



Czech  
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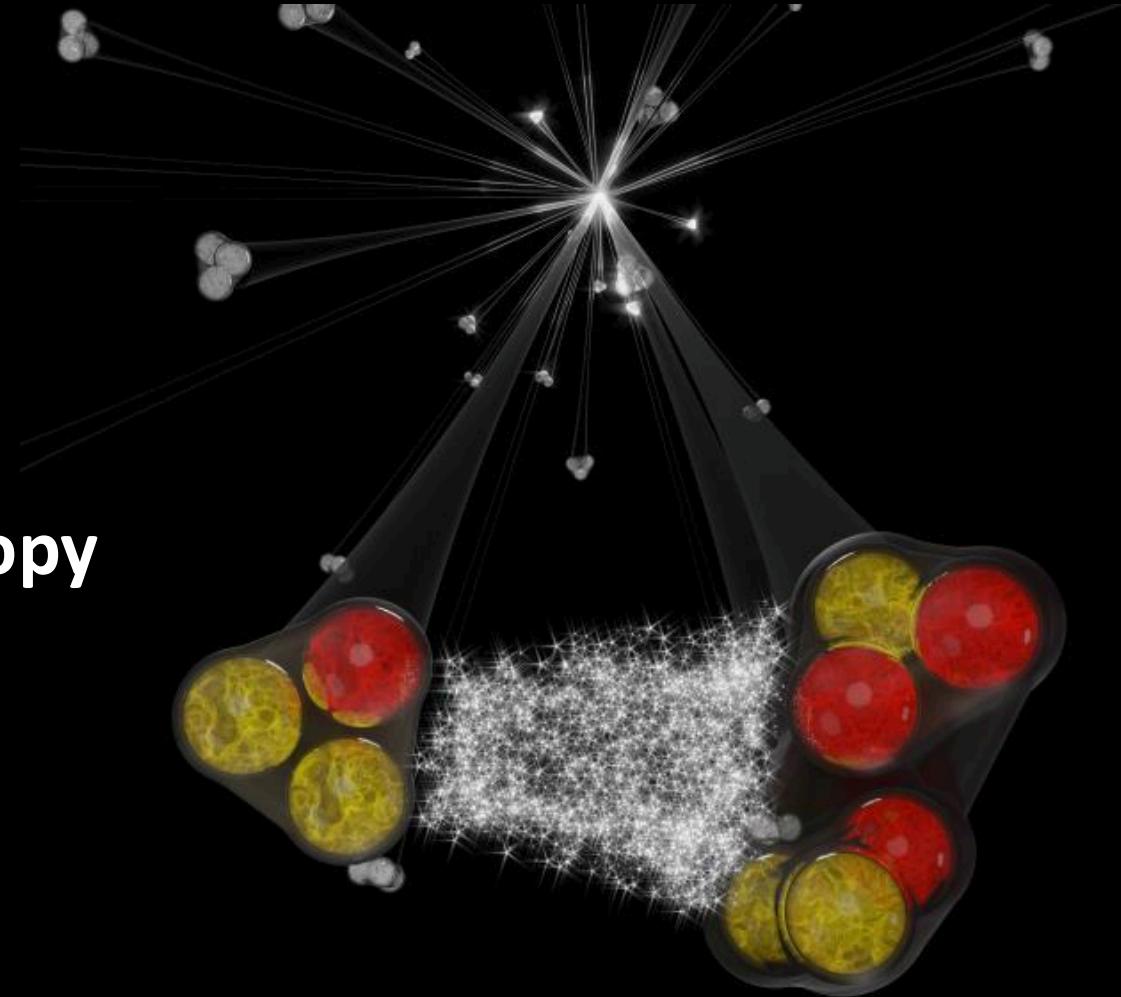
# Understanding two- and three-body hadronic interactions using femtoscopy

Raffaele Del Grande

Czech Technical University in Prague  
[raffaele.del.grande@fjfi.cvut.cz](mailto:raffaele.del.grande@fjfi.cvut.cz)

