QCD speed of sound and QCD thermalization

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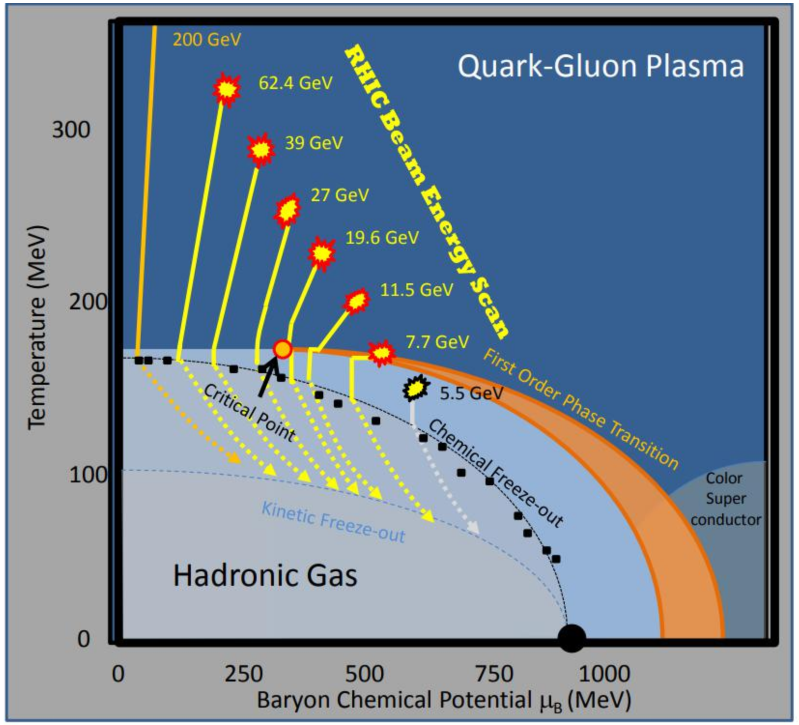
with Yu-Shan Mu, Jing-An Sun, and X-G. Huang, based on 2501.02777 and work in progress

Outline

- Motivation
- QCD thermalization: theory
- QCD thermalization: UCC experiment -- QCD speed of sound
- Extraction QCD speed of sound in UCC, Gaussianity of fluctuations.
- Probe of QCD thermalization in heavy-ion collisions.

Thermalization of a QCD matter is crucial for (almost) all current studies

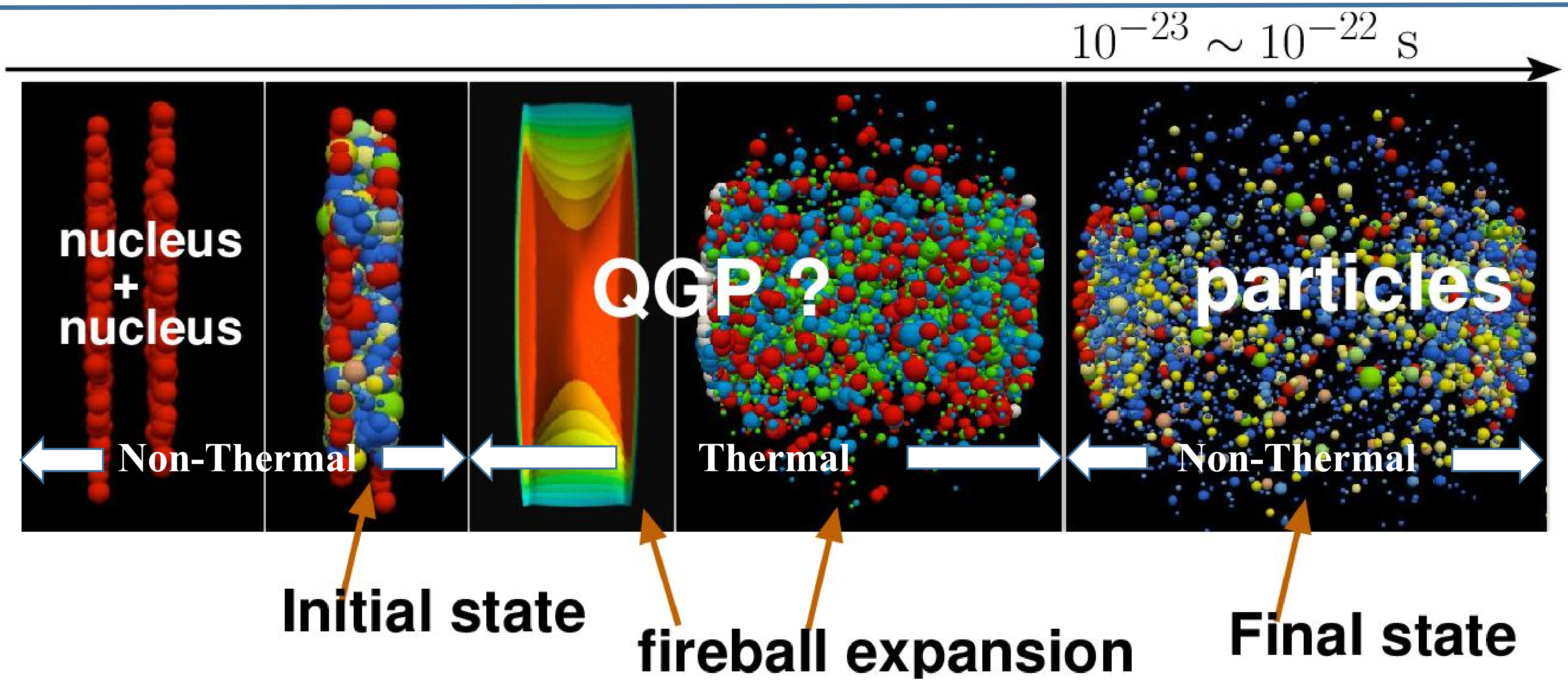


[STAR collaboration]

- QCD phases: QGP, EoS and critical point.
- QCD transport phenomena: eta/s, conductivity, etc.
- Topological and EM QCD effect: CME
- ...

- Does a strongly interacting quantum system thermalize? (QGP, cold atom, condensed matter, ...)
- Any direct probes of QCD thermalization in realistic heavy-ion collisions?

The standard modeling of heavy-ion collisions

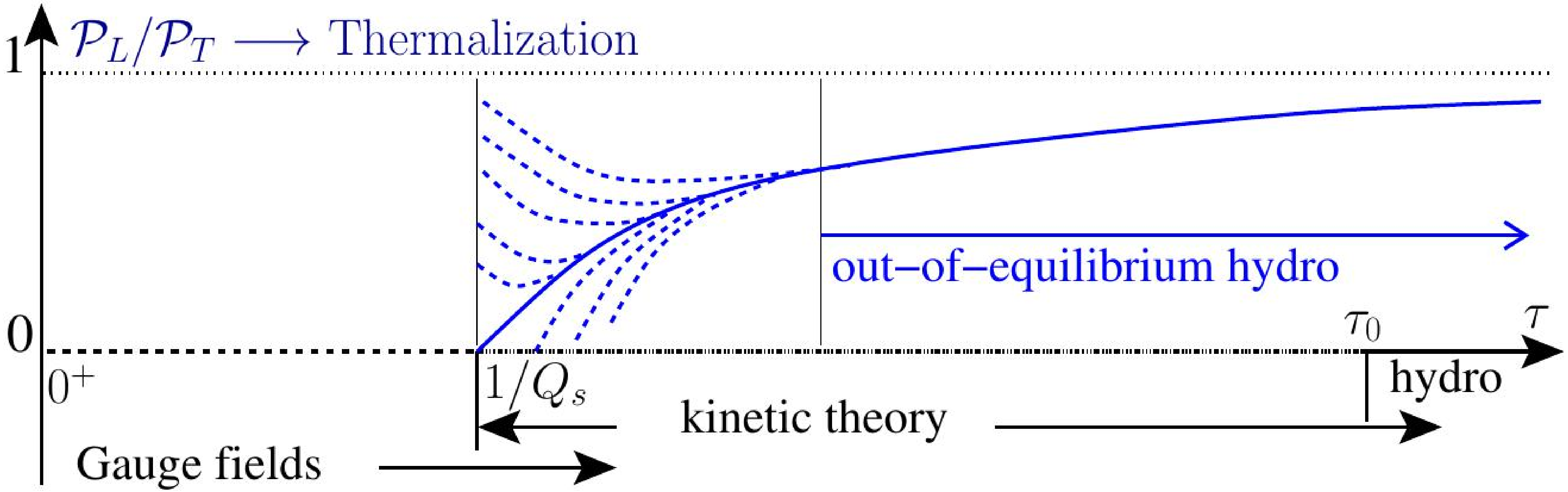


- Indirect signature of (transient) thermalization: collective flow in particle spectrum

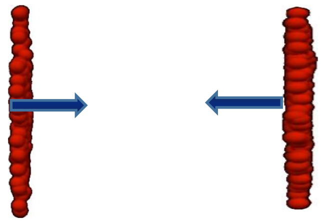
$$\text{Hydro response:} \quad V_n \propto \kappa(\text{EoS}, \eta/s, \ldots)\mathcal{E}_n$$

Thermalization of a QCD matter: semi-classical theory

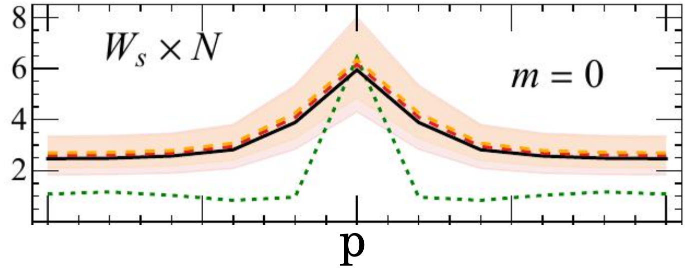
- The emergence of hydro attractor: hydrodynamics, kinetic theory [PRL115,072501(2015)]



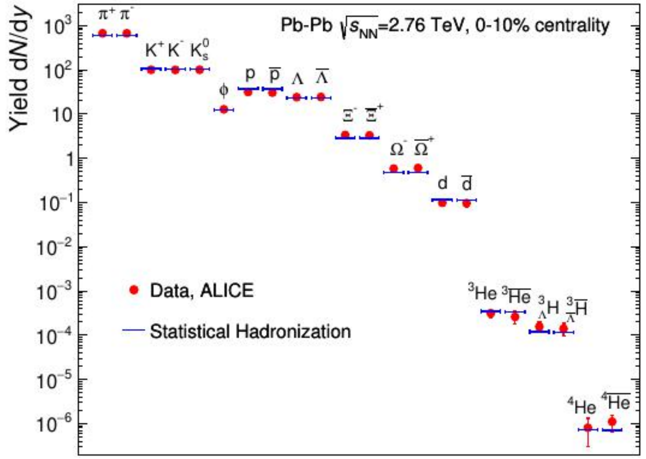
Thermalization of a QCD matter: quantum theory



pure state +
unitary interaction



quantum thermalized QGP



[Nature 561, 321(2018)]

$$\lim_{t\to\infty}\langle\Psi(t)|\hat{O}|\Psi(t)\rangle \propto \mathrm{Tr}[e^{-\hat{H}/T}\hat{O}]$$

- QGP is a high-energy quantum system obeying non-Abelian gauge theory.
- QGP thermalization is beyond perturbative QCD characterization.
- Lattice QCD needs to extend with time evolution.
- Quantum computation: Eigenstate Thermalization Hypothesis.

