The QCD Calculation for hadronic B decays

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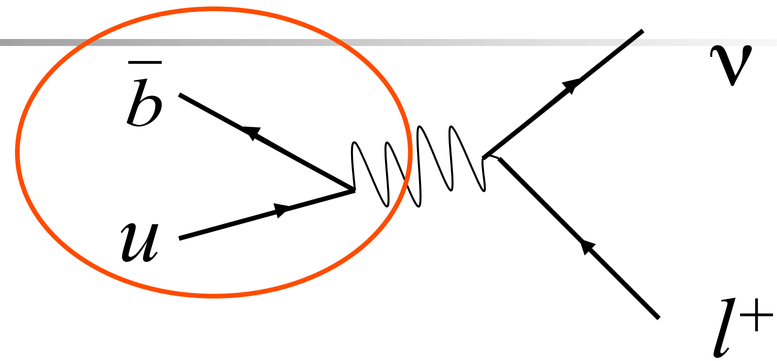
Pure leptonic decays

$$\langle P(p)|\bar{q}\gamma^{\mu}Lq'|0\rangle = if_P p^{\mu}.$$



- The decay constant is

the normalization of the meson

wave function i.e. the zero point of wave function

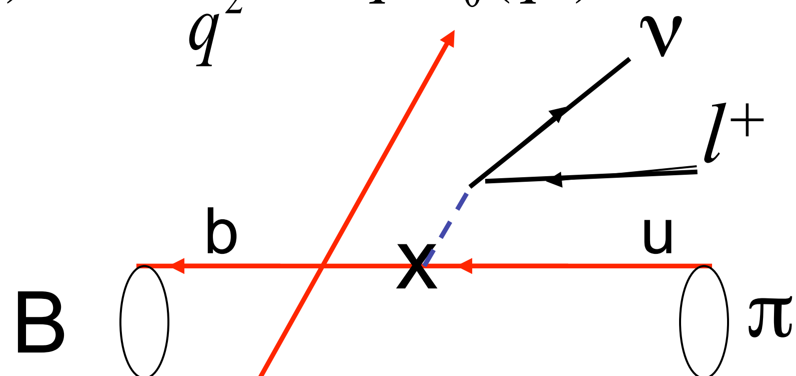
- The experimental measurement of pure leptonic decay can provide the product of decay constant and CKM matrix element.
- Theoretically decay constant can be calculated by QCD sum rule or Lattice QCD

We have two hadrons in semi-leptonic decays. It is described by form factors

$$\langle \pi | \bar{u}\gamma^{\mu} b | B \rangle = p_B{}^{\mu} f_1 + p_{\pi}{}^{\mu} f_2 \qquad q = p_B - p_{\pi}$$

$$= \left[(p_B + p_{\pi})^{\mu} - \frac{m_B{}^2 - m_{\pi}{}^2}{q^2} q^{\mu} \right] F_1(q^2) + \frac{m_B{}^2 - m_{\pi}{}^2}{q^2} q^{\mu} F_0(q^2)$$

Form factors can be calculated by lattice QCD, QCD sum rules, light cone sum rules etc.

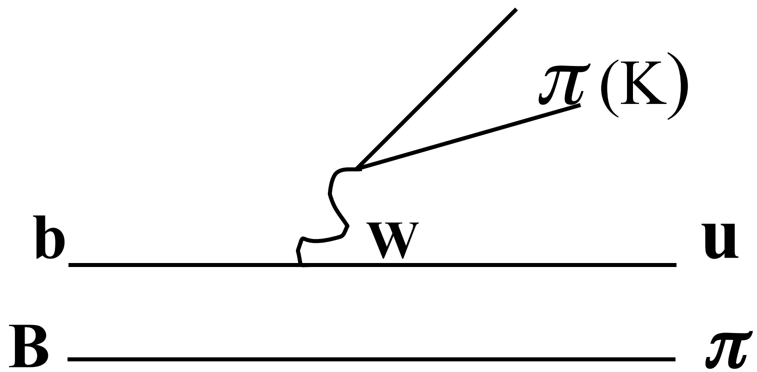


In the quark model, it is calculated by the overlap of two meson wave functions.

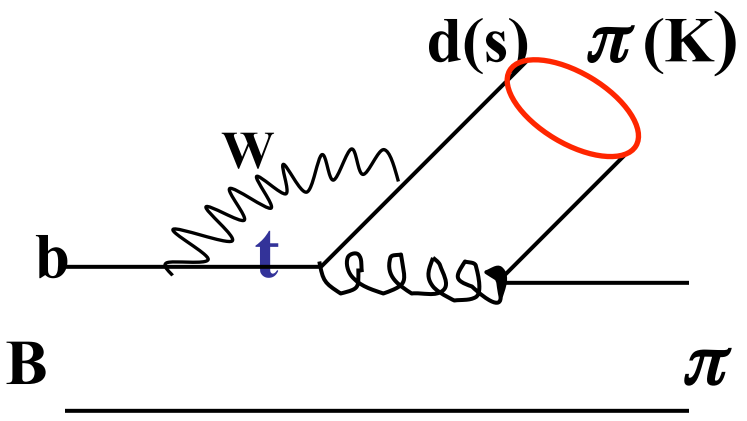
Not a constant but a **function**

Rich physics in hadronic B decays

CP violation, FCNC, sensitive to new physics contribution…



The standard model describes interactions amongst quarks and leptons

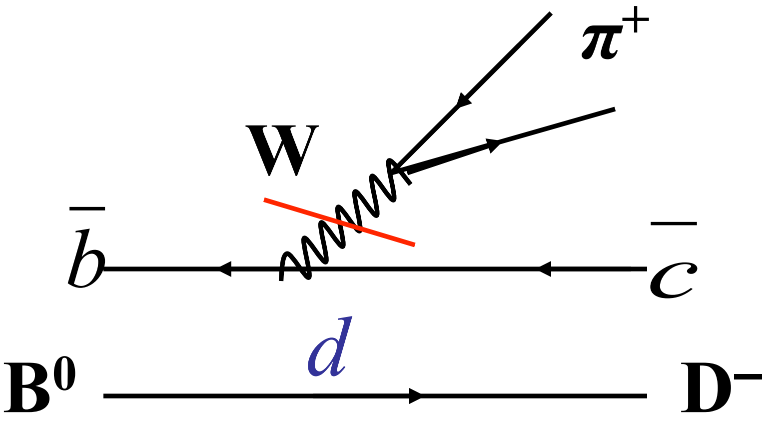


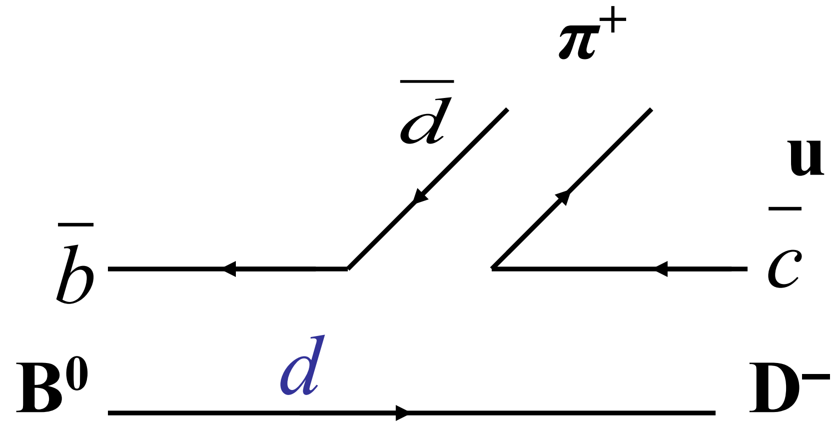
In experiments, we can only observe hadrons

pi K puzzle etc.

How can we test the standard model without solving QCD?

