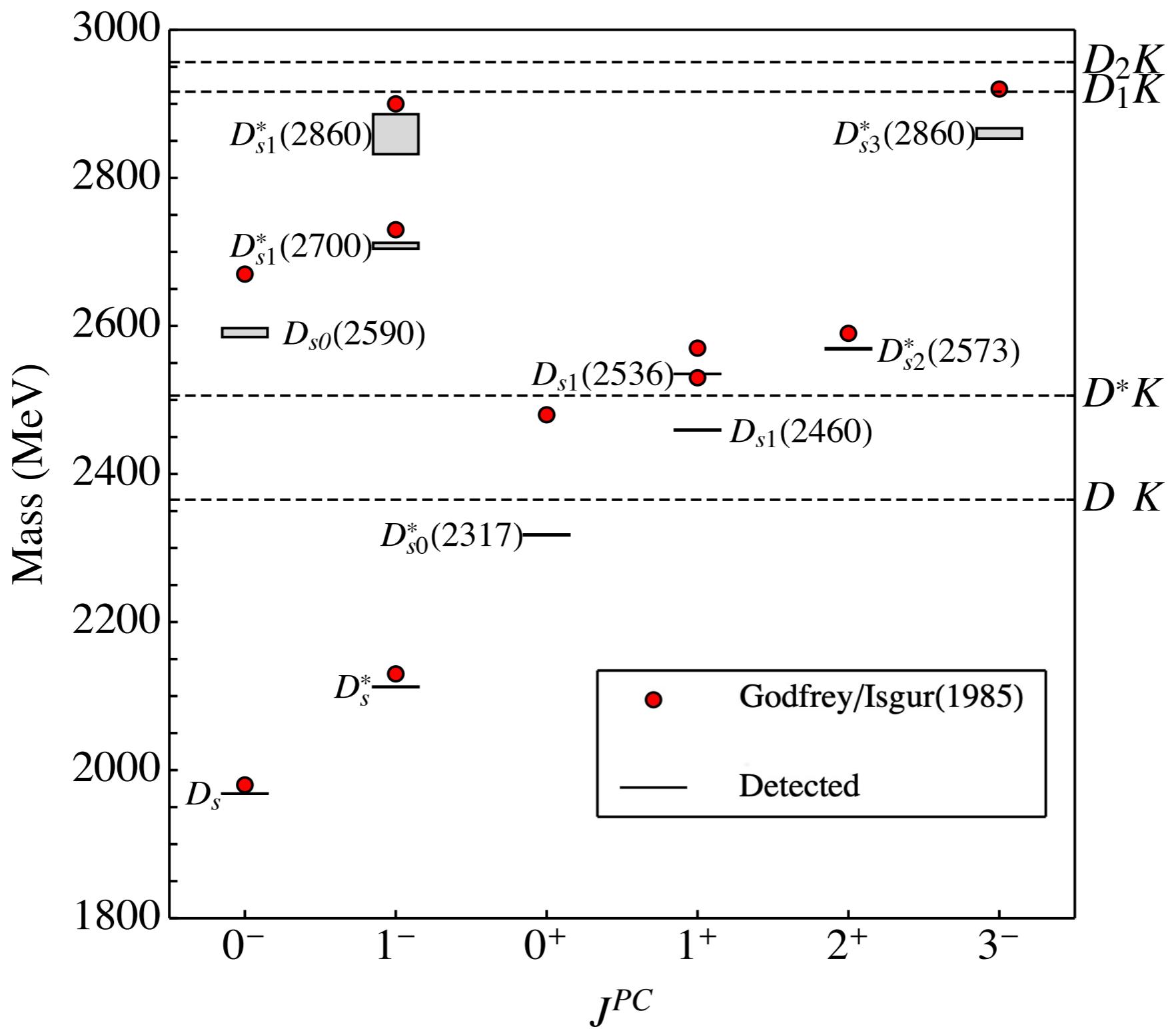
# Near-threshold Charm-Strange mesons from Lattice QCD and Hamiltonian Effective Field Theory

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Based on arXiv: 2107.04860  
In collaboration with 王广娟, 吴佳俊, Makoto Oka, 朱世琳

# $D_s$ mesons in quark model



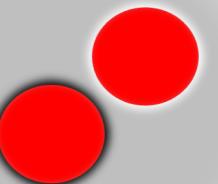
The relativized quark model

Godfrey, Isgur, **Phys. Rev. D 32, 189 (1985)**

# $D_s(2317)$ and $D_s(2460)$

## □ Quenched and unquenched quark model

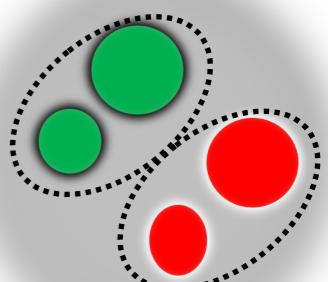
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  3. D. S. Hwang and D.-W. Kim, [Phys. Lett. B 601, 137 \(2004\)](#)
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  6. Q.-T. Song, D.-Y. Chen, X. Liu, and T. Matsuki, [Phys. Rev. D 91, 054031 \(2015\)](#)
  7. H.-Y. Cheng and F.-S. Yu, [Eur. Phys. J. C 77, 668 \(2017\)](#)
  8. S.-Q. Luo, B. Chen, X. Liu, and T. Matsuki, [Phys. Rev. D 103, 074027 \(2021\)](#)
  9. Z.-Y. Zhou and Z. Xiao, [Eur. Phys. J. C 81, 551 \(2021\)](#)
- .....



