Light Sneutrino and neutralino as dark matter candidates in U(1)xSSM

赵树民

河北大学

2021年8月16日—20日山东大学（青岛）

JHEP 02 (2020) 130;
Nucl. Phys. B 969 (2021) 115469

1、暗物质

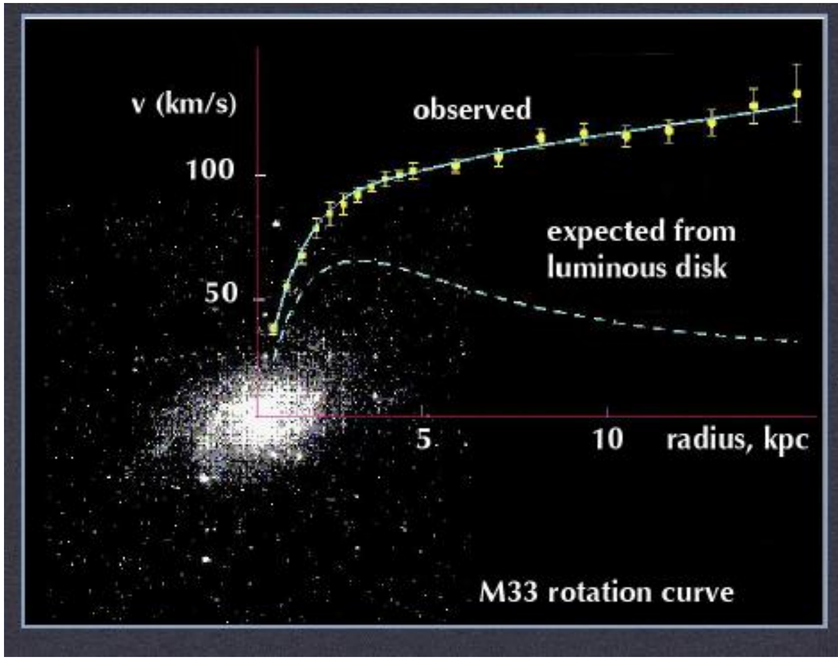
2、U(1)xSSM简介

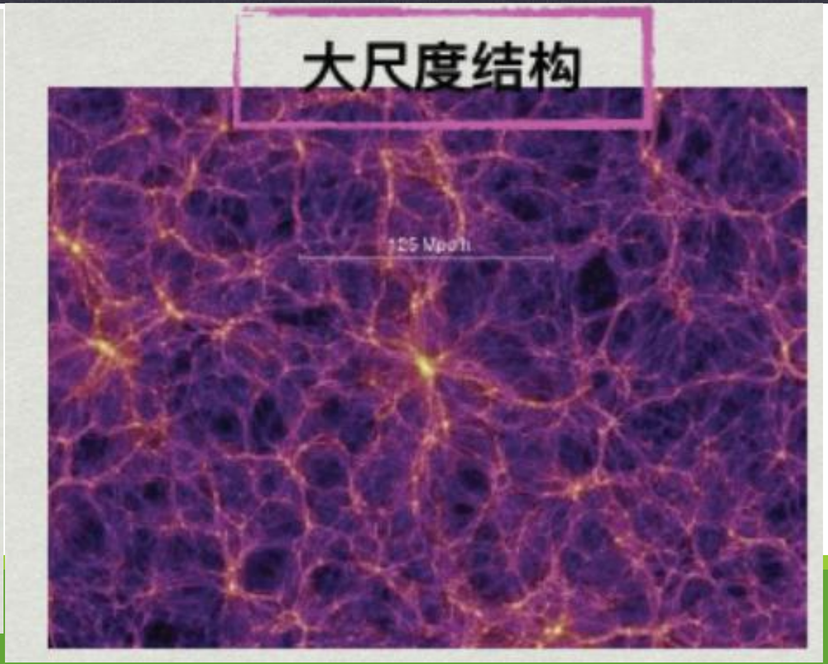
3、Sneutrino 充当暗物质

4、Neuralino 充当暗物质

5、Summary

1、暗物质


v (km/s)
observed
100
expected from
luminous disk
50
5
10
radius, kpc
M33 rotation curve


大尺度结构

暗物质的特点与探测

（1）暗物质参与引力相互作用，有质量。

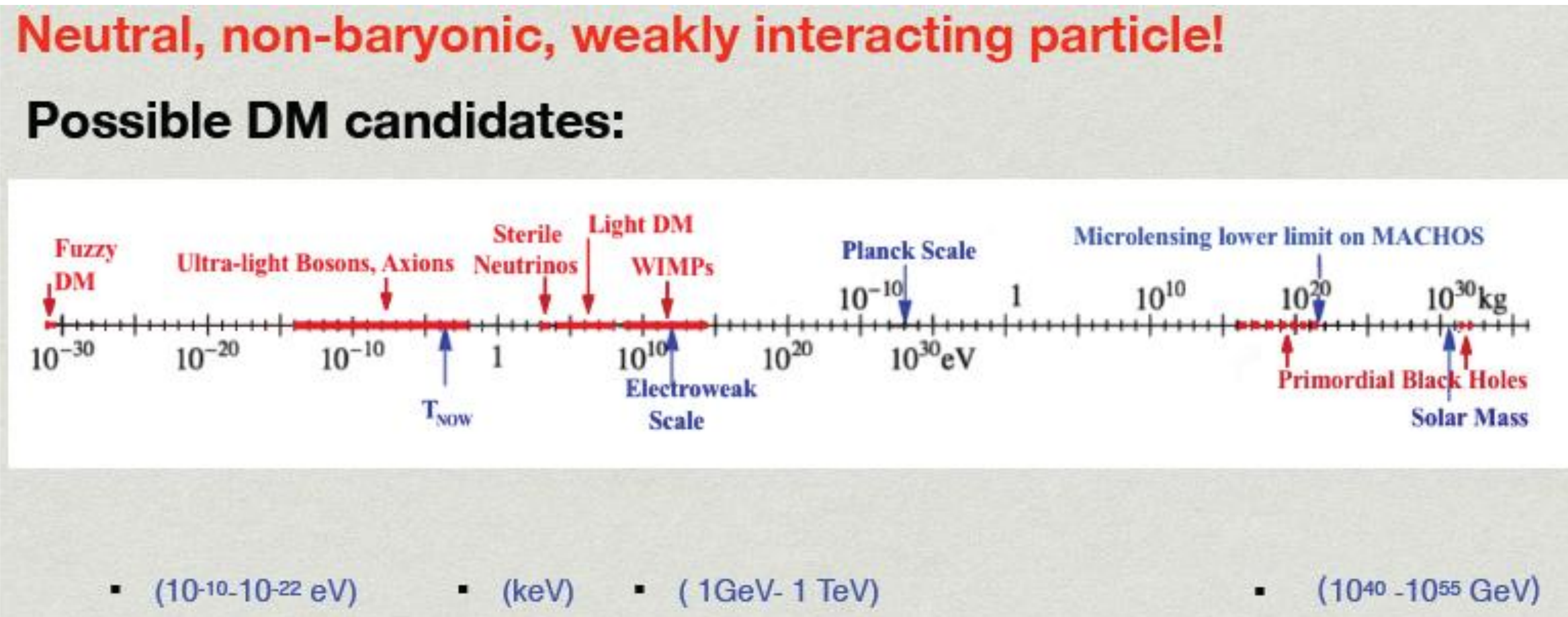
（2）暗物质应是高度稳定的。

（3）暗物质不参与电磁相互作用和强相互作用

（4）通过计算机模拟宇宙大尺度结构形成得知，大部分暗物质的运动速度应该是远低于光速，即“冷暗物质”。

直接探测，间接探测，对撞机探测

Dark matter candidates:
light neutrinos, axions,
sterile neutrinos, primordial black holes,
weakly interacting massive particles(WIMPs).



2 U(1)xSSM简介

The local gauge group

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
$\hat{Q}_i$	3	2	1/6	0
$\hat{u}_i^c$	$\bar{3}$	1	-2/3	-1/2
$\hat{d}_i^c$	$\bar{3}$	1	1/3	1/2
$\hat{L}_i$	1	2	-1/2	0
$\hat{e}_i^c$	1	1	1	1/2
$\hat{\nu}_i$	1	1	0	-1/2
$\hat{H}_u$	1	2	1/2	1/2
$\hat{H}_d$	1	2	-1/2	-1/2
$\hat{\eta}$	1	1	0	-1
$\hat{\bar{\eta}}$	1	1	0	1
$\hat{S}$	1	1	0	0

The superpotential

$$W = l_W \hat{S} + \mu \hat{H}_u \hat{H}_d + M_S \hat{S}\hat{S} - Y_d \hat{d}\hat{q}\hat{H}_d - Y_e \hat{e}\hat{l}\hat{H}_d + \lambda_H \hat{S}\hat{H}_u \hat{H}_d$$
$$+\lambda_C \hat{S}\hat{\eta}\hat{\bar{\eta}} + \frac{\kappa}{3}\hat{S}\hat{S}\hat{S} + Y_u \hat{u}\hat{q}\hat{H}_u + Y_X \hat{\nu}\hat{\bar{\eta}}\hat{\nu} + Y_\nu \hat{\nu}\hat{l}\hat{H}_u.$$

The Higgs superfields

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}\left(v_u + H_u^0 + iP_u^0\right) \end{pmatrix}, \qquad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_d + H_d^0 + iP_d^0\right) \\ H_d^- \end{pmatrix},$$
$$\eta = \frac{1}{\sqrt{2}}\left(v_\eta + \phi_\eta^0 + iP_\eta^0\right), \qquad \bar{\eta} = \frac{1}{\sqrt{2}}\left(v_{\bar{\eta}} + \phi_{\bar{\eta}}^0 + iP_{\bar{\eta}}^0\right),$$
$$S = \frac{1}{\sqrt{2}}\left(v_S + \phi_S^0 + iP_S^0\right).$$

Scalar neutrino superfields

$$\tilde{\nu}_L = \frac{1}{\sqrt{2}}\phi_l + \frac{i}{\sqrt{2}}\sigma_l, \qquad \tilde{\nu}_R = \frac{1}{\sqrt{2}}\phi_R + \frac{i}{\sqrt{2}}\sigma_R.$$

