中国物理学会高能物理分会第十三届全国粒子物理学术会议

Stabilizers of Finite Modular Groups and Leptonic CP Violation

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Based on **XW** and S. Zhou, JHEP 07 (2021) 093 [arXiv:2102.04358]

Content

Background and Motivation

Neutrino oscillation experiments have confirmed that

- Non-zero neutrino masses
- significant leptonic flavor mixing
- Leptonic CP violation?
  - —— *Connected to the cosmological matter-antimatter asymmetry*

How to understand neutrino masses, flavor mixing and CP violation?

Traditional method: Discrete flavor symmetry

$S_3, A_4, S_4, A_5, T', \Delta(27), \Delta(96)$ …

Requires a number of SM singlets (flavons)

A new method: *Modular symmetry !*

$$\text{modulus } \tau \xrightarrow{\gamma} \frac{a\tau + b}{c\tau + d}$$

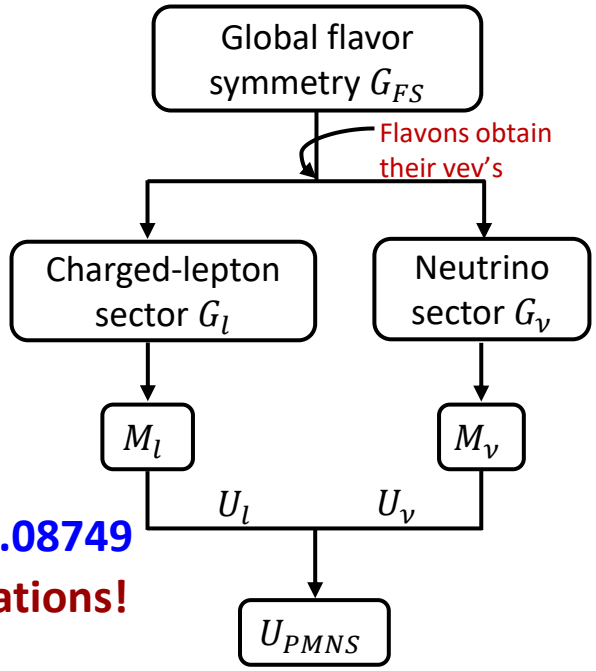
$a, b, c, d \in Z, ad - bc = 1$

$\mathbf{SL(2, Z)}$

Altarelli and Feruglio, NPB, 2006
Toorop, Feruglio and Hagedorn, NPB, 2012

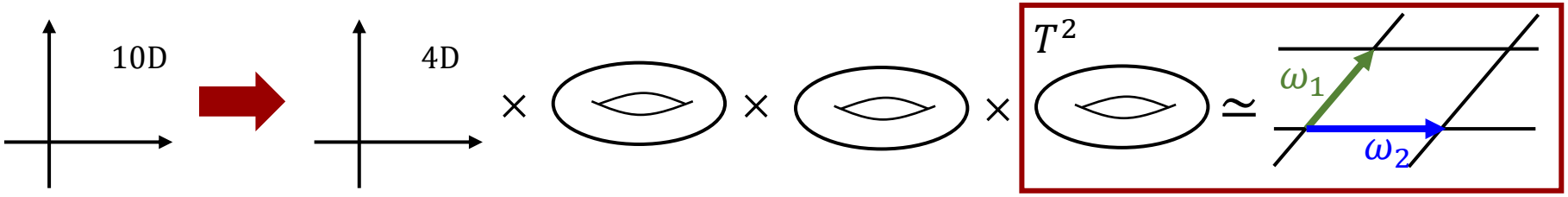
Feruglio, arXiv:1706.08749

127 Citations!



Background and Motivation

Orbifold compactification

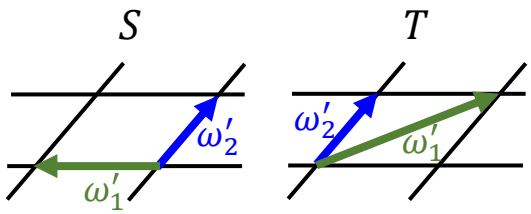


Modular transformation

$a, b, c, d \in Z, ad - bc = 1$

$\frac{\omega_1}{\omega_2} = \tau, \mathrm{Im}\,\tau > 0$

$$\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \to \begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$



S: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ T: $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Modular group $\Gamma \simeq \mathrm{SL}(2, Z)$ (homogeneous)

$\bar{\Gamma} = \Gamma / Z_2$ (inhomogeneous)

Principal congruence subgroups

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma, \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathrm{mod}\ N) \right\}$$

$\rho^{(I)}(\gamma)$: unitary representation of $\Gamma_N^{(\prime)}$

$\Gamma_N \equiv \bar{\Gamma} / \bar{\Gamma}(N)$ **Finite modular group**

$\Gamma_N' \equiv \Gamma / \Gamma(N)$ **Double covering of Γ_N**

e.g., $\Gamma_2^{(\prime)} \simeq S_3^{(\prime)}, \Gamma_3^{(\prime)} \simeq A_4^{(\prime)}, \Gamma_4^{(\prime)} \simeq S_4^{(\prime)}, \Gamma_5^{(\prime)} \simeq A_5^{(\prime)}$

$$\tau \to \gamma\tau = \frac{\omega_1'}{\omega_2'} = \frac{a\tau + b}{c\tau + d}$$

$$\chi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \chi^{(I)}$$

Background and Motivation

Modular forms

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \gamma \in \overline{\Gamma}(N)$$

$$f(\tau + N) = f(\tau)$$

Holomorphic functions of τ with weight k and level N.

$$\boxed{Y_{\mathbf{r}}^{(k)}(\gamma\tau) = (c\tau + d)^k \rho_{\mathbf{r}}(\gamma) Y_{\mathbf{r}}^{(k)}(\tau)\,, \quad \gamma \in \Gamma_N}$$

$Y_{\mathbf{r}}^{(k)} \equiv (f_1(\tau), f_2(\tau), \cdots)^{\mathrm{T}}$ transform as the irreducible representations of Γ_N.