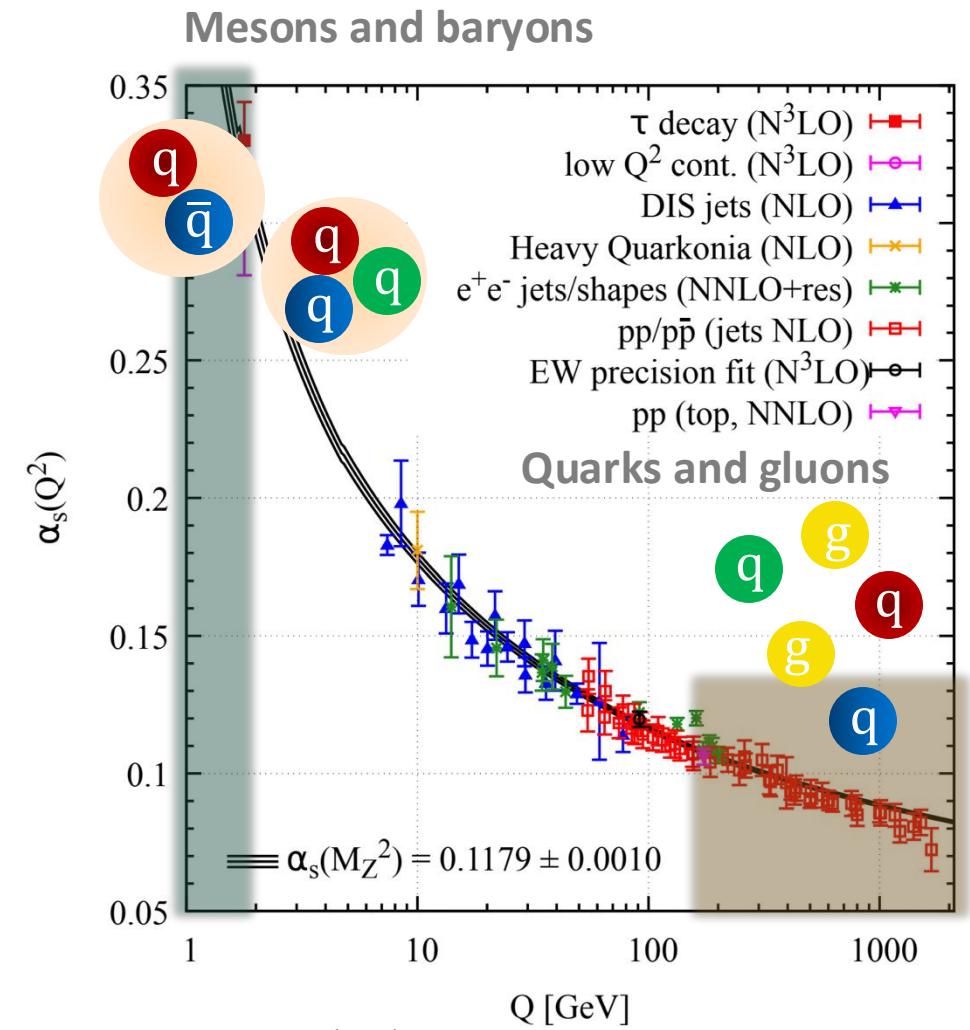
5 June 2025

HENPIC Seminar



# Hadronic interactions

Understand how QCD evolves from high-energy to low-energy regime

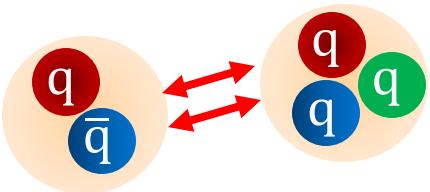


# Hadronic interactions

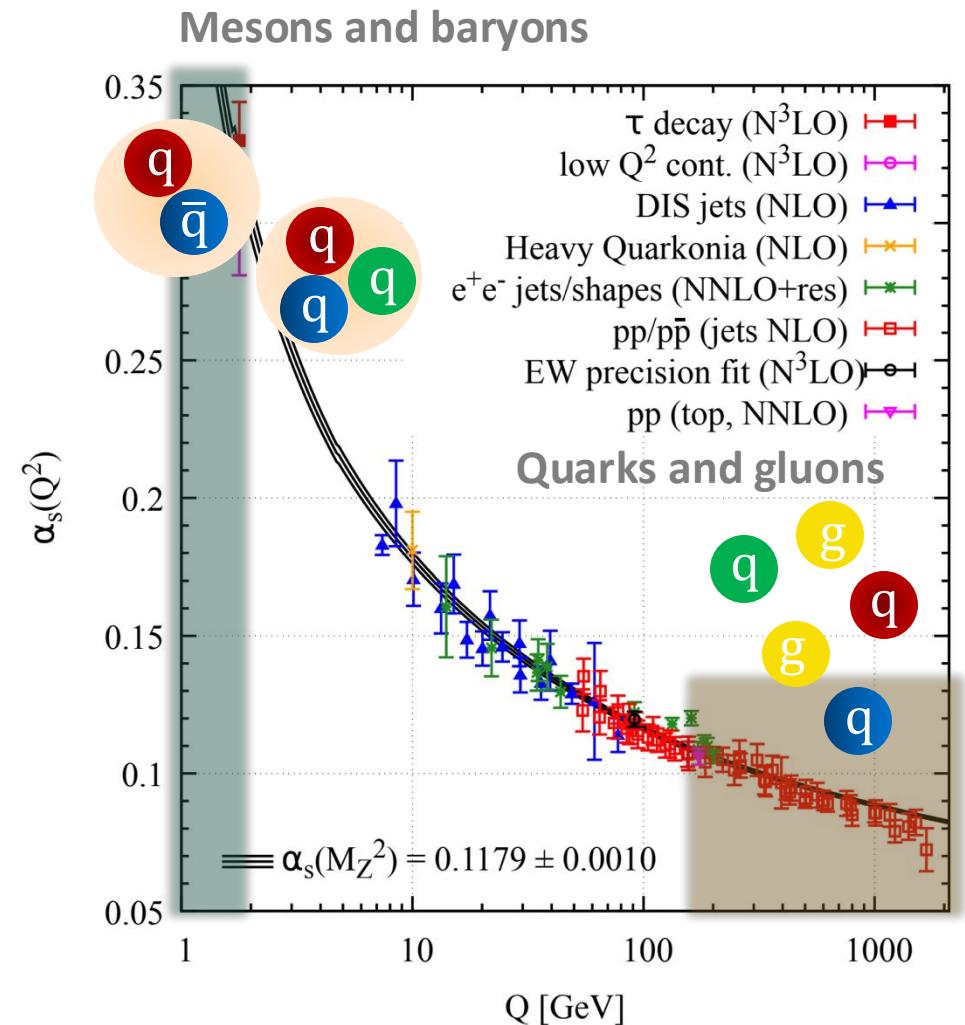
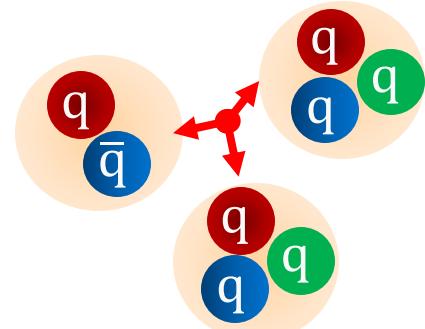
Understand how QCD evolves from high-energy to low-energy regime

How do hadrons interact?

