# Three-body B meson decays

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# Outline

# 01 Motivation

## Large amounts of data from LHCb, Belle, *BABAR*, CDF, D0 *etc*. Collaborations. (Most with BFs ~ $10^{-5}$)

**charmless**

$$B^- \to \pi^+\pi^-\pi^-, K^-\pi^+\pi^-, K^+K^-\pi^-, K^+K^-K^-, K^-K_SK_S$$

$$\overline{B}^0 \to \overline{K}^0\pi^+\pi^-, K^-\pi^+\pi^0, K^0K^-\pi^+, K^+K^-\pi^0, K^+K^-\overline{K}^0, K_SK_SK_S$$

**charm**

$$B^0_{(s)} \to D, \mathrm{X}_{c\bar{c}}(\pi\pi, KK, K\pi \ldots)$$

01 **They can help us to test the factorization approach, which have been used in two-body decays successfully.**

02 **They also help us to test SM by measuring the CKM phases.**

03 **They offer us opportunities to study the line-shape of intermediate states.**

04 **Combing with the two-body decays, we can try to understand the sources of strong phases.**

05 **The study of three-body decays help us to test the SU(3) asymmetry.**

# Using U spin to extract γ from charmless B→PPP decays

Bhubanjyoti Bhattacharya and David London

JHEP04(2015)154

$\mathrm{B_s^0 \rightarrow K_S\pi^+\pi^-\ (\bar{b} \rightarrow \bar{d});\ B_d^0 \rightarrow K_S K^+ K^-\ (\bar{b} \rightarrow \bar{s})}$

$\mathrm{B_s^0 \rightarrow K_S K^+ K^-\ (\bar{b} \rightarrow \bar{d});\ B_d^0 \rightarrow K_S\pi^+\pi^-\ (\bar{b} \rightarrow \bar{s})}$

## $\mathbf{B_s^0 \to K_S\pi^+\pi^-}$ $(\bar{\mathbf{b}} \to \bar{\mathbf{d}})$; $\mathbf{B_d^0 \to K_S K^+ K^-}$ $(\bar{\mathbf{b}} \to \bar{\mathbf{s}})$

$$f_d \equiv K_S(p_1)\pi^+(p_2)\pi^-(p_3) \qquad \bar{f}_d \equiv K_S(p_1)\pi^+(p_3)\pi^-(p_2)$$

$$\mathcal{A}_d = \mathcal{A}(B_s^0 \to f_d) \qquad \mathcal{A}_s = \mathcal{A}(B_d^0 \to f_s)$$

As these are three-body decays, $T_{d,s}$ and $P_{d,s}$ are all momentum-dependent. This means that $T_{d,s}$ $(P_{d,s})$ takes different values at different points of the Dalitz plot.

$$\mathcal{A}_d = V_{ub}^* V_{ud} T_d + V_{cb}^* V_{cd} P_d \,, \qquad \mathcal{A}_s = V_{ub}^* V_{us} T_s + V_{cb}^* V_{cs} P_s$$

For the CP-conjugate amplitudes:

$$\bar{\mathcal{A}}_d = V_{ub} V_{ud}^* \bar{T}_d + V_{cb} V_{cd}^* \bar{P}_d \,, \qquad \bar{\mathcal{A}}_s = V_{ub} V_{us}^* \bar{T}_s + V_{cb} V_{cs}^* \bar{P}_s$$

Because the final states in the CP-conjugate decays are not the same as in the decays ($p_2$and $p_3$ are exchanged), $T_{d,s} \neq \bar{T}_{d,s}, P_{d,s} \neq \bar{P}_{d,s}$

$$|\mathcal{A}_d|^2 - |\bar{\mathcal{A}}_d|^2 = 2\,\mathrm{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*)\,\mathrm{Im}(T_d P_d^* + \bar{T}_d^* \bar{P}_d)\,,$$
$$|\mathcal{A}_s|^2 - |\bar{\mathcal{A}}_s|^2 = 2\,\mathrm{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*)\,\mathrm{Im}(T_s P_s^* + \bar{T}_s^* \bar{P}_s)\,.$$

$$|\mathcal{A}_d|^2 - |\bar{\mathcal{A}}_d|^2 = 2\,\mathrm{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*)\,\mathrm{Im}(T_d P_d^* + \bar{T}_d^* \bar{P}_d)\,,$$
$$|\mathcal{A}_s|^2 - |\bar{\mathcal{A}}_s|^2 = 2\,\mathrm{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*)\,\mathrm{Im}(T_s P_s^* + \bar{T}_s^* \bar{P}_s)\,.$$

In the U-spin limit we have $T_d = T_s$, $P_d = P_s$, $\bar{T}_d = \bar{T}_s$, $\bar{P}_d = \bar{P}_s$:

$$-\frac{a_s^{CP}}{a_d^{CP}}\frac{\tau(B_d^0)b_s}{\tau(B_s^0)b_d} = 1$$

Here, $a_q^{CP}$ and $b_q$ are, respectively, the direct CP asymmetry and branching ratio defined locally, i.e., at a particular Dalitz-plot point. They are both momentum-dependent quantities.

$$\mathcal{A}_d = \mathcal{A}(B_s^0 \to \bar{f}_d) \qquad \mathcal{A}_s = \mathcal{A}(B_d^0 \to \bar{f}_s)$$

$$\mathcal{A}_d = V_{ub}^* V_{ud} \bar{T}_d + V_{cb}^* V_{cd} \bar{P}_d\,, \qquad \mathcal{A}_s = V_{ub}^* V_{us} \bar{T}_s + V_{cb}^* V_{cs} \bar{P}_s\,.$$
$$\bar{\mathcal{A}}_d = V_{ub} V_{ud}^* T_d + V_{cb} V_{cd}^* P_d\,, \qquad \bar{\mathcal{A}}_s = V_{ub} V_{us}^* T_s + V_{cb} V_{cs}^* P_s$$