Superpotential remains unchanged ➡

$$\mathcal{W} = \sum_n \sum_{\{I_1,\ldots,I_n\}} Y_{I_1\ldots I_n}(\tau)\chi^{(I_1)}\cdots\chi^{(I_n)}$$

$$Y_{I_1\ldots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho_Y(\gamma) Y_{I_1\ldots I_n}(\tau)$$

Yukawa couplings are **modular forms**!

A $\Gamma_3 \simeq A_4$ example

No flavon!

$$k = 2, Y_{\mathbf{3}}^{(2)} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$$

$$Y_1(\tau) = \frac{\mathrm{i}}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right]$$

$$Y_2(\tau) = \frac{-\mathrm{i}}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega^2\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]$$

$$Y_3(\tau) = \frac{-\mathrm{i}}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega^2\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]$$

$q = e^{2\pi\mathrm{i}\tau}$ **q-expansions**

$$Y_1(\tau) = 1 + 12q + 36q^2 + 12q^3 + \ldots$$

$$Y_2(\tau) = -6q^{1/3}\left(1 + 7q + 8q^2 + \ldots\right)$$

$$Y_3(\tau) = -18q^{2/3}\left(1 + 2q + 5q^2 + \ldots\right)$$

Modular forms with higher weights can be constructed using decomposition rules.

Background and Motivation

Generalized CP (gCP) symmetry

Novichkov *et al.*, JHEP, 2019
Baul *et al.*, PLB, 2019

$$\chi^{(I)}(x) \xrightarrow{\mathrm{CP}} X_{\mathrm{r}}\bar{\chi}^{(I)}(x_{\mathrm{P}})$$
$$\tau \xrightarrow{\mathrm{CP}} -\tau^*$$
$$Y_{\mathrm{r}}^{(k)}(\tau) \xrightarrow{\mathrm{CP}} Y_{\mathrm{r}}^{(k)}(-\tau^*) = \left[Y_{\mathrm{r}}^{(k)}(\tau)\right]^*$$

- All the coupling constants should be real.
- **Modulus τ is the only source of CP violation!**

Stabilizer and residual symmetry

$\tau_L = -1/2 + \sqrt{3}/2\mathrm{i}$ $\quad Z_3^{ST} = \{I, ST, (ST)^2\}$

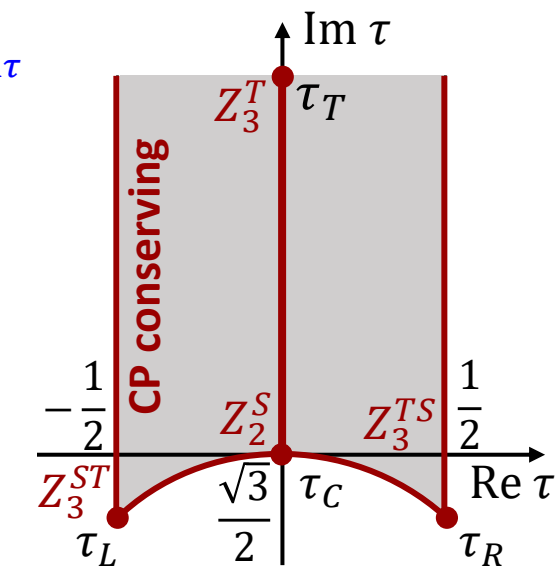
$\tau_R = +1/2 + \sqrt{3}/2\mathrm{i}$ $\quad Z_3^{TS} = \{I, TS, (TS)^2\}$

$\tau_C = \mathrm{i}$ $\quad Z_2^{S} = \{I, S\}$

$\tau_T = \mathrm{i}\infty$ $\quad Z_3^{T} = \{I, T, T^2\}$

Novichkov *et al.*, PLB, 2019
Ding *et al.*, JHEP, 2019
Okada *et al.*, JHEP, 2021
Novichkov *et al.*, JHEP, 2021
Feruglio *et al.*, JHEP, 2021

$q = e^{2\pi\mathrm{i}\tau}$



Fundamental domain

A small deviation of the modulus from its stabilizers may result in large CP violation. Novichkov *et al.*, JHEP, 2019; Okada *et al.*, JHEP, 2021

Background and Motivation

$\Gamma_2 \simeq S_3$: Kobayashi *et al.*, PRD, 2018
Kobayashi *et al.*, PLB, 2019
Okada and Orikasa, PRD, 2019

$\Gamma_3 \simeq A_4$: Criado *et al.*, SciPost Phys., 2018
Kobayashi *et al.*, JHEP, 2018
Okada and Tanimoto, PLB, 2019
Nomura *et al.*, PLB, 2020
Okada and Tanimoto, EPJC, 2021
Zhang, NPB, 2020
Ding, King and Liu, JHEP, 2019
Wang, NPB, 2020
S.J. King and S.F. King, JHEP, 2020
Yao, Lu and Ding, JHEP, 2021
Asaka *et al.*, JHEP, 2020

