

# Drell-Yan Rapidity Distribution at Next-to- Next-to-Next-to-Leading Order in QCD

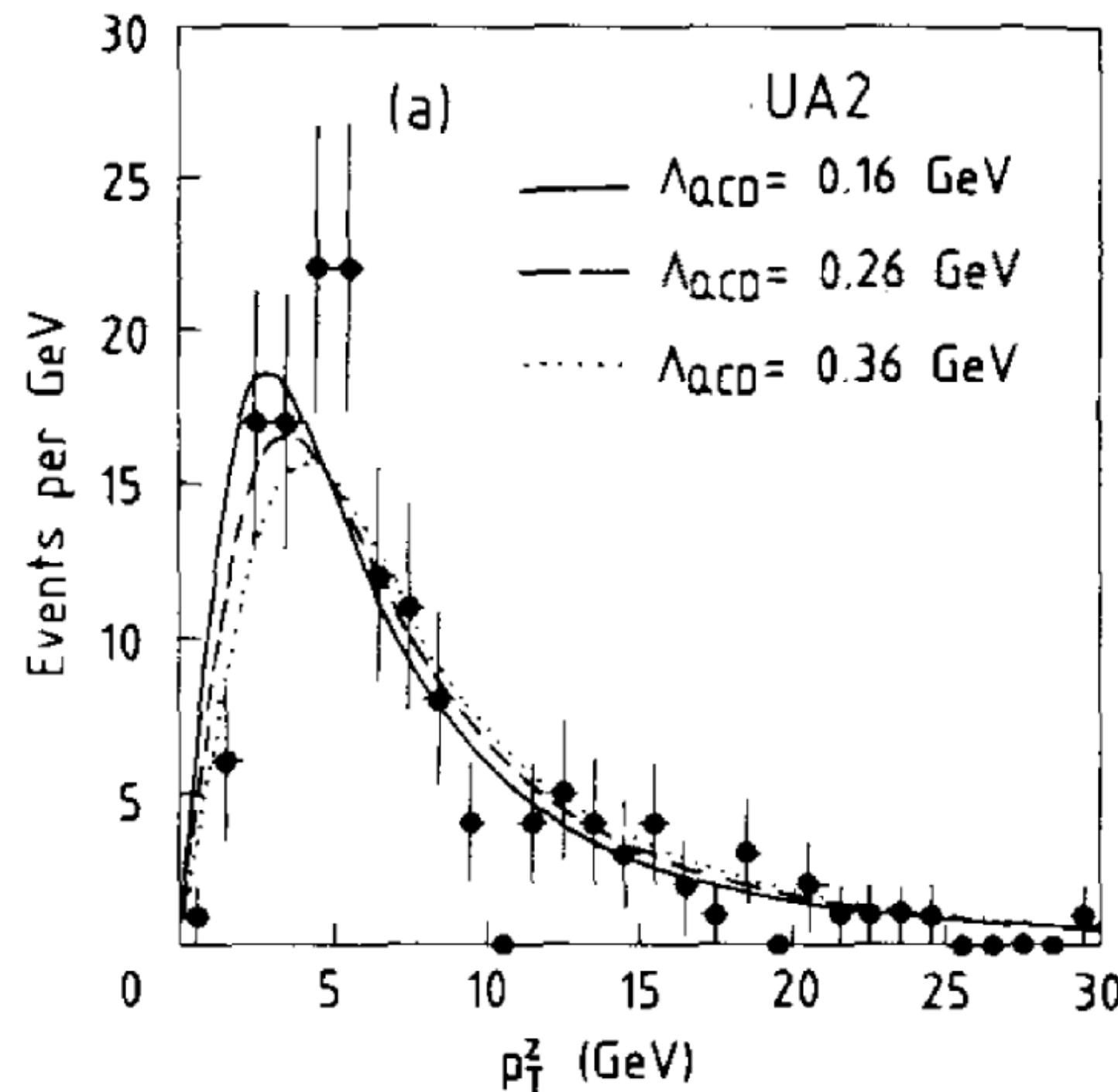
朱华星 (Hua Xing Zhu)  
Zhejiang University



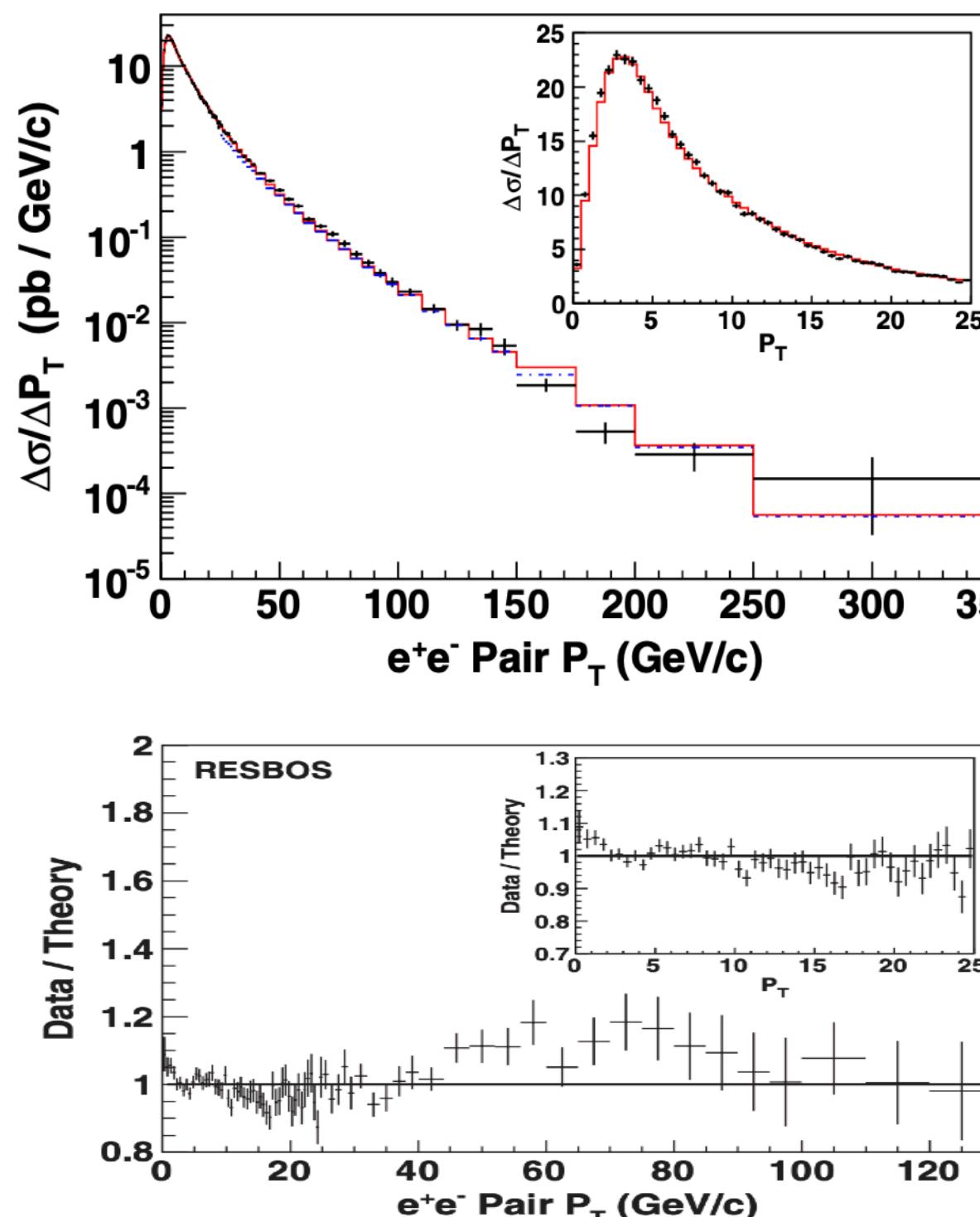
第十三届全国粒子物理学学术会议  
山东青岛，2021年8月16-20日

# The evolution of precision measurement at the hadron colliders

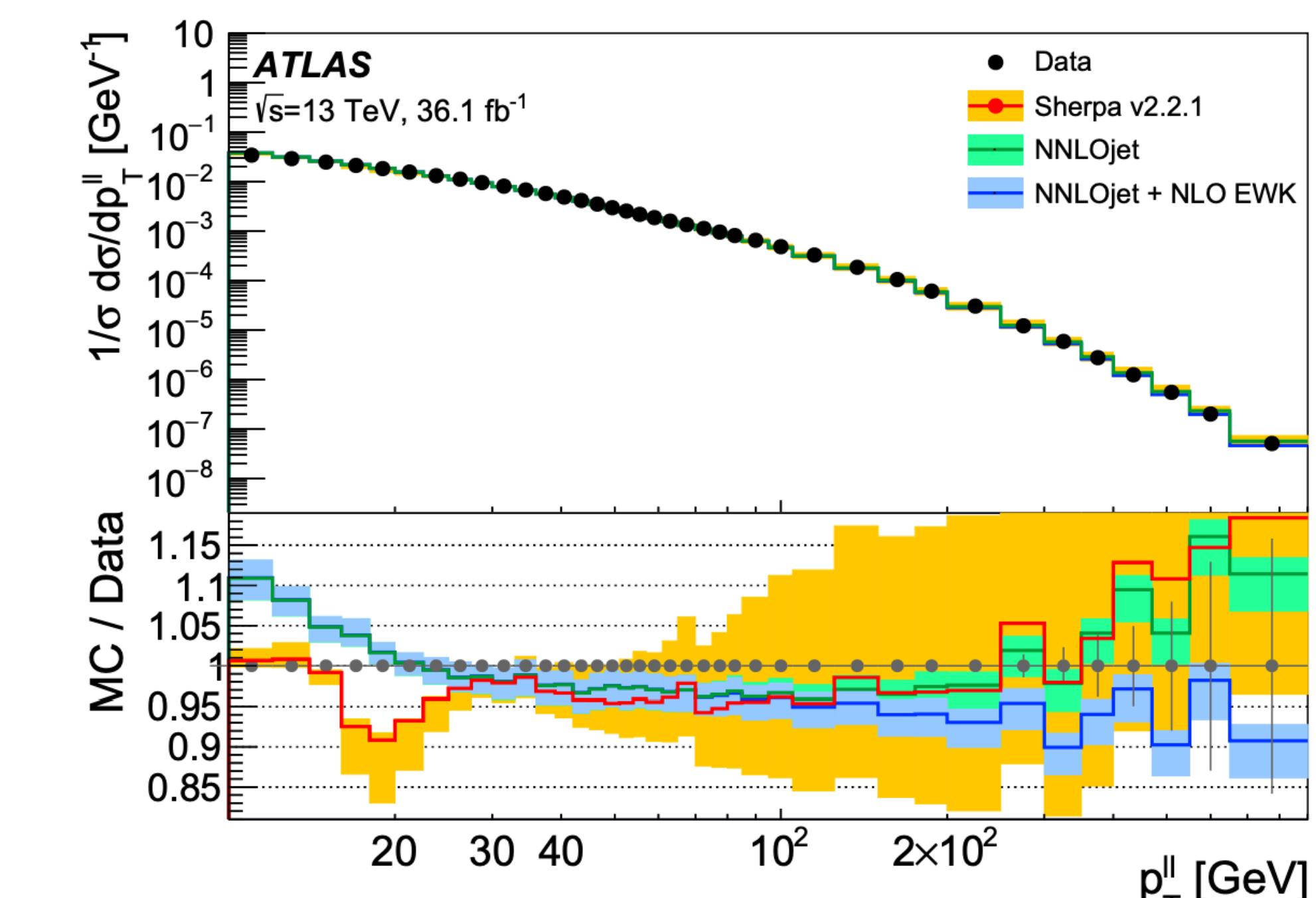
SppS (1981 - 1991)



Tevatron (1983 - 2011)

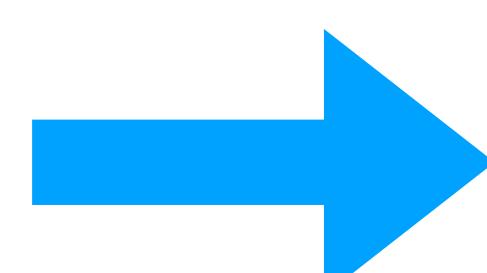


LHC (2011 - )

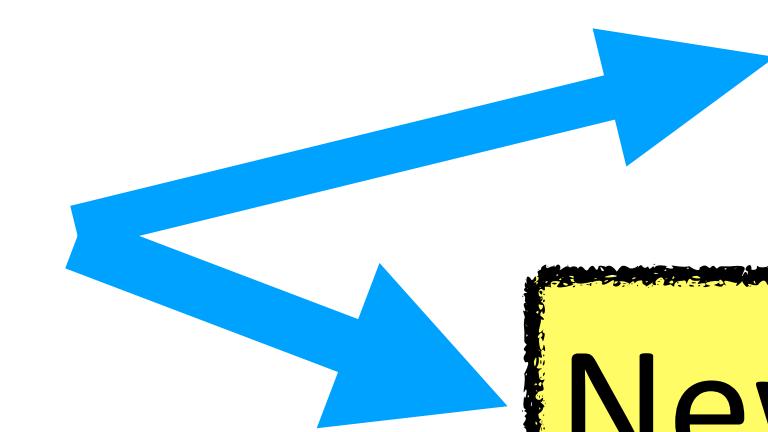


Precision measurement at the hadron colliders from dream to reality

Discovery



Precision



New discovery?