For three-body decays, there are two U-spin relations among the observables.

four decays $B^0, \bar{B}^0 \to f, \bar{f}$

$$\left|B^0_{\rm phys}(t)\right\rangle = f_+(t)\left|B^0\right\rangle + \frac{q}{p} f_-(t)\left|\overline{B}^0\right\rangle$$

$$\left|\overline{B}^0_{\rm phys}(t)\right\rangle = \frac{p}{q} f_-(t)\left|B^0\right\rangle + f_+(t)\left|\overline{B}^0\right\rangle$$

$$\langle f|B^0_{\rm phys}(t)\rangle = \langle f|B^0\rangle \left(f_+(t) + \lambda f_-(t)\right),$$

$$\langle \bar{f}|B^0_{\rm phys}(t)\rangle = \frac{q}{p}\left\langle \bar{f}|\overline{B}^0\right\rangle \left(f_+(t)\bar{\lambda} + f_-(t)\right)$$

$$\left\langle f|\overline{B}^0_{\rm phys}(t)\right\rangle = \frac{p}{q}\langle f|B^0\rangle \left(f_-(t) + \lambda f_+(t)\right),$$

$$\left\langle \bar{f}|\overline{B}^0_{\rm phys}(t)\right\rangle = \left\langle \bar{f}|\overline{B}^0\right\rangle \left(f_-(t)\bar{\lambda} + f_+(t)\right),$$

$$\begin{aligned}\Gamma(t) &= \frac{1}{2}\left(\Gamma(B^0_{\rm phys}(t)\to f) + \Gamma(\overline{B}^0_{\rm phys}(t)\to f)\right), \\ &= \frac{1}{2}\iint_{\rm bin} ds_{12}ds_{23}\,|A|^2 e^{-\Gamma t}\left[(1+|x|^2)\cosh(\Delta\Gamma t/2) + 2{\rm Re}(\lambda)\sinh(\Delta\Gamma t/2))\right]\end{aligned}$$

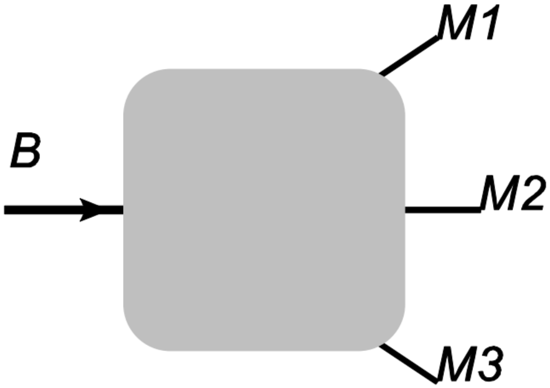
$$\begin{aligned}A_{CP}(t) &= \frac{\Gamma(B^0_{\rm phys}(t)\to f) - \Gamma(\overline{B}^0_{\rm phys}(t)\to f)}{\Gamma(B^0_{\rm phys}(t)\to f) + \Gamma(\overline{B}^0_{\rm phys}(t)\to f)}, \\ &= \frac{\iint_{\rm bin} ds_{12}ds_{23}\,|A|^2\left[(1-|x|^2)\cos(\Delta m t) - 2{\rm Im}(\lambda)\sin(\Delta m t)\right]}{\iint_{\rm bin} ds_{12}ds_{23}\,|A|^2\left[(1+|x|^2)\cosh(\Delta\Gamma t/2) + 2{\rm Re}(\lambda)\sinh(\Delta\Gamma t/2)\right]}.\end{aligned}$$

The number of observables is greater than the number of unknowns, $\gamma$ can be extracted.

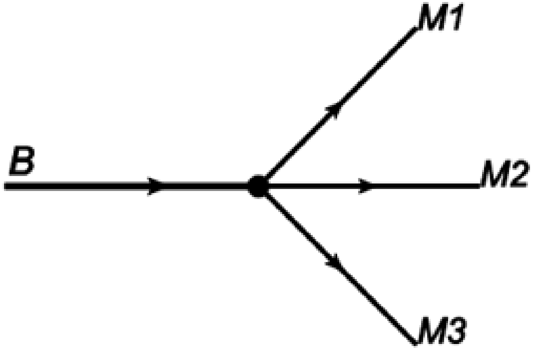
# 02 Framework

# Theoretical approach

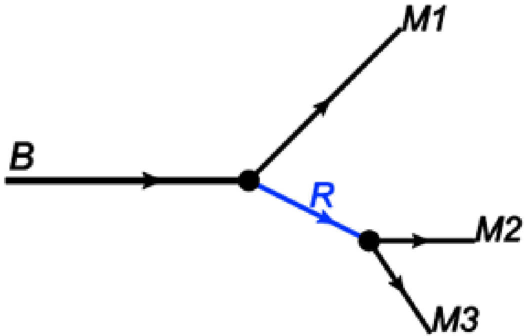
| | |
| --- | --- |
| **Factorization Approach** | H.Y. Cheng, C.K. Chua, Y. Li,… |
| **QCD Factorization** | A. Furman, B.El Bennich, R. Kaminski, T. Mannel, X.H. Guo, Y.D. Yang, Z.H. Zhang,… |
| **PQCD** | H.N. Li, C.D. Lü, Z.J. Xiao, W. Wang, W.F. Wang, R. Zhou, … |
| **Symmetry** | X.G. He, G.N. Li, D. Xu, J.L. Rosner, M. Gronau,… |
| **Sum rules** | Ulf-G. Meißner, Shan Cheng (Light-cone), A. Khodjamirian (QCD) |