Meson

## □ Tetraquark

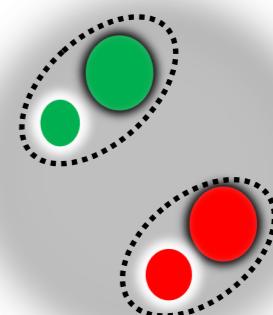
1. H.-Y. Cheng and W.-S. Hou, [Phys. Lett. B 566, 193 \(2003\)](#)
  2. Y.-Q. Chen and X.-Q. Li, [Phys. Rev. Lett. 93, 232001 \(2004\)](#)
  3. V. Dmitrasinovic, [Phys. Rev. Lett. 94, 162002 \(2005\)](#)
  4. H. Kim and Y. Oh, [Phys. Rev. D 72, 074012 \(2005\)](#)
  5. J.-R. Zhang, [Phys. Lett. B 789, 432 \(2019\)](#)
- .....



Compact multiquark

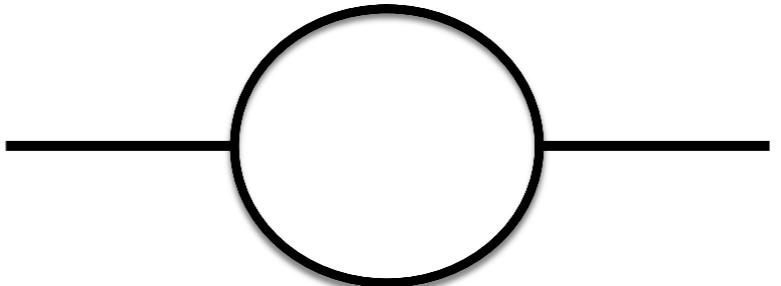
## □ Hadronic molecule

1. E. E. Kolomeitsev and M. F. M. Lutz, [Phys. Lett. B 582, 39 \(2004\)](#)
  2. A. P. Szczepaniak, [Phys. Lett. B 567, 23 \(2003\)](#)
  3. J. Hofmann and M. F. M. Lutz, [Nucl. Phys. A 733, 142 \(2004\)](#)
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  5. T. Barnes, F. E. Close, and H. J. Lipkin, [Phys. Rev. D 68, 054006 \(2003\)](#)
  6. D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, [Phys. Rev. D 76, 074016 \(2007\)](#)
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  10. A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, [Phys. Rev. D 76, 014005 \(2007\)](#)
  11. F.-K. Guo, C. Hanhart, and U.-G. Meissner, [Eur. Phys. J. A 40, 171 \(2009\)](#)
  12. Z.-X. Xie, G.-Q. Feng, and X.-H. Guo, [Phys. Rev. D 81, 036014 \(2010\)](#)
- .....



Hadronic molecule

# Coupled-channel effect



1. Yu. S. Kalashnikova, **Phys.Rev.D 72, 034010 (2005)**

☞ Charmonium

2. Z.-Y. Zhou and Z. Xiao, **Phys. Rev. D 84, 034023 (2011)**

☞ Charmed and charmed-strange spectra

3. Y. Lu, M. N. Anwar, B. S. Zou, **Phys.Rev.D 94, 034021 (2016)**

☞ Bottomonium

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- Coupled-channel effect due to hadron loop could cause sizable mass shift on the state in quark model.



