Axial kinetic theory for QED

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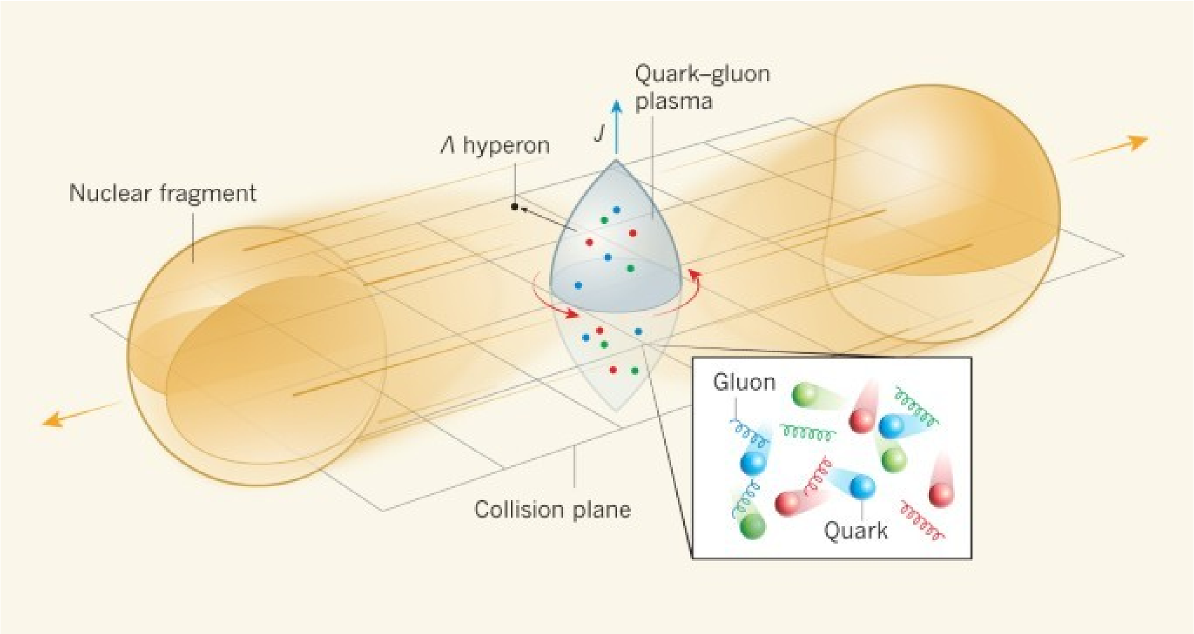
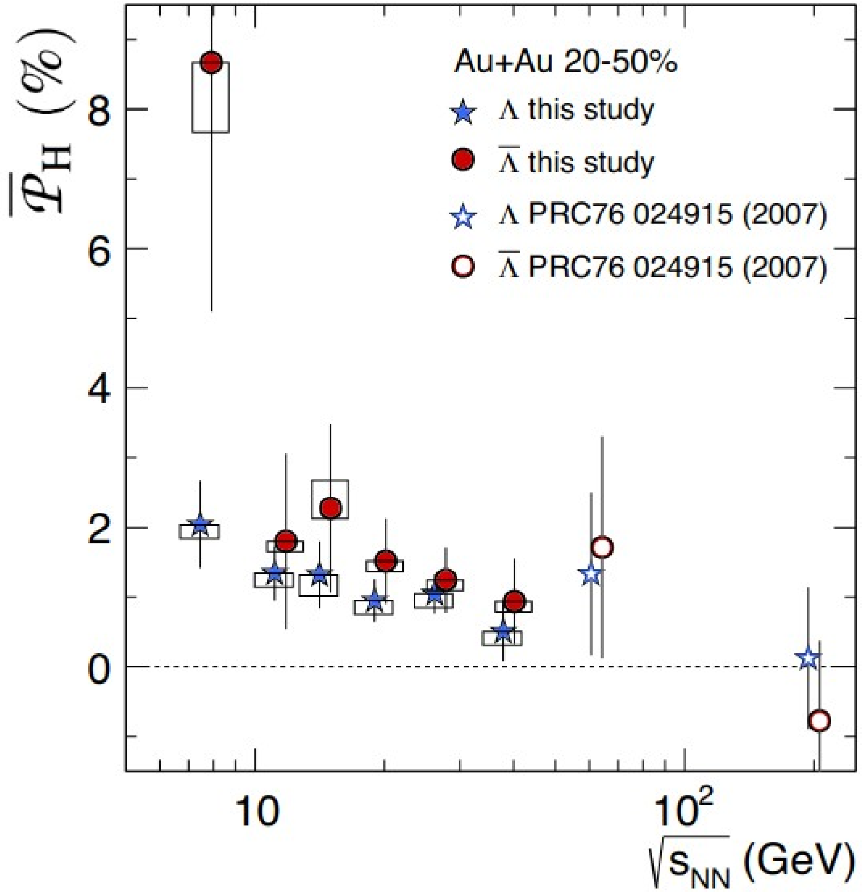
第十三届全国粒子物理学术会议，山东大学，青岛 2021

