

Lattice QCD Calculation of TMD Soft Function Through Large-Momentum Effective Theory

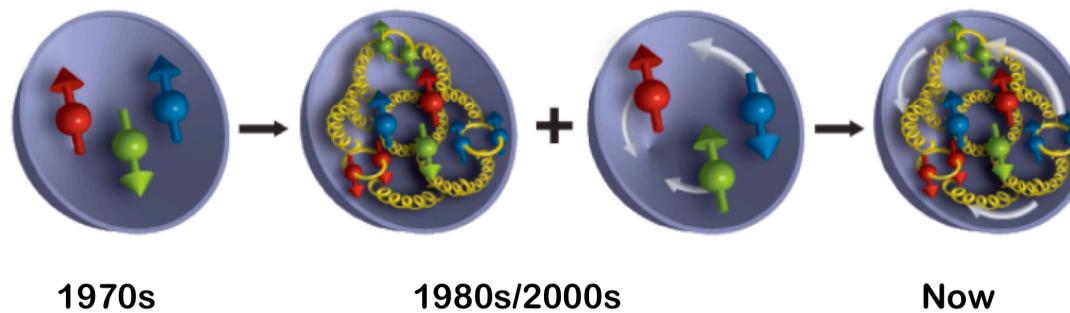
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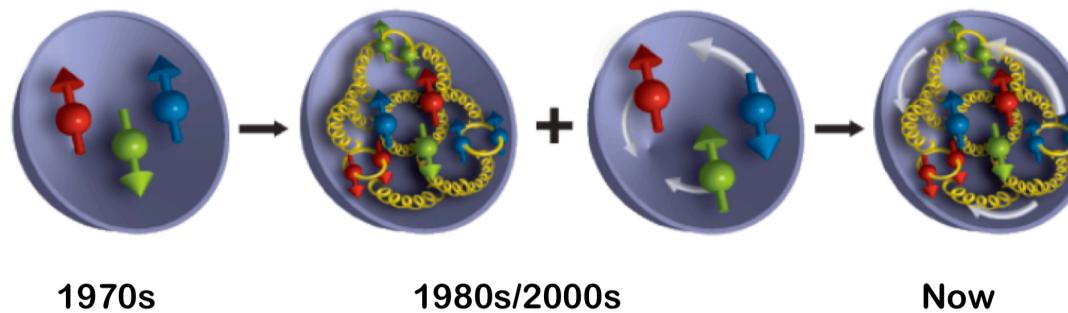
- **Motivation**
- **TMD soft function from lattice QCD**
- **Lattice setup and numerical results**
- **Summary**

➤ Our understanding of the internal structure of nucleon:



- **strongly interacting, relativistic bound state of quarks and gluons;**
- **Neither quarks nor gluons appear in isolation!**

➤ Our understanding of the internal structure of nucleon:

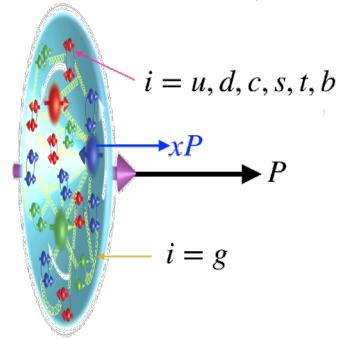


- **strongly interacting, relativistic bound state of quarks and gluons;**
- **Neither quarks nor gluons appear in isolation!**

How to probe nucleon structure without “seeing” quarks and gluons?

➤ Parton distribution function (PDF)

1969, R. Feynman



Parton probability
density with infinite
momentum

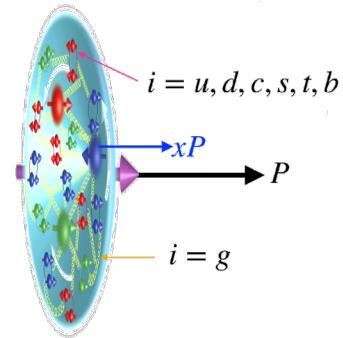
PDF

$f_{q/P}(x)$
longitudinal

Motivation

➤ Parton distribution function (PDF)

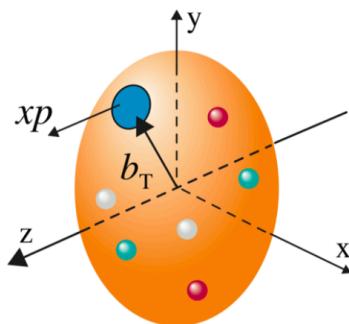
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➤ Transverse momentum dependent (TMD) parton distributions



