Three-dimensional Polarized Quark Distributions in the Nucleon

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How much do we understand our world?





Proton Spin Structure in Naïve Quark Model

Quark model:



M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Report No. TH-401 (1964).

ordinary baryons: $|qqq\rangle$, mesons: $|q\bar{q}\rangle$

Spin-flavor wave function of the proton:

$$\begin{split} \left| p_{\uparrow} \right\rangle &= \frac{1}{\sqrt{18}} \Big[2 \left| u_{\uparrow} d_{\downarrow} u_{\uparrow} \right\rangle + 2 \left| u_{\uparrow} u_{\uparrow} d_{\downarrow} \right\rangle + 2 \left| d_{\downarrow} u_{\uparrow} u_{\uparrow} \right\rangle - \left| u_{\uparrow} u_{\downarrow} d_{\uparrow} \right\rangle \\ &- \left| u_{\uparrow} d_{\uparrow} u_{\downarrow} \right\rangle - \left| u_{\downarrow} d_{\uparrow} u_{\uparrow} \right\rangle - \left| d_{\uparrow} u_{\downarrow} u_{\uparrow} \right\rangle - \left| d_{\uparrow} u_{\uparrow} u_{\downarrow} \right\rangle - \left| u_{\downarrow} u_{\uparrow} d_{\uparrow} \right\rangle \Big] \,. \\ \Delta u &= u_{\uparrow} - u_{\downarrow} = \frac{4}{3} \qquad \Delta d = d_{\uparrow} - d_{\downarrow} = -\frac{1}{3} \end{split}$$

The sum of quark spins gives the proton spin.



Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer: $Q \gg \Lambda_{\text{OCD}}$

- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization:

 $\sigma \propto H(Q) \otimes f_{i/P}(x,\mu^2)$

- overall corrections suppressed by $1/Q^n$
- indirectly "see" quarks, gluons and their dynamics
- predictive power relies on
- precision of the probe
- universality of $f_{i/P}(x, \mu^2)$

Modern "Rutherford" experiment.





Lepton-Hadron Deep Inelastic Scattering



H. Abramowicz et al., EPJC 78, 580 (2015).



A. Accardi et al., PRD 93, 114017 (2016).



Lepton-Hadron Deep Inelastic Scattering



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A successful story of QCD, factorization and evolution!

Nucleon Spin Structure

Proton spin puzzle

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$$

Spin decomposition

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$



JAM17: $\Delta\Sigma=0.36\pm0.09$

JAM Collaboration, PRL 119, 132001 (2017).

Quark spin only contributes a small fraction to the nucleon spin.

J. Ashman et al., PLB 206, 364 (1988); NP B328, 1 (1989).



Gluon spin from LQCD: $S_g = 0.251(47)(16)$

50% of total proton spin Y.-B. Yang *et al.* (χQCD Collaboration), PRL 118, 102001 (2017).





Wigner Rotation Effect

Melosh-Wigner rotation

quark spin in a rest proton \neq quark spin in a moving proton

If applying a *kinetic boost*, one may relate the spin states in *proton rest frame* to the spin states in *infinite momentum frame*

$$\chi_T^{\uparrow} = w \left[\left(k^+ + m \right) \chi_F^{\uparrow} - \left(k^1 + ik^2 \right) \chi_F^{\downarrow} \right] \qquad \qquad k^+ = k^0 + k^3$$
$$\chi_T^{\downarrow} = w \left[\left(k^+ + m \right) \chi_F^{\downarrow} + \left(k^1 - ik^2 \right) \chi_F^{\uparrow} \right] \qquad \qquad w = \left[2k^+ \left(k^0 + m \right) \right]^{-1/2}$$

E.P. Wigner, Ann. Math 40 (1939) 149; H.J. Melosh, Phys. Rev. D 9 (1974) 1095.

The effect on quark polarization

$$\Delta q = \int \mathrm{d}^3 \mathbf{k} \mathscr{M} \left[q^{\uparrow}(k) - q^{\downarrow}(k) \right] \qquad \qquad \mathscr{M} = \frac{(k^+ + m)^2 - k_T^2}{2k^+(k^0 + m)}$$

B.-Q. Ma, J. Phys. G 17 (1991) L53-L58; B.-Q. Ma, Q.-R. Zhang, Z. Phys. C 58 (1993) 479.