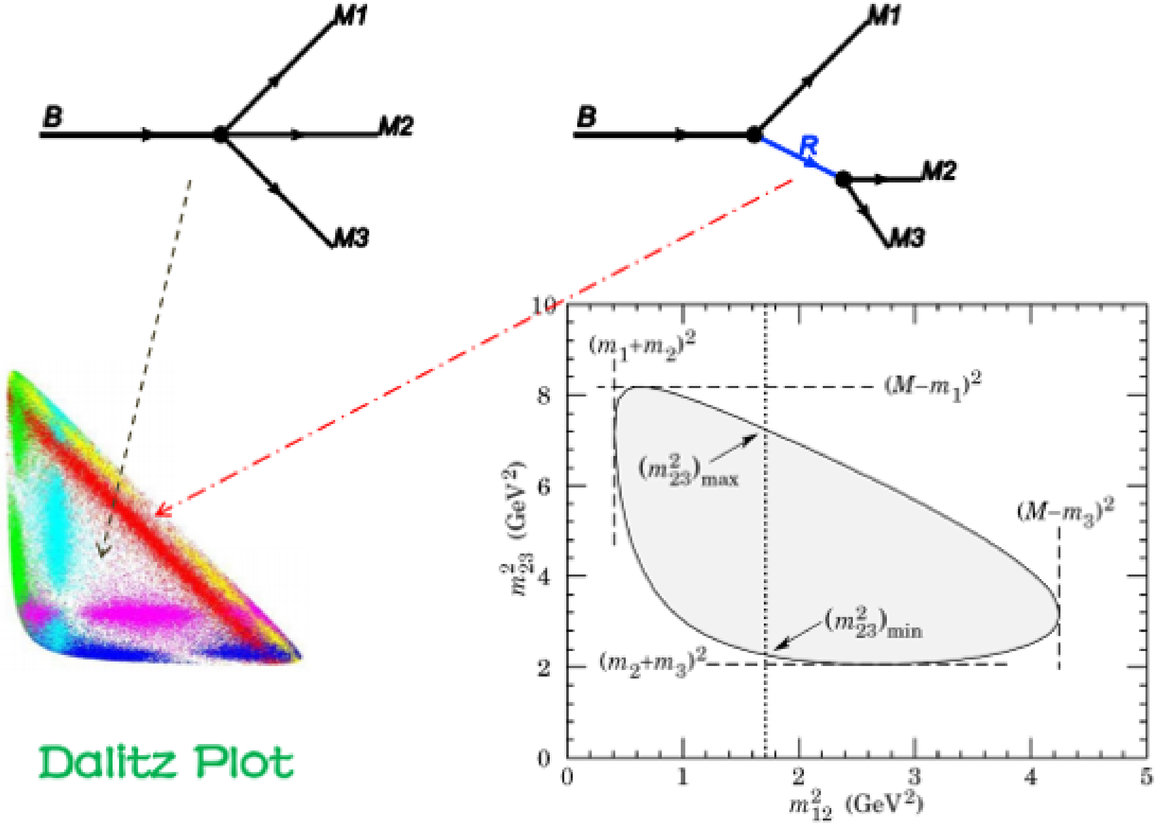

■ 1→3 decay modes




■ 1→2→3 decay mode

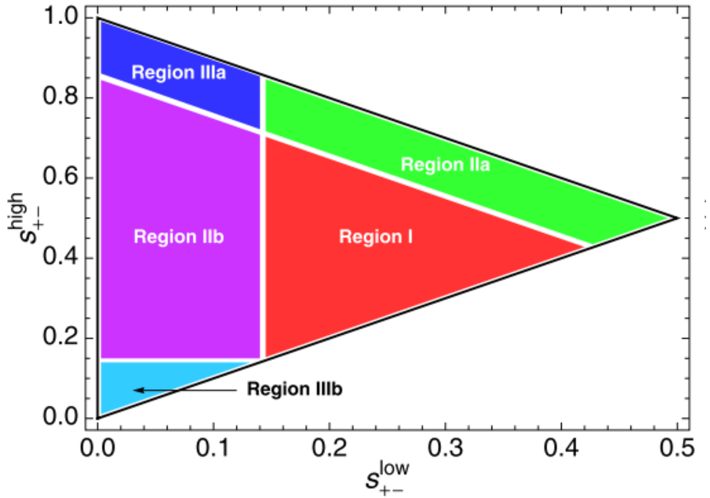


$$p_{ij} = p_i + p_j, \quad m_{ij}^2 = p_{ij}^2$$



**Thomas Mannel**, Susanne Kränkl, Javier Virto

Nuclear Physics B 899 (2015) 247



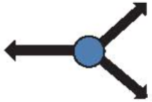

$$B^+(p) \to \pi^+(k_1)\, \pi^-(k_2)\, \pi^+(k_3)$$

$$s_{ij} \equiv \frac{(k_i + k_j)^2}{m_B^2} = \frac{2k_i \cdot k_j}{m_B^2} \quad (i \neq j)\,.$$

$$s_{12} \equiv s_{+-}^{\rm low},\; s_{13} \equiv s_{++} \text{ and } s_{23} \equiv s_{+-}^{\rm high}$$

$$s_{12} + s_{13} + s_{23} = 1 \text{ and } 0 \leq s_{ij} \leq 1.$$

**Region I:** $s_{++} \sim s_{+-}^{\rm low} \sim s_{+-}^{\rm high} \sim 1/3$



**Region IIa:** $s_{++} \sim 0\,, \quad s_{+-}^{\rm low} \sim s_{+-}^{\rm high} \sim 1/2$

**Region IIb:** $s_{+-}^{\rm low} \sim 0\,, \quad s_{++} \sim s_{+-}^{\rm high} \sim 1/2$

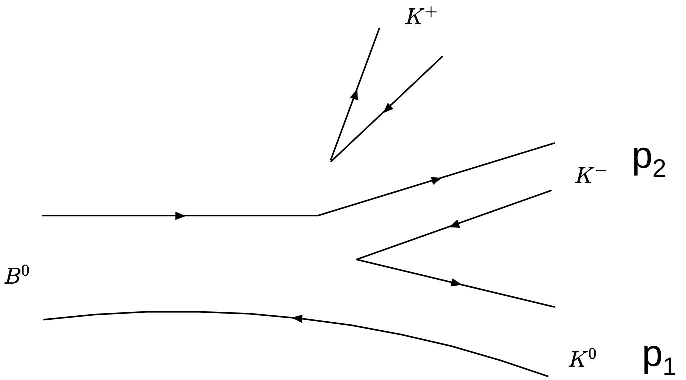
**Region IIIa:** $s_{++} \sim s_{+-}^{\rm low} \sim 0\,, \quad s_{+-}^{\rm high} \sim 1$