The soft breaking terms are

$$\mathcal{L}_{soft} = \mathcal{L}_{soft}^{MSSM} - B_S S^2 - L_S S - \frac{T_\kappa}{3} S^3 - T_{\lambda_C} S\eta\bar{\eta} + \epsilon_{ij} T_{\lambda_H} S H_d^i H_u^j$$
$$-T_X^{IJ}\bar{\eta}\tilde{\nu}_R^{*I}\tilde{\nu}_R^{*J} + \epsilon_{ij} T_\nu^{IJ} H_u^i \tilde{\nu}_R^{I*}\tilde{l}_j^J - m_\eta^2|\eta|^2 - m_{\bar{\eta}}^2|\bar{\eta}|^2$$
$$-m_S^2 S^2 - (m_{\tilde{\nu}_R}^2)^{IJ}\tilde{\nu}_R^{I*}\tilde{\nu}_R^J - \frac{1}{2}\Big(M_X\lambda_{\tilde{X}}^2 + 2M_{BB'}\lambda_{\tilde{B}}\lambda_{\tilde{X}}\Big) + h.c \quad .$$

The covariant derivatives of this model is shown in the general form

$$D_\mu = \partial_\mu - i \begin{pmatrix} Y, & X \end{pmatrix} \begin{pmatrix} g_Y, & g'_{YX} \\ g'_{XY}, & g'_X \end{pmatrix} \begin{pmatrix} A'^Y_\mu \\ A'^X_\mu \end{pmatrix} ,$$

$$\begin{pmatrix} g_Y, & g'_{YX} \\ g'_{XY}, & g'_X \end{pmatrix} R^T = \begin{pmatrix} g_1, & g_{YX} \\ 0, & g_X \end{pmatrix} ,$$

$$\begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\theta_W \cos\theta'_W & \cos\theta_W \cos\theta'_W & \sin\theta'_W \\ \sin\theta_W \sin\theta'_W & -\cos\theta'_W \sin\theta'_W & \cos\theta'_W \end{pmatrix} \begin{pmatrix} A^Y \\ V^3 \\ A^X \end{pmatrix} .$$

$$\sin^2\theta'_W = \frac{1}{2} - \frac{((g_{YX}+g_X)^2 - g_1^2 - g_2^2)v^2 + 4g_X^2\xi^2}{2\sqrt{((g_{YX}+g_X)^2 + g_1^2 + g_2^2)^2 v^4 + 8g_X^2((g_{YX}+g_X)^2 - g_1^2 - g_2^2)v^2\xi^2 + 16g_X^4\xi^4}}$$

with $v^2 = v_u^2 + v_d^2$ and $\xi^2 = v_\eta^2 + v_{\bar\eta}^2$.

$$\begin{aligned}
V = {} & \frac{1}{2}g_X(g_X+g_{YX})(|H_d^0|^2 - |H_u^0|^2)(|\eta|^2 - |\bar\eta|^2) + |\lambda_H|^2|H_u^0H_d^0|^2 + m_S^2|S|^2 \\
& + \frac{1}{8}\left(g_1^2 + g_2^2 + (g_X+g_{YX})^2\right)(|H_d^0|^2 - |H_u^0|^2)^2 + \frac{1}{2}g_X^2(|\eta|^2 - |\bar\eta|^2)^2 + \lambda_C^2|\eta\bar\eta|^2 \\
& + (|\mu|^2 + |\lambda_H|^2|S|^2 + 2\mathrm{Re}[\mu^*\lambda_H S])(|H_d^0|^2 + |H_u^0|^2) + |\lambda_C|^2|S|^2(|\eta|^2 + |\bar\eta|^2) \\
& + 2\mathrm{Re}[l_W^*(2M_S S + \lambda_C\eta\bar\eta - \lambda_H H_u^0H_d^0 + \kappa S^2)] + 4|M_S|^2|S|^2 + 2\mathrm{Re}[\lambda_C^*\kappa\eta^*\bar\eta^* S^2] \\
& + |\kappa|^2|S|^4 + 4\mathrm{Re}[M_S^*S^*(\lambda_C\eta\bar\eta - \lambda_H H_u^0H_d^0 + \kappa S^2)] - 2\mathrm{Re}[\lambda_C^*\lambda_H\eta^*\bar\eta^* H_u^0H_d^0] + |l_W|^2 \\
& - 2\mathrm{Re}[B_\mu H_d^0H_u^0] + 2\mathrm{Re}[L_S S] + \frac{2}{3}\mathrm{Re}[T_k S^3] + 2\mathrm{Re}[T_{\lambda_C}\eta\bar\eta S] - 2\mathrm{Re}[T_{\lambda_H}H_d^0H_u^0 S] \\
& - 2\mathrm{Re}[\lambda_H\kappa^* H_u^0H_d^0(S^2)^*] + m_\eta^2|\eta|^2 + m_{\bar\eta}^2|\bar\eta|^2 + m_{H_u^0}^2|H_u^0|^2 + m_{H_d^0}^2|H_d^0|^2 + 2\mathrm{Re}[B_S S^2].
\end{aligned}$$

The neutrino mass matrix is deduced in the base $(\nu_L, \bar{\nu}_R)$

$$M_\nu = \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}}(Y_\nu^T)^{IJ} \\ \frac{v_u}{\sqrt{2}}(Y_\nu)^{IJ} & \sqrt{2}v_{\bar{\eta}}(Y_X)^{IJ} \end{pmatrix},$$

The mass matrix for CP-even sneutrino (ϕ_l, ϕ_r) reads as

$$M_{\tilde{\nu}^R}^2 = \begin{pmatrix} m_{\phi_l\phi_l} & m_{\phi_r\phi_l}^T \\ m_{\phi_l\phi_r} & m_{\phi_r\phi_r} \end{pmatrix},$$

$$m_{\phi_l\phi_l} = \frac{1}{8}\Big((g_1^2 + g_{YX}^2 + g_2^2 + g_{YX}g_X)(v_d^2 - v_u^2) + g_{YX}g_X(2v_\eta^2 - 2v_{\bar{\eta}}^2)\Big)$$
$$+\frac{1}{2}v_u^2 Y_\nu^T Y_\nu + m_{\tilde{L}}^2,$$
$$m_{\phi_l\phi_r} = \frac{1}{\sqrt{2}}v_u T_\nu + v_u v_{\bar{\eta}} Y_X Y_\nu - \frac{1}{2}v_d(\lambda_H v_S + \sqrt{2}\mu)Y_\nu,$$
$$m_{\phi_r\phi_r} = \frac{1}{8}\Big((g_{YX}g_X + g_X^2)(v_d^2 - v_u^2) + 2g_X^2(v_\eta^2 - v_{\bar{\eta}}^2)\Big) + v_\eta v_S Y_X \lambda_C$$
$$+m_{\tilde{\nu}}^2 + \frac{1}{2}v_u^2|Y_\nu|^2 + v_{\bar{\eta}}(2v_{\bar{\eta}}Y_X Y_X + \sqrt{2}T_X).$$

The CP even Higgs $\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}}$ and ϕ_S mix together

in the basis: $(\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}}, \phi_s)$

$$m_h^2 = \begin{pmatrix} m_{\phi_d\phi_d} & m_{\phi_u\phi_d} & m_{\phi_\eta\phi_d} & m_{\phi_{\bar{\eta}}\phi_d} & m_{\phi_s\phi_d} \\ m_{\phi_d\phi_u} & m_{\phi_u\phi_u} & m_{\phi_\eta\phi_u} & m_{\phi_{\bar{\eta}}\phi_u} & m_{\phi_s\phi_u} \\ m_{\phi_d\phi_\eta} & m_{\phi_u\phi_\eta} & m_{\phi_\eta\phi_\eta} & m_{\phi_{\bar{\eta}}\phi_\eta} & m_{\phi_s\phi_\eta} \\ m_{\phi_d\phi_{\bar{\eta}}} & m_{\phi_u\phi_{\bar{\eta}}} & m_{\phi_\eta\phi_{\bar{\eta}}} & m_{\phi_{\bar{\eta}}\phi_{\bar{\eta}}} & m_{\phi_s\phi_{\bar{\eta}}} \\ m_{\phi_d\phi_s} & m_{\phi_u\phi_s} & m_{\phi_\eta\phi_s} & m_{\phi_{\bar{\eta}}\phi_s} & m_{\phi_s\phi_s} \end{pmatrix},$$

矩阵元的具体表达式不列出了，比较占地方

3、Sneutrino 充当暗物质

CP-even Higgs couples with CP-even sneutrinos

$$\mathcal{L}_{H\tilde{\nu}^R\tilde{\nu}^R} = H_i \tilde{\nu}_j^R \frac{i}{4}\Big\{ \sum_{a,b=1}^{3} \Big[-2\sqrt{2} Z^{R*}_{kb} Z^{R*}_{j3+a} (T_\nu)_{ab} Z^H_{i2} - 2\lambda_C v_S Z^{R*}_{k3+b} Z^{R*}_{j3+a} (Y^*_X)_{ab} Z^H_{i3}$$

$$-2\sqrt{2} Z^{R*}_{k3+b} Z^{R*}_{j3+a} (T_X)_{ab} Z^H_{i4} - 2\lambda_C v_\eta Z^{R*}_{k3+b} Z^{R*}_{j3+a} (Y_X)_{ab} Z^H_{i5} \Big] + \Big[j \leftrightarrow k \Big]$$

$$-16 v_{\bar{\eta}} \sum_{a,b,c=1}^{3} Z^{R*}_{k3+c} Z^{R*}_{j3+b} (Y_X)_{ac} (Y_X)_{ab} Z^H_{i4} + \sum_{a=1}^{3} Z^{R*}_{ka} Z^{R*}_{ja} \Big[(g_{YX} g_X + g_1^2$$

$$+ g_{YX}^2 + g_2^2)(-v_d Z^H_{i1} + v_u Z^H_{i2}) - 2 g_{YX} g_X (-v_{\bar{\eta}} Z^H_{i4} + v_\eta Z^H_{i3}) \Big]$$

$$+ \sum_{a=1}^{3} Z^{R*}_{k3+a} Z^{R*}_{j3+a} \Big[(g_{YX} g_X + g_X^2)(v_u Z^H_{i2} - v_d Z^H_{i1}) - 2 g_X^2 (v_\eta Z^H_{i3} - v_{\bar{\eta}} Z^H_{i4}) \Big] \Big\} \tilde{\nu}_k^{*R}.$$

We also deduce the vertexes of $\tilde{\nu}_k^R - \bar{e}_i - \chi_j^-$ and $\tilde{\nu}_k^R - \nu_i - \bar{\chi}_i^0$.

$$\mathcal{L}_{A\tilde{\nu}^I\tilde{\nu}^R} = A_i \tilde{\nu}_j^I \frac{i}{4} \sum_{a,b=1}^{3} \Big\{ \Big[2 v_S \lambda_C Z^{R*}_{k3+b} Z^{I*}_{j3+a} (Y_X)_{ab} Z^A_{i3} - 2\sqrt{2} Z^{R*}_{kb} Z^{I*}_{j3+a} (T_\nu)_{ab} Z^A_{i2}$$

$$-2\sqrt{2} Z^{R*}_{k3+b} Z^{I*}_{j3+a} (T_X)_{ab} Z^A_{i4} + 2 v_\eta \lambda_C Z^{R*}_{k3+b} Z^{I*}_{j3+a} (Y_X)_{ab} Z^A_{i5} \Big] + \Big[R \leftrightarrow I, j \leftrightarrow k \Big] \Big\} \tilde{\nu}_k^{*R}.$$

Relic density and cross section

$n_{\tilde{\nu}_1^R}$ is governed by the Boltzmann equation

$$\frac{dn_{\tilde{\nu}_1^R}}{dt} = -3Hn_{\tilde{\nu}_1^R} - \langle\sigma v\rangle_{SA}(n^2_{\tilde{\nu}_1^R} - n^2_{\tilde{\nu}_1^R eq}) - \langle\sigma v\rangle_{CA}(n_{\tilde{\nu}_1^R}n_\phi - n_{\tilde{\nu}_1^R eq}n_{\phi eq}).$$

$\tilde{\nu}_1^R$ can both self-annihilate and co-annihilate with another species ϕ.

