The contribution of QCD trace anomaly to hadron mass

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Outline

① **Introduction to trace anomaly of energy momentum tensor**

② **Hadron mass decomposition**

③ **Numerical results**

④ **Summary**

QCD Trace Anomaly

- **Scale transformation(dilatations):**

$$x \to x e^{-\sigma}$$
$$\phi(x) \to e^{-D\sigma}\phi(xe^{-\sigma})$$

D is the mass dimension of field ϕ

- **Mass term will break down scale symmetry** $\partial_\mu J^\mu = T^\mu_\mu = m_q \bar{q} q$

J_μ is the Norther current for scale transformation

T^μ_μ is the trace of energy momentum tensor

- **Scale symmetry is broken when quantum corrections are included** Peskin and Schroeder, An Introduction to QFT, Chapter 19

$$(T^\mu_\mu)^a = \frac{\beta_{QCD}}{2g} F^2 + \gamma_m m_q \bar{q} q$$

R. J. Crewther, PRL28(1972) 1421
M. S. Chanowitz PLB40(1972) 397
J.Collins et,al. PRD16(1977) 438
N. K. Nielsen, NPB120 (1977) 212

β_{QCD}: beta function of QCD

γ_m: Anomalous dimension of quark mass

- **Total trace term of QCD EMT**

$$(T^\mu_\mu) = (T^\mu_\mu)^a + m_q \bar{q} q = \frac{\beta_{QCD}}{2g} F^2 + (1+\gamma_m) m_q \bar{q} q$$

The Effect of Heavy quark

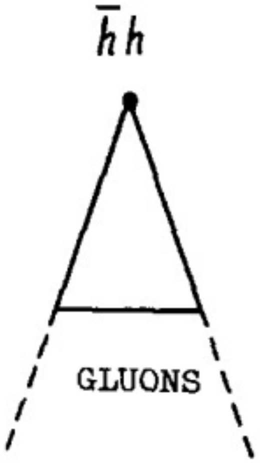
- **Trace term of ETM**

$$T^\mu_\mu = \frac{\beta_{QCD}}{2g}F^2 + \sum_l m_l(1+\gamma_{m,l})\bar{l}l + \sum_h m_h(1+\gamma_{m,h})\bar{h}h$$

- **The heavy quark mass term can be changed into**

M.A. SHIFMAN et,al. PLB78(1978)

$$m_h\bar{h}h \rightarrow -\frac{2}{3}\frac{\alpha}{8\pi}n_hF^2 + O(1/m_h)$$



- **The final expression of trace anomaly is**

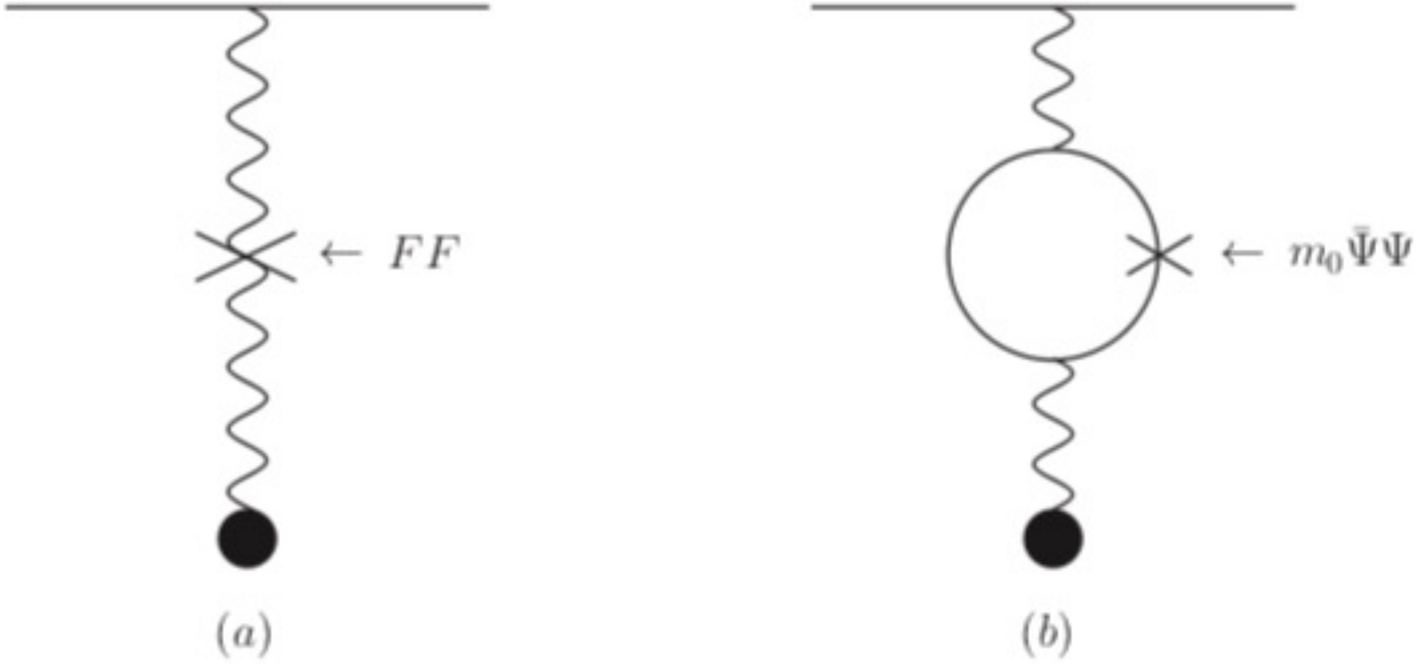
$$T^\mu_\mu = \frac{\tilde{\beta}}{2g}F^2 + \sum_l m_l(1+\gamma_{ml})\bar{l}l + O(1/m_h)$$

Trace Anomaly and Lamb Shift

B.D. Sun et,al. 2012.09443

- **Trace anomaly of QED can contribute a (small) part of the Lamb shift**

Talk By Baodong Sun, Tue 17th, 10:10



$$\Delta \mathrm{E} = 8\alpha_{em}^2\int d^3y\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{y}}\int_0^1 da\frac{a^2(1-a)^2}{m^2}\varphi_0^\dagger(y)\varphi_0(y)=\frac{-4\alpha_{em}^2}{15m^2}\varphi_0^\dagger(0)\varphi_0(0)$$

