Conserved charge number fluctuations and their correlations at nonzero magnetic fields

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based on EPJA 57(2021) 6,202 (arxiv: 2104.06843)

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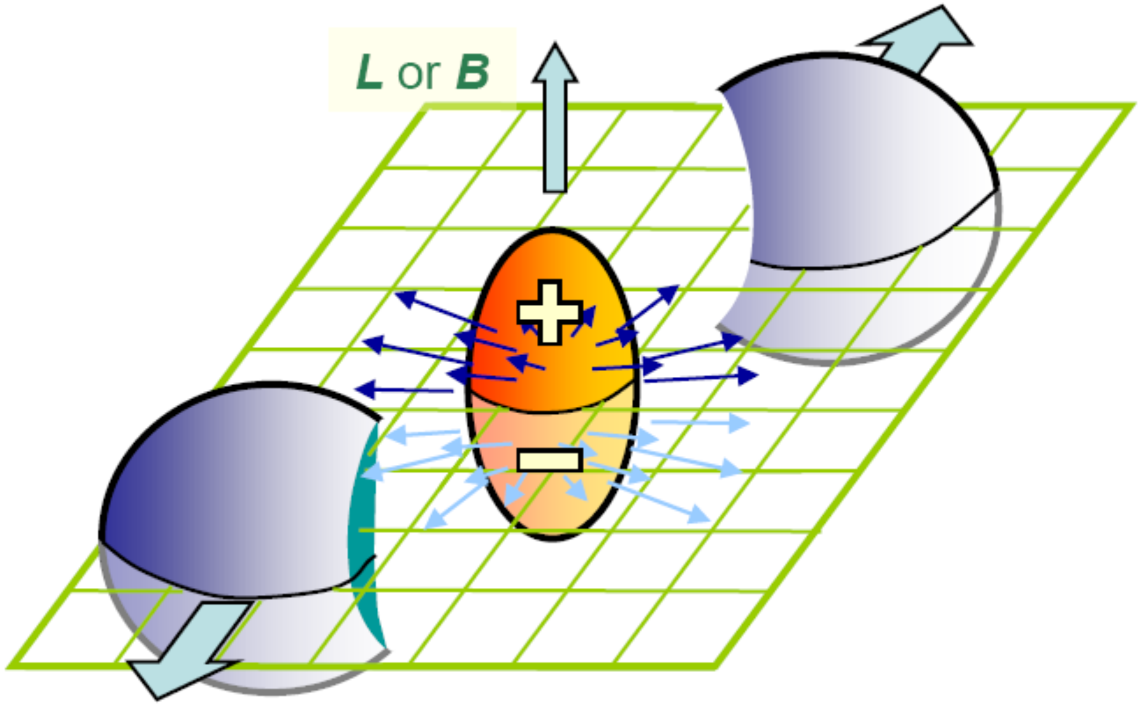
Outline

* Motivation
* Conserved number fluctuations and their correlations at finite eB
    * Lattice QCD calculation
    * Ideal gas limit
    * HRG model
* Lattice results
* Summary and outlook

Motivation

Heavy ion collision



STAR Collaboration (B.I. Abelev (Illinois U., Chicago) et al.). Phys.Rev. C81 (2010) 054908

RHIC: $eB \sim O(1)m_\pi^2$

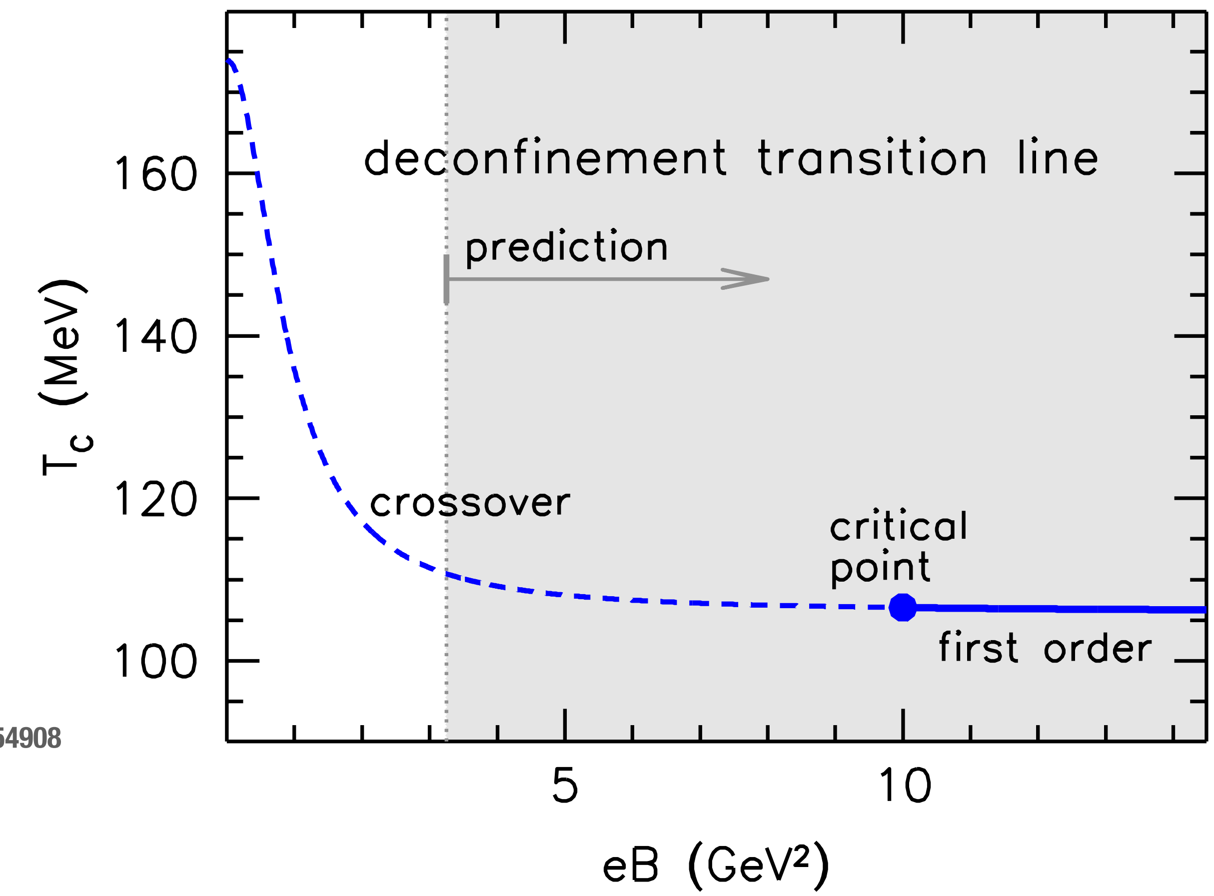
LHC: $eB \sim O(10)m_\pi^2$

V. Skokov, etc., Int.J.Mod.Phys.A 24 (2009) 5925-5932

A. Bzdak, etc., Phys. Rev. Lett. 110, 192301 (2013).

J. Bloczynski, etc., Phys. Lett. B 718, 1529-1535 (2013).

T-eB plane



Gergely Endrodi, JHEP 07 (2015) 173

① **Quantities to detect magnetic fields in Heavy-Ion collision**

② **Possible signatures for a critical end point in T-eB plane**

Conserved charge number fluctuations

$$Z(T,V) = \int [DU] \left(\det M_s[U]\right)^{1/4} \left(\det M_u[U]\right)^{1/4} \left(\det M_d[U]\right)^{1/4} e^{-S_G[U]}$$

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z\left(T, \mu_u, \mu_d, \mu_s\right)$$

$$\hat{\chi}_{ijk}^{uds} = \frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\Big|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{BQS} = \frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_B/T\right)^i \partial\left(\mu_Q/T\right)^j \partial\left(\mu_S/T\right)^k}\Big|_{\mu_{B,Q,S}=0}$$

Accessible
from lattice calculation

HotQCD Collaboration *Phys.Rev.D* 86 (2012) 034509

Ideal gas limit

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{20} + \sum_{f=u,d,s} \left[\frac{1}{2}\hat{\mu}_f^2 + \frac{1}{4\pi^2}\hat{\mu}_f^4 \right]$$

eB=0

Kapusta & Gale, Finite-temperature field theory: Principles and applications

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s} \frac{3|q_f|B}{\pi^2 T^2} \left[\frac{\pi^2}{12} + \frac{\hat{\mu}_f^2}{4} + p_f(B) \right]$$

eB≠0

$$p_f(B) = 2\frac{\sqrt{2\,|q_f|\,B}}{T} \sum_{l=1}^{\infty} \sqrt{l} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \cosh\left(k\hat{\mu}_f\right) \times \mathrm{K}_1\left(\frac{k\sqrt{2\,|q_f|\,Bl}}{T} \right)$$

