



Lattice QCD determination of Collins-Soper Kernel through TMD Wave Function in LaMET

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(Lattice Parton Collaboration)

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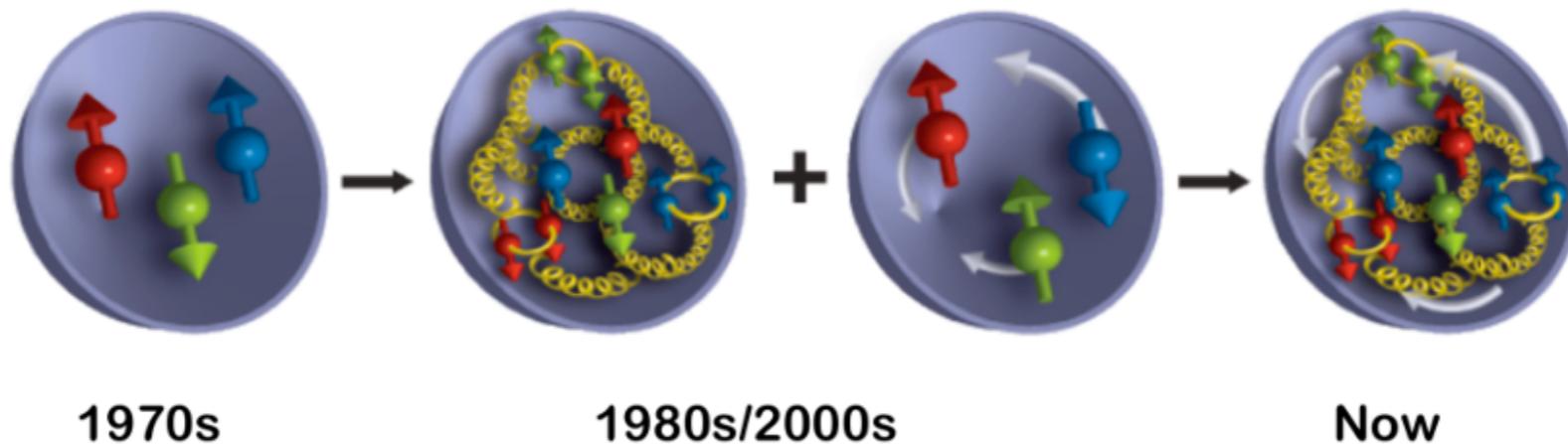
Outline

- Motivation
- Collins-Soper kernel from lattice QCD
- Computation includes numerical results
- Summary and outlook



Why Collins-Soper kernel ?

Our understanding of the internal structure of nucleon

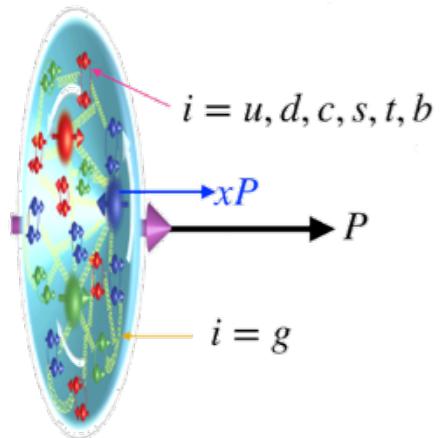


Strong interaction in boundary state of quarks and gluons.

How to probe nucleon structure without “seeing” quarks and gluons?



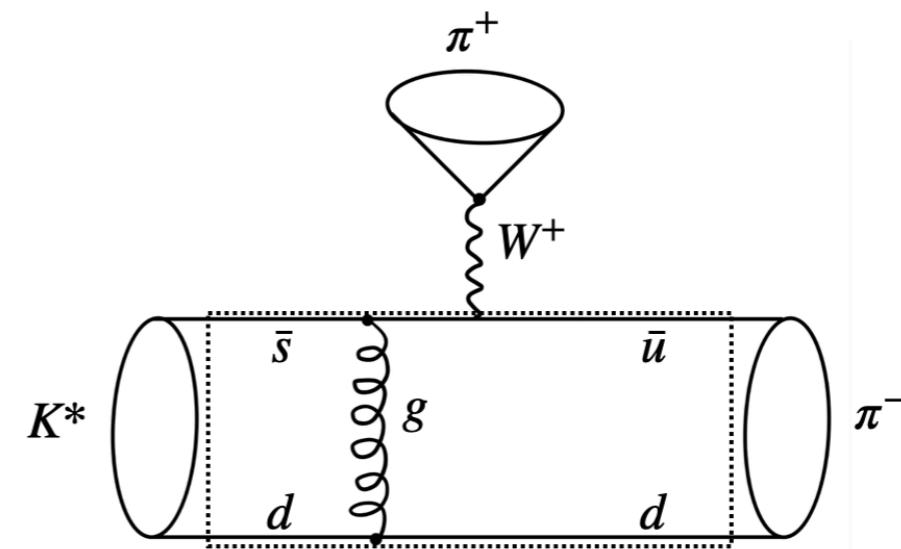
Why Collins-Soper kernel ?



Parton distributions
with momentum fraction x
in longitudinal direction.

PDF/DA: $f_{q/P}(x)$

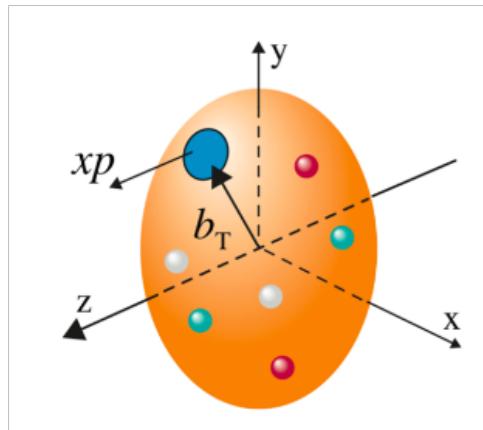
Example: $K^* \rightarrow \pi^+ + \pi^-$



$$\mathcal{A} = \langle \pi^+ \pi^- | K^* \rangle \sim \int d^4 k_1 d^4 k_2 d^4 k_3 \text{Tr}[C(t) \phi_{K^*}(p_1) \phi_{\pi^+}(p_2) \phi_{\pi^-}(p_3) H(k_1, k_2, k_3, t)]$$



Why Collins-Soper kernel ?