Two-body interactions



Three-body interactions

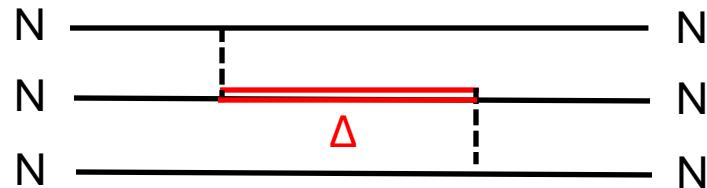


PDG, PTEP 2022, 083C01(2022)

# Three-body dynamics

Dynamics of baryons involves  
formation of hadronic excitations

H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)

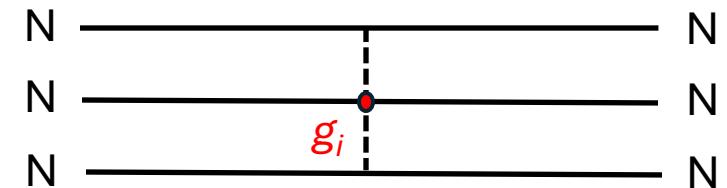


*Short-range  
dynamics*



Three-body forces in  
Effective Field Theories

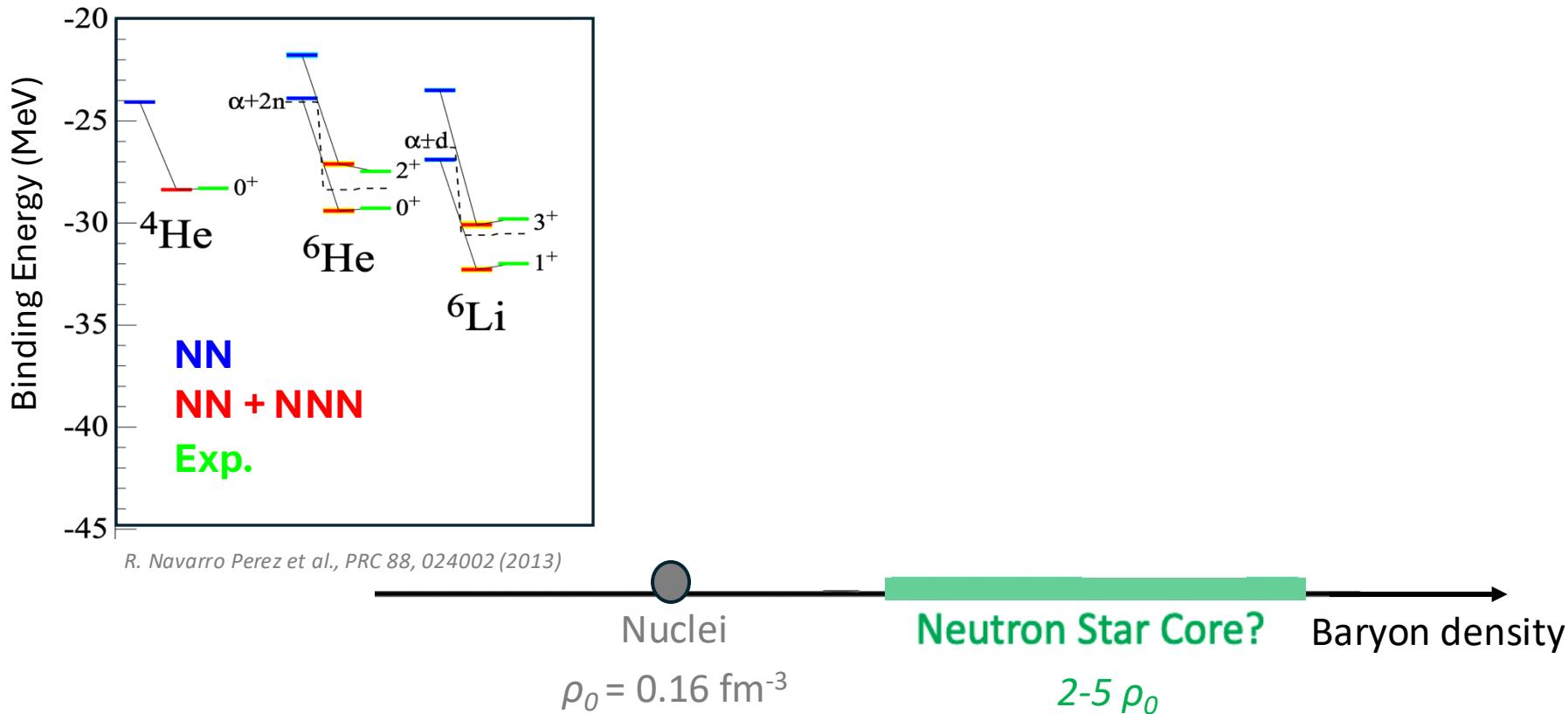
E. Epelbaum, H.-W. Hammer, U.-G. Meißner, RMP 81, 1773 (2009)



$g_i$  constants to be fixed by  
the experimental data

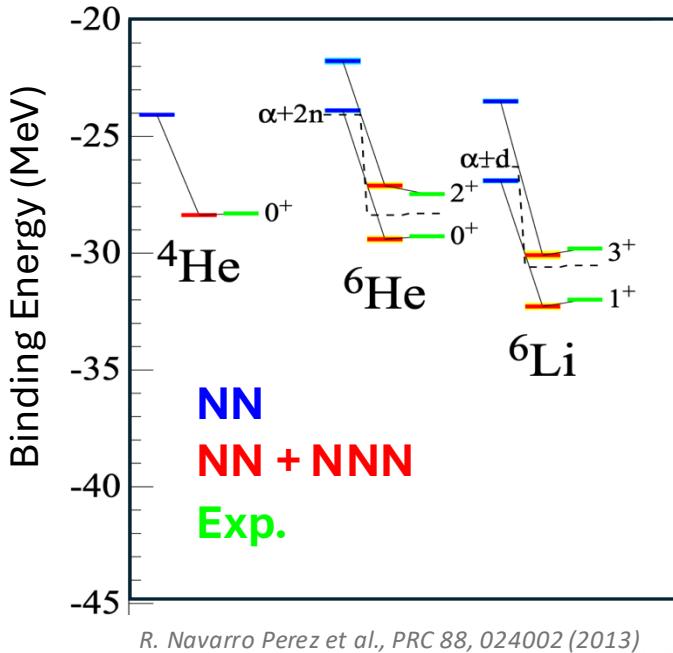
# Three-body forces

3BFs contribute 10-20% to the binding energies.