[S. Chen, Z. Shi and LY, 2412.00662] [PRA43, 2046 (1991); PRE50.888 (1994)]

QCD thermalization: Measurement of QCD speed of sound

System thermalization ⟷ Equation of State

$dP = sdT, \quad de = Tds$ constant volume

$$c_s^2 \equiv \frac{\partial P}{\partial e}\Big|_{\text{Adiabatic}} = \frac{d\ln T}{d\ln s}\Big|_{\text{Adiabatic}} = \frac{d\ln T}{d\ln S}\Big|_{\text{Adiabatic}}$$

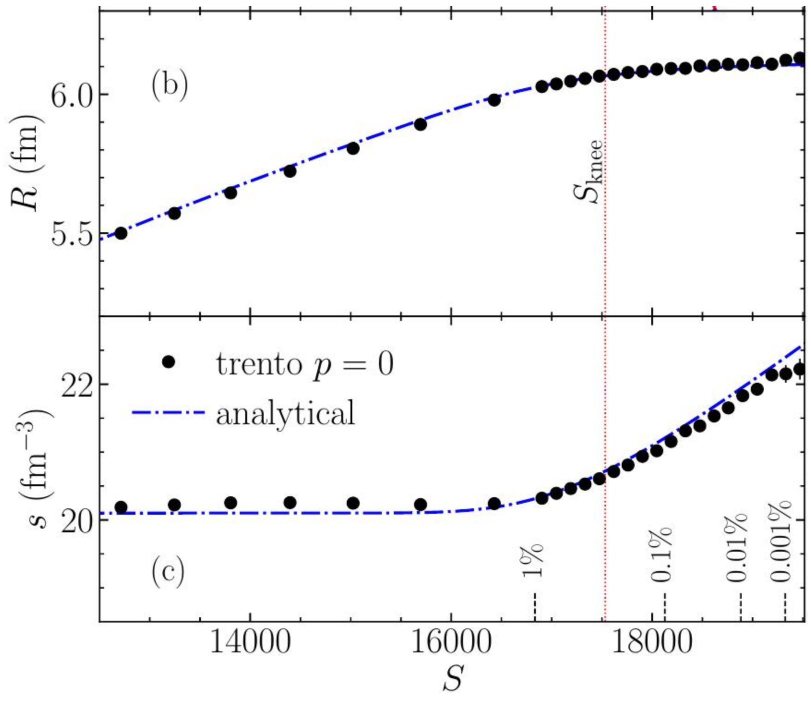
An ideal thought experiment:

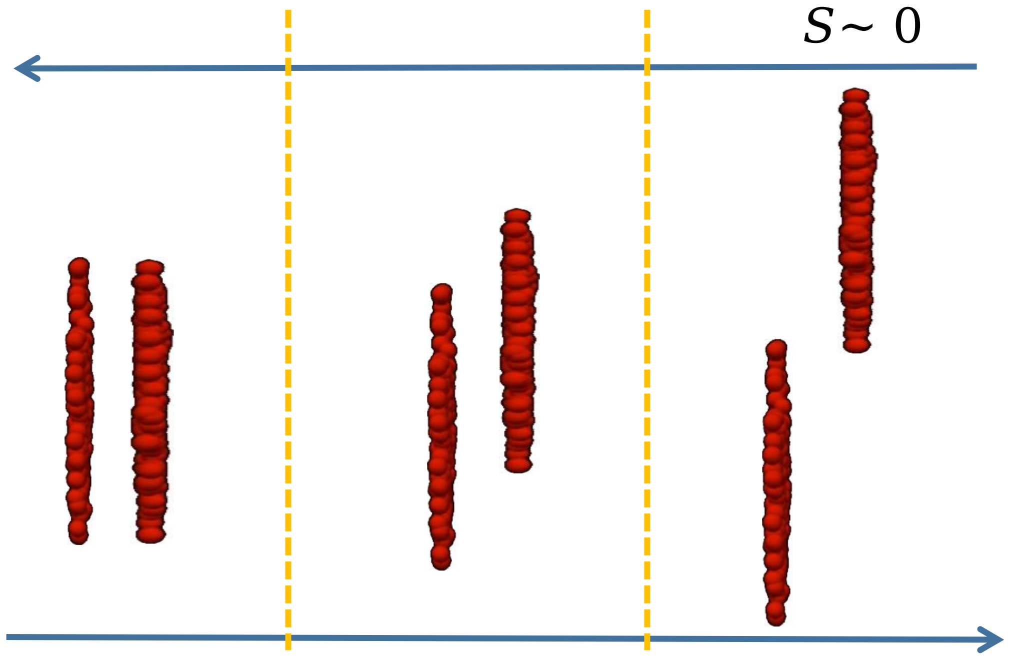
1. A homogeneous QGP system with fixed volume.
2. Injecting energy/entropy without heat transfer. (QM effect)
3. Measure change of temperature according to change of entropy, and take ratio => c_s^2

(S, T, V)

Ultra-central nucleus-nucleus collisions (UCC)

[F. Gardim et al , 1909.11609]





UCC: $b \sim 0$ fm,
~ 0% central

UPC: $b \gtrsim 2r$ fm,
100% central

- Size of the system saturates, volume is fixed.
- but (quantum) fluctuations still play a role: $\Delta S > 0$.
- Optimized collision events for the thermalization of QGP: largest entropy production.
- Ideal for studying the nuclear structure in heavy-ion collisions.

Measurement of QCD speed of sound in UCC

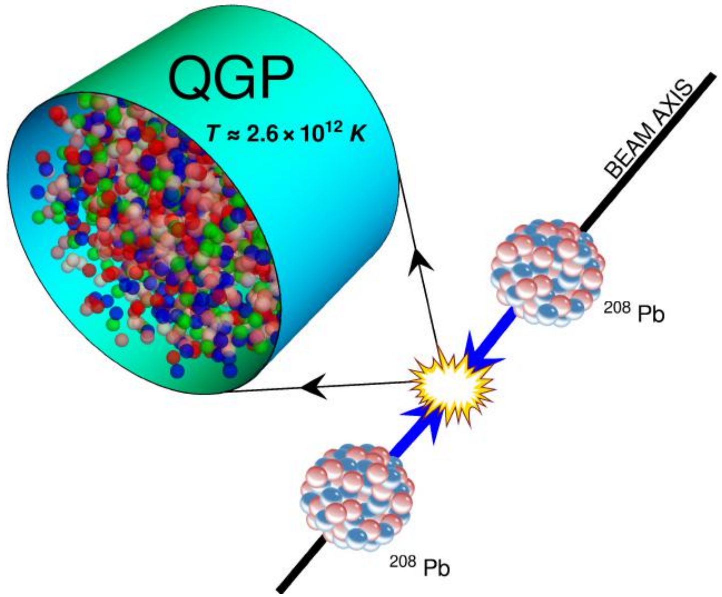
$dP = sdT, \quad de = Tds$ constant volume

$$c_s^2 \equiv \frac{\partial P}{\partial e}\bigg|_{\text{Adiabatic}} = \frac{d\ln T}{d\ln s}\bigg|_{\text{Adiabatic}} = \frac{d\ln T}{d\ln S}\bigg|_{\text{Adiabatic}}$$

Realistic QGP in Heavy-ion collisions:

1. Volume saturates in UCC.
2. Entropy increases due to QM fluctuations (e.g., nucleon scatttering).
3. Non-homogeneous QGP with fixed volume? How to measure temperature and entropy from particles ? Effect of quantum fluctuations?

$$T_{\text{eff}} \propto \langle p_T \rangle \qquad S \propto dN_{\text{ch}}/d\eta$$