$\Gamma_4 \simeq S_4$: Penedo and Petcov, NPB, 2019
Novichkov *et al.*, JHEP, 2019
Wang and Zhou, JHEP, 2020
Zhang and Zhou, arXiv: 2106.03433

$\Gamma_5 \simeq A_5$: Novichkov *et al.*, JHEP, 2019
Ding, King and Liu, PRD, 2019
Criado, Feruglio and King, JHEP, 2020

Γ_7 : Ding, King, Li and Zhou, JHEP, 2020

$\Gamma_3' \simeq A_4'$: Liu and Ding, JHEP, 2019

$\Gamma_4' \simeq S_4'$: Novichkov *et al.*, NPB, 2021
Liu, Yao and Ding, PRD, 2021

$\Gamma_5' \simeq A_5'$: **Wang, Yu and Zhou, PRD, 2021**
Yao, Liu and Ding, PRD, 2021

- Attempt to understand modular-symmetry models in an analytical way.
- Combine the modular symmetry with gCP symmetry, and analysis the behavior of CP violation near the stabilizers.

Modular A_5' Model

Superpotential

$$\mathcal{W}_l = \xi_1 \left[\widehat{L}_e \widehat{E}_1^{\mathrm{C}}\right]_{\mathbf{1}} \widehat{H}_{\mathrm{d}} + \xi_2 \left[\left(\widehat{L}_{\mu\tau} \widehat{E}_{23}^{\mathrm{C}}\right)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)}\right]_{\mathbf{1}} \widehat{H}_{\mathrm{d}} + \xi_3 \left[\left(\widehat{L}_{\mu\tau} \widehat{E}_{23}^{\mathrm{C}}\right)_{\mathbf{1}} Y_{\mathbf{1}}^{(4)}\right]_{\mathbf{1}} \widehat{H}_{\mathrm{d}}$$

$$\mathcal{W}_{\mathrm{D}} = g_1 \left[\left(\widehat{L}_e \widehat{N}^{\mathrm{C}}\right)_{\mathbf{3}'} Y_{\mathbf{3}'}^{(2)}\right]_{\mathbf{1}} \widehat{H}_{\mathrm{u}} + g_2 \left[\left(\widehat{L}_{\mu\tau} \widehat{N}^{\mathrm{C}}\right)_{\widehat{\mathbf{6}}} Y_{\widehat{\mathbf{6}},1}^{(3)}\right]_{\mathbf{1}} \widehat{H}_{\mathrm{u}} + g_3 \left[\left(\widehat{L}_{\mu\tau} \widehat{N}^{\mathrm{C}}\right)_{\widehat{\mathbf{6}}} Y_{\widehat{\mathbf{6}},2}^{(3)}\right]_{\mathbf{1}} \widehat{H}_{\mathrm{u}}$$

$$\mathcal{W}_{\mathrm{R}} = \frac{1}{2}\Lambda \left[\left(\widehat{N}^{\mathrm{C}} \widehat{N}^{\mathrm{C}}\right)_{\mathbf{5}} Y_{\mathbf{5}}^{(2)}\right]_{\mathbf{1}}$$

	SU(2)	A_5'	$-k_I$
$\widehat{L}_e$	2	$\mathbf{1}$	1
$\widehat{L}_{\mu\tau}$	2	$\widehat{\mathbf{2}}$	2
$\widehat{E}_{\mathbf{1}}^{\mathrm{C}}$	1	$\mathbf{1}$	-1
$\widehat{E}_{\mathbf{23}}^{\mathrm{C}}$	1	$\widehat{\mathbf{2}}$	2
$\widehat{N}^{\mathrm{C}}$	1	$\mathbf{3}'$	1
$\widehat{H}_{\mathrm{u,d}}$	2	$\mathbf{1}$	0

Mass matrices

Charged-lepton mass matrix

$$\boldsymbol{M_l} = \frac{v_{\mathrm{d}}}{\sqrt{2}} \begin{pmatrix} \xi_1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{3}\xi_2 \left(Y_{\mathbf{3}}^{(4)}\right)_2 & \xi_2 \left[\frac{\sqrt{6}}{6}\left(Y_{\mathbf{3}}^{(4)}\right)_1 - \frac{\sqrt{2}}{2}\widetilde{\xi} Y_{\mathbf{1}}^{(4)}\right] \\ 0 & \xi_2 \left[\frac{\sqrt{6}}{6}\left(Y_{\mathbf{3}}^{(4)}\right)_1 + \frac{\sqrt{2}}{2}\widetilde{\xi} Y_{\mathbf{1}}^{(4)}\right] & -\frac{\sqrt{3}}{3}\xi_2 \left(Y_{\mathbf{3}}^{(4)}\right)_3 \end{pmatrix}^*$$

$$\boldsymbol{M_{\mathrm{D}}} = \frac{\sqrt{6} g_1 v_{\mathrm{u}}}{12} \left[\begin{pmatrix} 2\left(Y_{\mathbf{3}'}^{(2)}\right)_1 & 2\left(Y_{\mathbf{3}'}^{(2)}\right)_3 & 2\left(Y_{\mathbf{3}'}^{(2)}\right)_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \widetilde{g}_2 \begin{pmatrix} 0 & 0 & 0 \\ -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_4 & -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_2 & \left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_1 - \left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_6 \\ -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_3 & \left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_1 + \left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_6 & -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},1}^{(3)}\right)_5 \end{pmatrix} \right.$$

$$\left. + \widetilde{g}_3 \begin{pmatrix} 0 & 0 & 0 \\ -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_4 & -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_2 & \left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_1 - \left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_6 \\ -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_3 & \left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_1 + \left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_6 & -\sqrt{2}\left(Y_{\widehat{\mathbf{6}},2}^{(3)}\right)_5 \end{pmatrix} \right]^*$$