Naïve Factorization (BSW model)





Bauer, Stech, Wirbel, Z. Phys. C29, 637 (1985); ibid 34, 103 (1987)

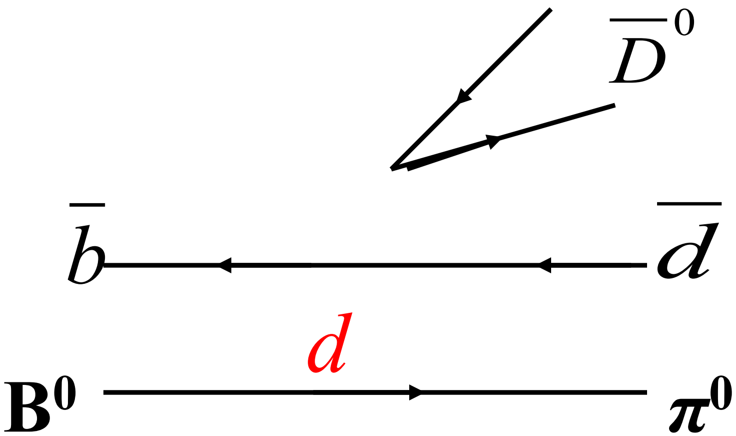
Hadronic parameters: Form factor and decay constant

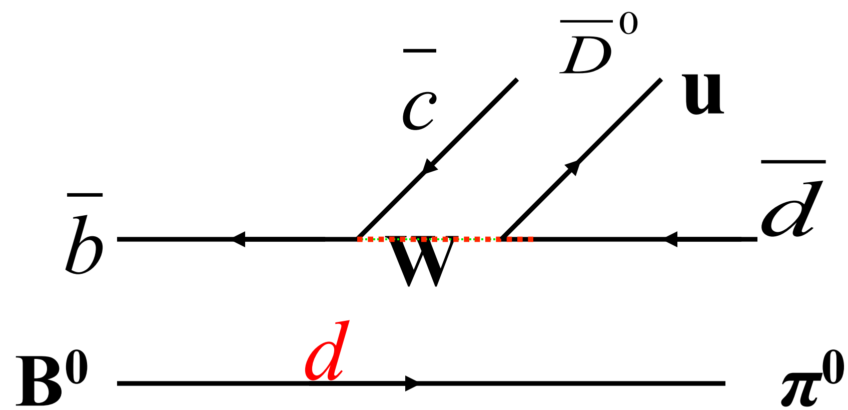
$$< \pi^+ D^- | H_{eff} | B > = a_1 \; \langle \pi | \bar{u} \gamma^\mu L d | 0 \rangle \; \langle D | \bar{b} \gamma_\mu L c | B \rangle$$

Form factors calculated from quark model

Generalized Factorization Approach

Ali, Kramer, Lu, Phys. Rev. D58, 094009 (1998)





$C_1 \sim -0.2$ $\sim$ $C_2(1/3 + s_8) \equiv C_2/N_c \sim +1/3$

$$\left\langle \pi^0 \overline{D}^0 \middle| H_{eff} \middle| B^0 \right\rangle = (C_1 + C_2/N_c)\, f_D\, F_0^{B\to\pi}$$

Non-factorizable contribution should be larger than expected, characterized by effective N_C

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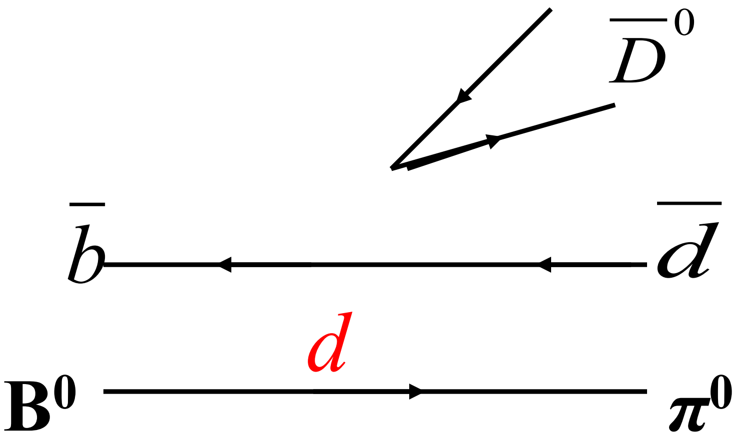

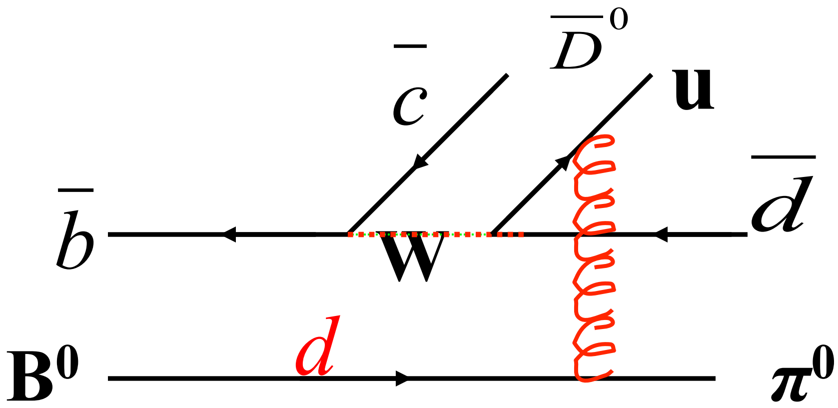

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Non-factorizable contribution should be larger than expected, characterized by effective N_C

QCD Penguin operators

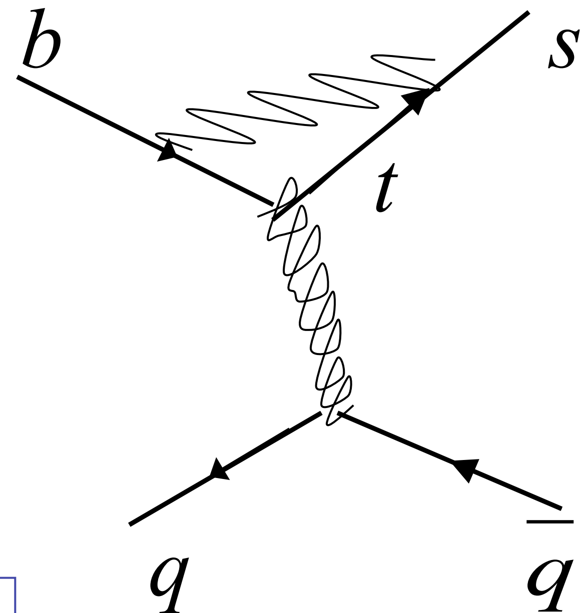
- **Wilson coefficients** $\propto \alpha_s$

$$O_3 = \bar{d}\gamma^{\mu}Lb \cdot \sum_q \bar{q}\gamma_{\mu}Lq$$

$$O_4 = \bar{d}_{\alpha}\gamma^{\mu}Lb_{\beta} \cdot \sum_q \bar{q}_{\beta}\gamma_{\mu}Lq_{\alpha}$$

$$O_5 = \bar{d}\gamma^{\mu}Lb \cdot \sum_q \bar{q}\gamma_{\mu}Rq$$

$$O_6 = \bar{d}_{\alpha}\gamma^{\mu}Lb_{\beta} \cdot \sum_q \bar{q}_{\beta}\gamma_{\mu}Rq_{\alpha}$$