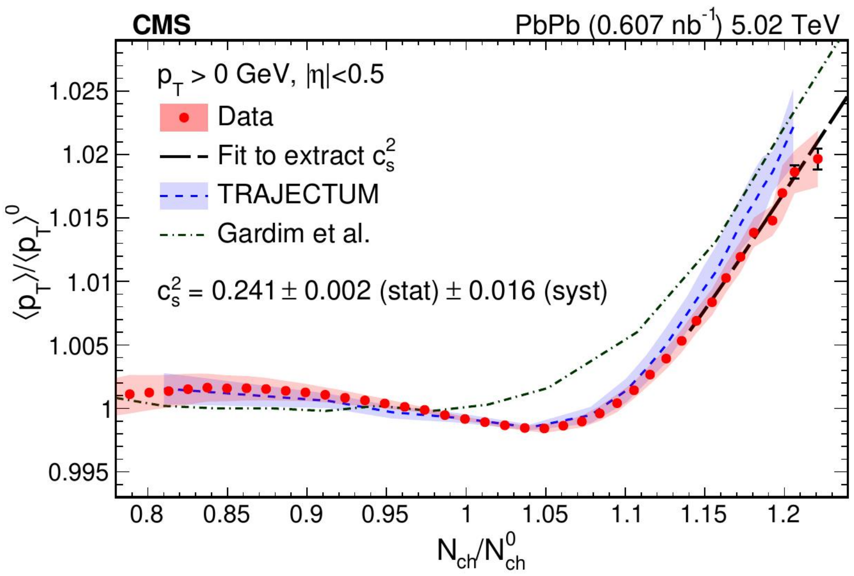


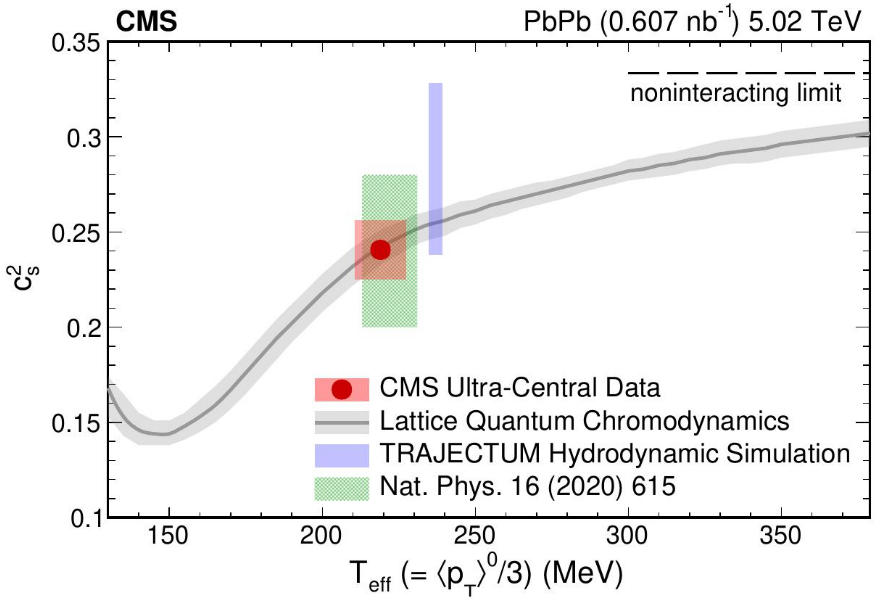
[F. Gardim et al , 1908.09728, Nature Physics]

c_s^2 from UCC experiments in mid-rapidity

$$\begin{cases} \langle p_T \rangle \propto T_{\text{eff}} \\ dN_{\text{ch}}/d\eta \propto S \end{cases} \quad + \quad c_s^2 = \frac{d\ln T}{d\ln S} \Rightarrow \quad \frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N}{N_0} \quad \text{with} \begin{cases} \Delta_p \equiv \langle p_T \rangle - \langle p_T \rangle_0 \\ \Delta_N \equiv N_{ch} - N_0 \end{cases}$$

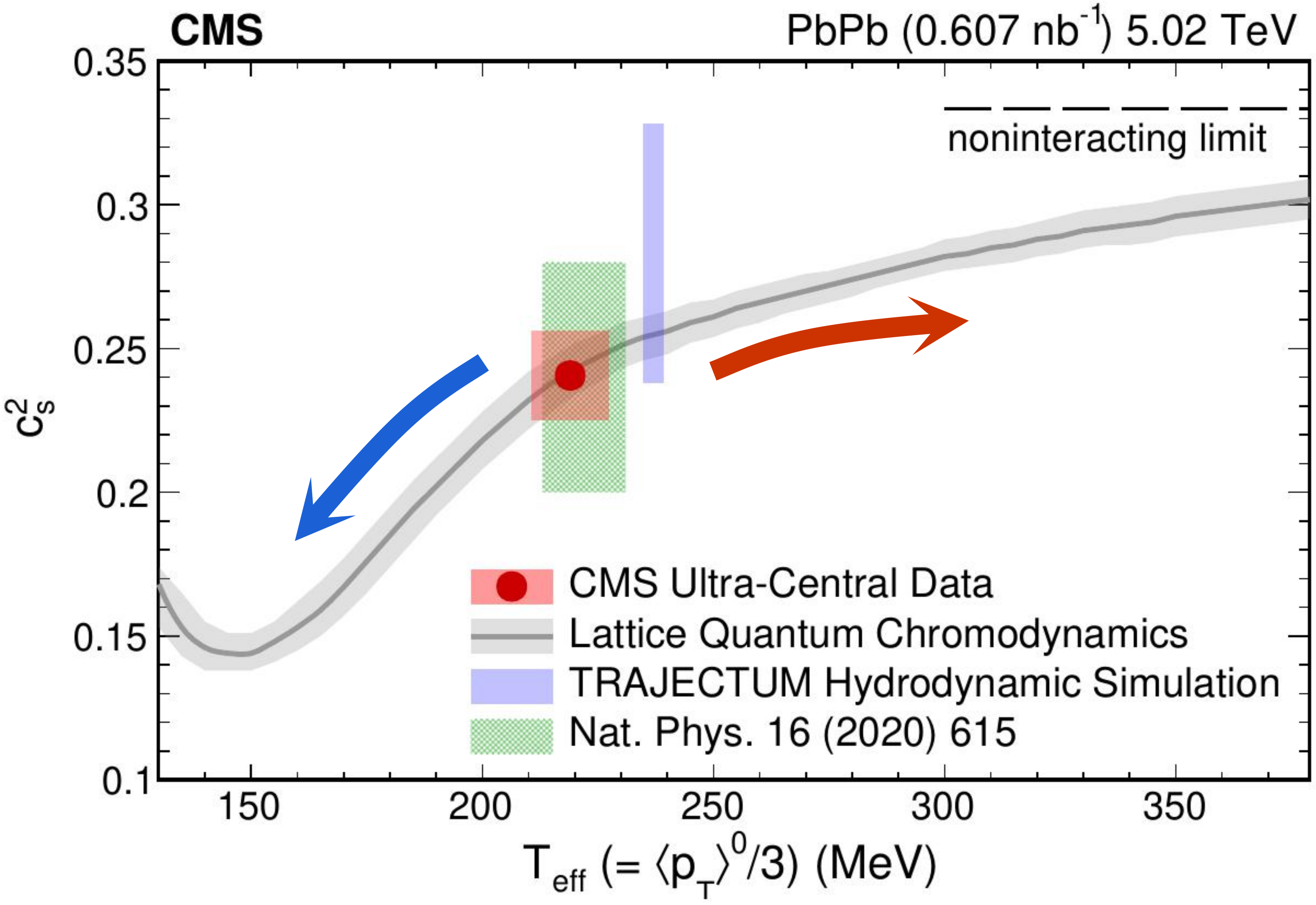
- QCD speed of sound implies a linear response relation: thermodynamic and deterministic.
- Extract c_s from sub-bin measurements: $\frac{\{\Delta_p\}_I}{\langle p_T \rangle_0} = c_s^2 \frac{\{\Delta_N\}_I}{N_0}$, with I labels sub-bin in central events





[CMS collaboration, 2401.06896]