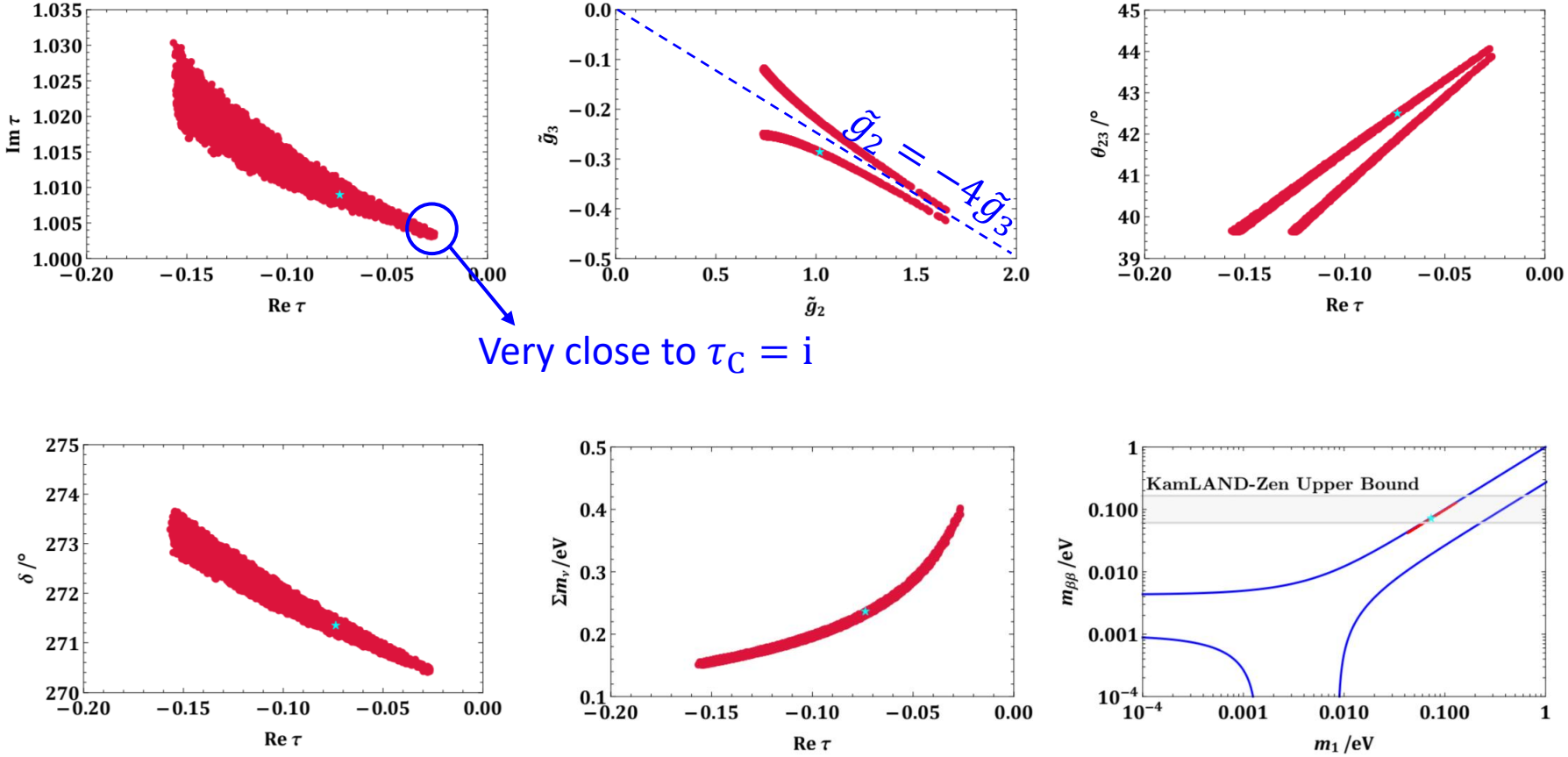
$$\boldsymbol{M_{\mathrm{R}}} = \frac{\Lambda}{4} \begin{pmatrix} 2\left(Y_{\mathbf{5}}^{(2)}\right)_1 & -\sqrt{3}\left(Y_{\mathbf{5}}^{(2)}\right)_4 & -\sqrt{3}\left(Y_{\mathbf{5}}^{(2)}\right)_3 \\ -\sqrt{3}\left(Y_{\mathbf{5}}^{(2)}\right)_4 & \sqrt{6}\left(Y_{\mathbf{5}}^{(2)}\right)_2 & -\left(Y_{\mathbf{5}}^{(2)}\right)_1 \\ -\sqrt{3}\left(Y_{\mathbf{5}}^{(2)}\right)_3 & -\left(Y_{\mathbf{5}}^{(2)}\right)_1 & \sqrt{6}\left(Y_{\mathbf{5}}^{(2)}\right)_5 \end{pmatrix}^*$$