# How important using lattice QCD data

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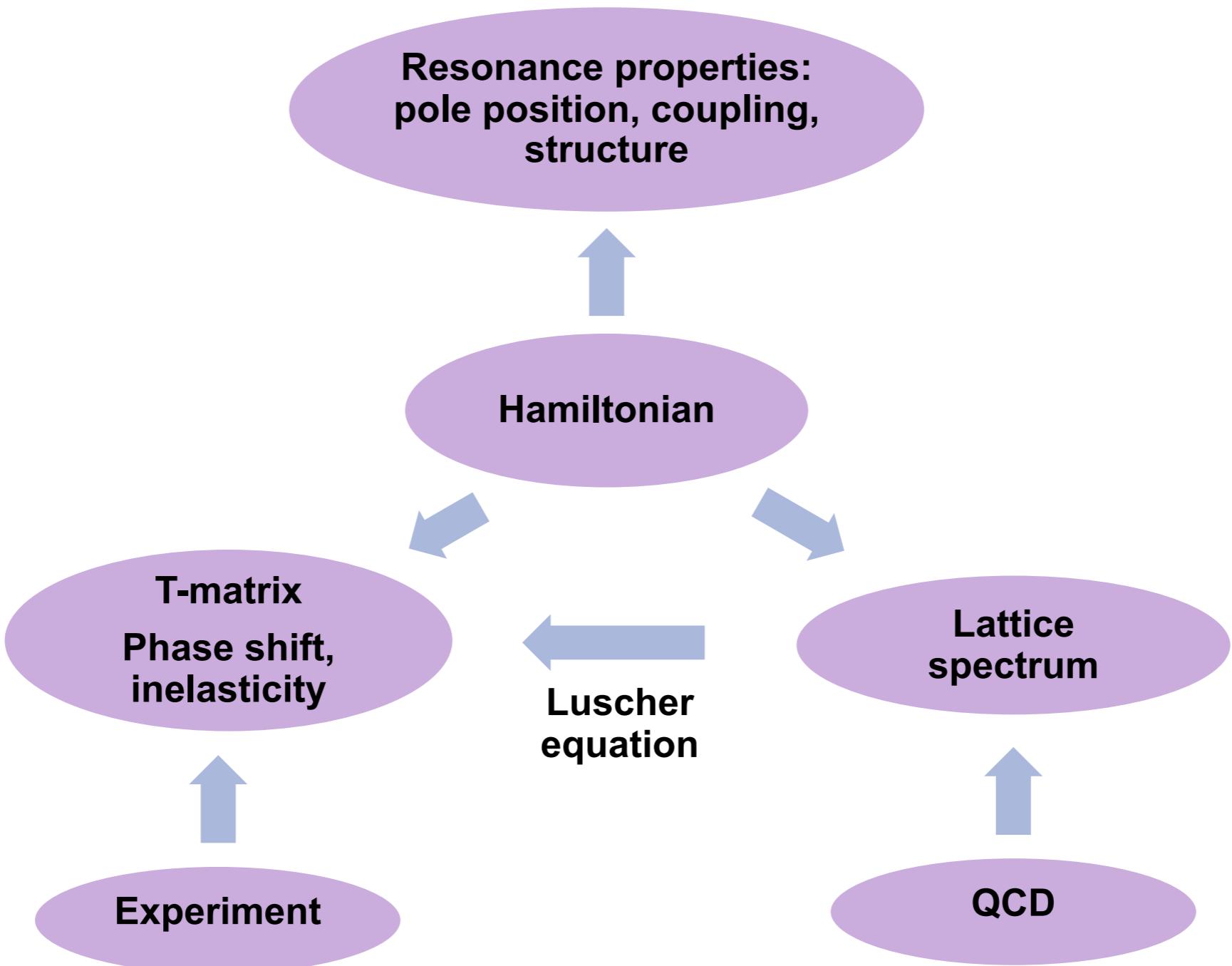
2→2 scattering is a good platform to study the coupled channel effect, for example

$$\begin{aligned} DK &\rightarrow DK \\ D^*K &\rightarrow D^*K \end{aligned}$$

However, this scattering data from experimental side is unavailable.

Thus, it is important to using lattice QCD data.

# Hamiltonian effective field theory





# Hamiltonian effective field theory

## 1. Finite-volume matrix Hamiltonian model for a $\Delta \rightarrow N\pi$ system

J.M.M. Hall, A.C.-P. Hsu, D.B. Leinweber, A.W.Thomas, R.D. Young

[Phys.Rev. D87 \(2013\) no.9, 094510](#)

## 2. Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD

Jia-Jun Wu, T.-S.H.Lee, A.W.Thomas, R.D. Young

[Phys.Rev. C90 \(2014\) no.5, 055206](#)

## 3. Hamiltonian effective field theory study of the $N^*(1535)$ resonance in lattice QCD

Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu

[Phys.Rev.Lett. 116 \(2016\) no.8, 082004](#)

## 4. Lattice QCD Evidence that the $\Lambda(1405)$ Resonance is an Antikaon-Nucleon Molecule

J.M.M. Hall, Waseem Kamleh, Derek B. Leinweber, Benjamin J. Menadue, Benjamin J. Owen, A.W.Thomas, R.D. Young

[Phys.Rev.Lett. 114 \(2015\) no.13, 132002](#)

## 5. Hamiltonian effective field theory study of the $N^*(1440)$ resonance in lattice QCD

Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu

[Phys.Rev. D95 \(2017\) no.3, 034034](#)

## 6. Structure of the $\Lambda(1405)$ from Hamiltonian effective field theory

Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu

[Phys.Rev. D95 \(2017\) no.1, 014506](#)

## 7. Nucleon resonance structure in the finite volume of lattice QCD

Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas

[Phys.Rev. D95 \(2017\) no.11, 114507](#)

## 8. Structure of the Roper Resonance from Lattice QCD Constraints

Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W. Thomas

[Phys.Rev. D97 \(2018\) no.9, 094509](#)

# Hamiltonian framework

The Hamiltonian reads

$$H = H_0 + H_I,$$

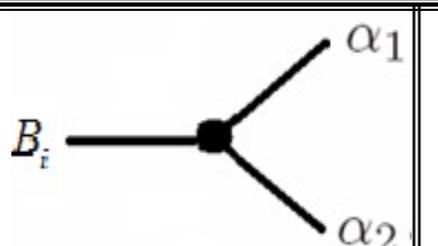
where the non-interacting one is

$$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_{\alpha} \int d^3 \vec{k} |\alpha(\vec{k})\rangle E_{\alpha}(\vec{k}) \langle \alpha(\vec{k})|.$$

