A modular S_4 model for neutrino mixing and dark matter

Xinyi Zhang (张忻怿)

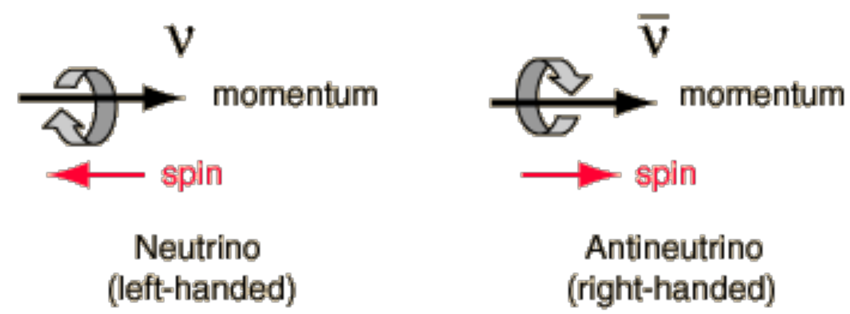
IHEP zhangxinyi@ihep.ac.cn

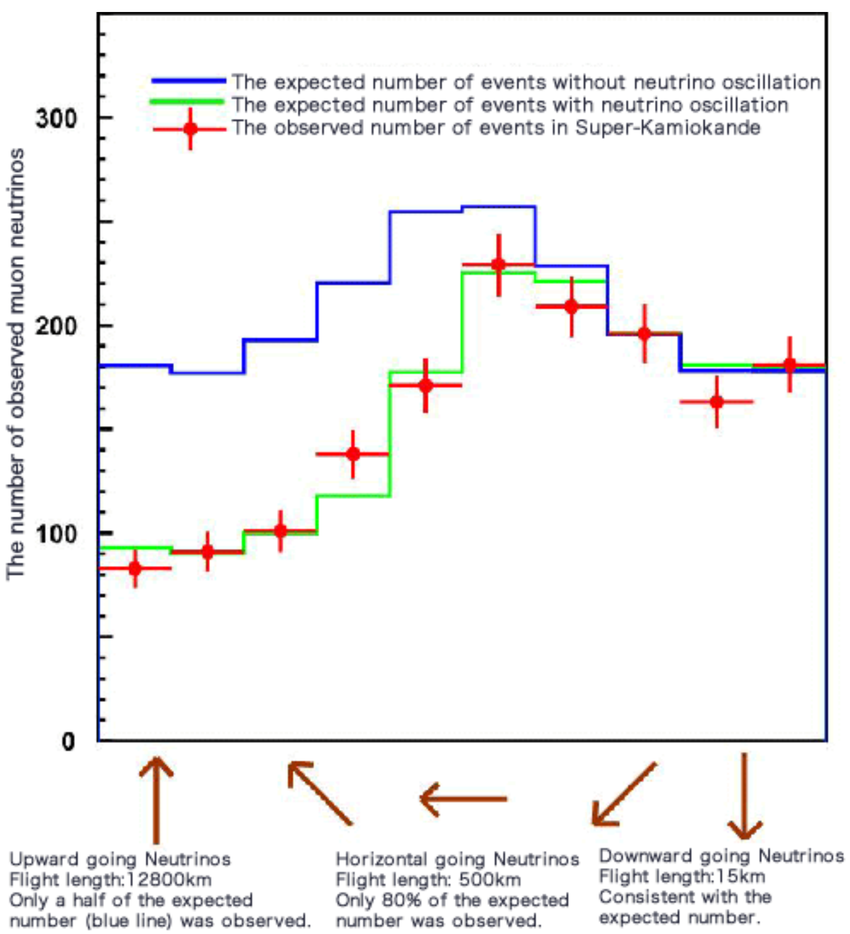
In collaboration with Shun Zhou, based on arXiv: 2106.03433

中国物理学会高能物理分会第十三届全国粒子物理学术会议, 2021年8月17日

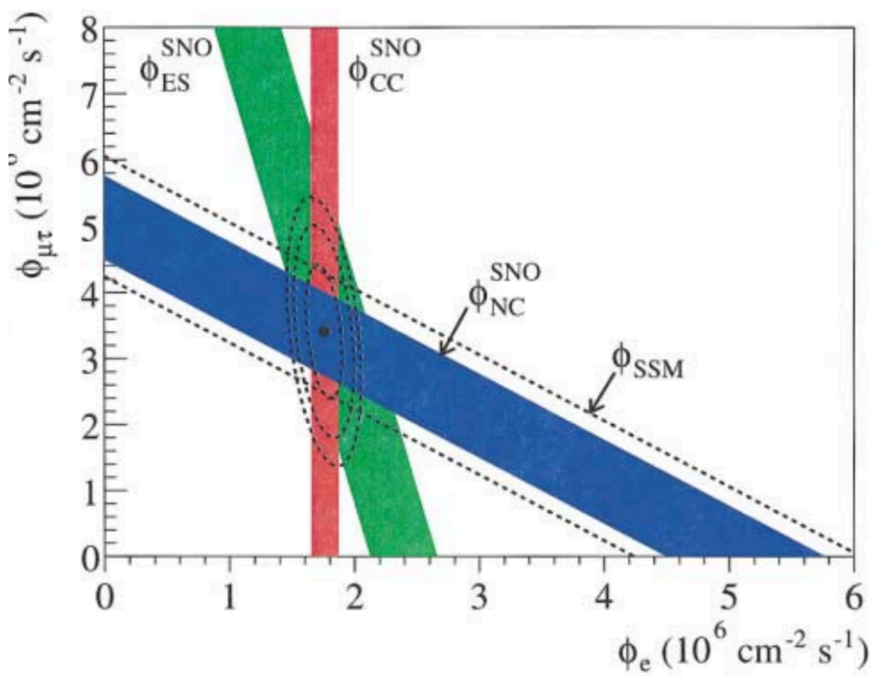
Oscillating neutrinos

Neutrinos in the Standard Model:
massless, interact only weakly





http://www.hyper-k.org/



Phys.Rev.Lett. 89 (2002), 011301

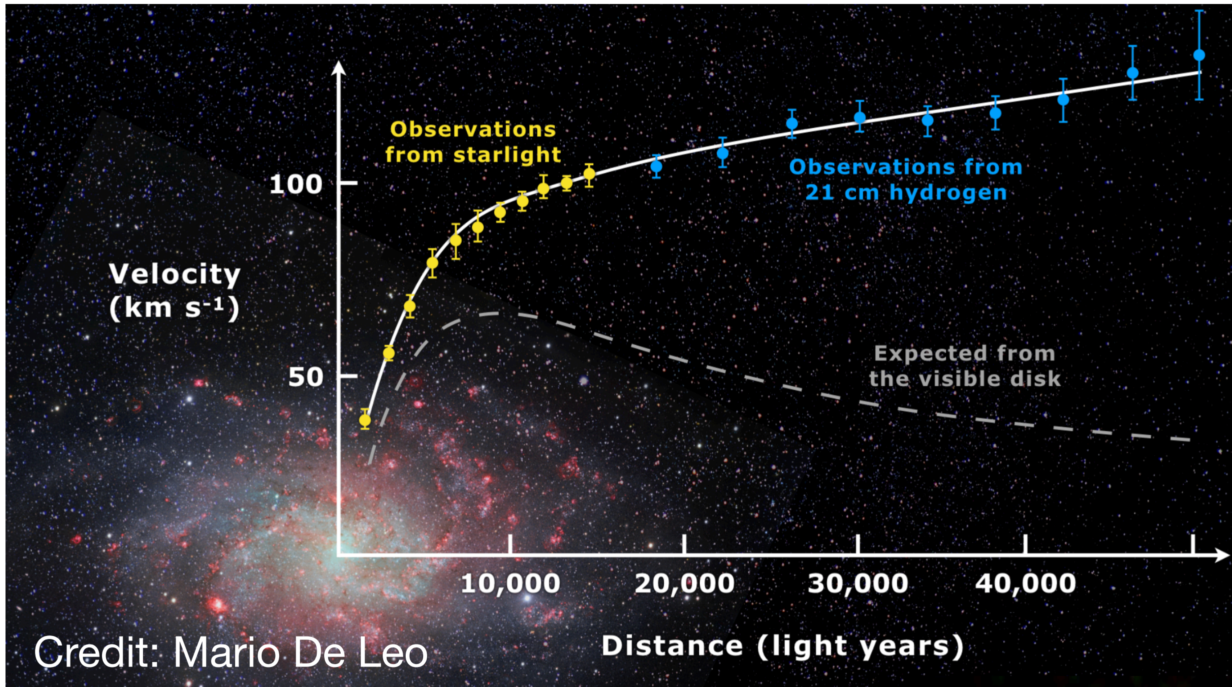
- Neutrinos have mass
- Mass eigenstates do not match flavor (weak) eigenstates
  → neutrino mixing