Dirac neutrino mass matrix

Majorana neutrino mass matrix

Modular A_5' Model

Numerical results



Very close to $\tau_C = \mathrm{i}$

$\delta \approx 270°$, nearly-maximal CP violation

Analytical Perturbations

$\tau = i$ exactly holds

$$\frac{\widehat{e}_2(\mathrm{i})}{\widehat{e}_1(\mathrm{i})} = \frac{\widehat{e}_3(\mathrm{i})}{\widehat{e}_2(\mathrm{i})} = \frac{\widehat{e}_4(\mathrm{i})}{\widehat{e}_3(\mathrm{i})} = \frac{\widehat{e}_5(\mathrm{i})}{\widehat{e}_4(\mathrm{i})} = \frac{\widehat{e}_6(\mathrm{i})}{\widehat{e}_5(\mathrm{i})} = A_0$$

$$A_0 = \sqrt{\sqrt{5}\phi - \phi} \qquad \phi = \frac{\sqrt{5}+1}{2}$$

$$Y_1(\mathrm{i}) = \widehat{e}_1(\mathrm{i})(1 - 3\,A_0^5)\,, \quad Y_2(\mathrm{i}) = 5\sqrt{2}\,\widehat{e}_1(\mathrm{i})A_0\,, \quad Y_3(\mathrm{i}) = 10\,\widehat{e}_1(\mathrm{i})A_0^2\,,$$
$$Y_4(\mathrm{i}) = 10\,\widehat{e}_1(\mathrm{i})A_0^3\,, \quad Y_5(\mathrm{i}) = 5\sqrt{2}\,\widehat{e}_1(\mathrm{i})A_0^4\,, \quad Y_6(\mathrm{i}) = -\widehat{e}_1(\mathrm{i})(3 + A_0^5)$$

Weight-one basis vectors

$$\widehat{e}_1 = 1 + 3q + 4q^2 + 2q^3 + \cdots\,,$$
$$\widehat{e}_2 = q^{1/5}\left(1 + 2q + 2q^2 + q^3 + \cdots\right)\,,$$
$$\widehat{e}_3 = q^{2/5}\left(1 + q + q^2 + q^3 \cdots\right)\,,$$
$$\widehat{e}_4 = q^{3/5}\left(1 + q^2 + q^3 \cdots\right)\,,$$
$$\widehat{e}_5 = q^{4/5}\left(1 - q + 2q^2 + \cdots\right)\,,$$
$$\widehat{e}_6 = q\left(1 - 2q + 4q^2 - 3q^3 + \cdots\right)$$

Wang, Yu and Zhou, PRD, 2021

Charged-lepton sector

$$U_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_l & -\sin\theta_l \\ 0 & \sin\theta_l & \cos\theta_l \end{pmatrix}$$

$\tan 2\theta_l = \sqrt{(\phi - 1)/\phi} \approx 0$

Approximately diagonal

Neutrino sector (In the M_l-diagonal basis)

$$\widetilde{M}_\nu = \frac{\widehat{g}_1^2 v_{\mathrm{u}}^2}{\mathrm{Det}(M_{\mathrm{R}})} \begin{pmatrix} 50.67 & 0 & \boxed{-17.19(\widehat{g}_2 + 4\widehat{g}_3)} \\ 0 & 0 & 0 \\ \boxed{-17.19(\widehat{g}_2 + 4\widehat{g}_3)} & 0 & \boxed{5.834(\widehat{g}_2 + 4\widehat{g}_3)^2} \end{pmatrix}$$

$\widehat{g}_1 = g_1|\widehat{e}_1(\tau)|^2\,, \quad \widehat{g}_{2,3} = \widetilde{g}_{2,3}|\widehat{e}_1(\tau)|$

$= 0$ if $\hat{g}_2 = -4\hat{g}_3$

Two-step perturbation

- Assume $\hat{g}_2 = -4\hat{g}_3$ holds and introduce perturbation $\boldsymbol{\tau = \mathrm{i} + \epsilon}$
- Break the above identity by requiring $\boldsymbol{\widehat{g}_3 = -\widehat{g}_2/4 + \kappa}$

Analytical Perturbations

$\tau = \mathrm{i} + \epsilon$

$\epsilon = \epsilon_{\mathrm{R}} + \mathrm{i}\epsilon_{\mathrm{I}}$

$$\widetilde{M}_\nu \approx -\frac{\widehat{g}_1^2 v_{\mathrm{u}}^2}{\Lambda\epsilon}\begin{pmatrix} 0.089\,\mathrm{i} - 0.331\epsilon & 0 & 0 \\ 0 & -0.823\,\mathrm{i}\widehat{g}_2^2\epsilon^2 & \widehat{g}_2^2\epsilon(1.258 + 5.731\,\mathrm{i}\epsilon) \\ 0 & \widehat{g}_2^2\epsilon(1.258 + 5.731\,\mathrm{i}\epsilon) & 2.332\,\mathrm{i}\widehat{g}_2^2\epsilon^2 \end{pmatrix}^*$$

- (2,3)-rotation $\sin\theta_{23} \approx \sqrt{2}/2 + 0.212\epsilon_{\mathrm{R}}$

$\mu_0 \equiv (\widehat{g}_1^2 v_{\mathrm{u}}^2)/(\Lambda|\epsilon|)$

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\mathrm{i} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

$$m_{1,0}^2 \approx (0.00793 - 0.0590\epsilon_{\mathrm{I}} - 0.0934\epsilon_{\mathrm{R}}^2)\mu_0^2$$
$$m_{2,0}^2 \approx (1.583 + 3.971\epsilon_{\mathrm{R}} - 14.42\epsilon_{\mathrm{I}} + 35.91\epsilon_{\mathrm{R}}^2)\widehat{g}_2^4\epsilon_{\mathrm{R}}^2\mu_0^2$$
$$m_{3,0}^2 \approx (1.583 - 3.971\epsilon_{\mathrm{R}} - 14.42\epsilon_{\mathrm{I}} + 35.91\epsilon_{\mathrm{R}}^2)\widehat{g}_2^4\epsilon_{\mathrm{R}}^2\mu_0^2$$