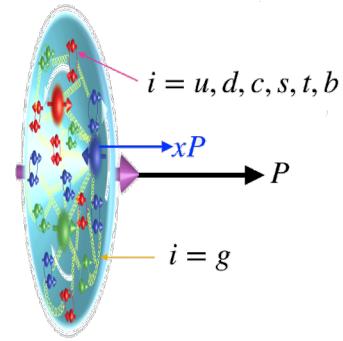
3-dimensional
structures of
nucleon

TMD
 $f_{q/P}(x, k_\perp)$
+ transverse

Motivation

➤ Parton distribution function (PDF)

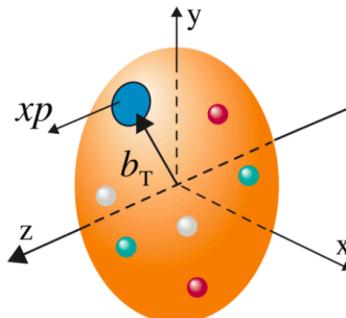
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Parton probability density with infinite momentum

PDF
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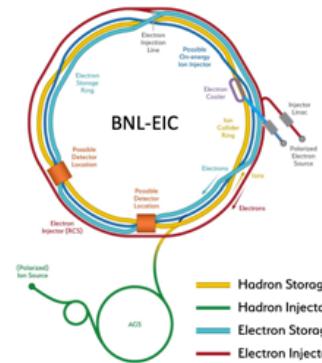
➤ Transverse momentum dependent (TMD) parton distributions



3-dimensional structures of nucleon

TMD
 $f_{q/P}(x, k_\perp)$
 + transverse

EIC



EicC

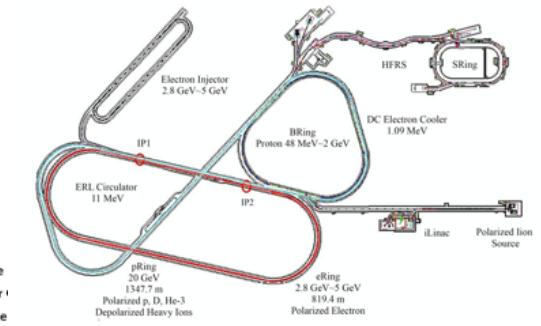
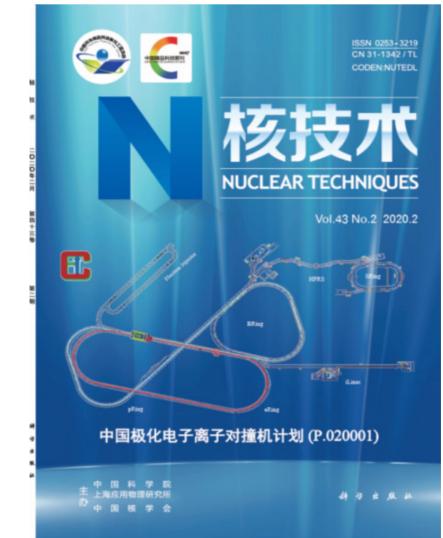
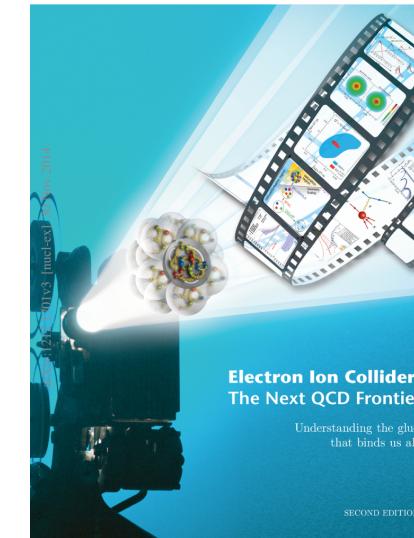
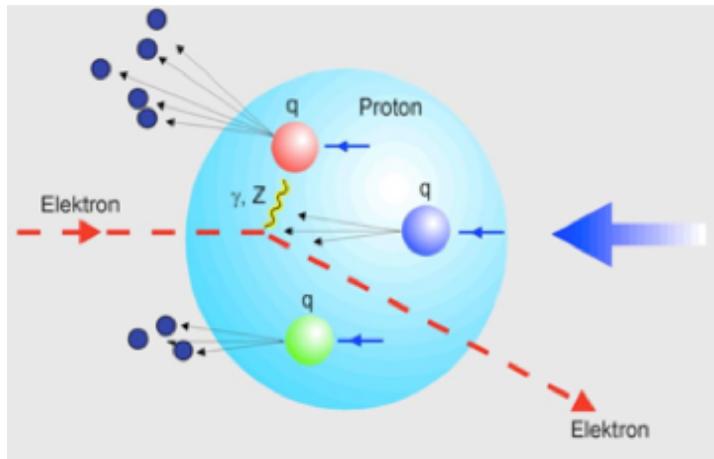