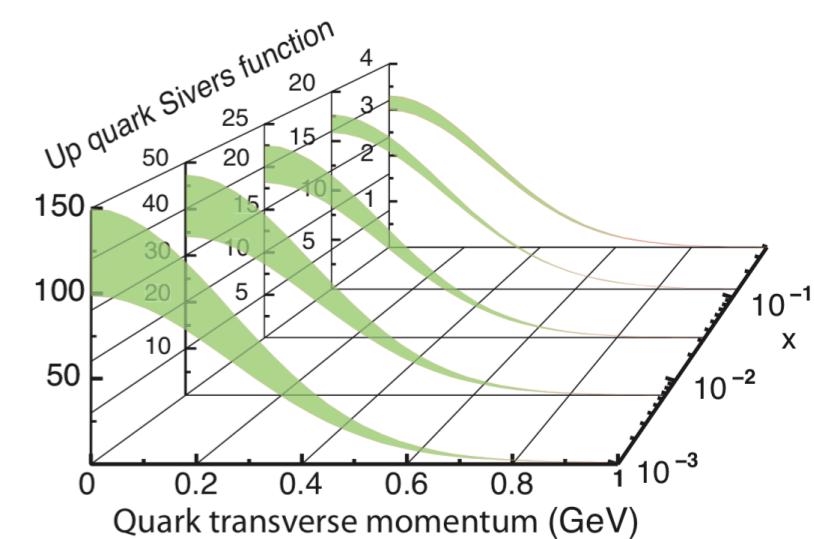
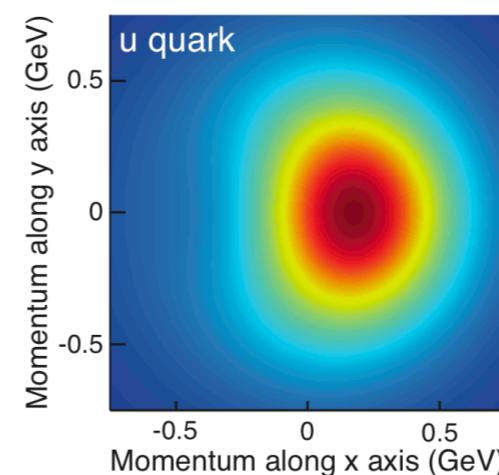
3-dimensional
structures of hadrons.



TMD PDF/DA: $f_{q/P}^{TMD}(x, k_\perp)$

N. G. Stefanis, Constantia Aleandrou et.al, arxiv 161203077(2016)

- Three dimensional nuclear imaging
- DIS/Drell Yan process
- ...



Lattice calculation of TMDWF is the calculation in first principle!



Why Collins-Soper kernel ?

The Collins-Soper kernel relates the evolution of transverse momentum-dependent parton distributions and hadron wave function.

- TMD Evolution:

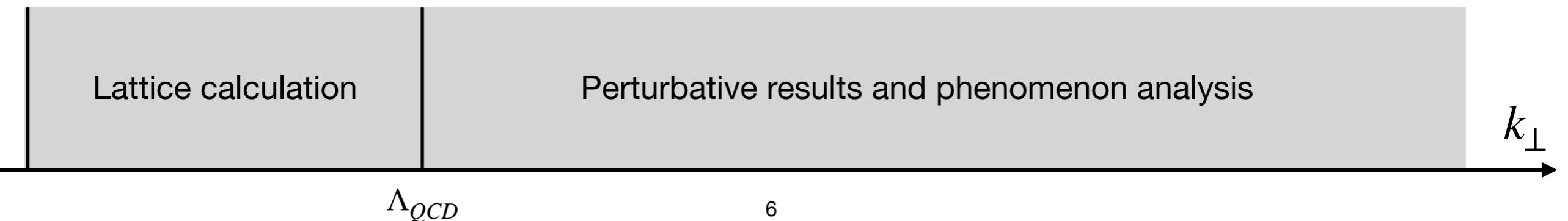
$$f^{TMD}(x, b_\perp, \mu, \zeta) = f^{TMD}(x, b_\perp, \mu_0, \zeta_0) \exp \left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma(\mu', \zeta_0) \right] \exp \left[\frac{1}{2} K(\mu, b_\perp) \ln \frac{\zeta}{\zeta_0} \right]$$

- LaMET matching for TMD:

$$\phi(b, x, \zeta) = H_1^{-1} \left(4x^2 P_z^2, 4(1-x)^2 P_z^2 \right) e^{-\frac{1}{2} \ln \left(\frac{4x^2 P_z^2}{\zeta} \right) K(b)} S_r^{\frac{1}{2}}(b) \tilde{\phi}(b, x, P_z)$$

Transverse momentum of parton k_\perp represents the scale of TMD physics.

Energy scale of parton in hadron





Collins-Soper kernel from lattice QCD

The P_z dependence of quasi-TMDWF is related to CS kernel:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$\phi(b, x, \zeta) = H_1^{-1} (4x^2 P_z^2, 4(1-x)^2 P_z^2) e^{-\frac{1}{2} \ln\left(\frac{4x^2 P_z^2}{\zeta}\right) K(b)} S_r^{\frac{1}{2}}(b) \tilde{\phi}(b, x, P_z)$$



$$K(b) = \frac{1}{\ln(P_2/P_1)} \ln \left[\frac{H_1(4x^2 P_1^2, 4(1-x)^2 P_1^2) \tilde{\phi}(b, x, P_2)}{H_1(4x^2 P_2^2, 4(1-x)^2 P_2^2) \tilde{\phi}(b, x, P_1)} \right]$$



$$K(b)_{tree} = \frac{1}{\ln(P_2/P_1)} \ln \left[\frac{\tilde{\phi}(b, z=0, P_2)}{\tilde{\phi}(b, z=0, P_1)} \right]$$

Tree level: $H_1 = 1$

Fourier transformation: $\int_0^1 \tilde{\phi}(b, x) dx = \tilde{\phi}(b, z=0)$

Matching kernel up to 1-loop level:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

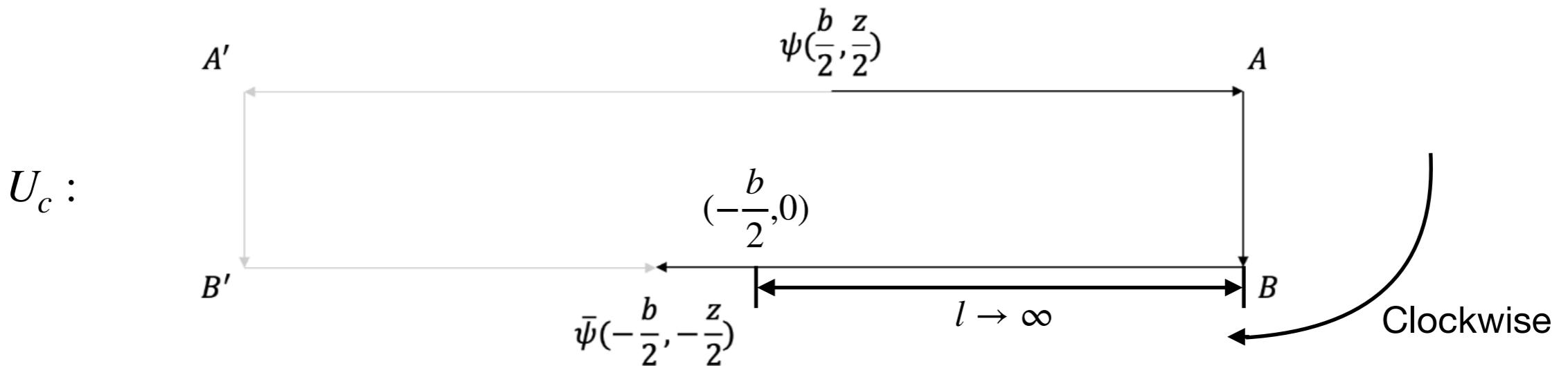
$$H_1(l, \bar{l}) = e^h, \quad h = \frac{\alpha_s C_F}{4\pi} \left[-\frac{5\pi^2}{6} - 4 + \ln \frac{-l - i0}{\mu^2} + \ln \frac{-\bar{l} - i0}{\mu^2} - \frac{1}{2} \left(\ln^2 \frac{-l - i0}{\mu^2} + \ln^2 \frac{-\bar{l} - i0}{\mu^2} \right) \right] + O(\alpha_s^2)$$