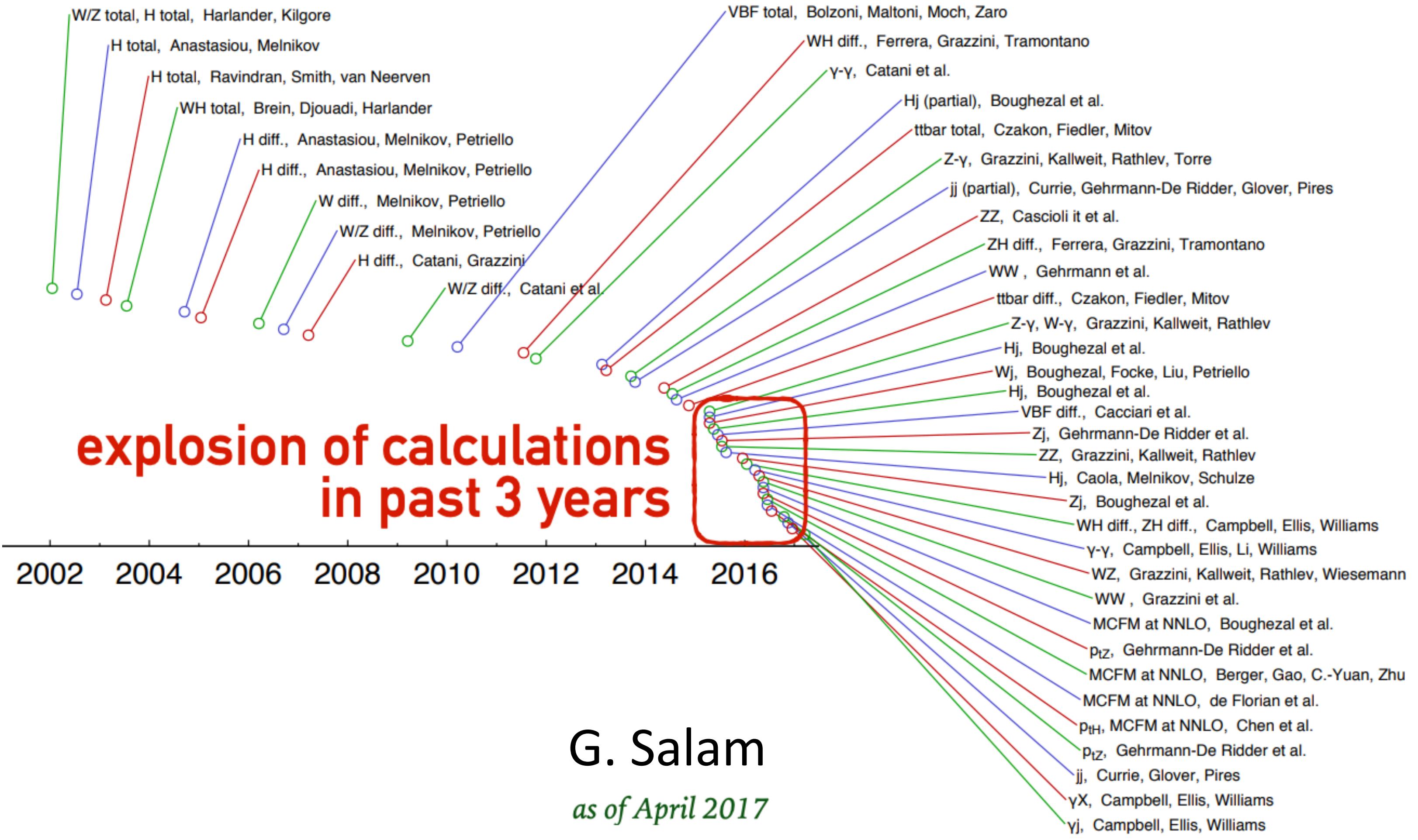
New understanding of QFT

# Continuous progress in precision theory

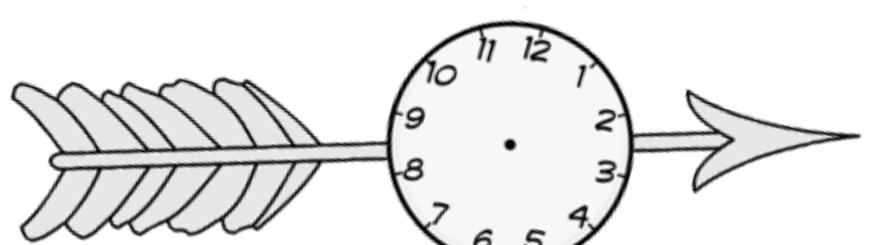
Process ( $V \in \{Z, W, \gamma\}$ )	Comments
Calculations completed since Les Houches 2005	
1. $pp \rightarrow VV$ jet	$WW$ jet completed by Dittmaier/Kallweit/Uwer [4, 5]; Campbell/Ellis/Zanderighi [6].
2. $pp \rightarrow \text{Higgs+2 jets}$	$\gamma$ -jet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7]
	NLO QCD to the $gg$ channel completed by Campbell/Ellis/Zanderighi [8]; NLO QCD+ $W$ to the VBF channel completed by Ciccolini/Denner/Dittmaier [9, 10]
	$ZZZ$ completed by Lazopoulos/Melnikov/Petriello [11] and $WWZ$ by Hankele/Zeppenfeld [12] (see also Binoth/Ossola/Papadopoulos/Pittau [13])
3. $pp \rightarrow VV V$	relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini [14, 15] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [16] calculated by the Blackhat/Sherpa [17] and Rockweller [18] collaborations
4. $pp \rightarrow t\bar{t} b\bar{b}$	
5. $pp \rightarrow V + 3 \text{ jets}$	
Calculations remaining from Les Houches 2005	
6. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek [19]
7. $pp \rightarrow VV b\bar{b}$ , 8. $pp \rightarrow VV + 2 \text{ jets}$	relevant for VBF $\rightarrow H \rightarrow VV$ , $t\bar{t}H$ relevant for VBF $\rightarrow H \rightarrow VV$ VBF contributions calculated by (Bozzi)/Jäger/Oleari/Zeppenfeld [20–22]
NLO calculations added to list in 2005	
9. $pp \rightarrow b\bar{b} b\bar{b}$	$q\bar{q}$ channel calculated by Golem collaboration [23]
NLO calculations added to list in 2009	
10. $pp \rightarrow V + 4 \text{ jets}$	top pair production, various new physics signatures
11. $pp \rightarrow Wbbj$	top, new physics signatures
12. $pp \rightarrow tt\bar{t}$	various new physics signatures
Calculations beyond NLO added in 2007	
13. $gg \rightarrow W^*W^* \mathcal{O}(\alpha^2 \alpha_s^3)$	backgrounds to Higgs
14. NNLO $pp \rightarrow t\bar{t}$	normalization of a benchmark process
15. NNLO to VBF and $Z/\gamma + \text{jet}$	Higgs couplings and SM benchmark
Calculations including electroweak effects	
16. NNLO QCD+NLO EW for $W/Z$	precision calculation of a SM benchmark