# Three-body forces

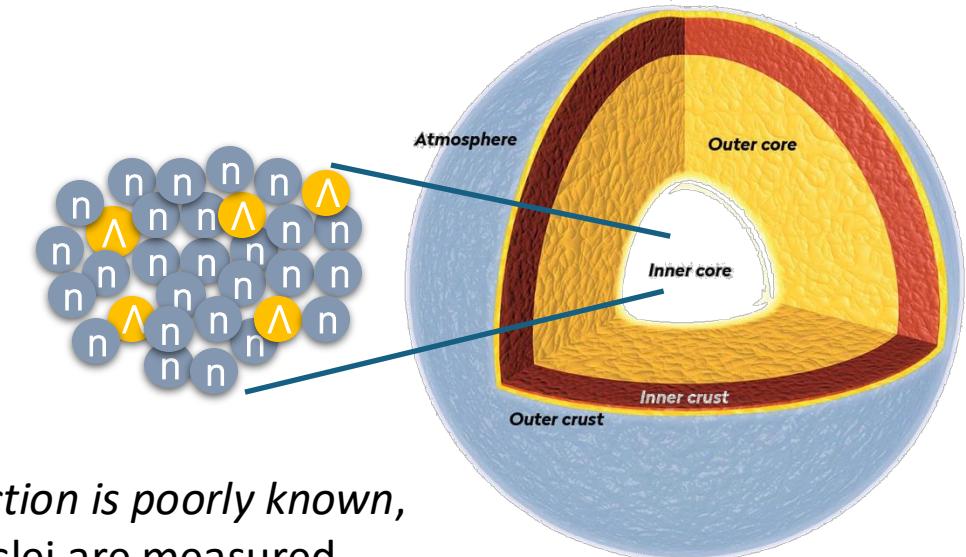
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R. Navarro Perez et al., PRC 88, 024002 (2013)



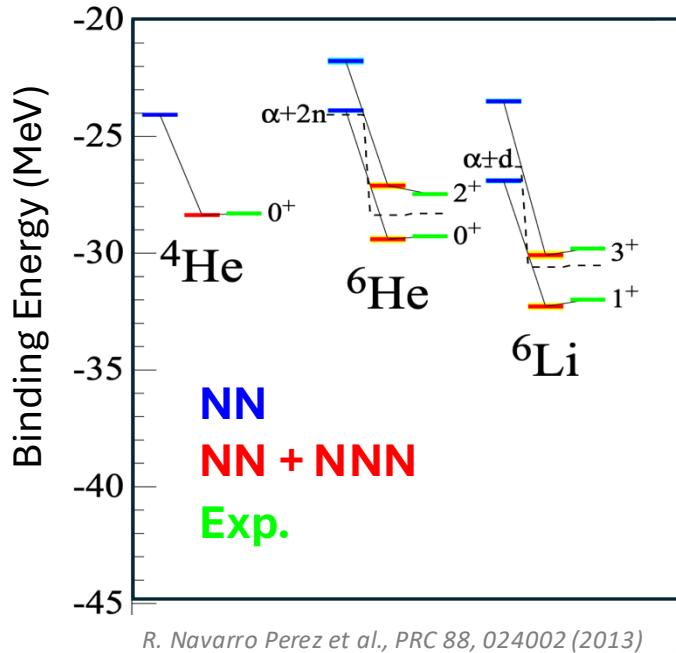
Stronger impact on dense nuclear matter?  
D. Lonardoni et al. PRL 114, 092301 (2015)



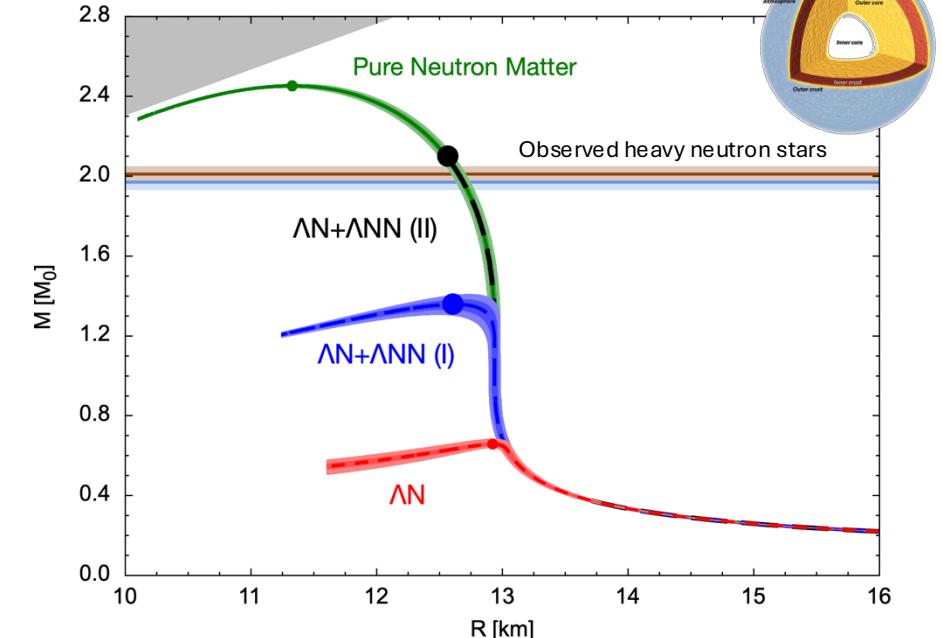
$\Lambda NN$  interaction is poorly known,  
38 hypernuclei are measured.

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3BFs contribute 10-20% to the binding energies.