It predicts decreasing polarization with k_T , which should be tested by data. This interpretation is based on a kinetic boost, but a complete boost including QCD dynamics is challenging.



Semi-inclusive Deep Inelastic Scattering

Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, ...
- a large momentum transfer *Q* provides a shortdistance probe
- an additional and adjustable momentum scale P_{hT}
- multidimensional imaging of the nucleon







SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



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Structure Functions of SIDIS





Leading Twist TMDs





Leading Twist TMDs





Longitudinal Double Spin Asymmetry

Longitudinal DSA in SIDIS

$$A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-} + \sigma_{--} - \sigma_{-+}}{\sigma_{++} + \sigma_{+-} + \sigma_{--} + \sigma_{-+}} = \frac{\sqrt{1 - \varepsilon^2} F_{LL} \left(x, z, P_{hT}^2, Q^2 \right)}{F_{UU} \left(x, z, P_{hT}^2, Q^2 \right)}$$

In TMD region:
$$F_{LL}\left(x, z, P_{hT}^2, Q^2\right) \sim g_{1L}(x, k_T^2) \otimes D_1(z, p_T^2)$$
$$F_{UU}\left(x, z, P_{hT}^2, Q^2\right) \sim f_1(x, k_T^2) \otimes D_1(z, p_T^2)$$



One needs P_{hT} -dependent DSA measurements to determine TMD helicity distributions.

 P_{hT} dependent DSA measurements

HERMES: proton (H_2) and deuteron (D_2) targets

HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

JLab CLAS: proton (NH₃) target

CLAS Collaboration, Phys. Lett. B 782 (2018) 662.



TMD Evolution

Evolution equations

$$\mu^{2} \frac{dF(x,b;\mu^{2},\zeta)}{d\mu^{2}} = \frac{\gamma_{F}(\mu,\zeta)}{2} F(x,b;\mu^{2},\zeta) \qquad -\zeta \frac{d\gamma_{F}(\mu,\zeta)}{d\zeta} = \mu \frac{d\mathscr{D}(\mu,b)}{d\mu} = \Gamma_{\text{cusp}}(\mu)$$

$$\zeta \frac{dF(x,b;\mu^{2},\zeta)}{d\zeta} = -\mathscr{D}(\mu,b)F(x,b;\mu^{2},\zeta) \qquad \gamma_{F}(\mu,\zeta) = \Gamma_{\text{cusp}}(\mu) \ln \frac{\mu^{2}}{\zeta} - \gamma_{V}(\mu)$$

$$F\left(x,b;\mu_{f},\zeta_{f}\right) = \exp\left[\int_{P}\left(\gamma_{F}(\mu,\zeta)\frac{d\mu}{\mu} - \mathscr{D}(\mu,b)\frac{d\zeta}{\zeta}\right)\right] F\left(x,b;\mu_{i},\zeta_{i}\right)$$

 ζ -prescription

equipotential lines: $\frac{d \ln \zeta_{\mu}(\mu, b)}{d \ln \mu^{2}} = \frac{\gamma_{F}\left(\mu, \zeta_{\mu}(\mu, b)\right)}{2\mathscr{D}(\mu, b)}$ $\mathscr{D}\left(\mu_{0}, b\right) = 0, \quad \gamma_{F}\left(\mu_{0}, \zeta_{\mu}\left(\mu_{0}, b\right)\right) = 0$ $F\left(x, b; Q, Q^{2}\right) = \left(\frac{Q^{2}}{\zeta_{Q}(b)}\right)^{-\mathscr{D}(Q,b)} F(x, b), \quad \mu_{f}^{2} = \zeta_{f} = Q^{2}$



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Parametrization

Parametrization of TMD helicity distributions

we parametrize the distributions in b space at the saddle point

$$g_{1L}(x,b) = \sum_{f'} \int_{x}^{1} \frac{d\xi}{\xi} \Delta C_{f \leftarrow f'}\left(\xi, b, \mu_{\text{OPE}}\right) g_{1L}^{f'}\left(\frac{x}{\xi}\right) g_{\text{NP}}(x,b),$$

$$g_{1L}^{f}(x) = N_{f} \frac{(1-x)^{\alpha_{f}} x^{\beta_{f}} \left(1+\epsilon_{f} x\right)}{n\left(\alpha_{f}, \beta_{f}, \epsilon_{f}\right)} g_{1}^{f}\left(x, \mu_{\text{OPE}}\right)$$