**Region IIIb:** $s_{+-}^{\rm high} \sim s_{+-}^{\rm low} \sim 0\,, \quad s_{++} \sim 1$

# Nonresonance

- **Heavy meson chiral pertuabation theory (HMChPT)**

$$A^{NR} = A^{HMChPT}_{transition}\, e^{-\alpha_{NR}\, p_B.(p_1-p_2)}\, e^{i\phi_{12}}$$




**--HMChPT is applicable only to soft mesons**

**--HMChPT is recovered in soft meson limit, $p_1$, $p_2 \rightarrow 0$**

**--The parameter $\alpha_{NR} \sim 1/(2m_B\Lambda_\chi)$ is constrained from $B^- \rightarrow \pi^+\pi^-\pi^-$**

# Nonresonance

- **U spin symmetry** (s $\leftrightarrow$ d) predictions for the relative signs between $K^-K^+K^-$ & $\pi^-\pi^+\pi^-$ and between $K^-\pi^+\pi^-$ & $\pi^-K^+K^-$ agree with experiment:

*U*-spin analysis of CP violation in $B^-$ decays into three charged light pseudoscalar mesons

Dong Xu [a], Guan-Nan Li [b], Xiao-Gang He [a,b,c,*]

Physics Letters B 728 (2014) 579 - 584

$$\frac{A_{CP}(B^- \to \pi^-\pi^+\pi^-)}{A_{CP}(B^- \to K^-K^+K^-)} = -\frac{\Gamma(B^- \to K^-K^+K^-)}{\Gamma(B^- \to \pi^-\pi^+\pi^-)}, \qquad \frac{A_{CP}(B^- \to \pi^-K^+K^-)}{A_{CP}(B^- \to K^-\pi^+\pi^-)} = -\frac{\Gamma(B^- \to K^-\pi^+\pi^-)}{\Gamma(B^- \to \pi^-K^+K^-)}$$

TABLE I. LHCb results of direct $CP$ asymmetries (in %) for various charmless three-body $B^-$ decays. The superscripts "incl" "low" and "resc"denote $CP$ asymmetries measured in full phase space, in the low invariant mass regions specified in Eq. (1.1) and in the rescattering regions with $1.0 < m_{\pi^+\pi^-,K^+K^-} < 1.5$ GeV, respectively. Data are taken from [6,7] for $\mathcal{A}_{CP}^{\rm low}$ and from [8] for $\mathcal{A}_{CP}^{\rm incl}$ and $\mathcal{A}_{CP}^{\rm resc}$.

| | $\pi^+\pi^-\pi^-$ | $K^+K^-\pi^-$ | $K^-\pi^+\pi^-$ | $K^-K^+K^-$ |
|---|---|---|---|---|
| $\mathcal{A}_{CP}^{\rm incl}$ | $5.8 \pm 0.8 \pm 0.9 \pm 0.7$ | $-12.3 \pm 1.7 \pm 1.2 \pm 0.7$ | $2.5 \pm 0.4 \pm 0.4 \pm 0.7$ | $-3.6 \pm 0.4 \pm 0.2 \pm 0.7$ |
| $\mathcal{A}_{CP}^{\rm low}$ | $58.4 \pm 8.2 \pm 2.7 \pm 0.7$ | $-64.8 \pm 7.0 \pm 1.3 \pm 0.7$ | $67.8 \pm 7.8 \pm 3.2 \pm 0.7$ | $-22.6 \pm 2.0 \pm 0.4 \pm 0.7$ |
| $\mathcal{A}_{CP}^{\rm resc}$ | $17.2 \pm 2.1 \pm 1.5 \pm 0.7$ | $-32.8 \pm 2.8 \pm 2.9 \pm 0.7$ | $12.1 \pm 1.2 \pm 1.7 \pm 0.7$ | $-21.1 \pm 1.1 \pm 0.4 \pm 0.7$ |

**References:**


R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 111, 101801 (2013) ,
Phys. Rev. Lett. 112, 011801 (2014) , Phys. Rev. D 90, 112004 (2014) .

# Nonresonance

- **PQCD** (with help of **two-meson distribution amplitudes**)

PHYSICAL REVIEW D **89**, 074031 (2014)

**Direct *CP* asymmetries of three-body *B* decays in perturbative QCD**

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

$$A_{CP}^{reg}(B^\pm \to \pi^+\pi^-\pi^\pm)$$

$$= 0.519^{+0.124}_{-0.219}(\omega_B)^{+0.108}_{-0.091}(a_2^\pi)^{+0.027}_{-0.032}(m_0^\pi),$$ PQCD

$$= 0.584 \pm 0.082 \pm 0.027 \pm 0.007$$ LHCb

LHCb PRL112-011801 (2014)

$$\begin{cases} F_\pi(w^2) = \dfrac{m^2 \exp[i\delta_1^1(w)]}{w^2 + m^2}, \\ F_t(w^2) = \dfrac{m_0^\pi m^2 \exp[i\delta_1^1(w)]}{w^3 + m_0^\pi m^2}, \\ F_s(w^2) = \dfrac{m_0^\pi m^2 \exp[i\delta_0^0(w)]}{w^3 + m_0^\pi m^2}, \end{cases}$$