H.T. Ding, S.-T. Li, Q. Shi and X.-D. Wang, EPJA 57(2021) 6,202

Ideal gas limit

$$\frac{\chi_2^{\mathrm{B}}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{2} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[\sqrt{2}\,\mathrm{K}_1\left(k\,\hat{b}\,\sqrt{2l}\right) + \mathrm{K}_1\left(k\,\hat{b}\,\sqrt{l}\right)\right]\right)$$

$$\frac{\chi_{11}^{\mathrm{BQ}}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{4} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[2\sqrt{2}\,\mathrm{K}_1\left(k\,\hat{b}\,\sqrt{2l}\right) - \mathrm{K}_1\left(k\,\hat{b}\,\sqrt{l}\right)\right]\right)$$

$$\hat{b} = \sqrt{2eB/3}/T$$

Quantity	Value
χ_2^u/eB	$1/\pi^2$
$\chi_2^{d/s/S}/eB$	$1/(2\pi^2)$
$\chi_{11}^{ud}/eB = \chi_{11}^{us}/eB = \chi_{11}^{ds}/eB{=}0$	0
χ_2^{B}/eB	$2/(9\pi^2)$
χ_2^{Q}/eB	$5/(9\pi^2)$
$\chi_{11}^{\mathrm{BQ}}/eB$	$1/(9\pi^2)$
$\chi_{11}^{\mathrm{QS}}/eB = -\chi_{11}^{\mathrm{BS}}/eB = \chi_2^{\mathrm{S}}/3eB$	$1/(6\pi^2)$

Taking derivatives with respect to chemical potentials, and set chemical potentials to zero, then take $\sqrt{eB}/T$ to infinity limit

Hadron resonance gas model

Starting point

$$p_n^{\mathrm{M/B}} = \mp \frac{d_i T}{2\pi^2} \int_0^\infty \mathrm{d}p |\vec{p}|^2 \ln \left[1 \mp e^{-(E_n - \mu_i)/T} \right]$$

$$E_n = \sqrt{m_i^2 + |\vec{p}|^2}$$

eB=0

+ : mesons
- : baryons

$$p_c^{\mathrm{M/B}} = \mp \frac{|q_i| BT}{2\pi^2} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \int_0^\infty \mathrm{d}p_z \ln \left[1 \mp e^{-(E_c - \mu_i)/T} \right]$$

$$E_c = \sqrt{p_z^2 + m_i^2 + 2|q_i| B(l + 1/2 - s_z)}$$

eB≠0

We consider eB≤0.3GeV², since π⁺ and K⁻ deviate from the above equation at eB ≥0.3 GeV²

H.T. Ding, S.T. Li, A. Tomiya, X.D. Wang and Y. Zhang, *Phys.Rev.D* 104 (2021) 014505

Take derivatives with respect to μ

$$\chi_2^X = \frac{B}{2\pi^2 T} \sum_i |q_i| X_i^2 \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} f(\varepsilon_0),$$

$$\chi_{11}^{XY} = \frac{B}{2\pi^2 T} \sum_i |q_i| X_i Y_i \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} f(\varepsilon_0), \quad f(\varepsilon_0) = \varepsilon_0 \sum_{k=1}^{\infty} (\pm 1)^{k+1} k \mathrm{K}_1 \left(\frac{k\varepsilon_0}{T} \right)$$

In our case, we incorporated all the hadrons listed in the PDG up to the mass of 2.5 GeV

Particle Data Group collaboration, Review of Particle Physics, Phys. Rev. D 98 (2018) 030001

Lattice Setup

- HISQ/tree action
- $m_s = m_s^{\text{phy}},\ m_l = m_s^{\text{phy}}/10,\ m_\pi \sim 220\ \text{MeV}$
- a is **fixed** to 0.117 fm, $T = \frac{1}{aN_\tau}$
- N_τ window: (6, 96)

  T window: (280 MeV, 17 MeV)
- The N_σ is **fixed** to 32, $N_\sigma = N_x = N_y = N_z$
- eB window: (0, 2.5 GeV^2)

NuclearScience Computing Center at CCNU

$$0 \leq N_b < \frac{N_x N_y}{4} \quad , \quad eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

N_τ	T [MeV]	N_b window	eB window $[GeV^2]$
6	280	[0, 48]	[0, 2.5]
8	210	[0, 48]	[0, 2.5]
10	168	[0, 48]	[0, 2.5]
12	140	[0, 48]	[0, 2.5]
14	120	[0, 48]	[0, 2.5]
16	105	[0, 48]	[0, 2.5]
18	94	[0, 48]	[0, 2.5]
24	70	[0, 48]	[0, 2.5]
96	17	[0, 64]	[0, 3.3]

H.T. Ding, S.T. Li, A. Tomiya, X.D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 014505

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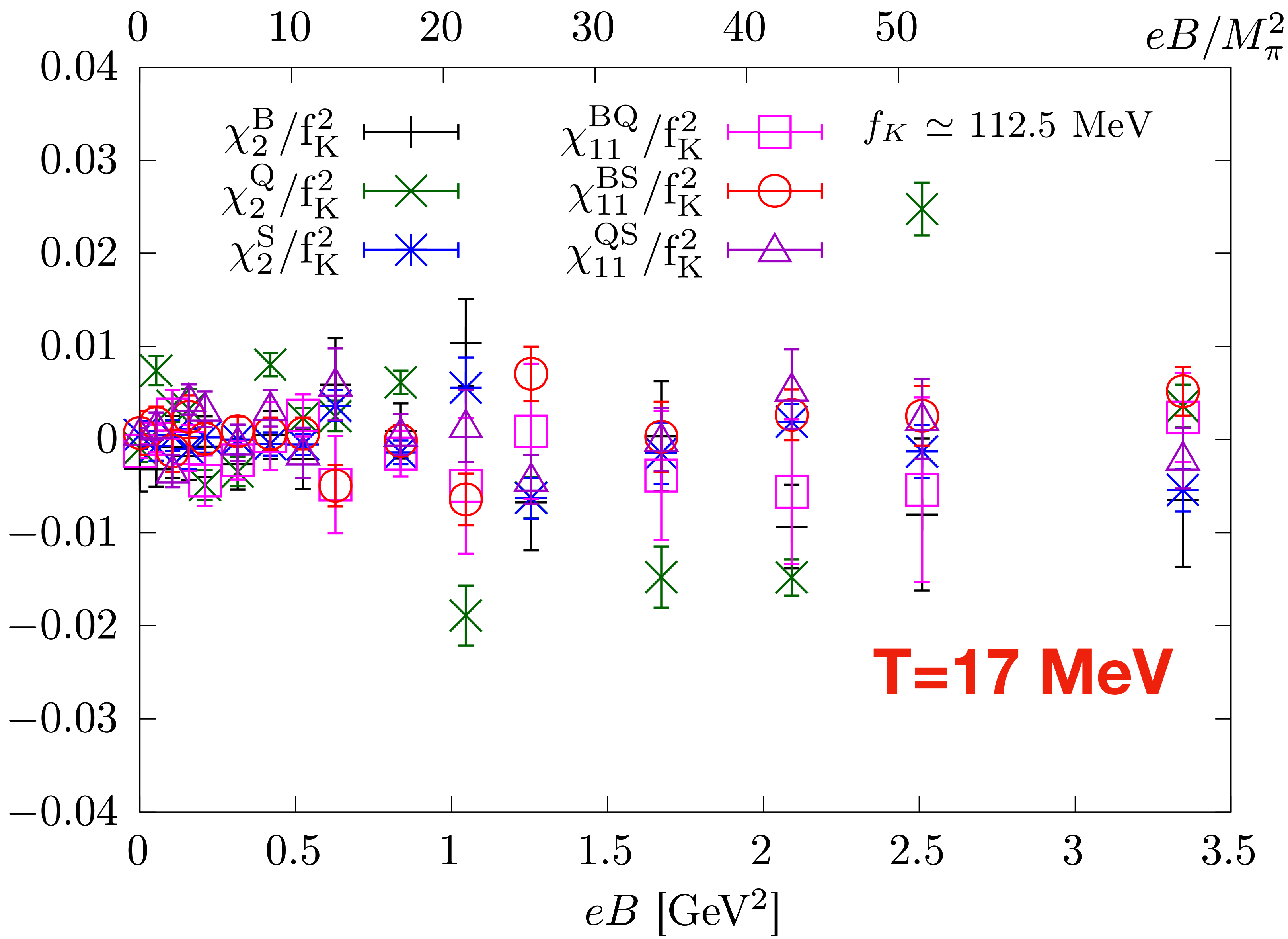
Conserved charge fluctuations and correlations at T=0