 $n(\alpha_f, \beta_f, \epsilon_f)$ is introduced to reduce the correlation between normalization and the shape.

$$g_{\rm NP}(x,b) = \exp\left[-\frac{\lambda_1(1-x) + \lambda_2 x + \lambda_5 x(1-x)}{\sqrt{1 + \lambda_3 x^{\lambda_4} b^2}}b^2\right]$$

the same form as adopted in the unpolarized distributions.

The *x*-dependent factor allows a variation from the collinear distribution. Such an *x*-shape modification can be removed by setting $\alpha_f = \beta_f = \epsilon_f = 0$.



First Extraction of TMD Helicity

NLO+NNLL analysis results



Nonzero signals for *u* and *d* quarks, while sea quarks and gluons are loosely constrained.

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

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Estimation of Uncertainties



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Estimation of Uncertainties

In each single fit:

minimize
$$\chi^2 = \sum_{\text{sets}} \sum_{i,j} (t_i - a_i) V_{ij}^{-1} (t_j - a_j)$$
 V_{ij} : covariant matrix

Dominant correlated uncertainties:

| Experiment | Process | Beam and target polarization | Dilution |
|------------|--------------------------|------------------------------|----------|
| HERMES | $e^{\pm}p \to e^{\pm}hX$ | 6.6% | 0 |
| HERMES | $e^{\pm}d \to e^{\pm}hX$ | 5.7% | 1.7% |
| CLAS | $e^-p \to e^-\pi^0 X$ | 4.5% | 5.8% |

In this study, 1000 replicas are generated according to data uncertainties and their correlations.



Parameter Values and Uncertainties

| Parameter | Value | Parameter | Value |
|--------------|-------------------------------------|--------------|-------------------------------------|
| N_u | $0.0223^{+0.0029}_{-0.0024}$ | $N_{ar{u}}$ | $-0.008^{+0.092}_{-0.035}$ |
| N_d | $0.0353\substack{+0.0051\\-0.0088}$ | $N_{ar{d}}$ | $0.006^{+0.032}_{-0.011}$ |
| N_s | $-0.022^{+0.043}_{-0.043}$ | N_g | $0.0220\substack{+0.0081\\-0.0706}$ |
| α_u | $2.78^{+0.45}_{-0.72}$ | $lpha_d$ | $4.28^{+0.38}_{-0.76}$ |
| β_u | $0.145^{+0.041}_{-0.194}$ | β_d | $1.16_{-0.40}^{+0.14}$ |
| ϵ_u | $7.4^{+2.3}_{-4.5}$ | ϵ_d | $-0.59^{+0.18}_{-0.20}$ |
| λ_1 | $0.240^{+0.062}_{-0.134}$ | λ_2 | $0.39^{+0.13}_{-0.33}$ |
| λ_3 | $0.92^{+12.17}_{-0.92}$ | λ_4 | $7.50^{+2.29}_{-0.78}$ |
| λ_5 | $-1.11_{-0.50}^{+0.87}$ | | |

Correlation Matrix of the Parameters





Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

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K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

0.5 < z < 0.8

Ō

₫

 $e^{\pm}p \rightarrow e^{\pm}\pi^{+}X$

0.14 < x < 0.2

 $e^{\pm}p \rightarrow e^{\pm}\pi^{-}X$

 $e^{\pm}d \to e^{\pm}\pi^+X$

 $e^{\pm}d \rightarrow e^{\pm}\pi^{-}X$

 $e^{\pm}d \to e^{\pm}K^+X$

 $e^{\pm}d \rightarrow e^{\pm}K^{-}X$

₹

0.2 0.5 0.8 1.1

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 P_{hT} (GeV)

δ

Compare with HERMES data





K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

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Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

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Compare with CLAS data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

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Transverse Momentum Dependent Polarization



 $g_{1L}(x, k_T^2) = q_{\uparrow}(x, k_T^2) - q_{\downarrow}(x, k_T^2)$ measures the absolute number density difference between spin-parallel and spinantiparallel quarks in a polarized proton.