- (1,3)-rotation

$$U_{13} = \begin{pmatrix} e^{\mathrm{i}\varphi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ 0 & 1 & 0 \\ \sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$$

$$\sin\theta_{13} \approx \frac{\mu_0^2|\kappa|\left(0.00761 - 0.108\,\widehat{g}_2^2\epsilon_{\mathrm{R}}\right)}{m_{3,0}^2 - m_{1,0}^2}$$

$$\varphi \approx \arctan\left(\frac{0.0705 - \widehat{g}_2^2\epsilon_{\mathrm{R}}}{\widehat{g}_2^2\epsilon_{\mathrm{I}}}\right)$$

- (1,2)-rotation

$$U_{12} = \begin{pmatrix} e^{-\mathrm{i}\varphi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

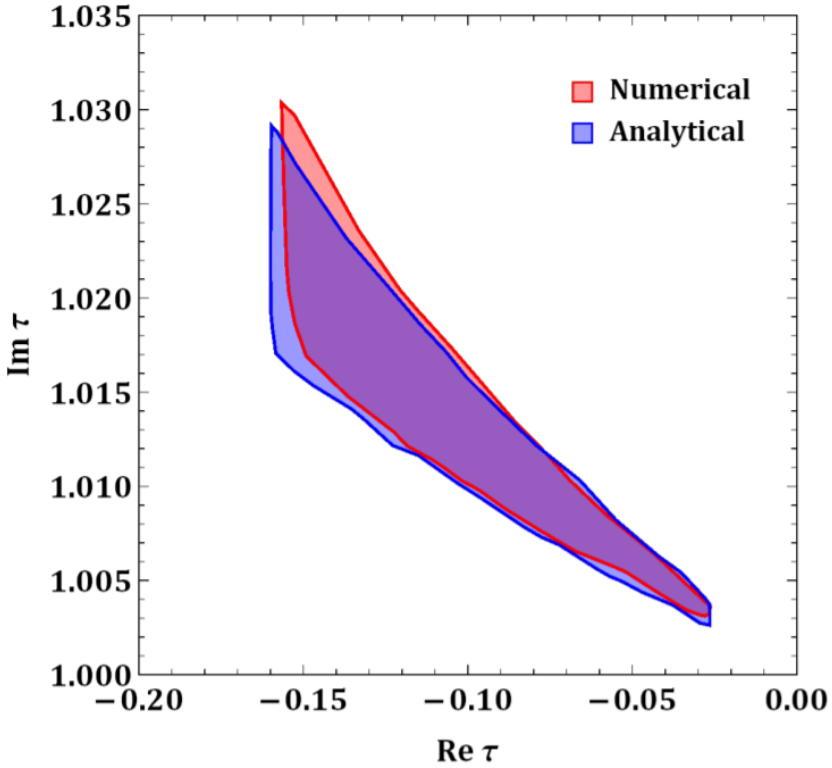
$$\sin^2\theta_{12} \approx \frac{0.108\,\mu_0^2\widehat{g}_2^2\epsilon_{\mathrm{I}}|\kappa|}{m_{2,0}^2 - m_{1,0}^2 + 0.216\,\mu_0^2\widehat{g}_2^2\epsilon_{\mathrm{I}}|\kappa|}$$

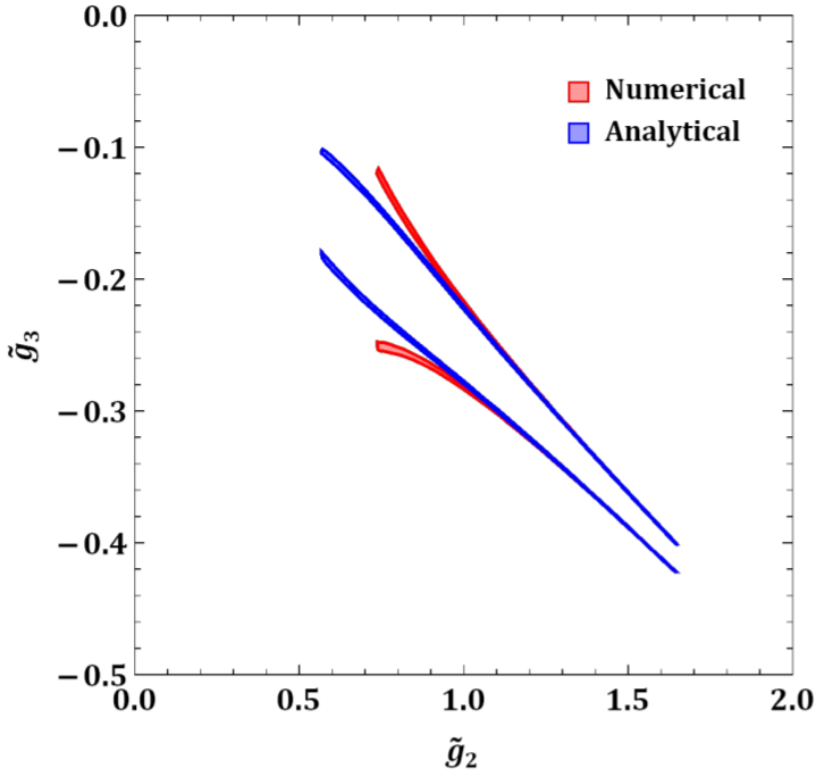
Neutrino masses

$$m_1^2 \approx m_{1,0}^2 - 0.108\,\mu_0^2\widehat{g}_2^2\epsilon_{\mathrm{I}}|\kappa|$$
$$m_2^2 \approx m_{2,0}^2 + 0.108\,\mu_0^2\widehat{g}_2^2\epsilon_{\mathrm{I}}|\kappa| \quad m_3^2 \approx m_{3,0}^2$$