Les Houches  
2005 – 2009

Closed



From NLO revolution

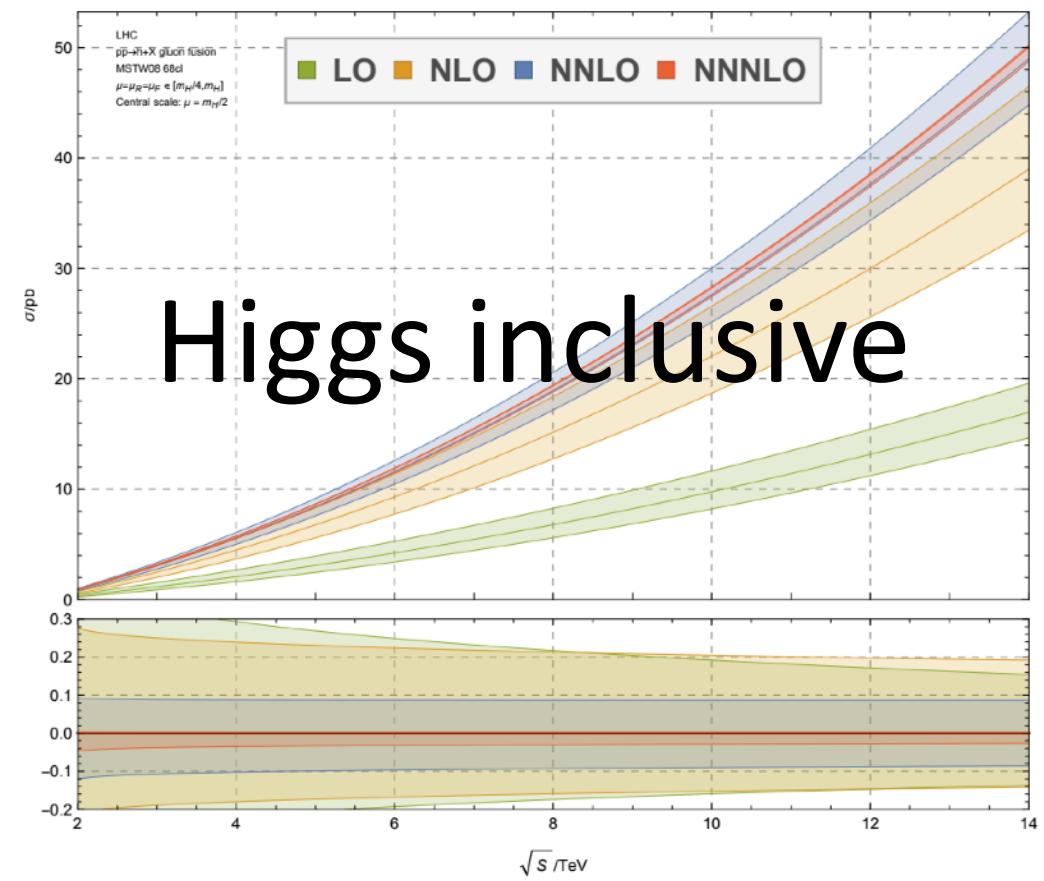


to NNLO revolution

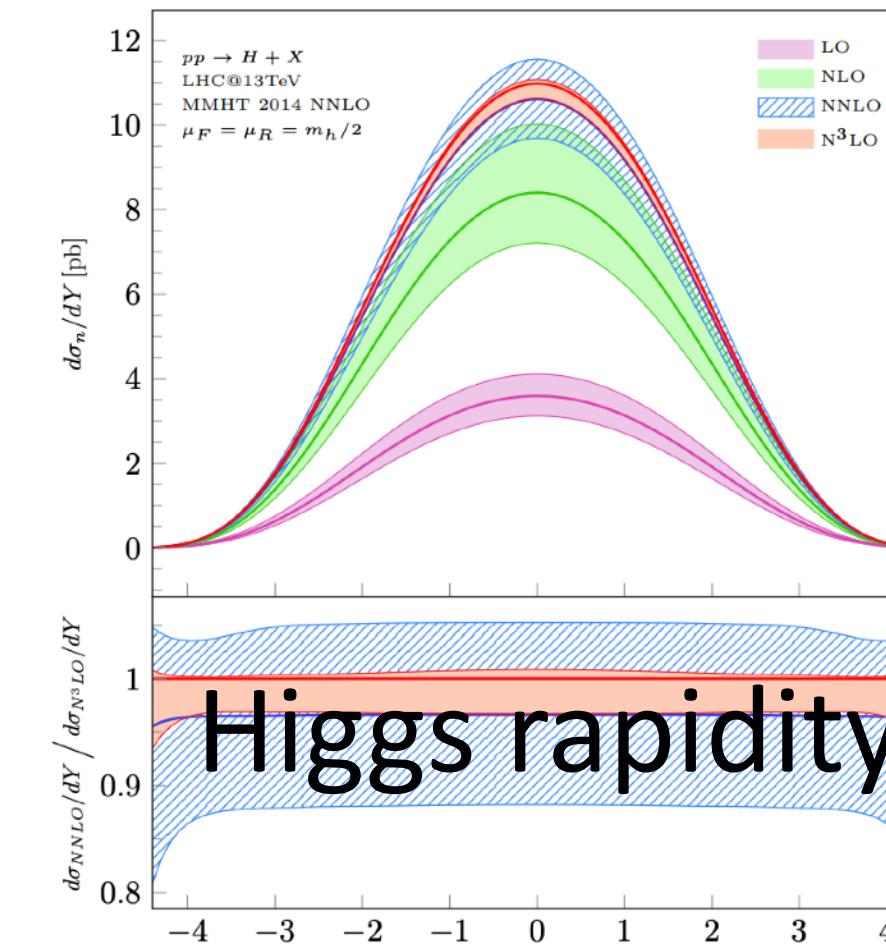
See also Jun Gao's plenary talk

# Precision frontier at N3LO

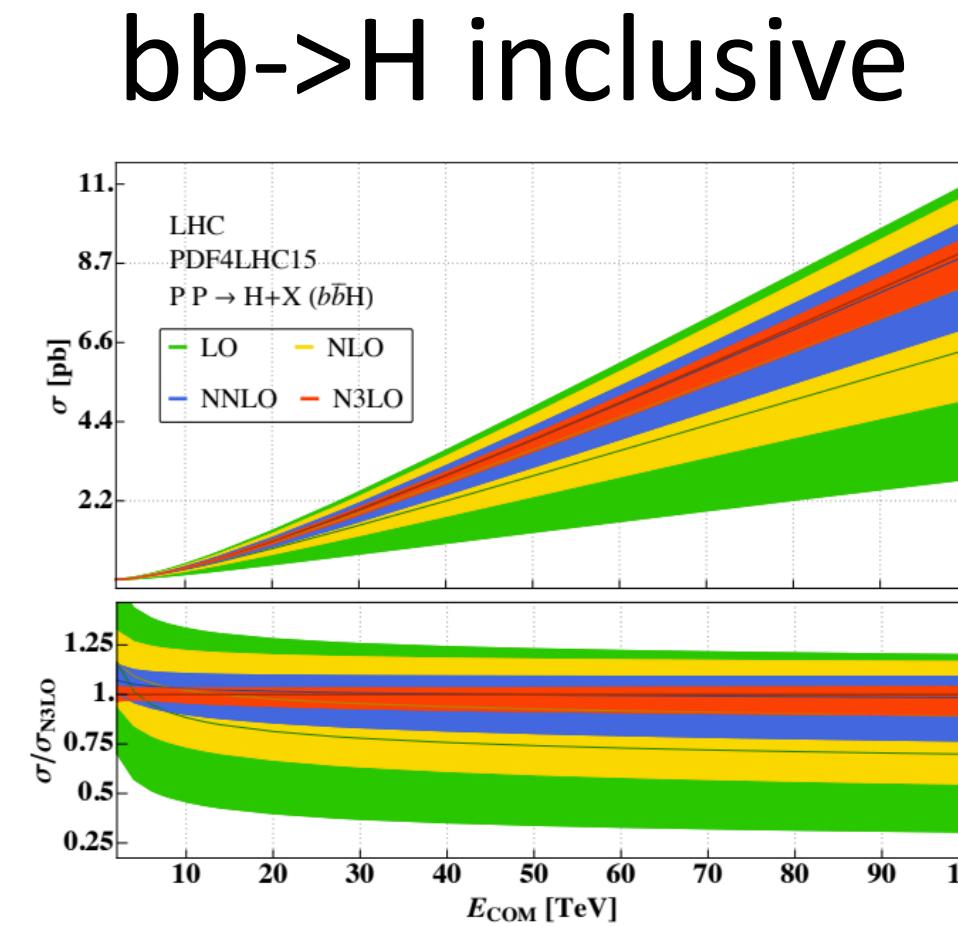
Few processes at hadron colliders are now available at N3LO



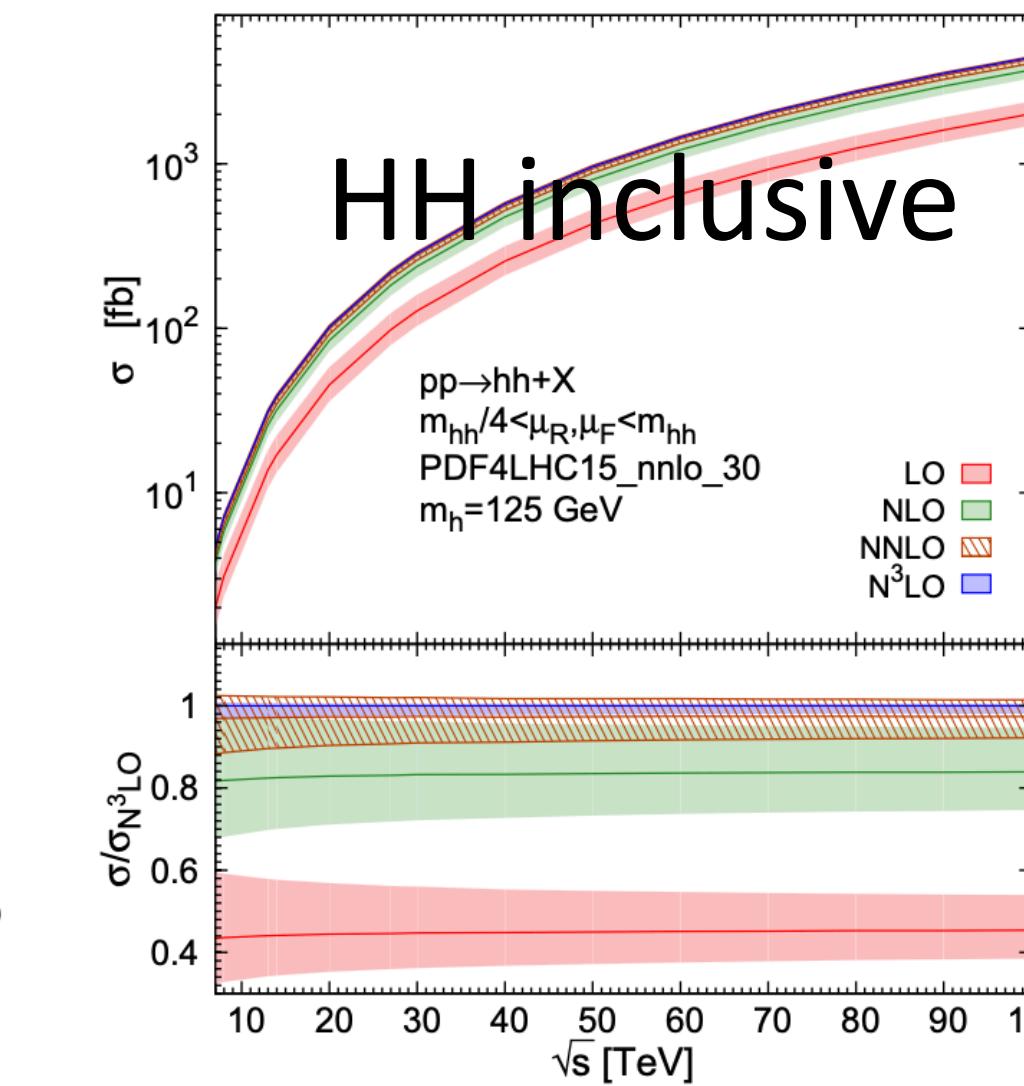
Anastasiou et al.;  
Mistlberger



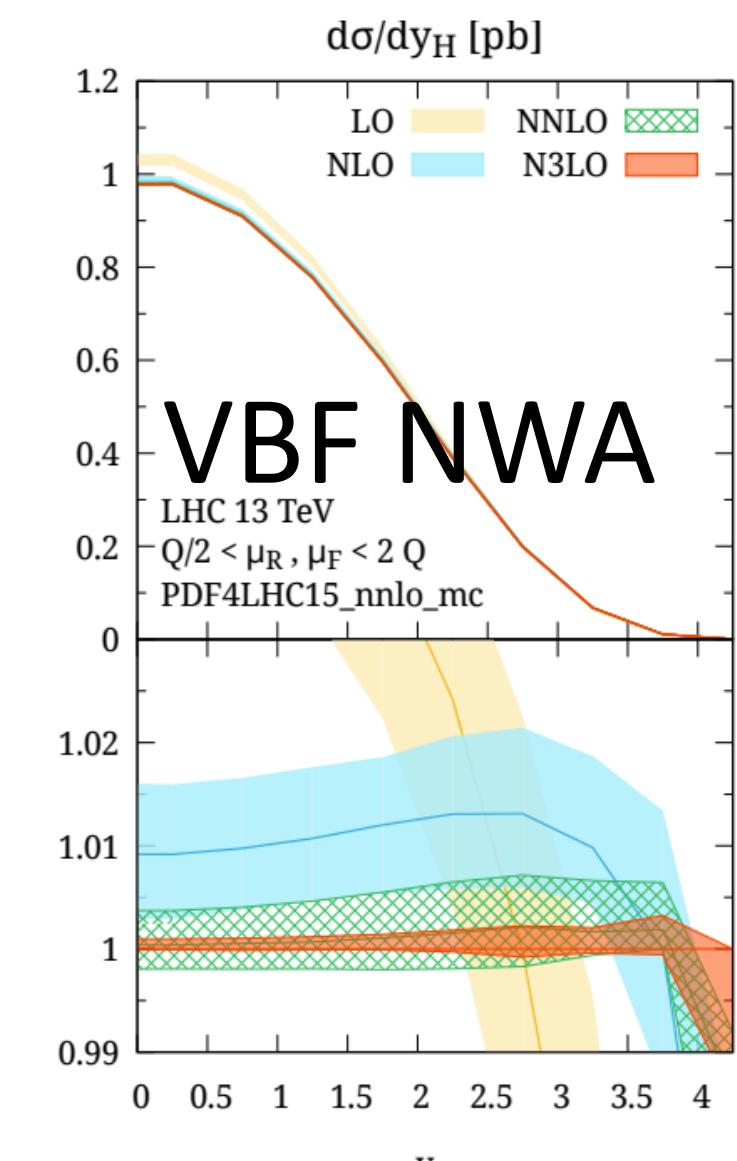
Chen et al.;  
Dulat, Mistlberger, Pelloni