 $\frac{g_{1L}(x,k_T^2)}{f_1(x,k_T^2)} = \frac{q_{\uparrow}(x,k_T^2) - q_{\downarrow}(x,k_T^2)}{q_{\uparrow}(x,k_T^2) + q_{\downarrow}(x,k_T^2)}$ measures the polarization rate of quarks.

 At large x, where valence components dominate, the polarization decreases with increasing k_T
 Qualitatively consistent with kinetic Wigner rotation effects



Transverse Momentum Dependent Polarization



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• At low x, where the valence component is no longer adequate, distributions are highly driven by complex QCD dynamics The polarization is found increasing with k_T



Examine the Positivity Bound



Positivity bound: $|g_{1L}| \le f_1$ based on probability interpretation

It *should not* be imposed during the fit, which will introduce bias to results.

It can be examined. No breaking of the bound is observed according to current uncertainties.

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Test the Sensitivity to FFs



Input of TMD FFs: $D_1(z, p_T^2)$ SV19-DSS parametrization

The polarization distributions, i.e. the ratio between g_{1L} and f_1 , are not sensitive to the choice of input FFs.



Flexible vs. Fixed x-dependence



Recall the parametrization:

$$g_{1L}^{f}(x) = N_{f} \frac{(1-x)^{\alpha_{f}} x^{\beta_{f}} \left(1 + \epsilon_{f} x\right)}{n\left(\alpha_{f}, \beta_{f}, \epsilon_{f}\right)} g_{1}^{f}\left(x, \mu_{\text{OPE}}\right)$$

By setting $\alpha_f = \beta_f = \epsilon_f = 0$, one can fix it to the collinear distributions.

The results from these two choices are consistent within current uncertainties.

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

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Comparison between theoretical calculations and HERMES data





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Comparison between theoretical calculations and HERMES data





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Transverse Nucleon Tomography Collaboration





Transverse Nucleon Tomography Collaboration

Sivers distribution functions:

Chunhua Zeng, Tianbo Liu, Peng Sun, Yuxiang Zhao, Phys. Rev. D 106 (2022) 094039.

Transversity distribution functions and Collins fragmentation functions:

Chunhua Zeng, Hongxin Dong, Tianbo Liu, Peng Sun, Yuxiang Zhao, Phys. Rev. D 109 (2024) 056002. Trans-helicity worm-gear distribution functions:

Ke Yang, Tianbo Liu, Peng Sun, Yuxiang Zhao, Bo-Qiang Ma, Phys. Rev. D 110 (2024) 034036. Transverse momentum dependent helicity distribution functions:

Ke Yang, Tianbo Liu, Peng Sun, Yuxiang Zhao, Bo-Qiang Ma, Phys. Rev. Lett. 134 (2025) 121902.
Sivers, transversity, and Collins functions including DY and new COMPASS data: Chunhua Zeng, Hongxin Dong, Tianbo Liu, Peng Sun, Yuxiang Zhao, Phys. Lett. B (2025).
Helicity distributions and azimuthal modulations in longitudinal DSA: Ke Yang, Tianbo Liu, Peng Sun, Yuxiang Zhao, Bo-Qiang Ma, in preparation.



Summary and Outlook

- Spin always surprises since its discovery 100 years ago
- Nucleon spin structure is still not well understood
- Rich information is contained in multidimensional imagings, TMDs as well as GPDs
- First extraction of TMD helicity distributions is obtained by analyzing transverse momentum dependent SIDIS DSA data

- at large-x region, where valence components dominate, quark polarization decreases with increasing k_T , supporting the kinetic Wigner rotation effects;

- in low-x region, increasing quark polarization in dependence on k_T is observed, indicating the essential role of QCD dynamics;

- sea quarks and gluon distributions are loosely constrained by existing SIDIS data.
- Transverse momentum dependent measurements of W production in polarized pp collisions at RHIC may help constrain sea quark distributions.
- Opportunities from existing experiments at RHIC, JLab12, BESIII, BelleII, and future facilities, EIC, EicC, STCF, to understand nucleon spin structures and fragmentation functions.

Thank you!