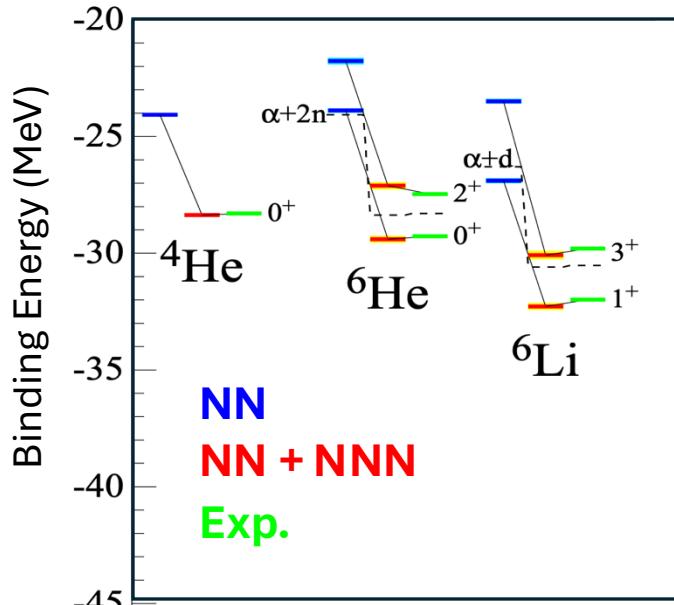


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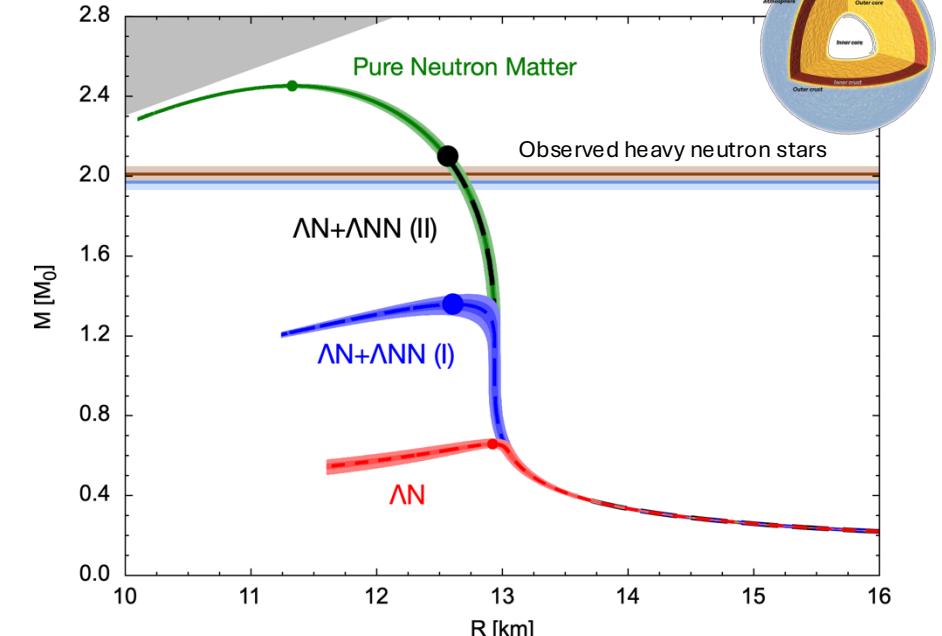
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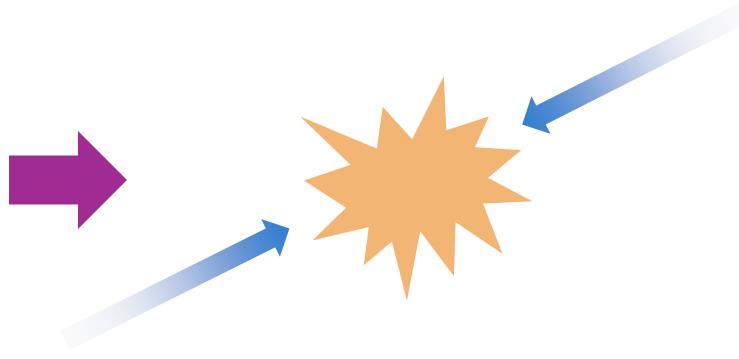