图31 EicC 装置总体布局
Fig.31 The layout of EicC

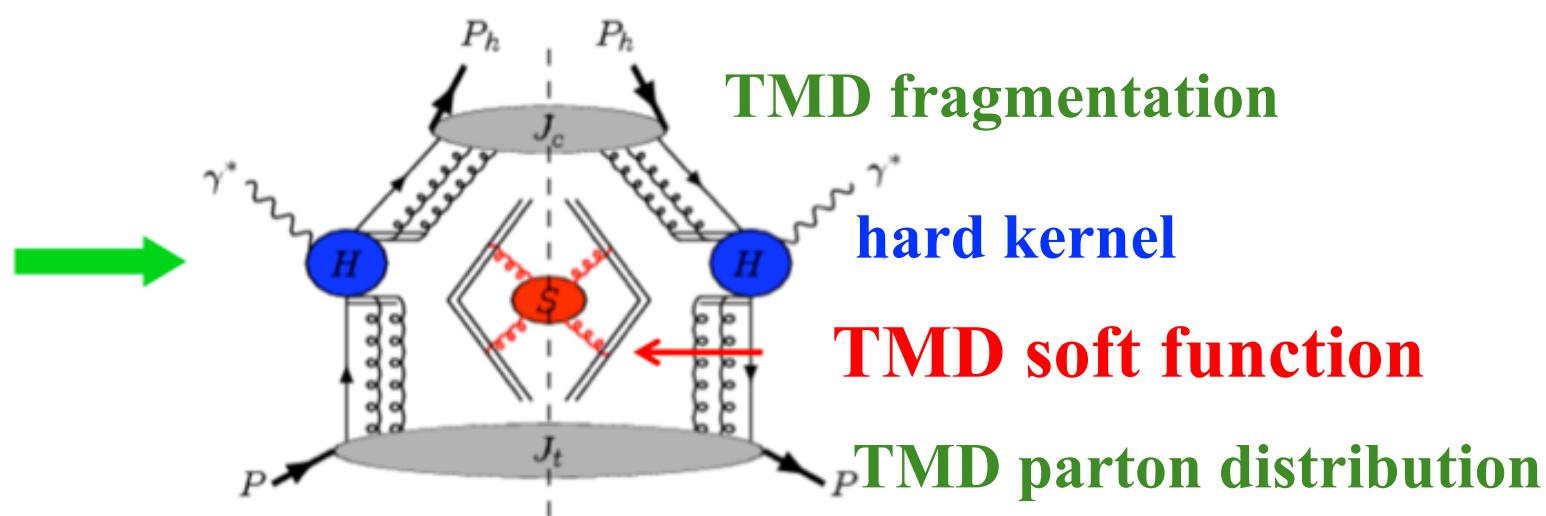
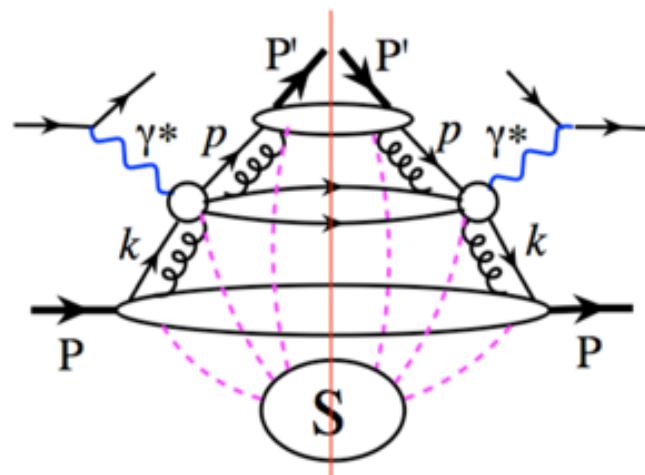


➤ Semi-inclusive DIS (SIDIS):



TMD factorization:

$$\sigma_{SIDIS} = \sum_i \hat{H}(Q, \mu) \otimes f_i^{TMD}(x, k_\perp, \mu, \zeta) \\ \otimes D_i(x', p_\perp, \mu, \zeta') \otimes S(k_{s\perp}, \mu, Y, Y') + \mathcal{O}\left(\frac{p_{h\perp}^2}{Q^2}, \frac{\Lambda_{QCD}^2}{Q^2}\right)$$