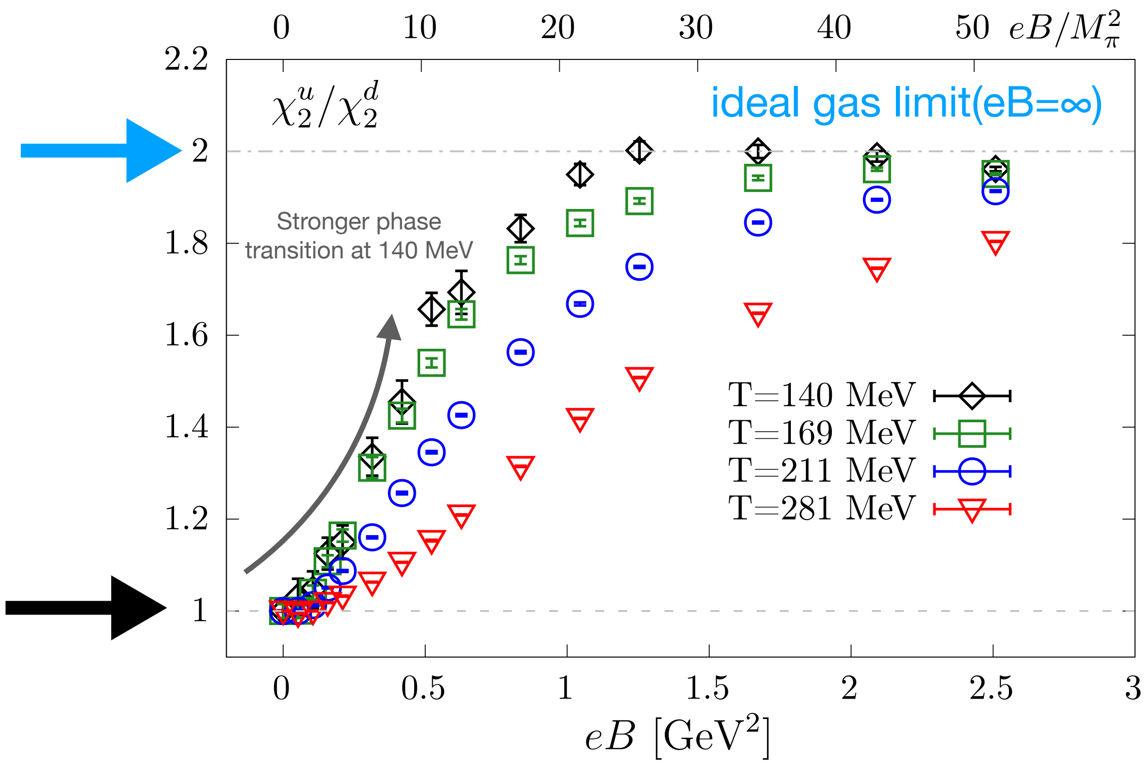
Heng-Tong Ding, Sheng-Tai Li, Qi Shi, Xiao-Dan Wang, EPJA 57(2021) 6,202