And the interacting one includes two parts

$$H_I = g + v$$

bare state core  $\rightarrow$  channel :



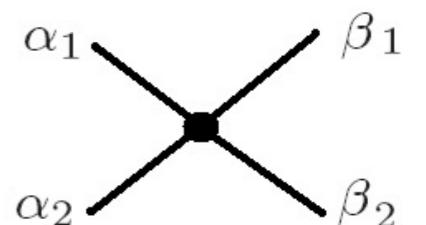
$$g = \sum_{\alpha, B} \int d^3 \vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

P. G. Ortega, et al,  
**Phys. Rev. D 94, 074037 (2016)** truncate the hard vertices  
 given by usual QPC

channel  $\rightarrow$  channel :



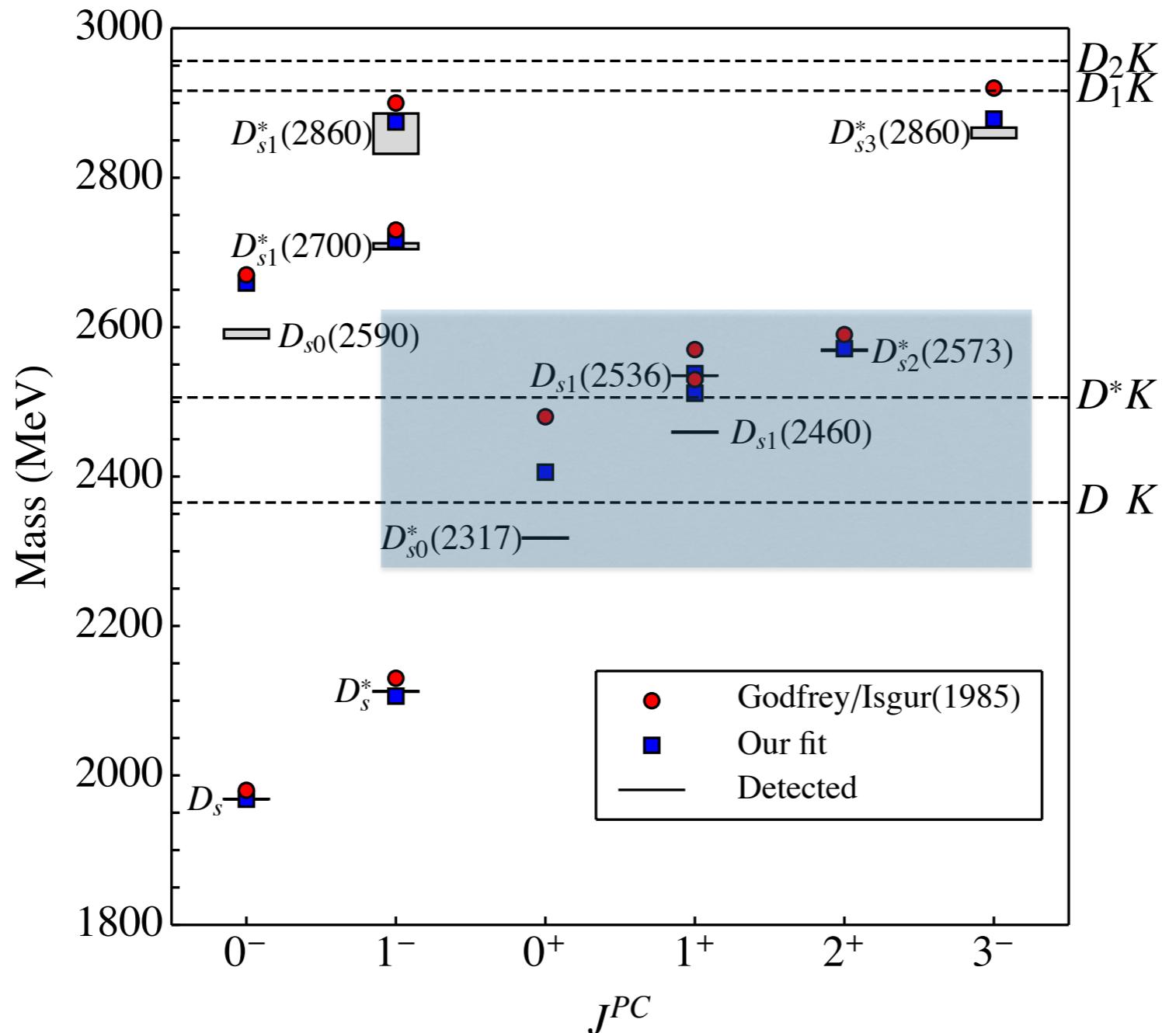
$$v = \sum_{\alpha, \beta} \int d^3 \vec{k} d^3 \vec{k}' |\alpha(\vec{k})\rangle V_{\alpha, \beta}^L(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Effective Lagrangian: (exchanging  $\rho/\omega$ )

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{PPV} + \mathcal{L}_{VVV} \\ &= ig_v \text{Tr}(\partial^{\mu} P[P, V_{\mu}]) + ig_v \text{Tr}(\partial^{\mu} V^{\nu}[V_{\mu}, V_{\nu}]) \end{aligned}$$

Form factor:  $\left( \frac{\Lambda^2}{\Lambda^2 + p_f^2} \right)^2 \left( \frac{\Lambda^2}{\Lambda^2 + p_i^2} \right)^2$

# $D_s$ mesons in quark model



- Fit with updated masses with low-lying states away from thresholds,
- Our fit is more consistent with observation.