The species freeze out at the temperature T_F.

$$\langle\sigma v\rangle_{SA}n_{\tilde{\nu}_1^R} + \langle\sigma v\rangle_{CA}n_\phi \sim H(T_F).$$

With the supposition $M_\phi > M_{\tilde{\nu}_1^R}$

$$n_\phi = \left(\frac{M_\phi}{M_{\tilde{\nu}_1^R}}\right)^{3/2}\text{Exp}[(M_{\tilde{\nu}_1^R} - M_\phi)/T]n_{\tilde{\nu}_1^R}.$$

Then it becomes

$$\Big[\langle\sigma v\rangle_{SA} + \langle\sigma v\rangle_{CA}\Big(\frac{M_\phi}{M_{\tilde{\nu}^R}}\Big)^{3/2}\texttt{Exp}[(M_{\tilde{\nu}_1^R} - M_\phi)/T]\Big] n_{\tilde{\nu}_1^R} \sim H(T_F).$$

The freeze-out temperature(T_F)

$$x_F = \frac{m_D}{T_F} \simeq \ln[\frac{0.038 M_{Pl} m_D (a + 6b/x_F)}{\sqrt{g_* x_F}}].$$

$$\Omega_D h^2 \simeq \frac{1.07 \times 10^9 x_F}{\sqrt{g_*} M_{PL}(a + 3b/x_F)\texttt{GeV}} .$$

its value should be $\Omega_D h^2 = 0.1186 \pm 0.0020$

从夸克算符转换成核子算符

The operators $\tilde{\nu}^{R*}\tilde{\nu}^{R}\bar{q}q$ and $\tilde{\nu}^{R*}\partial_\mu\tilde{\nu}^{R}\bar{q}\gamma^\mu q$ at the quark level

$$a_q m_q \bar{q}q \to f_N m_N \bar{N}N, \qquad f_N = \sum_{q=u,d,s} f_{Tq}^{(N)} a_q + \frac{2}{27} f_{TG}^{(N)} \sum_{q=c,b,t} a_q,$$

$$\langle N|m_q\bar{q}q|N\rangle = m_N f_{Tq}^{(N)}, \qquad f_{TG}^{(N)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(N)}.$$

convert the operator $b_q\tilde{\nu}^{R*}\partial_\mu\tilde{\nu}^{R}\bar{q}\gamma^\mu q$ to $b_N\tilde{\nu}^{R*}\partial_\mu\tilde{\nu}^{R}\bar{N}\gamma^\mu N$

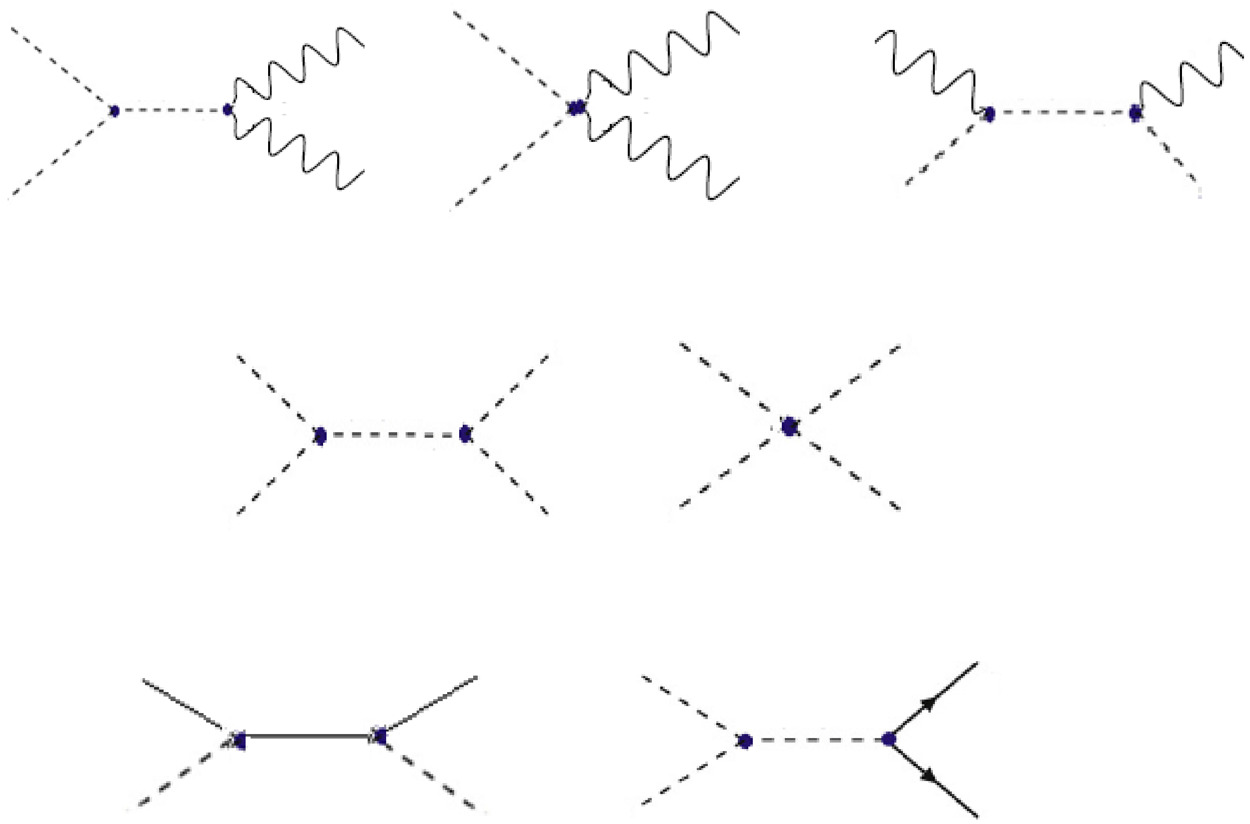
$$b_p = 2b_u + b_d, \quad b_n = 2b_d + b_u.$$

With the obtained f_N, one gets the nucleon cross section

$$\sigma = \frac{1}{\pi}\mu^2[Zf_p + (A-Z)f_n]^2.$$

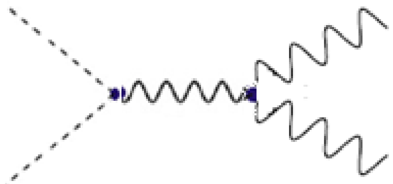
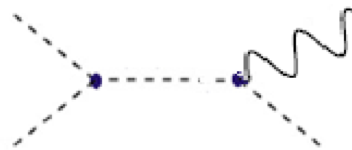
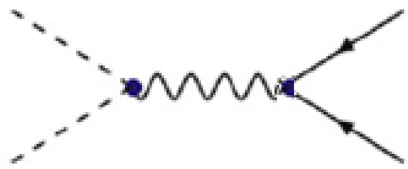
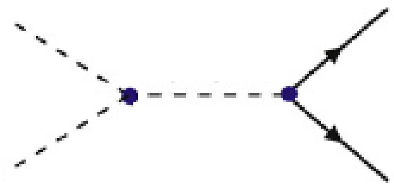
self-annihilation

$$\tilde{\nu}_1^R+\tilde{\nu}_1^R \to \{(W+W),\ (Z+Z),\ (h^0+h^0),$$
$$(\bar{u}_i+u_i),\ (\bar{d}_i+d_i),\ (\bar{l}_i+l_i),\ \ (\bar{\nu}_i+\nu_i)\}$$



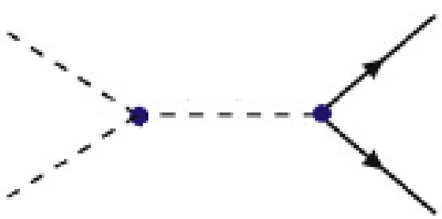
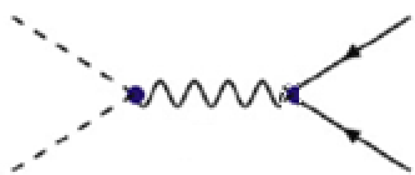
co-annihilation

$$\tilde{\nu}_1^R + \tilde{\nu}_1^I \to \{(W+W),\ (Z+h^0),\ (\bar{u}_i + u_i),$$
$$(\bar{d}_i + d_i),\ (\bar{l}_i + l_i),\ \ (\bar{\nu}_i + \nu_i)\}$$

scattering off nucleon

The main scattering processes of CP-even sneutrinos off nucleons are $\tilde{\nu}^R + q \to \tilde{\nu}^R + q$ and $\tilde{\nu}^R + q \to \tilde{\nu}^I + q$.