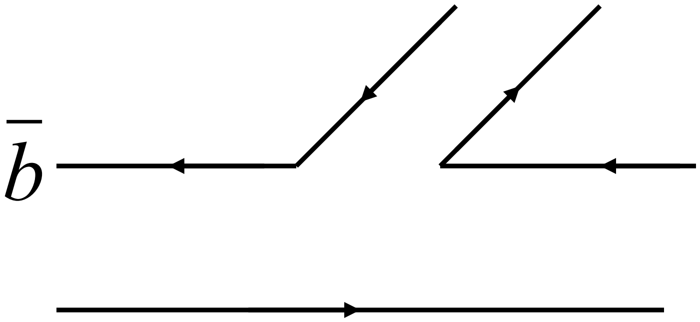


$R=1+\gamma^5$

Chiral enhanced penguin

Fiertz transformation gives a

Chiral enhanced factor m_π^2/m_d



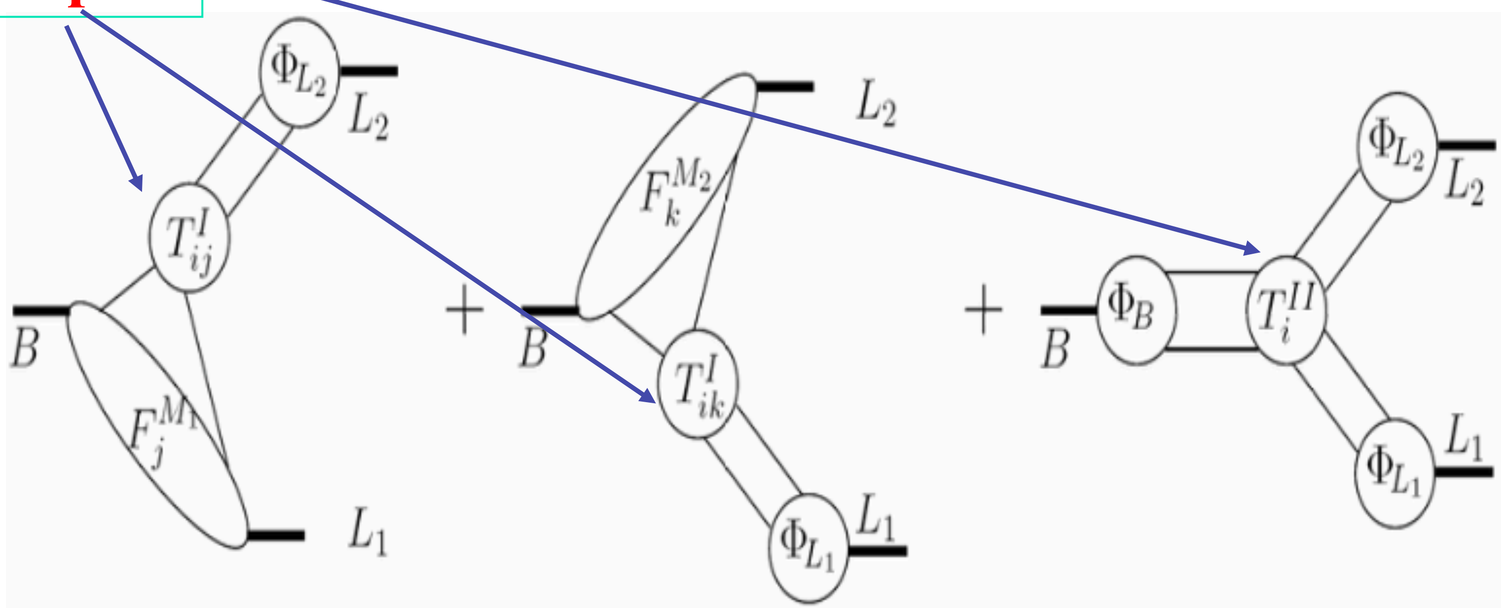
This makes $Br(B \rightarrow \pi^+ K^-) > Br(B \rightarrow \pi^+ \pi^-)$

Previously in BSW model it is the inverse case

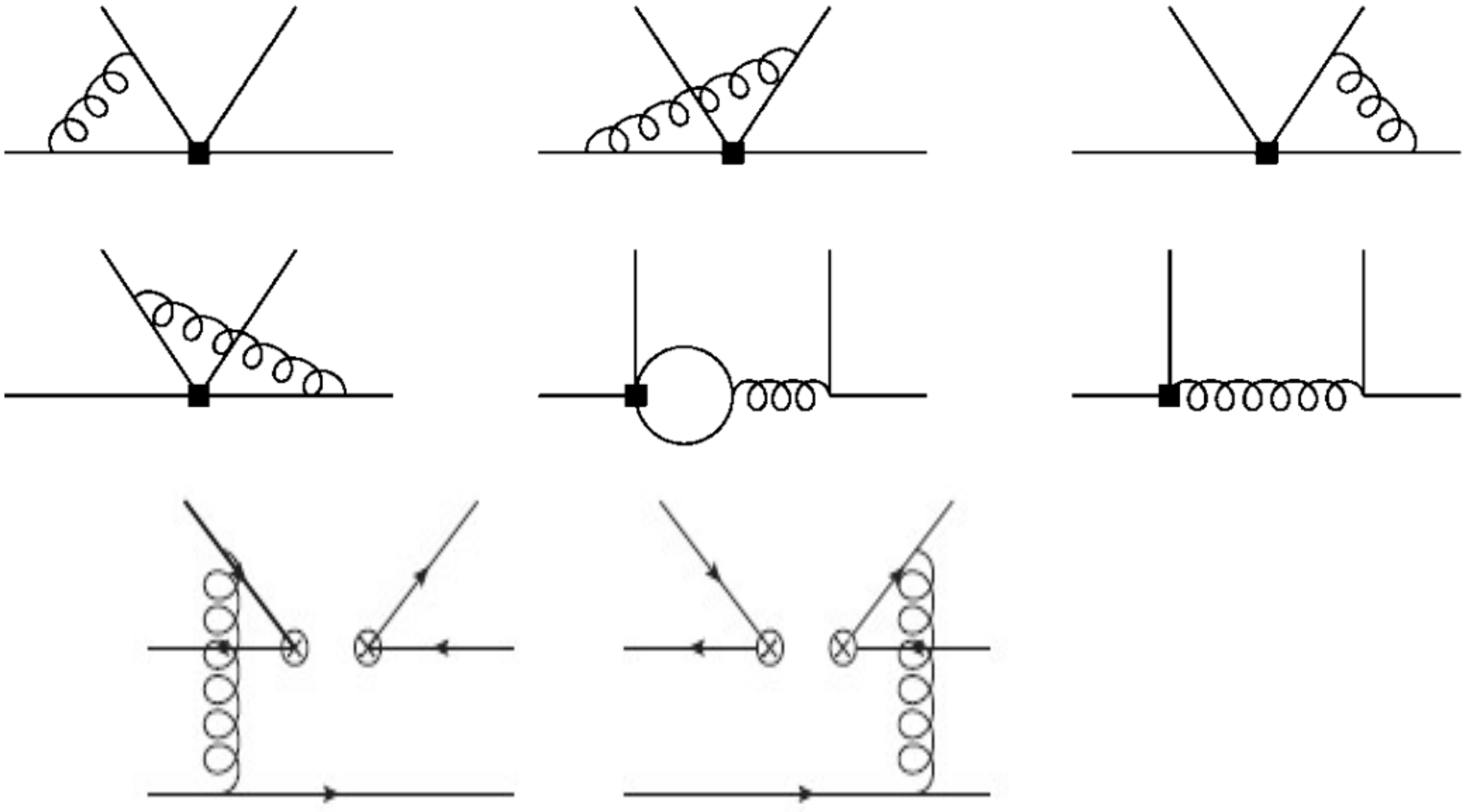
QCD factorization by BBNS: **PRL 83 (1999) 1914; NPB591 (2000) 313**

$$
\begin{aligned}
-\langle L_1 L_2 | Q_i | \bar{B} \rangle \;=\; & \sum_j F_j^{B\to L_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u)\, \Phi_{L_2}(u) \\
& + \sum_k F_k^{B\to L_2}(m_1^2) \int_0^1 dv\, T_{ik}^I(v)\, \Phi_{L_1}(v), \\
& + \int_0^1 d\xi du dv\, T_i^{II}(\xi,u,v)\, \Phi_B(\xi)\, \Phi_{L_1}(v)\, \Phi_{L_2}(u)
\end{aligned}
$$

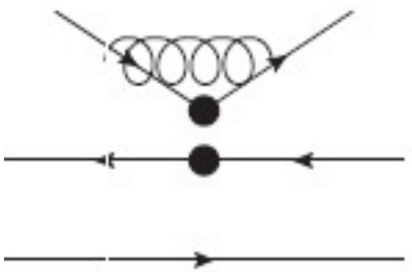
hard part



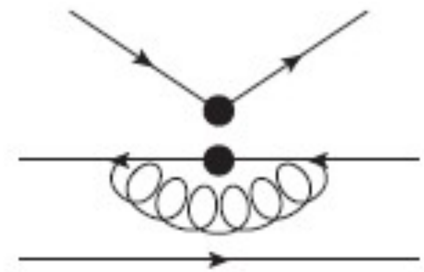
α_s corrections to the hard part T



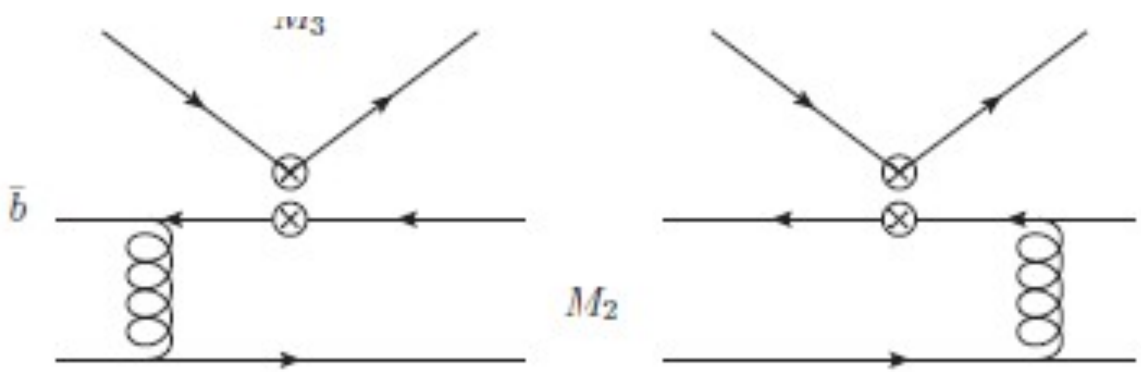
The missing diagrams, which contribute to the renormalization of decay constant or form factors



(e)