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Hardon Mass Decomposition

X. Ji, PRL 74,1071(1995)

- **Hadron energy can be decomposed into**

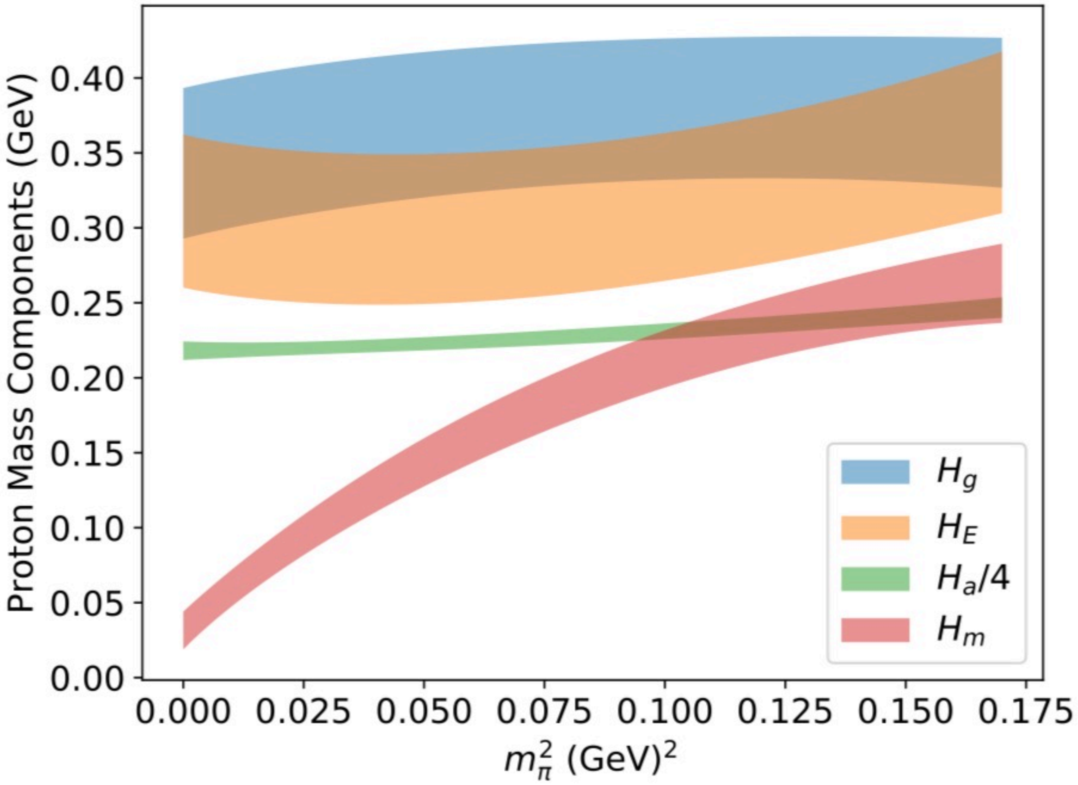
$$M_H = T^{00} = H_g + H_E + H_m + \frac{1}{4}H_a$$

H_g: **Gluon kinematic Energy**

H_E: **Quark kinematic Energy**

H_m: **Quark mass**

H_a: **Trace anomaly**



Y.B. Yang et,al. (χQCD Collaboration)PRL121(2018)

- **Hadron invariant mass can be decomposed as**

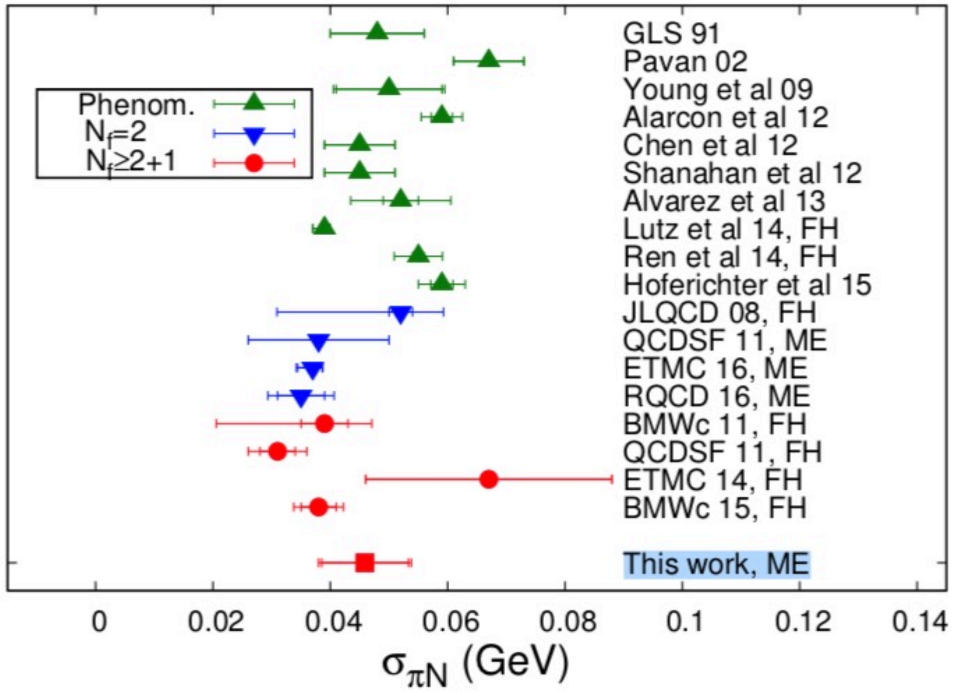
M.A. SHIFMAN et,al. PLB78(1978)

$$M = -\langle \hat{T}_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle.$$

Quark Mass Contribution

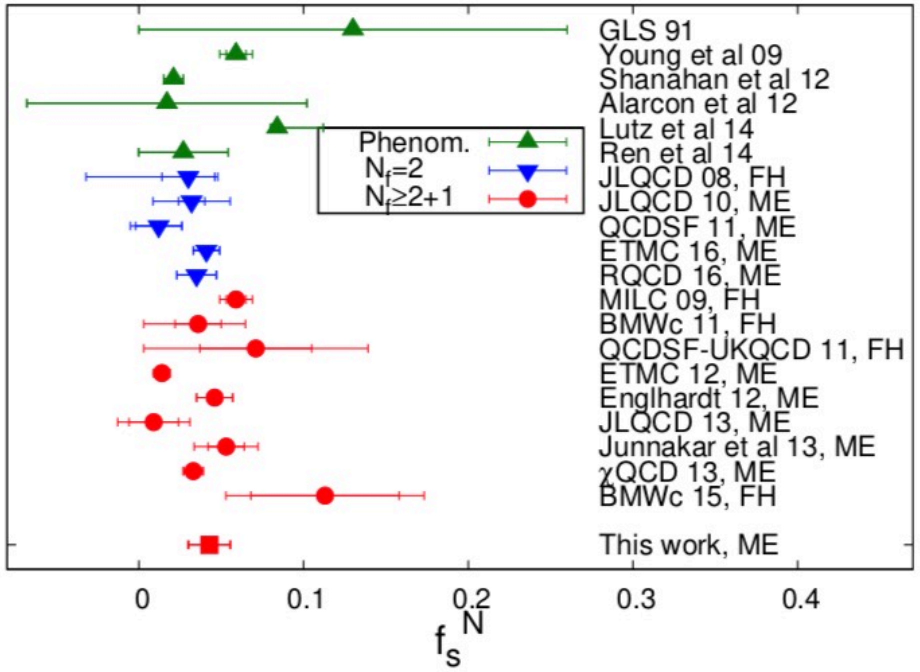
- **Quark mass contribution to proton mass (sigma terms)**