based on 2108.XXXXX

Λ Global Polarization at RHIC

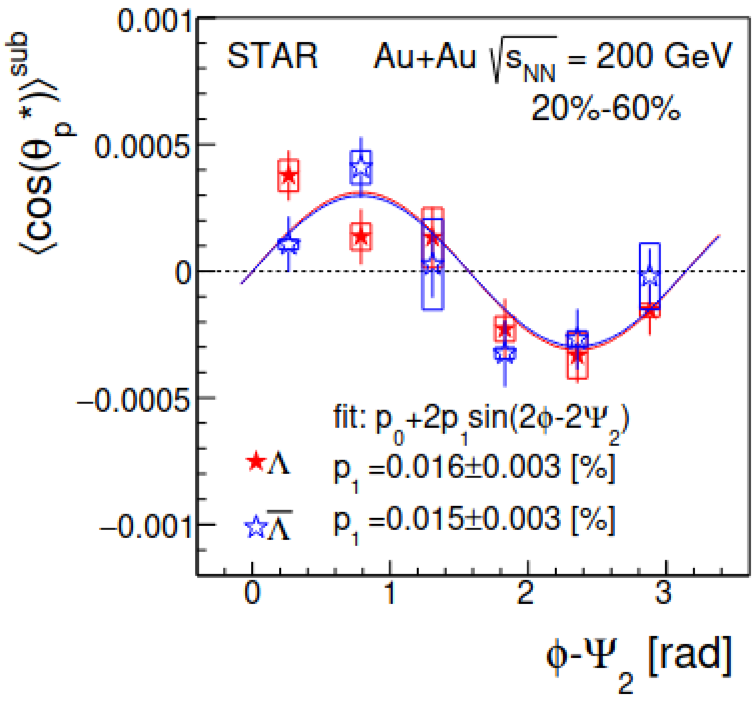


Liang, Wang, PRL 2005

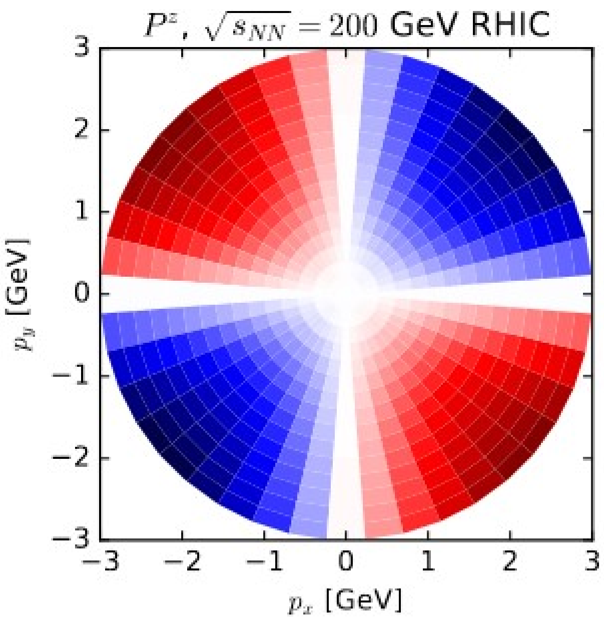


STAR collaboration, Nature 2017

Λ Local polarization: sign puzzle



STAR collaboration, PRL 2019



Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Liu, Yin 2103.09200. Fu, Liu, Pang, Song, Yin, 2103.10403

thermal vorticity induced

Becattini, et al, PLB 2021, 2103.14621
Yi, Pu, Yang, 2106.00238

+thermal shear induced etc

Dynamical models

- Spin hydrodynamics
  - pros: macroscopic, less dofs
  - cons: assumed hierarchy of relaxation times: $\tau_{\rm hydro} \gtrsim \tau_{\rm spin} \gg \tau_{\rm other}$
- Axial kinetic theory
  - pros: microscopic, no assumption needed
  - cons: quasi-particle description