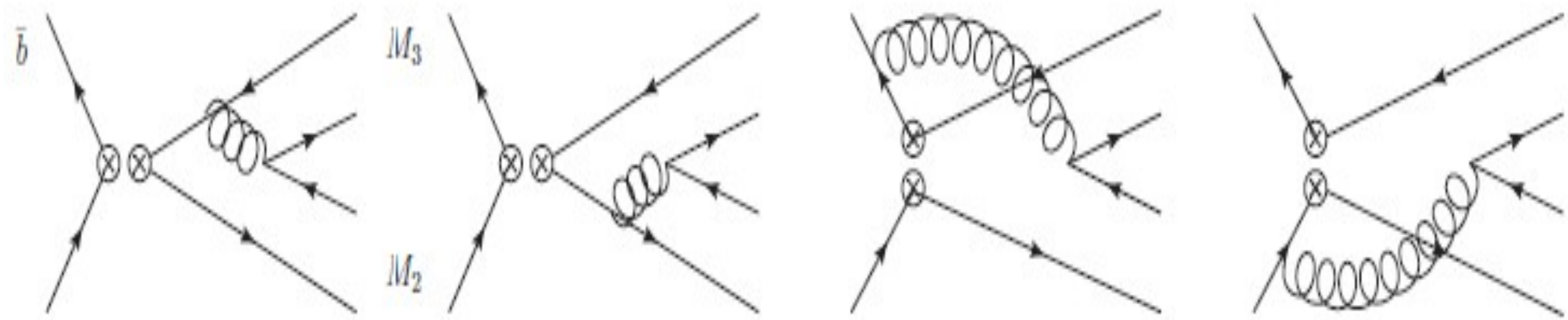


(f)



Endpoint divergence appears in these calculations

The annihilation type diagrams are important to the source of strong phases



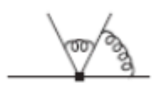
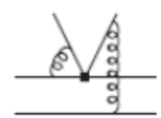
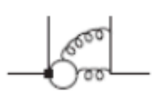
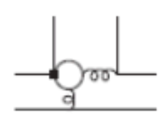
- **However, these diagrams are similar to the form factor diagrams, which have endpoint singularity, not perturbatively calculable.**
- **These divergences are not physical, can only be treated in QCDF as free parameters, which makes CP asymmetry not predictable:**

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

Status of NNLO QCD factorization calculations

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\rm I}(\mu_h,\mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h,\mu_I)}_{1+\ldots} \star \underbrace{J^{\rm II}(\mu_I,\mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 in progress	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

Analyses of complete sets of final states

- **PP, PV**
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237
- **VV**
  MB, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237
- **AP, AV, AA**
  Cheng, Yang, 0709.0137, 0805.0329
- **SP, SV**
  Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403
- **TP, TV**
  Cheng, Yang, 1010.3309

Based on NLO hard-scattering functions.

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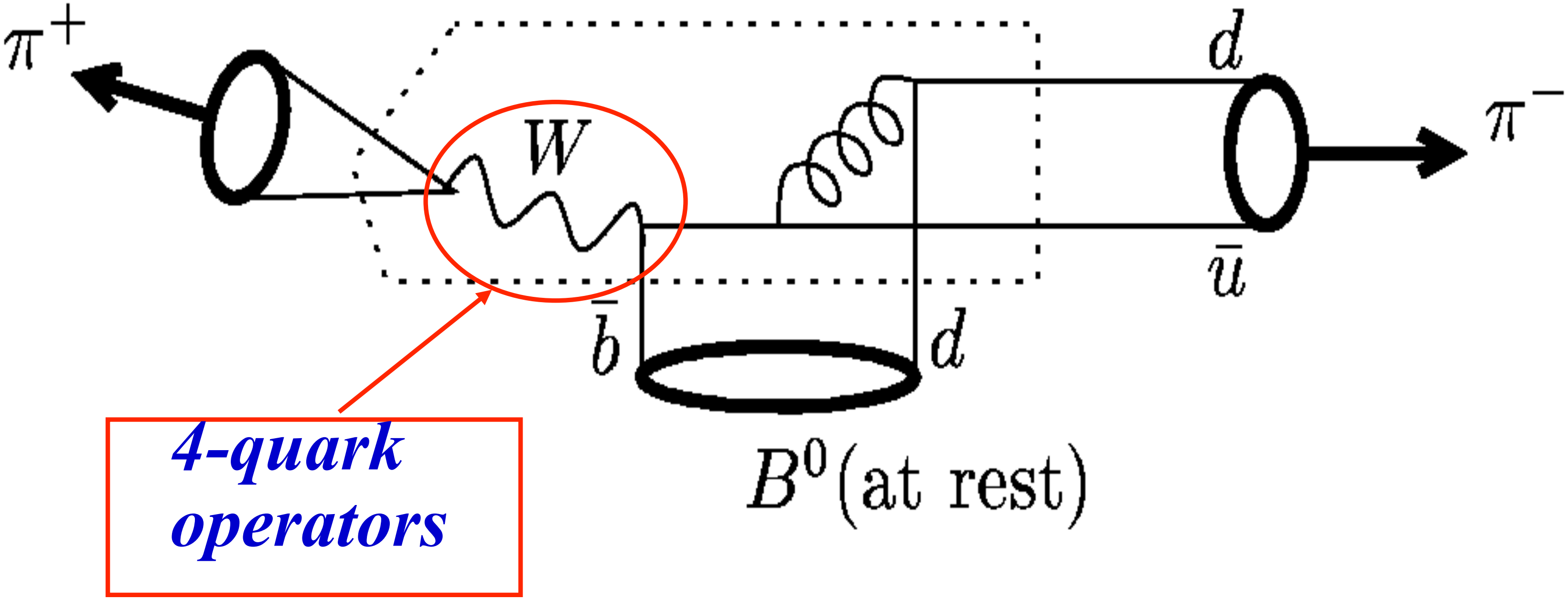
- TP, TV
  Cheng, Yang, 1010.3309

Following: amplitudes and phenomenology with NNLO results (except polarization)

With more and more precise data, power corrections are urgently needed

Based on NLO hard-scattering functions.