Y.B. Yang et,al.(χQCD Collaboration)PRD(2016),054503



$H_{m,u+d} = 45.9(7.4)(2.8)MeV$

Y.B. Yang et,al. (χQCD Collaboration)PRL121(2018)



$H_{m,s} = 40.2(11.7)(3.5)MeV$

- **The three light quark mass will contribute less than 100MeV to proton mass，according to sum rule：** $M_p = \sum_q < H_{m,q} > + H_a$ **Most of proton mass is contributed by trace anomaly!**

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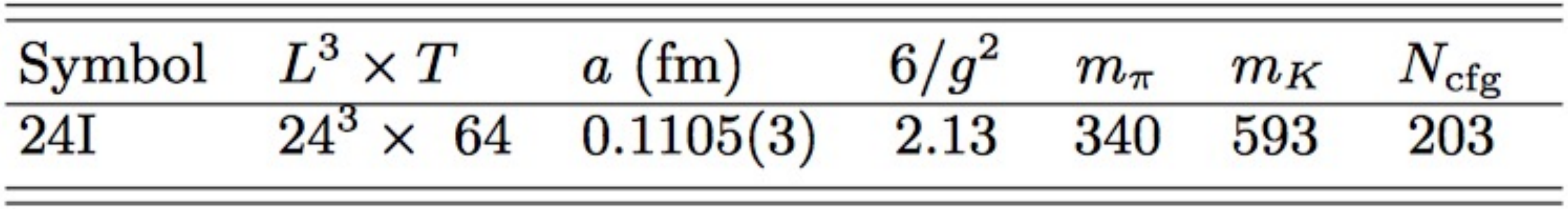


Symbol	$L^3 \times T$	a (fm)	$6/g^2$	m_π	m_K	$N_{\rm cfg}$
24I	$24^3 \times\ 64$	0.1105(3)	2.13	340	593	203

④ **Summary**

Calculation Procedure

- **To verify the mass sum rule** $M_H = < m_q \bar{q} q >_H + \gamma_m < m_q \bar{q} q >_H + \frac{\beta}{2g} < F^2 >_H$

Our calculation is divided into the following steps:

① Calculate the hadron mass and the matrix elements of quark mass, gluon condensate in different hadron, such as in pseudoscalar meson , vector meson and nucleon…

② Determine the values of γ_m and β. Since their values are independent of hadron state and quark mass, we can obtain them by solving the following equations

$$M_{PS} - (1 + \gamma_m) < m_q \bar{q} q >_{PS} - \frac{\beta}{2g^3} < g^2 F^2 >_{PS} |_{m_v = 0.48GeV} = 0$$

$$M_V - (1 + \gamma_m) < m_q \bar{q} q >_V - \frac{\beta}{2g^3} < g^2 F^2 >_V |_{m_v = 0.48GeV} = 0$$

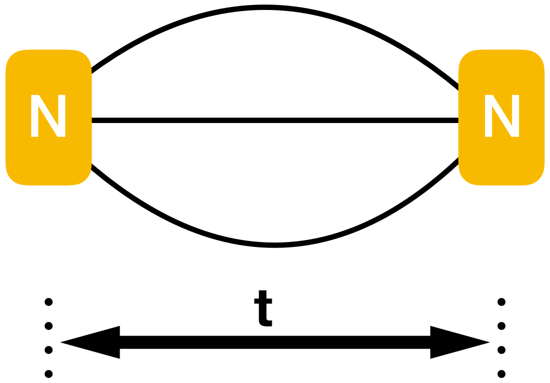
PS: Pseudoscalar meson
V: Vector meson

③ Check the mass sum in different hadron state for different quark mass.

Hadron Mass

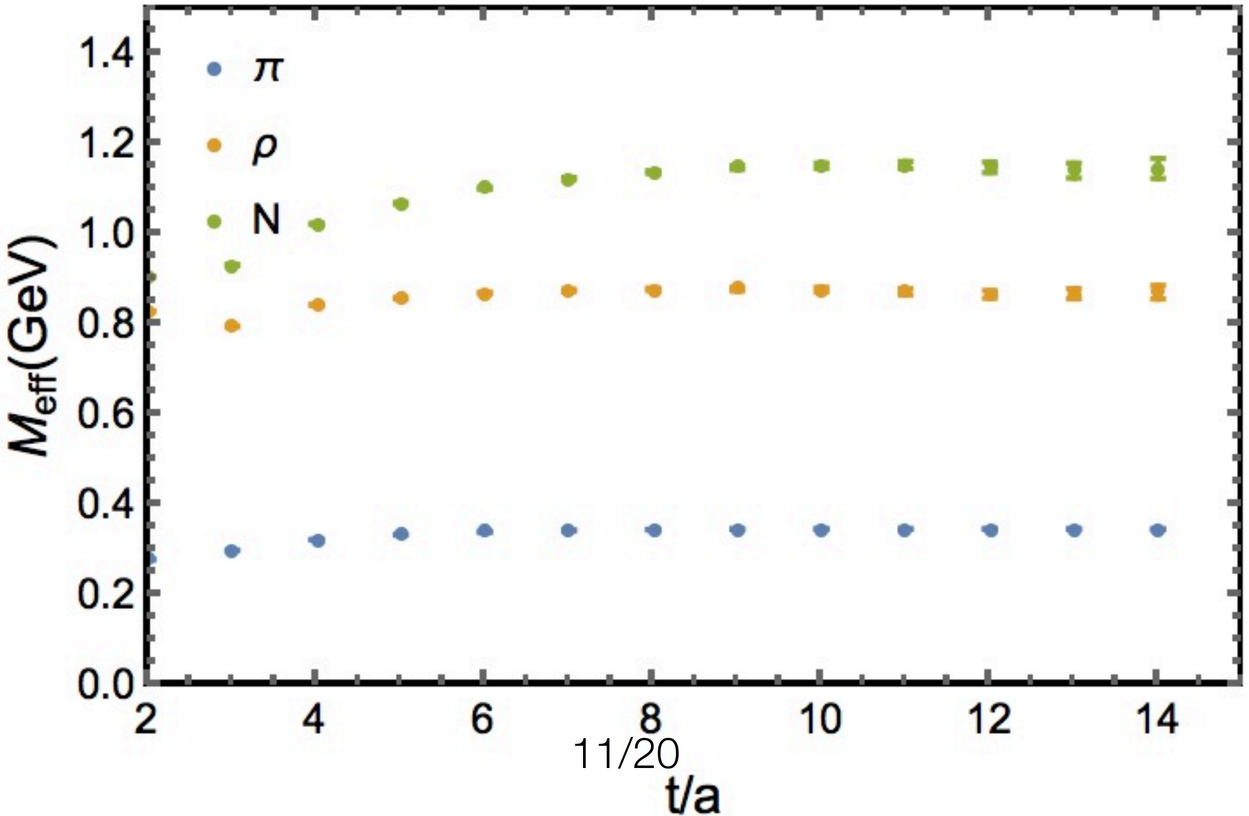
- **The effective mass can be extracted through two point correlation function**