# Resonance

- **Factorization Approach:**

PHYSICAL REVIEW D **94**, 094015 (2016)

**Direct *CP* violation in charmless three-body decays of *B* mesons**

Hai-Yang Cheng,[1] Chun-Khiang Chua,[2] and Zhi-Qing Zhang[3]

$$\langle P_1(p_1)P_2(p_2)|(\bar{q}b)_{V-A}|B\rangle^R = \sum_i \langle P_1P_2|V_i\rangle \frac{1}{s_{12}-m_{V_i}^2+im_{V_i}\Gamma_{V_i}}\langle V_i|(\bar{q}b)_{V-A}|B\rangle + \sum_i \langle P_1P_2|S_i\rangle \frac{-1}{s_{12}-m_{S_i}^2+im_{S_i}\Gamma_{S_i}}\langle S_i|(\bar{q}b)_{V-A}|B\rangle,$$

$$\langle P_1P_2|\bar{q}_1\gamma_\mu q_2|0\rangle^R = \sum_i \langle P_1P_2|V_i\rangle \frac{1}{s_{12}-m_{V_i}^2+im_{V_i}\Gamma_{V_i}}\langle V_i|\bar{q}_1\gamma_\mu q_2|0\rangle, + \sum_i \langle P_1P_2|S_i\rangle \frac{-1}{s_{12}-m_{S_i}^2+im_{S_i}\Gamma_{S_i}}\langle S_i|\bar{q}_1\gamma_\mu q_2|0\rangle,$$

$$\langle P_1P_2|\bar{q}_1 q_2|0\rangle^R = \sum_i \langle P_1P_2|S_i\rangle \frac{-1}{s_{12}-m_{S_i}^2+im_{S_i}\Gamma_{S_i}}\langle S_i|\bar{q}_1 q_2|0\rangle,$$

**$V_i$ = $\phi,\rho,\omega$,… ,**
**$S_i$ =$f_0(980)$, $f_0(1370)$, $f_0(1500)$,…**
**for $P_1P_2=\pi^+\pi^-$;**

**$V_i$ = $K^*(892)$,$K^*(1410)$,… ,**
**$S_i$ =$K_0^*(1430)$,… for $P_1P_2$= $K^\pm\pi^\mp$**

# Resonance

- **PQCD:**

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Physics Letters B 561 (2003) 258–265

PHYSICS LETTERS B

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## Three-body nonleptonic $B$ decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li

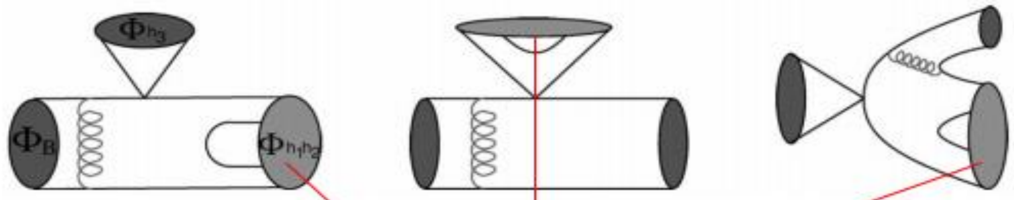
**Abstract**

We develop perturbative QCD formalism for three-body nonleptonic $B$ meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

Physics Letters B 561 (2003) 258–265

# Three-body nonleptonic $B$ decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li




two-meson distribution amplitudes

**TMDA**

$$B \to h_1 h_2 h_3$$

$$\Rightarrow \quad \mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1h_2} \otimes \Phi_{h_3}$$

$$\Phi^{\mathrm{P}}_{\pi\pi} = \frac{1}{\sqrt{2N_c}}\left[\not{p}\,\Phi^{I=1}_{v\nu=-}(z,\zeta,\omega^2) + \omega\Phi^{I=1}_{s}(z,\zeta,\omega^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{w(2\zeta-1)}\Phi^{I=1}_{t\nu=+}(z,\zeta,\omega^2)\right],$$

# 03 Puzzle

# CP violation in $B^+ \to \pi^+\rho^0 \to \pi^+\pi^-\pi^+$

## Quasi-two-body decays $B_{(s)} \to P\rho \to P\pi\pi$ in the perturbative QCD approach

Ya Li,[1,†] Ai-Jun Ma,[1,‡] Wen-Fei Wang,[2,*] and Zhen-Jun Xiao[1,3,§]

| Modes | | Quasi-two-body results | Experiment |
|---|---|---|---|
| $B^+ \to \pi^+(\rho^0 \to)\pi^+\pi^-$ | $\mathcal{B}(10^{-6})$ | $8.84^{+1.48}_{-1.24}(\omega_B)^{+0.12}_{-0.13}(a^t_{2\rho})^{+1.17}_{-1.11}(a^s_{2\rho})^{+0.25}_{-0.26}(a^0_{2\rho})$ | $8.30 \pm 1.20$ |
| | $\mathcal{A}_{CP}(\%)$ | $-27.5^{+2.3}_{-3.1}(\omega_B)^{+0.9}_{-1.0}(a^t_{2\rho}) \pm 1.4(a^s_{2\rho}) \pm 0.9(a^0_{2\rho})$ | $18.0^{+9.0}_{-17.0}$ |

**LHCb and BABAR measurements for this quantity, however, prefer a positive CP asymmetry in the m($\pi^+\pi^-$) region peaked at $m_\rho$ .**