Numerical results

The used parameters

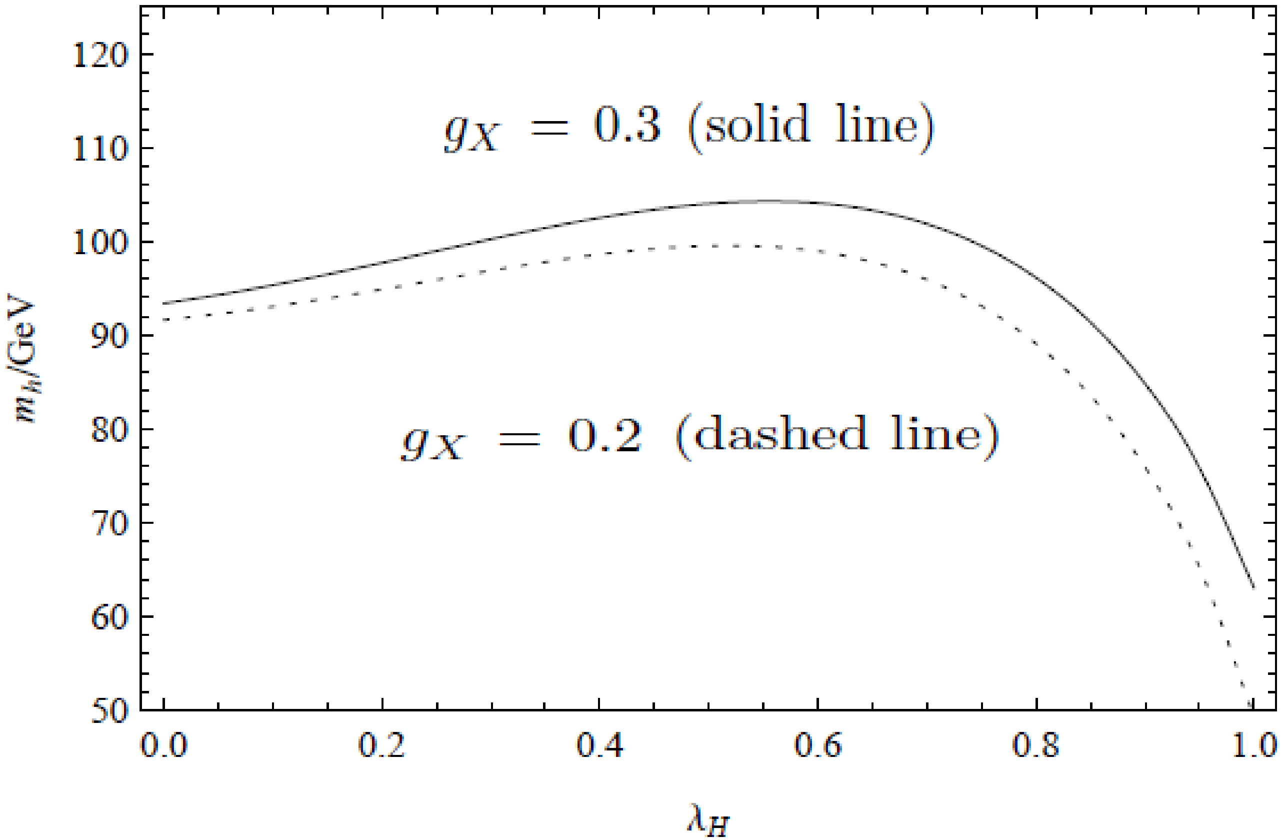
$$\mu = M_S = T_\kappa = M1 = M2 = 1\ \text{TeV}, \quad \tan\beta = 5, \quad \upsilon_\eta = 2\ \text{TeV}, \quad \upsilon_{\bar{\eta}} = 0.9\ \text{TeV},$$

$$\lambda_C = -0.5, \quad \upsilon_S = 3\ \text{TeV}, \quad T_{\lambda_H} = 2\ \text{TeV}, \quad L_W = B_\mu = B_S = m_S^2 = 1\text{TeV}^2,$$

$$T_{\nu 22} = 2\ \text{TeV}, \quad T_{\nu 33} = 3\ \text{TeV}, \quad M_{\nu 11} = 1176^2\ \text{GeV}^2, \quad T_{\lambda_C} = 1.2\ \text{TeV},$$

$$M_{\nu 22} = 2\ \text{TeV}^2, \quad M_{\nu 33} = 3\ \text{TeV}^2, \quad T_{x22} = T_{x33} = 1.1\ \text{TeV}, \quad \kappa = 0.7,$$

$$M_L = M_E = \delta_{ij} 10\ \text{TeV}^2 \text{ and } T_e = \delta_{ij}\text{TeV} \text{ and } Y_x = \delta_{ij}0.5, \ (i, j = 1, 2, 3).$$

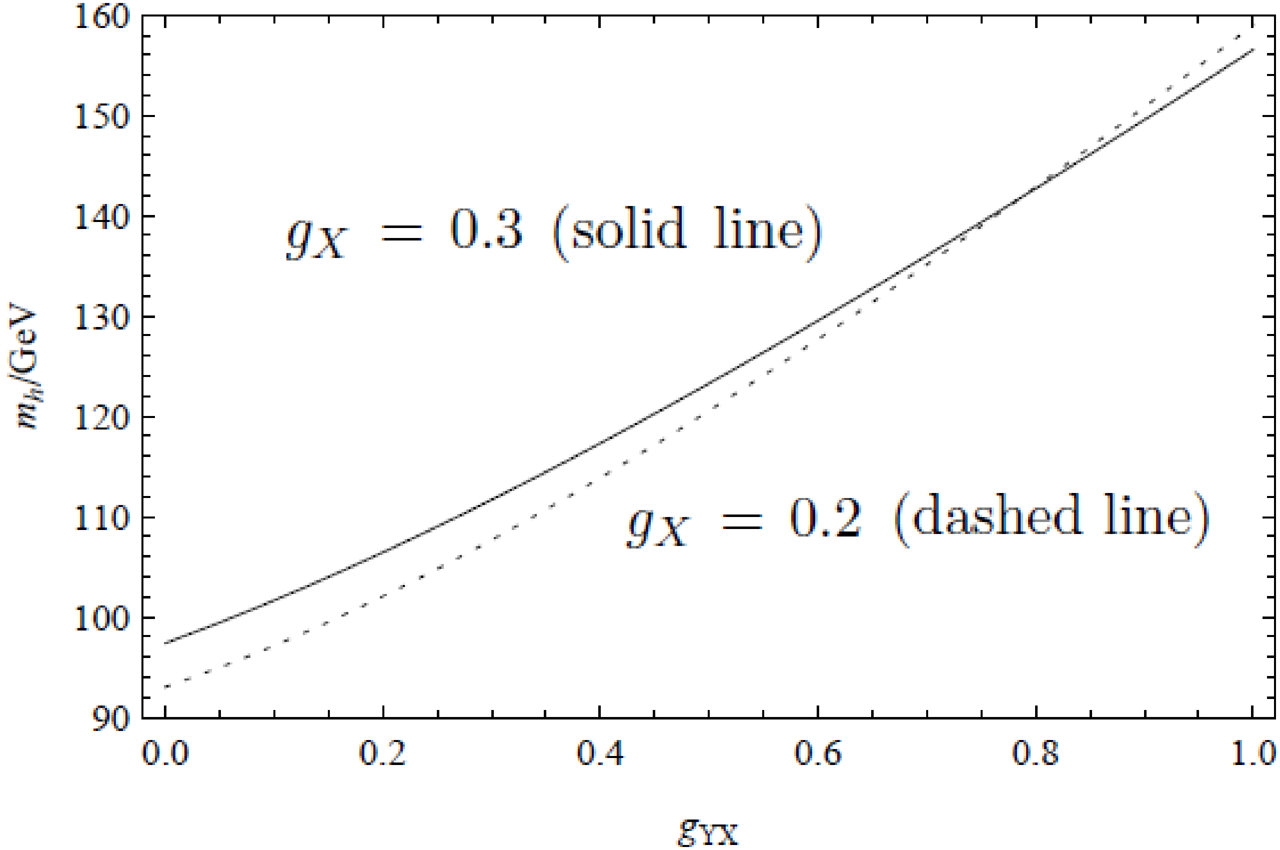


$g_X = 0.2$

$m_{H^1}(6611\sim8283)$ GeV, $m_{H^2}(5220 \sim 6609)$GeV, $m_{H^3} \sim 2036$GeV and $m_{H^4} \sim 640$GeV.

$g_X = 0.3$

$m_{H^1}(6611\sim8293)$ GeV, $m_{H^2}(5220 \sim 6609)$GeV, $m_{H^3} \sim 2066$GeV and $m_{H^4} \sim 725$GeV