$$C_2(t) = \langle N(t)N(0)\rangle \propto e^{-M_{eff}t} \qquad C_2(t) =$$



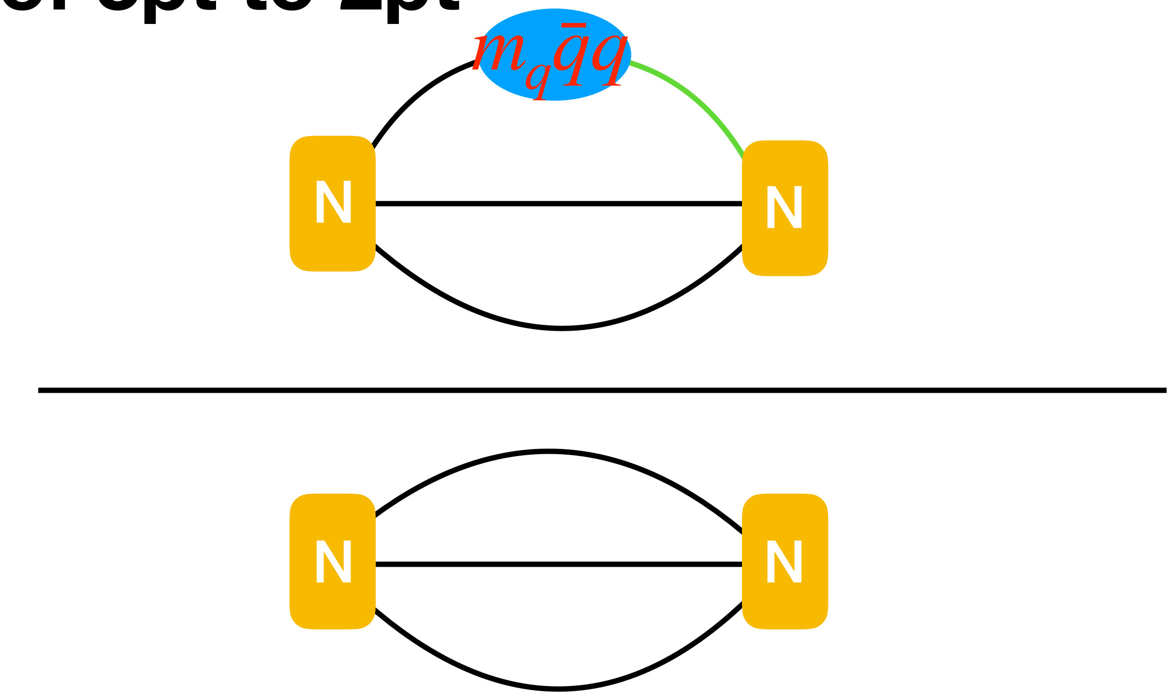
where $C_2(t)$ is two point correlation function

and t is time separation between initial and final state



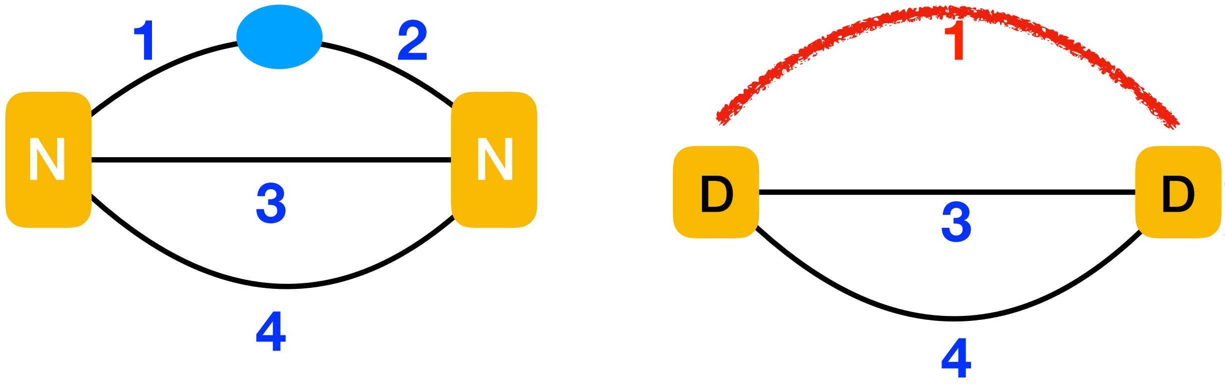
Quark mass term $\langle m_q \bar{q} q \rangle_H$

- The quark mass contribution can be obtained by the ratio of 3pt to 2pt (Large t limit)



= $\langle m_q \bar{q} q \rangle_H$

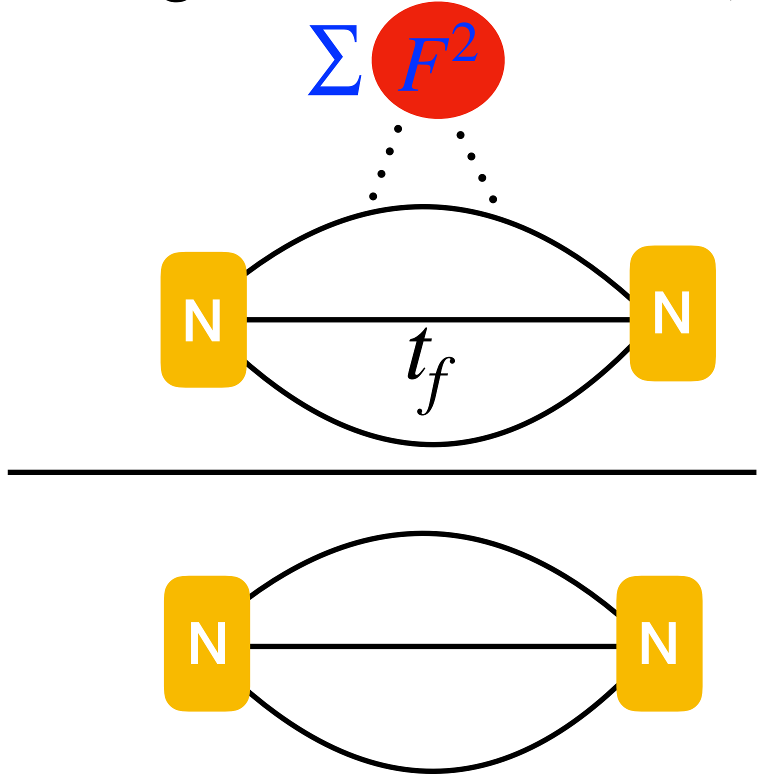
- Feynman–Hellmann propagator C.C. Chang et,al. Nature(2018),558



$$R(t_f, O) = \frac{\langle SC_3(t_f, O) \rangle}{\langle C_2(t_f) \rangle} - \frac{\langle SC_3(t_f - 1, O) \rangle}{\langle C_2(t_f - 1) \rangle} = \langle H|O|H \rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Gluon Condensate $\langle F^2 \rangle_H$

