Radiative neutrino masses, lepton flavor mixing and muon g − 2 in a leptoquark model

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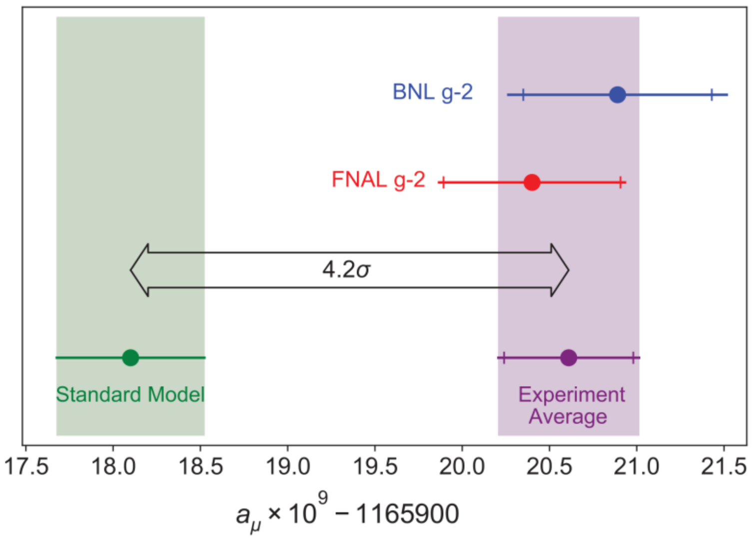
Motivation

Charged-lepton sector:



SM prediction for muon g-2:

$$a_\mu^{\mathrm{SM}} = 116591810\,(43) \times 10^{-11}$$ T. Aoyama et al, 2020

The combined experimental average for muon g-2:

$$a_\mu^{\mathrm{exp}} = 116592061\,(41) \times 10^{-11}$$ G. W. Bennett et al, 2006; B. Abi et al, 2021

The difference between the experimental value and SM prediction:

$$\boxed{\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 251\,(59) \times 10^{-11}}$$ 4.2σ

Beyond the SM

Neutrino sector:

Global-fit results

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{\mathrm{CP}}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

I. Esteban et al, 2020; P. F. de Salas et al, 2020

The upp[illegible] n anti-neutrino mass:

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} < 0.8\ \mathrm{eV}$$ M. Aker et al, 2021

Two messages:

- **Neutrinos are massive but their masses are tiny**
- **Lepton flavor mixing exists**

Motivation

Anomaly of muon g-2

New interaction for charged leptons

Neutrino masses

New interaction for neutrinos

Is it possible to connect these two issues?

- **(Type-I, II, III) Seesaw mechanisms: simple and natural** A. Abada et al, 2007; Z. Z. Xing, S. Zhou, 2010

$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{\ell_{\alpha \rm L}} \tilde{H} \tilde{H}^T \ell^c_{\beta \rm L} + {\rm h.c.} & ({\rm Type~I}) \,, \\ -\frac{\lambda_\Delta}{M_\Delta}\left(Y_\Delta\right)_{\alpha\beta} \overline{\ell_{\alpha \rm L}} \tilde{H} \tilde{H}^T \ell^c_{\beta \rm L} + {\rm h.c.} & ({\rm Type~II}) \,, \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{\ell_{\alpha \rm L}} \tilde{H} \tilde{H}^T \ell^c_{\beta \rm L} + {\rm h.c.} & ({\rm Type~III}) \,. \end{cases}$$

SSB

$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_{\rm R}} Y_\nu^T & ({\rm Type~I}) \,, \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & ({\rm Type~II}) \,, \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & ({\rm Type~III}) \,. \end{cases}$$

But they lead to a small and/or negative Δa_μ ✗ **Anomaly of muon g-2**

C. Biggio, 2008; S. Zhou, 2021

- **Radiative neutrino masses: generating neutrino masses at loop level** Y. Cai et al, 2017; K. S. Babu et al, 2019

One kind of radiative neutrino mass models: extending the SM with leptoquarks (LQs)

- **Scalar LQ $\widetilde{R}_2(3,2,1/6)$ with either $S_1\left(\bar{3},1,1/3\right)$ or $S_3\left(\bar{3},3,1/3\right)$ can generate neutrino masses at the one-loop level** C. K. Chua, X. G. He, W-Y. P. Hwang, 1999; U. Mahanta, 2000; D. A. Sierra, M. Hirsch, S. G. Kovalenko, 2007; I. Dorsner, S. Fajfer, O Sumensari, 2019
- **Single scalar LQ $S_1\left(\bar{3},1,1/3\right)$ or $R_2(3,2,7/6)$ with both the left-handed and right-handed chiral couplings can account for Δa_μ** K. M. Cheung, 2001; S. Saad, 2020

$\widetilde{R}_2(3,2,1/6)$ and $S_1\left(\bar{3},1,1/3\right)$ may give a combined explanation of neutrino masses, lepton flavor mixing and the anomaly of muon anomalous magnetic moment