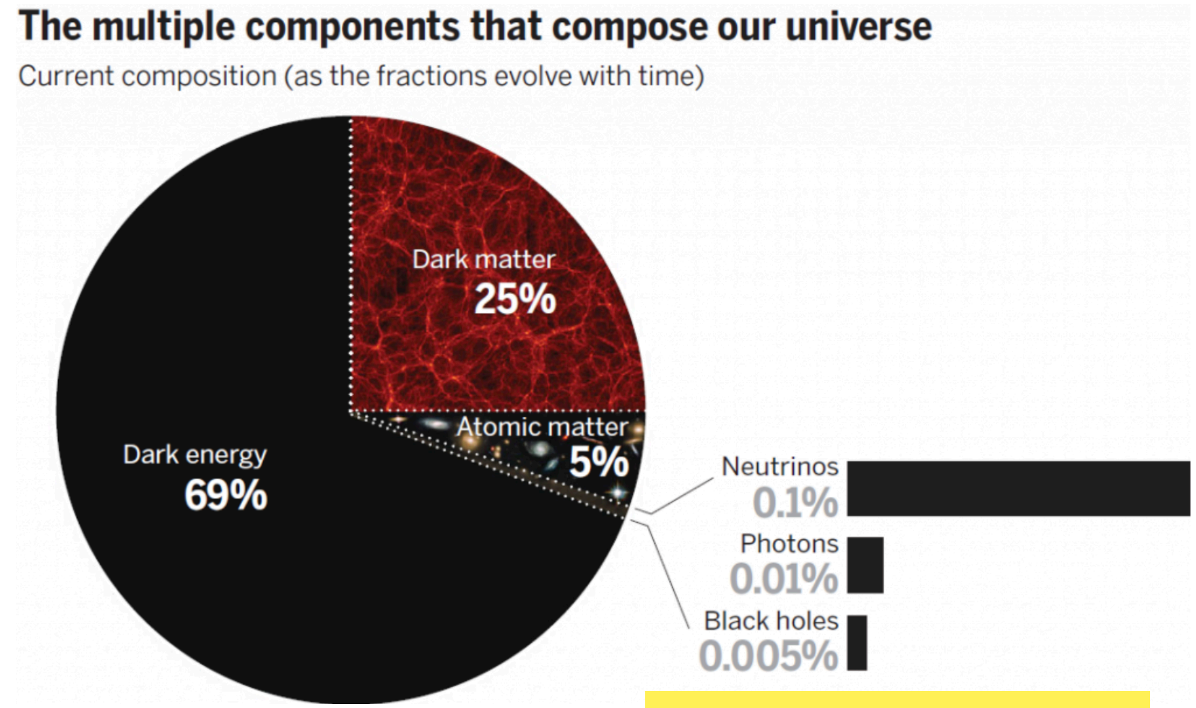
$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right)$$

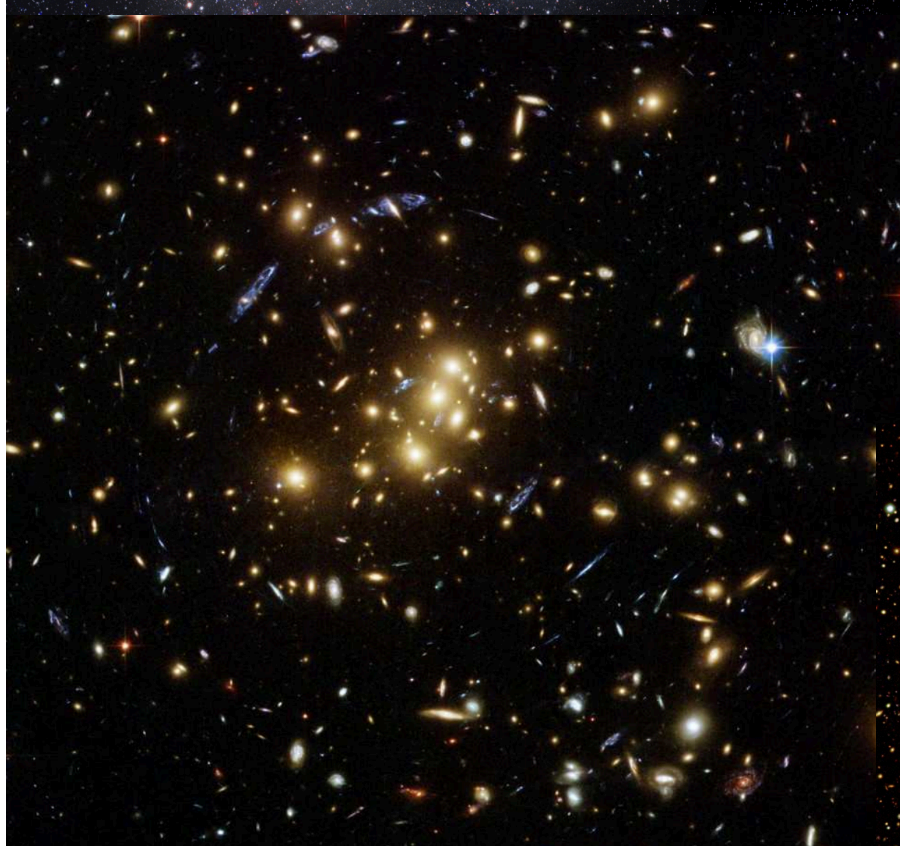
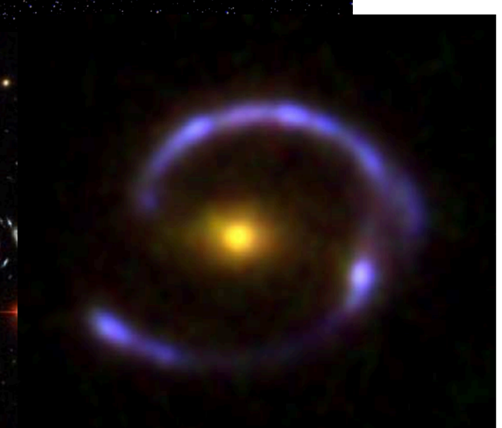
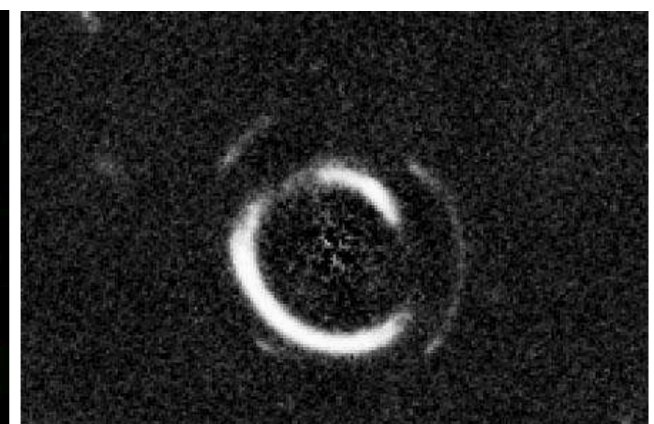
Takaaki Kajita and Arthur B. McDonald shared the 2015 Nobel Prize in Physics,
"for the discovery of neutrino oscillations, which shows that neutrinos have mass".