- For the gluon condensate, the ratio of 3pt to 2pt is defined as



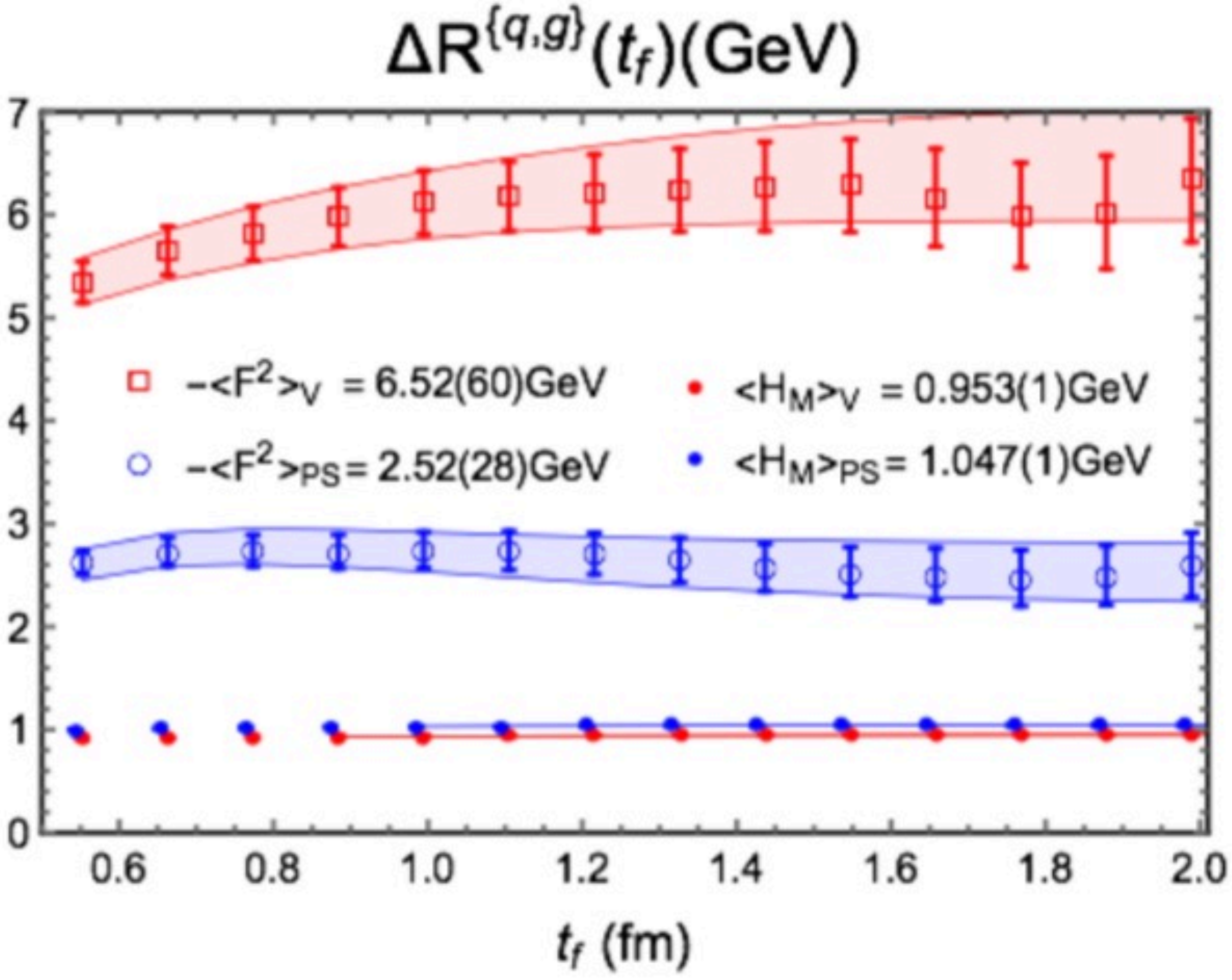
$$= \frac{\langle N(t_f) \sum_{t_f>t>0} F^2(t) N(0) \rangle - \langle \sum_{t_f>t>0} F^2(t) \rangle \langle N(t_f) N(0) \rangle}{\langle N(t_f) N(0) \rangle}$$

- We can extract the matrix element of gluon condensate from 3pt over 2pt ratio

$$\tilde{R}(t_f, \tilde{O}) = \frac{\sum_{t_f>t>0} \langle C_3(t_f, t, \tilde{O}) \rangle}{\langle C_2(t_f) \rangle} - \frac{\sum_{t_f-1>t>0} \langle C_3(t_f - 1, \tilde{O}) \rangle}{\langle C_2(t_f - 1) \rangle} = \langle H | \tilde{O} | H \rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Numerical Results

- **Numerical results for quark mass terms and gluon condensate in PS and V meson with** $m_q = 0.48 GeV$