➤ TMD soft function:

- *J. Collins, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 32, 1 (2011).*
- *Ji, Liu, Liu, Nucl. Phys. B 955 (2020) 115054*

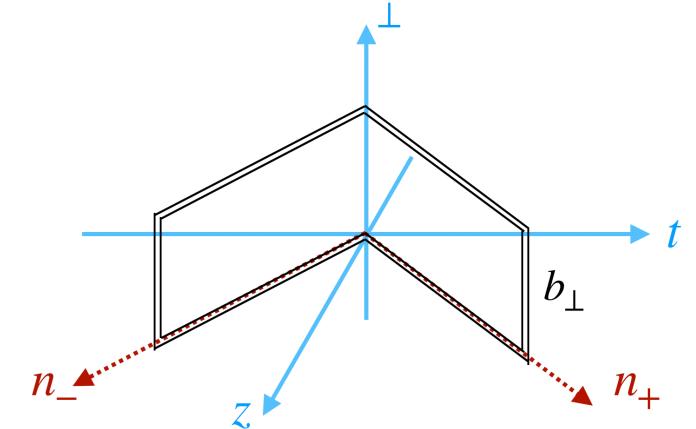
Defined by two conjugate light-like Wilson lines

$$S(\mathbf{b}_\perp, \mu, Y, Y') = \frac{1}{N_c} \text{tr} \langle 0 | \bar{\mathcal{T}} [U_{n_+}^\dagger(-\infty, \mathbf{b}_\perp)_{Y'} U_{n_-}^\dagger(\pm\infty, \mathbf{b}_\perp)_Y \\ \mathcal{T} [U_{n_-}(\pm\infty, 0)_Y U_{n_+}(-\infty, 0)_{Y'}] | 0 \rangle$$

➤ The intrinsic, rapidity independent, soft function:

$$S(\mathbf{b}_\perp, \mu, Y, Y') = e^{(Y+Y')K(\mathbf{b}_\perp, \mu)} S_I(\mathbf{b}_\perp, \mu)$$

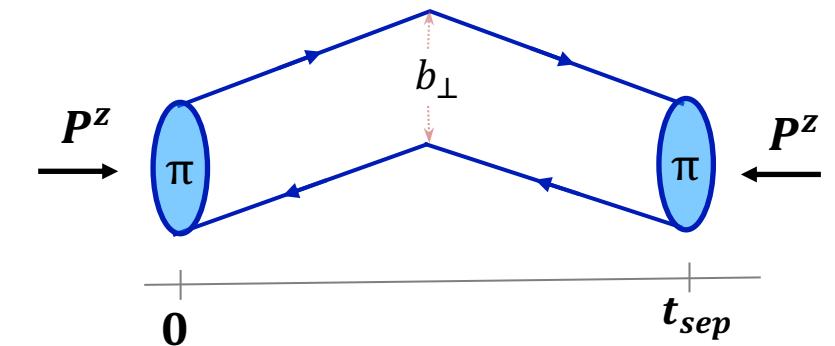
- Rapidity evolution can be described by the Collins-Soper kernel $K(\mathbf{b}_\perp, \mu)$.



- *Ji, Liu, Liu, Nucl. Phys. B 955 (2020) 115054*

➤ Four-quark form factor with large P^z :

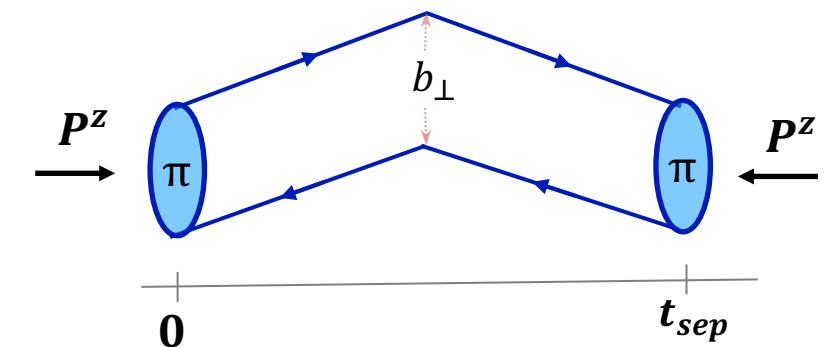
$$F(b_\perp, P^z) = \langle \pi(-P^z) | (\bar{q}_1 \Gamma q_1)(b_\perp) (\bar{q}_2 \Gamma q_2)(0) | \pi(P^z) \rangle$$