TMD wave function

leading twist for pion: $\Gamma_1 = \gamma^t \gamma_5, \gamma^z \gamma_5$

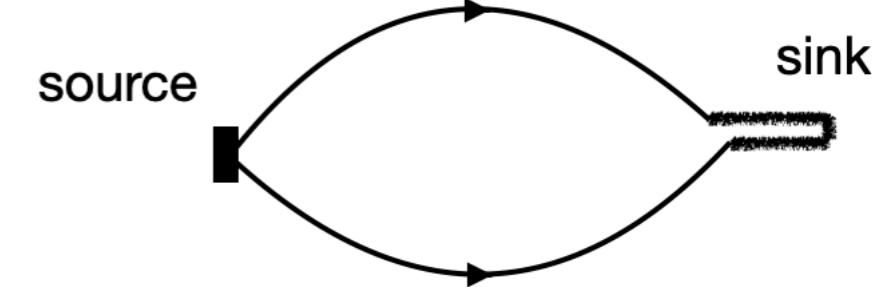
$$\text{TMDWF: } \tilde{\Phi}(b, z, \vec{P}) = \lim_{l \rightarrow \infty} \langle 0 | \bar{\psi}_1\left(-\frac{b}{2}\hat{x} - \frac{z}{2}\hat{z}\right) \Gamma_1 U_c \psi_2\left(\frac{b}{2}\hat{x} + \frac{z}{2}\hat{z}\right) | \pi(\vec{P}) \rangle$$



Two point correlation function:

$$C_2(b, z, t, \vec{P}) = \int d^3x e^{-i\vec{P}\cdot\vec{x}} \langle 0 | \chi(\vec{x}, t, b, z, \Gamma_1) \bar{\chi}(0, 0, 0, 0, \gamma_5) | 0 \rangle$$

$$\chi(\vec{x}, t, b, z) = \bar{\psi}_1\left(\vec{x} - \frac{b}{2}\hat{x} - \frac{z}{2}\hat{z}\right) \Gamma_1 U_c \psi_2\left(\vec{x} + \frac{b}{2}\hat{x} + \frac{z}{2}\hat{z}\right)$$

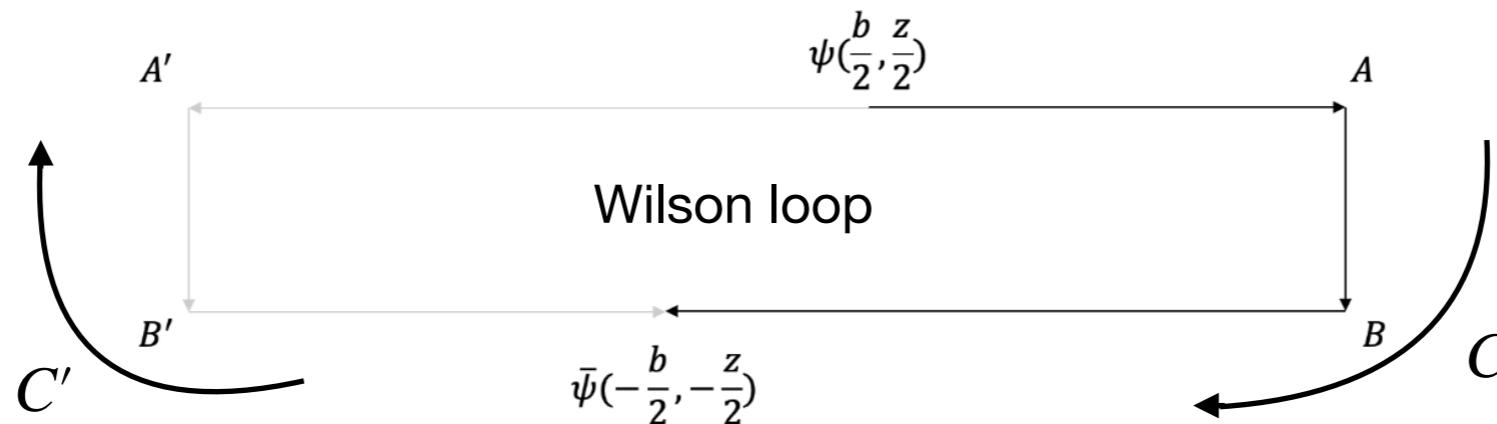




Computation

Subtracted quasi-TMDWF:

$$\tilde{\phi}(z, b, P_z)_{sub} = \lim_{l \rightarrow \infty} \frac{\tilde{\phi}(l, z, b, P_z)}{\sqrt{U_{loop}}} = \lim_{l \rightarrow \infty} \frac{\langle 0 | \bar{\psi}_1(-\frac{b}{2}\hat{x} - \frac{z}{2}\hat{z}) \Gamma_1 U_c \psi_2(\frac{b}{2}\hat{x} + \frac{z}{2}\hat{z}) | \pi(\vec{P}) \rangle}{\langle 0 | \bar{\psi}_1(0) \Gamma_1 \psi_2(0) | \pi(\vec{P}) \rangle \sqrt{\langle 0 | U_{c+c'} | \pi \rangle}}$$



Lattice setup: MILC configurations with $a = 0.12\text{fm}$ and choose three hadron momenta $P_z = \{1.72, 2.15, 2.58\} \text{ GeV}$. We used 373 configurations and in each we measured 4 times, so the total measurement is $4 \times 373 = 1492$