# Four near-threshold $D_s$ states in quark model



Transformation from physical bases to the heavy quark limit bases

$c\bar{s}$ cores		channel			
	$B( ^{2S+1}L_J\rangle)$	$B(\text{mass})$	$\alpha$	$L$	
$D_{s0}^*(2317)$	$ {}^3P_0\rangle$	2405.9	$DK$	$S$	
$D_{s1}^*(2460)$	$0.68 {}^1P_1\rangle - 0.74 {}^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	$D^*K$	$S$	$D$
$D_{s1}^*(2536)$	$-0.74 {}^1P_1\rangle - 0.68 {}^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	$D^*K$	$S$ , $D$	
$D_{s2}^*(2573)$	$ {}^3P_2\rangle$	2571.2	$DK, D^*K$	$D$	

- $D_s(2317)$  and  $D_s(2460)$  are much heavier than detected,
- The bare  $1^+$  states are almost purely given by the states with heavy-quark spin bases.

# Connecting the lattice data

- Hamiltonian in finite volume with discrete momentum

Continuous	$\int d\vec{k}$	and	$ \alpha(\vec{k}_\alpha)\rangle$	and	$\langle \beta(\vec{k}_\beta)   \alpha(\vec{k}_\alpha) \rangle = \delta_{\alpha\beta} \delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓	↓	↓	↓	↓
Discrete	$\sum_i (2\pi/L)^3$	and	$(2\pi/L)^{-3/2}  \vec{k}_i, -\vec{k}_i\rangle_\alpha$	and	${}_\beta \langle \vec{k}_j, -\vec{k}_j   \vec{k}_i, -\vec{k}_i \rangle_\alpha = \delta_{\alpha\beta} \delta_{ij}$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha,i} |\vec{k}_i, -\vec{k}_i\rangle_\alpha \left[ \sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2} \right]_\alpha \langle \vec{k}_i, -\vec{k}_i |$$

$$H_I = \sum_j (2\pi/L)^{3/2} \sum_\alpha \sum_{i=1,n} \left[ |\vec{k}_j, -\vec{k}_j\rangle_\alpha g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha}^- \langle \vec{k}_j, -\vec{k}_j | \right]$$

$$+ \sum_{i,j} (2\pi/L)^3 \sum_{\alpha,\beta} |\vec{k}_i, -\vec{k}_i\rangle_\alpha v_{\alpha,\beta}^- \langle \vec{k}_j, -\vec{k}_j |$$