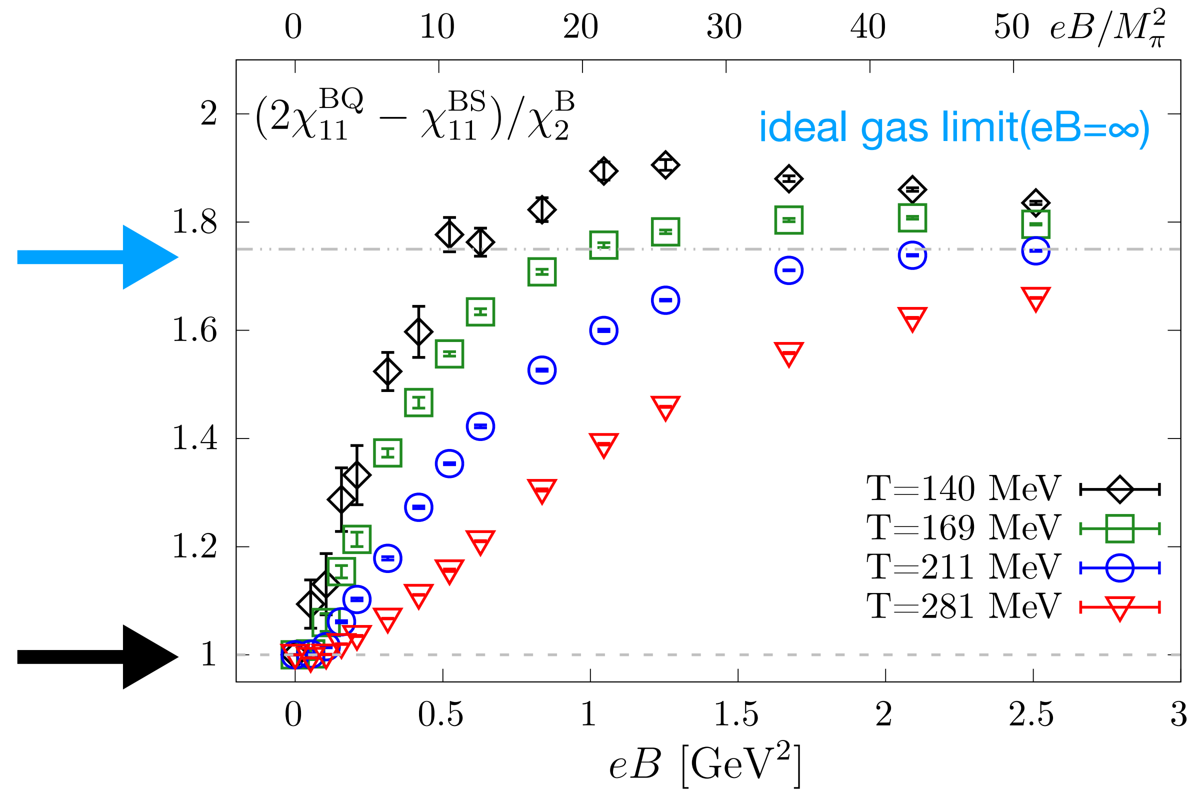
ρ is a boson

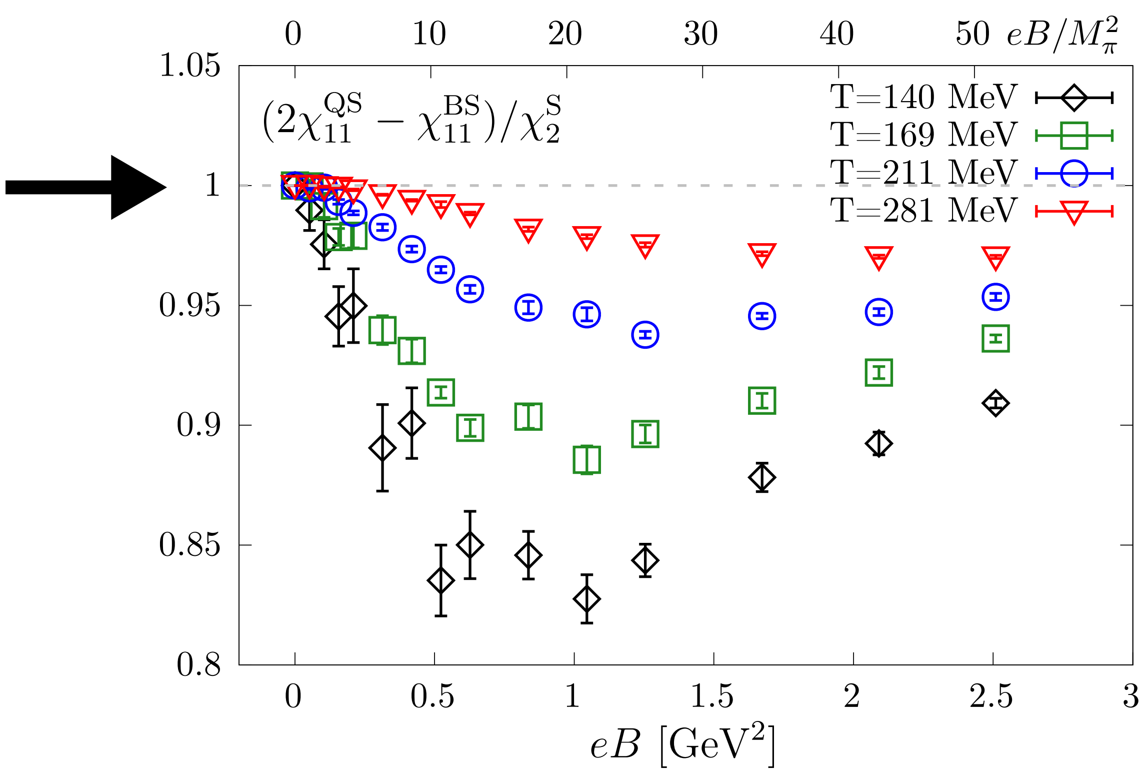
$$f(E) = \frac{1}{e^{E/kT} - 1}$$

No evidence (χ_2^Q is not divergent) for a superconducting phase at T=0 is found

Isospin symmetry breaking

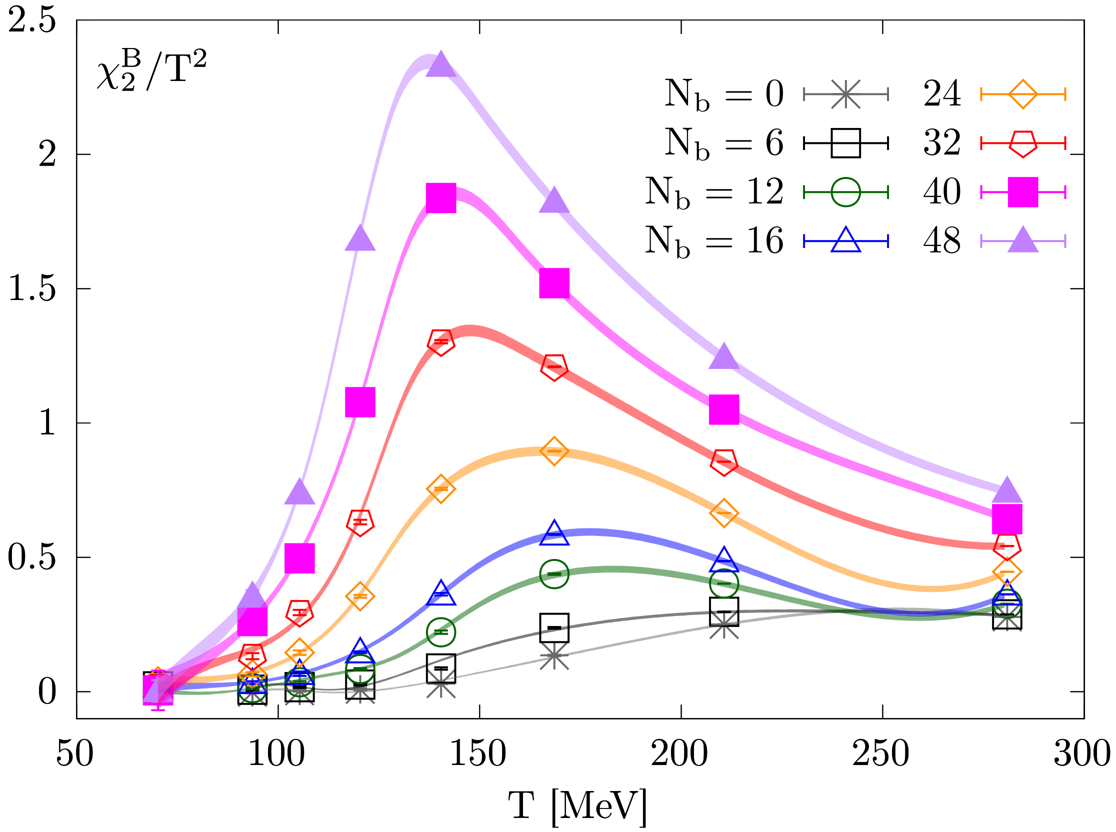


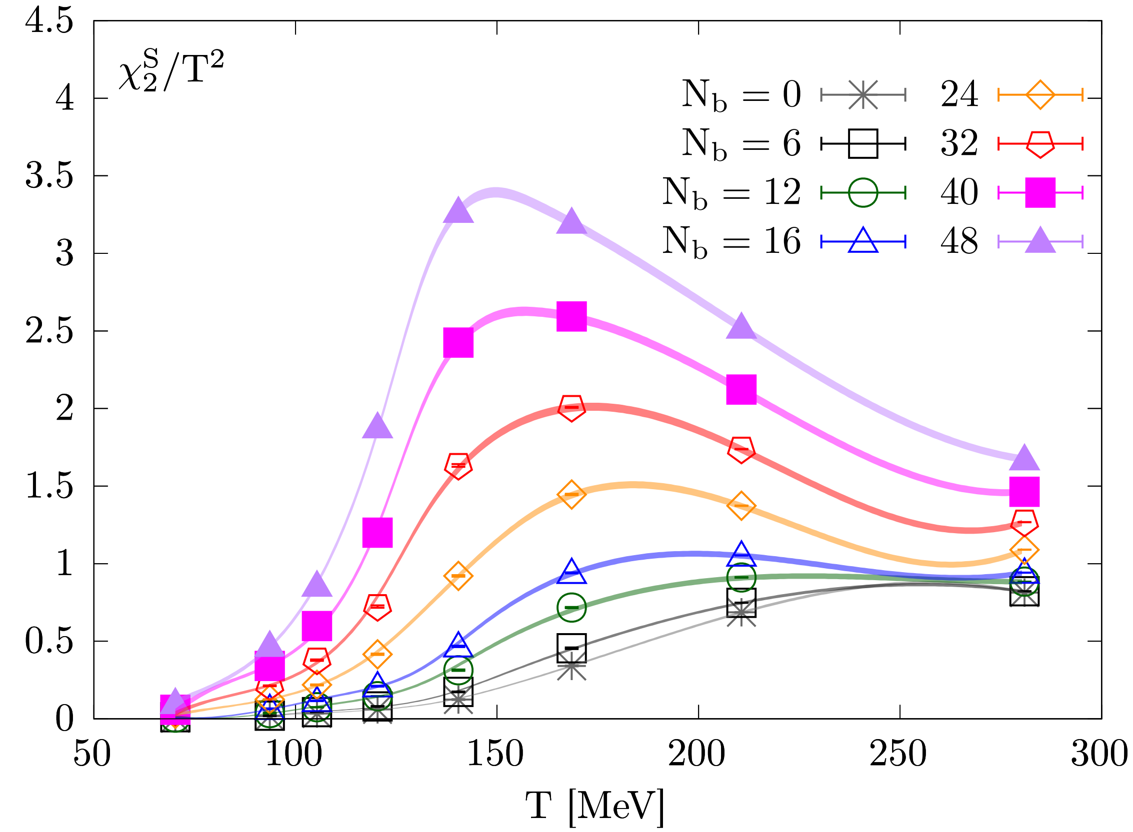


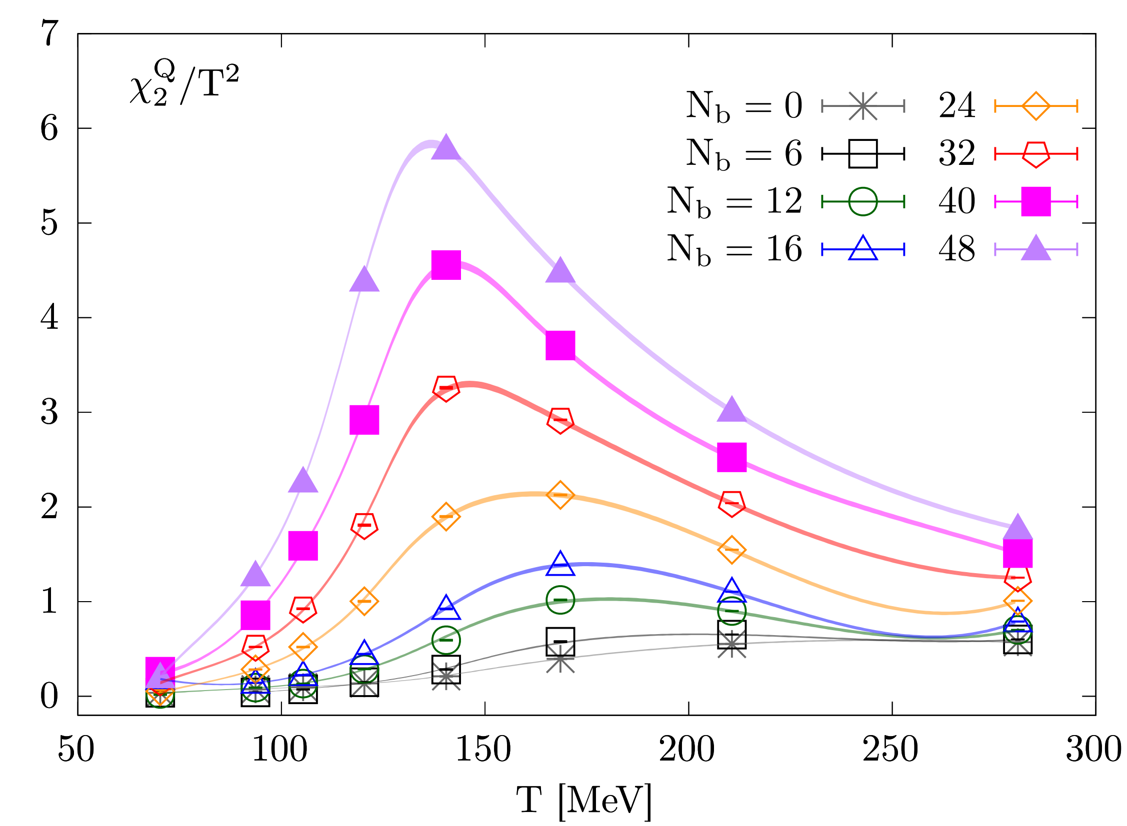


- ★ → **Denotes the isospin symmetric phase**
- ★ → **Denotes the ideal gas limits**
- ★ **Stronger isospin symmetry breaking at lower T**
- ★ $\left(2\chi_{11}^{\rm BQ} - \chi_{11}^{\rm BS}\right)/\chi_2^{\rm B}$ **and** $\left(2\chi_{11}^{\rm QS} - \chi_{11}^{\rm BS}\right)/\chi_2^{\rm S}$ **could be useful in experiment to observe the isospin symmetry breaking**