Outline

- Recent development on axial kinetic theory with collisions
- Structure of kinetic equations and solutions
- Elastic/inelastic collisions from self-energies
- Spin polarizaion from first order solution
- Summary & Outlook

Works on collision term

CKT with collisions
Yang, Hattori, Hidaka JHEP 2020 (general framework for fermion)
Hattori, Hidaka, Yamamoto, Yang JHEP 2021 (general framework for photon)
Hidaka, Pu, Yang, PRD 2017 (QED Coulomb scattering)
Li, Yee, PRD 2019 (QCD Coulomb scattering)
Carignano, Manuel, Torre-Rincon, PRD 2020 (QED Coulomb scattering)
Hou, SL, PLB 2021 (QED/QCD Coulomb scattering)
Weickgnnant, Speranza, Sheng, Q. Wang, Rischke, 2103.04896 ($2\rightarrow2$ elastic scattering)
Sheng, Weickgnnant, Speranza, Rischke, Q. Wang 2103.10636 (Yukawa theory)
Z. Wang, Guo, Zhuang, 2009.10930, Z. Wang, Zhuang 2105.00915 ($2\rightarrow2$ NJL)

This talk: derive collision term for QED based on Kadanoff-Baym equation, both $2\rightarrow2$ elastic scattering and $1\rightarrow2$ inelastic scattering

New ingredients: Compton/annihilation + collinear inelastic scatterings

DOFs of kinetic theory

Massive fermion: f_V^e f_A^e a_μ

Hattori, Hidaka, Yang, PRD 2019
Gao, Liang, PRD 2019
Weickgenannt et al, PRD 2019

Photon: f_V^γ f_A^γ

Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

Assumptions & simplifications

Massive fermion: f_V^e $\cancel{f_A^e \quad a_\mu}$

Hattori, Hidaka, Yang, PRD 2019
Gao, Liang, PRD 2019
Weickgenannt et al, PRD 2019

Photon: f_V^γ $\cancel{f_A^\gamma}$

Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

assume system parity invariant and no EM field

same DOF as in the classical kinetic theory by Arnold, Moore, Yaffe

Arnold, Moore, Yaffe, JHEP 2003

Kadanoff-Baym equations for fermions

$$S^{<}_{\alpha\beta}(X,P) = -\int d^4(x-y) e^{iP\cdot(x-y)/\hbar} \langle \bar{\psi}_\beta(y)\psi_\alpha(x)\rangle$$

$$\frac{i\hbar}{2}\not{\partial} S^{<} + \not{P} S^{<} - mS^{<} = \frac{i\hbar}{2}\left(\Sigma^{>}S^{<} - \Sigma^{<}S^{>}\right)$$

$$S^{<} = S^{<(0)} + \hbar S^{<(1)}$$

$S^{<(0)}$ Classical, describe momentum distribution

$S^{<(1)}$ Quantum correction, contains spin dynamics

Kadanoff-Baym equations for photons

$$D^{\mu\nu<}(X,P) = \int d^4(x-y) e^{iP\cdot(x-y)/\hbar} \langle A_\nu(y) A_\mu(x) \rangle$$

$$\left(-P^2 g^{\mu\nu} + P^\mu P^\nu - \frac{1}{\xi} P^{\mu\alpha} P^{\nu\beta} P_\alpha P_\beta + i\hbar \left(-\frac{1}{2} P\cdot\partial g^{\mu\nu} + \frac{1}{2}\partial^\mu P^\nu - \frac{1}{2\xi} P^{\mu\alpha} P^{\nu\beta} \partial_\alpha P_\beta + \mu \leftrightarrow \nu\right)\right) D^<_{\nu\rho}$$

$$= \frac{i\hbar}{2} \left(\Pi^{\mu\nu>} D^<_{\nu\rho} - \Pi^{\mu\nu<} D^>_{\nu\rho}\right)$$