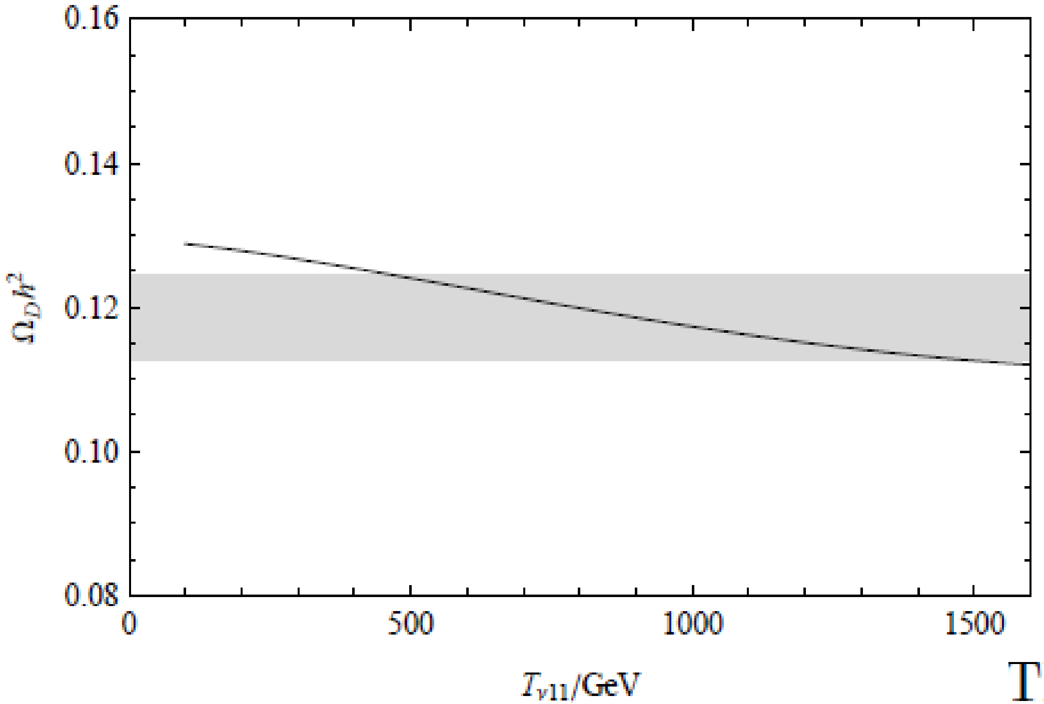


$g_X = 0.2$

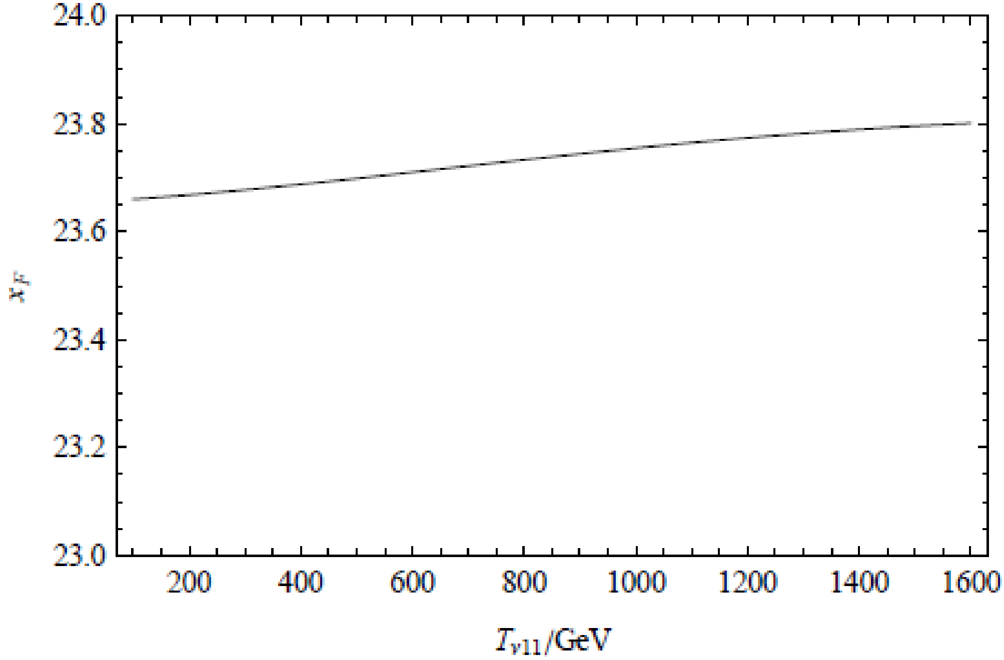
$m_{H^1} \sim 6930$ GeV, $m_{H^2} \sim 6602$ GeV, $m_{H^3} \sim 2036$ GeV, $m_{H^4} \sim 640$ GeV

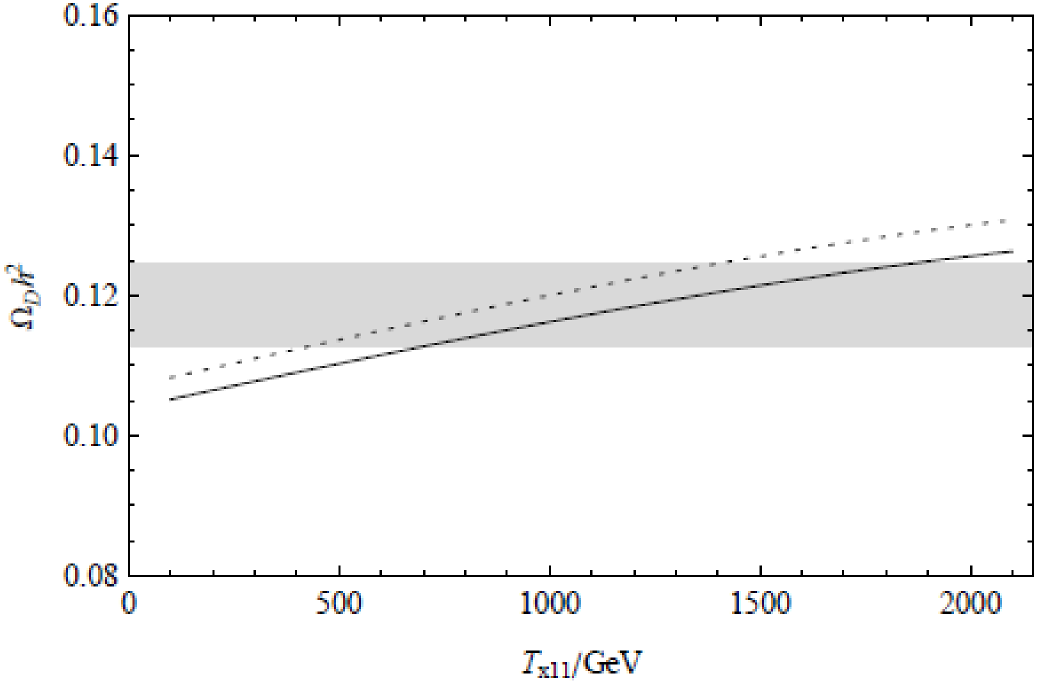
$g_X = 0.3$

$m_{H^1} \sim 6931$ GeV, $m_{H^2} \sim 6602$ GeV, $m_{H^3} \sim 2066$ GeV, $m_{H^4} \sim 725$ GeV.



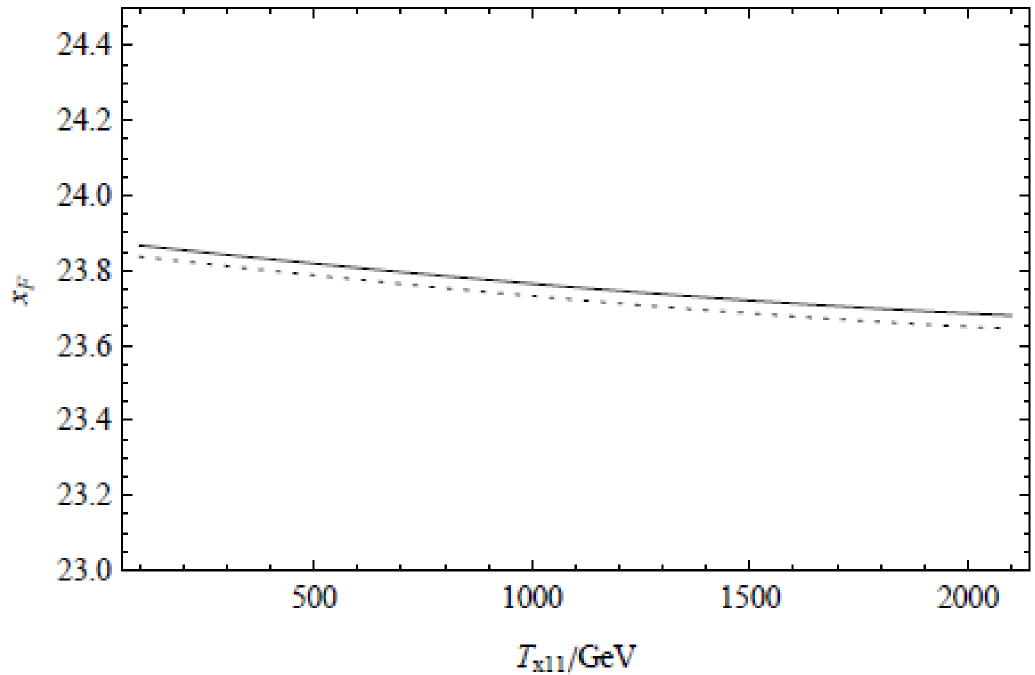
The relic density and x_F versus $T_{\nu 11}$.

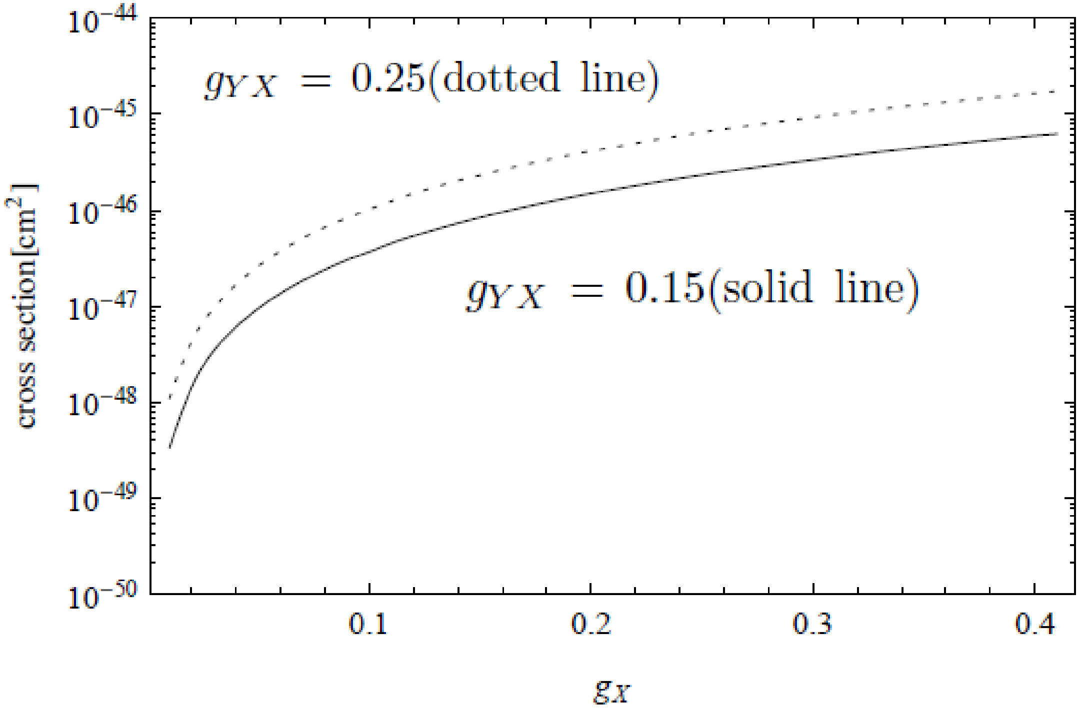


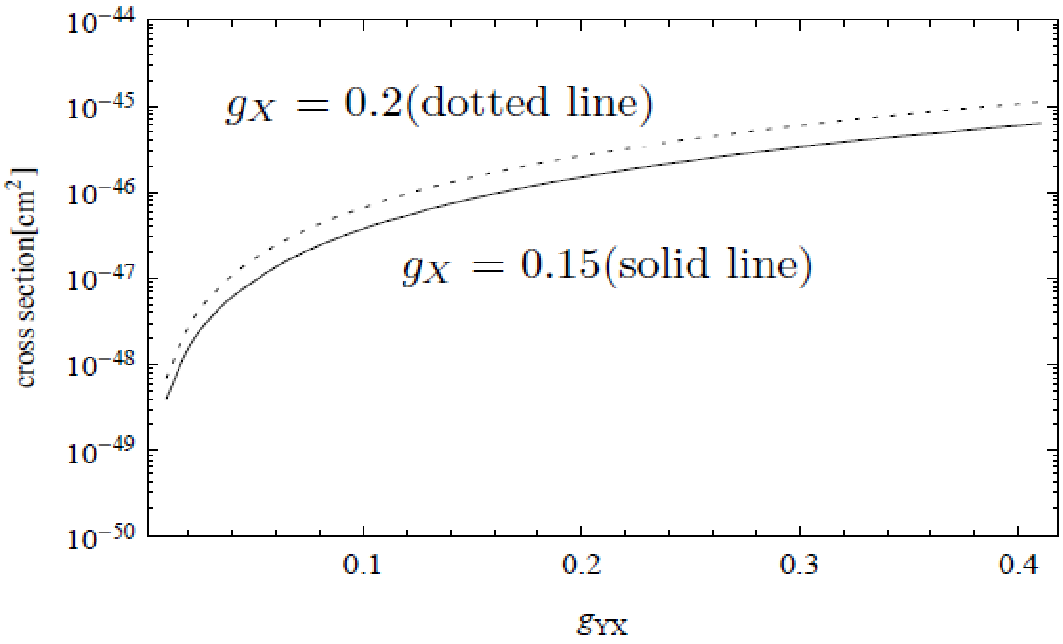


$M_{BB'}$=100GeV (solid line)