$\chi_2^B, \chi_2^Q, \chi_2^S$ at finite eB

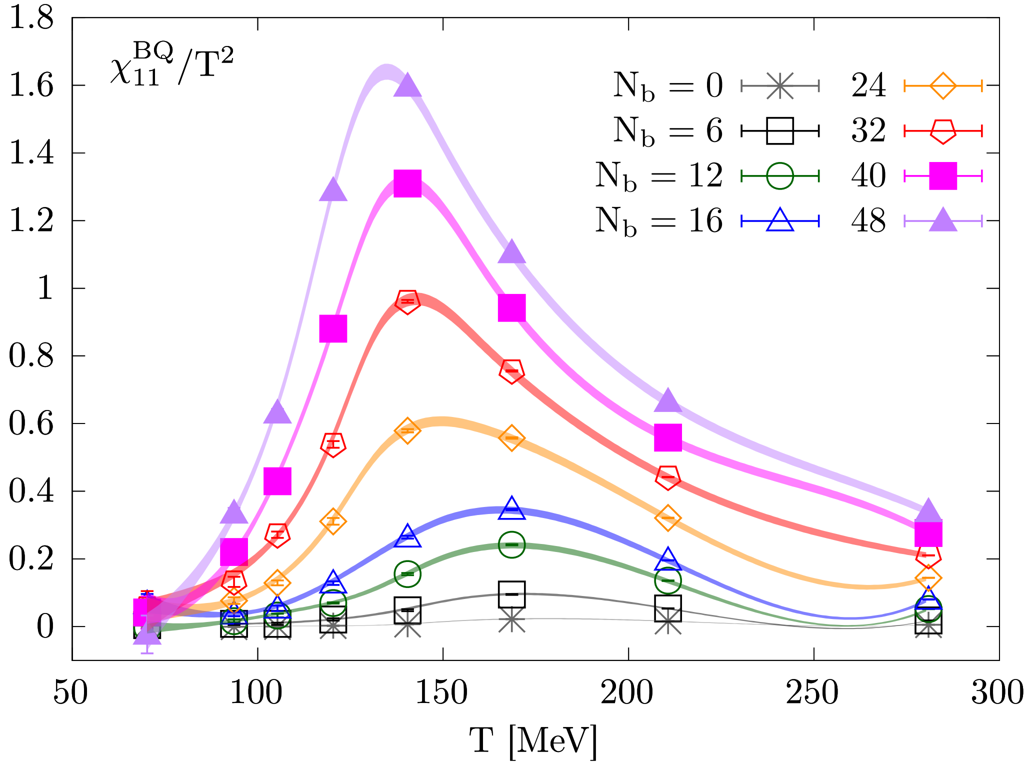


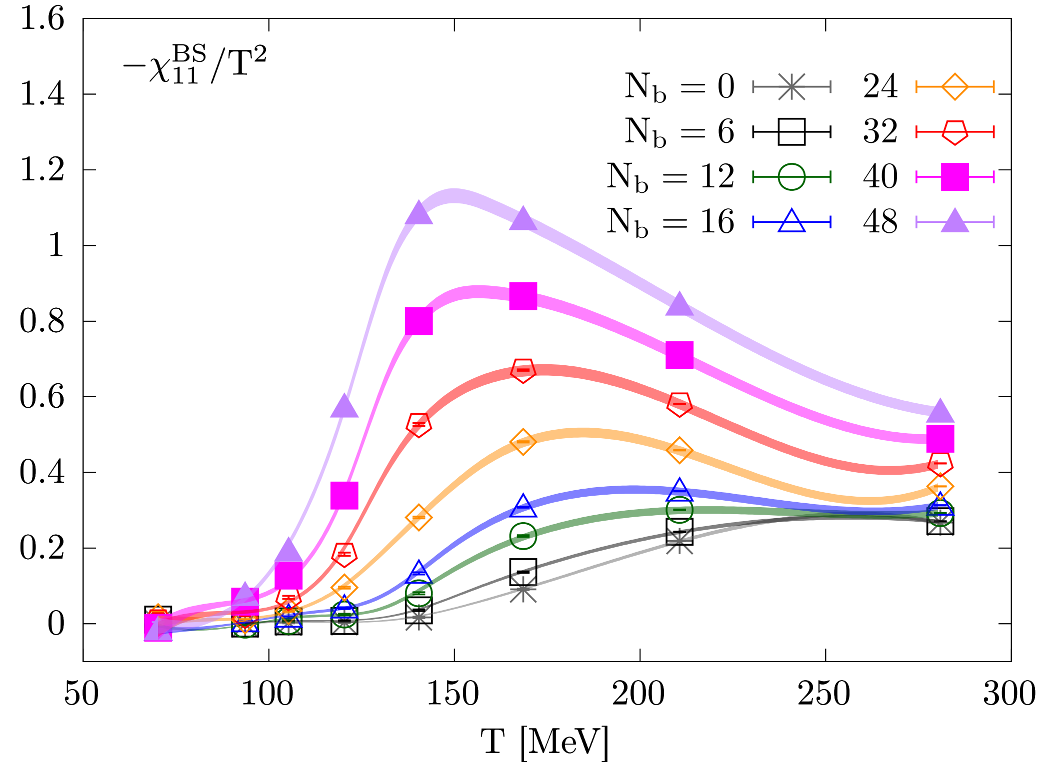


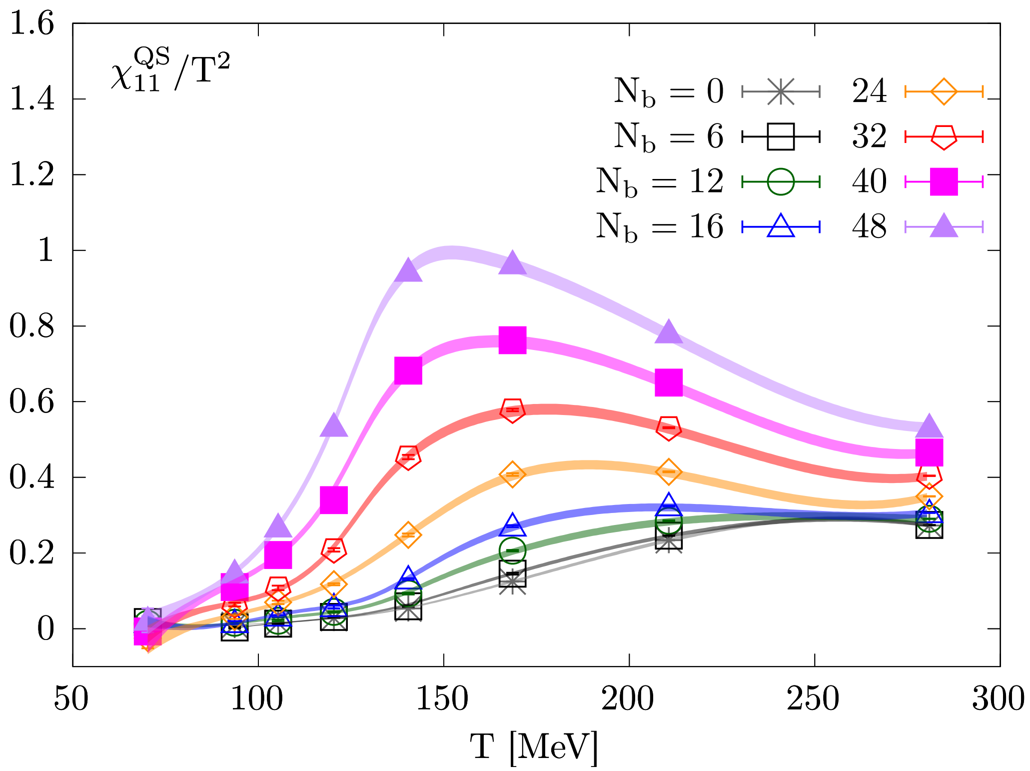


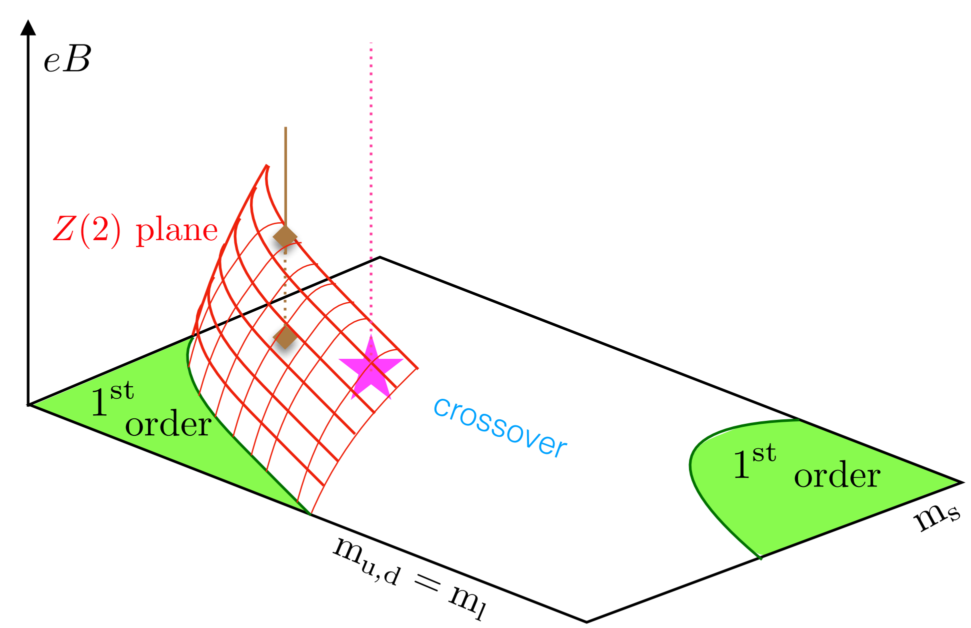
- ☆ $\chi_2^B, \chi_2^Q, \chi_2^S$ **become larger and develop a peak structure as N_b grows**
- ☆ **The peak location/inflection points shift to lower temperatures as N_b↑, it indicates that T_{pc}↓ as N_b↑**