# Femtoscopy technique at the Large Hadron Collider

ALICE at the LHC

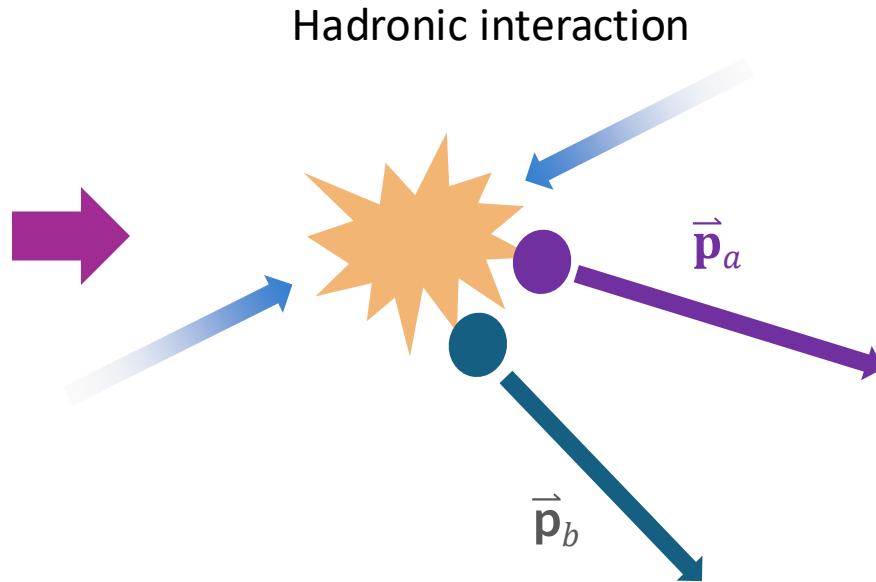


Hadronic interaction



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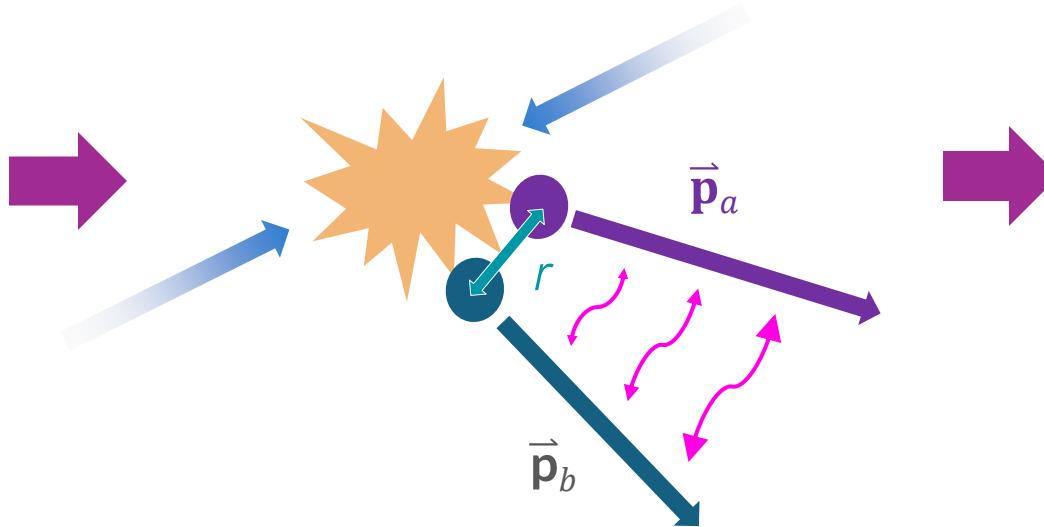


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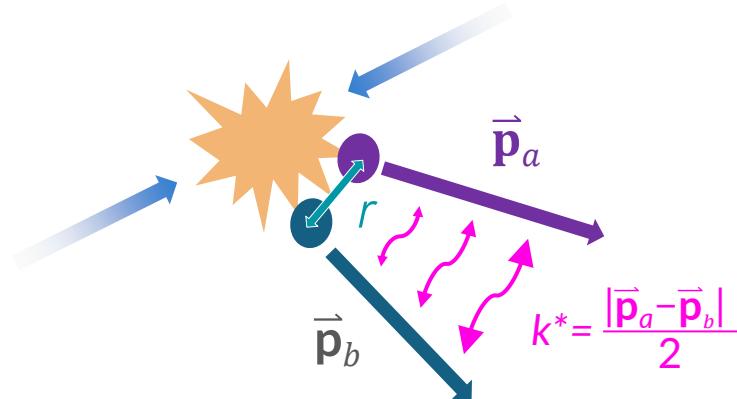
Hadronic interaction



Femtoscopy technique

$$C(\vec{p}_a, \vec{p}_b) \equiv \frac{P(\vec{p}_a, \vec{p}_b)}{P(\vec{p}_a) P(\vec{p}_b)}$$

# Correlation function

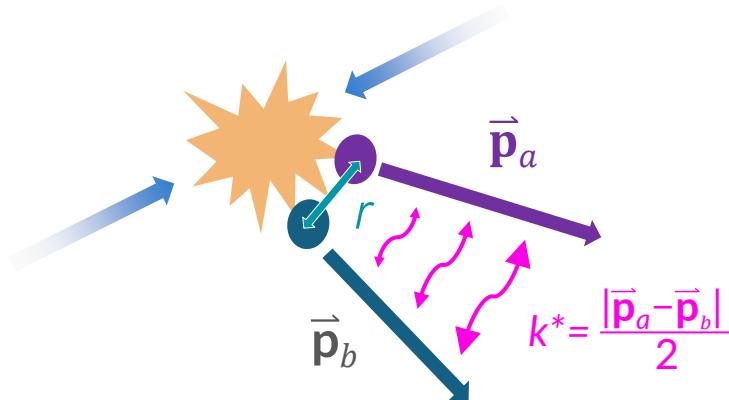


Emission source  $S(\vec{r})$

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d\vec{r} = \mathcal{N}(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

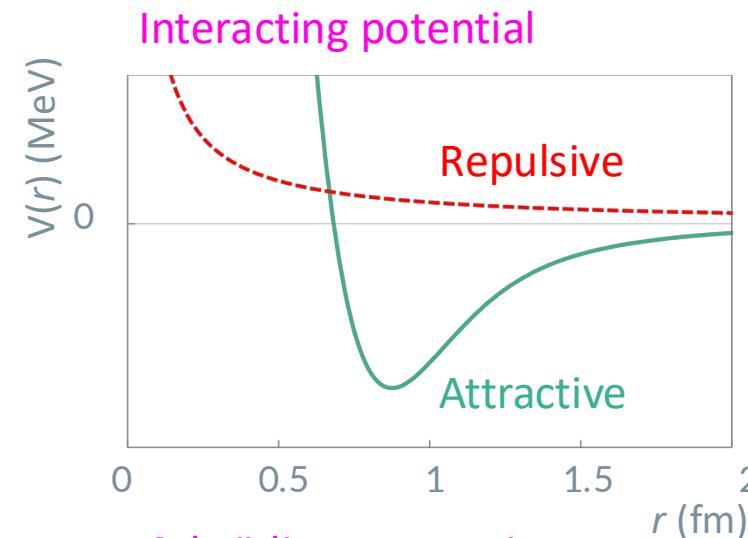
M.Lisa, S. Pratt et al., ARNPS 55 (2005), 357-402  
L.Fabbietti et al., ARNPS 71 (2021), 377-402