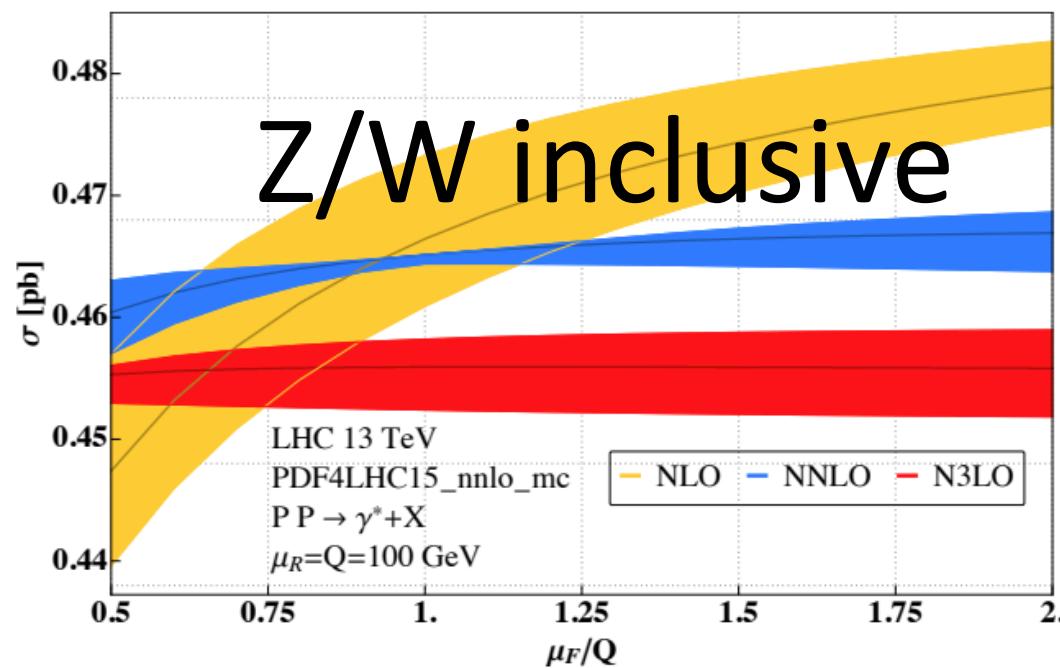
Duhr, Dulat, Mistlberger



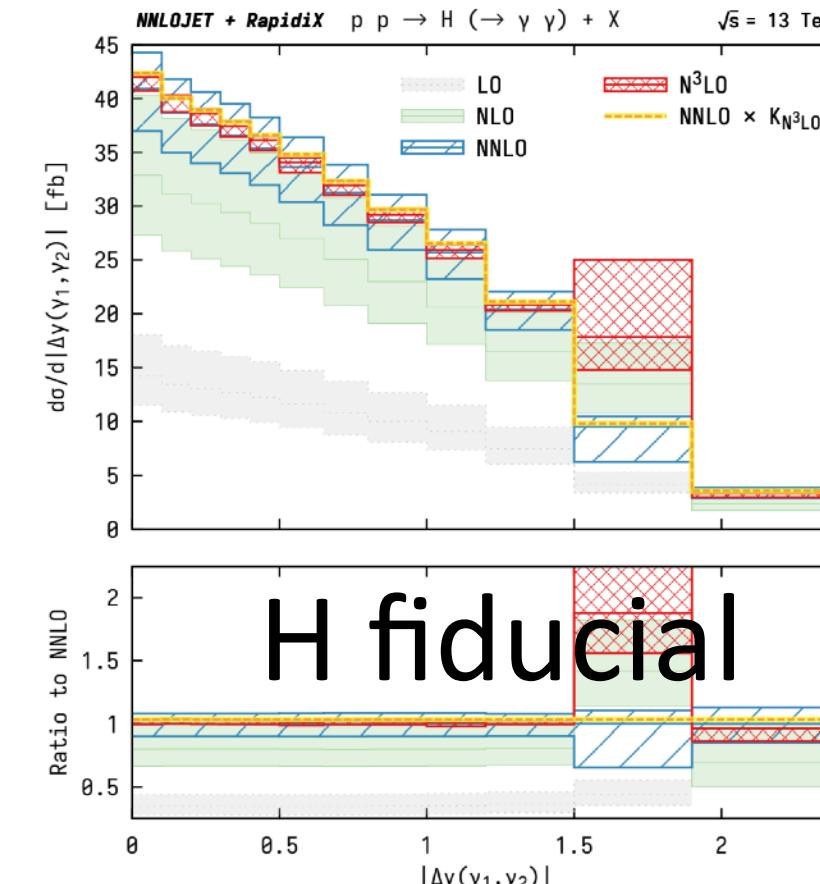
Chen, Li, Shao, Wang



Dreyer, Karlberg



Duhr, Dulat, Mistlberger



Chen et al.

- All available results based on analytic N3LO coefficient functions
- Difficult to apply to more complex processes

# Di-lepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD

Xuan Chen,<sup>1, 2, 3,\*</sup> Thomas Gehrmann,<sup>1,†</sup> Nigel Glover,<sup>4,‡</sup>  
Alexander Huss,<sup>5,§</sup> Tong-Zhi Yang,<sup>1,¶</sup> and Hua Xing Zhu<sup>6,\*\*</sup>

arXiv: 2107.09085

## First N3LO corrections to Drell-Yan rapidity distribution

### Method generalizable to more complex processes, e.g. diboson production

based upon earlier works:

3-loop soft function (Y. Li, HXZ, 2016), 3-loop TMD PDFs (MX Luo, TZ Yang, HXZ, YJ Zhu, 2019), NNLOJet (Gehrmann, Glover, et al.)

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## Standard Model theory: highlights

First  $2 \rightarrow 3$  NNLO results are becoming available. N3LO for some  $2 \rightarrow 2$  processes within reach

**pp  $\rightarrow$   $\gamma\gamma$  at NNLO**  
Precision probes of QCD dynamics  
[Chawdhry, Czakon, Mitov, Poncelet '21]

**pp  $\rightarrow$  jjj at NNLO**  
"Tour de force in Quantum Chromodynamics"  
[Czakon, Mitov, Poncelet '21]

**DY@N3LO: differential**  
N3LO not covered by the NNLO band  
[SCT+NNLOJET, pp to  $\gamma^*$ , LHC 13 TeV]  
Ratio to NNLO vs  $|y_\gamma|$

Towards @ N3LO  $2 \rightarrow 2$   
Recent progress on virtual 3 loop integrals and 3 loop amplitudes  
Very new ideas: finite field arithmetic, algebraic geometry for special functions, intersection theory, multi-variate partial fractioning, new flexible semi-numerical methods...

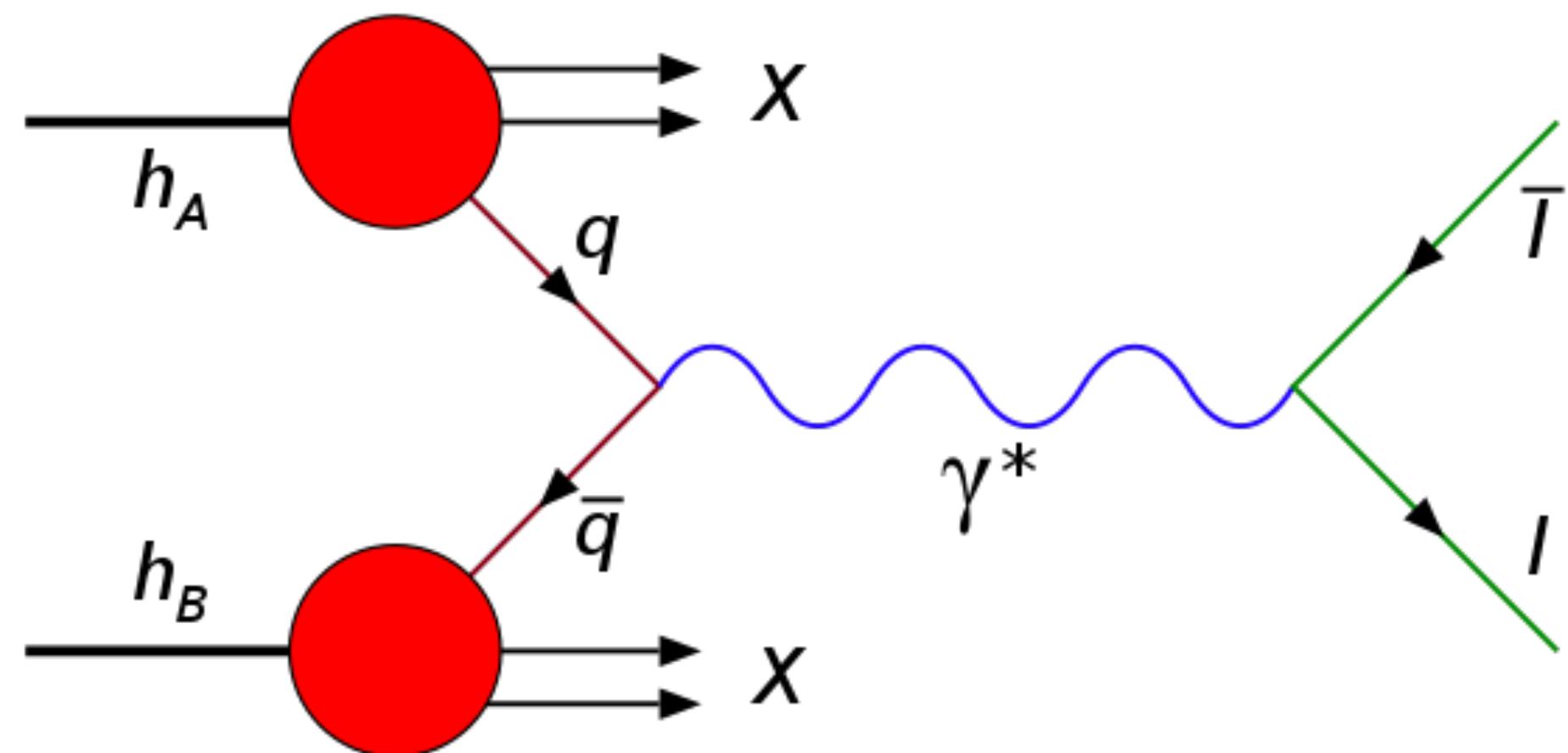
Steven Weinberg (1933 - 2021) one of most important contributors to the construction of the Standard Model

Florencia Canelli

Participants 179 Chat Share Screen Live Transcript Reactions Unmute Start Video Leave

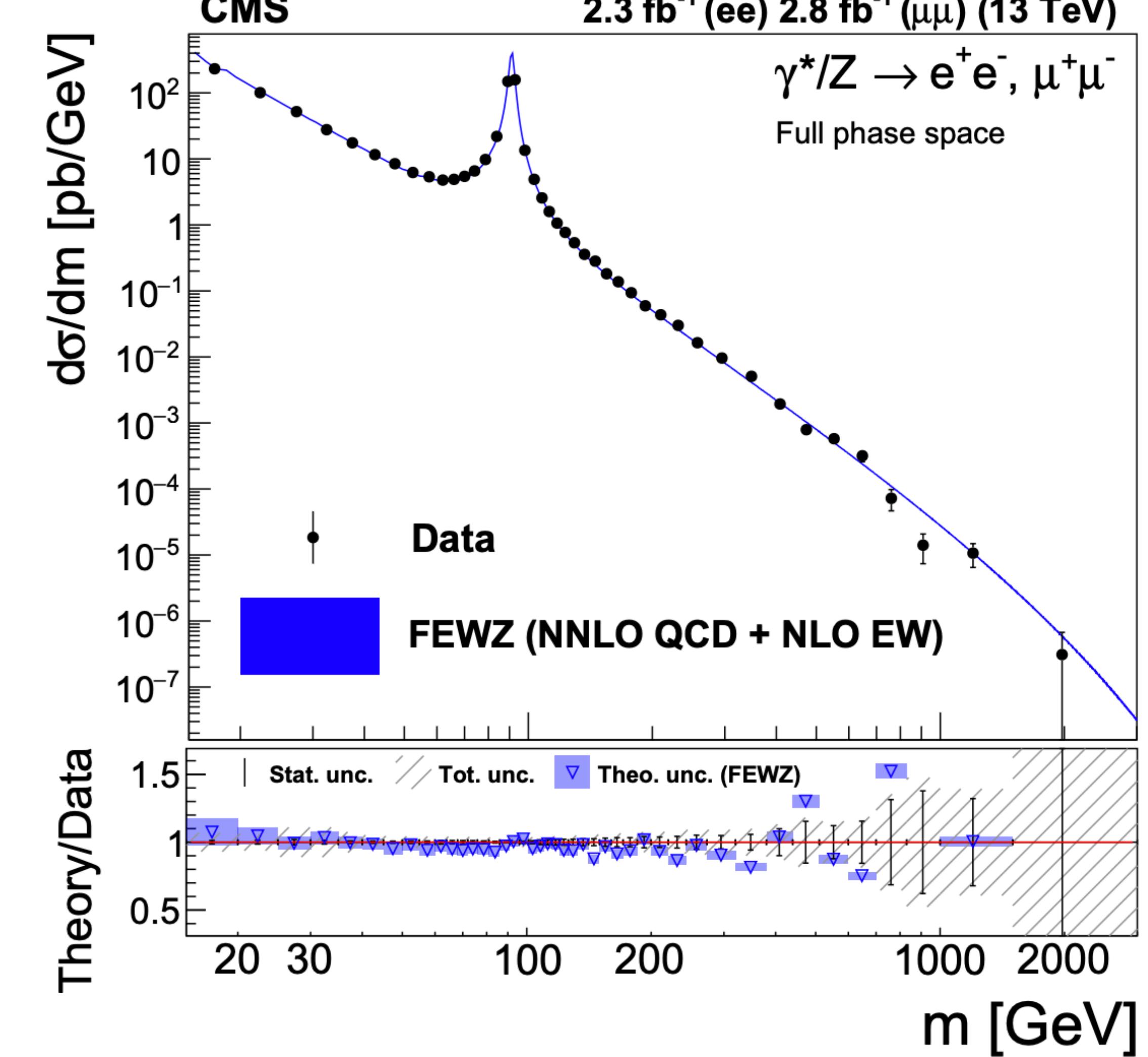
Highlighted in the summary talk at EPS 2021

# The Drell-Yan process



The “Standard Candle” process at hadron collider. Important for:

- PDFs global fit
- Beam calibration
- SM EW parameter determination
- TMD physics
- BSM search
- Dominant uncertainties from luminosity.  
Normalized cross section < 1% uncertainty!