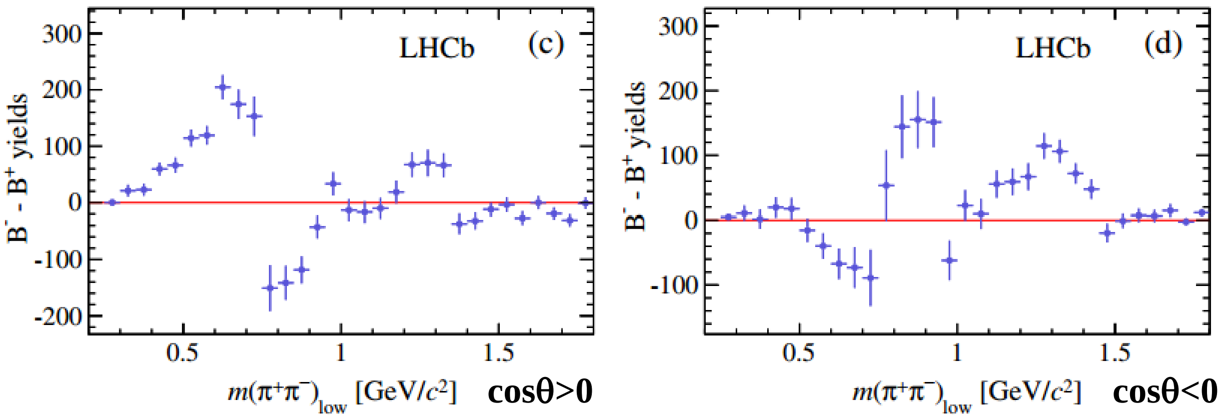
**The theoretical predictions based on the QCDF, PQCD and SCET all give a negative CP asymmetry of order -0.20 for $B^+ \to \rho^0\pi^+$ .**

PHYSICAL REVIEW D **90,** 112004 (2014)

## Measurements of *CP* violation in the three-body phase space of charmless $B^{\pm}$ decays

R. Aaij *et al.*[*]
(LHCb Collaboration)
(Received 25 August 2014; published 11 December 2014)

LHCb has measured CP asymmetries in regions dominated by vector resonances




**Summing over regions I-IV yields CP asymmetry consistent with zero with slightly positive central value**

**I: $0.47 < m(\pi^+\pi^-)_{low} < 0.77$ GeV, $\cos\theta > 0$,**
**II: $0.77 < m(\pi^+\pi^-)_{low} < 0.92$ GeV, $\cos\theta > 0$,**
**III: $0.47 < m(\pi^+\pi^-)_{low} < 0.77$ GeV, $\cos\theta < 0$,**
**IV: $0.77 < m(\pi^+\pi^-)_{low} < 0.92$ GeV, $\cos\theta < 0$.**

**$A_{CP}$ changes sign at $m(\pi^+\pi^-) \sim m_\rho$**

# Observation of several sources of $CP$ violation in $B^+ \to \pi^+\pi^+\pi^-$ decays

LHCb collaboration **arXiv:1909.05211**

| Contribution | Fit fraction ($10^{-2}$) | $A_{CP}$ ($10^{-2}$) | $B^+$ phase (°) | $B^-$ phase (°) |
|---|---|---|---|---|
| Isobar model | | | | |
| $\rho(770)^0$ | $55.5 \pm 0.6 \pm 2.5$ | $+0.7 \pm 1.1 \pm 1.6$ | — | — |
| K-matrix | | | | |
| $\rho(770)^0$ | $56.5 \pm 0.7 \pm 3.4$ | $+4.2 \pm 1.5 \pm 6.4$ | — | — |
| QMI | | | | |
| $\rho(770)^0$ | $54.8 \pm 1.0 \pm 2.2$ | $+4.4 \pm 1.7 \pm 2.8$ | — | — |

**A quasi-two-body CP asymmetry is consistent with zero.**

# Possible reasons:

- **Interference between ρ and $f_0(500)$**

PHYSICAL REVIEW D **87,** 076007 (2013)

***CP* violation in $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ in the region with low invariant mass of one $\pi^{+}\pi^{-}$ pair**

Zhen-Hua Zhang,[1,*] Xin-Heng Guo,[2,†] and Ya-Dong Yang[1,‡]

- **The fraction of tree and penguin contributions varys across the phase space**

PHYSICAL REVIEW D **88,** 114014 (2013)

**Branching fractions and direct *CP* violation in charmless three-body decays of *B* mesons**

Hai-Yang Cheng[1] and Chun-Khiang Chua[2]

# 04 Outlook

# Goals

- Develop a systematic theoretical approach to 3-body hadronic B decays in the whole phase space (both resonance and nonresonance)

- Three-body B decays receive sizable NR contributions. In general, NR contributions alone yield large CP-violating effects.  How to resolve the NR contributions reliably?

- It is important to pin down the mechanism responsible for regional CP asymmetries.

# Extraction of the SM parameters

## Determination of ϕ1 ($\beta$) :

$$\phi_1 \equiv \arg[-V_{cb}^{*}V_{cd}/(V_{tb}^{*}V_{td})]$$

**sin 2 ϕ1 from $b \to c\bar{c}s$**

$\mathbf{B \to J/\psi\phi,\ \phi \to KK}$
$\mathbf{B \to J/\psi K^{*},\ K^{*} \to K_S^0\pi^0}$
$\mathbf{B \to J/\psi f_0(980),\ f_0(980) \to \pi^+\pi^-}$

## Determination of ϕ2 ($\alpha$):

$$\phi_2 \equiv \arg[-V_{tb}^{*}V_{td}/V_{ub}^{*}V_{ud}]$$

$\mathbf{B \to \rho\pi \to \pi^+\pi^-\pi^0}$
$\mathbf{B \to \rho\rho,\ \rho \to \pi\pi}$