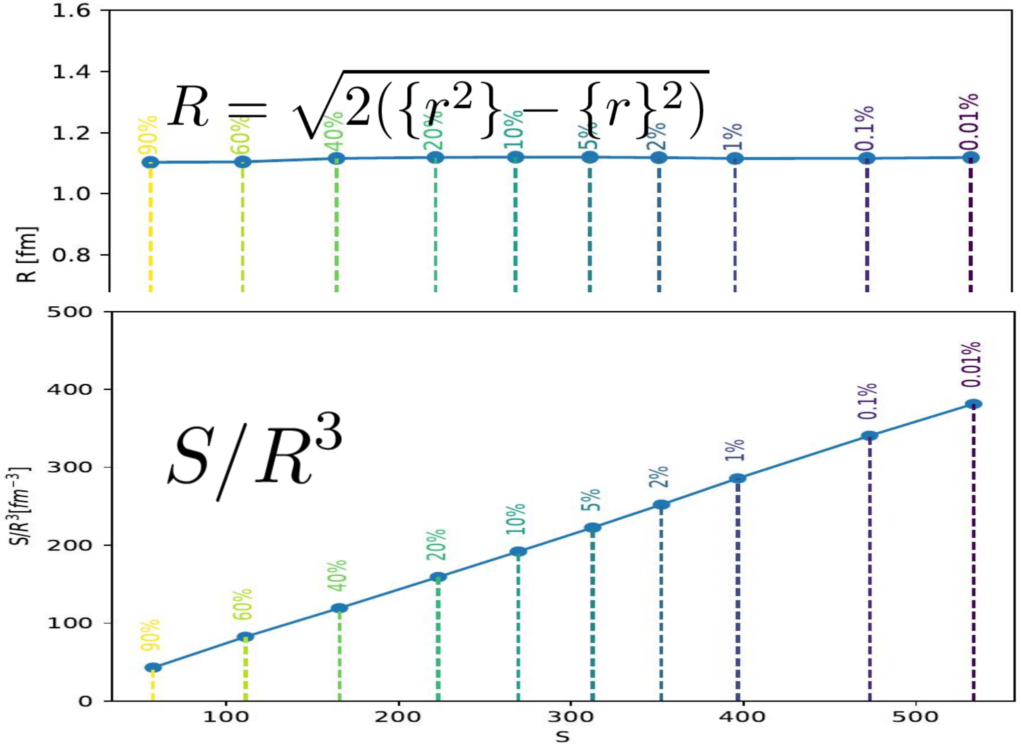
UCC in Small systems: pPb

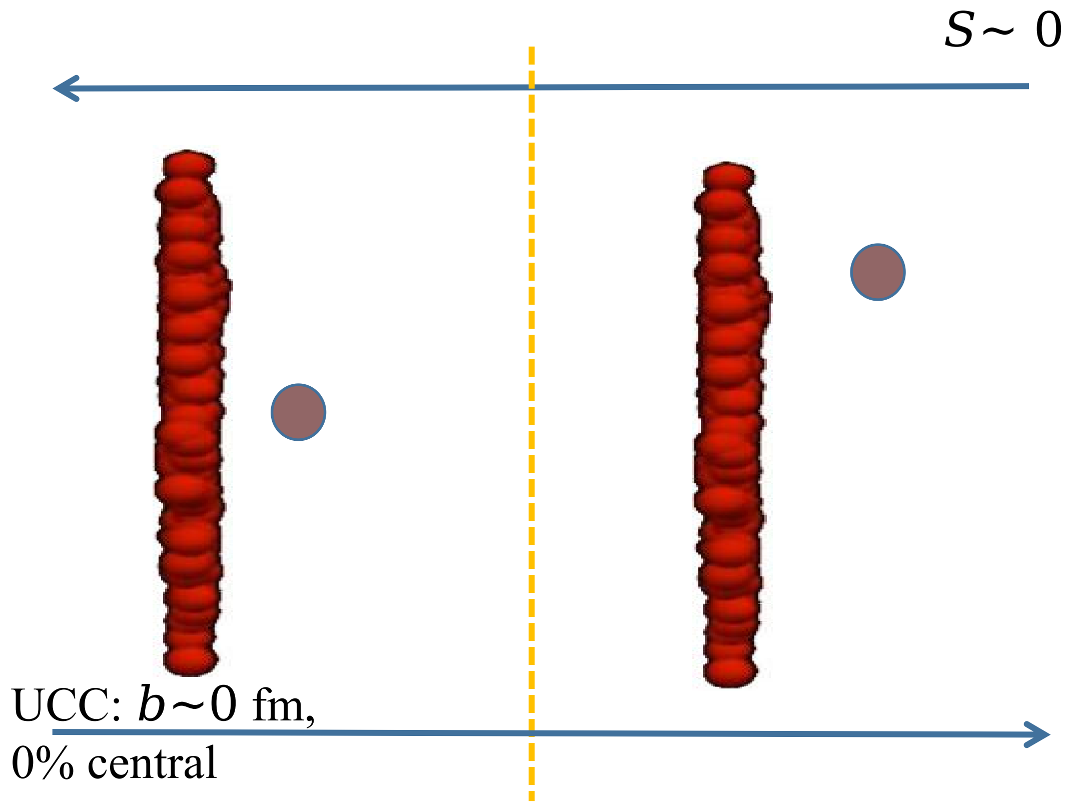


$$\frac{T_{\rm eff}(\rm pPb)}{T_{\rm eff}(\rm PbPb)} \sim \left(\frac{R(\rm PbPb)}{R(\rm pPb)}\right)^{1/3} \sim 1.8$$

Small systems: pPb

Trento 3D and mid-rapidity

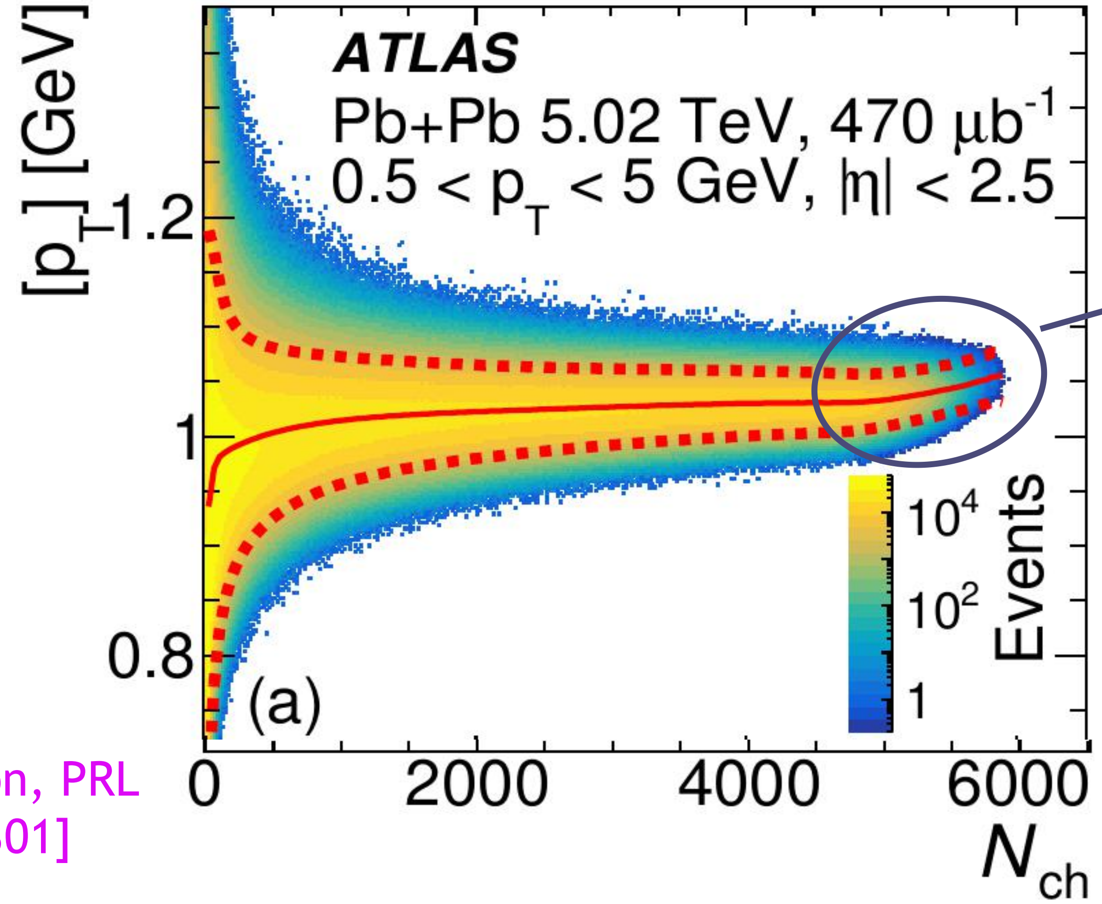




System size saturates (determined by proton size), while entropy always increases towards UCC.

- Significant contributions from **quantum fluctuations** ==> contaminate the extraction of speed of sound.
- No clear evidence of QGP thermalization for all centralities, e.g., 10%?

Realistic measurement involving fluctuations



UCC: tilted tip

[ATLAS collaboration, PRL 133 (2024) 25, 252301]

Event-by-event hybrid hydro modeling, and HIJING and PYTHIA

Tharmal model -- Hybrid hydro modeling:

- We run standard hybrid hydro by fixing mean pT and Nc with respect to experiment UCC:

Trento3D + (3+1D MUSIC + LEOS) + UrQMD

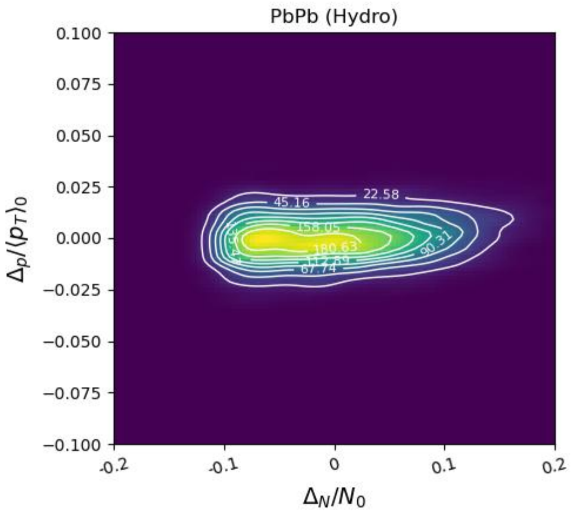
- Trento3D: I.C. quantum noise
- (3+1D MUSIC + LEOS): thermalized QCD system
- UrQMD: non-thermal system

- We focus on mid-rapidity and mean pT cut as in experiments.

Non-thermal model -- HIJING and PYTHIA

When fluctuations are included: two-dimensional joint probability $\mathcal{P}(\Delta_p, \Delta_N)$

PbPb (Hydro)



- Hydro: thermodynamic response and quantum noise

$$\frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \quad \longleftrightarrow \quad \text{thermodynamic resp.} + \text{quantum noise}$$

When fluctuations are included: two-dimensional joint probability $\mathcal{P}(\Delta_p, \Delta_N)$

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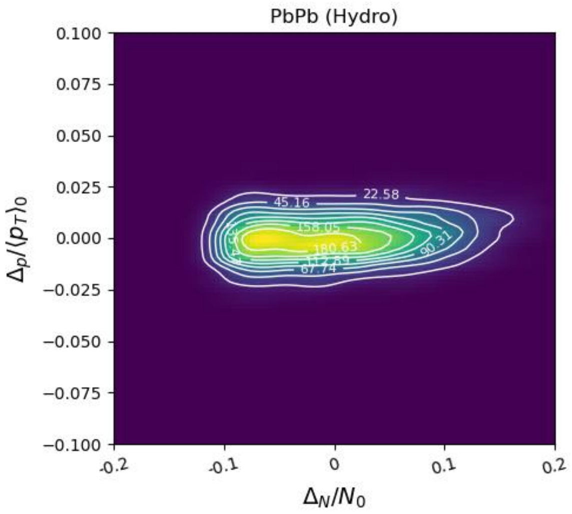
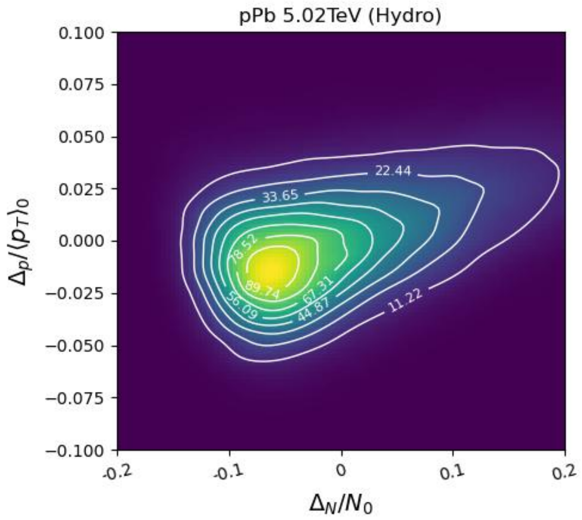


pPb (Hydro)