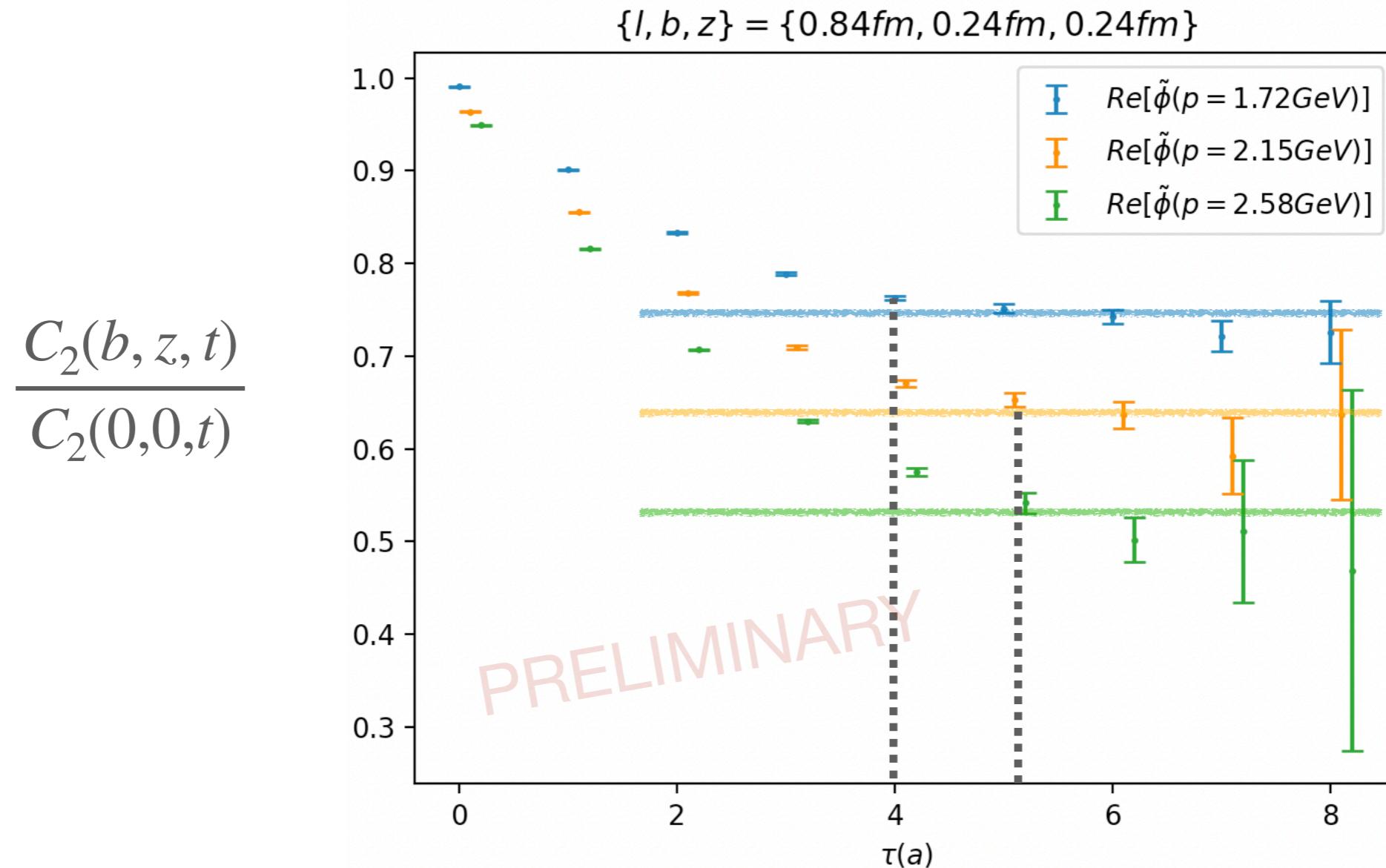
Ensemble	$a(\text{fm})$	$L^3 \times T$	$m_{\pi,sea}(\text{MeV})$	$m_{\pi,val}(\text{MeV})$	momentum(γ)
a12m130	0.12	48×64	140	670	$1.72\text{GeV}(2.57), 2.15\text{GeV}(3.21), 2.58\text{GeV}(3.85)$
a12m130	0.12	48×64	140	310	$1.72\text{GeV}(5.55), 2.15\text{GeV}(6.93), 2.58\text{GeV}(8.32)$



Computation: Presetting

Two point correlation function $\overrightarrow{P} = (P_0, 0, 0, P_z)$, $\Gamma_1 = \gamma^t \gamma_5$

$$\frac{C_2(l, b, z, t)}{C_2(0, 0, 0, t)} \sim \tilde{\phi}(l, b, z)_{sub}(1 + A(l, b, z)e^{-\Delta E \tau} + \dots)$$