More aspects of LQs can be seen in the review for leptoquarks: I. Dorsner et al, 2016

The Leptoquark Model

Enforcing baryon number conservation, the Lagrangian with two scalar LQs S_1 and $\widetilde{R}_2$ is

$$\mathcal{L}_{\rm LQ} = \underline{\lambda^{\rm L}_{i\alpha}\overline{Q^c_{i\rm L}}\epsilon\ell_{\alpha\rm L}S_1 + \lambda^{\rm R}_{i\alpha}\overline{u^c_{i\rm R}}E_{\alpha\rm R}S_1} + \lambda_{i\alpha}\overline{d_{i\rm R}}\widetilde{R}^T_2\epsilon\ell_{\alpha\rm L} + {\rm h.c.} + \left(D_\mu S_1\right)^\dagger\left(D^\mu S_1\right) + \left(D_\mu \widetilde{R}_2\right)^\dagger\left(D^\mu \widetilde{R}_2\right) - V_{\rm LQ}\,,$$

$$\widetilde{R}_2 = \begin{pmatrix} \widetilde{R}_2^{+\frac{2}{3}} \\ \widetilde{R}_2^{-\frac{1}{3}} \end{pmatrix}$$

where the scalar potential is given by

$$V_{\rm LQ} = \mu_S^2 S_1^\dagger S_1 + \mu_R^2 \widetilde{R}_2^\dagger\widetilde{R}_2 + \boxed{\left(\lambda_{\rm mix}S_1^*\widetilde{R}_2^\dagger H + {\rm h.c.}\right)} + \lambda_{HS}\left(H^\dagger H\right)\left(S_1^\dagger S_1\right) + \lambda_{HR}^{(1)}\left(H^\dagger H\right)\left(\widetilde{R}_2^\dagger\widetilde{R}_2\right)$$
$$+\lambda_{HR}^{(3)}\left(H^\dagger\tau^I H\right)\left(\widetilde{R}_2^\dagger\tau^I\widetilde{R}_2\right) + \lambda_S\left(S_1^\dagger S_1\right)^2 + \lambda_R^{(1)}\left(\widetilde{R}_2^\dagger\widetilde{R}_2\right)^2 + \lambda_R^{(8)}\left(\widetilde{R}_2^\dagger T^A\widetilde{R}_2\right)^2$$
$$+\lambda_{SR}^{(1)}\left(S_1^\dagger S_1\right)\left(\widetilde{R}_2^\dagger\widetilde{R}_2\right) + \lambda_{SR}^{(8)}\left(S_1^\dagger T^A S_1\right)\left(\widetilde{R}_2^\dagger T^A\widetilde{R}_2\right)\,,$$

- **The scalar LQ S_1 has both the left-handed and right-handed couplings with quarks and leptons, which can chirally enhance the contributions to muon g-2**
- **The LQ-Higgs interaction $\lambda_{\rm mix}S_1^*\widetilde{R}_2^\dagger H$ plays an important role in radiatively generating neutrino masses, and has some effects on the contributions from LQs to muon g-2**

After spontaneous gauge symmetry breaking, the mass matrix for S_1 and $\widetilde{R}_2^{-\frac{1}{3}*}$ is found to be

$$M^2_{\rm mix} = \begin{pmatrix} m_S^2 & \frac{v}{\sqrt{2}}\lambda_{\rm mix} \\ \frac{v}{\sqrt{2}}\lambda^*_{\rm mix} & m_R^2 \end{pmatrix} \quad \textbf{with} \quad m_S^2 = \mu_S^2 + \frac{v^2}{2}\lambda_{HS}\,, \quad m_R^2 = \mu_R^2 + \frac{v^2}{2}\left(\lambda_{HR}^{(1)} + \lambda_{HR}^{(3)}\right)$$

which is not diagonal leading to a mixing between S_1 and $\widetilde{R}_2^{-\frac{1}{3}*}$, and we assume that all coupling constants involved in above mass matrix are real

The Leptoquark Model

To work in the basis of the physical LQs, one can make following transformation

$$\begin{pmatrix} S_1 \\ \widetilde{R}_2^{-\frac{1}{3}*} \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} S_1 \\ \widetilde{R}_2^{-\frac{1}{3}*} \end{pmatrix}$$

with

$$\tan 2\theta = \frac{\sqrt{2}\lambda_{\mathrm{mix}} v}{m_R^2 - m_S^2}$$

Mixing angle

- **The masses of the physical LQs are given by**

$$M_{1,2}^2 = \frac{1}{2}\left[m_S^2 + m_R^2 \pm \sqrt{\left(m_S^2 - m_R^2\right)^2 + 2\lambda_{\mathrm{mix}}^2 v^2}\right], \quad M_3^2 = \mu_R^2 + \frac{v^2}{2}\left(\lambda_{HR}^{(1)} - \lambda_{HR}^{(3)}\right)$$

where M_1 , M_2 and M_3 are the masses of physical S_1, $\widetilde{R}_2^{-\frac{1}{3}}$ and $\widetilde{R}_2^{+\frac{2}{3}}$, respectively