$\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS}$ at finite eB

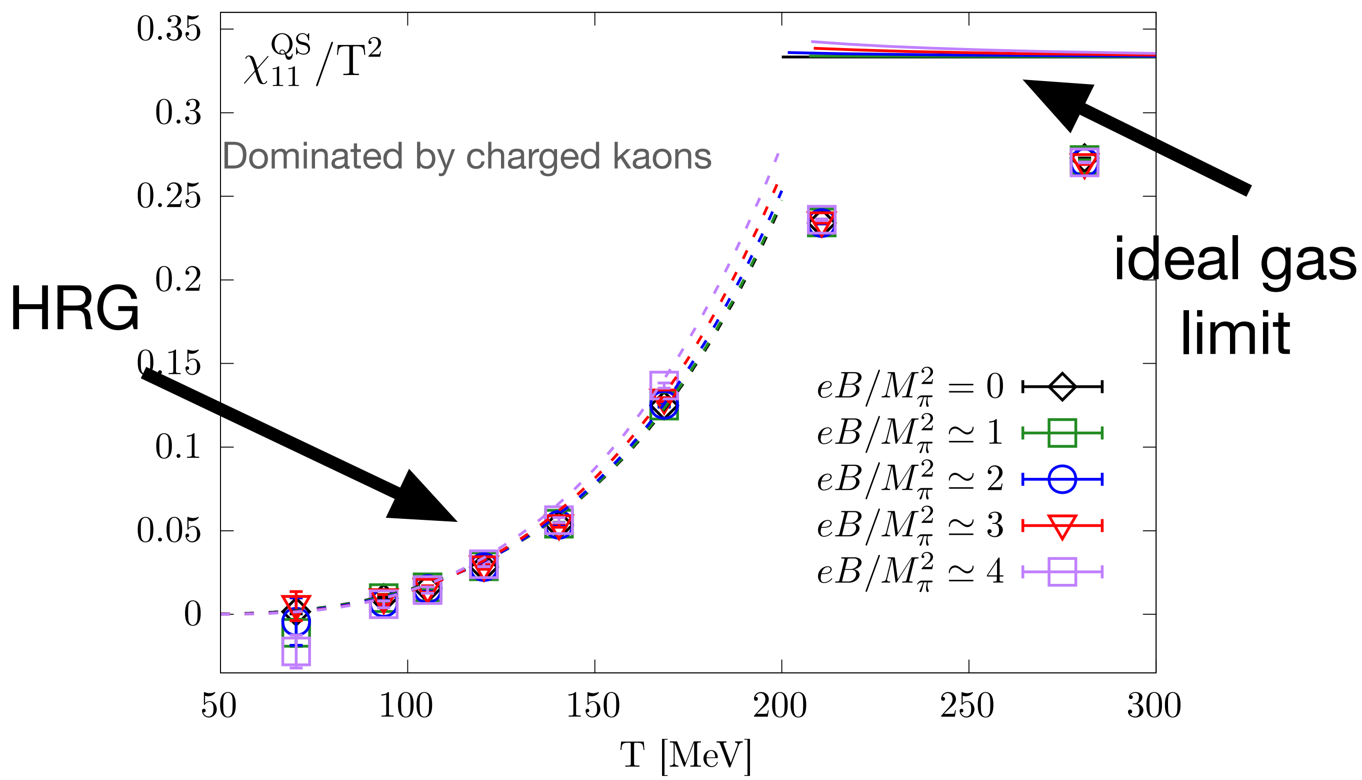


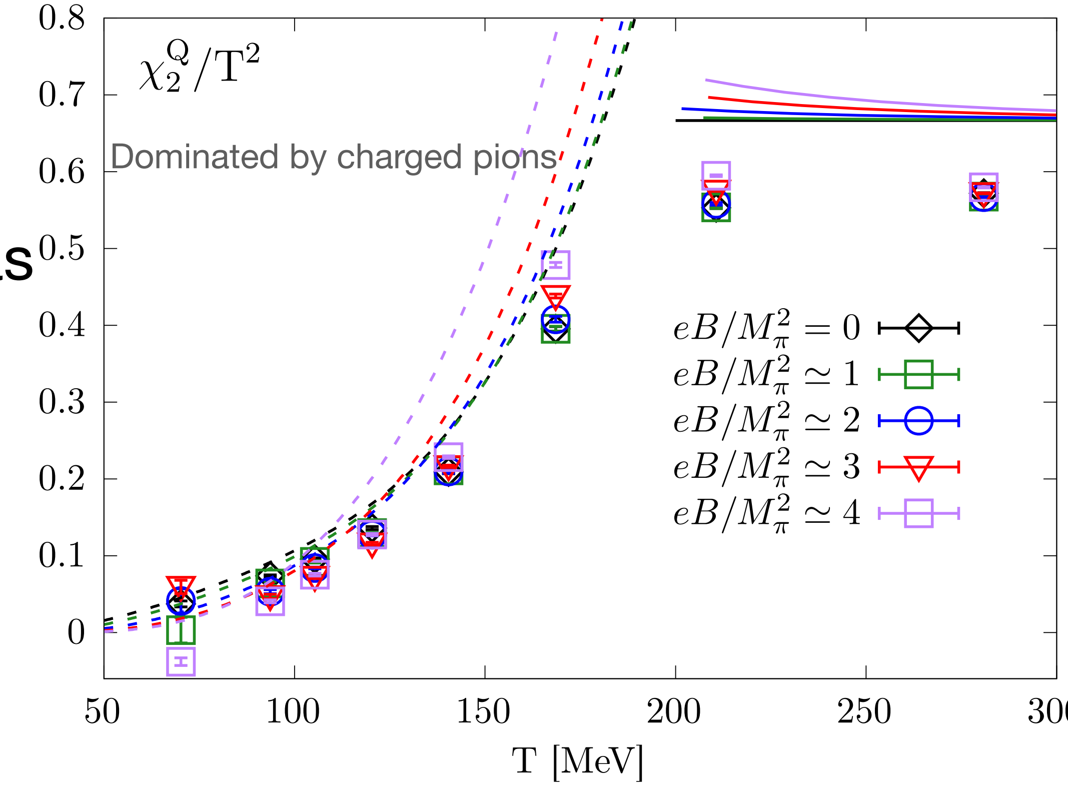


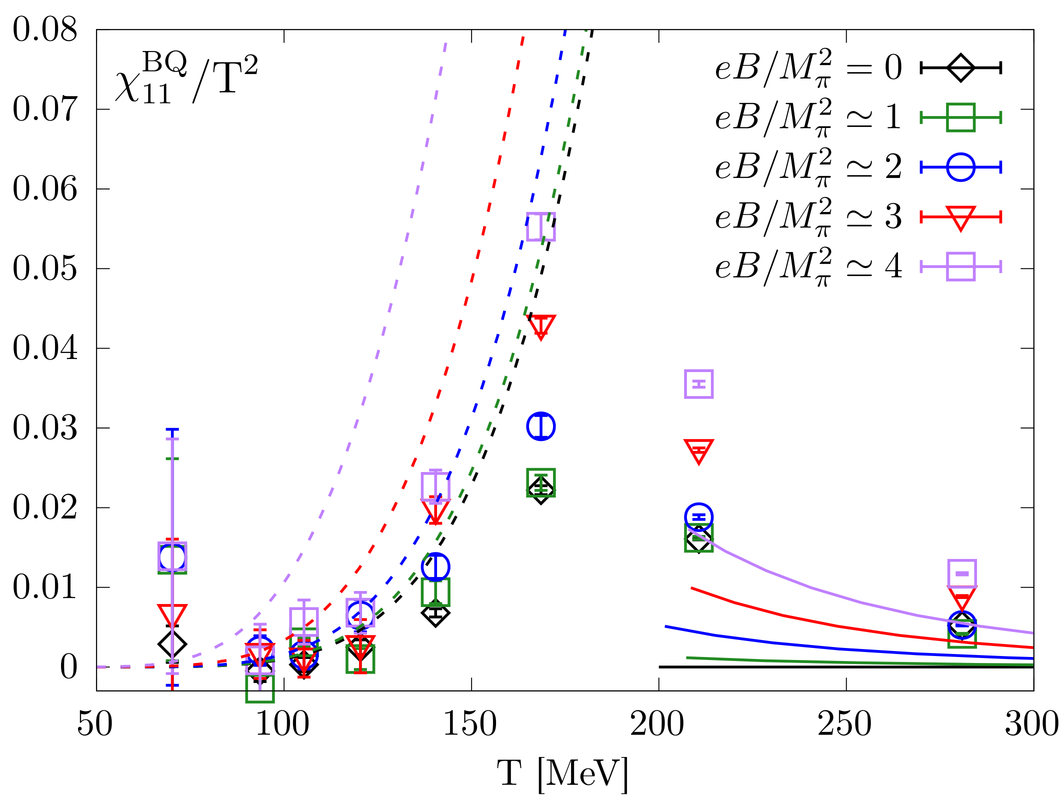




Comparisons to HRG & free limit



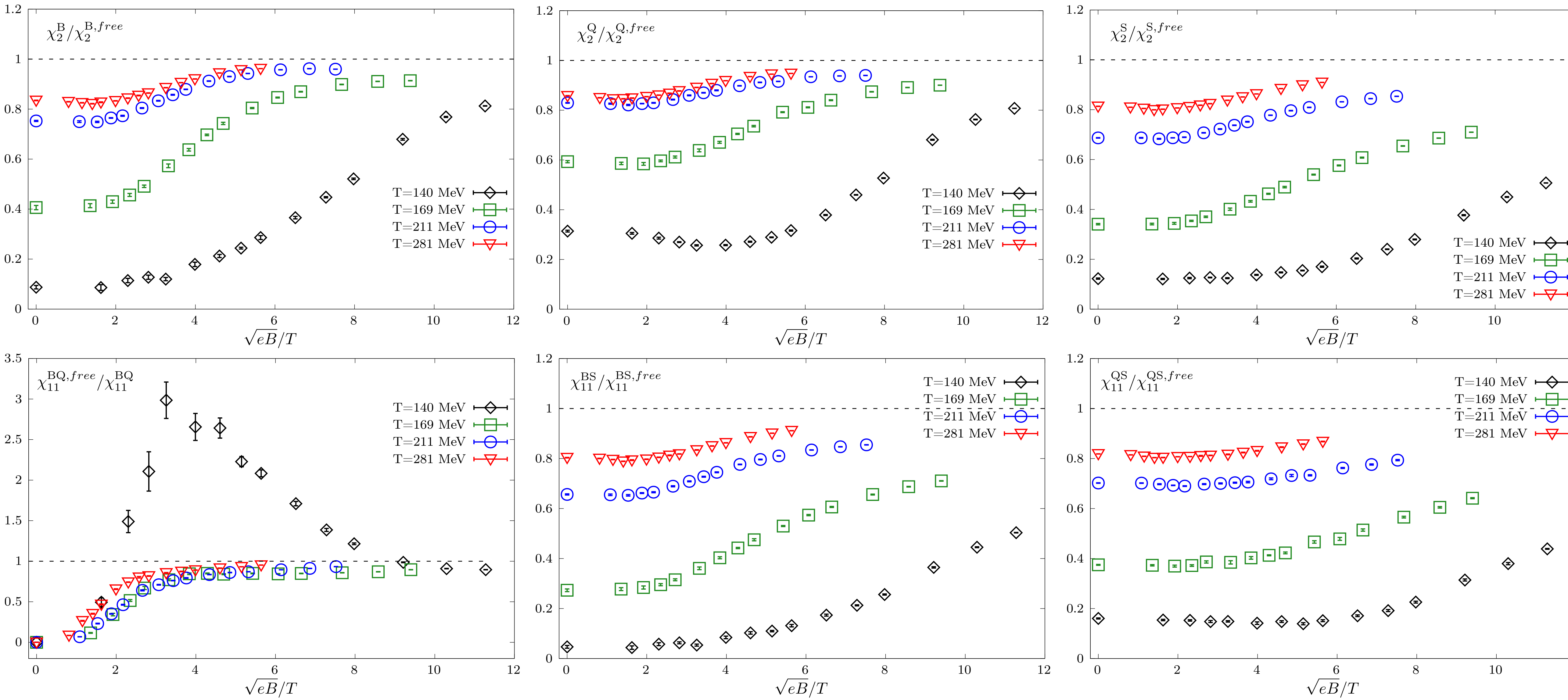