Solid beyond Standard Model physics

Dark matter

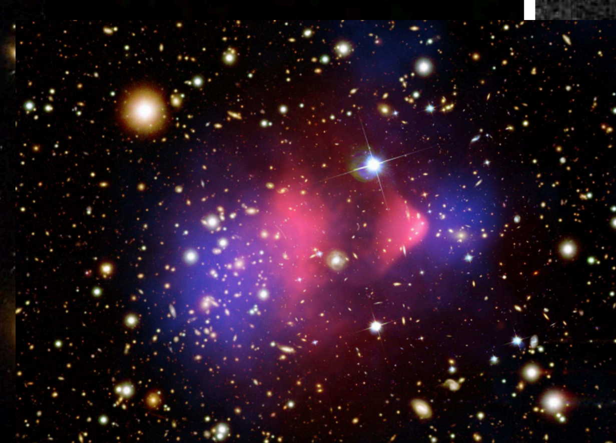
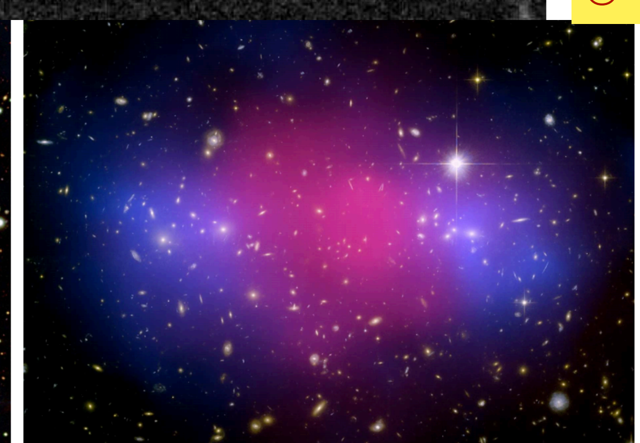