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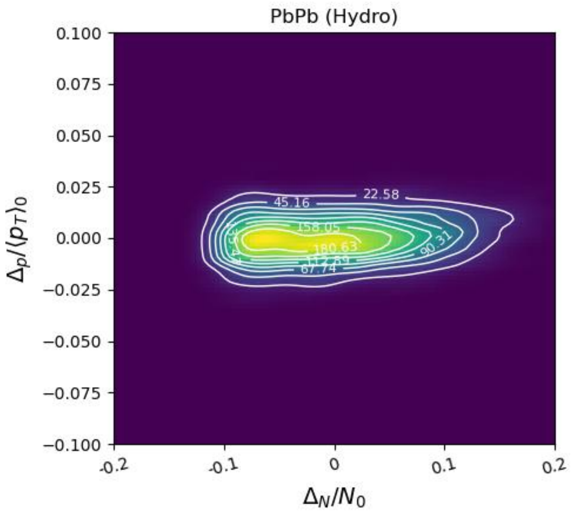
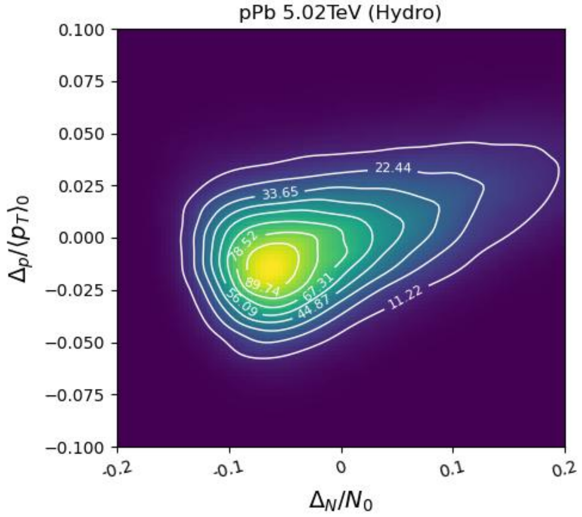
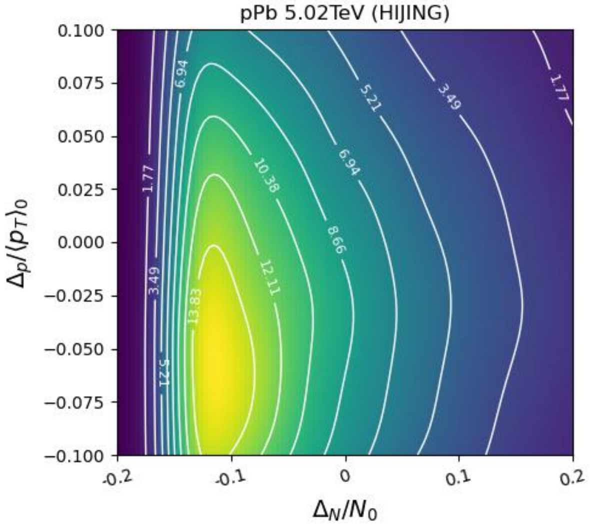


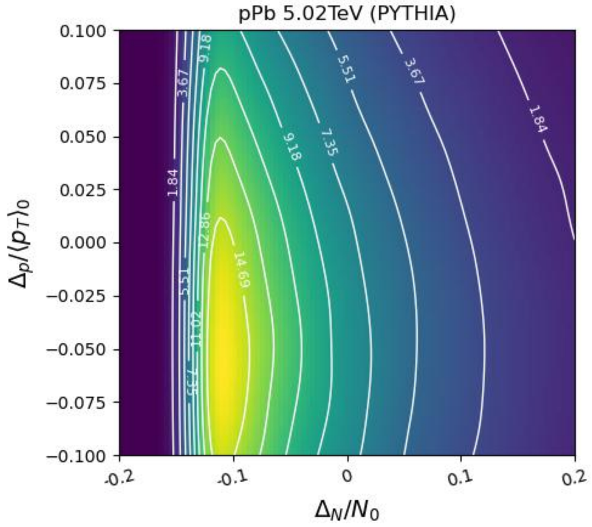
pPb (Hydro)



pPb (HIJING)



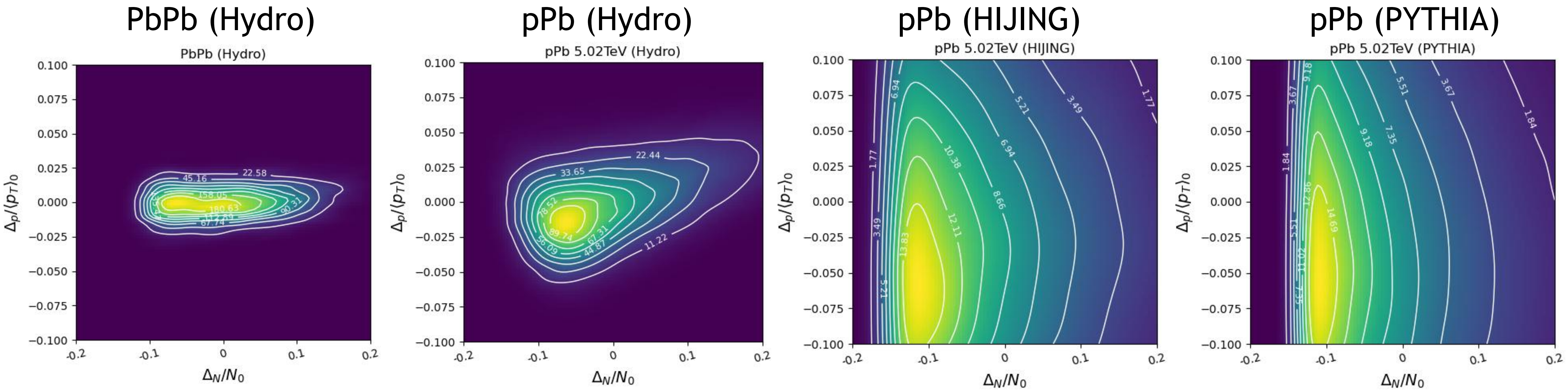
pPb (PYTHIA)



- Hydro: thermodynamic response and quantum noise

$$\frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \quad \longleftrightarrow \quad \text{thermodynamic resp.} + \text{quantum noise}$$

When fluctuations are included: two-dimensional joint probability $\mathcal{P}(\Delta_p, \Delta_N)$



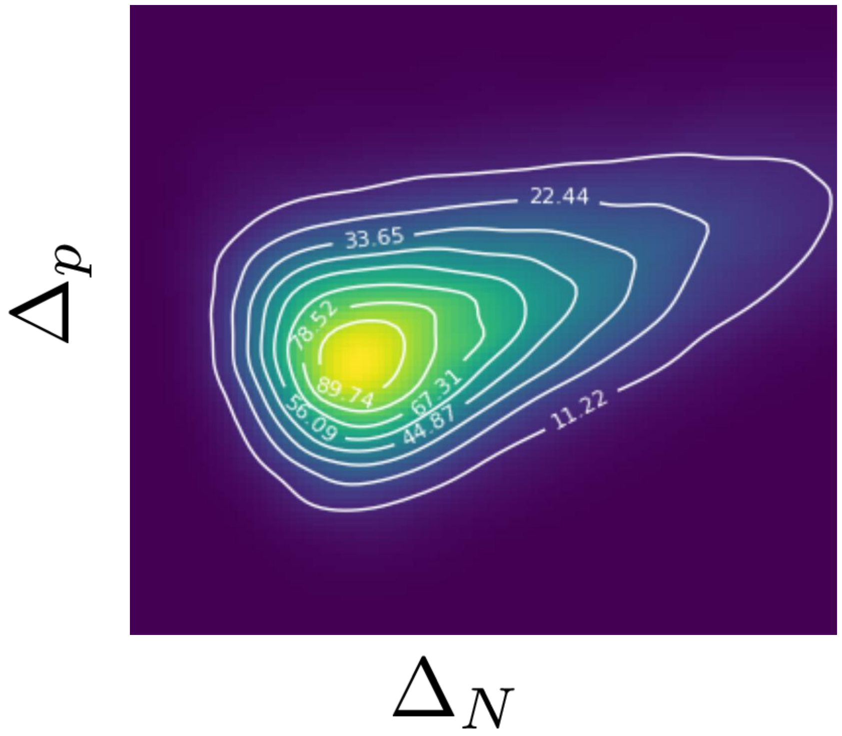
- Hydro: thermodynamic response and quantum noise

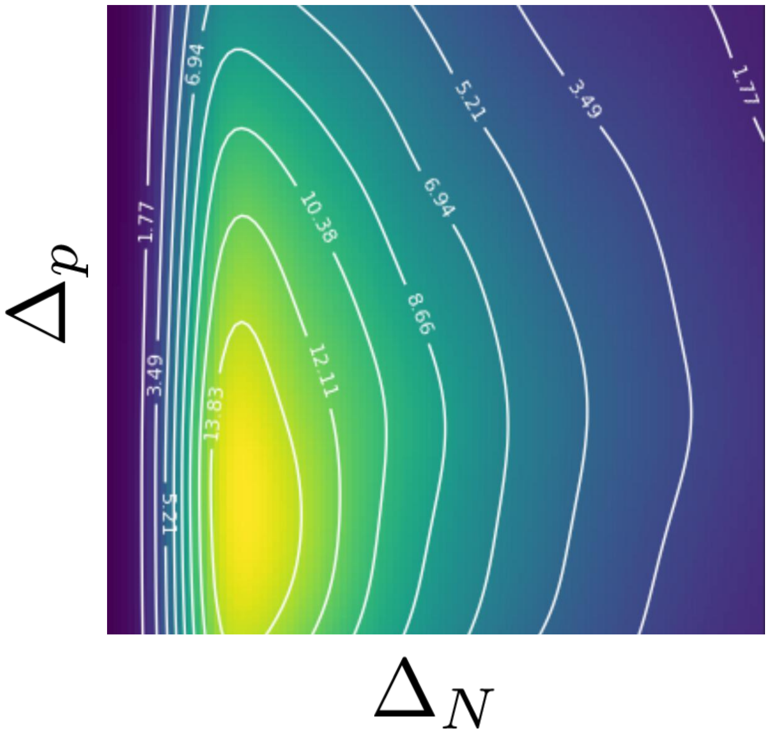
$$\frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \quad \longleftrightarrow \quad \text{thermodynamic resp.} + \text{quantum noise}$$

- Non-thermal models: Quantum response relation (e.g., multi-parton scatterings) and quantum noise

$$\frac{\Delta_p}{\langle p_T\rangle_0} = \kappa \frac{\Delta_N + \delta}{N_0} \quad \longleftrightarrow \quad \text{quantum resp.} + \text{quantum noise}$$