Complex eB dependence of charged baryons

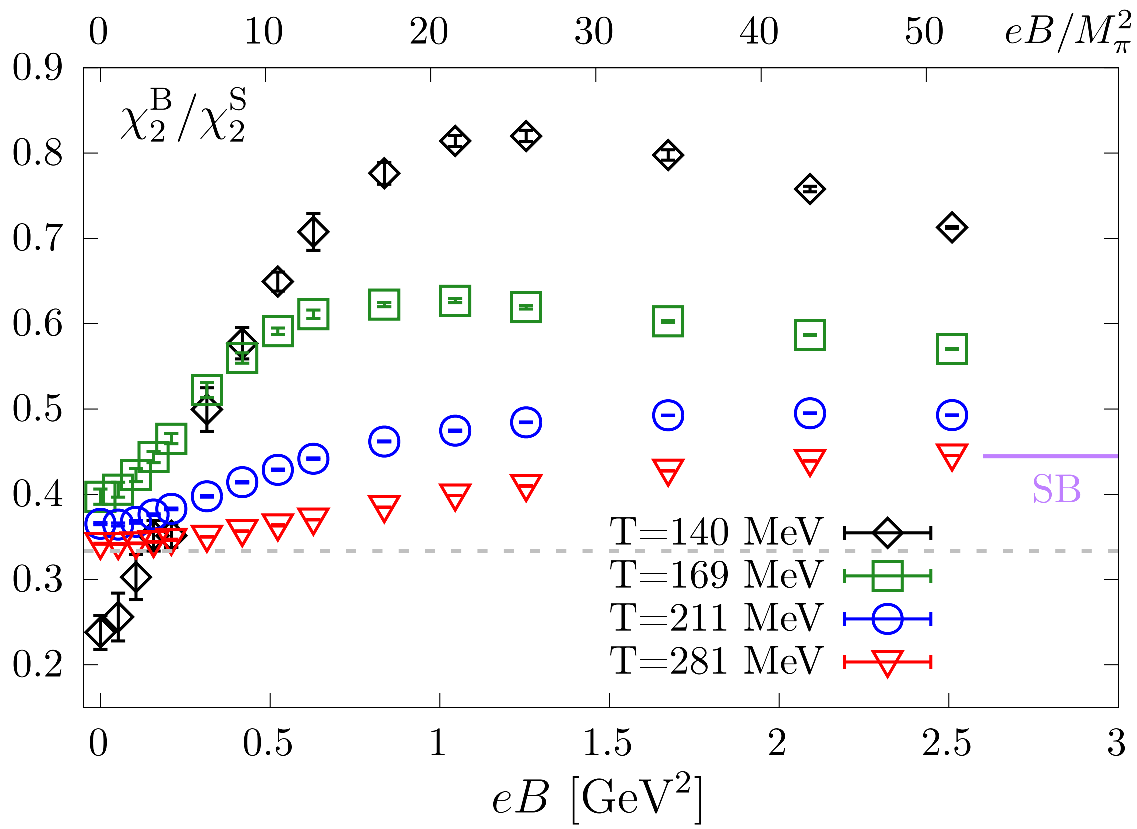
- **Pion masses are more affected comparing to kaons by eB**
- **At low T, HRG failed to describe χ_{11}^{BQ}, χ_2^Q with increasing eB, this agrees with the fact that T_{pc} decreases as eB grows**