**Isospin symmetry**

# Determination of $\phi 3$ ($\gamma$) : $\phi_3 \equiv -\arg\left(V_{ub}^* V_{ud}/V_{cb}^* V_{cd}\right)$

**$B \to PPP, P = \pi, K$**

**U-spin**

**Diagrammatic analysis**

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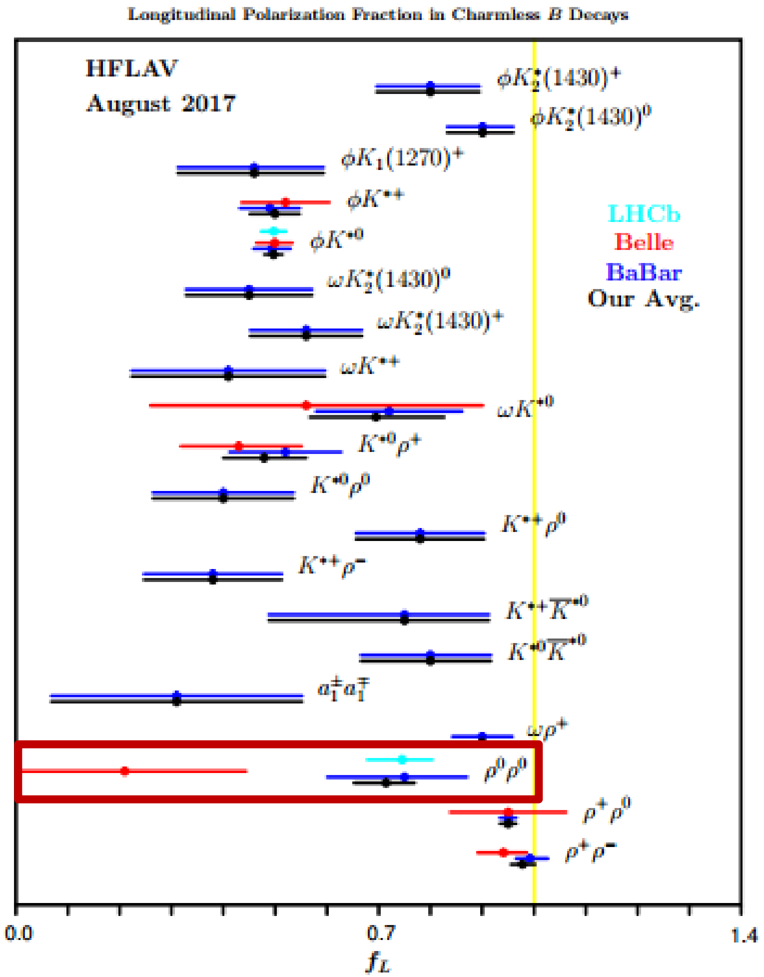
**$B \to DP$**

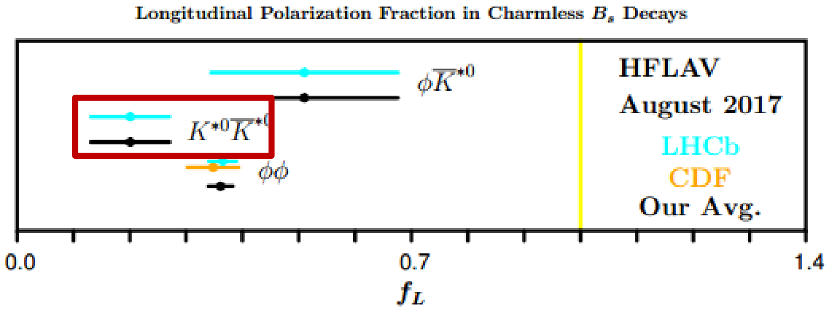
Table 100: Methods and $D$ decay modes used in $B^- \to DK^-$ and $B^- \to D^*K^-$ measurements. Those in parentheses have not been published by Belle.

| Type of $D$ decay | Method name | $D$ final states studied |
|---|---|---|
| $CP$-eigenstates | GLW | $CP$-even: $K^+K^-$, $\pi^+\pi^-$; $CP$-odd $K_S^0\pi^0$, $K_S^0\eta$ |
| CF and DCS | ADS | $K^\pm\pi^\mp$, $K^\pm\pi^\mp\pi^0$, $(K^\pm\pi^\mp\pi^+\pi^-)$ |
| Self-conjugate | GGSZ | $K_S^0\pi^+\pi^-$, $(K_S^0K^+K^-)$, $(\pi^+\pi^-\pi^0)$, $(K^+K^-\pi^0)$, $(\pi^+\pi^-\pi^+\pi^-)$ |
| SCS | GLS | $(K_S^0K^\pm\pi^\mp)$ |

# $B_{(s)} \to VV$



Longitudinal Polarization Fraction in Charmless B Decays
HFLAV
August 2017
LHCb
Belle
BaBar
Our Avg.
0.0
0.7
1.4
fL

Longitudinal Polarization Fraction in Charmless Bs Decays
HFLAV
August 2017
LHCb
CDF
Our Avg.
0.0
0.7
1.4
fL

**These decay modes with large transverse polarization fraction (around 50%) will provide further insight into the QCD dynamics that governs the different helicity amplitudes .**

$B \to K^*\phi,\ B \to K^*\rho,\ B^0 \to \rho^0\omega, B^0 \to \omega\omega, B^0 \to \rho^0\rho^0,$
$B^0 \to K^{*+}K^{*-}, B^- \to \phi\rho^-, B^0 \to \phi\rho^0,$
$B_s \to K^*\phi, B_s \to \phi\phi$