General consensus on the existence of dark matter

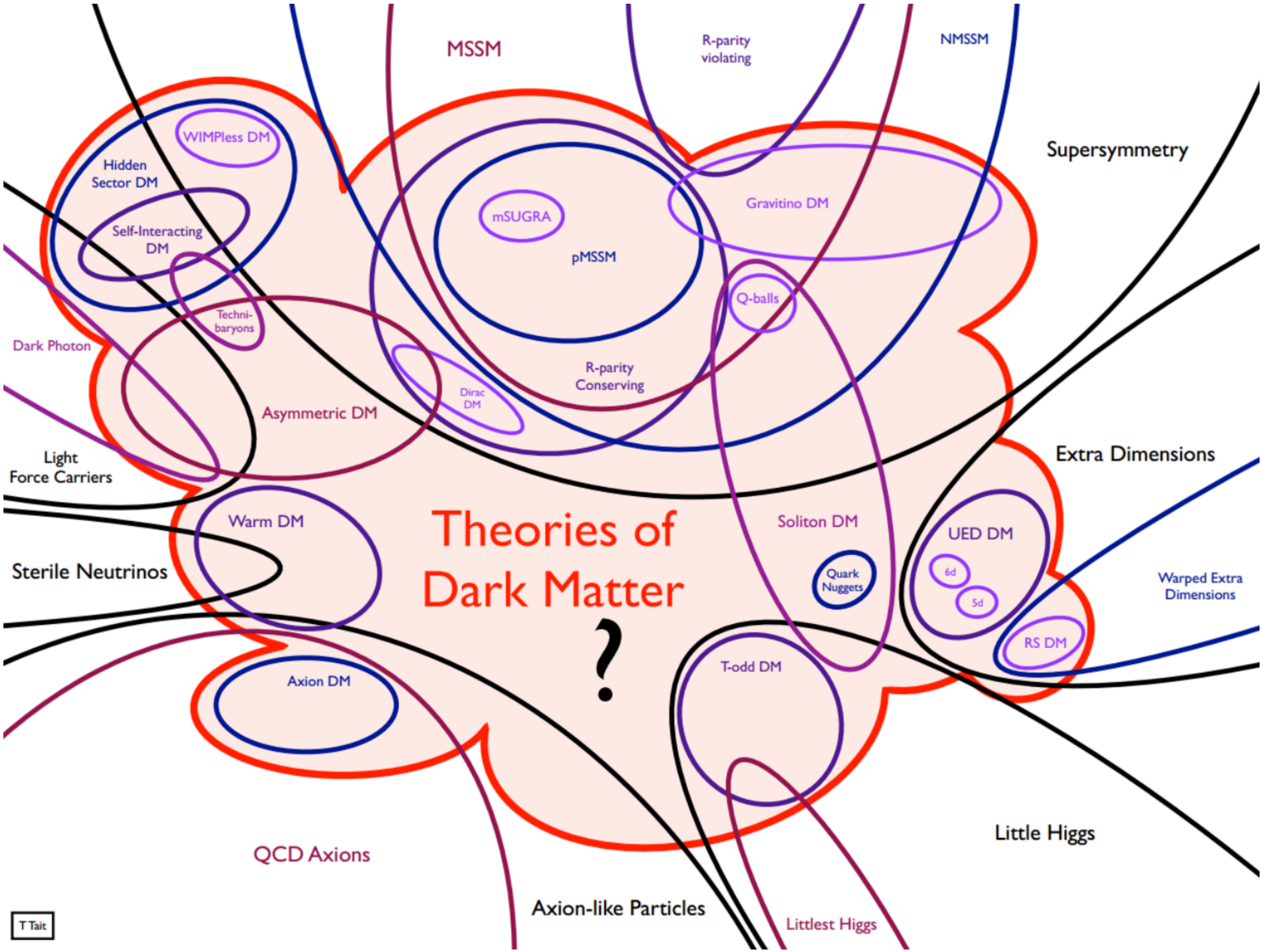
- Non-baryonic
- Cold
- Collision-less

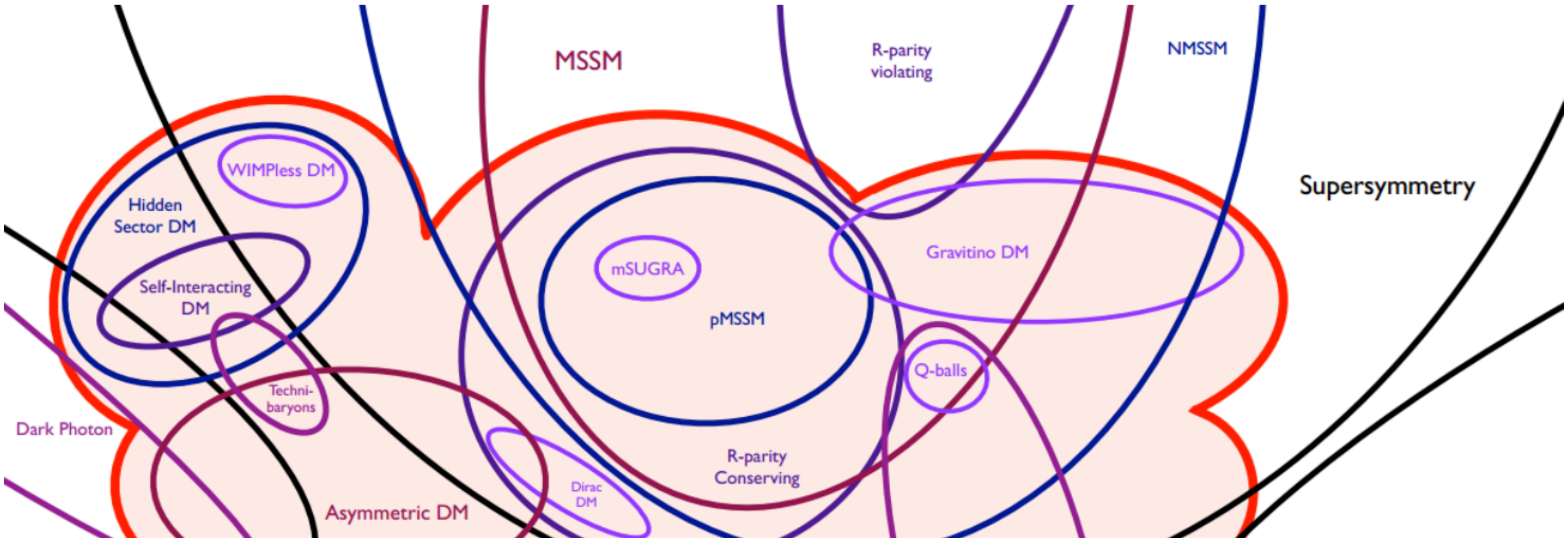
R. Massey, T. Kitching, RPP 2010

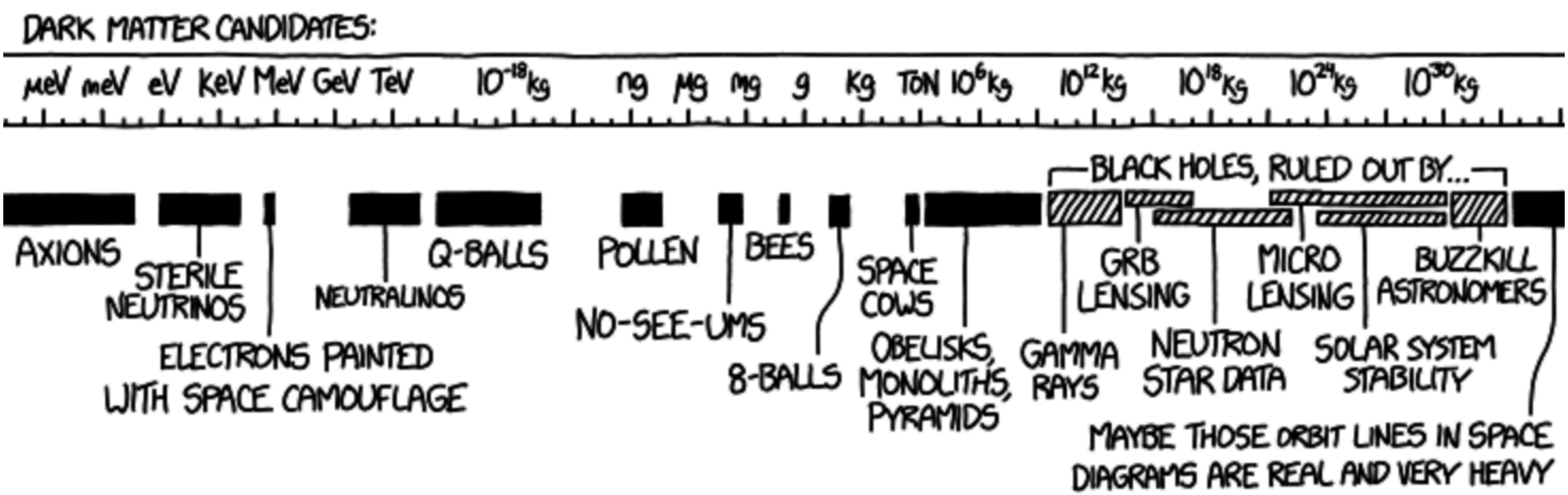
Another beyond Standard Model physics

A closer look at dark matter: theoretical candidates



A closer look at dark matter: theoretical candidates





https://explainxkcd.com/wiki/index.php/2035:_Dark_Matter_Candidates

A closer look at dark matter

Require:

- a single particle DM candidate (not a dark sector)
- a relation with neutrino physics

DM candidates：

- [x] **Neutrinos** **0.5% - 1.5%**
- [x] **keV sterile neutrinos** This work
- [ ] **Majoron**
- [ ] **Exotic fermions**

Possible, but another story, not tell it here

The framework explains two BSM physics at the same time

A closer look at neutrino mass & mixing

Why it is massive?

Why it is so light?

With only SM fields, S. Weinberg, 1980

$$\frac{1}{\Lambda}\bar{L}^c \otimes \Phi \otimes \Phi \otimes L$$

$$\underbrace{\underbrace{\bar{L}^c \otimes \Phi}_{1} \otimes \underbrace{\Phi \otimes L}_{1}}_{\text{Type I}}, \quad \underbrace{\underbrace{\bar{L}^c \otimes L}_{3} \otimes \underbrace{\Phi \otimes \Phi}_{3}}_{\text{Type II}}, \quad \underbrace{\underbrace{\bar{L}^c \otimes \Phi}_{3} \otimes \underbrace{\Phi \otimes L}_{3}}_{\text{Type III}}$$

Seesaw paradise (mostly Majorana neutrino)

Tree-level realization of Weinberg operator: Type I, II, III & …

Loop-level realization of Weinberg operator: radiative seesaw

Low-scale seesaw: linear, inverse, …

Other variants: Dirac seesaw

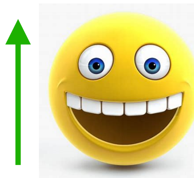
P. Minkowski,1977; T. Yanagida, ,1979; J. Schechter and J. W. F. Valle, 1980, …

Why mixing so?

Flavour (horizontal) **symmetry**

Constant mixing @LO:

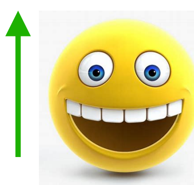
bimaximal mixing, tribimaximal mixing, …



Discrete flavor symmetry:

$A_4, S_3, S_4, A_5, \ldots$

Flavons # & vacuum aligments



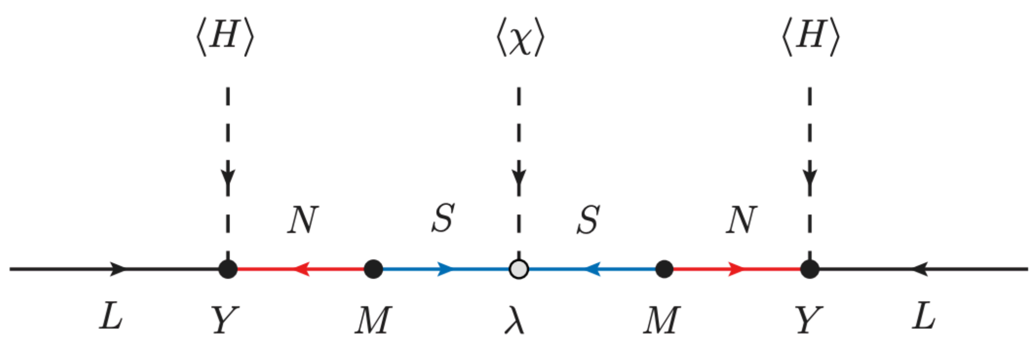
Modular symmetry:

$\Gamma_2 \simeq S_3, \Gamma_3 \simeq A_4, \Gamma_4 \simeq S_4, \Gamma_5 \simeq A_5$