- *Ji, Liu, Liu, Nucl. Phys. B 955 (2020) 115054*

➤ Four-quark form factor with large P^z :

$$F(\mathbf{b}_\perp, \mathbf{P}^z) = \langle \pi(-\mathbf{P}^z) | (\bar{q}_1 \Gamma q_1)(\mathbf{b}_\perp) (\bar{q}_2 \Gamma q_2)(\mathbf{0}) | \pi(\mathbf{P}^z) \rangle$$



➤ Factorization of form factor in the framework of LaMET:

$$F(\mathbf{b}_\perp, \mathbf{P}^z) = \int dx dx' H(x, x', \mathbf{P}^z) \frac{\tilde{\phi}(x, \bar{Y}, \mathbf{P}^z, \mathbf{b}_\perp)}{S(Y, \bar{Y}, \mathbf{b}_\perp)} \frac{\tilde{\phi}^\dagger(x', \bar{Y}', \mathbf{P}^z, \mathbf{b}_\perp)}{S(Y', \bar{Y}', \mathbf{b}_\perp)} S(Y, Y', \mathbf{b}_\perp)$$

subtracted quasi-TMD wave function

perturbative hard kernel:

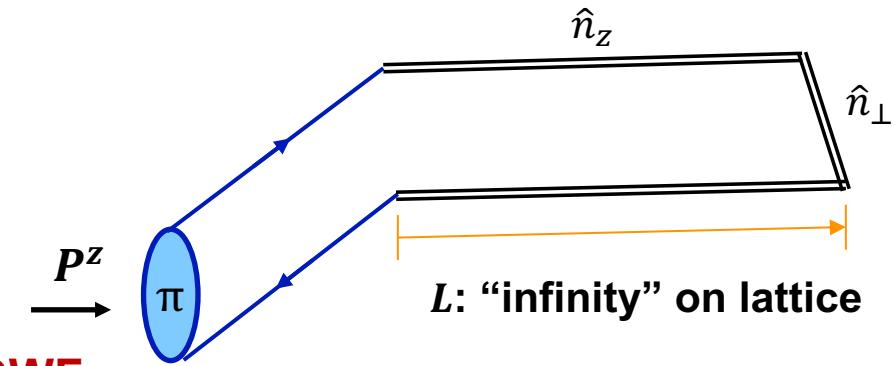
$$H(x, x', \mathbf{P}^z) = \frac{1}{2N_c} + \mathcal{O}(\alpha_s)$$

➤ Subtracted quasi-TMD wave function:

$$\phi(z, P^z, b_\perp) = \lim_{L \rightarrow \infty} \frac{\phi_L(z, P^z, b_\perp, L)}{\sqrt{Z_E(2L, b_\perp)}}$$

unsubtracted quasi-TMDWF

$$= \lim_{L \rightarrow \infty} \frac{\left\langle 0 \left| \bar{q}_1 \left(\frac{z}{2} \hat{n}_z + b_\perp \hat{n}_\perp \right) \gamma^t \gamma_5 q_2 \left(-\frac{z}{2} \hat{n}_z \right) \right| \pi(P^z) \right\rangle}{\sqrt{Z_E(2L, b_\perp)}}$$



$Z_E(2L, b_\perp)$: vacuum expectation value of Wilson loop, removes the pinch-pole singularity and Wilson-line self-energy in bare quasi-TMDWF

- Intrinsic soft function: *Ji, Liu, Liu, Nucl. Phys. B 955 (2020) 115054*

$$\begin{aligned} S_I(\mathbf{b}_\perp) &\equiv \frac{S(Y, Y', \mathbf{b}_\perp)}{S(Y, \mathbf{0}, \mathbf{b}_\perp)S(\mathbf{0}, Y', \mathbf{b}_\perp)} \\ &= \frac{F(\mathbf{b}_\perp, \mathbf{P}^z)}{\int dx dx' H(x, x', \mathbf{P}^z) \tilde{\phi}(x, \bar{Y}, \mathbf{P}^z, \mathbf{b}_\perp) \tilde{\phi}^\dagger(x', \bar{Y}', \mathbf{P}^z, \mathbf{b}_\perp)} \end{aligned}$$

leading order matching:

$$S_I(\mathbf{b}_\perp) = 2N_c \frac{F(\mathbf{b}_\perp, \mathbf{P}^z)}{|\phi(\mathbf{0}, \mathbf{b}_\perp, \mathbf{P}^z)|^2} + \mathcal{O}(\alpha_s, 1/(\mathbf{P}^z)^2)$$