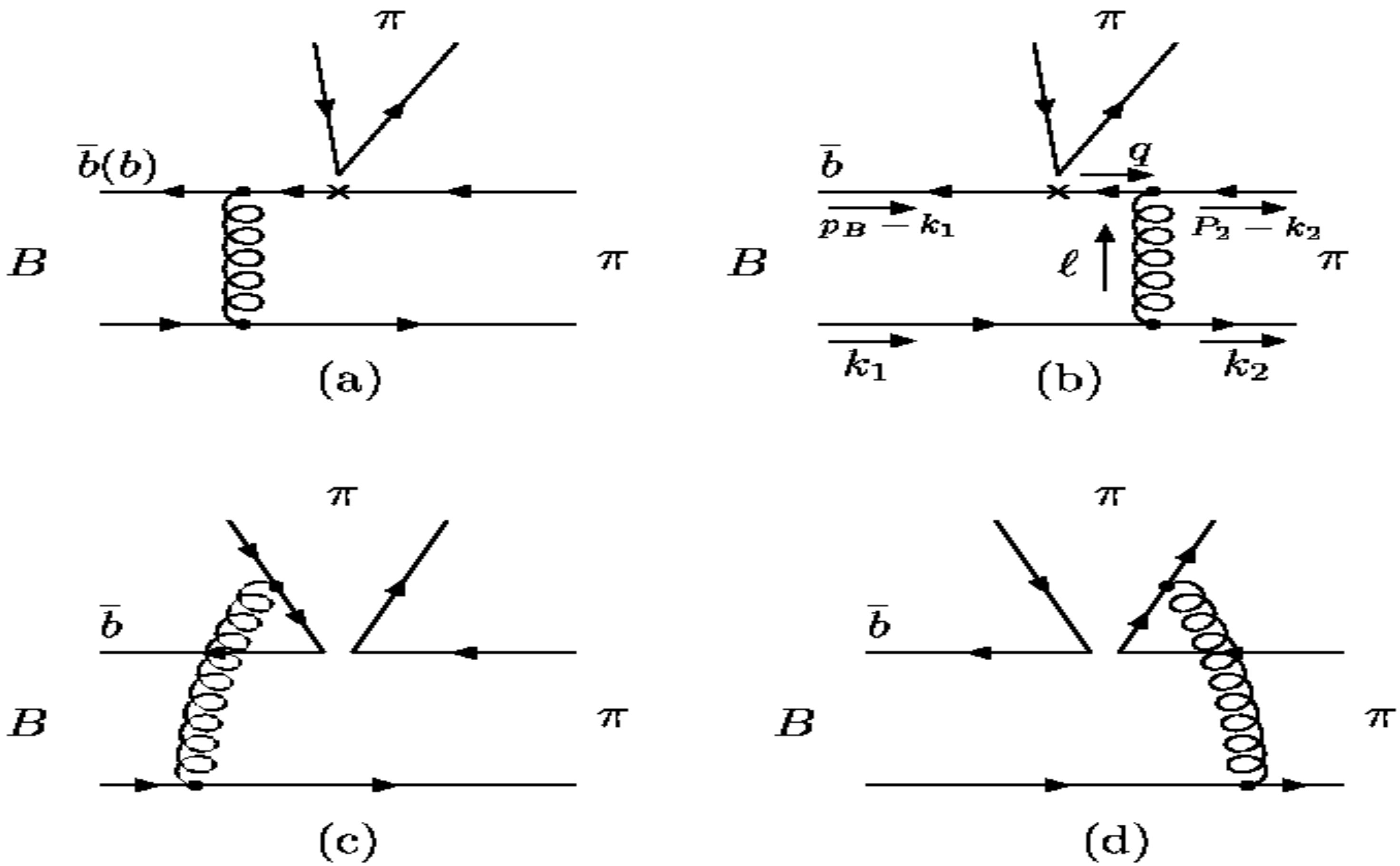
Picture of PQCD Approach



Keum, Li, Sanda, **Phys.Rev. D63 (2001) 054008;**

Lu, Ukai, Yang, **Phys.Rev. D63 (2001) 074009**

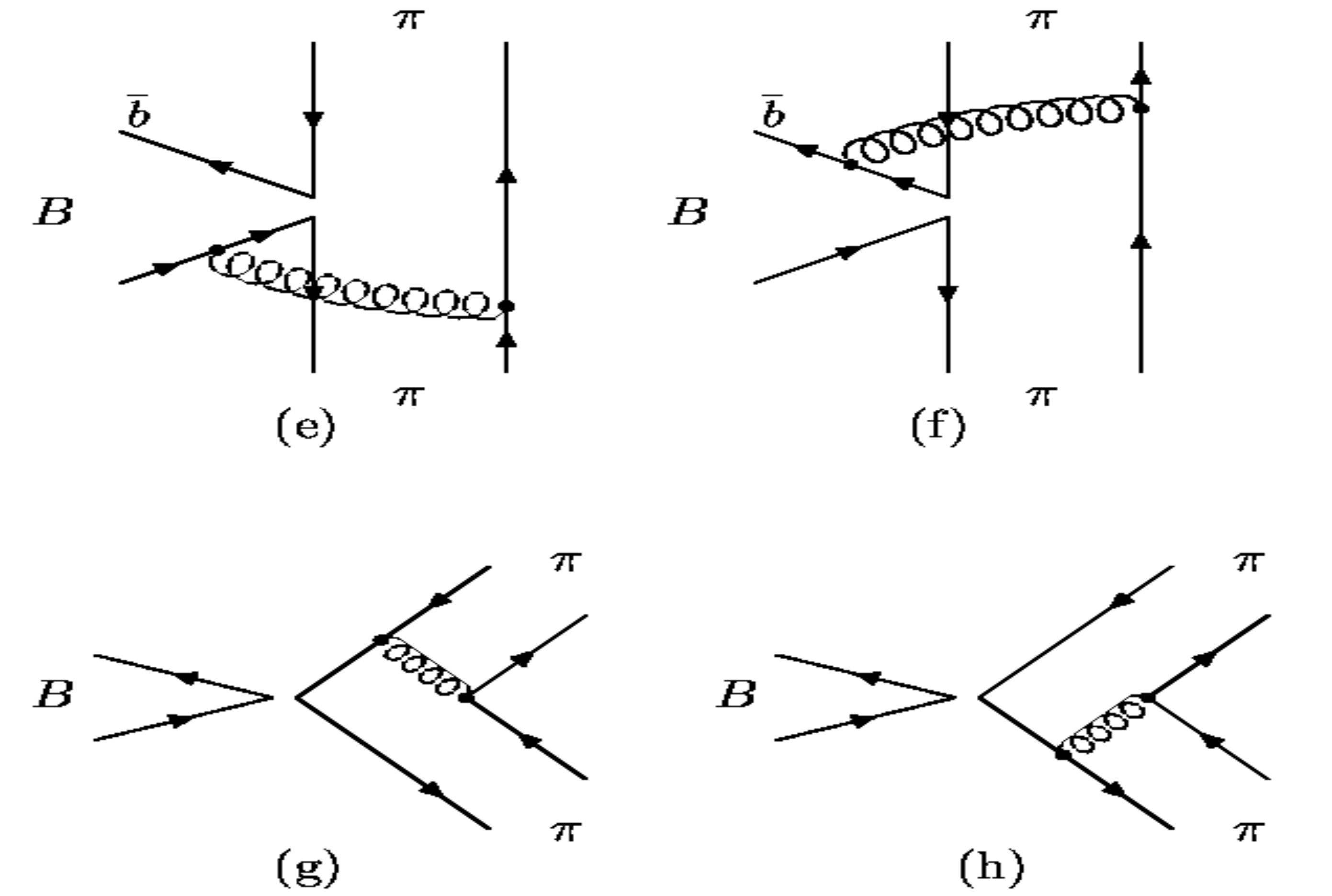
The leading order emission Feynman diagram in PQCD approach



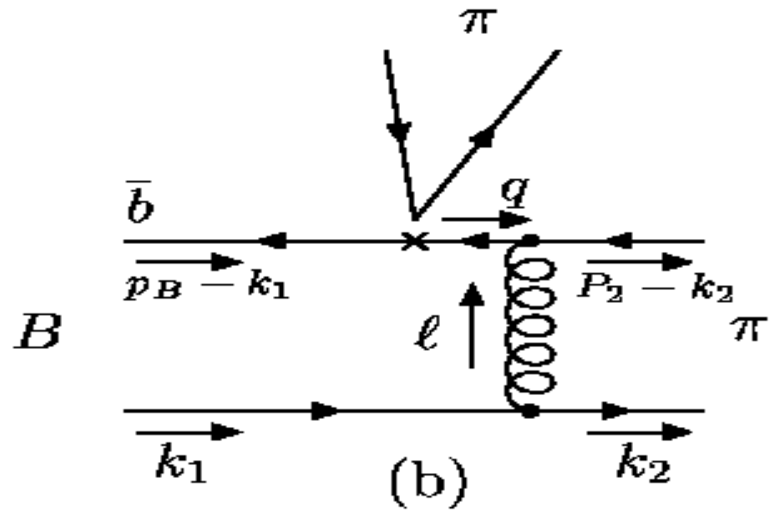
Form factor diagram

Hard scattering diagram

The leading order Annihilation type Feynman diagram in PQCD approach



Endpoint singularity



- **Gluon propagator**

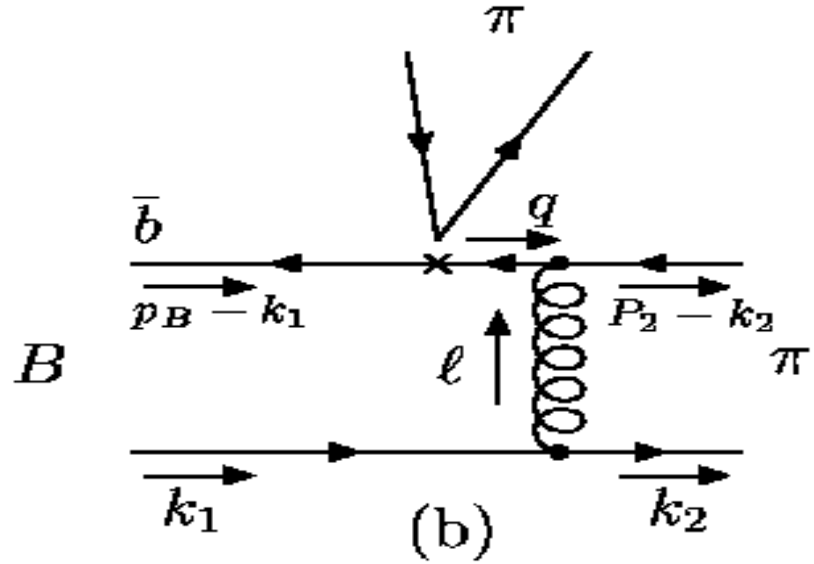
$$\frac{i}{(k_1 - k_2)^2} = \frac{i}{-2xym_B^2}$$