Two questions

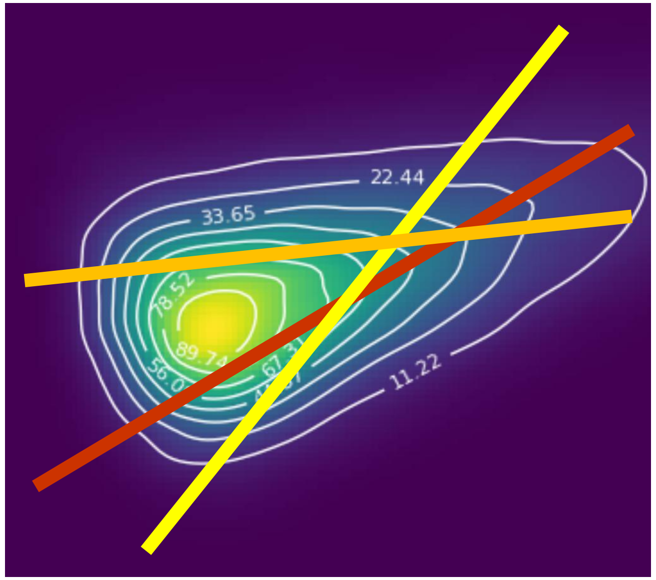


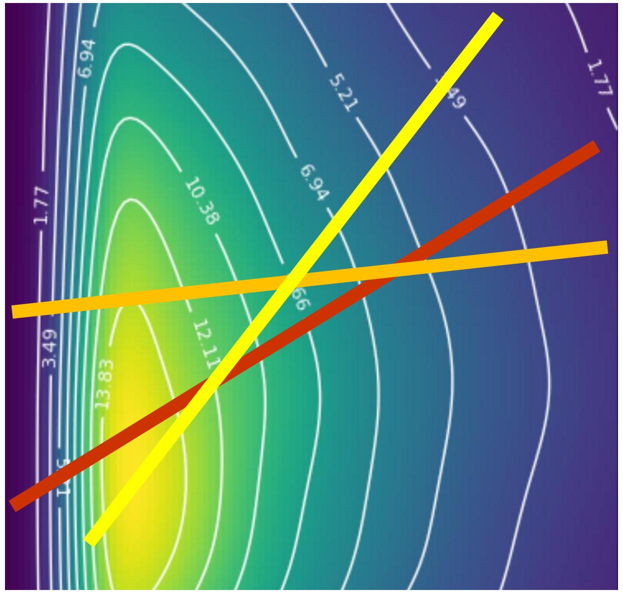


- How would one know if the system is thermalized?
- How to extract the speed of sound in the presence of fluctuations , given $\mathcal{P}(\Delta_p, \Delta_N)$, $\mathcal{P}_P(\Delta_p)$ and $\mathcal{P}_N(\Delta_N)$?

$$\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \qquad \frac{\Delta_p}{\langle p_T \rangle_0} = \kappa \frac{\Delta_N + \delta}{N_0} \qquad \rightarrow \rho(\delta | c_s^2)$$

Disentangle quantum fluctuations from thermodynamic response





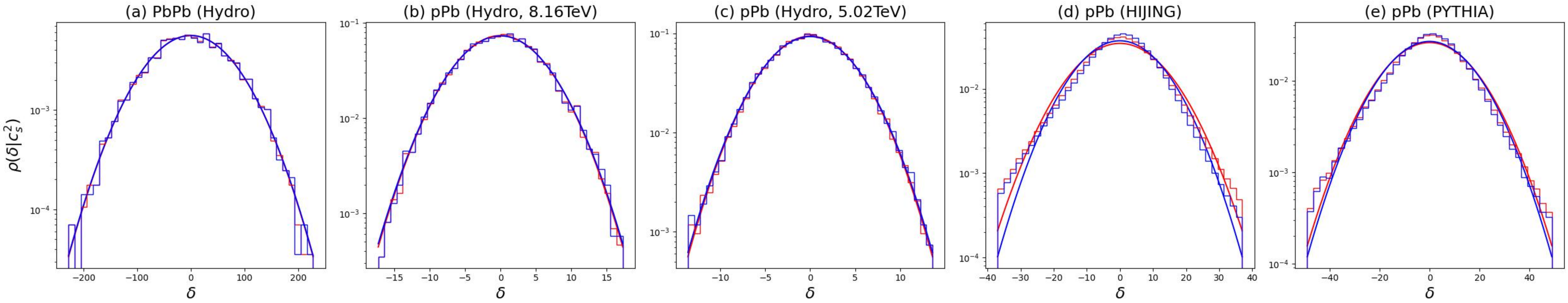
- Thermalized QGP: $\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0}$ ⟷ thermodynamic resp. + quantum noise
- Quantum noise is independent from the thermodynamic response: Gaussian (CLT)

[PRC 109, L051902 (2024)]

- Expectation:

$$\rho(\delta|c_s^2) \sim \begin{cases} \text{Gaussian, if system is thermalized and } c_s^2 \text{ takes physical value} \\ \text{non-Gaussian, otherwise} \end{cases}$$

Verify Gaussianity condition of quantum fluctuations



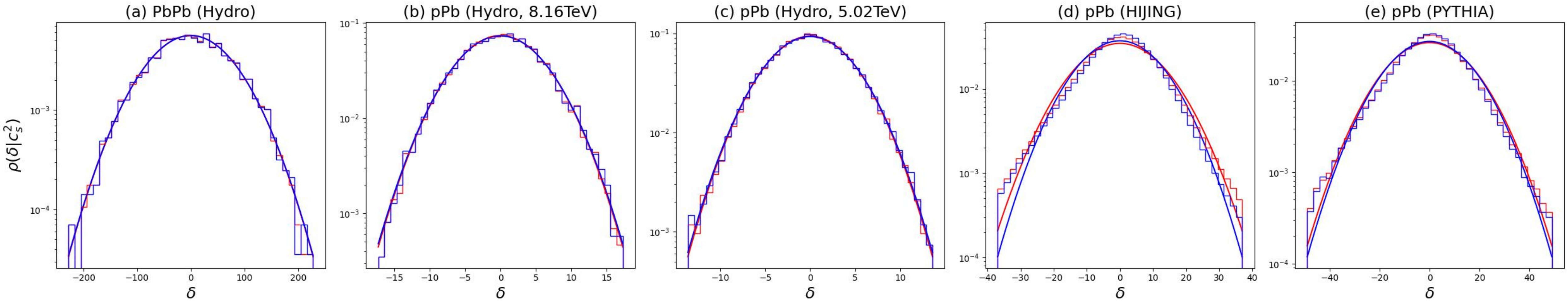
1. Gaussianity leads to the zero skewness condition: solve c_s^2 as the root of equation

$$\{\delta^3\}_c = \{\delta^3\} = 0 \quad \rightarrow \quad (c_s^2)^3 \frac{\{\Delta_N^3\}}{N_0^3} - 3(c_s^2)^2 \frac{\{\Delta_N^2 \Delta_p\}}{N_0^2 \langle p_T \rangle_0} + 3c_s^2 \frac{\{\Delta_N \Delta_p^2\}}{N_0 \langle p_T \rangle_0^2} - \frac{\{\Delta_p^3\}}{\langle p_T \rangle_0^3} = 0\,,$$

Exp. measurables: mixed skewness of transverse momentum and charged multiplicity

Note that mean of quantum fluctuations vanishes by construction: $\{\delta\} \equiv \frac{1}{N_{\rm eve}} \sum_{\text{event i}} \delta_i = 0$

Verify Gaussianity condition of quantum fluctuations

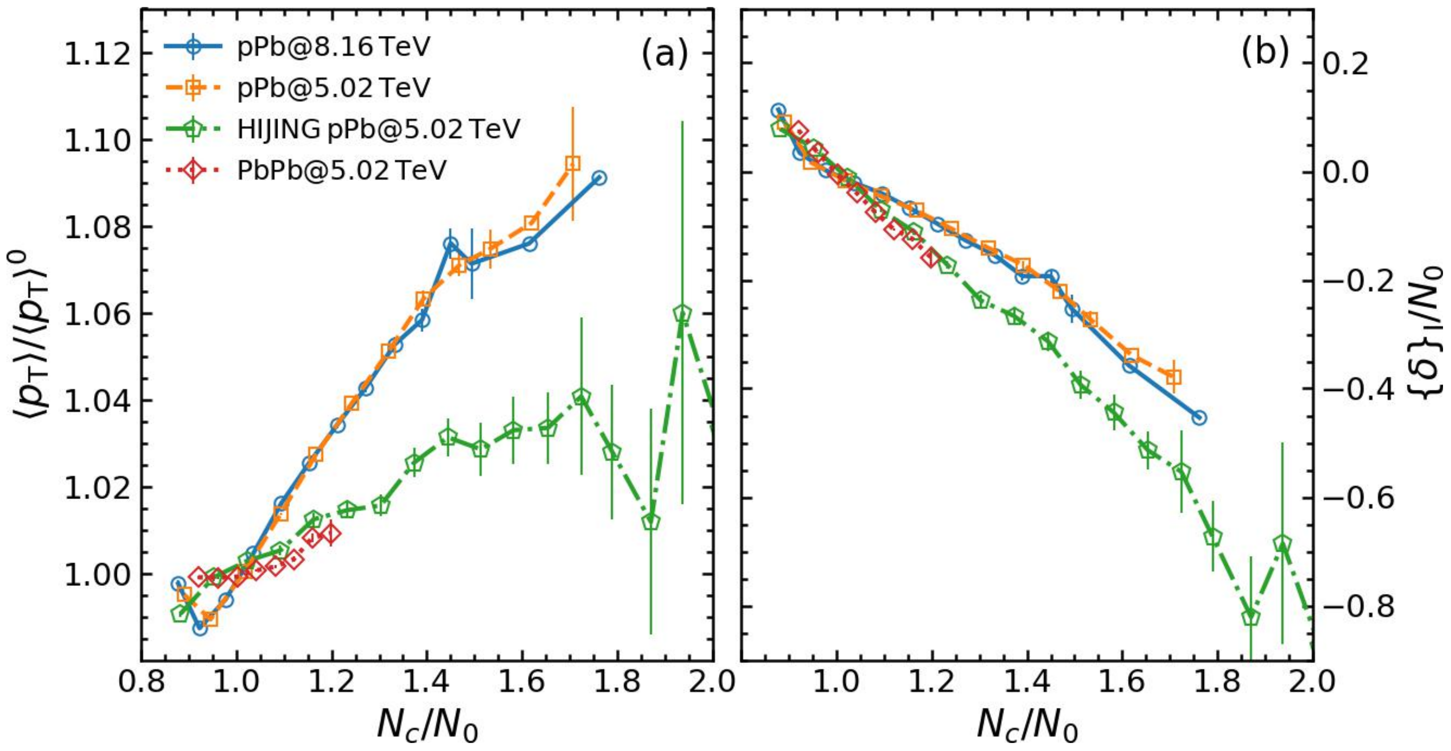


2. Solve the probability distribution with respect to c_s^2: $\rho(\delta|c_s^2)$
3. Compare it to gaussian distribution.
4. A test of convergence: repeat with respect to 5th order cumulant:

$$\{\delta^5\}_c = \{\delta^5\} - 10\{\delta^3\}\{\delta^2\} = 0$$

5. Statistical corrections: $\sim \dfrac{1}{N_0^a}$

Quantum fluctuations are correlated with N_{ch} (or mean pT)



- Simple extraction from the linear fit gets contamination (correction) from quantum fluctuations

$$\frac{\{\Delta_p\}_I}{\langle p_T\rangle_0} = c_s^2 \frac{\{\Delta_N\}_I + \{\delta\}_I}{N_0} \qquad \{\delta\}_I \propto \{\Delta_N\}_I$$

[2407.05570]

- One should be careful when extracting cs2 from the slope, unless correction can be well accounted.