- **The Yukawa coupling interactions involving the physical LQs after SSB are**

$$\boxed{\begin{aligned} \mathcal{L}_{\mathrm{Y}} = {} & \overline{\nu_\alpha}\left(\lambda_{i\alpha}^{*}\sin\theta P_{\mathrm{R}} - \lambda_{i\alpha}^{\mathrm{L}}\cos\theta P_{\mathrm{L}}\right) d_i S_1 + \overline{l_\alpha^c}\left(\underline{\lambda_{i\alpha}^{\prime\mathrm{L}}} P_{\mathrm{L}} + \lambda_{i\alpha}^{\mathrm{R}} P_{\mathrm{R}}\right)\cos\theta u_i S_1 \\ & - \overline{\nu_\alpha}\left(\lambda_{i\alpha}^{*}\cos\theta P_{\mathrm{R}} + \lambda_{i\alpha}^{\mathrm{L}}\sin\theta P_{\mathrm{L}}\right) d_i \widetilde{R}_2^{-\frac{1}{3}*} + \overline{l_\alpha^c}\left(\underline{\lambda_{i\alpha}^{\prime\mathrm{L}}} P_{\mathrm{L}} + \lambda_{i\alpha}^{\mathrm{R}} P_{\mathrm{R}}\right)\sin\theta u_i \widetilde{R}_2^{-\frac{1}{3}*} \\ & + \lambda_{i\alpha}\overline{d_i} P_{\mathrm{L}} l_\alpha \widetilde{R}_2^{+\frac{2}{3}} + \mathrm{h.c.}, \end{aligned}}$$

in which $\lambda^{\prime\mathrm{L}} = V^T \lambda^{\mathrm{L}}$ with *V* being the CKM matrix

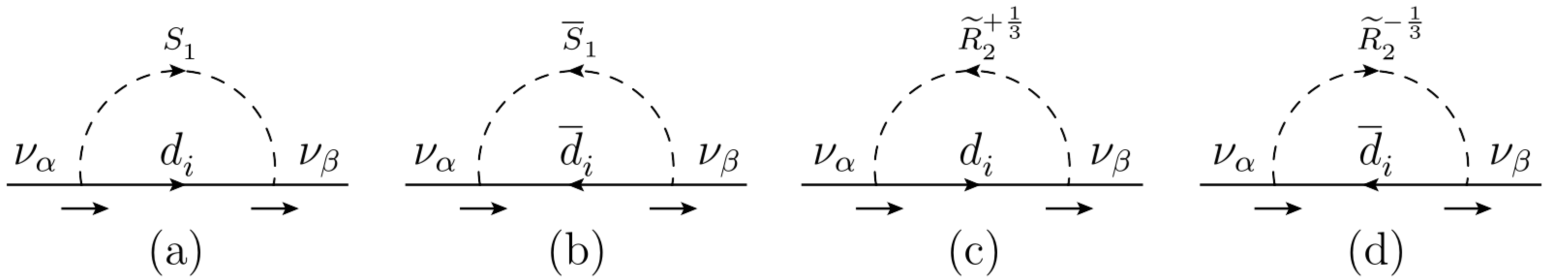
Similar interactions for S_1 and $\widetilde{R}_2^{-\frac{1}{3}}$

Similar contributions from S_1 and $\widetilde{R}_2^{-\frac{1}{3}}$ to neutrino masses and muon g-2

Radiative Neutrino Masses

No right-handed neutrinos in this LQ model → **Neutrinos are massless at tree level**

Neutrino masses are generated at one-loop level in the present LQ model



Calculating above self-energy diagrams, one can obtain the radiative neutrino mass matrix:

$$(M_\nu)_{\alpha\beta} = -\frac{N_c}{2(4\pi)^2}\left(\lambda^*_{i\alpha}\lambda^{\mathrm{L}*}_{i\beta} + \lambda^{\mathrm{L}*}_{i\alpha}\lambda^*_{i\beta}\right)\hat{m}_i \sin 2\theta \left[\frac{M_2^2 \ln\frac{\hat{m}_i^2}{M_2^2}}{M_2^2-\hat{m}_i^2} - \frac{M_1^2 \ln\frac{\hat{m}_i^2}{M_1^2}}{M_1^2-\hat{m}_i^2}\right]$$

$$N_c = 3 \qquad \hat{m} = (m_\mathrm{d}, m_\mathrm{s}, m_\mathrm{b})$$

$$\simeq \boxed{\frac{3\sin 2\theta}{32\pi^2}\ln\frac{M_2^2}{M_1^2}\left[\left(\lambda^\dagger\right)_{\alpha i}\hat{m}_i\left(\lambda^{\mathrm{L}*}\right)_{i\beta} + \left(\lambda^{\mathrm{L}\dagger}\right)_{\alpha i}\hat{m}_i\left(\lambda^*\right)_{i\beta}\right]},$$