Analytical Perturbations

Comparison between analytical and numerical results





Analytical Perturbations

Leptonic CP violation

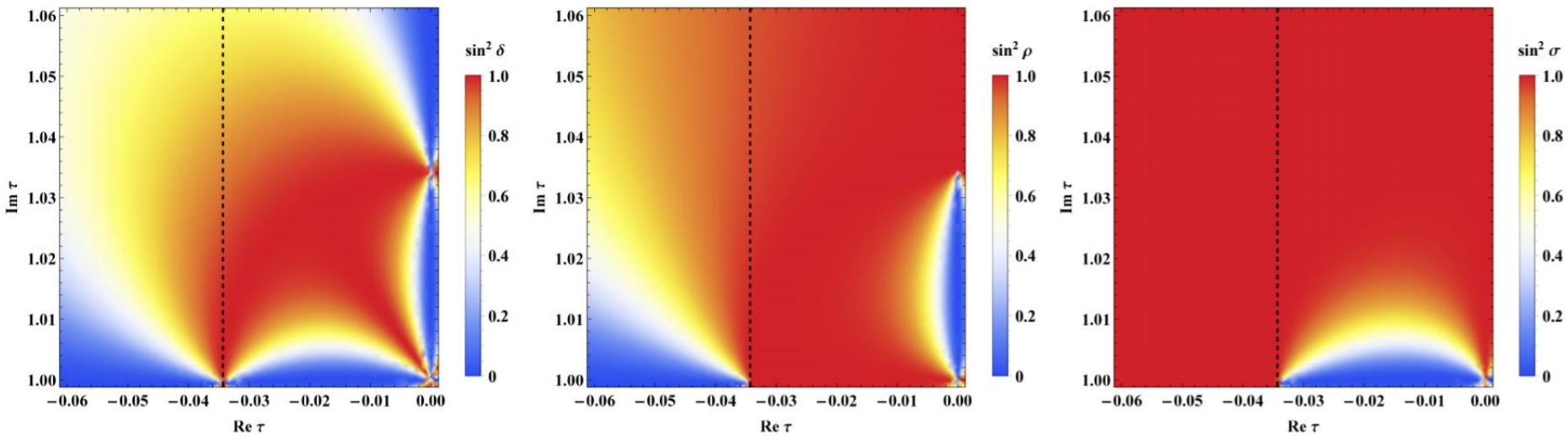
CP-violating phases

$$\delta = 2\pi - \varphi \approx 2\pi - \arctan\left(\frac{0.0705 - \widehat{g}_2^2 \epsilon_{\rm R}}{\widehat{g}_2^2 \epsilon_{\rm I}}\right)$$

$$\rho \approx \frac{\pi}{2} + \arctan\left(\frac{\epsilon_{\rm I}}{\epsilon_{\rm R}}\right)$$

$$\sigma \approx \frac{\pi}{2} - \arctan\left(\frac{\epsilon_{\rm I}}{\epsilon_{\rm R}}\right)$$

$\epsilon_{\rm I} \ll \epsilon_{\rm R}$, nearly-maximal CP violation even if τ is close to i.



Radiative Corrections

One-loop RG equations

Machacek and Vaughn, NPB, 1984
Arason *et al.*, PRD, 1992
Castano *et al.*, PRD, 1994
Chankowski and Pluciennik, PLB, 1993
Babu *et al.*, PLB, 1993
Antusch *et al.*, PLB, 2001

$$16\pi^2 \frac{\mathrm{d}\widetilde{Y}_l}{\mathrm{d}t} = \left[\alpha_l + 3\left(\widetilde{Y}_l \widetilde{Y}_l^\dagger\right)\right]\widetilde{Y}_l$$

$$16\pi^2 \frac{\mathrm{d}\mathcal{M}}{\mathrm{d}t} = \alpha_\nu \mathcal{M} + \left[\left(\widetilde{Y}_l \widetilde{Y}_l^\dagger\right)\mathcal{M} + \mathcal{M}\left(\widetilde{Y}_l \widetilde{Y}_l^\dagger\right)^{\mathrm{T}}\right]$$

$$Y_\nu = \sqrt{2} M_{\mathrm{D}}/v_{\mathrm{u}}$$

$$\mathcal{M} \equiv -U_l^\dagger Y_\nu M_{\mathrm{R}}^{-1} Y_\nu^{\mathrm{T}} U_l^*$$

$$\mathcal{M}(m_Z) = I_\nu \begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix} \mathcal{M}(\Lambda_{\mathrm{SS}}) \begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix}$$

Mei and Xing, PRD, 2004

$$I_\nu = \exp\left[-\frac{1}{16\pi^2}\int_0^{\ln(\Lambda_{\mathrm{SS}}/m_Z)} \alpha_\nu(t)\mathrm{d}t\right]$$

I_ν affects the absolute scale of neutrino masses.

$$I_\alpha = \exp\left[-\frac{1}{16\pi^2}\int_0^{\ln(\Lambda_{\mathrm{SS}}/m_Z)} y_\alpha^2(t)\mathrm{d}t\right]$$

I_α modify both neutrino masses and flavor mixing parameters. $I_e \approx I_\mu \approx 1$, I_τ is dominant.

$$m_1^2(m_Z) \approx m_{1,0}^2 - 0.108\, I_\tau \mu_0^2 \widehat{g}_2^2 \epsilon_{\mathrm{I}} |\kappa|$$