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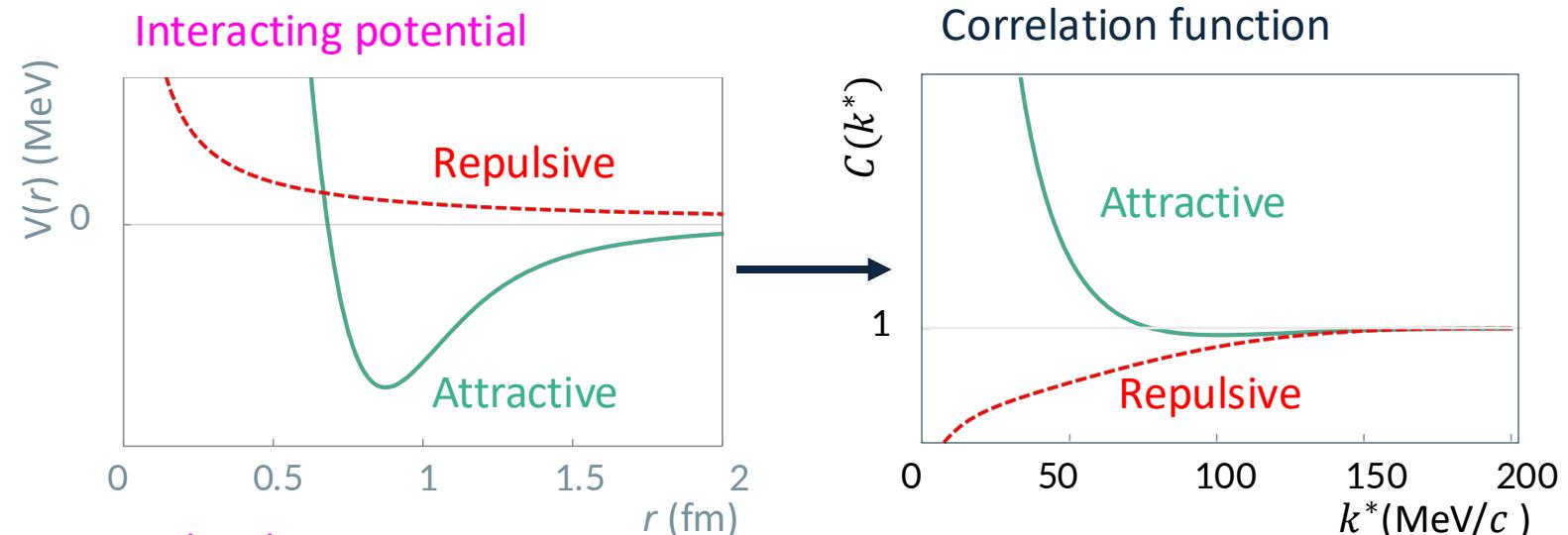
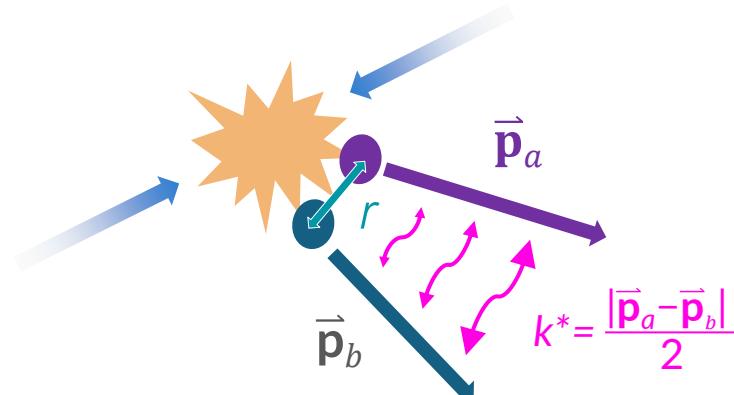
Schrödinger equation

D. Mihaylov et al., EPJC 78 (2018), 5, 394

Two-particle wave function

M. Lisa, S. Pratt et al., ARNPS 55 (2005), 357-402  
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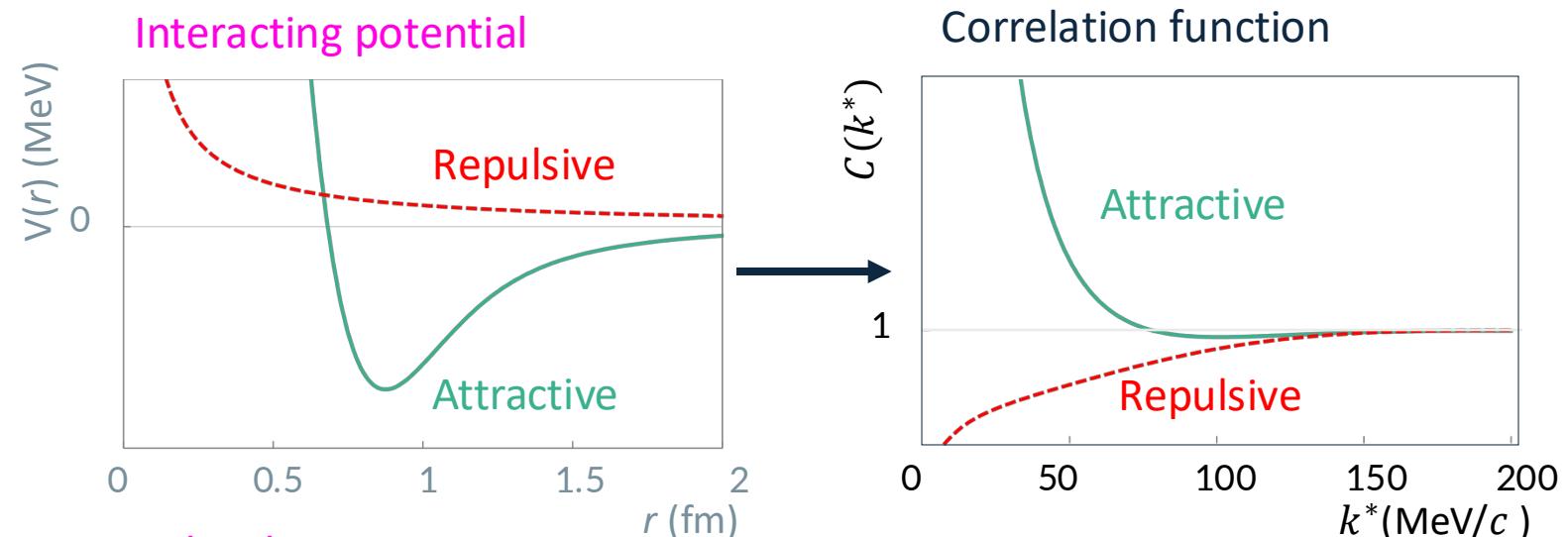
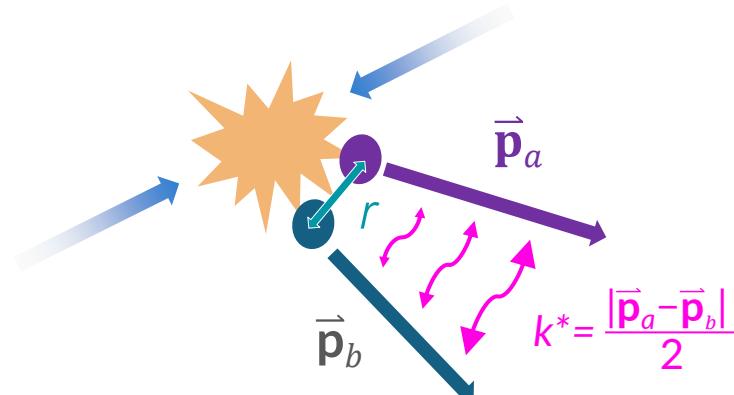
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# Correlation function