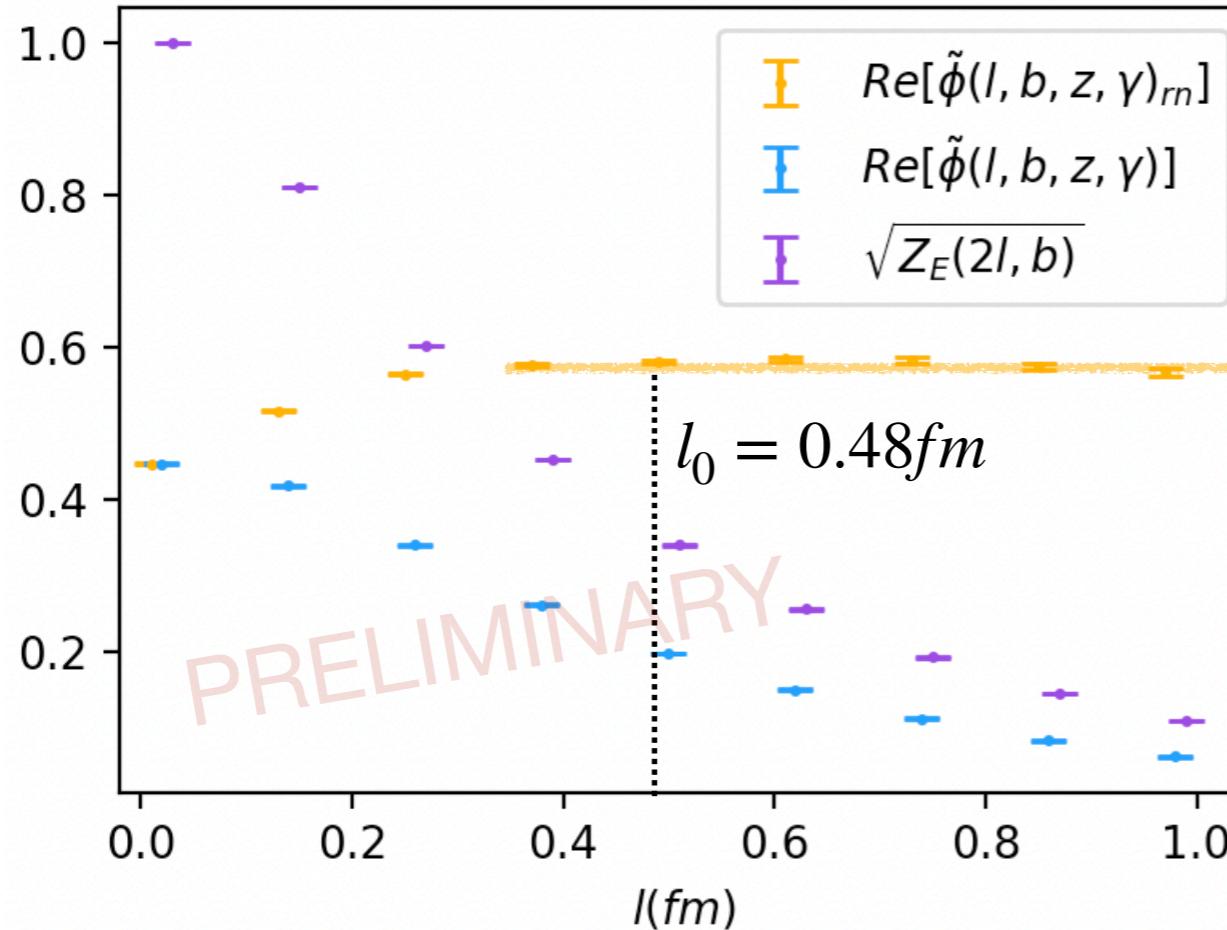




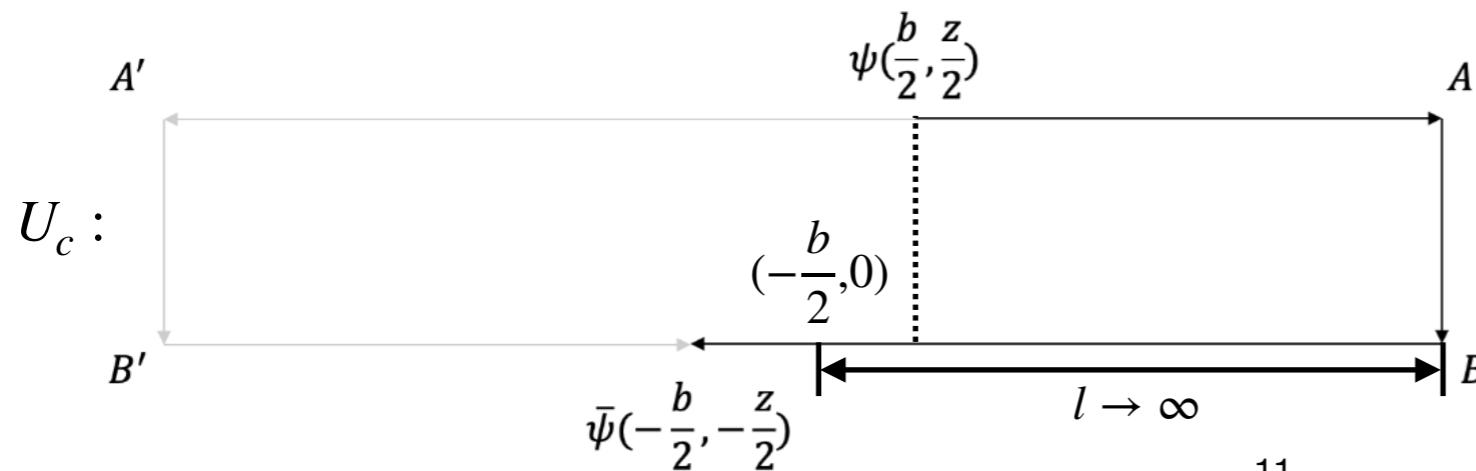
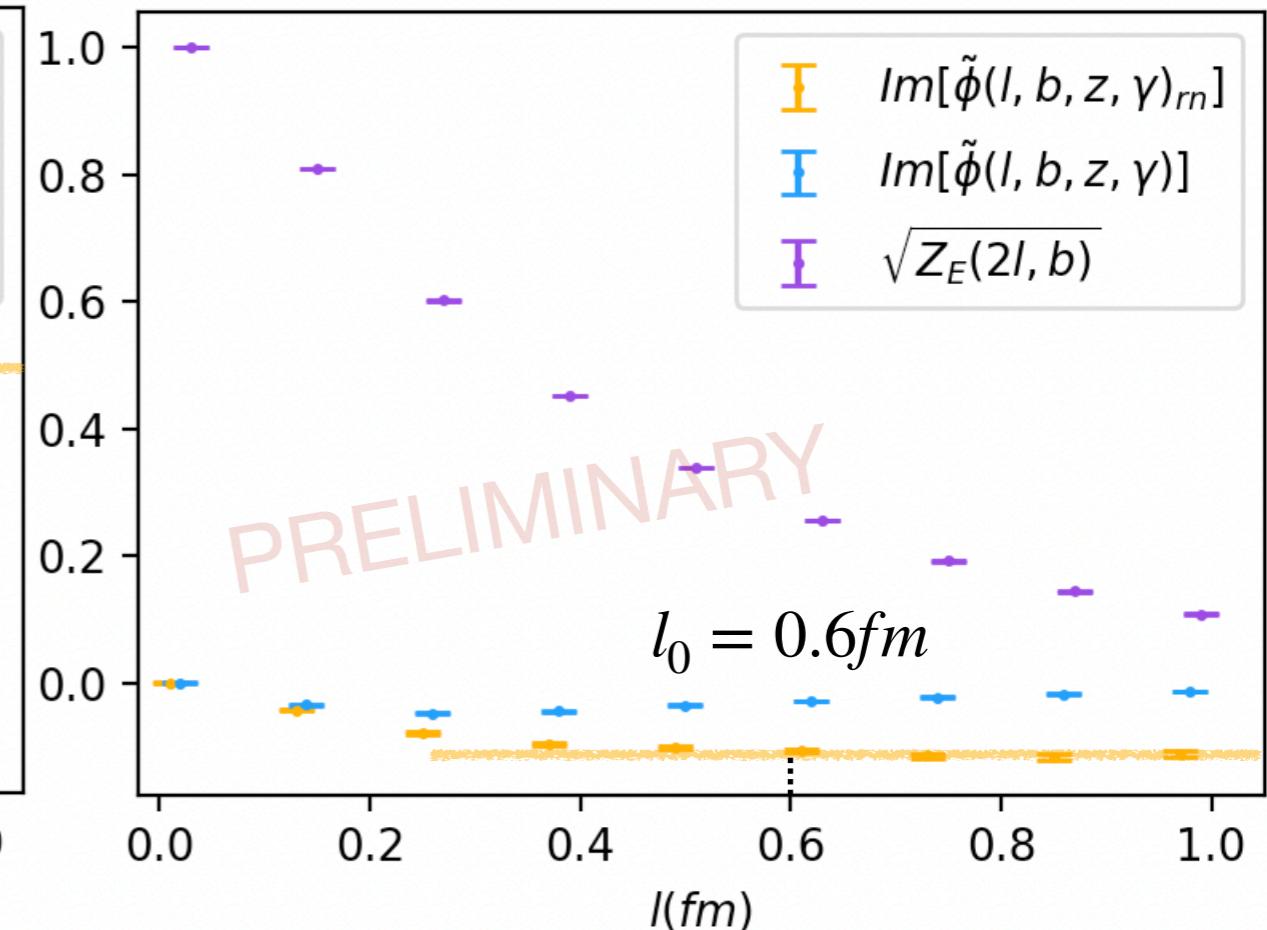
Computation: Presetting