calculable on lattice

- **CLS configuration: A654**
 - **2+1flavor clover fermions and tree-level Symanzik gauge action;**
 - **Coulomb gauge fixed wall source propagators;**
 - **$P^z = 1.05, 1.58, 2.11 \text{ GeV}$;**
 - **Use $m_\pi = 547 \text{ MeV}$ instead of 333 MeV to get a better signal;**
 - **Physically, the soft function becomes independent of the meson mass for large P^z .**
-
- | β | $L^3 \times T$ | a (fm) | c_{sw} | κ_l^{sea} | $m_\pi^{\text{sea}} (\text{MeV})$ |
|---------|------------------|--------|----------|-------------------------|-----------------------------------|
| 3.34 | $24^3 \times 48$ | 0.098 | 2.06686 | 0.13675 | 333 |
-
- | N_{cfg} | κ_l^v | $m_\pi^v (\text{MeV})$ |
|-----------|--------------|------------------------|
| 868 | 0.13622 | 547 |

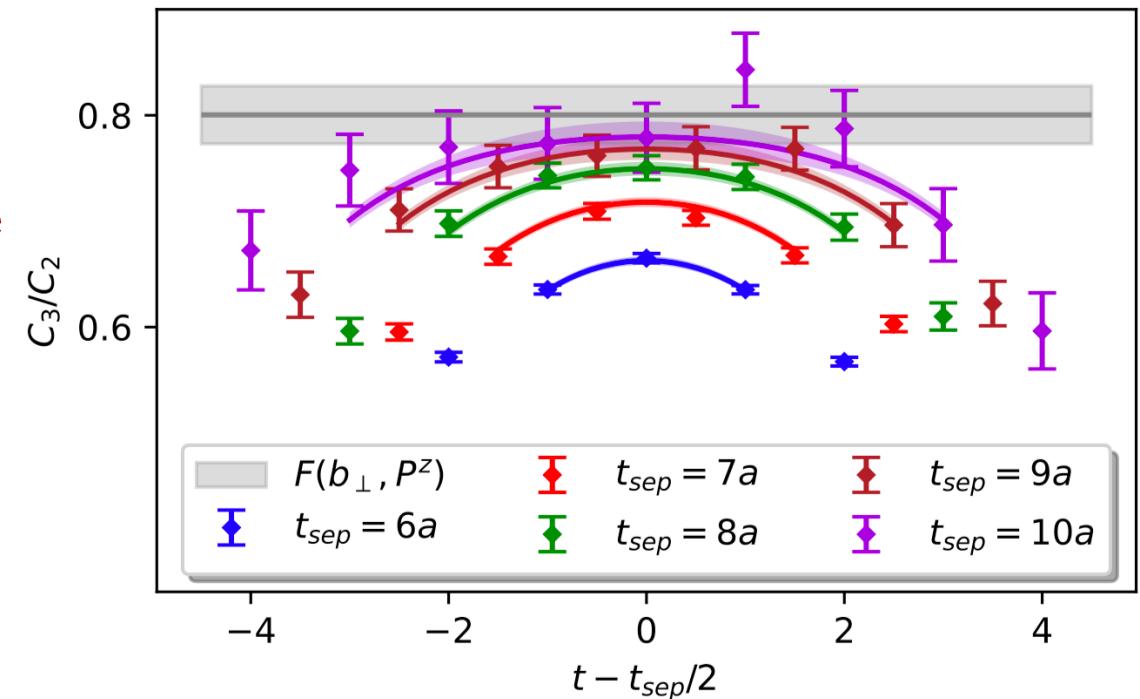
➤ Four-quark form factor: ratio of 3-point and 2-point function:

$$C_3(b_\perp, t, t_{sep}) \sim \langle O_\pi(t_{sep}) O_{4\text{-quark}}(t) \bar{O}_\pi(0) \rangle$$

$$= \frac{A_w^2}{(2E)^2} e^{-Et_{sep}} [F + c_1(e^{-\Delta Et} + e^{-\Delta E(t_{sep}-t)}) + c_2 e^{-\Delta Et_{sep}}]$$

$$C_2(t) \sim \langle O_\pi(t) \bar{O}_\pi(0) \rangle = \frac{A_w A_p}{2E} e^{-Et} (1 + c_0 e^{-\Delta Et})$$

- ✓ Fitted data at $t_{sep} = 0.6 \sim 1 \text{ fm}$;
- ✓ Form factor corresponds to the ground state contribution at $t_{sep} \rightarrow \infty$;
- ✓ $\chi^2/d.o.f = 0.6$, data agree with the fit function.