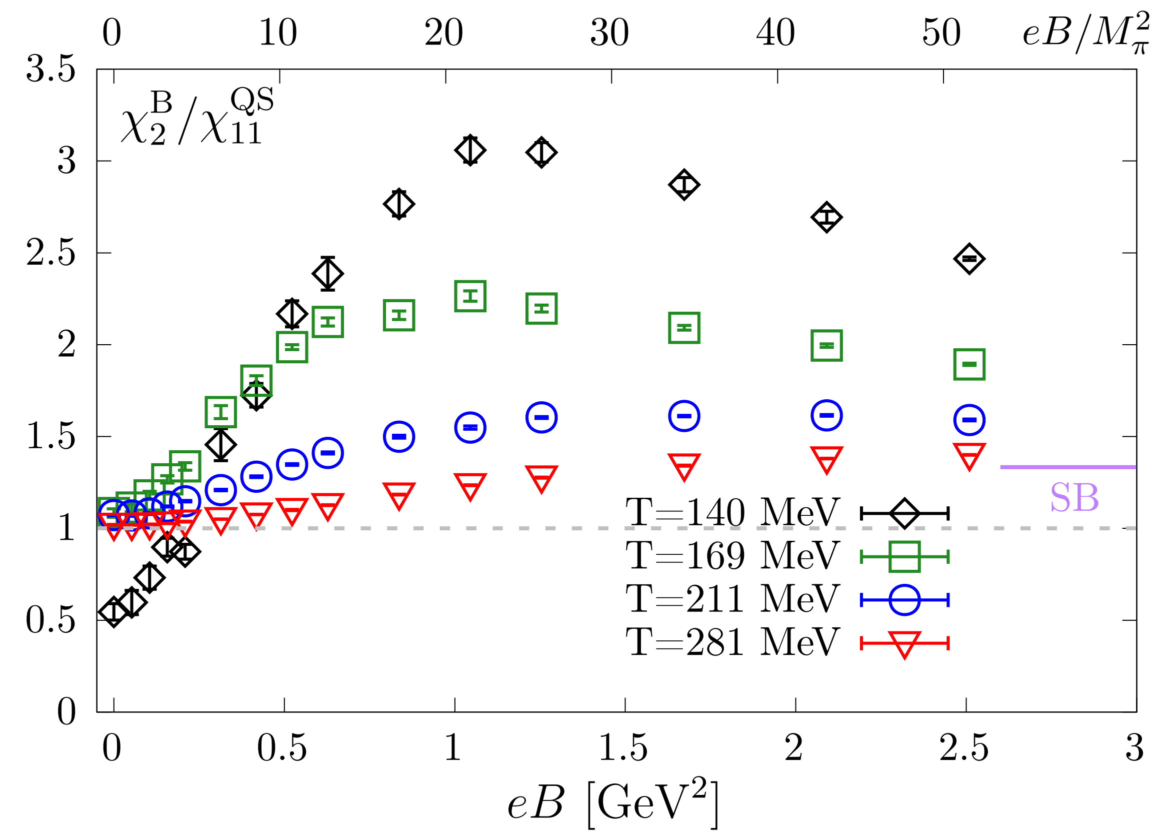
Comparisons to HRG & free limit

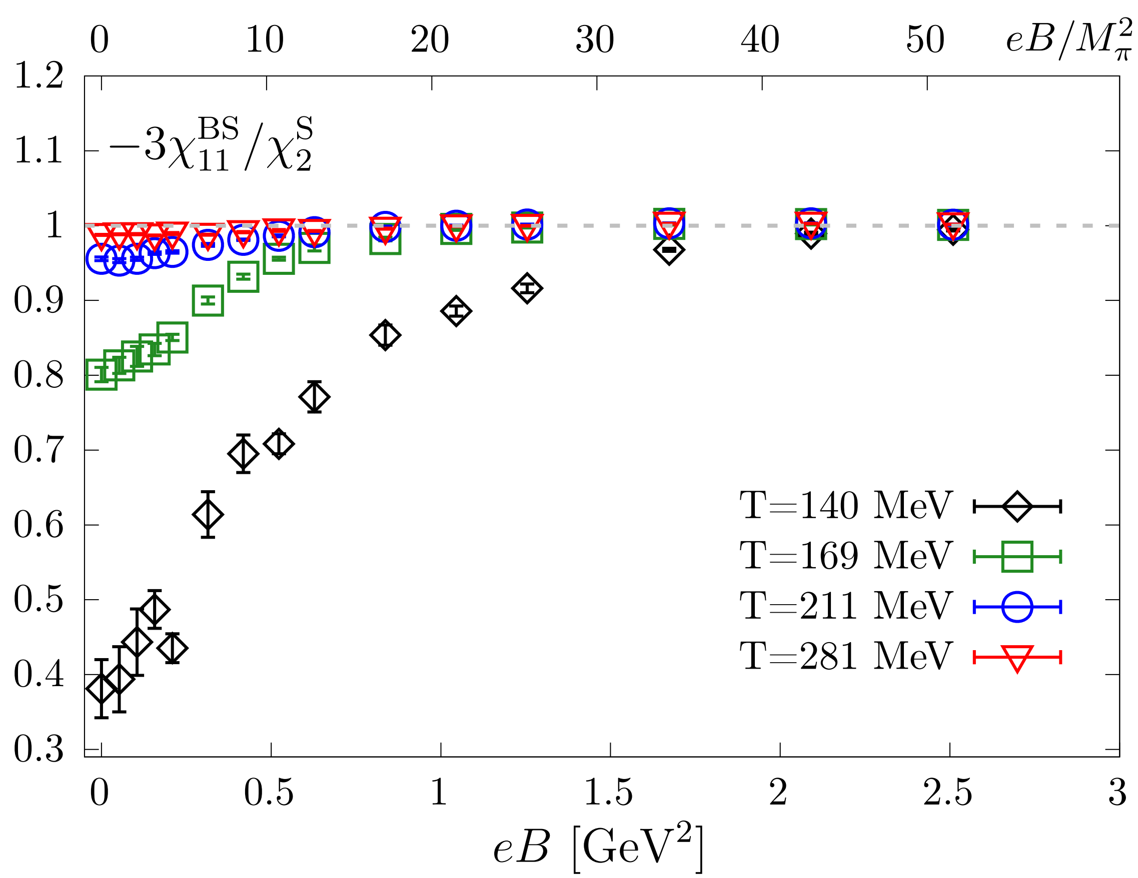


- ✱ T_{pc} becomes lower at larger eB, indicating that free theory works at larger eB for fixed T
- ✱ At lower T, degree of freedom in the system changes dramatically from confined hadron phase to deconfined QGP phase

The ratios: χ_2^B/χ_2^S , χ_2^B/χ_{11}^{QS} and $-3\chi_{11}^{BS}/\chi_2^S$







- There exists non-monotonous behavior in eB at two lowest T in χ_2^B/χ_2^S & χ_2^B/χ_{11}^{QS}
- For lower T, the quantities increase more rapidly with increasing eB
- For $-3\chi_{11}^{BS}/\chi_2^S$, it has a stronger eB dependence at lower T—-> magnetic field fosters the transition

Summary and outlook

- [x] No superconducting phase is found in our eB window
- [x] eB fosters the phase transition
- [x] At lower T, as eB grows, the degree of freedom changes faster
- [x] Several quantities might be useful to probe eB in Heavy ion collision experiments
- [ ] A possible Z(2) second order phase transition at sufficiently high eB
- [ ] The study at finite eB with physical pion mass is in progress