# Large local CP asymmetries

TABLE I. LHCb results of direct $CP$ asymmetries (in %) for various charmless three-body $B^-$ decays. The superscripts "incl" "low" and "resc" denote $CP$ asymmetries measured in full phase space, in the low invariant mass regions specified in Eq. (1.1) and in the rescattering regions with $1.0 < m_{\pi^+\pi^-,K^+K^-} < 1.5$ GeV, respectively. Data are taken from [6,7] for $\mathcal{A}_{CP}^{\rm low}$ and from [8] for $\mathcal{A}_{CP}^{\rm incl}$ and $\mathcal{A}_{CP}^{\rm resc}$.

| | $\pi^+\pi^-\pi^-$ | $K^+K^-\pi^-$ | $K^-\pi^+\pi^-$ | $K^-K^+K^-$ |
|---|---|---|---|---|
| $\mathcal{A}_{CP}^{\rm incl}$ | $5.8 \pm 0.8 \pm 0.9 \pm 0.7$ | $-12.3 \pm 1.7 \pm 1.2 \pm 0.7$ | $2.5 \pm 0.4 \pm 0.4 \pm 0.7$ | $-3.6 \pm 0.4 \pm 0.2 \pm 0.7$ |
| $\mathcal{A}_{CP}^{\rm low}$ | $58.4 \pm 8.2 \pm 2.7 \pm 0.7$ | $-64.8 \pm 7.0 \pm 1.3 \pm 0.7$ | $67.8 \pm 7.8 \pm 3.2 \pm 0.7$ | $-22.6 \pm 2.0 \pm 0.4 \pm 0.7$ |
| $\mathcal{A}_{CP}^{\rm resc}$ | $17.2 \pm 2.1 \pm 1.5 \pm 0.7$ | $-32.8 \pm 2.8 \pm 2.9 \pm 0.7$ | $12.1 \pm 1.2 \pm 1.7 \pm 0.7$ | $-21.1 \pm 1.1 \pm 0.4 \pm 0.7$ |

**References:**

R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 111, 101801 (2013) ,
Phys. Rev. Lett. 112, 011801 (2014) , Phys. Rev. D 90, 112004 (2014) .

# Final-state (KK↔ $\pi\pi$ ) rescattering

It has been conjectured that CPT theorem & final-state rescattering of $\pi^+\pi^-$ $\leftrightarrow$ $K^+K^-$ may play important roles to explain the CP correlation observed by LHCb. Consider $\pi^+\pi^-$ & $K^+K^-$ rescattering and neglect possible interactions with 3rd meson

Bediaga et al, Phys. Rev. D 89, 094013 (2014)

$$\begin{pmatrix} A(B^- \to \pi^+\pi^- P^-) \\ A(B^- \to K^+K^- P^-) \end{pmatrix}_{FSI} = S^{1/2} \begin{pmatrix} A(B^- \to \pi^+\pi^- P^-) \\ A(B^- \to K^+K^- P^-) \end{pmatrix} \quad \text{with } P = \pi, K$$

Suzuki, Wolfenstein Phys. Rev. D 60, 074019 (1999)

$$S = \begin{pmatrix} \eta e^{2i\delta_{\pi\pi}} & i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi}+\delta_{K\bar{K}})} \\ i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi}+\delta_{K\bar{K}})} & \eta e^{2i\delta_{K\bar{K}}} \end{pmatrix}$$

$\eta$: inelasticity, assuming $\delta_{K\underline{K}} = \delta_{\pi\pi}$

$$S^{1/2} = e^{i\delta_{\pi\pi}} \begin{pmatrix} \cos\phi & i\sin\phi \\ i\sin\phi & \cos\phi \end{pmatrix} \quad \text{with} \quad 2\phi = \tan^{-1}\frac{\sqrt{1-\eta^2}}{\eta}$$

# KK↔ $\pi\pi$ rescattering

PHYSICAL REVIEW D **94**, 094015 (2016)

## Direct *CP* violation in charmless three-body decays of *B* mesons

Hai-Yang Cheng,[1] Chun-Khiang Chua,[2] and Zhi-Qing Zhang[3]

| | Expt (%) | NR + Res | NR+RES+FSI |
|---|---|---|---|
| $(\pi^+\pi^-\pi^-)_{incl}$ | 5.8±1.4 | $8.3^{+1.7}_{-1.9}$ | -15.6 |
| $(K^+K^-\pi^-)_{incl}$ | -12.3±2.2 | $4.9^{+1.1}_{-1.0}$ | 8.1 |
| $(K^-\pi^+\pi^-)_{incl}$ | 2.5±0.9 | $-0.8^{+0.9}_{-0.6}$ | 0.7 |
| $(K^+K^-K^-)_{incl}$ | -3.6±0.8 | $-6.0^{+2.0}_{-1.5}$ | -6.1 |
| | | | |
| $(\pi^+\pi^-\pi^-)_{low}$ | 58.4±8.7 | $21.9^{+3.0}_{-3.3}$ | -17.6 |
| $(K^+K^-\pi^-)_{low}$ | -64.8±7.2 | $4.6^{+0.9}_{-1.0}$ | 13.2 |
| $(K^-\pi^+\pi^-)_{low}$ | 67.8±8.5 | $40.7^{+5.9}_{-8.9}$ | 2.3 |
| $(K^+K^-K^-)_{low}$ | -22.6±2.2 | $-16.8^{+4.5}_{-3.9}$ | -16.7 |

**Final-state $\pi^+\pi^-$ ↔ $K^+K^-$ rescattering seems to head in wrong direction**

**Understand data and predict direct CP asymmetries of 3-body decay modes in localized regions of phase space.**

$B^0 \rightarrow \mathrm{K}^+ K^- K_S^0, \mathrm{K}^+ K^- \pi^0, \mathrm{K}^+ \pi^0 \pi^0 \ldots$
$B^+ \rightarrow K_S^0 \pi^+ \pi^0, K_S^0 K_S^0 \mathrm{K}^+, K_S^0 K_S^0 \pi^+ \ldots$

# Thank you

# 欢迎您的批评指正……