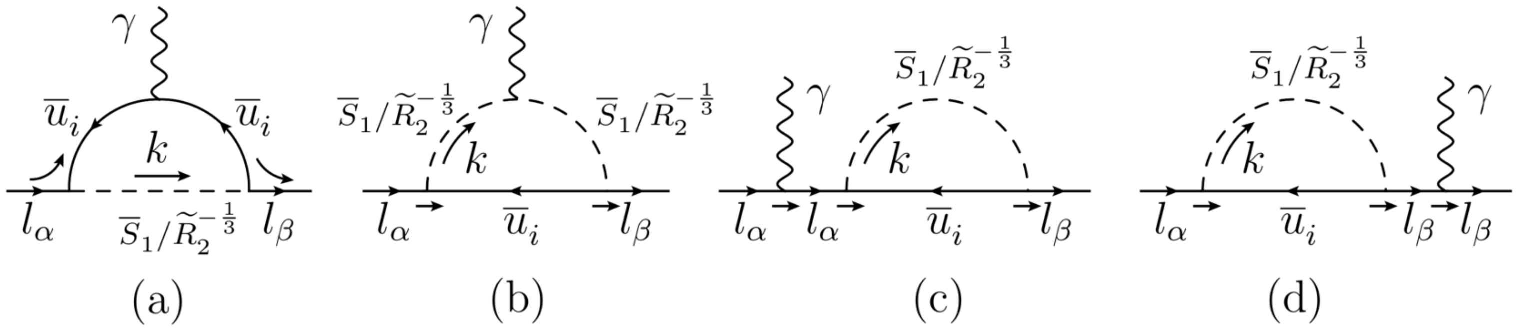
$$\hat{m}_i \ll M_{1,2}$$

- **Only coupling matrices λ and λ^L are involved, λ^R does not contribute**
- **The mixing between S_1 and $\widetilde{R}_2^{-\frac{1}{3}*}$ is important to radiatively generate neutrino masses**
- **If the masses of the physical S_1 and $\widetilde{R}_2^{-\frac{1}{3}}$ are exactly degenerate, neutrinos will remain massless at one-loop level**

$\theta = 0$ **or** $M_1 = M_2$ → $M_\nu = 0$

Muon anomalous magnetic moment

Contributions from S_1 and $\widetilde{R}_2^{-\frac{1}{3}}$ to muon g-2 or radiative decays of charged leptons:



Calculating above diagrams, the total amplitude mediated by S_1 is:

$$\boxed{\mathrm{i}\mathcal{M}^{S_1} = -\frac{\mathrm{i}e}{(4\pi)^2}\epsilon_\mu^*(q)\,\overline{u}(p-q)\left(\mathcal{A}_\mathrm{L}^{S_1}P_\mathrm{L} + \mathcal{A}_\mathrm{R}^{S_1}P_\mathrm{R}\right)\mathrm{i}\sigma^{\mu\nu}q_\nu u(p)}$$

$$\tilde{m} = (m_\mathrm{u}, m_\mathrm{c}, m_\mathrm{t})$$
$$m = \left(m_e, m_\mu, m_\tau\right)$$

$$\mathcal{A}_\mathrm{L}^{S_1} = \frac{N_c\cos^2\theta}{12M_1^2}\left[2\tilde{m}_i\lambda_{i\alpha}^{\prime\mathrm{L}}\lambda_{i\beta}^{\mathrm{R}*}\mathcal{F}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) - \left(m_\beta\lambda_{i\alpha}^{\prime\mathrm{L}}\lambda_{i\beta}^{\prime\mathrm{L}*} + m_\alpha\lambda_{i\alpha}^{\mathrm{R}}\lambda_{i\beta}^{\mathrm{R}*}\right)\mathcal{G}\left(\frac{\tilde{m}_i^2}{M_1^2}\right)\right] \qquad \mathcal{F}(x) = \frac{7-8x+x^2+2(2+x)\ln x}{(1-x)^3}$$

$$\mathcal{A}_\mathrm{R}^{S_1} = \frac{N_c\cos^2\theta}{12M_1^2}\left[2\tilde{m}_i\lambda_{i\alpha}^{\mathrm{R}}\lambda_{i\beta}^{\prime\mathrm{L}*}\mathcal{F}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) - \left(m_\alpha\lambda_{i\alpha}^{\prime\mathrm{L}}\lambda_{i\beta}^{\prime\mathrm{L}*} + m_\beta\lambda_{i\alpha}^{\mathrm{R}}\lambda_{i\beta}^{\mathrm{R}*}\right)\mathcal{G}\left(\frac{\tilde{m}_i^2}{M_1^2}\right)\right] \qquad \mathcal{G}(x) = \frac{1+4x-5x^2+2x(2+x)\ln x}{(1-x)^4}$$