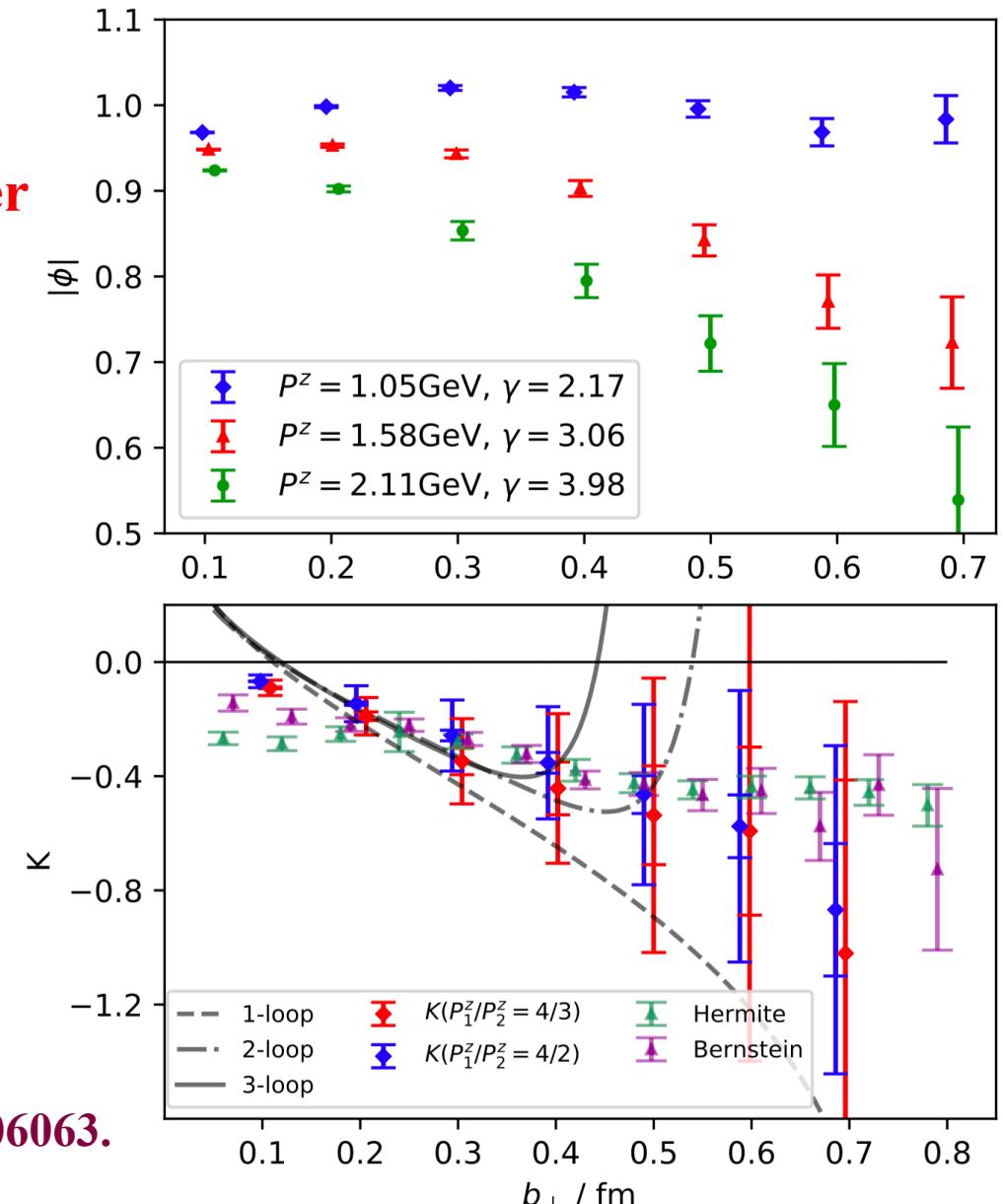
- Subtracted quasi-TMD wave function:
- ✓ The P^z -dependence related to the **Collins-Soper kernel** (tree-level):

$$K(b_\perp, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left| \frac{\phi(0, b_\perp, P_1^z)}{\phi(0, b_\perp, P_2^z)} \right| + \mathcal{O}(\alpha_s, (1/P^z)^2)$$

- ✓ Extracted Collins Soper kernel, compare with perturbative calculations up to 3-loops* and results from quenched lattice calculations of TMDPDF**.