# Connecting the lattice data

- Hamiltonian matrix

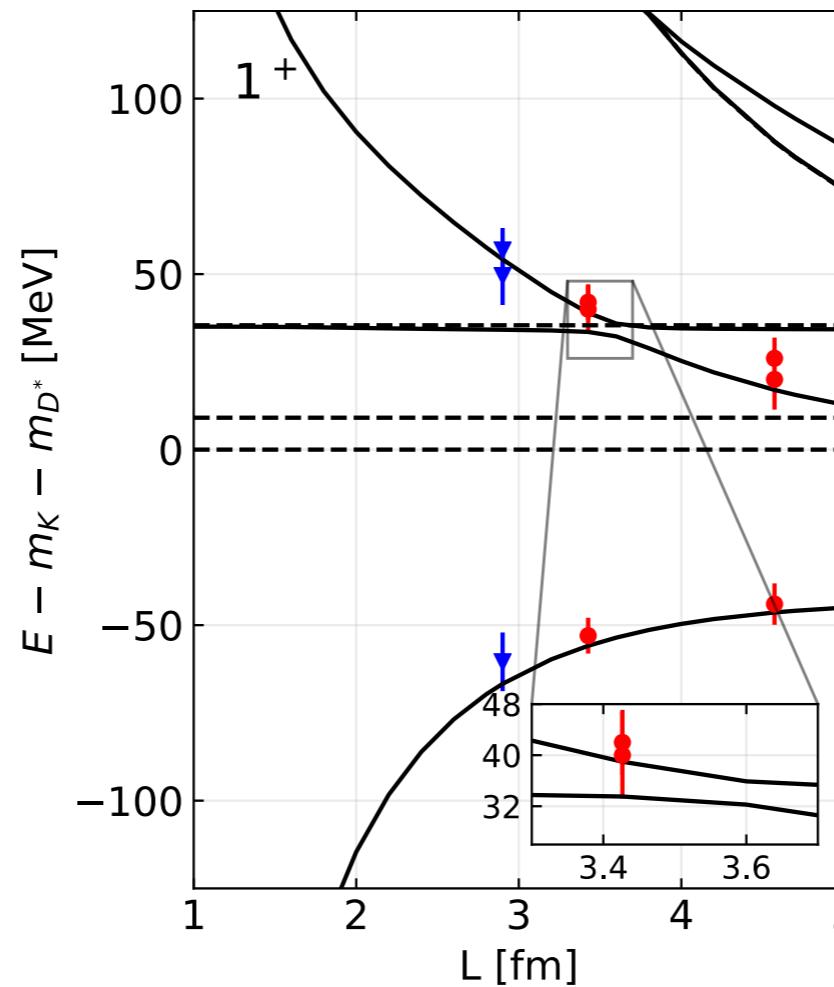
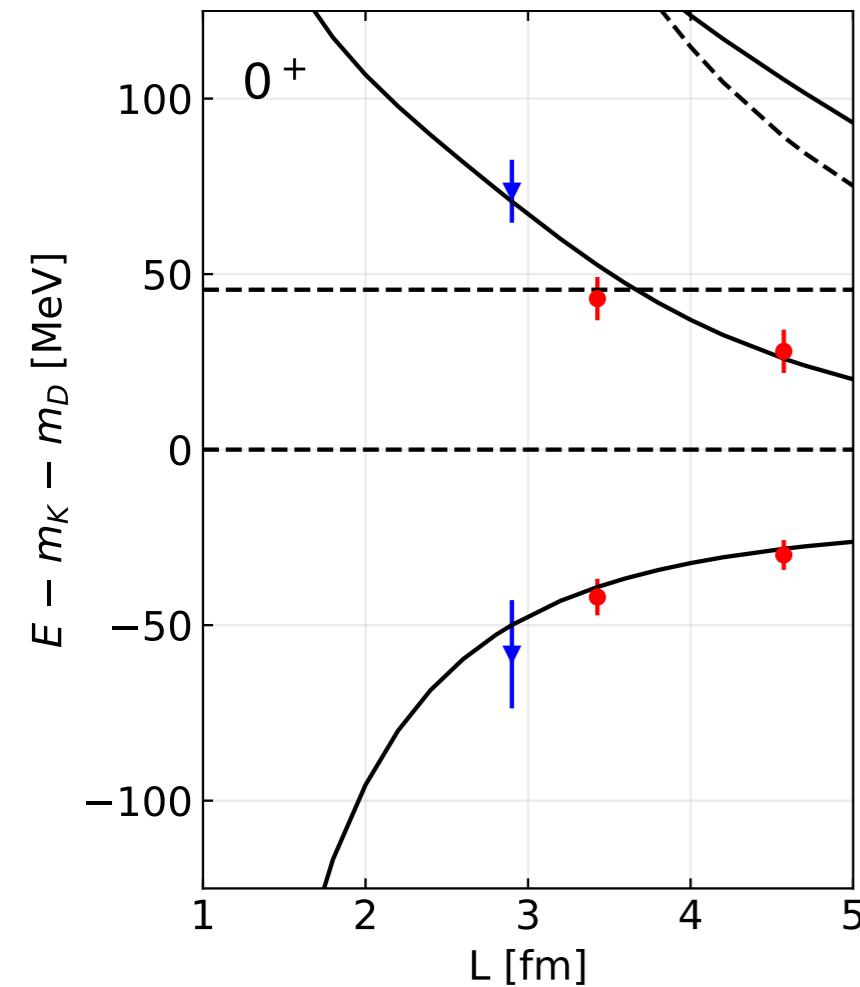
$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0, k_0) & v_{1,2}^V(k_0, k_0) & \cdots & v_{1,n_c}^V(k_0, k_0) & v_{1,1}^V(k_0, k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0, k_0) & v_{2,2}^V(k_0, k_0) & \cdots & v_{2,n_c}^V(k_0, k_0) & v_{2,1}^V(k_0, k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0, k_0) & v_{n_c,2}^V(k_0, k_0) & \cdots & v_{n_c,n_c}^V(k_0, k_0) & v_{n_c,1}^V(k_0, k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1, k_0) & v_{1,2}^V(k_1, k_0) & \cdots & v_{1,n_c}^V(k_1, k_0) & v_{1,1}^V(k_1, k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