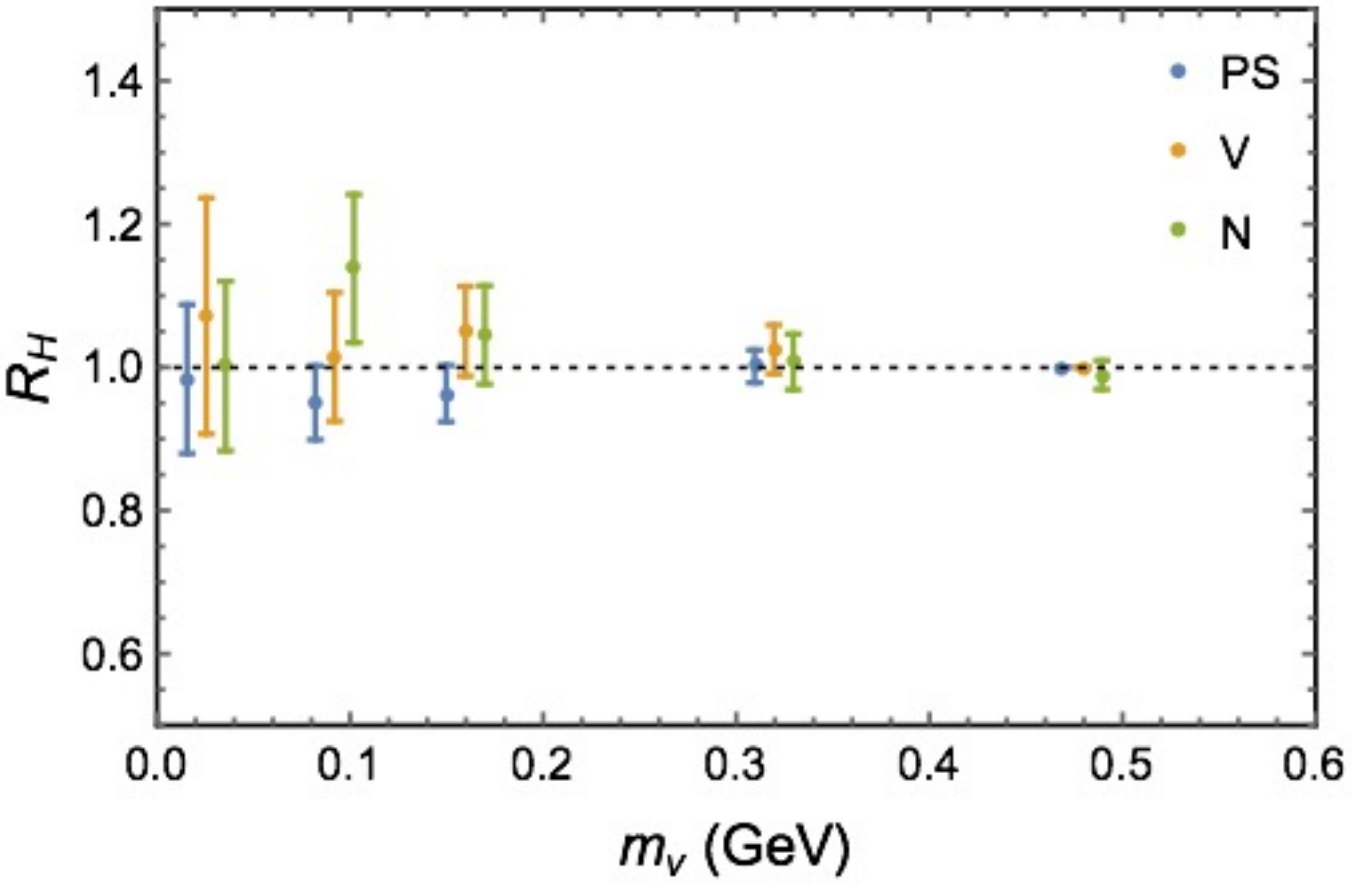
The Values of γ_m and β

- **Comparison of γ_m and $\frac{\beta}{g^3}$ between our results and other results**

γ_m		$\frac{\beta}{g^3}$	
Our result	4-loop results(MSbar) J.Vermaseren. PLB, 405(1997) 327	Our result	regularization independent leading order
0.38(3)	0.33(1) $(\mu = 1/a = 1.78GeV)$	-0.056(6)	$\frac{-11+\frac{2N_f}{3}}{(4\pi)^2} = -0.057$

Numerical Results

- **Verify Sum rules:** $M_H = < H_m >_H + \gamma_m < H_m >_H + \frac{\beta}{2g} < F^2 >_H$

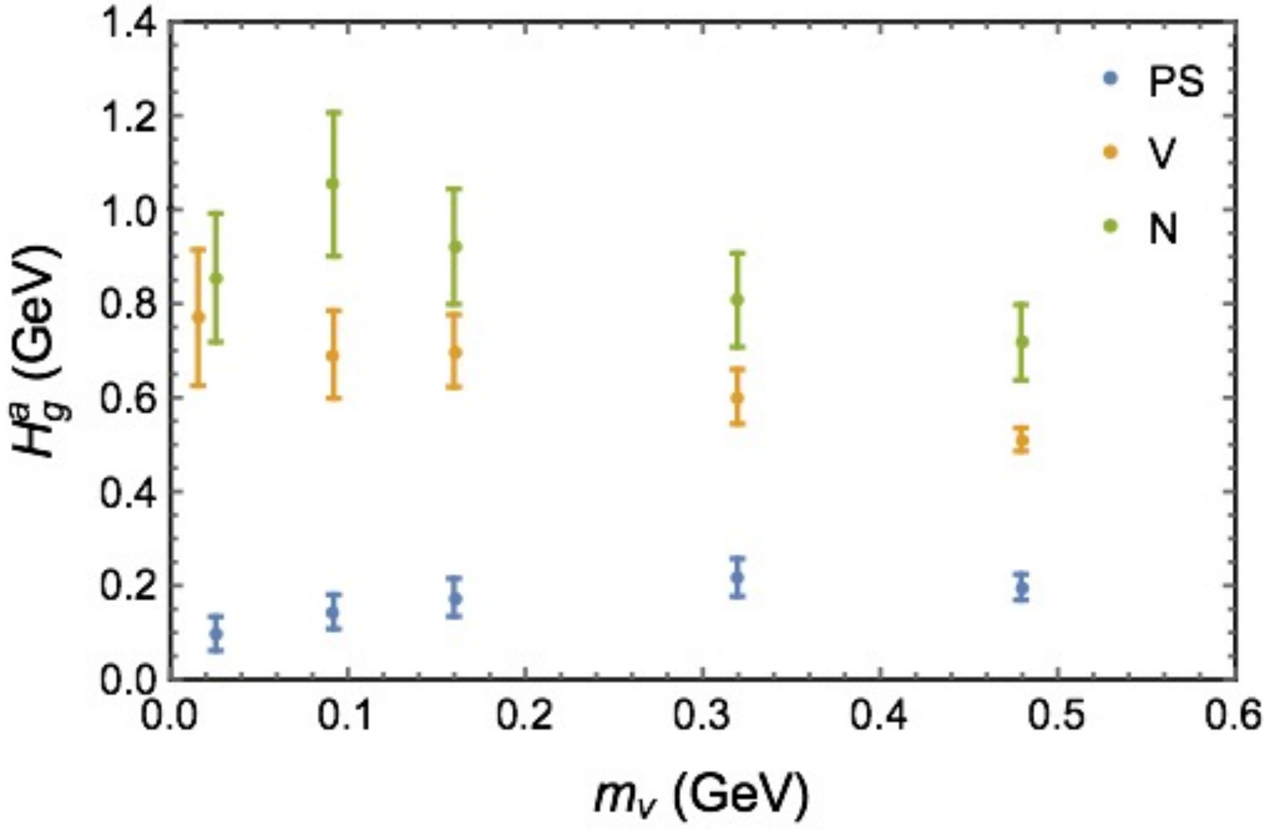


$$R_H = \frac{(1+\gamma_m) < H_m >_H + \frac{\beta}{2g} < F^2 >_H}{M_H}$$

We checked the trace anomaly sum rule. The ratio of sum rules to hadron mass is plotted, We can see that the R_H all the cases are consistent with one within the uncertainties.