- **x,y Integrate from 0→1, that is endpoint singularity**
- **The reason is that, one neglects the transverse momentum of quarks, which is not applicable at endpoint.**
- **If we pick back the transverse momentum, the divergence disappears**

$$\frac{i}{(k_1 - k_2)^2} = \frac{i}{-2xym_B^2 - (k_1^T - k_2^T)^2}$$

Endpoint singularity

- **It is similar for the quark propagator**

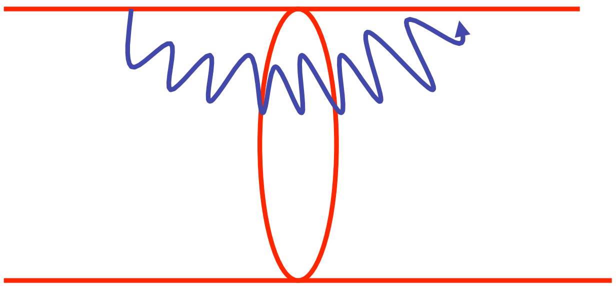


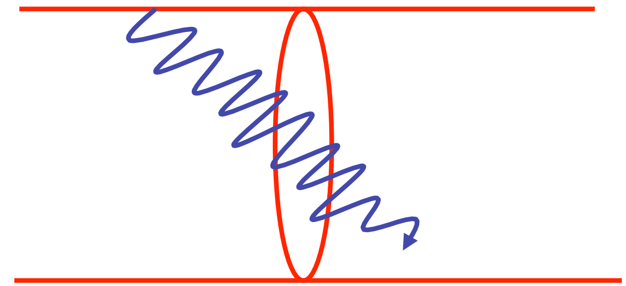
$$\int_0^1 \frac{1}{x} dx = \ln \frac{1}{\varepsilon}$$

$$\int_0^1 \frac{1}{x+k} dx dk = \int dk \left[\ln(x+k) \right]_0^1 = \int dk \left[\ln(1+k) - \ln k \right]$$

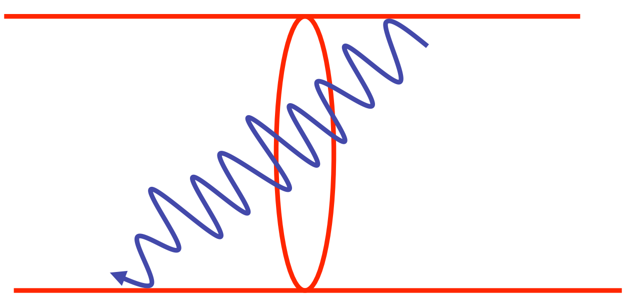
The logarithm divergence disappear if one has an extra dimension

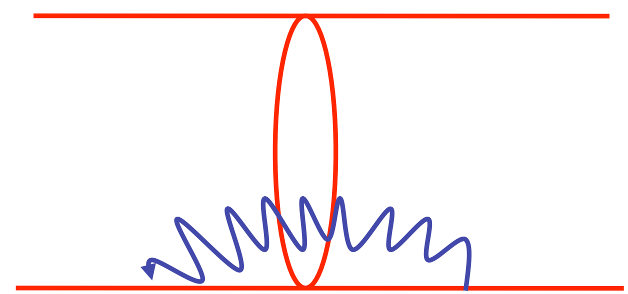
However, with transverse momentum, means one extra energy scale





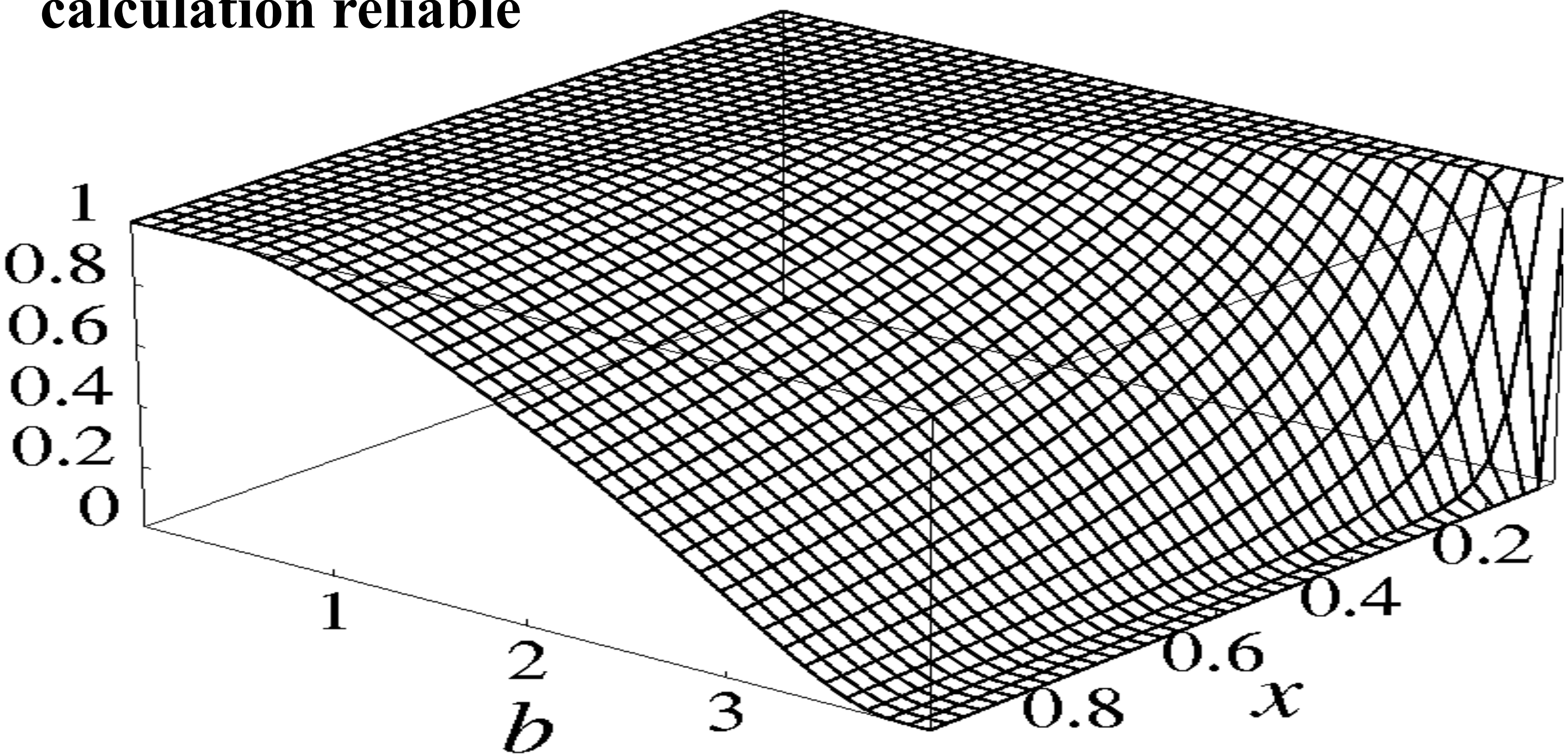
The overlap of Soft and collinear divergence will give double logarithm $ln^2 Pb$, which is too big to spoil the perturbative expansion. We have to use renormalization group equation to resum all of the logs to give the so called Sudakov Form factor





Sudakov Form factor $\exp\{-S(x,b)\}$

This factor exponentially suppresses the contribution at the endpoint (small k_T), makes our perturbative calculation reliable



CP Violation in $B \to \pi\pi (K)$ *(real prediction before exp.)*

CP(%)	FA	BBNS	PQCD *(2001)*	Exp *(2004)*
$\pi^+ K^-$	+9±3	+5±9	–17±5	–11.5±1.8
$\pi^0 K^+$	+8 ± 2	7 ±9	–13 ±4	+4 ± 4
$\pi^+ K^0$	1. 7± 0. 1	1 ±1	–1. 0±0.5	–2 ±4
$\pi^+ \pi^-$	–5±3	–6±12	+30±10	+37±10

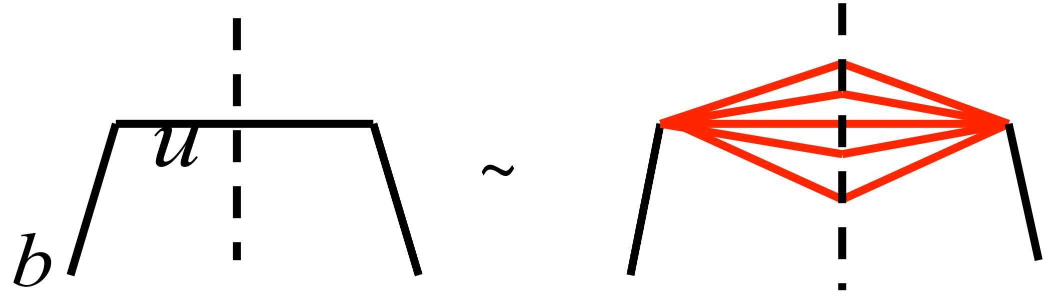
CP Violation in $B \to \pi\pi(K)$

Including large annihilation fixed from exp.