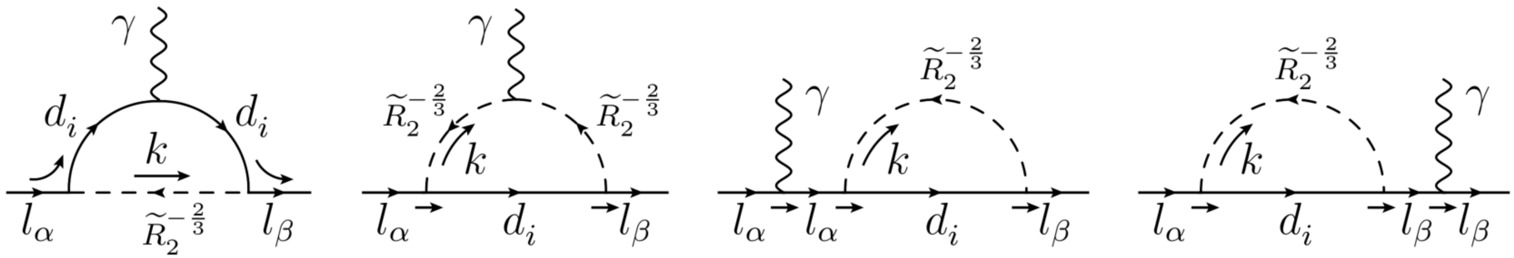
The total amplitude mediated by $\widetilde{R}_2^{-\frac{1}{3}}$ can be obtained by making following replacements:

$\cos^2\theta \to \sin^2\theta$ **and** $M_1 \to M_2$ $\Rightarrow$ $\boxed{\mathrm{i}\mathcal{M}^{\widetilde{R}_2^{-\frac{1}{3}}} = \mathrm{i}\mathcal{M}^{S_1}(M_1 \to M_2)\tan^2\theta}$

All divergences are canceled out

Muon anomalous magnetic moment

Contributions from $\widetilde{R}_2^{+\frac{2}{3}}$ to muon g-2 or radiative decays of charged leptons :



(e) (f) (g) (h)

Calculating above diagrams, the total amplitude mediated by $\widetilde{R}_2^{+\frac{2}{3}}$ is:

$$\boxed{\mathrm{i}\mathcal{M}^{\widetilde{R}_2^{+\frac{2}{3}}} = -\frac{\mathrm{i}e}{(4\pi)^2}\epsilon_\mu^*(q)\,\overline{u}(p-q)\left(\mathcal{A}_{\mathrm{L}}^{\widetilde{R}_2^{+\frac{2}{3}}}P_{\mathrm{L}} + \mathcal{A}_{\mathrm{R}}^{\widetilde{R}_2^{+\frac{2}{3}}}P_{\mathrm{R}}\right)\mathrm{i}\sigma^{\mu\nu}q_\nu u(p)}$$

$$\mathcal{A}_{\mathrm{L}}^{\widetilde{R}_2^{+\frac{2}{3}}} = -\frac{N_c m_\beta}{12M_3^2}\lambda_{i\alpha}\lambda_{i\beta}^*\mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right), \quad \mathcal{A}_{\mathrm{R}}^{\widetilde{R}_2^{+\frac{2}{3}}} = -\frac{N_c m_\alpha}{12M_3^2}\lambda_{i\alpha}\lambda_{i\beta}^*\mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right) \qquad \mathcal{I}(x) = \frac{x\left[5-4x-x^2+(2+4x)\ln x\right]}{(1-x)^4}$$

All divergences are canceled out

The branching ratios of the radiative decays of charged leptons:

$$\boxed{\mathcal{B}\left(l_\alpha^- \to l_\beta^- + \gamma\right) = \frac{\alpha_{\mathrm{em}}\left(m_\alpha^2 - m_\beta^2\right)^3}{4(4\pi)^4 m_\alpha^3\Gamma_\alpha}\left(\left|\mathcal{A}_{\mathrm{L}}^{S_1} + \mathcal{A}_{\mathrm{L}}^{\widetilde{R}_2^{-\frac{1}{3}}} + \mathcal{A}_{\mathrm{L}}^{\widetilde{R}_2^{+\frac{2}{3}}}\right|^2 + \left|\mathcal{A}_{\mathrm{R}}^{S_1} + \mathcal{A}_{\mathrm{R}}^{\widetilde{R}_2^{-\frac{1}{3}}} + \mathcal{A}_{\mathrm{R}}^{\widetilde{R}_2^{+\frac{2}{3}}}\right|^2\right)}$$

Strong constraints on the Yukawa couplings, especially that from $\tau^- \to \mu^- + \gamma$