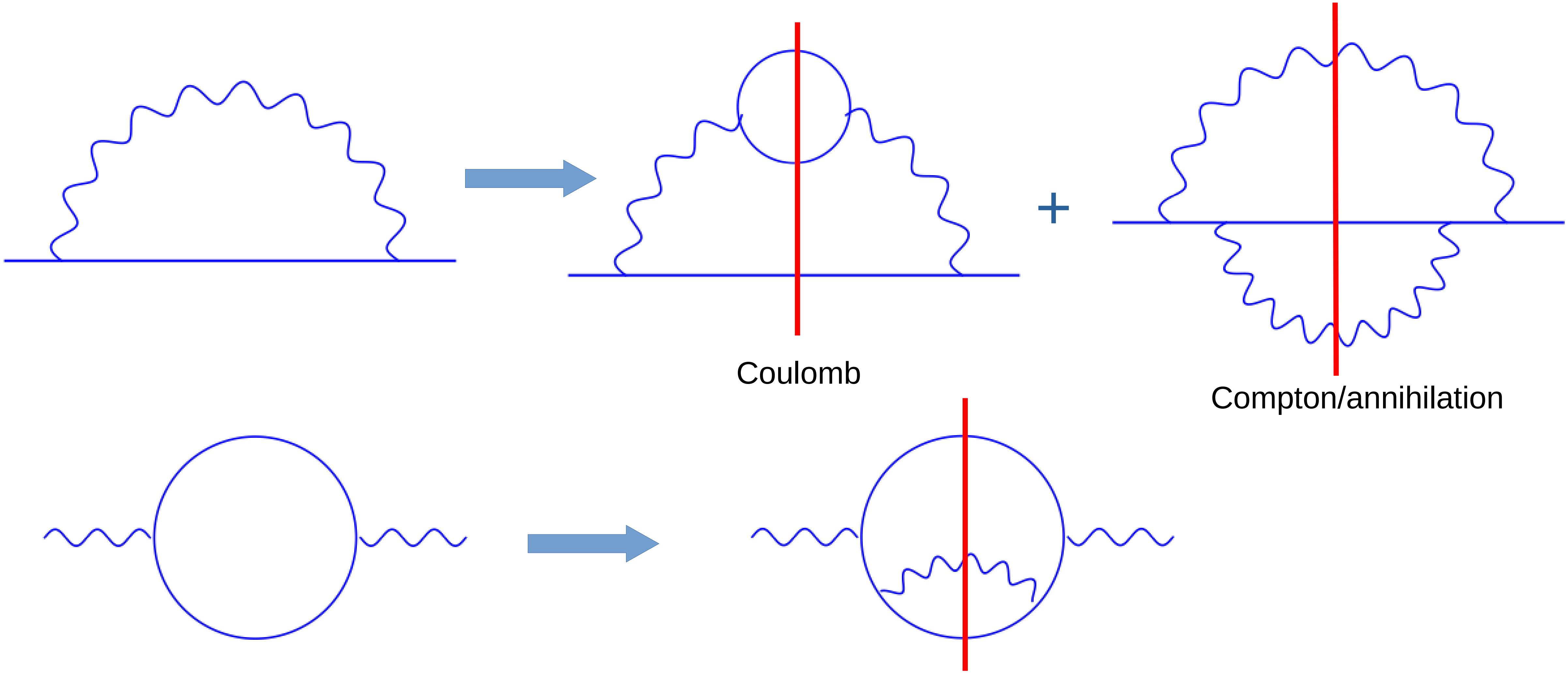
Coulomb gauge $\xi = 0$ $\longrightarrow$ $P^{\mu\alpha} \left(\frac{i\hbar}{2}\partial_\alpha + P_\alpha\right) D^<_{\mu\nu} = 0$