* Li and Zhu, Phys. Rev. Lett. 118(2017)2, 022004;

** Shanahan and Wagman and Zhao, arXiv:hep-lat/2003.06063.

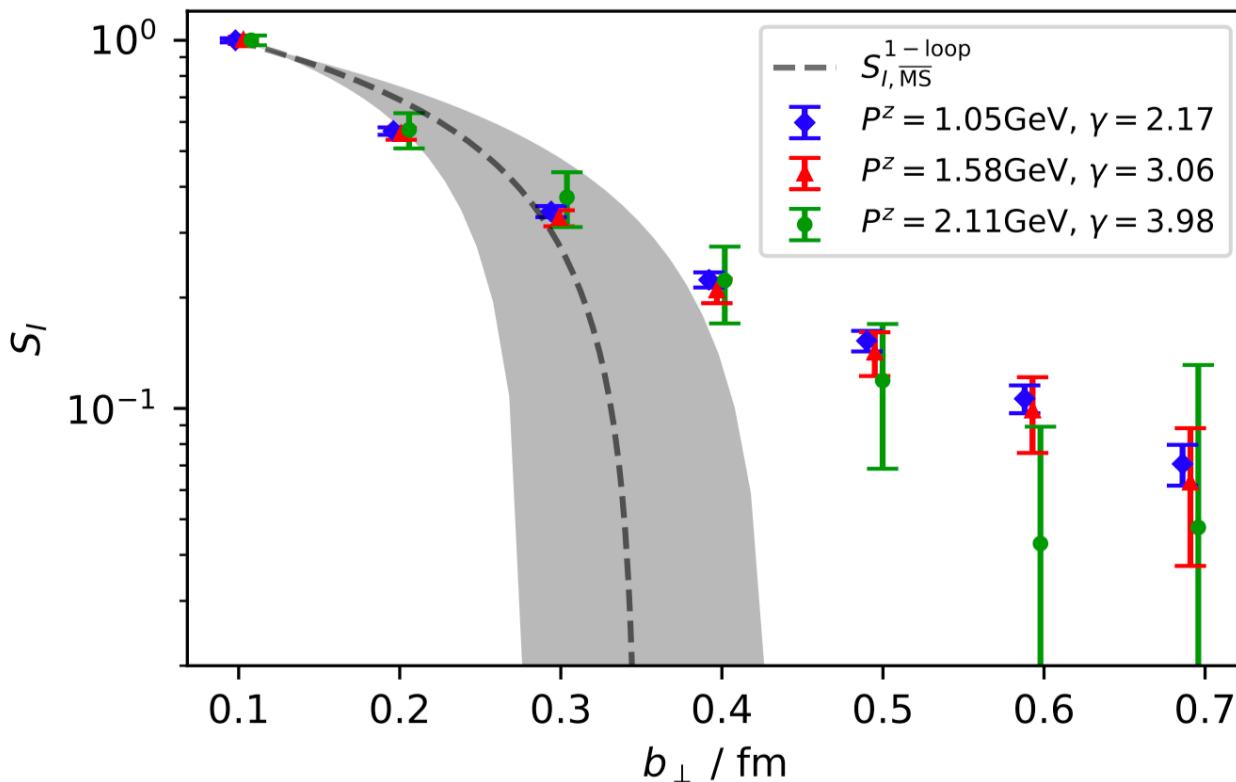


Numerical Results

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➤ Intrinsic soft function (leading order):

$$S_I(b_\perp) = 2N_c \frac{F(b_\perp, P^z)}{|\phi(0, b_\perp, P^z)|^2} + \mathcal{O}(\alpha_s, 1/(P^z)^2)$$



- The results are **consistent with each other with different P^z** ;
- The systematic uncertainty from the **operator mixing has been taken into account**;
- Theoretical result up to 1-loop:

$$S_{I,\overline{MS}}(b_\perp, \mu) = 1 - \frac{\alpha_s C_F}{\pi} \ln \frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}} + \mathcal{O}(\alpha_s)$$

- This work present an **exploratory** lattice calculation of the intrinsic soft function;
- The Collin-Soper kernel obtained from our quasi-TMDWF agrees with the perturbative results and previous quenched lattice calculations;
- Our results of the intrinsic soft function are almost **independent** of the hadron momentum, and **consistent with** the 1-loop perturbative calculation;
- This work paves the way towards the first principle predictions of physical cross sections for, e.g., Drell-Yan and Higgs productions at small transverse momentum.

Thank you for your attention!

Operator mixing