Muon anomalous magnetic moment

Muon anomalous magnetic moment in the LQ model:

$$\boxed{\Delta a_\mu = \Delta a_\mu^{S_1} + \Delta a_\mu^{\widetilde{R}_2^{-\frac{1}{3}}} + \Delta a_\mu^{\widetilde{R}_2^{+\frac{2}{3}}}}$$

$$\Delta a_\mu^{S_1} = -\frac{N_c m_\mu \cos^2\theta}{6\,(4\pi)^2 M_1^2}\left[\underline{2\tilde{m}_i \mathrm{Re}\left(\lambda_{i\mu}^{\prime\mathrm{L}} \lambda_{i\mu}^{\mathrm{R}*}\right)} \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) - m_\mu \left(\left|\lambda_{i\mu}^{\prime\mathrm{L}}\right|^2 + \left|\lambda_{i\mu}^{\mathrm{R}}\right|^2\right) \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_1^2}\right)\right],$$

$$\Delta a_\mu^{\widetilde{R}_2^{-\frac{1}{3}}} = -\frac{N_c m_\mu \sin^2\theta}{6\,(4\pi)^2 M_2^2}\left[\underline{2\tilde{m}_i \mathrm{Re}\left(\lambda_{i\mu}^{\prime\mathrm{L}} \lambda_{i\mu}^{\mathrm{R}*}\right)} \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_2^2}\right) - m_\mu \left(\left|\lambda_{i\mu}^{\prime\mathrm{L}}\right|^2 + \left|\lambda_{i\mu}^{\mathrm{R}}\right|^2\right) \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_2^2}\right)\right],$$

$$\Delta a_\mu^{\widetilde{R}_2^{+\frac{2}{3}}} = \frac{N_c m_\mu^2 \left|\lambda_{i\mu}\right|^2}{6\,(4\pi)^2 M_3^2} \mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right).$$

$$\boxed{\tilde{m} = (m_\mathrm{u}, m_\mathrm{c}, m_\mathrm{t}) \qquad \hat{m} = (m_\mathrm{d}, m_\mathrm{s}, m_\mathrm{b})}$$

$$\boxed{\tilde{m}_i, \hat{m}_j \ll M_k} \Longrightarrow \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_{1,2}^2}\right) \simeq 7 + 4\ln\frac{\tilde{m}_i^2}{M_{1,2}^2}, \quad \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_{1,2}^2}\right) \simeq 1, \quad \mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right) \simeq \frac{\hat{m}_i^2}{M_3^2}\left(5 + 2\ln\frac{\hat{m}_i^2}{M_3^2}\right)$$

$$\boxed{\Delta a_\mu \simeq \frac{4 m_\mu \tilde{m}_i}{(4\pi)^2} \mathrm{Re}\left(\lambda_{i\mu}^{\prime\mathrm{L}} \lambda_{i\mu}^{\mathrm{R}*}\right)\left[\frac{\cos^2\theta}{M_1^2}\left(\ln\frac{M_1^2}{\tilde{m}_i^2} - \frac{7}{4}\right) + \frac{\sin^2\theta}{M_2^2}\left(\ln\frac{M_2^2}{\tilde{m}_i^2} - \frac{7}{4}\right)\right]}$$

- **Contributions from S_1 and $\widetilde{R}_2^{-\frac{1}{3}}$ are enhanced by masses of up-type quarks**
- **Contribution from $\widetilde{R}_2^{+\frac{2}{3}}$ is strongly suppressed by M_3^{-4}, thus is usually omitted**
- **Coupling matrices λ^{L}, λ^{R} and λ are involved, but effects of λ are suppressed**
- **If $\theta = 0$ or $M_1 = M_2$, this result for muon g-2 will recover the results caused by S_1 all alone**

Textures for Yukawa Coupling Matrices

Assuming following textures for the Yukawa coupling matrices:

$$\lambda = \begin{pmatrix} 0 & 0 & 0 \\ b(1+\varepsilon) & 0 & c(1+\varepsilon) \\ \frac{m_s}{m_b} b^* & \frac{m_s}{m_b} c^* & d \end{pmatrix}, \quad \lambda^{\rm L} = a\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^{\rm R} = a\begin{pmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Breaks μ-τ symmetry **Neutrino masses** **Muon g-2** **Suppresses $\tau^- \to \mu^-\gamma$**

- **Neutrino masses and flavor mixing**

$$M_\nu = \frac{3\sin 2\theta}{32\pi^2}\ln\left(\frac{M_2^2}{M_1^2}\right) a m_b \begin{pmatrix} 0 & \frac{m_s}{m_b} b & \frac{m_s}{m_b} b^*(1+\varepsilon) \\ \frac{m_s}{m_b} b & 2\frac{m_s}{m_b} c & d \\ \frac{m_s}{m_b} b^*(1+\varepsilon) & d & 2\frac{m_s}{m_b} c^*(1+\varepsilon) \end{pmatrix}$$

> $(M_\nu)_{ee} = 0$ ➡ **NMO**

> $\begin{cases} m_1 \sim 2.2\ \text{meV for}\ (\rho,\sigma)\sim(\pi/2,0) \\ m_1 \sim 6.3\ \text{meV for}\ (\rho,\sigma)\sim(0,\pi/2) \end{cases}$