$M_{BB'}$=400GeV (dashed line)




$g_{YX} = 0.25$(dotted line)
$g_{YX} = 0.15$(solid line)
cross section[cm^2]
10^{-44}
10^{-45}
10^{-46}
10^{-47}
10^{-48}
10^{-49}
10^{-50}
0.1
0.2
0.3
0.4
g_X


$g_X = 0.2$(dotted line)
$g_X = 0.15$(solid line)
cross section[cm^2]
10^{-44}
10^{-45}
10^{-46}
10^{-47}
10^{-48}
10^{-49}
10^{-50}
0.1
0.2
0.3
0.4
g_{YX}

4、Neuralino 充当暗物质

The mass matrix for neutralinos in the basis: $(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_{\tilde{X}}, \tilde{\eta}, \tilde{\bar{\eta}}, \tilde{s})$

$$M_{\tilde{\chi}^0} = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^T & \mathcal{C} \end{pmatrix}$$

The concrete forms of $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are

$$\mathcal{A} = \begin{pmatrix} M_1 & 0 & -\frac{g_1}{2}v_d & \frac{g_1}{2}v_u \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u \\ -\frac{g_1}{2}v_d & \frac{1}{2}g_2 v_d & 0 & m_{\tilde{H}_u^0\tilde{H}_d^0} \\ \frac{g_1}{2}v_u & -\frac{1}{2}g_2 v_u & m_{\tilde{H}_d^0\tilde{H}_u^0} & 0 \end{pmatrix} \quad \mathcal{B} = \begin{pmatrix} M_{BB'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m_{\lambda_{\tilde{X}}\tilde{H}_d^0} & 0 & 0 & -\frac{\lambda_H v_u}{\sqrt{2}} \\ m_{\lambda_{\tilde{X}}\tilde{H}_u^0} & 0 & 0 & -\frac{\lambda_H v_d}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{C} = \begin{pmatrix} M_{BL} & -g_X v_\eta & g_X v_{\bar\eta} & 0 \\ -g_X v_\eta & 0 & \frac{1}{\sqrt{2}}\lambda_C v_S + \mu_\eta & \frac{1}{\sqrt{2}}\lambda_C v_{\bar\eta} \\ g_X v_{\bar\eta} & \frac{1}{\sqrt{2}}\lambda_C v_S + \mu_\eta & 0 & \frac{1}{\sqrt{2}}\lambda_C v_\eta \\ 0 & \frac{1}{\sqrt{2}}\lambda_C v_{\bar\eta} & \frac{1}{\sqrt{2}}\lambda_C v_\eta & m_{\tilde{s}\tilde{s}} \end{pmatrix},$$

$$m_{\tilde{H}_d^0 \tilde{H}_u^0} = -\frac{1}{\sqrt{2}}\lambda_H v_S - \mu, \qquad m_{\tilde{H}_d^0 \lambda_{\tilde{X}}} = -\frac{1}{2}\Big(g_{YX} + g_X\Big)v_d,$$

$$m_{\tilde{H}_u^0 \lambda_{\tilde{X}}} = \frac{1}{2}\Big(g_{YX} + g_X\Big)v_u, \qquad m_{\tilde{s}\tilde{s}} = 2M_S + \sqrt{2}\kappa v_S.$$

This matrix is diagonalized by N

$$N^* M_{\tilde{\chi}^0} N^\dagger = M_{\tilde{\chi}^0}^{diag}.$$

With some supposition, we can deduce the lightest neutralino mass and eigenvector approximately. Comparing with $\mathcal{A}$ and $\mathcal{C}$, the matrix $\mathcal{B}$ is small. In this condition, we can use Z_N^T to simplify $M_{\tilde{\chi}^0}$ with matrix ζ

$$Z_N^T.M_{\tilde{\chi}^0}.Z_N \sim \begin{pmatrix} \mathcal{A} & 0 \\ 0 & \mathcal{C} \end{pmatrix}$$

In this work, we suppose the lightest neutralino $m_{\chi_1^0}$ is different from the MSSM condition, that is to say $m_{\chi_1^0}$ dominantly comes from the matrix $\mathcal{C}$ and MSSM neutralinos in the matrix $\mathcal{A}$ are heavy. Considering the constraint that $\tan\beta_\eta$ is near 1, we use $s_m = v_{\bar{\eta}} - v_\eta$ with the relation $v_\eta \gg s_m$. So, $\mathcal{C}$ turns to

$$\mathcal{C} = \begin{pmatrix} M_{BL} & -g_X v_\eta & g_X v_\eta + g_X s_m & 0 \\ -g_X v_\eta & 0 & \lambda'_C v_S + \mu_\eta & \lambda'_C v_\eta + \lambda'_C s_m \\ g_X v_\eta + g_X s_m & \lambda'_C v_S + \mu_\eta & 0 & \lambda'_C v_\eta \\ 0 & \lambda'_C v_\eta + \lambda'_C s_m & \lambda'_C v_\eta & m_{\tilde{S}\tilde{S}} \end{pmatrix},$$

with $\lambda'_C = \lambda_C/\sqrt{2}$.

The eigenvalues of $\mathcal{C}$ are deduced to the leading order according to s_m.

$$m^{(0)}_{\chi^0_a} = \frac{1}{2}\left(M_{BL} - \lambda'_C v_S - \mu_\eta + \sqrt{8g_X^2 v_\eta^2 + (\lambda'_C v_S + \mu_\eta + M_{BL})^2}\right),$$
$$m^{(0)}_{\chi^0_b} = \frac{1}{2}\left(-M_{BL} + \lambda'_C v_S + \mu_\eta + \sqrt{8g_X^2 v_\eta^2 + (\lambda'_C v_S + \mu_\eta + M_{BL})^2}\right),$$
$$m^{(0)}_{\chi^0_c} = \frac{1}{2}\left(\lambda'_C v_S + \mu_\eta + m_{\tilde{s}\tilde{s}} + \sqrt{8(\lambda'_C)^2 v_\eta^2 + (m_{\tilde{s}\tilde{s}} - \lambda'_C v_S - \mu_\eta)^2}\right),$$
$$m^{(0)}_{\chi^0_d} = \frac{1}{2}\left(\lambda'_C v_S + \mu_\eta + m_{\tilde{s}\tilde{s}} - \sqrt{8(\lambda'_C)^2 v_\eta^2 + (m_{\tilde{s}\tilde{s}} - \lambda'_C v_S - \mu_\eta)^2}\right).$$

M_{BL} and $m_{\tilde{s}\tilde{s}}$ are positive parameters. v_η is large and bigger than v_S. $m_{\chi^0_a}$ and $m_{\chi^0_c}$ are large values. Using $m_{\tilde{s}\tilde{s}} \gg M_{BL}$ and μ_η, $m_{\chi^0_d}$ is smaller than $m_{\chi^0_b}$, so $m_{\chi^0_d}$ is the lightest neutralino mass $m_{\chi^0_1}$.

$$m^{(1)}_{\chi^0_d} = -\frac{2(\lambda'_C)^2 s_m v_\eta}{\sqrt{8(\lambda'_C)^2 v_\eta^2 + (m_{\tilde{s}\tilde{s}} - \lambda'_C v_S - \mu_\eta)^2}}.$$

At the leading order, the eigenvector of $m_{\chi_d^0}$ is

$$V^{(0)}_{\chi_d^0} = \frac{1}{\sqrt{2+a_0^2}}\Big(0, 1, 1, a_0\Big) \sim \Big(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\Big),$$

Here, a_0 is a small parameter. The s_m correction to eigenvector $V^{(0)}_{\chi_d^0}$ is $V^{(1)}_{\chi_d^0}$

$$V^{(1)}_{\chi_d^0} = \frac{s_m}{v_\eta}\frac{1}{\sqrt{b_1^2+c_1^2}}\Big(0,\ 0,\ b_1,\ c_1\Big),$$

$$b_1 = \frac{\sqrt{8(\lambda'_C)^2 v_\eta^2 + m_{\tilde{s}\tilde{s}}^2} + m_{\tilde{s}\tilde{s}}}{4\lambda'_C v_\eta}, \quad c_1 = \frac{1}{2}\Big(\frac{m_{\tilde{s}\tilde{s}}}{\sqrt{8(\lambda'_C)^2 v_\eta^2 + m_{\tilde{s}\tilde{s}}^2}} - 1\Big).$$

c_1 is much smaller than b_1, then $V^{(1)}_{\chi_d^0}$ can be simplified as

$$V^{(1)}_{\chi_d^0} \sim \frac{s_m}{v_\eta}\Big(0,\ 0,\ 1,\ 0\Big).$$

In the whole, the eigenvector of the lightest neutralino is dominantly composed by the linear combination of $\tilde{\eta}$ and $\tilde{\bar{\eta}}$.

The self-annihilation processes are dominant in general condition.

$\chi_1^0 + \chi_1^0 \to A + B$, A and B represent final states $(Z,\ h)$, $(W,\ W)$, $(Z,\ Z)$, $(h,\ h)$, $(\bar{u}_i, u_i)$, $(\bar{d}_i,\ d_i)$, $(\bar{l}_i,\ l_i)$, $(\bar{\nu}_i,\ \nu_i)$. Here $i = 1,\ 2,\ 3$ and h represents the lightest CP-even Higgs.