S. F. King, 2017; S. T. Petcov, 2018;Z. z. Xing, 2019.

Inverse seesaw & dark matter

R.N. Mohapatra & J.W.F. Valle, 1986
M. C. Gonzalez-Garcia & J. W. F. Valle, 1989



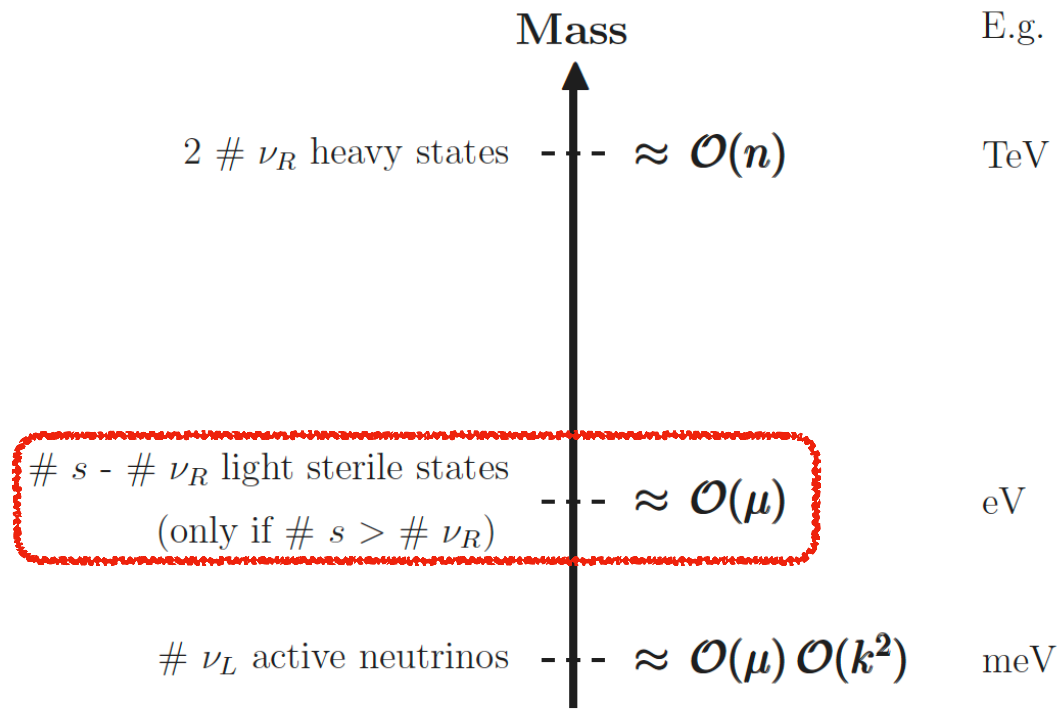
S. C. Chulia et al, 2020

$$\mathcal{L}_m = \begin{pmatrix} \bar{\nu}^c & \bar{N}^c & \bar{S}^c \end{pmatrix} \begin{pmatrix} 0 & Y v & 0 \\ Y^T v & \mu' & M^T \\ 0 & M & \mu \end{pmatrix} \begin{pmatrix} \nu \\ N \\ S \end{pmatrix}$$

LNV terms are small - naturalness

$$\mu, \mu' \ll Y v \ll M \,, \;\; m_\nu = -Y^2 \frac{v^2 \mu}{M^2} \,,$$

Neutrinos are light due to the smallness of LNV effects



A. Abada & M. Lucente,1401.1507

Appearance of an intermediate mass state!

$$\left(\frac{m_\nu}{0.1 \;\; \text{eV}}\right) = \left(\frac{M_D}{100 \;\; \text{GeV}}\right)^2 \left(\frac{\mu}{\text{keV}}\right) \left(\frac{M_{NS}}{10^4 \; \text{GeV}}\right)^{-2}$$

This mass state @ keV fits ν mass!

Warm dark matter

For reviews, see, e.g.,
K. Abazajian, G. M. Fuller and M. Patel, 2001;
A. Kusenko, 2009; A. Merle, 2013, 2017.

Models

Basic idea

Inverse seesaw → keV dark matter
Modular symmetry → light neutrino masses & mixings

Guiding principle: **Simplicity & Generality**

modular form multiplets with weights ≤ 4

all allowed couplings included

	L	$H_{\rm u}$	$H_{\rm d}$	$E_1^{\rm C}$	$E_2^{\rm C}$	$E_3^{\rm C}$	$N^{\rm C}$	S
SU(2)	2	2	2	1	1	1	1	1
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{3}$

→Minimal group: S_4

Charged lepton sector $W_l = \alpha \left(LE_1^{\rm C}\right)_{\mathbf{3}'} Y_{\mathbf{3}'} H_{\rm d} + \beta \left(LE_2^{\rm C}\right)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)} H_{\rm d} + \gamma \left(LE_3^{\rm C}\right)_{\mathbf{3}'} Y_{\mathbf{3}'}^{(4)} H_{\rm d}$

Neutrino sector

A1: $g\,(SS)_{\mathbf{1}}\,, \quad k_S = 0\,;$

A2: $g\,(SS)_{\mathbf{2}} Y_{\mathbf{2}}, \quad k_S = -1\,;$

A3: $g\left[(SS)_{\mathbf{1}} Y_{\mathbf{1}}^{(4)} + r_{g_1} e^{{\rm i}p_{g_1}} (SS)_{\mathbf{2}} Y_{\mathbf{2}}^{(4)} + r_{g_2} e^{{\rm i}p_{g_2}} (SS)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)}\right], \quad k_S = -2\,,$

B1: $\Lambda \left(SN^{\rm C}\right)_{\mathbf{3}'} Y_{\mathbf{3}'}, \quad k_{N^{\rm C}} = -2 - k_S\,;$

B2: $\Lambda \left[\left(SN^{\rm C}\right)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)} + r_\Lambda e^{{\rm i}p_\Lambda} \left(SN^{\rm C}\right)_{\mathbf{3}'} Y_{\mathbf{3}'}^{(4)}\right], \quad k_{N^{\rm C}} = -4 - k_S\,;$

C1: $y \left(LN^{\rm C}\right)_{\mathbf{3}'} Y_{\mathbf{3}'}, \quad k_L = -2 - k_{N^{\rm C}}\,;$

C2: $y \left[\left(LN^{\rm C}\right)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)} + r_y e^{{\rm i}p_y} \left(LN^{\rm C}\right)_{\mathbf{3}'} Y_{\mathbf{3}'}^{(4)}\right], \quad k_L = -4 - k_{N^{\rm C}}\,,$

We have $3 \times 2 \times 2$ models:

AiBjCk

XYZ and S. Zhou, 2106.03433

Models After symmetry breaking

Basic idea

Inverse seesaw → keV dark matter
Modular symmetry → light neutrino masses & mixings

Guiding principle: **Simplicity & Generality**

modular form multiplets with weights ≤ 4

all allowed couplings included

	L	$H_{\rm u}$	$H_{\rm d}$	$E_1^{\rm C}$	$E_2^{\rm C}$	$E_3^{\rm C}$	$N^{\rm C}$	S
SU(2)	2	2	2	1	1	1	1	1
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$	$\mathbf{3}$

→Minimal group: S_4

Charged lepton sector $W_l = \alpha \left(L E_1^{\rm C}\right)_{\mathbf{3'}} Y_{\mathbf{3'}} H_{\rm d} + \beta \left(L E_2^{\rm C}\right)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)} H_{\rm d} + \gamma \left(L E_3^{\rm C}\right)_{\mathbf{3'}} Y_{\mathbf{3'}}^{(4)} H_{\rm d}$

Neutrino sector

μ

A1: $g\,(SS)_{\mathbf{1}}\,, \quad k_S = 0\,;$

A2: $g\,(SS)_{\mathbf{2}} Y_{\mathbf{2}}, \quad k_S = -1\,;$

A3: $g\left[(SS)_{\mathbf{1}} Y_{\mathbf{1}}^{(4)} + r_{g_1} e^{{\rm i}p_{g_1}} (S\ldots\right.$

M_S

B1: $\Lambda\,(SN^{\rm C})_{\mathbf{3'}} Y_{\mathbf{3'}}, \quad k_{N^{\rm C}} = -2 - k_S\,,$

B2: $\Lambda\left[(SN^{\rm C})_{\mathbf{3}} Y_{\mathbf{3}}^{(4)} + r_\Lambda e^{{\rm i}p_\Lambda} (SN^{\rm C})_{\mathbf{3'}} Y_{\mathbf{3'}}^{(4)}\right], \quad k_{N^{\rm C}} = -4 - k_S\,;$

$M_{\rm D}$

C1: $y\,(LN^{\rm C})_{\mathbf{3'}} Y_{\mathbf{3'}}, \quad k_L = -2 - k_{N^{\rm C}}\,;$

C2: $y\left[(LN^{\rm C})_{\mathbf{3}} Y_{\mathbf{3}}^{(4)} + r_y e^{{\rm i}p_y} (LN^{\rm C})_{\mathbf{3'}} Y_{\mathbf{3'}}^{(4)}\right], \quad k_L = -4 - k_{N^{\rm C}}\,,$

$$\mathcal{M} = \begin{pmatrix} \mathbf{0}_{3\times3} & [M_{\rm D}]_{3\times2} & \mathbf{0}_{3\times3} \\ [M_{\rm D}^{\rm T}]_{2\times3} & \mathbf{0}_{2\times2} & [M_{\rm S}^{\rm T}]_{2\times3} \\ \mathbf{0}_{3\times3} & [M_{\rm S}]_{3\times2} & [\mu]_{3\times3} \end{pmatrix}$$