# Drell-Yan as the laboratory for cutting-edge tech.

X. Chen, Gehrmann, Glover, Huss, T.Z.  
Yang, HXZ, 2021

N3LO rapidity

First hadron collider  
fudicial NNLO

Catani, Grazzini, 2007

Anastasiou, Dixon, Melnikov, Petriello, 2004  
NNLO rapidity

First hadron collider NNLO calculation

Altarelli, Ellis, Martinelli, 1978  
NLO

Duhr, Dulat, Mistlberger, 2020  
Inclusive N3LO

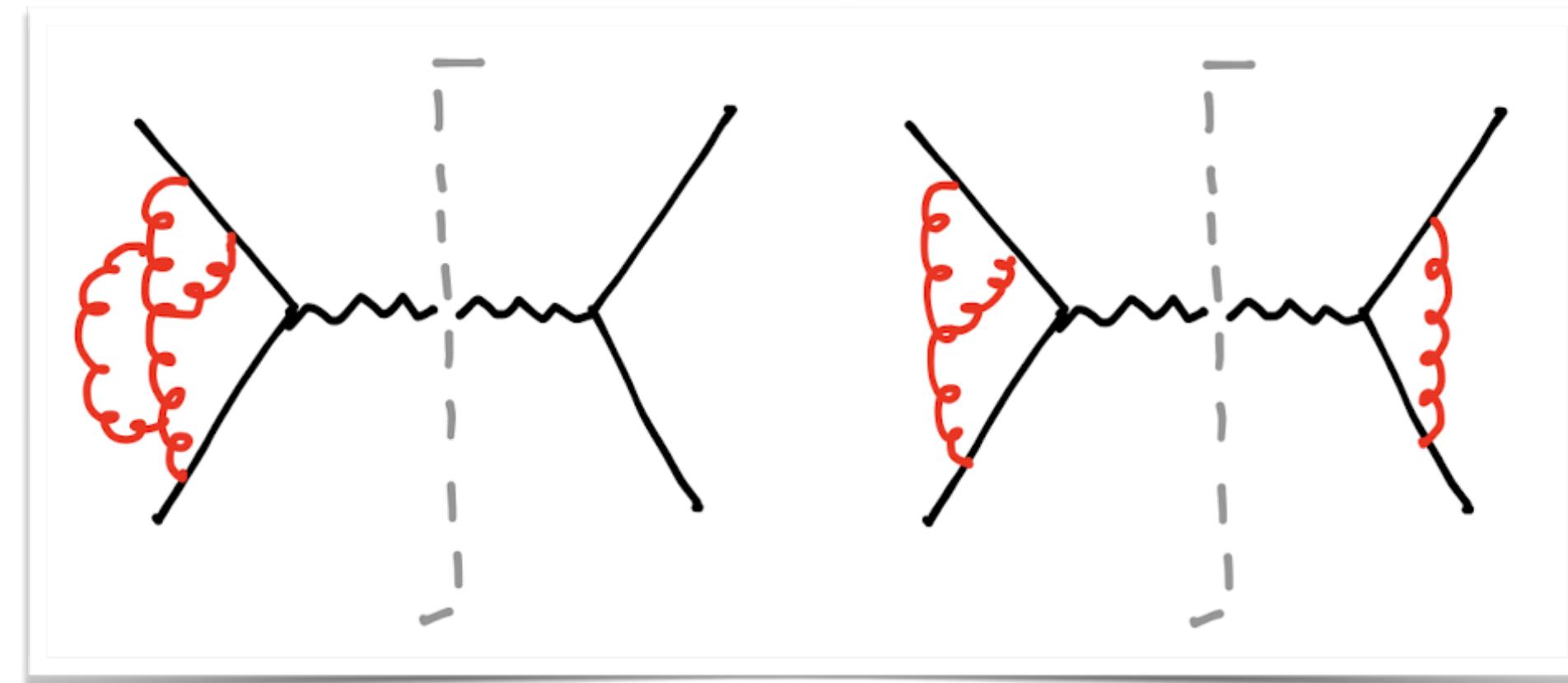
Melnikov, Petriello, 2006  
Full differential NNLO

First hadron collider differential NNLO

Hamberg, van Neerven, Matsuura, 1991  
Inclusive NNLO

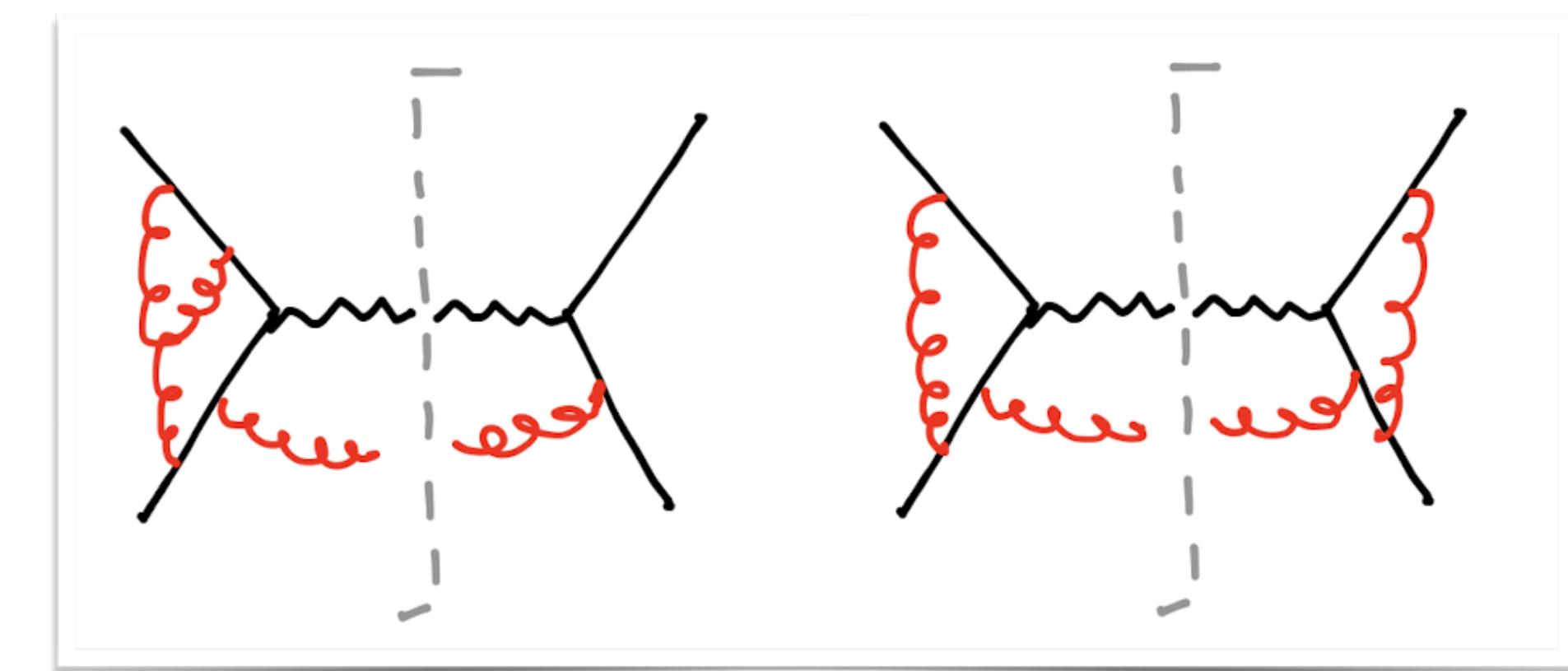
First hadron collider NLO calculation

# Anatomy for QCD corrections at N3LO



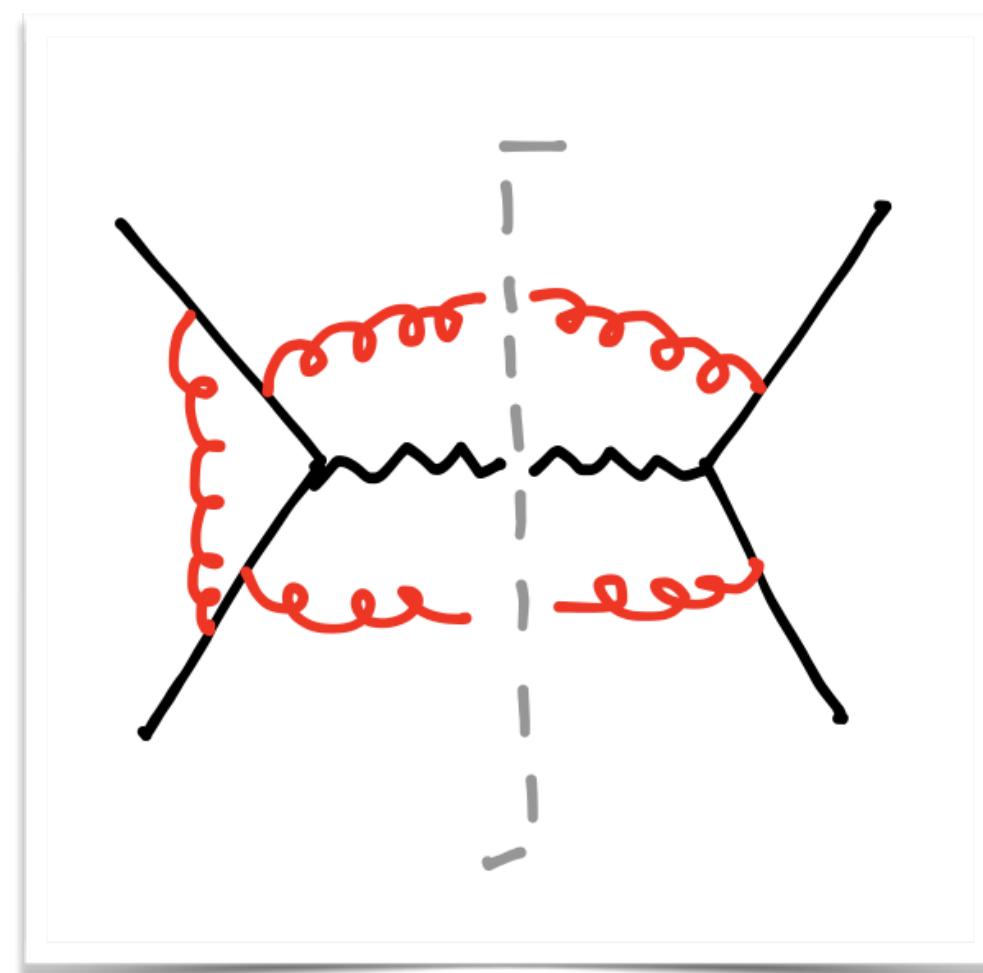
Pure virtual

$$\frac{1}{\epsilon^6}$$



double-virtual real

$$\int \text{PS}_1 \frac{1}{\epsilon^4} \sim \frac{1}{\epsilon^6}$$



Virtual-real-real

$$\int \text{PS}_2 \frac{1}{\epsilon^2} \sim \frac{1}{\epsilon^6}$$

Triple real

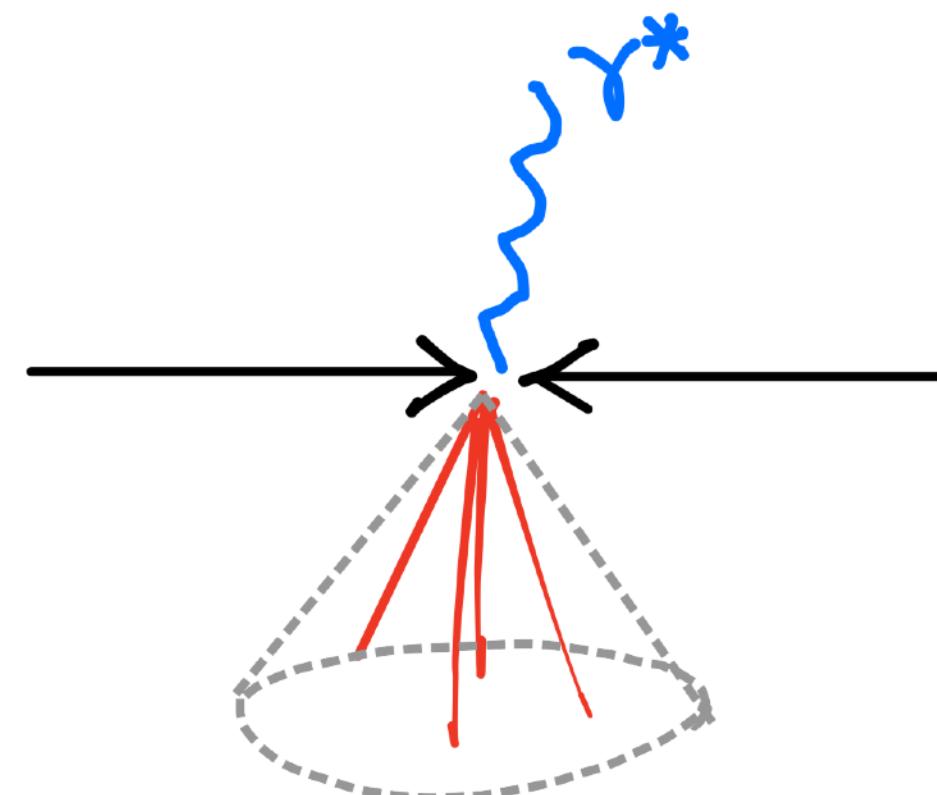
$$\int \text{PS}_3 \sim \frac{1}{\epsilon^6}$$

- Virtual diagrams known since 2009  
(Baikov et al., 2009; Gehrmann et al., 2010)
- Main challenging: cancelling of infrared singularities from phase space integrals and loop integrals

# A NNLO solution: qT subtraction/slicing method

Catani, Grazzini, 2007

Cluster all QCD partons into a single jet



$$\vec{p}_T^{\gamma^*} = -\vec{p}_T^{\text{jet}}$$

The most singular infrared region resides at  $p_T^\gamma = 0$

$$\frac{d^2\sigma_{\gamma^*}}{dQ^2dy} = \int_0^{q_T^{\text{cut}}} d^2\mathbf{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\mathbf{q}_T dQ^2 dy} + \int_{q_T^{\text{cut}}} d^2\mathbf{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\mathbf{q}_T dQ^2 dy}$$

Contains the most singular region

largely universal

Degree of divergence reduce by one order

process dependent

$$d\sigma(< p_T^{\text{cut}}) = d\sigma(\text{tot}) - d\sigma(> p_T^{\text{cut}})$$

Known to N3LO: NNLOJet/MCFM

Hard to get the  $< p_T^{\text{cut}}$  contribution beyond NNLO!