For co-annihilation processes, if the mass of another particle is almost equal to mass of χ_1^0, they give considerable contributions to the annihilation cross section.

a. The lightest neutralino χ_1^0 annihilates with heavier neutralinos $\chi_k^0 (k = 2 \ldots 8)$, whose final states are same as those produced by self-annihilation processes.
b. $\chi_1^0 + \chi^- \to \{(\nu, l^-), (\bar{u}, d), (W^-, Z), (W^-, \gamma), (W^-, h^0)\}$. The corresponding processes are obtained by the charge conjugate transformation.
c. $\chi_1^0 + \tilde{L}^- \to \{(\gamma, l^-), (Z, l^-), (h^0, l^-)\}$. Similar as the condition b, condition c also has charge conjugate processes.
d. $\chi_1^0 + \tilde{\nu}^R(\tilde{\nu}^I) \to \Big\{(\nu, Z), (l^-, W^+), (l^+, W^-)\Big\}$.

The co-annihilation between χ_1^0 and scalar quarks $(\tilde{U},\ \tilde{D})$ are neglected, because scalar quark masses are very heavy and much larger than $m_{\chi_1^0}$.

The lightest neutralino scatters off nucleus, and the process is $\chi_1^0 + q \to \chi_1^0 + q$. The exchanged particles can be CP-even Higgs H_i^0, CP-odd Higgs A_i^0, gauge bosons Z, Z'. Because neutralino is Majorana particle, the operator $\bar{\chi}_1^0 \gamma_\mu \chi_1^0 \bar{q} \gamma^\mu q$ disappears.

For the CP-odd Higgs A_j^0 contribution, there are two suppression factors:

1. The Yukawa coupling Y_q of light quark;
2. The operators $\bar{\chi}_1^0 \chi_1^0 \bar{q} \gamma_5 q$ and $\bar{\chi}_1^0 \gamma_5 \chi_1^0 \bar{q} \gamma_5 q$ are suppressed by q^2 and q^4.

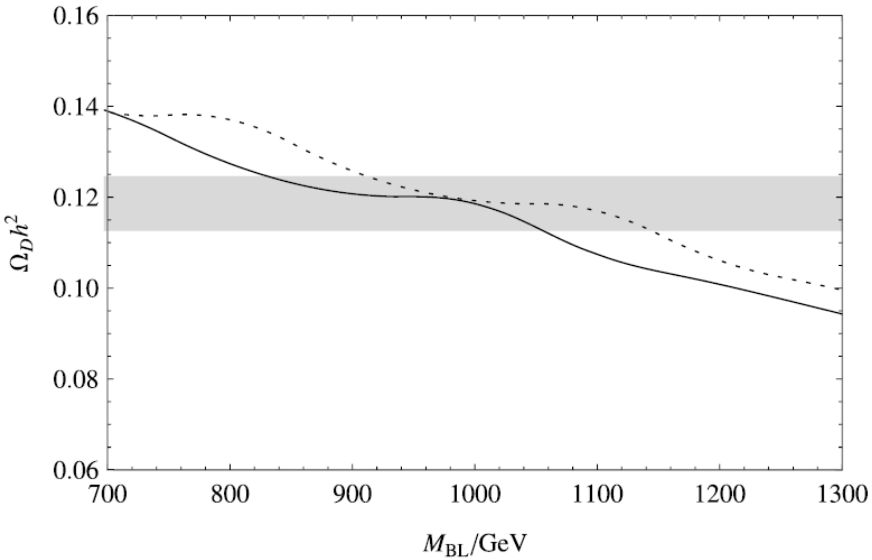
The dominant operators are $\bar{\chi}_1^0 \chi_1^0 \bar{q} q$ and $\bar{\chi}_1^0 \gamma_\mu \gamma_5 \chi_1^0 \bar{q} \gamma^\mu \gamma_5 q$ obtained from CP-even Higgs and vector bosons Z, Z' contributions.

The parameters

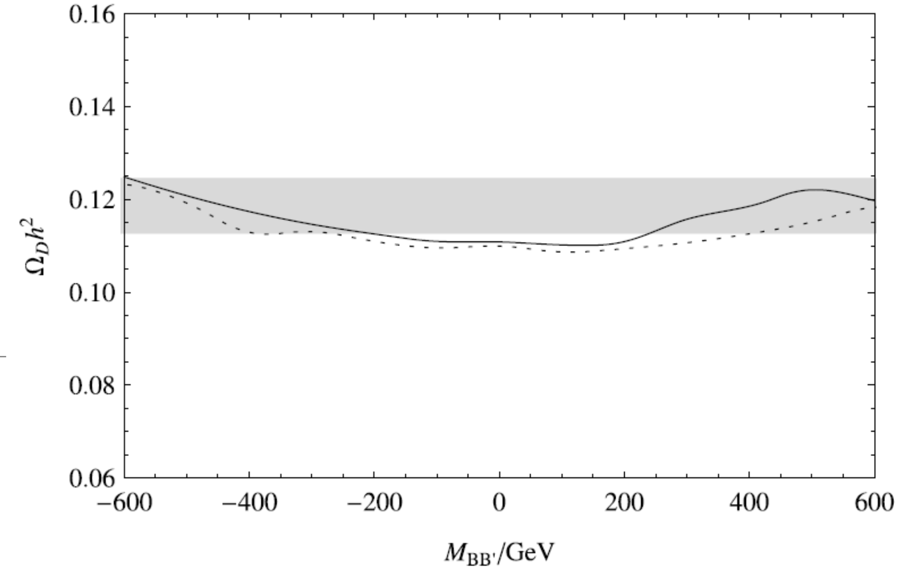
$M_S = 2.7$ TeV, $T_\kappa = 1.6$ TeV, $g_{YX} = 0.2$, $g_X = 0.3$, $\lambda_C = -0.08$, $\lambda_H = 0.1$,
$\upsilon_\eta = 15.5 \times \cos\beta_\eta$ TeV, $\upsilon_{\bar\eta} = 15.5 \times \sin\beta_\eta$ TeV, $Y_{X11} = Y_{X22} = 0.5$, $Y_{X33} = 0.4$,
$T_{\lambda_H} = 0.3$ TeV, $T_{X11} = T_{X22} = T_{X33} = -1$ TeV, $T_{e11} = T_{e22} = T_{e33} = -3$ TeV,
$T_{\lambda_C} = -0.1$ TeV, $\mu_\eta = 10$ GeV, $M^2_{U11} = M^2_{U22} = 10$ TeV^2, $M^2_{Q33} = M^2_{U33} = 3.5$ TeV^2,
$l_W = 4$ TeV^2, $M^2_{\nu11} = M^2_{\nu22} = M^2_{\nu33} = 0.5$ TeV^2, $T_{u11} = T_{u22} = T_{u33} = -2$ TeV, $\kappa = 1$,
$T_{d12} = T_{d21} = 0.2$ TeV, $M^2_{L11} = M^2_{L22} = M^2_{L33} = 1.9$ TeV^2, $B_\mu = B_S = m_S^2 = 1$ TeV^2,
$M^2_{E11} = M^2_{E22} = M^2_{E33} = 3.54$ TeV^2, $\tan\beta_\eta = 0.8$, $T_{\nu11} = T_{\nu22} = T_{\nu33} = 0.5$ TeV.

T_ν, T_X, T_u etc. are supposed as diagonal matrices and we use the supposition

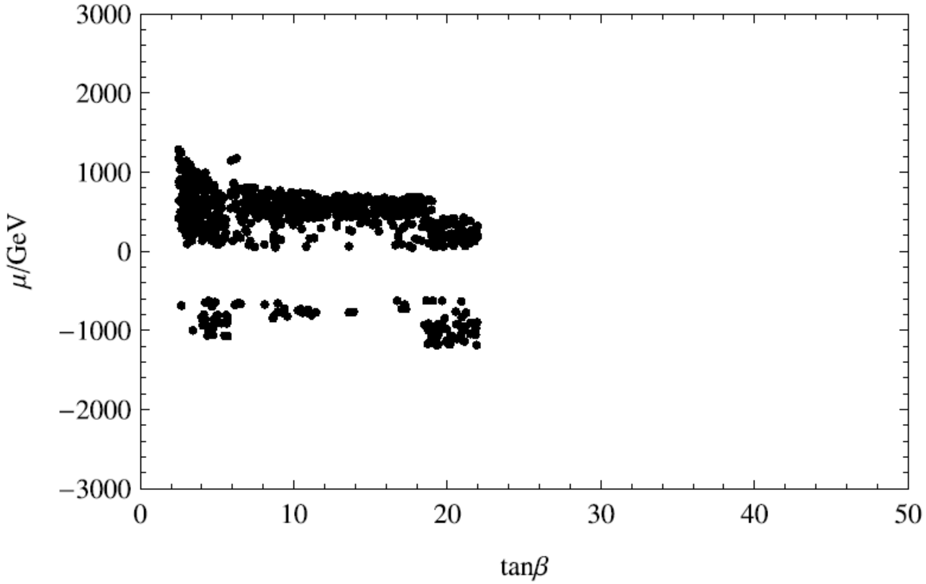
$$M^2_{D11} = M^2_{D22} = M^2_{D33} = M^2_D, \qquad T_{d11} = T_{d22} = T_{d33} = T_d.$$



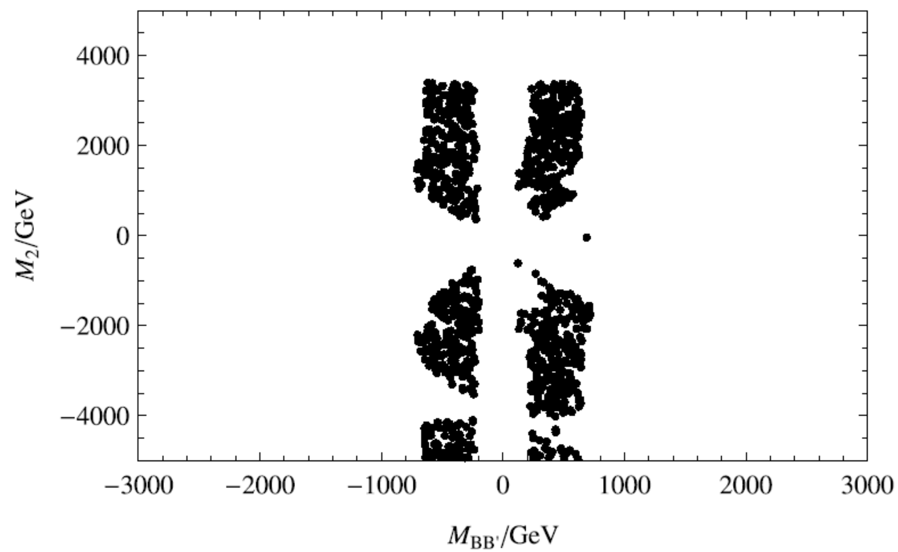
The relic density $\Omega_D h^2$ versus M_{BL} is plotted by the solid line (dotted line) $\tan\beta$=9(5).