We have $3\times2\times2$ models:

AiBjCk

$$M_\nu = -M_{\rm D}\left(M_{\rm S}^{\rm T} M_{\rm S}\right)^{-1} M_{\rm S}^{\rm T} \mu M_{\rm S} \left(M_{\rm S}^{\rm T} M_{\rm S}\right)^{-1} M_{\rm D}^{\rm T}\,.$$

XYZ and S. Zhou, 2106.03433

The mass matrices

For example,

$$\mathbf{B1}: M_{\rm S} = \Lambda^* \begin{pmatrix} 0 & -Y_3 \\ \frac{\sqrt{3}}{2}Y_4 & \frac{1}{2}Y_5 \\ \frac{\sqrt{3}}{2}Y_5 & \frac{1}{2}Y_4 \end{pmatrix}^*$$

*Modular form

Recall that

$$\mathcal{M} = \begin{pmatrix} \mathbf{0}_{3\times 3} & [M_{\rm D}]_{3\times 2} & \mathbf{0}_{3\times 3} \\ [M_{\rm D}^{\rm T}]_{2\times 3} & \mathbf{0}_{2\times 2} & [M_{\rm S}^{\rm T}]_{2\times 3} \\ \mathbf{0}_{3\times 3} & [M_{\rm S}]_{3\times 2} & [\mu]_{3\times 3} \end{pmatrix}$$

Basis in the space of the lowest-weight modular forms

$$Y_1 = -3\pi\left(\frac{1}{8} + 3q + 3q^2 + 12q^3 + 3q^4 + 18q^5 + 12q^6 + 24q^7 + 3q^8 + 39q^9\right) \;;$$
$$Y_2 = 3\sqrt{3}\pi q^{1/2}\left(1 + 4q + 6q^2 + 8q^3 + 13q^4 + 12q^5 + 14q^6 + 24q^7 + 18q^8 + 20q^9\right) \;;$$
$$Y_3 = \pi\left(\frac{1}{4} - 2q + 6q^2 - 8q^3 + 6q^4 - 12q^5 + 24q^6 - 16q^7 + 6q^8 - 26q^9 + 38q^{10}\right) \;;$$
$$Y_4 = -\sqrt{2}\pi q^{1/4}\left(1 + 6q + 13q^2 + 14q^3 + 18q^4 + 32q^5 + 31q^6 + 30q^7 + 48q^8 + 38q^9\right) \;;$$
$$Y_5 = -4\sqrt{2}\pi q^{3/4}\left(1 + 2q + 3q^2 + 6q^3 + 5q^4 + 6q^5 + 10q^6 + 8q^7 + 12q^8 + 14q^9\right) \;,$$

$$q \equiv e^{{\rm i}2\pi\tau}$$

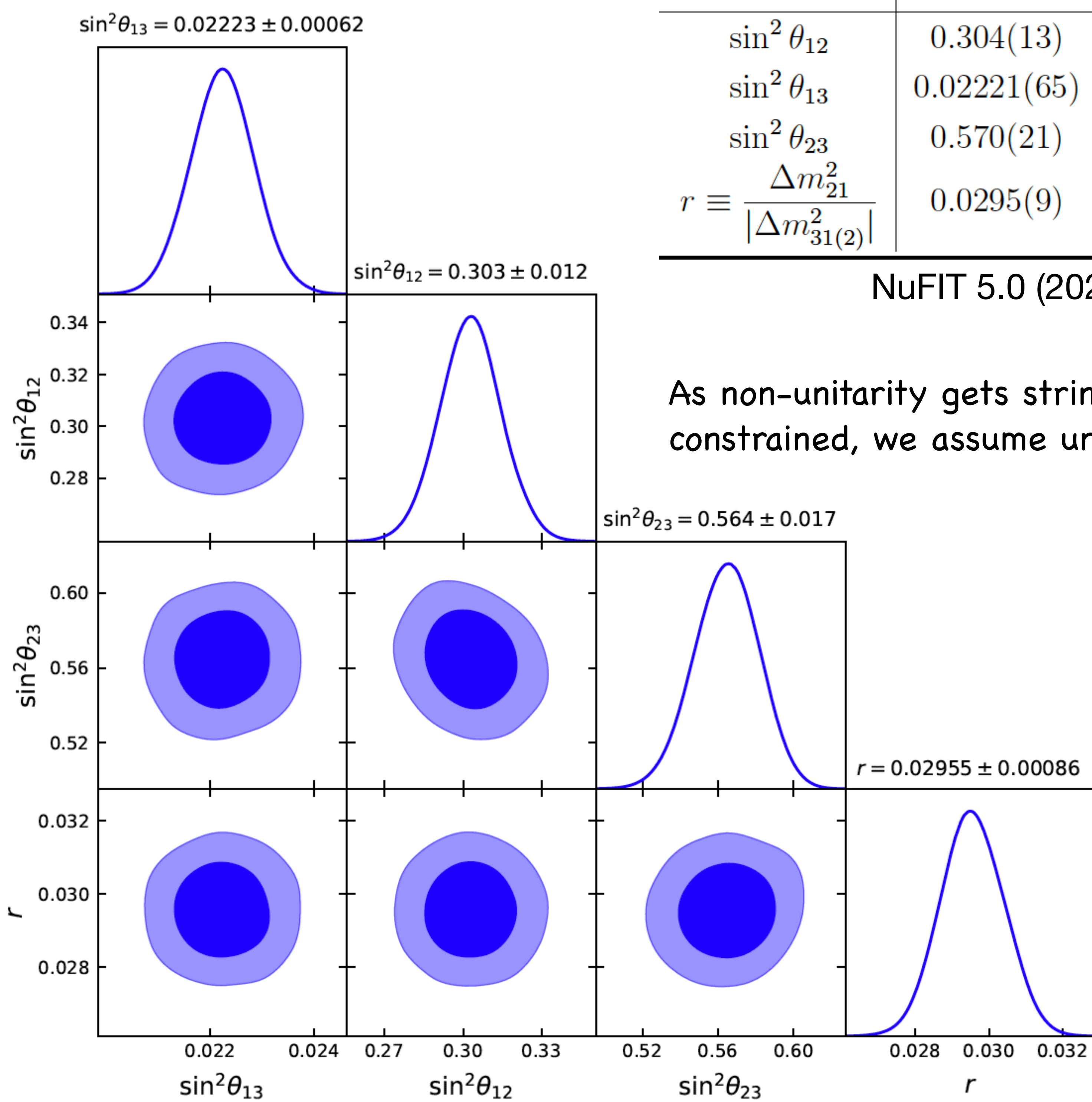
Moduli

Highly non-linear system
Difficult to get a constant LO approximation (except for special values of τ)

XYZ and S. Zhou, 2106.03433

Confronting oscillation data: fitting results

Model: A1B2C2



	NO	IO
$\sin^2\theta_{12}$	$0.304(13)$	$0.304(13)$
$\sin^2\theta_{13}$	$0.02221(65)$	$0.02240(62)$
$\sin^2\theta_{23}$	$0.570(21)$	$0.575(19)$
$r \equiv \dfrac{\Delta m^2_{21}}{\|\Delta m^2_{31(2)}\|}$	$0.0295(9)$	$0.0297(9)$

NuFIT 5.0 (2020)

As non-unitarity gets stringently constrained, we assume unitarity at LO.