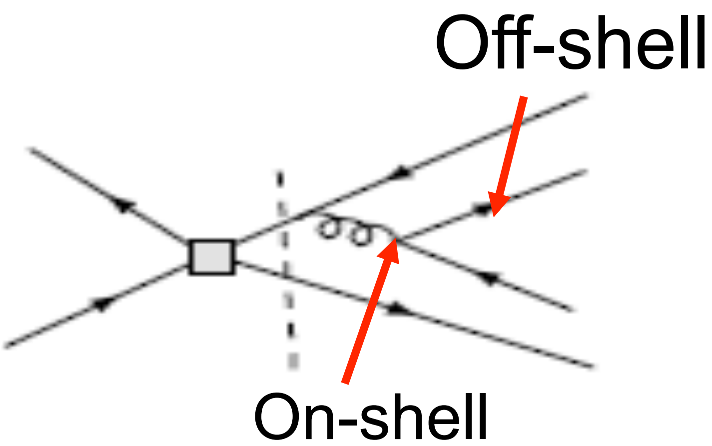
CP(%)	FA	Cheng,HY	PQCD (2001)	Exp
$\pi^+ K^-$	+9±3	-7.4 ± 5.0	–17±5	–9.7±1.2
$\pi^0 K^+$	+8 ± 2	0.28±0.10	–13 ±4	4. 7 ± 2.6
$\pi^+ K^0$	1. 7± 0. 1	4.9 ± 5.9	–1. 0±0.5	0.9 ±2.5
$\pi^+ \pi^-$	–5±3	17 ± 1.3	+30±10	+38±7

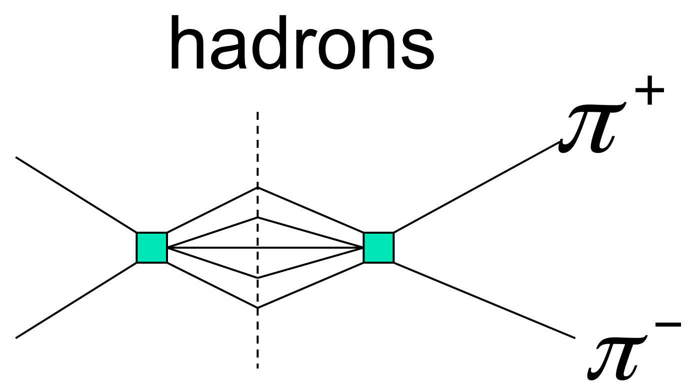
Inclusive Decay and B meson annihilation decay

$B \to X_u l\nu$ π, ρ, p, n



Cut quark diagram ~ Sum over final-state hadrons





Large strong phase

QCD-methods based on factorization work well for the leading power of $1/m_b$ expansion

collinear QCD Factorization approach
[Beneke, Buchalla, Neubert, Sachrajda, 99']

Perturbative QCD approach based on k_T factorization
[Keum, Li, Sanda, 00'; Lu, Ukai, Yang, 00']

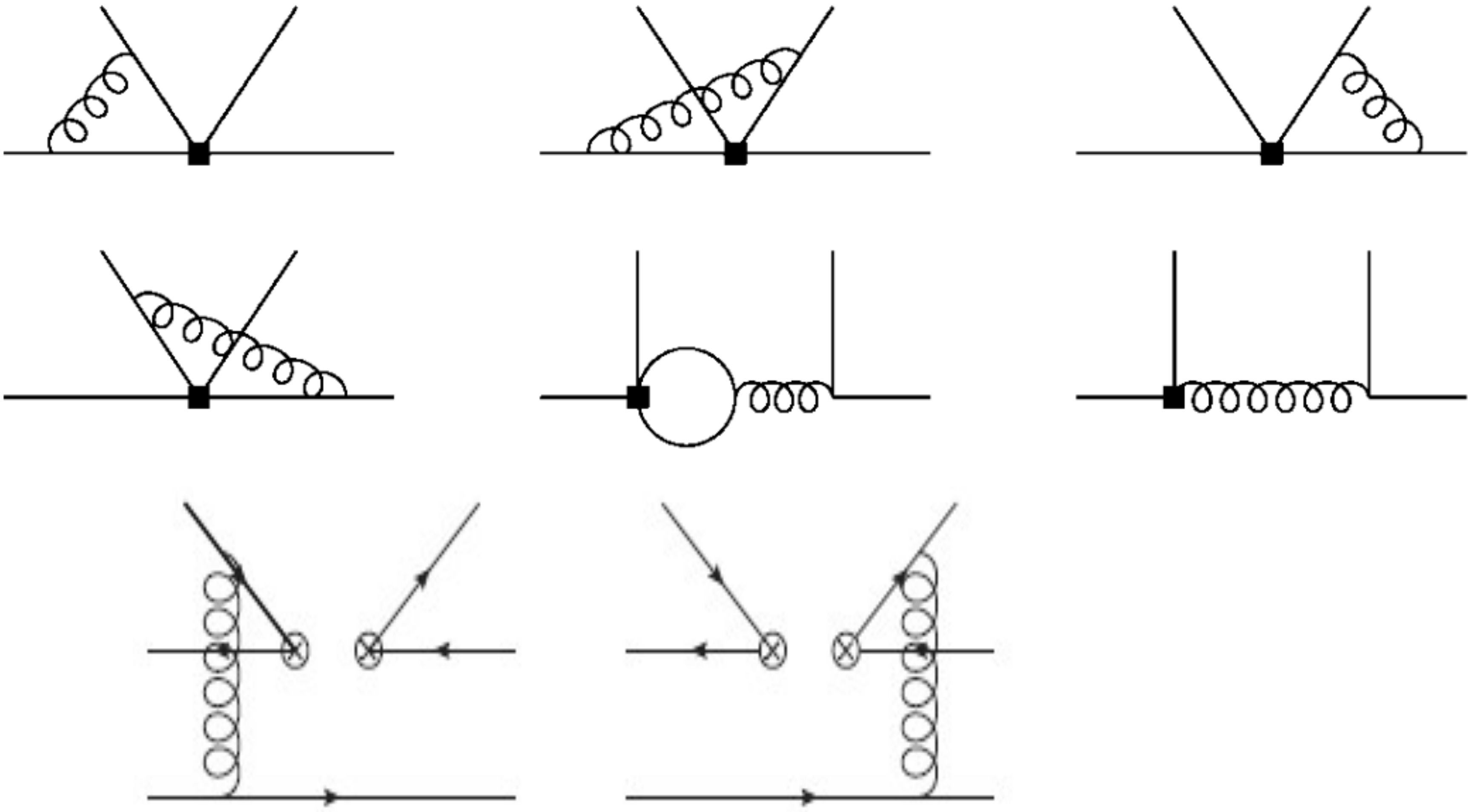
Soft-Collinear Effective Theory
Bauer, Fleming, **Pirjol, Stewart, Phys.Rev. D63 (2001) 114020**

- **Work well for most of charmless B decays, except for ππ, πK puzzle etc.**

For the charming penguin, an additional scale m_c is involved

- $1/m_c$, m_c/m_b expansion is needed
- QCDF and PQCD work well at only the leading order of these power expansion
- SCET parameterize this contribution, since factorization breaks down at the next-to-leading power correction.
- The main source of strong phase needed for direct CP violation, comes from here in SCET

α_s corrections to the hard part T



Factorization can only be proved in power expansion by operator product expansion. To achieve that, we need a hard scale Q

- In the certain order of 1/Q expansion, the hard dynamics characterized by Q factorize from the soft dynamics
- Hard dynamics is process-dependent, but calculable
- Soft dynamics are universal (process-independent) ⇨ predictive power of factorization theorem
- Factorization theorem holds up to all orders in α_s, but to certain power in 1/Q

QCD Light-Cone Sum Rules

- the general outline of the method:

vacuum-to-hadron correlator of currents in the spacelike region

↙ ↘

light-cone OPE = hadronic dispersion relation

quark-hadron duality

↙

sum rule for hadronic transition amplitude

- applications to $B \to h$ semileptonic form factors

$$F_a^{B\to h}(q^2) = \langle h(p)|\bar{q}\,\Gamma_a b|B(p+q)\rangle \qquad q = u, d, s, c,$$

- valid at $q^2 \ll (m_B - m_h)^2$ (large recoil of h)

the correlation function

$$\Pi_\mu(n\cdot p,\bar{n}\cdot p) = \int d^4x\; e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\not{n}\,\gamma_5\, q(x),\;\bar{q}(0)\,\Gamma_\mu\, b(0)\right\}|\bar{B}(p+q)\rangle$$