TMDWF: $\tilde{\phi}(b, z) = \lim_{l \rightarrow \infty} \tilde{\phi}(l, b, z)$

$$\{P_z, b, z\} = \{2.58\text{GeV}, 0.24\text{fm}, 0.24\text{fm}\}$$



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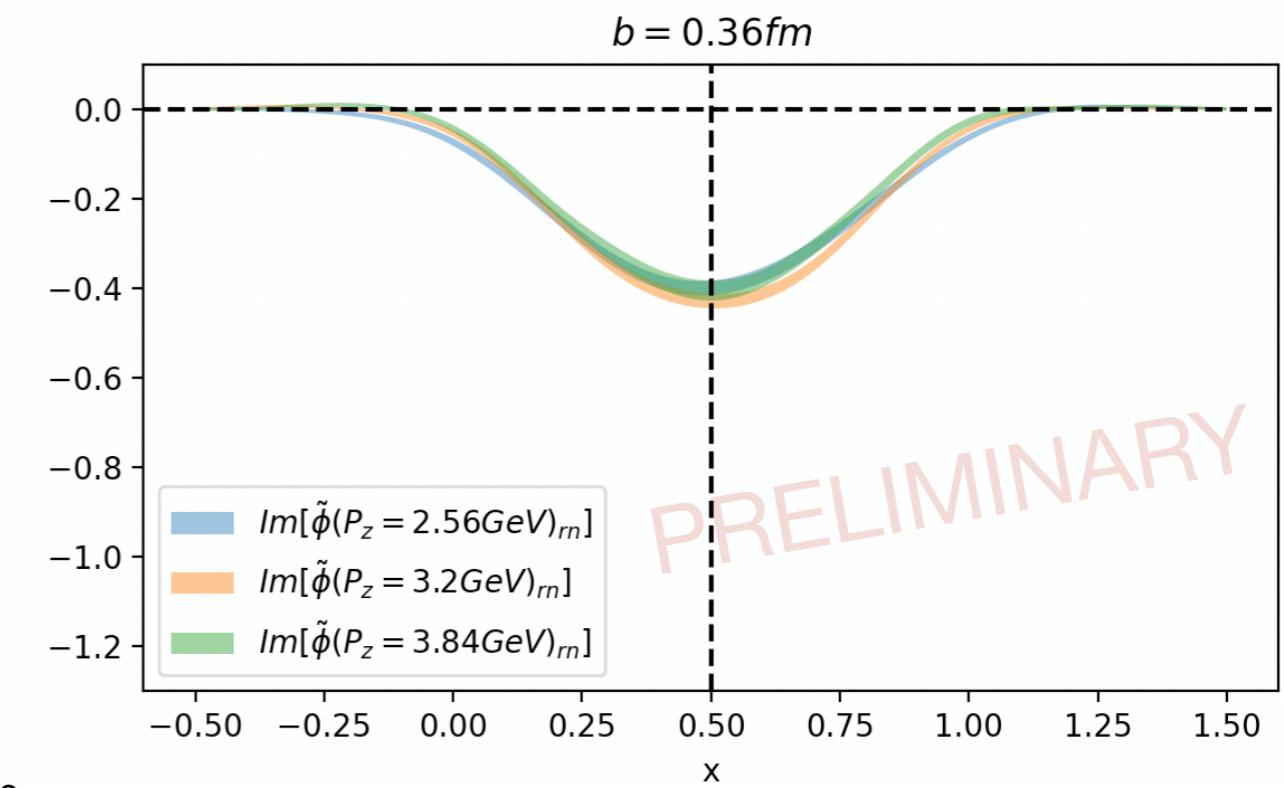