$$D^{\mu\nu<} = D^{\mu\nu<(0)} + \hbar D^{\mu\nu<(1)}$$

$D^{\mu\nu<(0)}$ Classical, describe momentum distribution

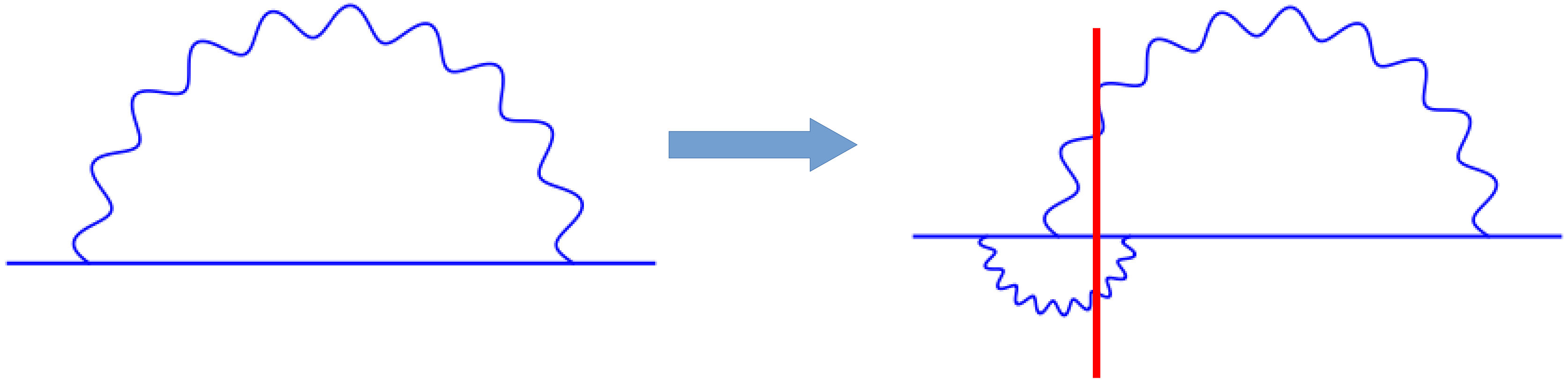
$D^{\mu\nu<(1)}$ Quantum correction, contains spin dynamics

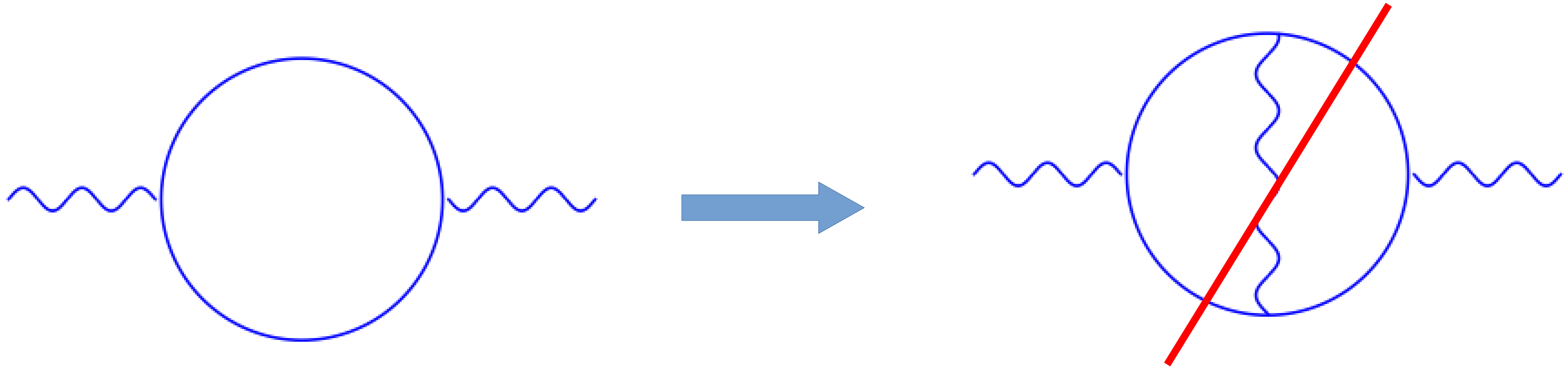
Self-energies: correction to propagators



Compton/annihilation

Self-energies: correction to vertices (elastic $2 \to 2$)