Numerical Results

- **The contribution of gluon to hadron mass ($H_g^a = \frac{\beta}{2g} < F^2 >_H$)**



Hadron mass at the unitary point:

$$M_N = 1.16GeV$$
$$M_V = 0.881GeV$$
$$M_{PS} = 0.34GeV$$

1. **The contribution of gluon part in trace anomaly to the pseudoscalar meson mass is always much smaller than that in the other hadrons, especially around the chiral limit.**
2. **The ration of gluon trace anomaly in pseudoscalar meson is also smaller.**

The density of gluon trace anomaly

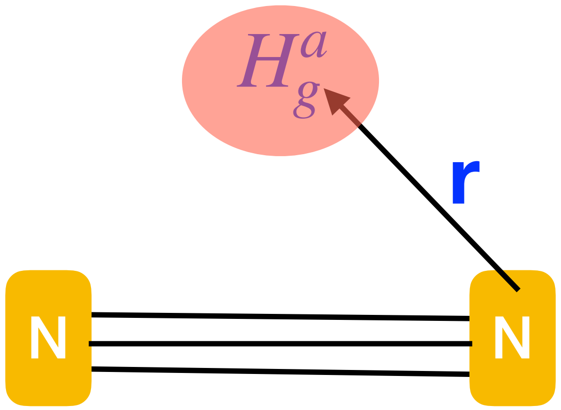
- **The density of gluon trace anomaly can be expressed as**

$$\rho_H(|r|) = \frac{\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) H_a^g(t, \vec{y} + \vec{r}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle}{\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle} |_{t, t_f - t \to \infty},$$

Cluster decomposition method:
K. F. Liu et al., PRD,97(2018) 034507

C. Bouchard et al., PoS Lattoce2016,170
X. Feng et al., PRD,101(2020) 051502

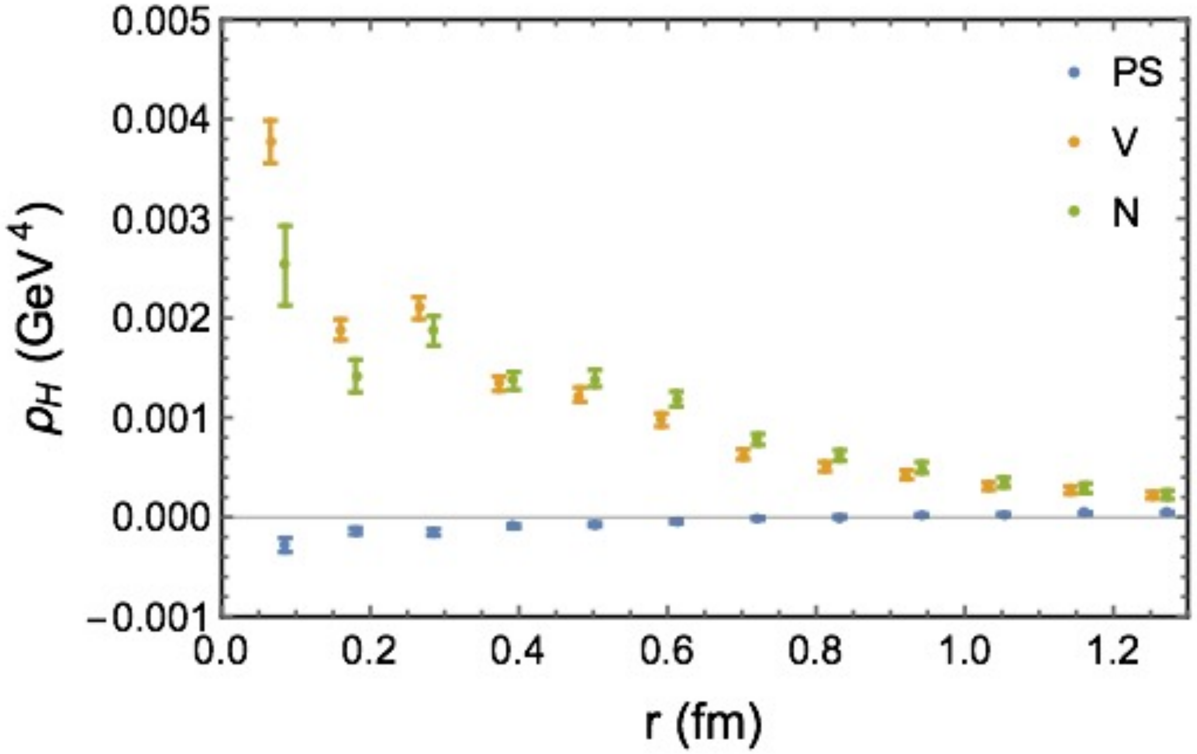
- **Schematic description**



Numerical Results

F. He, P. Sun and Y. Yang, arXiv：2101.04942.

- The density of gluon trace anomaly ($\int d^3r\bar{\rho}_H(r) = \frac{\beta}{2g}\langle F^2\rangle_H$) at the unitary point ($M_\pi = 340 MeV$)



- The density of gluon trace anomaly in pseudoscalar meson is negative, make the total trace anomaly of gluon is smaller than that in other hadron.

Summary

- **Summary**

① **We calculate the contribution of quark mass and trace anomaly in different hadron, we also verify the mass sum rule, the hadron mass obtained from sum rule is consistent with its ground state mass.**

② **We determine the values of γ_m and β/g^3, γ_m is comparable with the perturbative result, β/g^3 is perfectly consistent with the regularization independent leading ordering term.**

③ **We find the gluon trace anomaly contribute most of the hadron masss, except the pion case.**