> $\theta_{23} \sim \frac{\pi}{4} - \frac{\varepsilon}{4}\,,$ $\quad \delta_{\rm CP} \sim \pm\frac{\pi}{2} \pm \frac{\varepsilon\left[1 - r_{12} - (1+r_{12})\cos 2\theta_{12}\right]\cot 2\theta_{12}}{4(1-r_{12})\theta_{13}}$

Approximate μ-τ reflection symmetry ($\varepsilon \ll 1$)

> ${\rm Tr}\left(M_\nu M_\nu^\dagger\right)$ ➡ $r \sim \frac{16\pi^2}{3am_b}\sqrt{\frac{1}{2d^2}\left(3m_1^2 + \Delta m_{21}^2 + \Delta m_{31}^2\right)} \sim 8\times 10^{-7}$ ⬅ $a \sim 0.085\quad d \sim \mathcal{O}(0.01)\quad m_1 \sim \mathcal{O}(0.01\,{\rm eV})$

- **Muon g-2**

$M_1 = M_2/(1+r) = M_3 = 2\,{\rm TeV}$, $\theta = \pi/4$ **with** $r \equiv (M_2 - M_1)/M_1$

$$\Delta a_\mu \simeq \frac{m_\mu m_t a^2}{4\pi^2 M_1^2}{\rm Re}\left(V_{tb} - \frac{V_{cb}V_{ts}}{V_{cs}}\right)\left(\ln\frac{M_1^2}{m_t^2} - \frac{7}{4}\right) \sim 251\times 10^{-11}\left(\frac{a}{0.085}\right)^2$$

⬅ $$x = -\frac{m_t V_{cb}^*\left(7 + \ln\frac{m_t^2}{M_1^2}\right)}{m_c V_{cs}^*\left(7+\ln\frac{m_c^2}{M_1^2}\right)}$$

$a \sim 0.085$ can accommodate the experimental value of Δa_μ

Numerical Calculations

Assuming $M_1 = M_2/(1+r) = M_3 = 2\,\mathrm{TeV}$ **,** $\theta = \pi/4$ **with** $r \equiv (M_2 - M_1)/M_1$

Eight parameters are left: $\{r, a, \mathrm{Re}(b), \mathrm{Im}(b), \mathrm{Re}(c), \mathrm{Im}(c), d, \varepsilon\}$

- **Accommodate the experimental values of** $\{\Delta m^2_{21}, \Delta m^2_{31}, \sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, \Delta a_\mu\}$
- **Predict the values of** $\{m_1, \delta_{\rm CP}, \rho, \sigma\}$
- **Satisfy the constraints from the radiative decays of charged leptons**

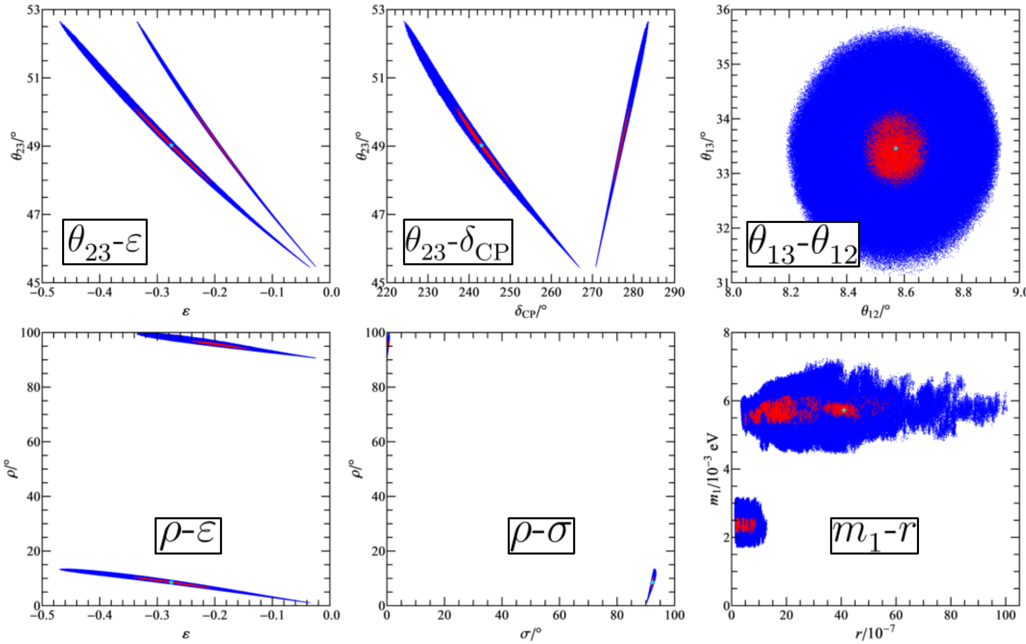
Best-fit values of the model parameters and predicted observables:

$$\boxed{\begin{gathered} a = 8.37\times 10^{-2}\ ,\ d = 1.47\times 10^{-3}\ ,\ \mathrm{Re}(b) = -4.73\times 10^{-4}\ ,\ \mathrm{Im}(b) = 2.46\times 10^{-2} \\ \mathrm{Re}(c) = 2.80\times 10^{-4}\ ,\ \mathrm{Im}(c) = 3.67\times 10^{-2}\ ,\ r = 4.10\times 10^{-6}\ ,\ \varepsilon = -0.276 \end{gathered}}$$

$$\boxed{\begin{gathered} m_1 = 5.73\ \mathrm{meV}\ ,\ \Delta m^2_{21} = 7.42\times 10^{-5}\ \mathrm{eV}^2\ ,\ \Delta m^2_{31} = 2.51\times 10^{-3}\ \mathrm{eV}^2 \\ \theta_{12} = 33.46^\circ\ ,\ \theta_{13} = 8.57^\circ\ ,\ \theta_{23} = 49.03^\circ \\ \delta_{\rm CP} = 242.99^\circ\ ,\ \rho = 8.58^\circ\ ,\ \sigma = 92.27^\circ \\ \Delta a_\mu = 251.10\times 10^{-11}\ ,\ \mathcal{B}(\tau^- \to \mu^- + \gamma) = 1.17\times 10^{-18} \end{gathered}}$$

Successfully give a combined explanation of neutrino masses, lepton flavor mixing and the muon anomalous magnetic moment

Numerical Calculations

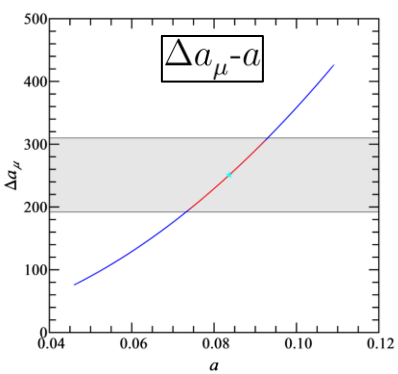


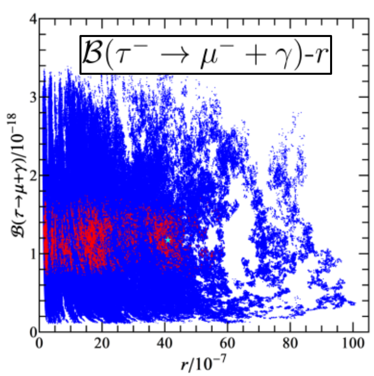
$$\theta_{23} \sim \frac{\pi}{4} - \frac{\varepsilon}{4}$$

$$\delta_{\rm CP} \sim -\frac{\pi}{2} - \frac{\varepsilon\left[1 - r_{12} - (1 + r_{12})\cos 2\theta_{12}\right]\cot 2\theta_{12}}{4(1 - r_{12})\theta_{13}}$$

$$\begin{cases} m_1 \sim 2.2 \text{ meV for } (\rho, \sigma) \sim (\pi/2, 0) \\ m_1 \sim 6.3 \text{ meV for } (\rho, \sigma) \sim (0, \pi/2) \end{cases}$$

$$r \sim \frac{16\pi^2}{3am_{\rm b}}\sqrt{\frac{1}{2d^2}\left(3m_1^2 + \Delta m_{21}^2 + \Delta m_{31}^2\right)}$$





$$\Delta a_\mu \simeq \frac{m_\mu m_{\rm t} a^2}{4\pi^2 M_1^2}{\rm Re}\left(V_{\rm tb} - \frac{V_{\rm cb}V_{\rm ts}}{V_{\rm cs}}\right)\left(\ln\frac{M_1^2}{m_{\rm t}^2} - \frac{7}{4}\right) \sim 251 \times 10^{-11}\left(\frac{a}{0.085}\right)^2$$

$$\mathcal{B}\left(\tau^- \to \mu^- + \gamma\right) \simeq \frac{\alpha_{\rm em} m_\tau^3 m_{\rm t}^2 \left|V_{\rm cb}\right|^2 a^4 r^2}{16\pi^4 M_1^4 \Gamma_\tau}\left(\frac{\ln\frac{m_{\rm t}^2}{m_{\rm c}^2}}{7 + 4\ln\frac{m_{\rm c}^2}{M_1^2}}\right)^2 \sim 1 \times 10^{-6} a^4 r^2$$

See DZ, JHEP 07 (2021) 069 for more details

Summary

- **The LQ-Higgs interaction plays a greatly important role in generating neutrino masses at one-loop level**

- **The mixing between S_1 and $\widetilde{R}_2^{-\frac{1}{3}*}$ has significant effects on the contributions from the LQs to muon g-2**

- **With specific textures for the LQ Yukawa coupling matrices, the neutrino mass matrix possesses an approximate μ-τ reflection symmetry with $(M_\nu)_{ee} = 0$**

- **The present LQ model can successfully explain neutrino masses, lepton flavor mixing and the anomaly of muon g-2**

THANKS FOR YOUR ATTENTION