# Effective Field Theory assisted Slicing method

$$d\sigma(< p_T^{\text{cut}}) = d\sigma(\text{tot}) - d\sigma(> p_T^{\text{cut}})$$

Admit a factorization description

Can be directly calculated in Soft-Collinear Effective Theory

- First application of EFT idea to NNLO fixed-order calculation: top-quark decay@NNLO (J. Gao, C.S. Li, HXZ, 2012)
- Can be generalized to other processes:
  - ttbar production at hadron colliders (C.S. Li, H.T. Li, D.Y. Shao, L.L. Yang, HXZ, 2012-2013; Catani, Grazzini, Torre, 2014; Catani, Devoto, Grazzini, Kallweit, Mazzitelli, 2019)
  - N-jettiness subtraction (Boughezal, Focke, X.H. Liu, Petriello, 2015; Gaunt, Stahlhofen, Tackmann, Walsh, 2015)

$$d\sigma(< p_T^{\text{cut}}) \simeq HB \otimes B \otimes S + \mathcal{O}\left(\frac{(p_T^{\text{cut}})^2}{Q^2}\right)$$



TMD beam function



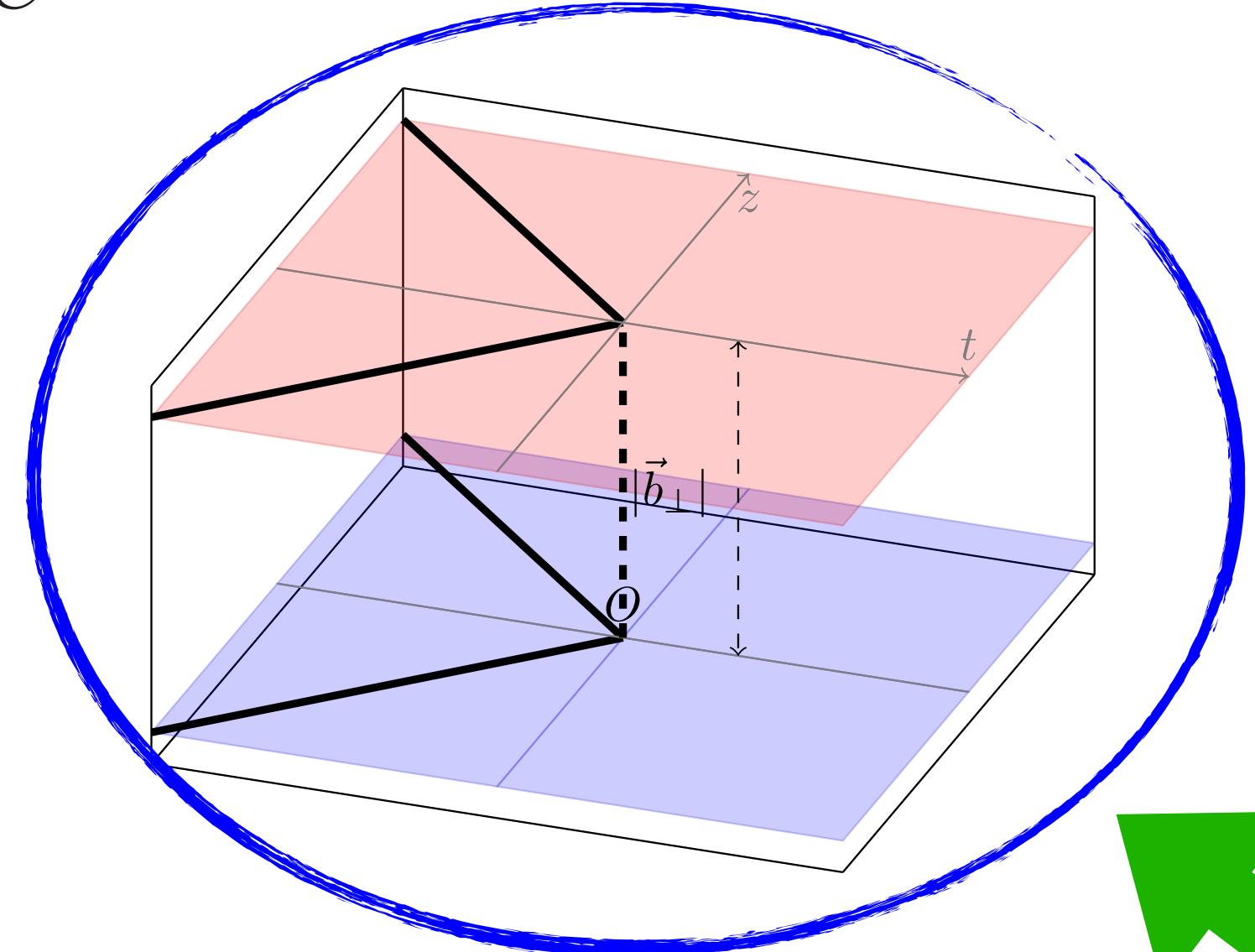
TMD soft function

Main challenge:

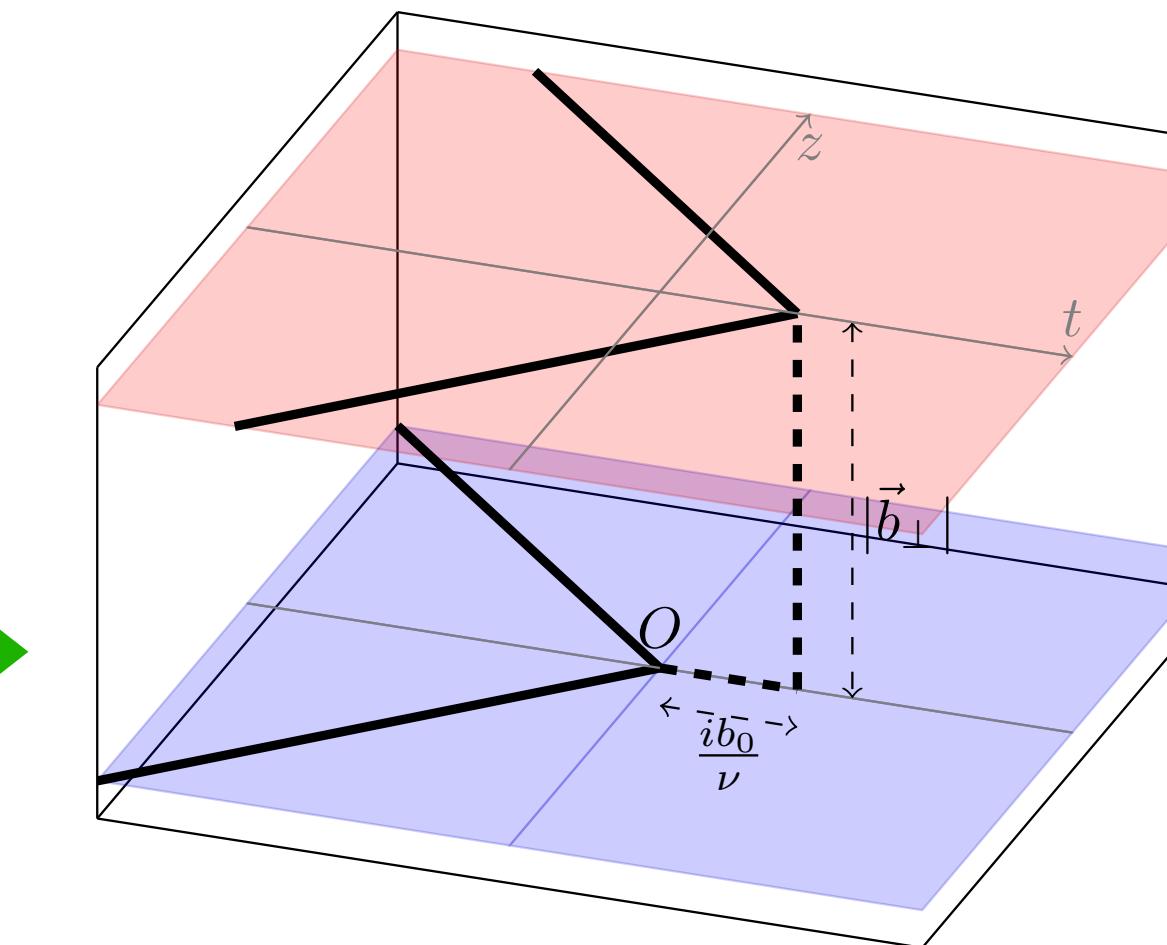
- N3LO effective theory phase space integrals
- Rapidity divergence not regularized by dimensional regularization

# TMD soft function at N3LO

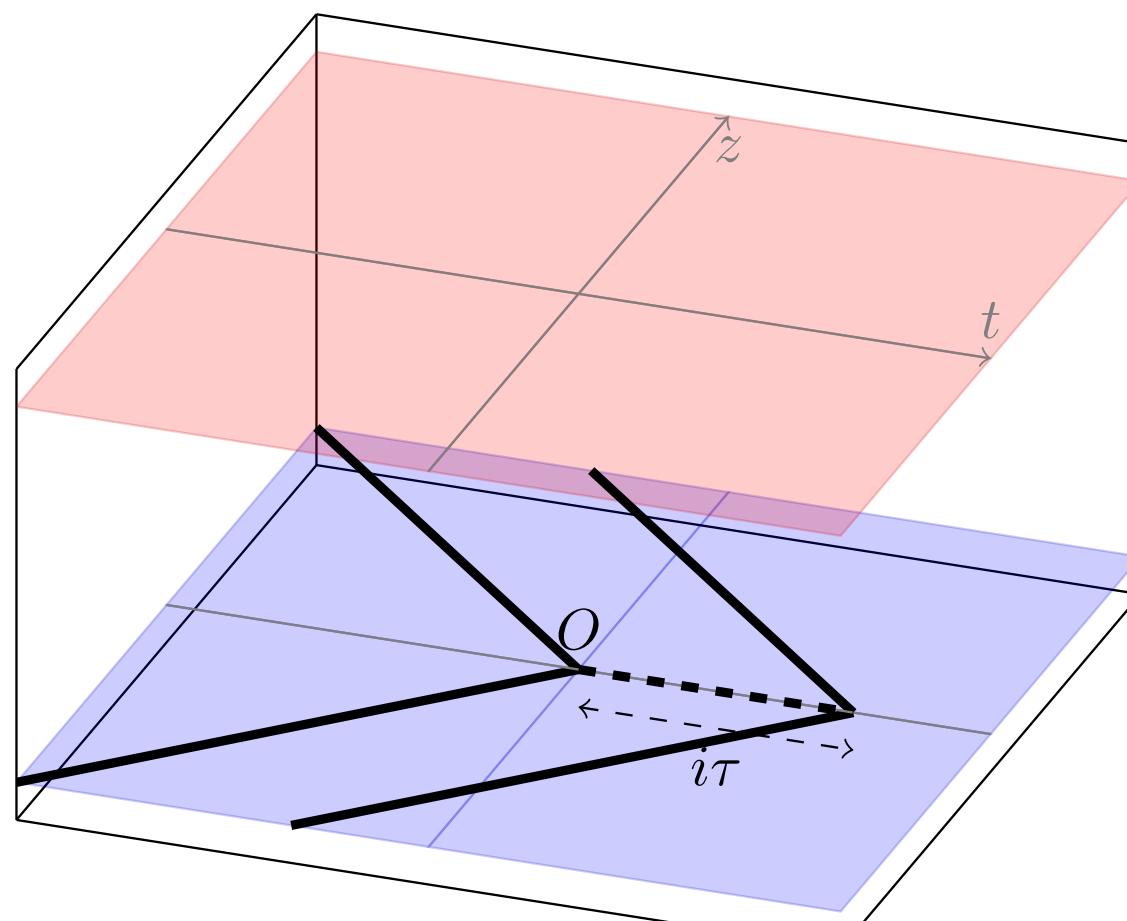
$$S_{\perp} = \frac{\text{Tr}}{C} \langle 0 | T\{S_{\bar{n}}^{\dagger} S_n(0,0,0)\} \bar{T}\{S_n^{\dagger} S_{\bar{n}}(0,0,\vec{b}_{\perp})\} | 0 \rangle$$