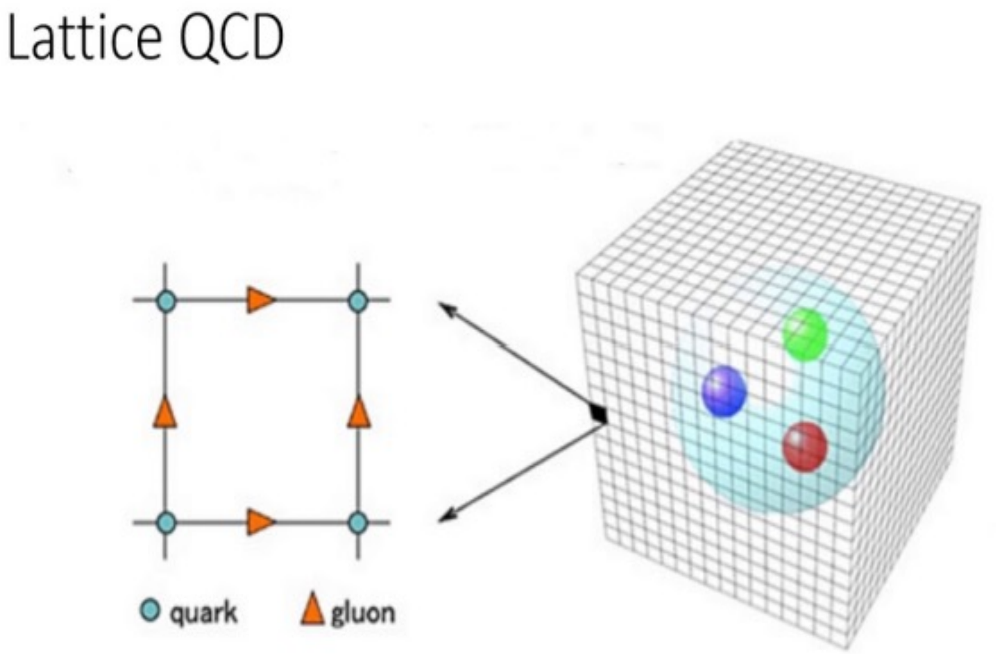
④ **The density of gluon trace anomaly in pseudoscalar meson is negative near the center and the magnitude is smaller than that in other hadron.**

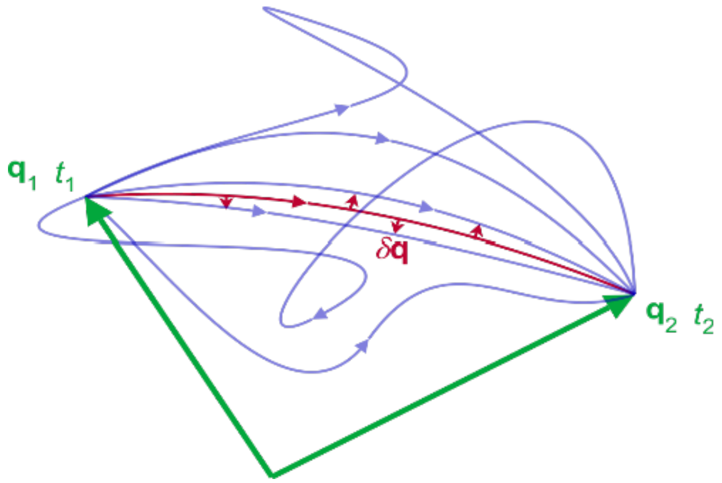
Thanks for your attention!

Backup

Lattice QCD

- **Lattice QCD is well-established non-perturbative method to solve QCD problem.**





$$\langle\langle \Gamma[\phi] \rangle\rangle \equiv \frac{1}{\mathcal{Z}} \int \mathrm{e}^{-S[\phi]}\,\Gamma[\phi] \prod_{x_j \,\epsilon\, \mathrm{grid}} d\phi(x_j) \;\rightarrow \frac{1}{N}\sum_{i=1}^{N} \Gamma[\phi_i]$$

Energy Momentum Tensor of QCD

- **The energy momentum tensor of QCD:**

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} \psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha},$$

- $T^{\mu\nu}$ **can be decomposition into two parts:**

$$T^{\mu\nu} = \underbrace{\frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} \psi - \frac{1}{4} g^{\mu\nu} m \bar{\psi}\psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}}_{\text{Traceless}} + \underbrace{\frac{1}{4} g^{\mu\nu} m \bar{\psi}\psi}_{\text{Trace term}}$$

The relation of hardon mass and trace of EMT

$$M_H = < T^{\mu}_{\mu} >_H = \sum_l m_l < \bar{l} l >_H + \sum_h m_h < \bar{h} h >_H$$ ❓

- **Puzzles:**
  1. **Does heavy quark dominate contribute to hadron mass?**
  2. **All hadron mass will become zero at chiral limit**

Overlap Fermion

- **Clover Fermion will introduce extra term to energy momentum tensor of QCD**

$$\left(D_w + c_{sw}\sigma^{\mu\nu}F_{\mu\nu} + (m_c + m_q)\right)\psi = 0$$

- **The definition of Overlap operator**

$$D_{ov} = \rho(1 + \gamma_5\epsilon_{ov}(\gamma_5 D_w))$$

Where $\epsilon_{ov}(\gamma_5 D_w)$ is the sign function of Wilson operator D_w

- **Overlap operator satisfies Ginsparg-Wilson Relation**

$$D_{ov}\gamma_5 + \gamma_5 D_{ov} = \frac{1}{\rho} D_{ov}\gamma_5 D_{ov}$$

Anomalous Breaking of Scale Invariance

- **The coupling constant and quark mass will vary with scale**

$$\frac{dlog(m)}{dlog(\mu)} = -\gamma_m$$

$$\mu\frac{dg}{d\mu} = \beta$$

$$\xrightarrow{\mu \to \mu + \sigma\mu}$$

$$g \to g + \sigma\beta(g)$$

$$m \to m - \sigma m\gamma_m$$

- **Corresponding change in the Lagrangian is**

$$\sigma\beta(g)\frac{\partial L}{\partial g} - \sigma\gamma_m m\frac{\partial L}{\partial m}$$

- **Trace anomaly can be obtained**

$$\sigma\partial_\mu D^\mu = \sigma T^\mu_\mu = \sigma\beta(g)\frac{\partial L}{\partial g} - \sigma\gamma_m m\frac{\partial L}{\partial m} \longrightarrow T^\mu_\mu = \frac{\beta}{2g}F^2 + m\gamma_m\bar{\psi}\psi$$

Trace anomaly in perturbation theory

- The trace of ETM in d dimension

$$T^{\alpha}_{\alpha} = -2\epsilon\frac{F^2}{4} + \bar{\psi} i \overleftrightarrow{\not{D}} \psi = -2\epsilon\frac{F^2}{4} + m\bar{\psi}\psi.$$

In d dimension

- Renormalization of FF.

$$F^2 = \left(1 - \frac{\beta}{g}\frac{1}{\epsilon}\right)(F^2)_R - 2\frac{\gamma_m}{\epsilon}(m\bar{\psi}\psi)_R$$

The bare operator FF is divergent

$$\begin{aligned} T^{\mu}_{\mu} &= -2\epsilon\frac{F^2}{4} + m\bar{q}q \\ &= \underbrace{\frac{\beta(g)}{2g}(F^2)_R + \gamma_m(m\bar{q}q)_R}_{\text{from } (T_g)^{\mu}_{\mu}} + \underbrace{(m\bar{q}q)_R}_{\text{from } (T_q)^{\mu}_{\mu}} \end{aligned}$$

For the bare ETM, the anomaly entirely comes from the gluon part