Extract QCD speed of sound

c_s^2	sub-bin slope	$\{\delta^3\}_c = 0$	$\{\delta^5\}_c = 0$	LEOS
PbPb (Hydro, 5.02TeV)	0.123 ± 0.035	0.217 ± 0.032	0.216 ± 0.041	0.222-0.242
pPb (Hydro, 8.16 TeV)	0.176 ± 0.004	0.292 ± 0.013	0.287 ± 0.012	0.282-0.309
pPb (Hydro, 5.02 TeV)	0.197 ± 0.004	0.318 ± 0.011	0.313 ± 0.008	0.269-0.304
pPb (PYTHIA, 5.02 TeV)	-0.032 ± 0.002	1.178 ± 0.006	1.352 ± 0.019	0.227-0.278
pPb (HIJING, 5.02 TeV)	0.079 ± 0.003	1.104 ± 0.019	1.171 ± 0.053	0.206-0.271

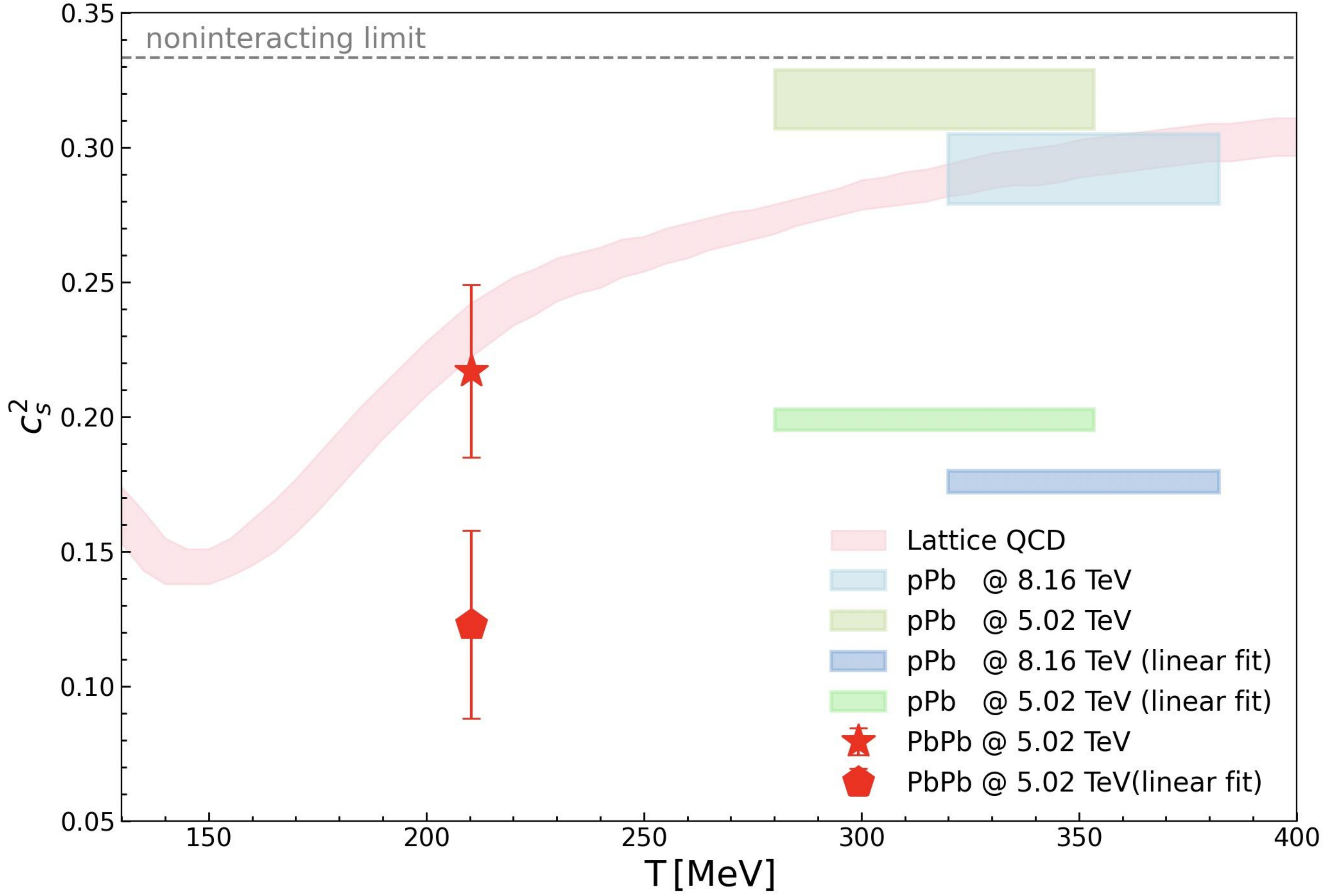
- Simple extraction from the slope does not correspond to physical values.
- LEOS relis on evaluations of effective temperature, which is somehow model dependent.
- The "speed of sound" extracted from non-thermal models violates causality.

$$c_s^2 > \text{causality bound} \sim \begin{cases} \frac{1}{3}: & \mu = 0 \\ 0.781: & \mu \neq 0 \end{cases}$$

[PRD, 80:066003(2009)]

[PLB 860 (2025) 139184]

Extract QCD speed of sound



Probe of QCD thermalization in realistic heavy-ion collisions

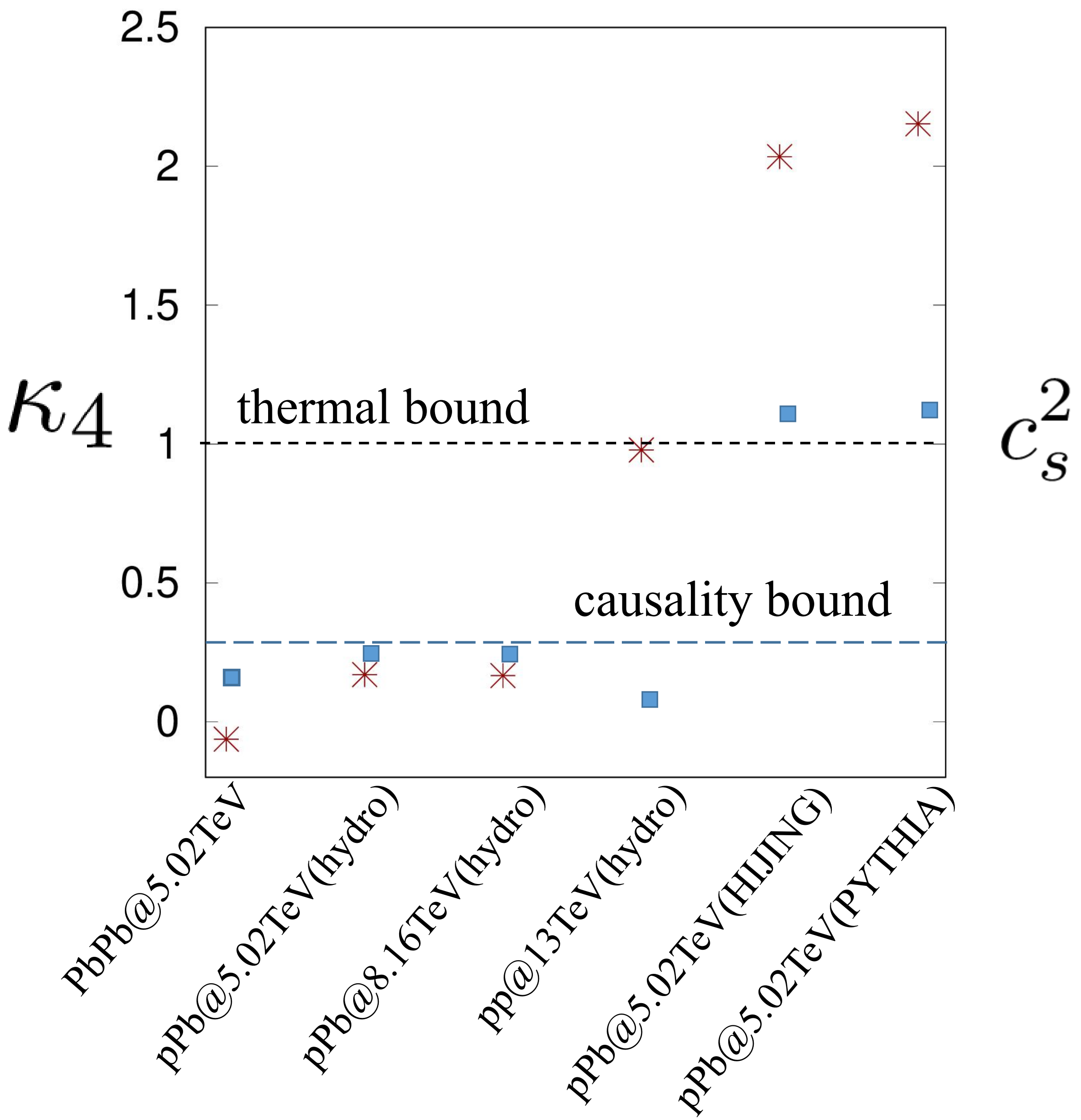
- The realistic system created in heavy-ion collisions is partially thermalized, depending on collision energy, system size, centrality, etc.
- The realistic observables from heavy-ion collisions are from thermal contributions (hydro) + non-thermal contributions (e.g., initial hard scatterings, jet, hadron scatterings).
- Probe of QCD thermalization: Simultaneous measurement of c_s^2 and δ

E.g., standardized kurtosis of δ,

$$\kappa_4 = \frac{\{\delta^4\}}{\{\delta^2\}^2} - 3 : \quad \begin{cases} 0: & \text{absolute thermalization} \\ [0,1]: & \text{partial thermalization} \\ \gg 1: & \text{non-thermal} \end{cases}$$

$$c_s^2 : \quad \begin{cases} [0, \text{causality bound}] : \text{thermalization} \\ > \text{causality bound}: \text{non-thermal} \end{cases}$$

Quantify realistic QCD thermalization: hydro vs. non-hydro



- Even from hydro modeling, the system only achieves partial thermalization, due to, e.g., hadron scatterings.
- From larger (PbPb) to small (pp) systems, the system becomes less thermalized.
- HIJING and PYTIHA do NOT generate thermalized system, as expected.