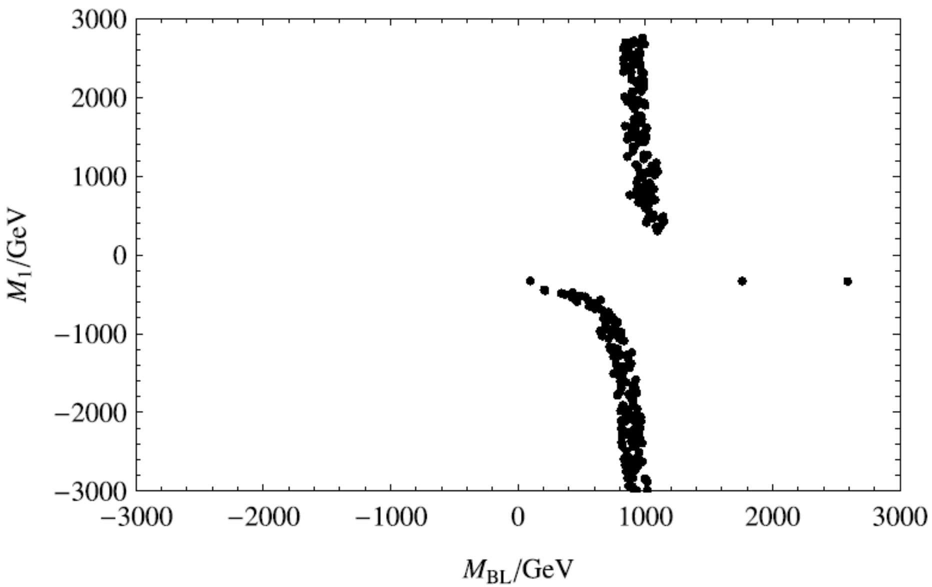
The relic density $\Omega_D h^2$ versus $M_{BB'}$ is plotted by the solid line (dotted line) μ=0.5(0.4) TeV.



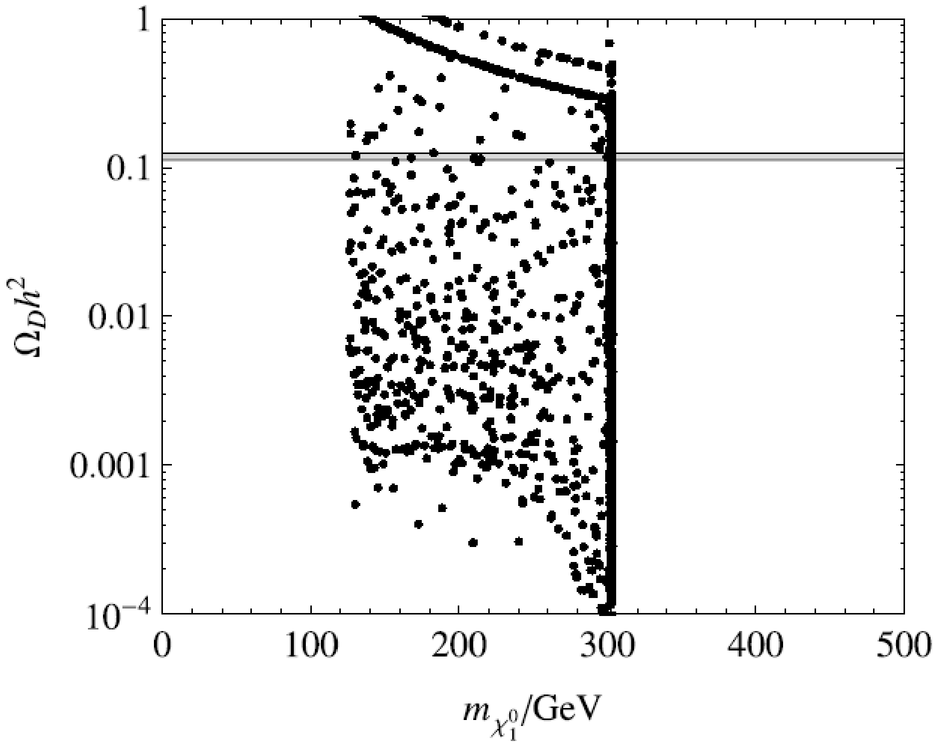
The allowed parameters in the plane of $\tan\beta$ and μ.



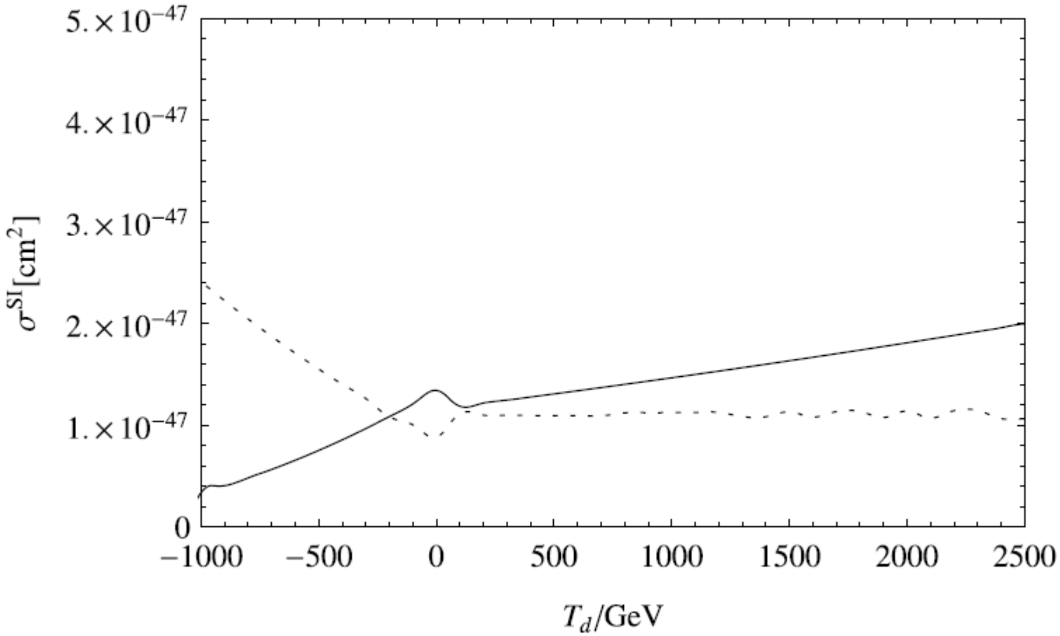
The allowed parameters in the plane of $M_{BB'}$ and M_2.



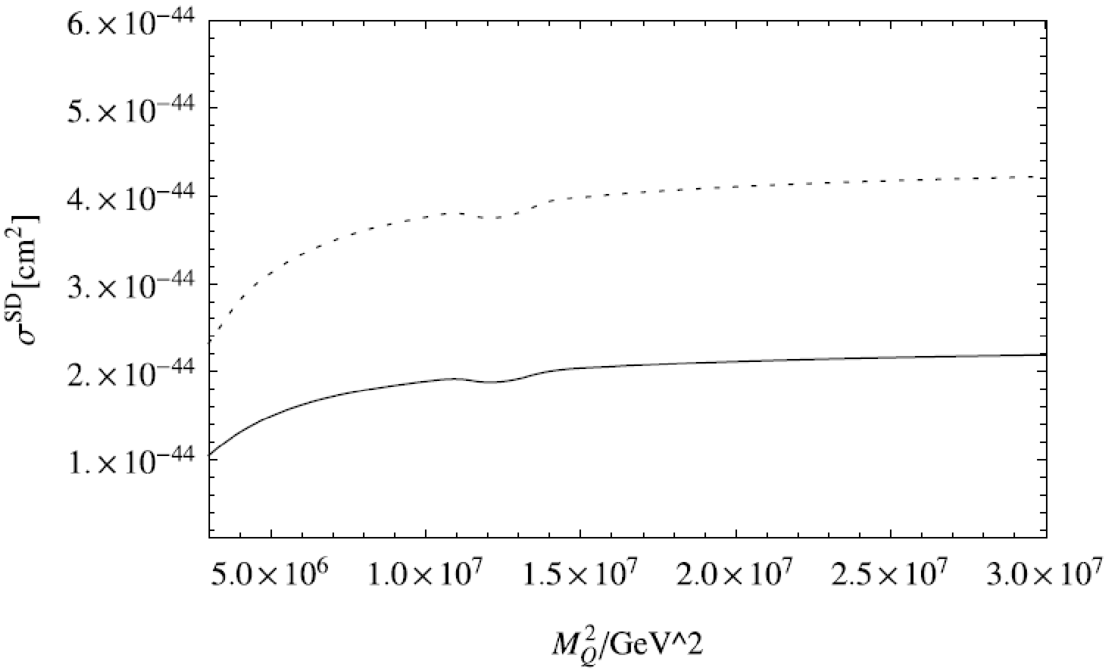
The allowed parameters in the plane of M_{BL} and M_1.



The relic density $\Omega_D h^2$ versus the lightest neutralino mass $M_{\chi_1^0}$.



The spin-independent cross section versus T_d, the solid line corresponds to M_D^2=6 TeV^2 and the dotted line corresponds to M_D^2=5 TeV^2.



The spin-dependent cross section versus M_Q^2 is plotted by the solid line (dotted line) with $M_D^2 = 10(5)$ TeV^2.

Summary

1 The $U(1)_X$SSM is the extension of MSSM, whose local gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. The Higgs superfields $\hat{\eta}$, $\hat{\bar{\eta}}$, $\hat{S}$ are added to the MSSM.

2 With the assumption that the lightest CP-even sneutrino can be a cold dark matter candidate, the relic density of dark matter and the cross section of dark matter scattering off nucleon are both studied.

3 The virtual Higgs contributions to both the relic density and the scattering cross section are dominant. The numerical results imply that the parameters $M^2_{\tilde{\nu}33}, M^2_{\tilde{L}33}, T_{\nu 33}$ and μ are all important.

4 From our numerical results, we find that $M_{BB'}$ and M_{BL} in the neutralino mass matrix are sensitive. The reason is that both $M_{BB'}$ and M_{BL} affect neutralino mixing. Large parameter space supports $m_{\chi_1^0} \sim 300$ GeV, though $m_{\chi_1^0}$ can be smaller with reasonable parameter space.

从满足暗物质丰度和直接探测的限制来说，Sneutrino比neutralino有优越性！


谢谢