Parameters	Predicted value
$\sin^2\theta_{12}$	0.305
$\sin^2\theta_{13}$	0.02227
$\sin^2\theta_{23}$	0.571
$\dfrac{\Delta m^2_{21}}{\Delta m^2_{31}}$	0.0296
y_e	2.7774×10^{-6}
y_μ	5.8505×10^{-4}
y_τ	9.9372×10^{-3}
δ [°]	108.9
α_{21} [°]	25.8
α_{31} [°]	52.7

Our results

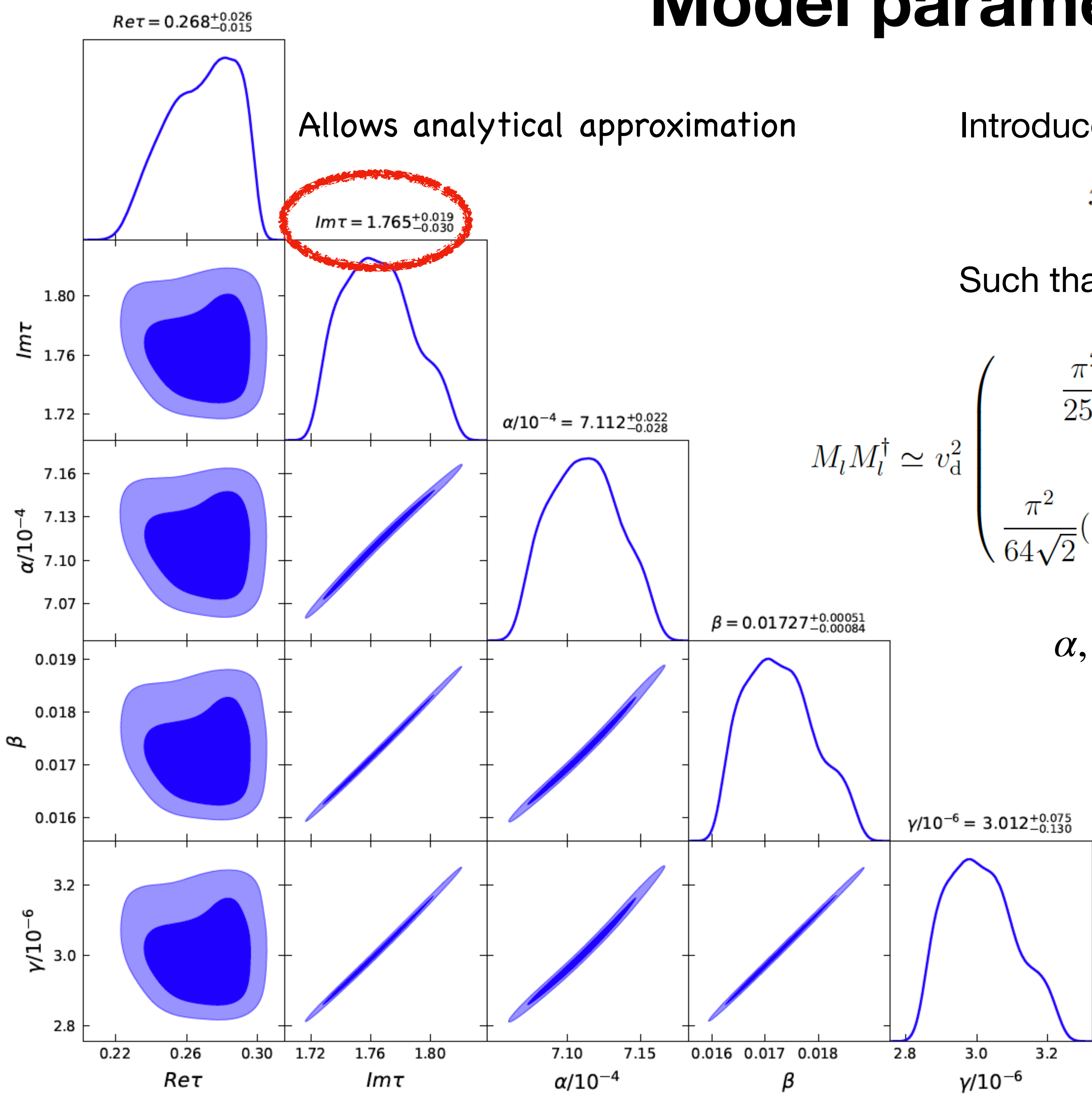
A good fit.

How many parameters & observables?

XYZ and S. Zhou, 2106.03433

Model: A1B2C2

Model parameters



Allows analytical approximation

Introduce

$$x \equiv \exp(-\pi \mathrm{Im}\tau/2), \quad y \equiv \pi \mathrm{Re}\tau/2 \,,$$

Such that $x \simeq 0.063$ can be a good perturbative parameter

$$M_l M_l^\dagger \simeq v_d^2 \begin{pmatrix} \frac{\pi^2}{256}(16\alpha^2 + 9\pi^2\gamma^2) & 0 & \frac{\pi^2}{64\sqrt{2}}(-32\alpha^2 + 9\pi^2\gamma^2) x e^{iy} \\ 0 & \frac{27}{32}\pi^4\beta^2 x^2 & 0 \\ \frac{\pi^2}{64\sqrt{2}}(-32\alpha^2 + 9\pi^2\gamma^2) x e^{-iy} & 0 & \frac{\pi^2}{32}(64\alpha^2 + 9\pi^2\gamma^2) x^2 \end{pmatrix}$$

α, β, γ, x are correlated by charged lepton masses

$$\tan 2\theta_{13}^l = \frac{2|H_{13}|}{H_{33} - H_{11}} \simeq 8\sqrt{2}x \,,$$

$$\theta_{13}^l \simeq 18^\circ$$

Neutrino mass matrix: complicated…

9 parameters, 3 strongly correlated

Fit to 7 observables (data)

XYZ and S. Zhou, 2106.03433

keV sterile neutrino as warm dark matter

Production Mechanism:

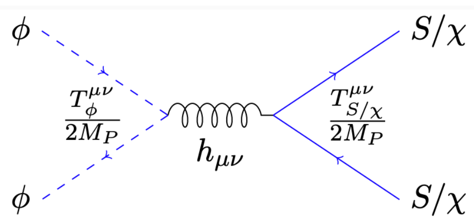
Our model

- Dodelson-Widrow (DW) mechanism **S. Dodelson and L. M. Widrow, 1994**
  through active-sterile mixing at $T \sim 100\ \mathrm{MeV}$
- Shi-Fuller (SF) mechanism **X. D. Shi and G. M. Fuller, 1998**
  resonant production by a preexisting lepton asymmetry Rely on DW
- Particle decay: e.g., inflaton decay
- Diluted thermal overproduction: charged under BSM gauge group
- Gravitational production 2102.06214

Non-thermal

Thermal

Depends



Constraints for being a dark matter candidate:

✳ Relic density:

$$\Omega_{\rm DM} h^2 = 1.1 \times 10^7 \sum_\alpha C_\alpha(m_s) |U_{\alpha s}|^2 \left(\frac{m_s}{\rm keV}\right)^2, \quad \alpha = e, \mu, \tau$$

✳ X-ray line search: Chandra, XMM-Newton, NuSTAR, … Model-independent

$$\Gamma_\gamma(m_s, \sin^2 2\theta) \approx 1.36 \times 10^{-30}\,{\rm s}^{-1} \left(\frac{\sin^2 2\theta}{10^{-7}}\right)\left(\frac{m_s}{1\,\rm keV}\right)^5$$