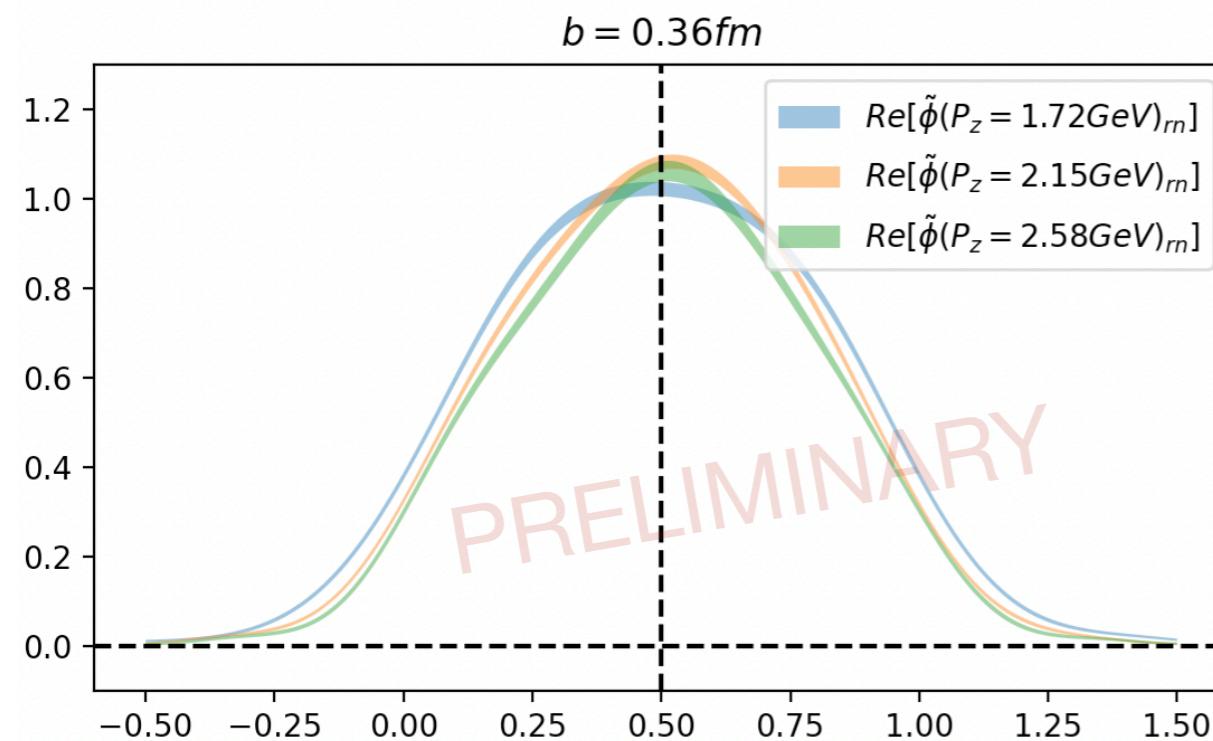
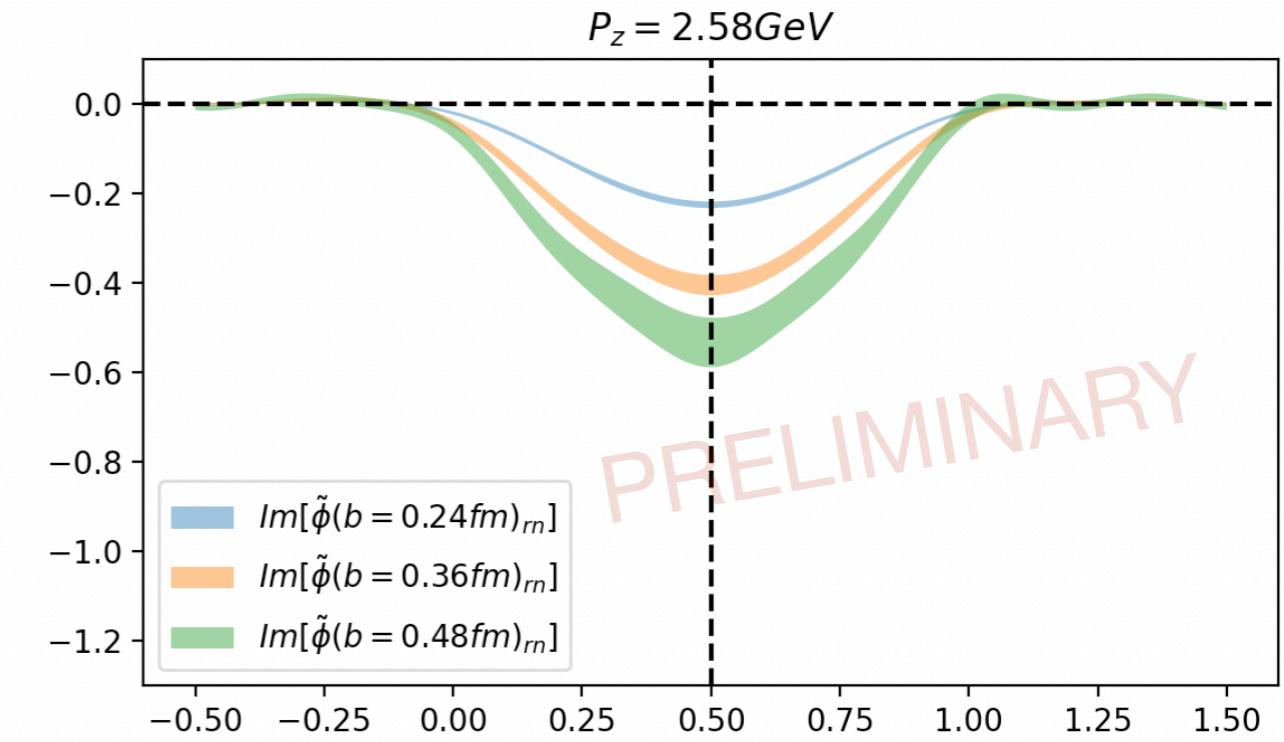
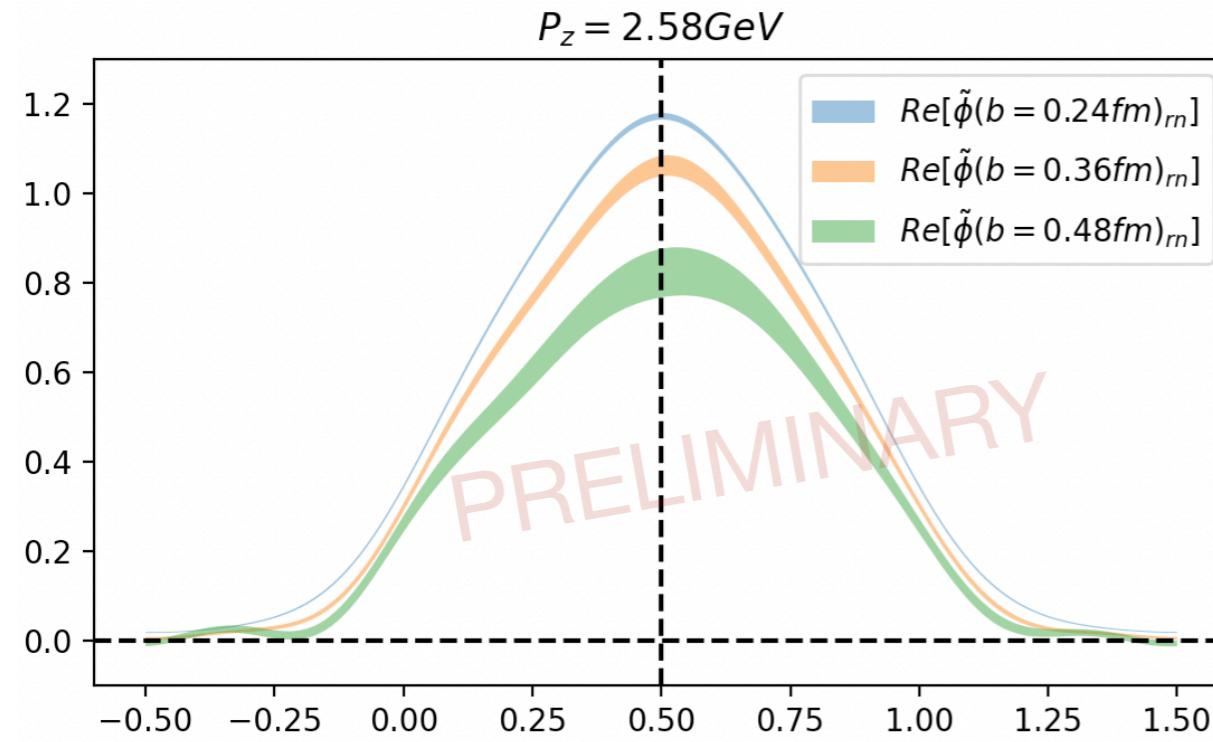
Due to $l_0 \geq \frac{z}{2}$, we choose $l_0 = \frac{z_{max}}{2}$