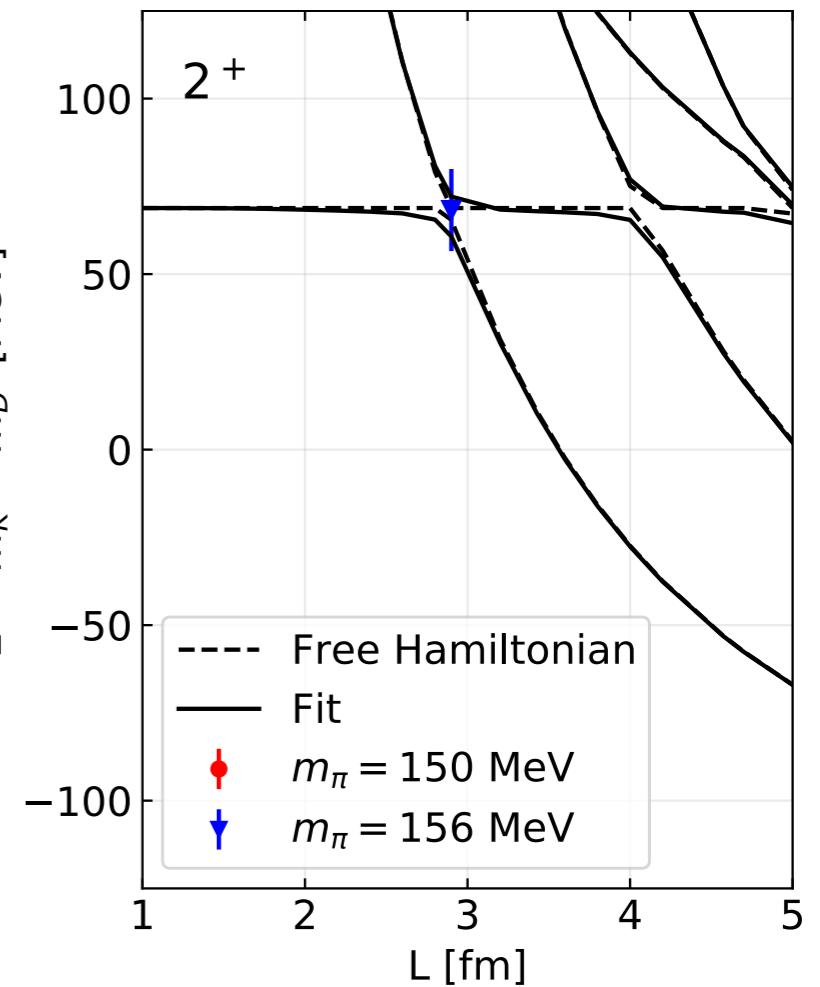
$(H_0 + H_I)|\Psi\rangle = E|\Psi\rangle$       Eigenvalue            Lattice levels

# Fit the lattice data

Fit



Postpredict



	$B(\text{mass})$	$\alpha$	$L$
$D_{s0}^*(2317)$	2405.9	$DK$	$S$
$D_{s1}^*(2460)$	2511.5	$D^*K$	$S, D$
$D_{s1}^*(2536)$	2537.8	$D^*K$	$S, D$
$D_{s2}^*(2573)$	2571.2	$DK, D^*K$	$D$

Lattice data from: C. B. Lang et al., [Phys. Rev. D 90, 034510 \(2014\)](#);  
G. S. Bali et al., [Phys. Rev. D 96, 074501 \(2017\)](#)

- Fit  $D_s(2317, 2460, 2536)$
- With fixed  $\Lambda = 1.0 \text{ GeV}$ ,  $\chi^2/\text{dof} = 0.95$

$$g_c = 4.2^{+2.2}_{-3.1}, \Lambda' = 0.323^{+0.033}_{-0.031} \text{ GeV}$$

$$\gamma = 10.3^{+1.1}_{-1.0}$$



# Component and pole mass

- Component

$$(H_0 + H_I)|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi_E\rangle = C_0|B\rangle + \sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}} C_E(\vec{k}_n)|\alpha(\vec{k}_n)\rangle$$

Eigenvector Component

- Pole mass

In the infinite volume, the scattering T-matrix reads

$$T_{\alpha,\beta}(k, k'; E) = \mathcal{V}_{\alpha,\beta}(k, k'; E) + \sum_{\alpha'} \int q^2 dq \frac{\mathcal{V}_{\alpha,\alpha'}(k, q; E) T_{\alpha,\beta}(q, k'; E)}{E - E_{\alpha'}(q) + i\epsilon}$$

where the effective potential reads

$$\mathcal{V}_{\alpha,\beta}(k, k'; E) = \sum_B \frac{g_{\alpha B}(k) g_{\beta B}^*(k')}{E - m_B} + V_{\alpha,\beta}^L(k, k').$$

T-matrix Pole mass

# Component and pole mass

state	L=4.57 fm	Pole mass at $L \rightarrow \infty$	
	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	$2317.8 \pm 0.5$
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	$2459.5 \pm 0.6$
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	$2535.11 \pm 0.06$
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	$2569.1 \pm 0.8$

A. M. Torres, E. Oset, S. Prelovsek, and A. Ramos  
**JHEP 05, 153 (2015)**

$P(KD) = 72 \pm 13 \pm 5 \%$ , for the  $D_{s0}^*(2317)$

$P(KD^*) = 57 \pm 21 \pm 6 \%$ , for the  $D_{s1}(2460)$

# Component and pole mass

$D_s(2317)$

- Both the bare  $c\bar{s}$  core and DK components are significant and essential.