✳ Lyman-α forest data: SDSS-III/BOSS, XQ-100, HIRES, MIKE,… Thermal, DW

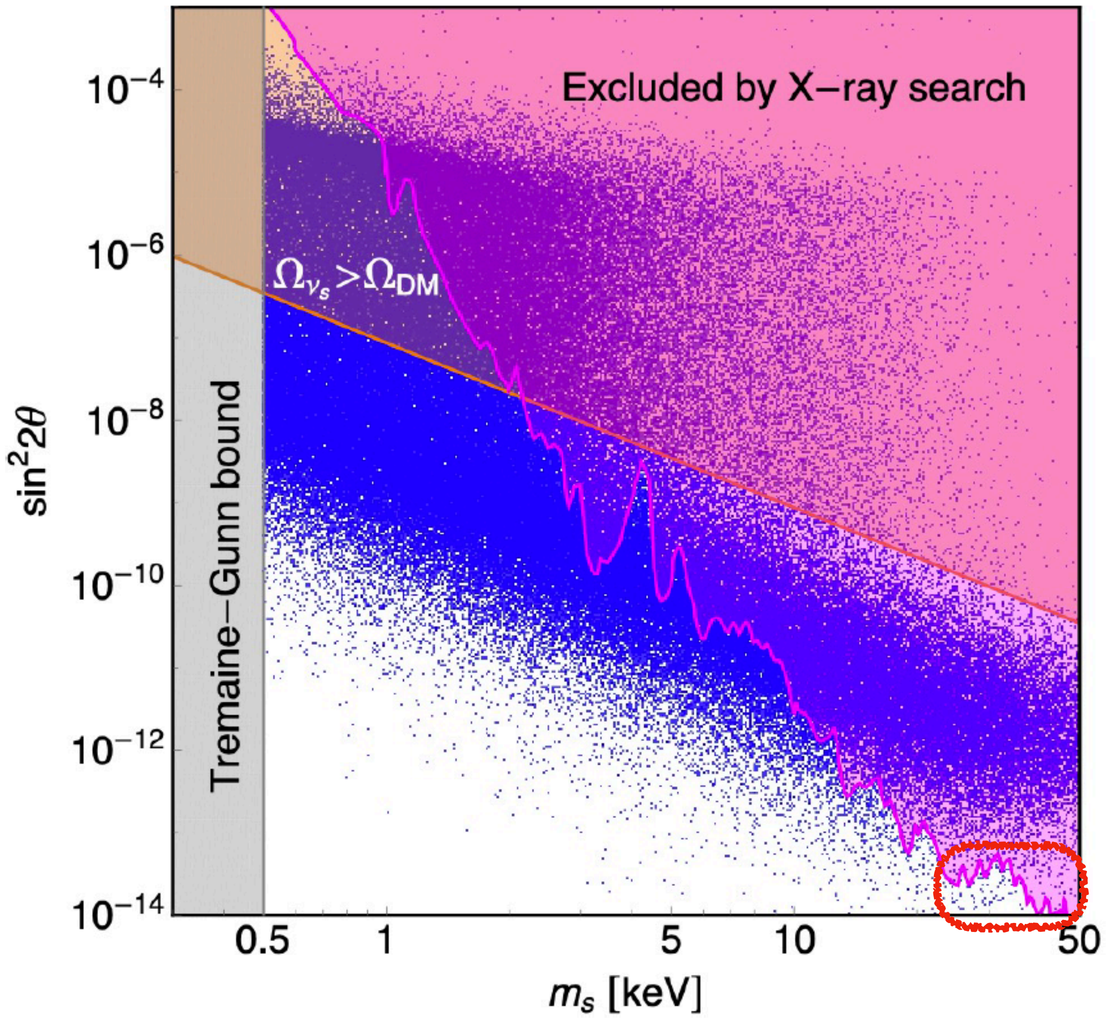
$$m_{\rm WDM} \gtrsim (1.9 - 5.3)\ \ {\rm keV}$$

Fermi-Dirac distribution

→ Rescaling in DW production

$$\frac{m_s}{3.9\ {\rm keV}} = \left(\frac{m_{\rm WDM}}{\rm keV}\right)^{1.294} \left(\frac{0.25 \times 0.7^2}{\Omega_{\rm DM} h^2}\right)^{1/3}$$

Confronting DM constraints: results



$m_s \simeq m_4$

$m_5 \simeq m_6 \in [1, 10^3]$ GeV

$m_7 \simeq m_8 \in [10, 10^4]$ GeV

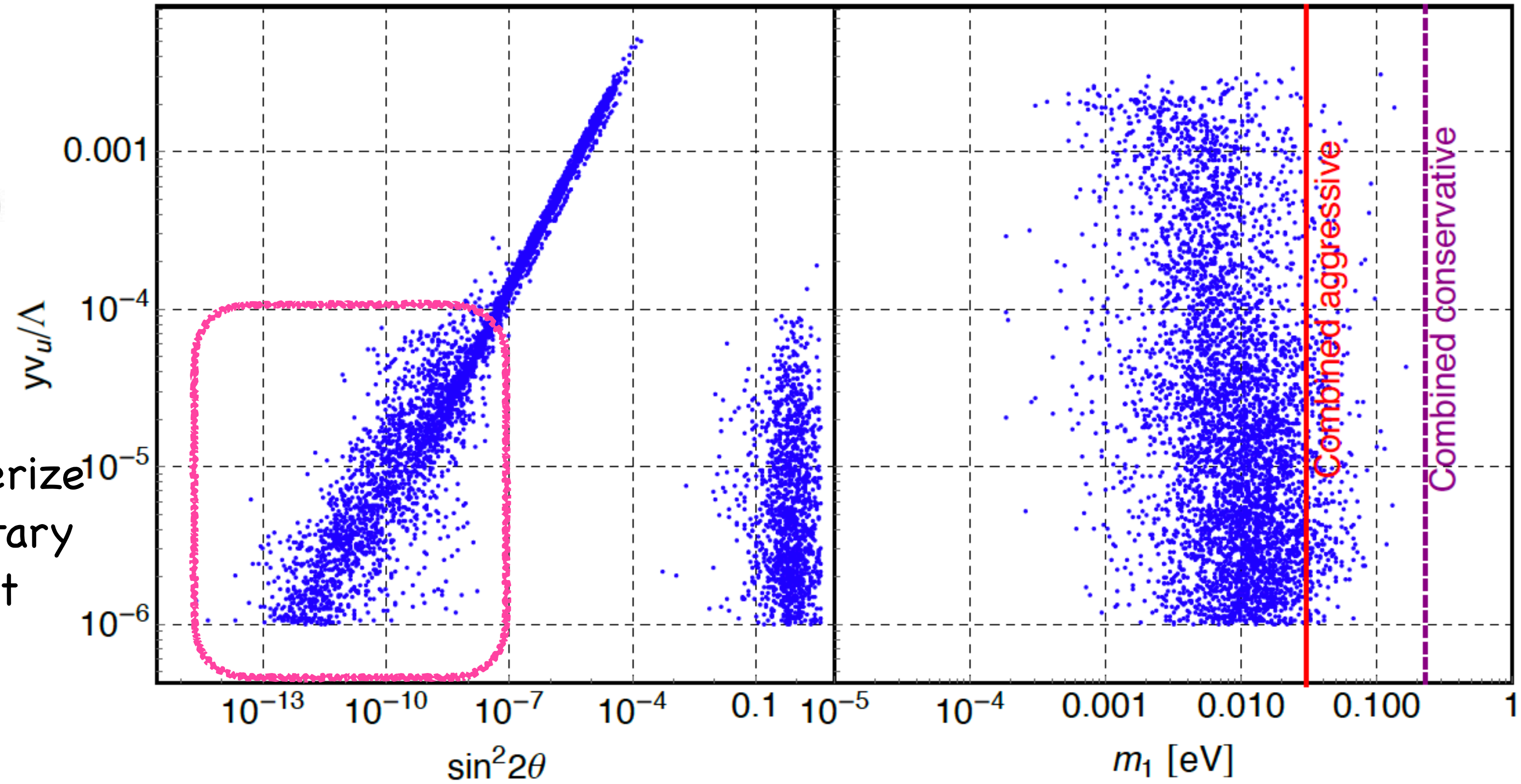
Tremaine-Gunn bound S. Tremaine and J. E. Gunn, 1979

$$m_s \gtrsim 0.5 \text{ keV}$$

Lyman-α bound $m_s \gtrsim (8.9 - 33.8)$ keV.

- Produced from DW, contribute only a fraction of DM
- DW production + entropy dilution from heavy states
- SF production: loose Lyman-α; small mixing - loose X-ray

Characterize non-unitary effect



XYZ and S. Zhou, 2106.03433

Concluding remarks

- By natural and simple construction, we find one model among all the possibilities in excellent agreement with neutrino masses and mixings and it provides a viable dark matter candidate.
- The model is highly predictive (whole mass spectrum & mixing) and can be tested in future oscillation experiments and cosmological observations.

☑ keV sterile neutrino can still be 100% dark matter (very constrained)

- ☑ Compatible with oscillation constraints
- ❖ But not produced by simplest DW mechanism
- ☐ Other production mechanisms?

谢谢大家！