In our data, it is $l_0 = 6a = 0.72\text{fm}$



Computation: Fourier Transformation

TMDWF: $\tilde{\phi}(l = l_0, b, z, t = t_0) \rightarrow \tilde{\phi}(b, z) \xrightarrow{\text{Fourier transformation}} \tilde{\phi}(b, x)$



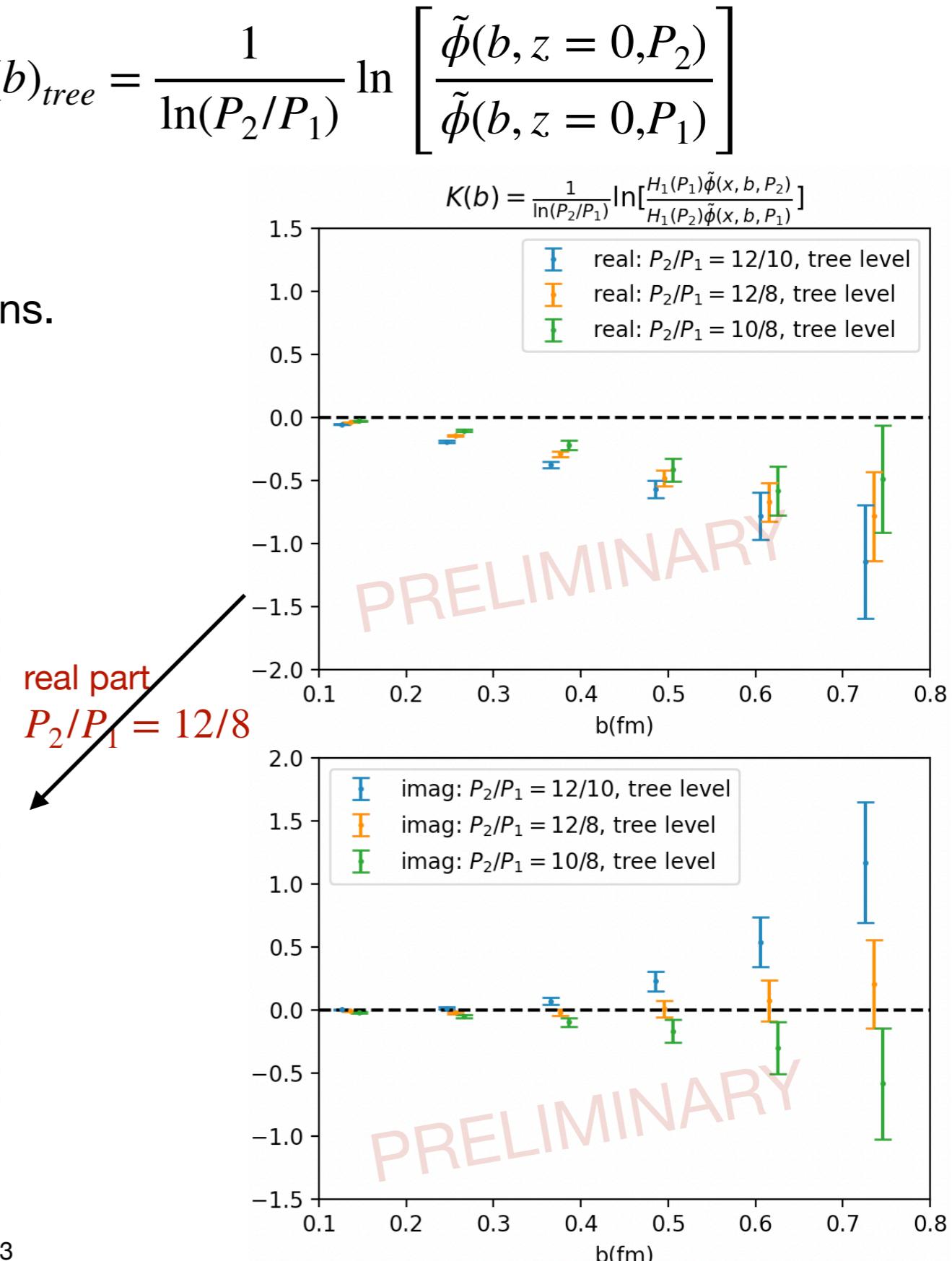
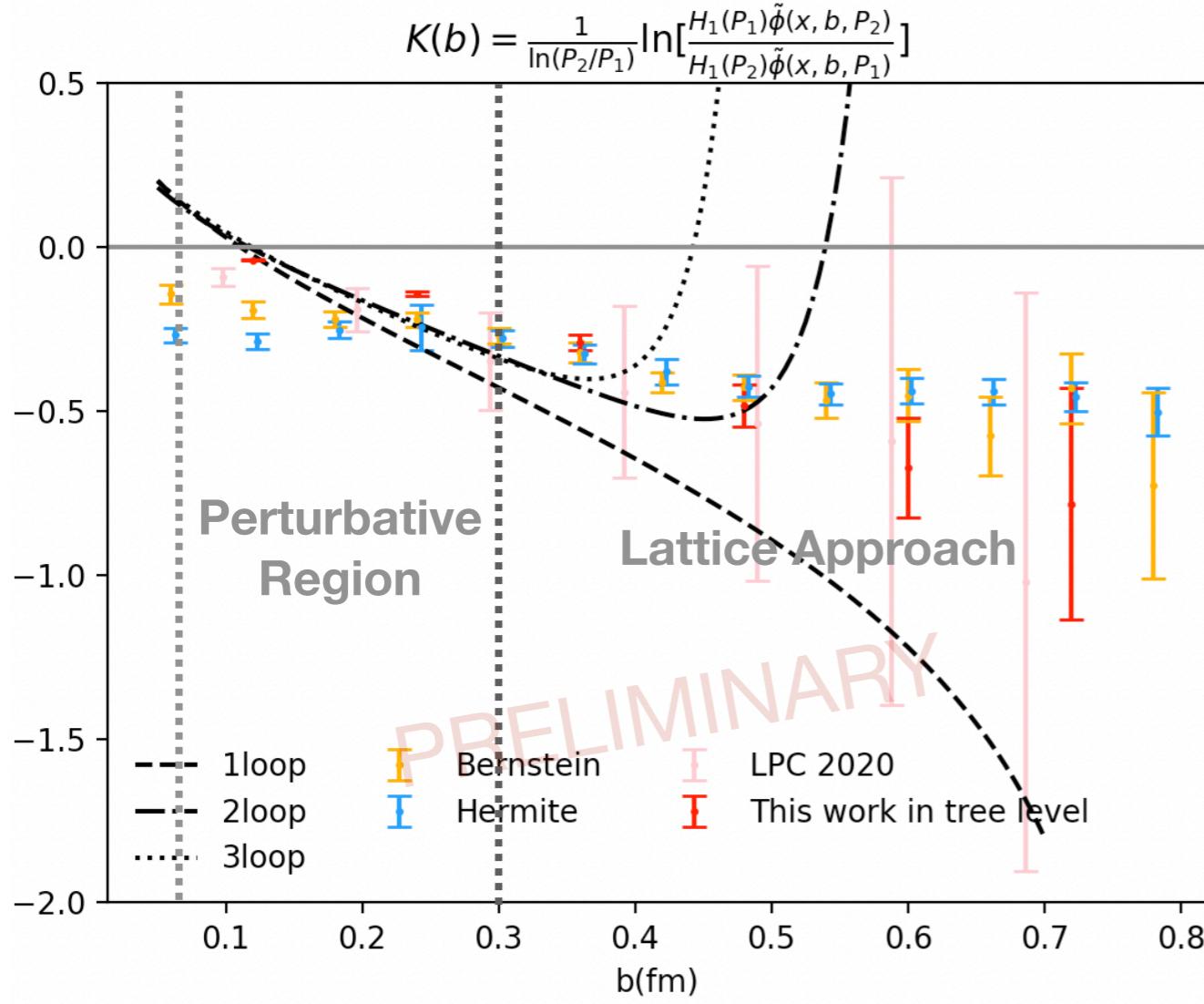


Computation: Numerical Results

Collins-Soper kernel (tree level):

$$K(b)_{tree} = \frac{1}{\ln(P_2/P_1)} \ln \left[\frac{\tilde{\phi}(b, z=0, P_2)}{\tilde{\phi}(b, z=0, P_1)} \right]$$

- Collins-Soper kernel $K(b_\perp)$ is **almost real**.
- **Stable** with different momentum combinations.





Computation: Promote to 1-loop (in progress)

- Collins-Soper kernel:

$$K(b_\perp) = \frac{1}{\ln(P_2/P_1)} \ln \left[\frac{H_1(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\phi}(b, x, P_2)}{H_1(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\phi}(b, x, P_1)} \right]$$

- Matching kernel up to 1-loop level:

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$$H_1(l, \bar{l}) = e^h, \quad h = \frac{\alpha_s C_F}{4\pi} \left[-\frac{5\pi^2}{6} - 4 + \ln \frac{-l - i0}{\mu^2} + \ln \frac{-\bar{l} - i0}{\mu^2} - \frac{1}{2} \left(\ln^2 \frac{-l - i0}{\mu^2} + \ln^2 \frac{-\bar{l} - i0}{\mu^2} \right) \right] + O(\alpha_s^2)$$

Kernel H_1 is given with an imaginary part, how to deal with it numerically.

- Momentum fraction dependence:

No momentum fraction dependence in tree level, what about 1-loop level?



Summary and outlook

- **Collins-Soper kernel can be extracted from TMDWF non-perturbatively.**
- **We are trying to compute Collins-Soper kernel through TMDWF with 1-loop matching kernel.**
- **Promote our results to continuum limit and physical pion mass.**
- **Collins-Soper kernel from the first principle can be used in global fit of the TMDPDFs. It will reveal the internal structure of hadrons.**

Thanks for your attention!