$D_s(2460)$ ,  $D_s(2536)$

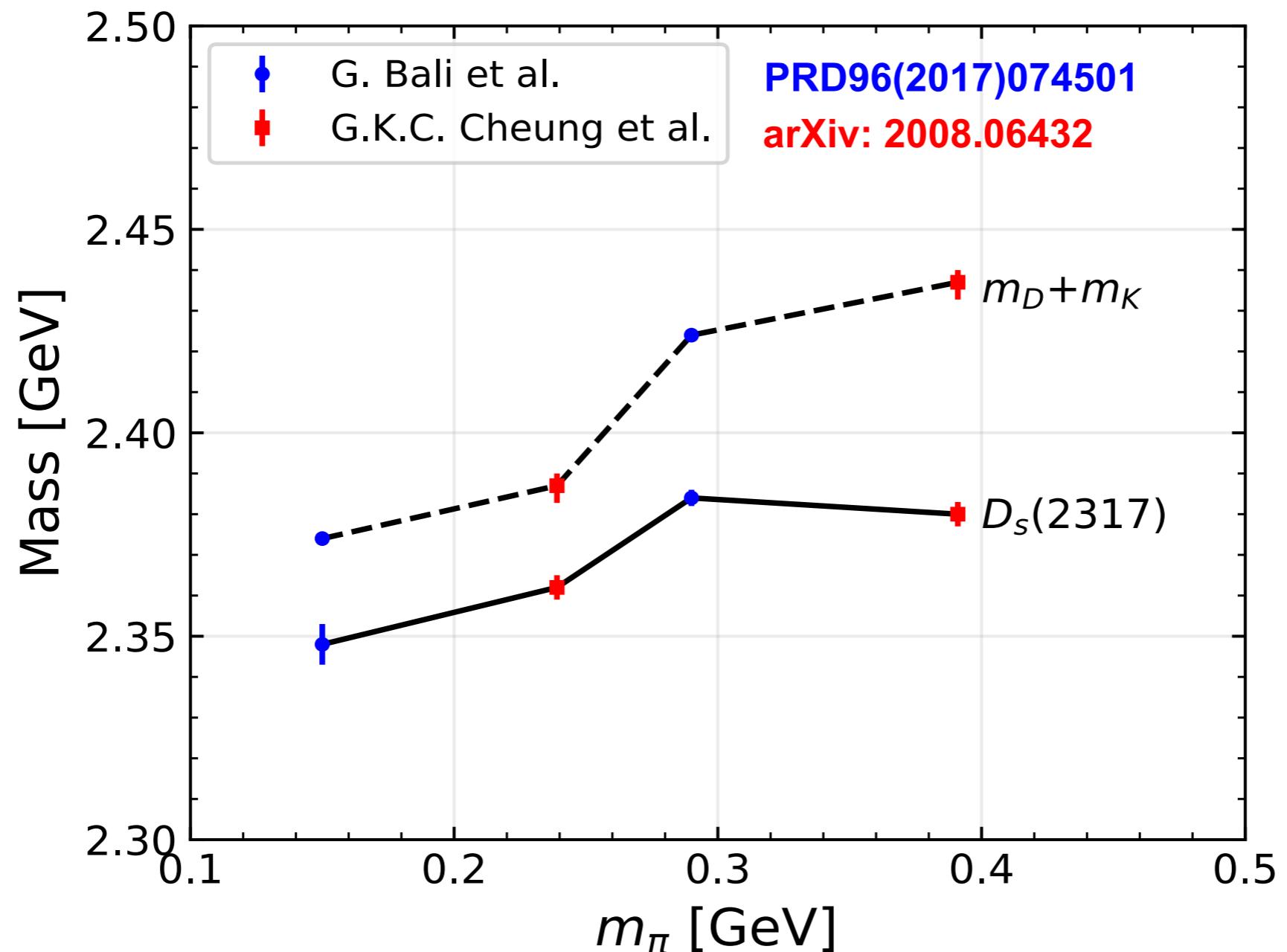
- Dominated by  $\phi_s$  and  $\phi_d$ , respectively; mainly couple with S- and D-wave  $D^*K$ , respectively.
- Then significant mass shift happen to the bare core for  $D_s(2460)$ ;  $c\bar{s}$  core and DK components are significant.
- Almost a pure  $c\bar{s}$  state for  $D_s(2536)$ .

Postdicted  $D_s(2573)$

- One  $c\bar{s}(J^P = 2^+)$  core in QM and D-wave  $DK$  &  $D^*K$  channels.
- Almost a pure  $c\bar{s}$  state.

	$P(c\bar{s})[\%]$
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$

# Pion mass dependence



1. Molecule: tends to become larger with larger  $m_\pi$ .
2. Bare state ( $c\bar{s}$ ): be stable with larger  $m_\pi$ .
3. Need more lattice data.



# Summary

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- A new framework connecting quark model and lattice QCD is constructed.
- Components and pole masses of the physical  $D_s(2317)$ ,  $D_s(2460)$  ,  
 $D_s(2536)$  and  $D_s(2573)$  are derived.
- Improve experimental data analysis! Such as B->DDK.
- This framework can be extended to study other state lying close to thresholds, for example the exotic XYZ states.

Thank you!