Exponential regularization



Analytic ansatz from a space of HPLs



Bootstrap from threshold limit

$$\begin{aligned} & \mathbb{N} c^3 \left( \frac{1}{90} \pi^4 c_{23} H_{0,1}[x] + c_{21} \text{Zeta}[3] H_{0,0,1}[x] + c_{22} \text{Zeta}[3] H_{0,1,1}[x] + \right. \\ & \frac{1}{6} \pi^2 c_{17} H_{0,0,0,1}[x] + \frac{1}{6} \pi^2 c_{18} H_{0,0,1,1}[x] + \frac{1}{6} \pi^2 c_{19} H_{0,1,0,1}[x] + \\ & \frac{1}{6} \pi^2 c_{20} H_{0,1,1,1}[x] + c_1 H_{0,0,0,0,0,1}[x] + c_2 H_{0,0,0,0,1,1}[x] + \\ & c_3 H_{0,0,0,1,0,1}[x] + c_4 H_{0,0,0,1,1,1}[x] + c_5 H_{0,0,1,0,0,1}[x] + c_6 H_{0,0,1,0,1,1}[x] + \\ & c_7 H_{0,0,1,1,0,1}[x] + c_8 H_{0,0,1,1,1,1}[x] + c_9 H_{0,1,0,0,0,1}[x] + c_{10} H_{0,1,0,0,1,1}[x] + \\ & c_{11} H_{0,1,0,1,0,1}[x] + c_{12} H_{0,1,0,1,1,1}[x] + c_{13} H_{0,1,1,0,0,1}[x] + \\ & \left. c_{14} H_{0,1,1,0,1,1}[x] + c_{15} H_{0,1,1,1,0,1}[x] + c_{16} H_{0,1,1,1,1,1}[x] \right) + \dots \end{aligned}$$

Anastasiou, Duhr,  
Dulat, Mistlberger;  
Y. Li, HXZ;  
Duhr, Gehrmann;  
Y. Li, von  
Manteuffel,  
Scharbinger, HXZ;  
HXZ

Y. Li, HXZ, 1604.01404

See also recent lattice calculation:

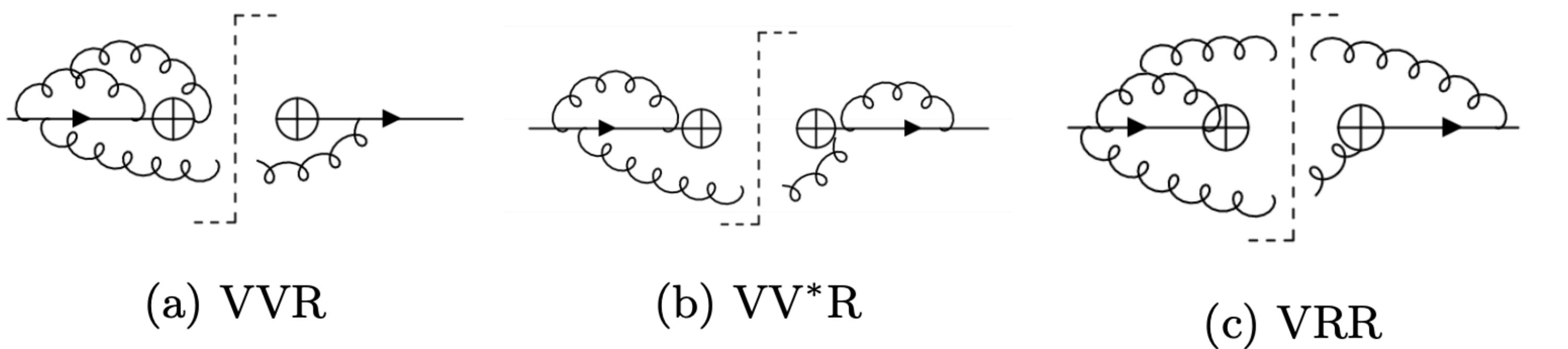
Q.A. Zhang, X.D. Ji, P. Sun, W. Wang, Y.B. Yang et al., 2005.14572

Y. Li, X. Feng, C. Liu, et al., 2106.13027

# TMD Beam function at N3LO

M.X. Luo, T.Z. Yang, HXZ, Y.J. Zhu, 1912.05778  
see also Ebert, Mistlberger, Vita, 2006.05329

$$\mathcal{B}_{q/N}^{\text{bare}}(x, b_\perp) = \int \frac{db_-}{2\pi} e^{-ixb^- P^+} \langle q(P) | \bar{\chi}_n(0, b^-, b_\perp) \sum_{X_n} | X_n \rangle \langle X_n | \frac{\bar{n} \cdot \gamma}{2} \chi_n(0) | q(P) \rangle$$



Main complexity:  
rapidity regularization

$$\int \frac{d^3 p_i}{2E_i} \exp(-\tau b_0 E_i)$$

$$z = x$$

$$J_i(z, \tau, \epsilon) \stackrel{\tau \rightarrow 0}{=} \sum_j \sum_n \sum_{k=0} J_i^{(j,n,k)}(z, \epsilon) \tau^{j+n\epsilon} \ln^k \tau$$



$$\begin{aligned} \frac{\partial \vec{J}}{\partial \tau} &= A(\tau, z) \vec{J}, \\ \frac{\partial \vec{J}}{\partial z} &= B(\tau, z) \vec{J}, \end{aligned}$$



$$\frac{\partial J_i^{(j,n,k)}}{\partial z} = \sum_{a,b,c,d} C(z, \epsilon)_{ijnk,abcd} J_a^{(b,c,d)}$$

$$\begin{aligned} \text{Left side: } & \frac{\partial J_i^{(j,n,k)}}{\partial z} = \dots \\ \text{Right side: } & \sum_{a,b,c,d} C(z, \epsilon)_{ijnk,abcd} J_a^{(b,c,d)} = \dots \end{aligned}$$

+ 8 more pages

# Merging EFT and full theory calculation

$$\frac{d^2\sigma_{\gamma^*}}{dQ^2dy} = \int_0^{q_T^{\text{cut}}} d^2\mathbf{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\mathbf{q}_T dQ^2 dy} + \int_{q_T^{\text{cut}}} d^2\mathbf{q}_T \frac{d^4\sigma_{\gamma^*}}{d^2\mathbf{q}_T dQ^2 dy}$$

The Feynman diagram illustrates the process  $S \rightarrow B_i/A + B_i/B$ . A red circle labeled  $S$  emits a wavy line labeled  $H$ , which then splits into two green ovals labeled  $B_{i/A}$  and  $B_{i/B}$ . The incoming particles are labeled  $i$  and  $\bar{i}$ .

SCET + power corrections



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*****
* NNLOJET: A multiprocess parton level event generator at O(alpha_s^3)
* X.Chen, J.Cruz-Martinez, J.Currie, R.Gauld, A.Gehrmann-De Ridder,
* T.Gehrmann, N.Glover, M.Hofer, A.Huss, M.Jaquier, I.Majer,
* M.Marcoli, J.Mo, T.Morgan, J.Niehues, J.Pires, R.Schurmann,
* G.Stagnitto, D.Walker, S.Wells, J.Whitehead
* ****
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In the following I will present the results for  $\gamma^*$  only.

Since QCD corrections are largely insensitive to underlying EW vertex, results are a good reference for  $Z/W^\pm$ .

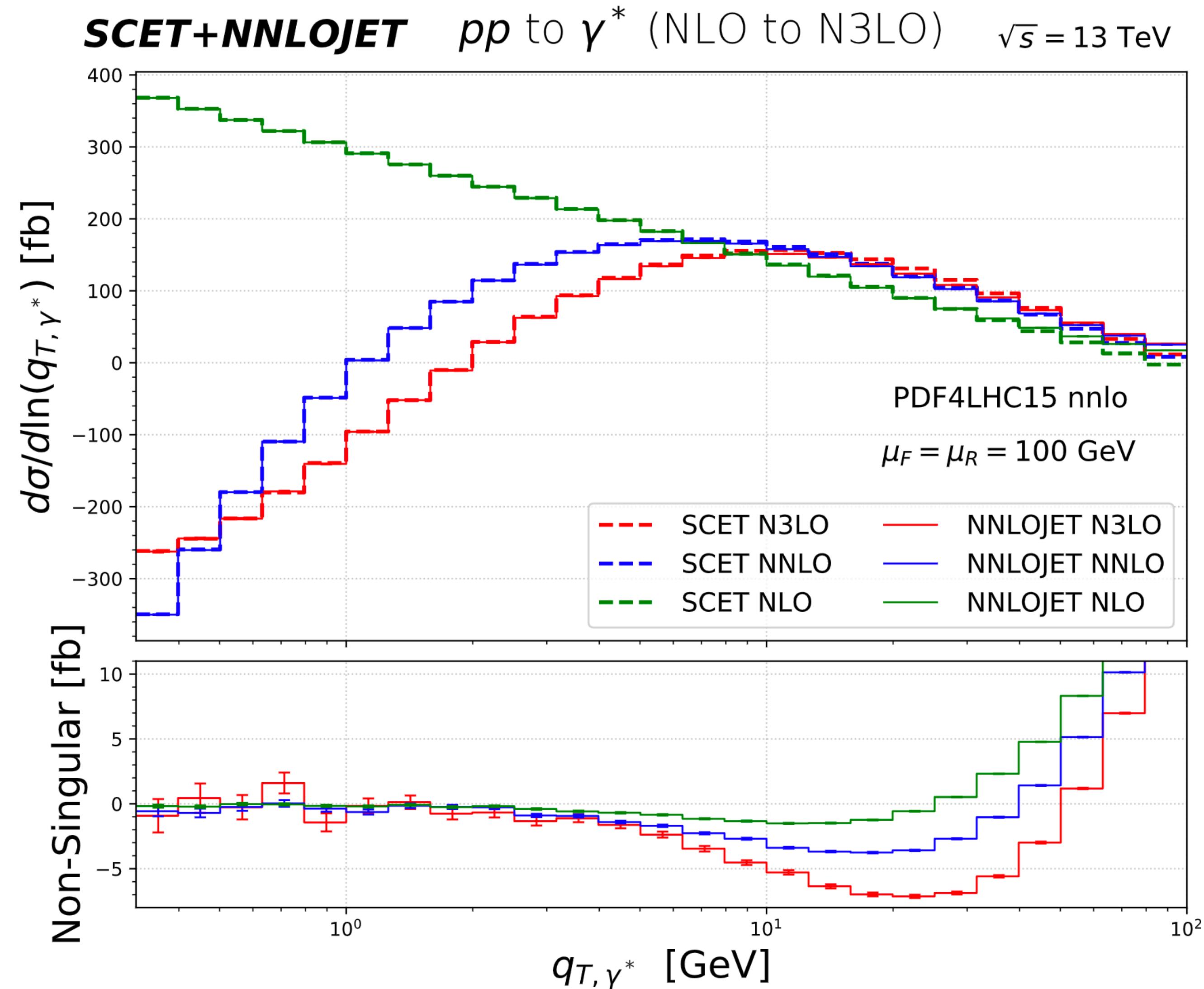
# Validating qT factorization

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, HXZ, 2107.09085

Factorization prediction from SCET  
valid at small pT region

The difference between full QCD and  
SCET (non-singular) should vanishes  
at small pT