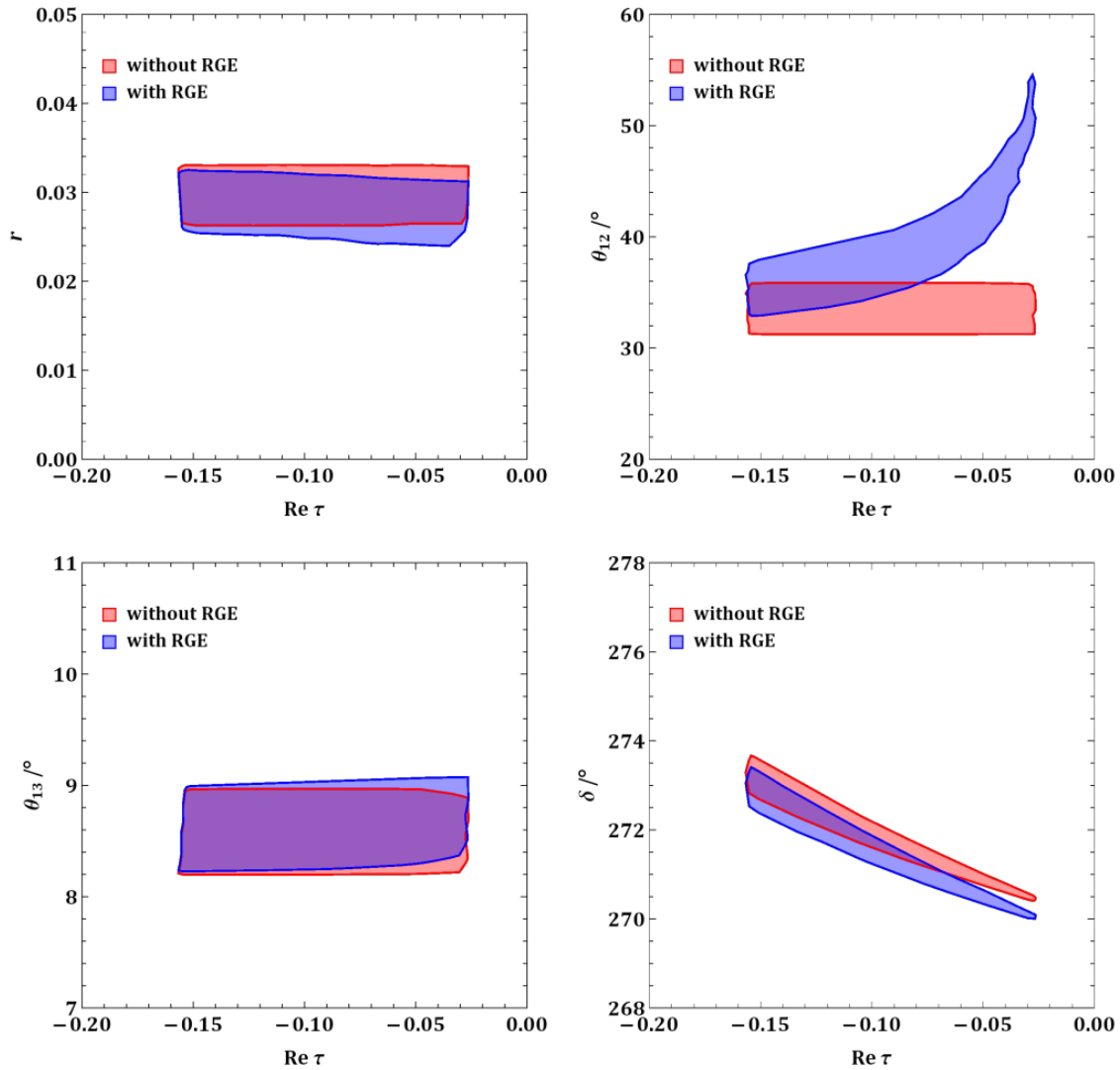
$$m_2^2(m_Z) \approx I_\tau^2 m_{2,0}^2 + 0.108\, I_\tau \mu_0^2 \widehat{g}_2^2 \epsilon_{\mathrm{I}} |\kappa|$$

$$m_3^2(m_Z) \approx I_\tau^2 m_{3,0}^2$$

$$\sin^2\theta_{12}(m_Z) \approx \frac{0.108\, I_\tau \mu_0^2 \widehat{g}_2^2 \epsilon_{\mathrm{I}} |\kappa|}{\underline{I_\tau^2 m_{2,0}^2 - m_{1,0}^2} + 0.216\, I_\tau \mu_0^2 \widehat{g}_2^2 \epsilon_{\mathrm{I}} |\kappa|}$$

$m_{2,0}^2 \approx m_{1,0}^2$, $I_\tau^2 m_{2,0}^2 - m_{1,0}^2$ can lead to large corrections to $\sin^2\theta_{12}$ even if $I_\tau - 1$ is small.

Radiative Corrections



Summary

- It is interesting to explore the basic properties of modular-symmetry models with a modulus τ in the vicinity of the stabilizers.
- we construct a feasible lepton flavor model based on the modular A_5' group combined with the gCP symmetry, and explain why it predicts a nearly-maximal CP-violating phase within the neighbourhood of $\tau = \mathrm{i}$ in an analytical way.
- The RG running effect turns out to be significant in our model due to the mass degeneracy between m_1 and m_2.
- It does shed some light on the common features of the modular-symmetry models that seem to be "unstable" around the stabilizers.

Thank you!