Quark level

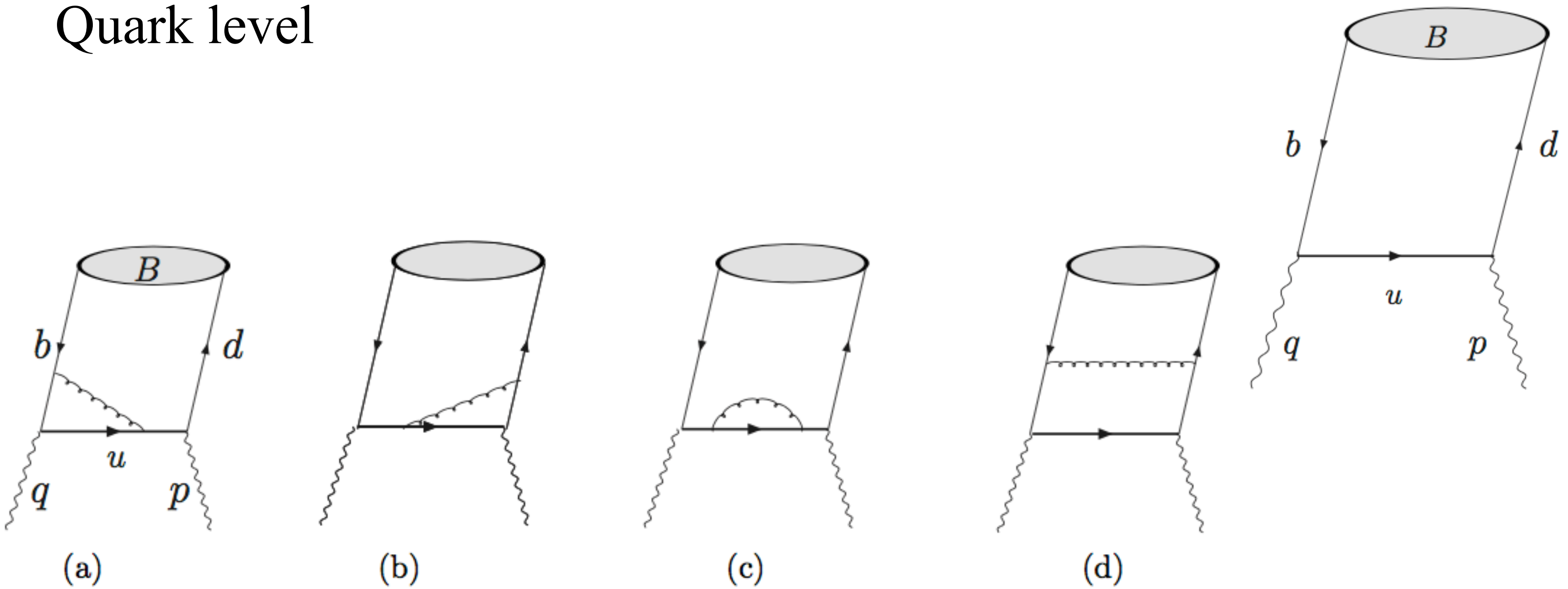


Figure 2: Diagrammatical representation of the correlation function $\Pi_\mu(n\cdot p,\bar{n}\cdot p)$ at $\mathcal{O}(\alpha_s)$.

the correlation function

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \int d^4x\, e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\not{n}\,\gamma_5\, q(x),\ \bar{q}(0)\,\Gamma_\mu\, b(0)\right\}|\bar{B}(p+q)\rangle$$

Hadron level

Quark hadron duality

Insert a complete set of hadronic states Σ|n><n|

$$\langle \pi(p)|\bar{u}\gamma_\mu b|\bar{B}(p_B)\rangle = f^{+}_{B\pi}(q^2) + f^{0}_{B\pi}(q^2)$$

$$\langle \pi(p)|\bar{d}\,\not{n}\,\gamma_5\, u|0\rangle = -i\, n\cdot p\, f_\pi$$

□ Form factors from LCSRs with light hadron DAs

- $B \to \pi$: gradual improvements of OPE

[V.Belyaev, A.K., R.Rückl (1993)]; [V.Belyaev, V.M.Braun, A.K., R.Rückl (1995)]
[A.K., R.Rückl, S.Weinzierl, O.I.Yakovlev (1997)]; [E.Bagan,P.Ball, V.M. Braun (1997)]
[P.Ball, R.Zwicky (2004)]; [G.Duplancic, A.K., T.Mannel, B.Melic, N.Offen (2008)]
[A.K., T.Mannel, N.Offen, Y.M. Wang (2011)]
[A. Bharucha (2012)], [A.Rusov (2016)]

- $D \to \pi, K$: byproduct of $B \to \pi$ LCSR [A.K., C.Klein, T.Mannel, N.Offen, (2009)]

- $B \to K$, $B_s \to K$: $SU(3)$ breaking: $m_s \neq 0$, in kaon DAs, $f_{B_s} \neq f_B$

the latest update in [A.K., A.Rusov (2017)]

- $B_{(s)} \to \rho, \omega, K^*, \phi$: with (zero-width) ρ, K^* DAs

[P.Ball, R. Zwicky (2004)], [A.Bharucha, D.Straub, R.Zwicky (2015)]

- $\Lambda_b \to p$: with nucleon DAs, no NLO corrections yet

[AK, Th.Mannel, Ch. Klein, Y.-M. Wang (2011)]

□ *B*-meson DAs

- definition of two-particle DA in HQET:

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle = -\frac{if_B m_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

$\oplus$ higher twists

- key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

  - possible to extract λ_B from $B\to\gamma\ell\nu_\ell$ using QCDF$\oplus$LCSR

    [M.Beneke, V.M. Braun, Y.Ji, Y.B. Wei (2018)]

  - current limit from Belle measurement (2018): $\lambda_B > 240$ MeV
  - QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460\pm 110$ MeV

    [V.Braun, D.Ivanov, G.Korchemsky (2004)]

- higher twists DAs recently worked out [V. Braun, Y. Ji and A. Manashov (2017)]

Current status of B → $\gamma l \nu$

- Factorization properties at leading power [Korchemsky, Pirjol and Yan, 2000; Descotes-Genon and Sachrajda, 2002; Lunghi, Pirjol and Wyler, 2003; Bosch, Hill, Lange and Neubert, 2003].

- Leading power contributions at NLL and (partial)-subleading power corrections at tree level [Beneke and Rohrwild, 2011].

- Subleading power corrections from the dispersion technique:
  - Soft two-particle correction at tree level [Braun and Khodjamirian, 2013].
  - Soft two-particle correction at one loop [Wang, 2016].
  - Three-particle *B*-meson DA's contribution at tree level [Wang, 2016; Beneke et al, 2018].
  - Subleading effective current and twist-5 and 6 corrections at tree level. [Beneke et al, 2018].

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- Subleading power corrections from the direct QCD approach:
  - Hadronic photon corrections at tree level up to the twist-4 accuracy [Khodjamirian, Stoll, Wyler, 1995; Ali, Braun, 1995; Eilam, Halperin, Mendel, 1995].
  - Hadronic photon corrections of twist-two at one loop and of higher-twist at tree level [Ball and Kou, 2003; Wang and Shen, 2018].

LCSRs with B meson DA's

- $B \to h$ form factors:
  - $B \to \pi, K, K^*, \rho$ [A.K., T.Mannel, N.Offen (2007)]
  - $B \to D, D^*$ [S.Faller,A.K., C.Klein,T.Mannel (2009)]
  - NLO corrections to $B \to \pi$ FFs [Y-M. Wang, Y-L.Shen (2015)
  - higher twists in OPE, $B \to \pi, K$ [C-D.Lü,Y.L. Shen,Y-M. Wang,Y-B. Wei (2018)]
  - all $B \to \pi, K, D, \rho, K^*, D^*$ form factors [N.Gubernari, A.Kokulu, D. van Dyk, (2018)
- Heavy baryon form factors:
  - $\Lambda_b \to \Lambda$ [T.Feldmann, M. Yip (2012)]

Which LCSR method is better?

taking $B \to \pi$ form factor $f^+(q^2)$ as a sample

LCSR	method 1 (pion DAs)	method 2 (B DAs)
input for DAs exp. data	sufficient for π DAs pion FFs	λ_B,$\lambda_{H,E}$ uncertainty $B \to \ell \nu_\ell \gamma$
OPE twist expansion α_s expansion	$\leq$ tw 6 (N)NLO tw 2, NLO tw3	$\leq$ tw 6 NLO tw 2
disp.relation/duality	s_0^B	s_0^π

- the method 1 needs a set of DAs for every light hadron

the method 2 more flexible (changing the current),

- B_s form factors not available in method 2, need λ_{B_s}

Summary/Challenges

- Hadronic B Decays are important in the test of standard model and search for signals of new physics.
- Power corrections in QCDF are very important that need to be calculated precisely
- Such as The annihilation type diagrams are the key point in explaining the K pi puzzle and large direct CP asymmetry found in B decays
- Next-to-leading order perturbative calculations is needed to explain the more and more precise experimental data