Inteference terms

Screening effect

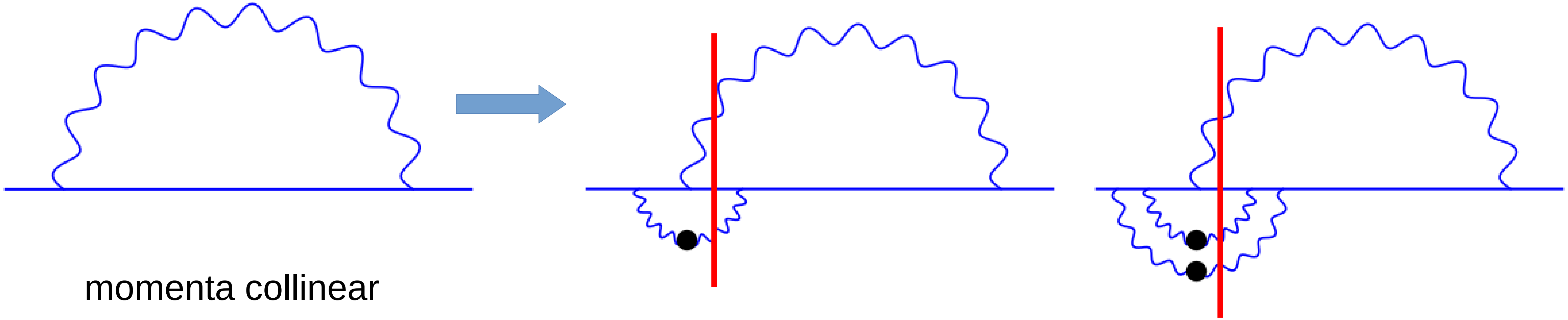
Potential IR divergence rendered finite by screening effect:
fermion/photon gain thermal mass through interacting with medium

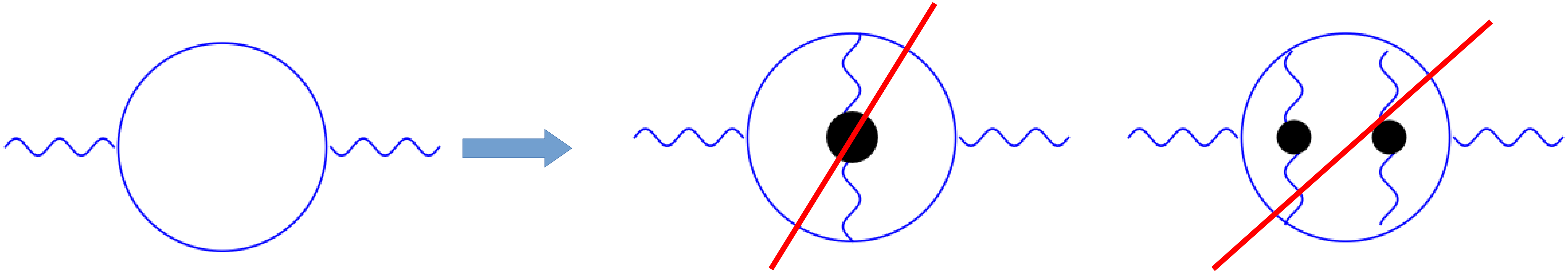
$$\delta m_e^2 = e^2 \int \frac{d^3p}{(2\pi)^3} \frac{8}{2p} \left(f_e(p) + f_\gamma(p)\right)$$

$$m_\gamma^2 = e^2 \int \frac{d^3p}{(2\pi)^3} \frac{4}{E_p} f_e(E_p)$$

$\delta m_e, m_\gamma \sim eT$ provides cutoff for IR divergence

Self-energies: correction to vertices (inelastic $1 \rightarrow 2$)





Two scenarios for fermion mass

	$1 \to 2$ processes	$2 \to 2$ processes
$m \gg eT$	suppressed, too heavy to radiate	screening needed for Coulomb only
$m \sim eT$	modified rate	screening needed for all

Zeroth order solution

$$S^{<(0)} = -2\pi\epsilon(P\cdot u)\delta(P^2 - m^2)\left((\not{P} + m)f_e\right)$$

$$D^{<(0)}_{\mu\nu} = 2\pi\epsilon(P\cdot n)\delta(P^2)P^T_{\mu\nu}f_\gamma$$

$$\partial_t f_e - \mathbf{v}_e\cdot\nabla f_e = C^e[2\to 2] + C^e[1\to 2]$$

$$\partial_t f_\gamma - \mathbf{v}_\gamma\cdot\nabla f_\gamma = C^\gamma[2\to 2] + C^\gamma[1\to 2]$$

massive generalization of the Boltzmann equation written down by Arnold, Moore, Yaffe