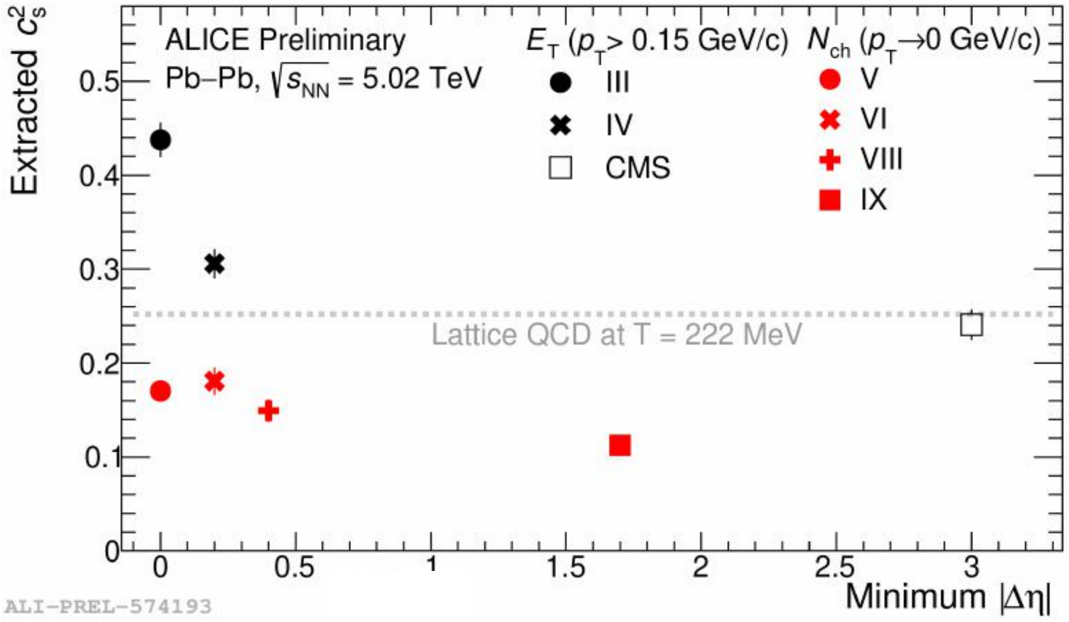
Summary and conclusion

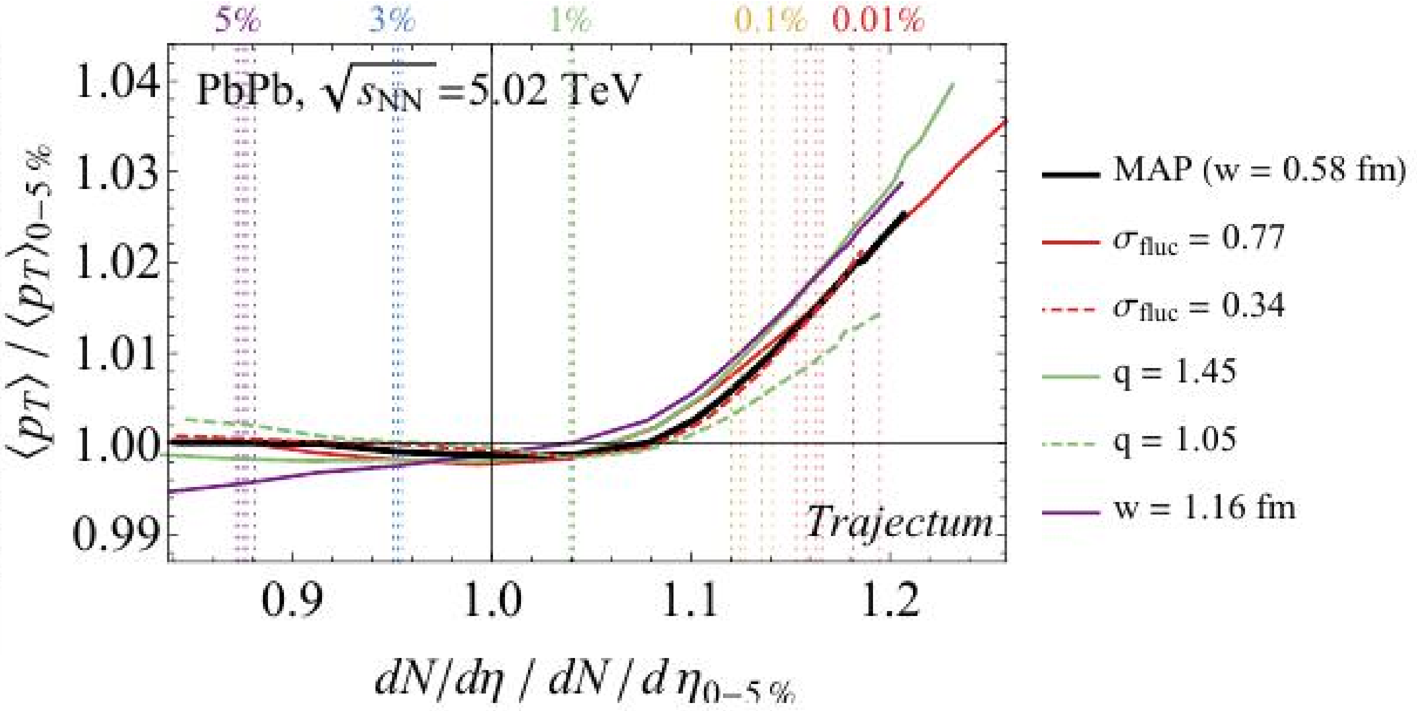
- QCD speed of sound is strongly related to QCD thermalization.
- Measurement of QCD speed of sound in UCC is affected by quantum fluctuations.
- (non-)Gaussianity of the quantum fluctuations indicates QCD thermalization.

Dependence on models and kinematic selections in experiment ?

- The slope (extracted cs2) varies in exp. and in model simulations:

[G. Nijs and W. van der Schee, 2312.04623]

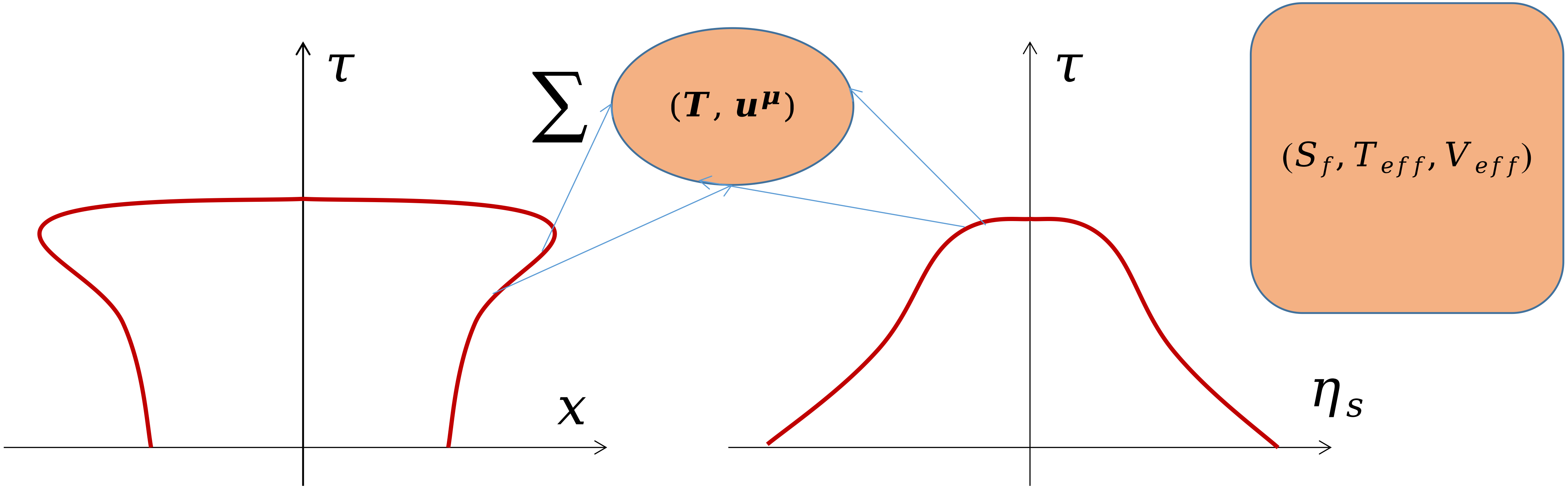




[ALICE collaboration, SQM '24]

- Note: speed of sound is a physical quantity related to QCD, it should NOT depend on initial state model, nor on kinematic selections.

Prescription: effective QCD fireball from freeze out



- Fireball determined from final-state freeze-out hypersurface:

$$E_f \equiv \int_\Sigma d\sigma_\mu T^{\mu 0} = \underbrace{e_{\text{eff}}(T_{\text{eff}})}_{\text{LEOS}} V_{\text{eff}} \qquad S_f \equiv \int_\Sigma d\sigma_\mu s^\mu = \underbrace{s_{\text{eff}}(T_{\text{eff}})}_{\text{LEOS}} V_{\text{eff}}$$

[F. Gardim et al , 1908.09728, Nature Physics]