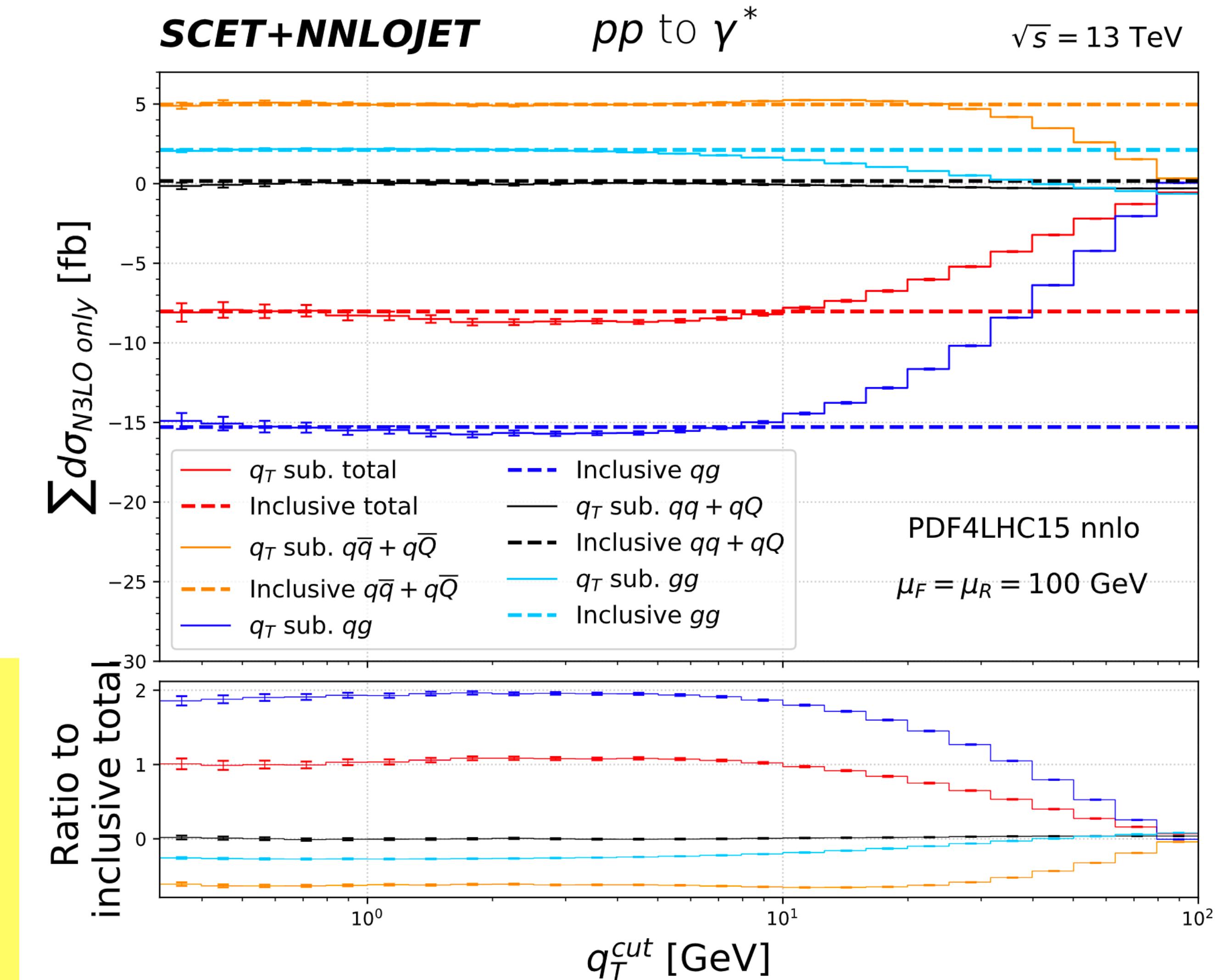
Non-trivial confirmation of the  
validity of perturbative factorization  
up to NNNLO



# N3LO calculation DY from qT subtraction

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, HXZ, 2107.09085

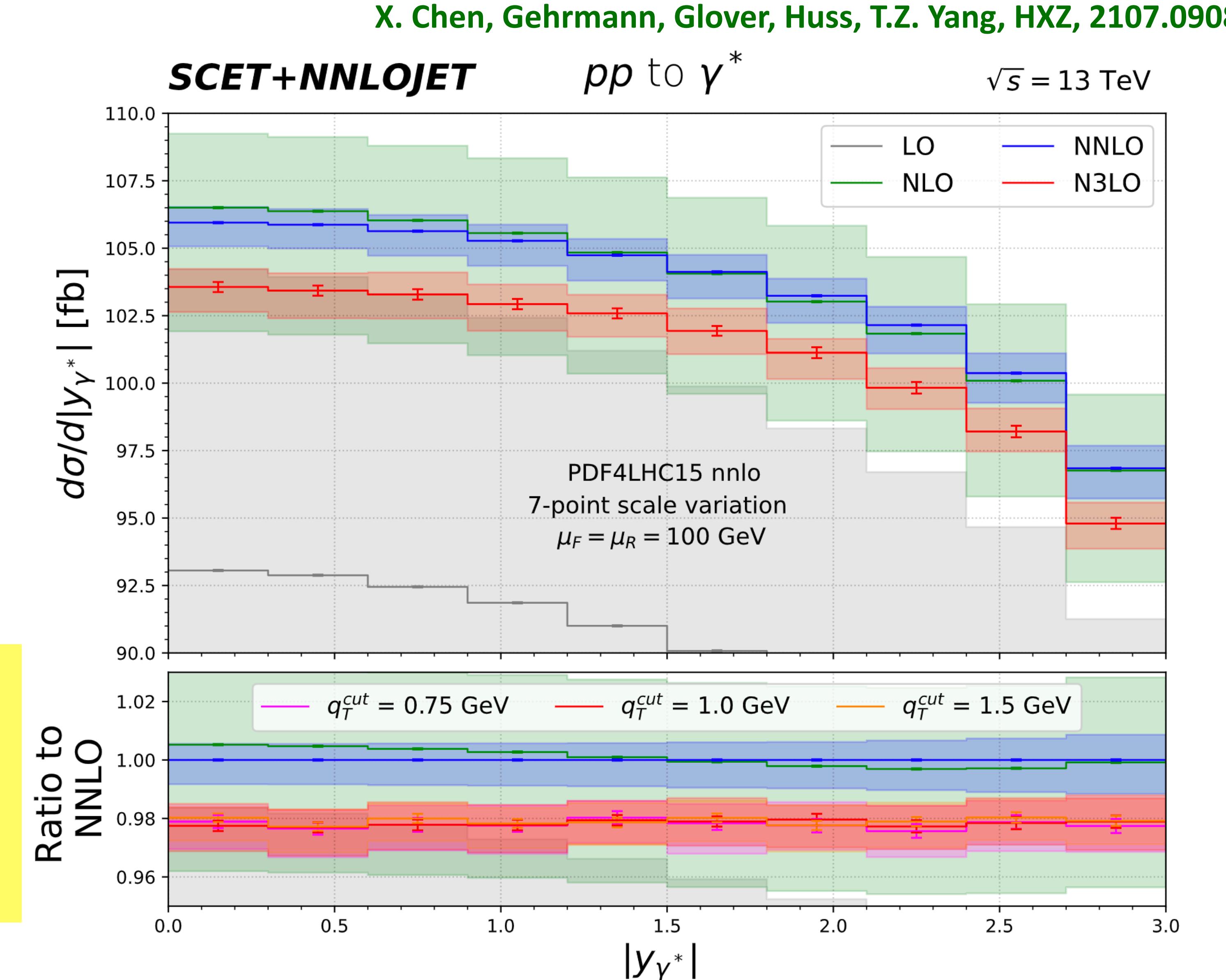
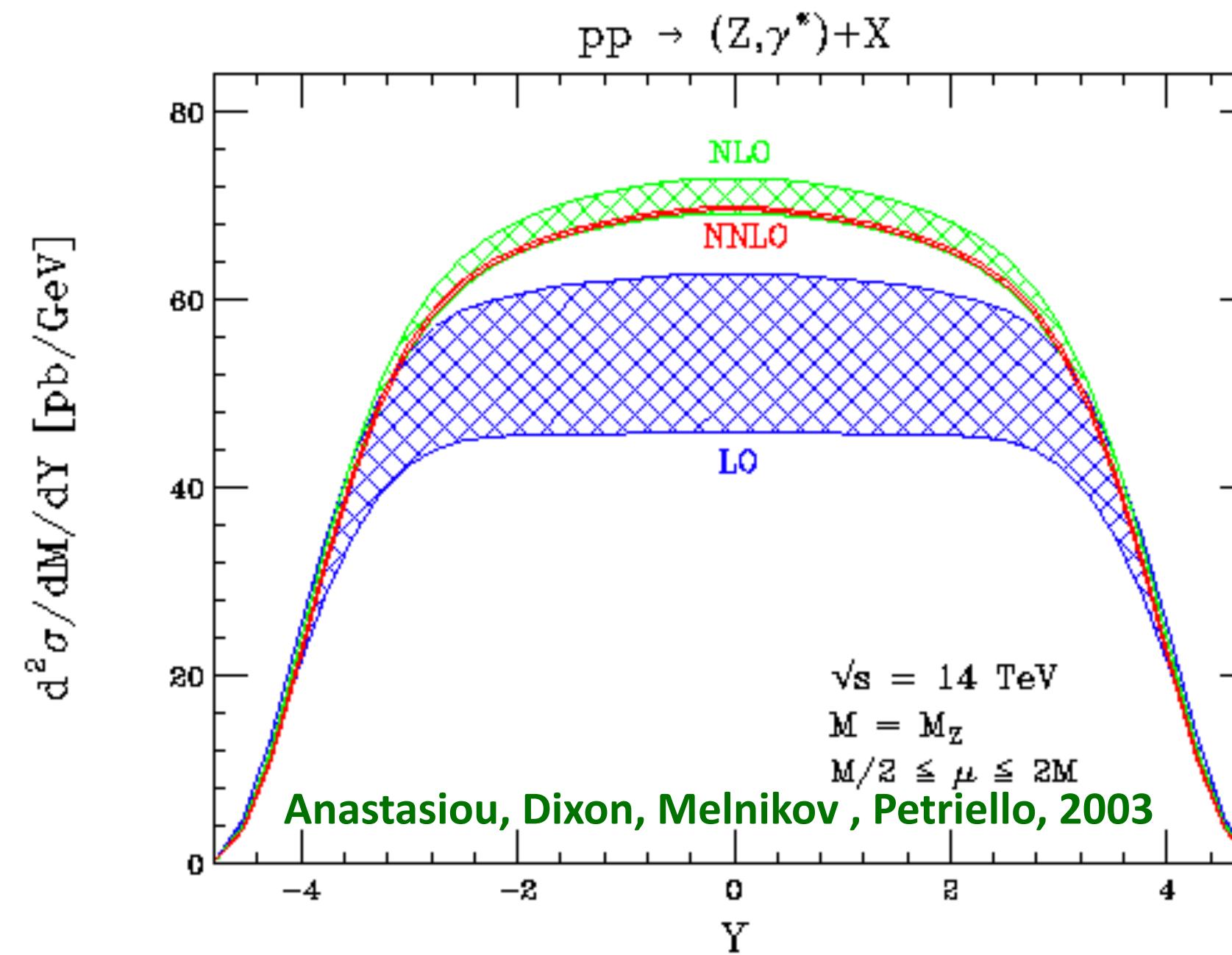
Fixed Order	$\sigma_{pp \rightarrow \gamma^*} (\text{fb})$	
LO		$339.62^{+34.06}_{-37.48}$
NLO		$391.25^{+10.84}_{-16.62}$
NNLO		$390.09^{+3.06}_{-4.11}$
N <sup>3</sup> LO		$382.08^{+2.64}_{-3.09}$ from [14]
N <sup>3</sup> LO only	$q_T$ -subtraction	Results from [14]
$qg$	$-15.32(32)$	$-15.29$
$q\bar{q} + q\bar{Q}$	$+5.08(11)$	$+4.97$
$gg$	$+2.17(6)$	$+2.12$
$qq + qQ$	$+0.09(13)$	$+0.17$
Total	$-7.98(36)$	$-8.03$



- Accidentally small NNLO corrections due to large cancellation between qq and qg channel
- N3LO corrections comparable with NNLO ones, can't be ignored!

# First results on N3LO differential distribution

X. Chen, Gehrmann, Glover, Huss, T.Z. Yang, HXZ, 2107.09085



- Large N3LO corrections to  $\gamma^*$  distribution compared with NNLO
- Scale uncertainty band do not overlap between NNLO and N3LO!

# Summary and outlook

- Remarkable progress in precision program at the LHC from both experiment and theory sides
- Many phenomenology important processes have been computed to NNLO.  
Automation?
- A few landmark processes have been achieved N3LO accuracy. This talk: Drell-Yan rapidity distribution at N3LO
- Methods can be applied to more complex processes, such as diboson production
- New opportunities: precision SM phenomenology, precision QCD studies at N3LO