Arnold, Moore, Yaffe, JHEP 2003
field theory derivation by Gagnon, Jeon, PRD 2007

First order solution: Non-dynamical part

$$S^{<(1)} = \gamma^5 \gamma^\mu \frac{\epsilon_{\mu\nu\rho\sigma} P^\nu u^\rho \left(-\partial^\sigma f_e + \mathcal{C}^\sigma\right)}{2P \cdot u} \delta(P^2)$$

$$\mathcal{C}^\mu = \Sigma^{>\mu} f_e - \Sigma^{<\mu}(1 - f_e)$$

Chen et al, PRL 2014, PRL 2015
Hidaka, Pu, Yang, PRD 2017, PRD 2018

Green term: included in phenomenological studies
Blue term: should also be included

First order solution: complete

$$\frac{i\hbar}{2}\not{\partial} S^< + \not{P} S^< = \frac{i\hbar}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right) - \frac{\hbar^2}{4}\boxed{\left(\{\Sigma^>, S^<\}_{\mathrm{PB}} - \{\Sigma^<, S^>\}_{\mathrm{PB}}\right)}$$

$$\frac{i\hbar}{2}\not{\partial} S^{<(0)} + \not{P} \boxed{S^{<(1)}_{\text{non-dyn}}} = \frac{i\hbar}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right)^{(0)}$$

$$\frac{i\hbar}{2}\not{\partial} \boxed{S^{<(1)}_{\text{dyn}}} + \not{P} S^{<(2)} = \frac{i\hbar}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right)^{(1)} - \frac{\hbar^2}{4}\left(\{\Sigma^>, S^<\}_{\mathrm{PB}} - \{\Sigma^<, S^>\}_{\mathrm{PB}}\right)^{(0)}$$

- Non-dynamical part required by frame independence
- Dynamical part from quantum correction to collision term

Both needed for spin polarization phenomenology!

Summary

- Derived axial kinetic equation for QED including elastic & inelsatic processes
- Zeroth order solution generalizes Boltzmann equation to massive case
- (Non-dynamical part of) first order solution required by side-jump

Outlook

- (Dynamical part of) first order solution needs quantum correction in Collision kernel
- More complete study of spin polarization

Thank you!

First order solution: photons

$$D^{\mu\nu<(1)} = i\frac{P^{\mu\alpha}P^{\nu\beta}\left(P_\alpha\partial_\beta - P_\beta\partial_\alpha\right)}{2(P\cdot u)^2}\delta(P^2)f_\gamma$$

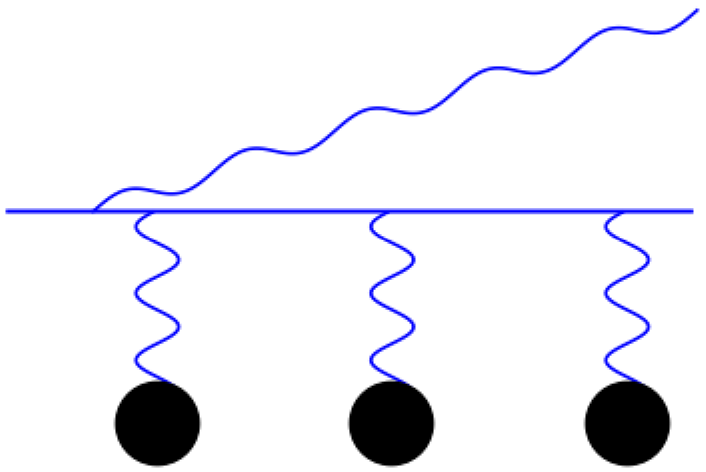
Non-dynamical contribution

First order correction necessary to satisfy gauge condition

$$P^{\mu\alpha}\left(\frac{i\hbar}{2}\partial_\alpha + P_\alpha\right)D^<_{\mu\nu} = 0$$

Huang, Mitkin, Sadofyev, Speranza, JHEP 2020
Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

Two scenarios for mass



medium kicks: $\sim eT$

fermion mass: $\sqrt{m^2 + m_{\mathrm{th}}^2}$ $m_{\mathrm{th}} \sim eT$

1 → 2 processes irrelevant for mass regime

$m \gg eT$ heavy fermion barely radiates!

$m \sim eT$ Modified 1 → 2 collision term