Emission source  $S(\vec{r})$

$$C(k^*) = \int \underbrace{S(\vec{r})}_{\text{Emission source}} \underbrace{\left| \psi(\vec{k}^*, \vec{r}) \right|^2}_{\text{Two-particle wave function}} d\vec{r} = \mathcal{N}(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

Measuring  $C(k^*)$ , fixing the source  $S(\vec{r})$ , study the interaction

M. Lisa, S. Pratt et al., ARNPS 55 (2005), 357-402  
L. Fabbietti et al., ARNPS 71 (2021), 377-402

# Source function in pp collisions at the LHC

- Emitting source function anchored to p-p correlation function

$$C(k^*) = \int \underset{\text{measured}}{S(\vec{r})} \left| \psi(\vec{k}^*, \vec{r}) \right|^2 d^3\vec{r}$$

known interaction

- Gaussian parametrization

$$S(r) = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right) \times \text{Effect of short lived resonances (c}\tau \sim 1 \text{ fm)}$$

*ALICE Coll., PLB, 811 (2020), 135849*

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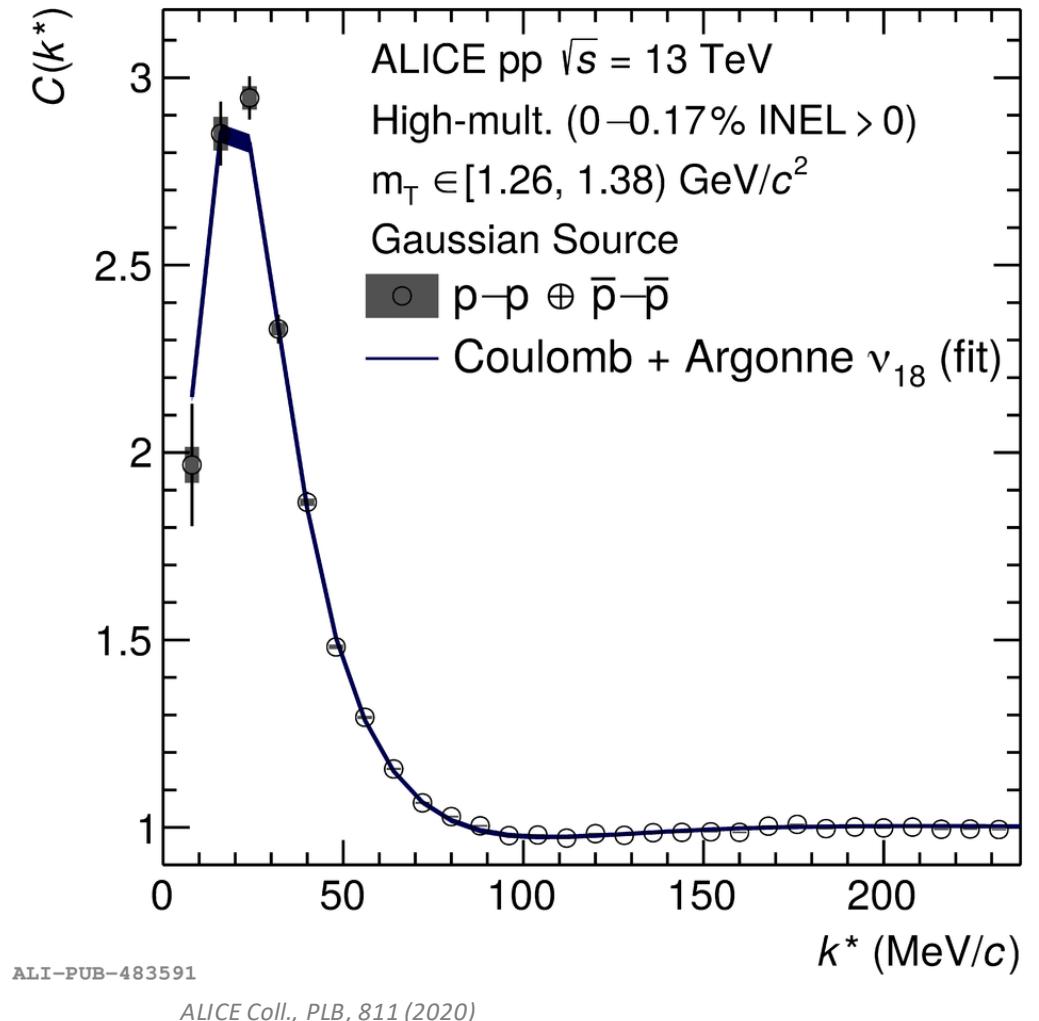
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measured    known interaction

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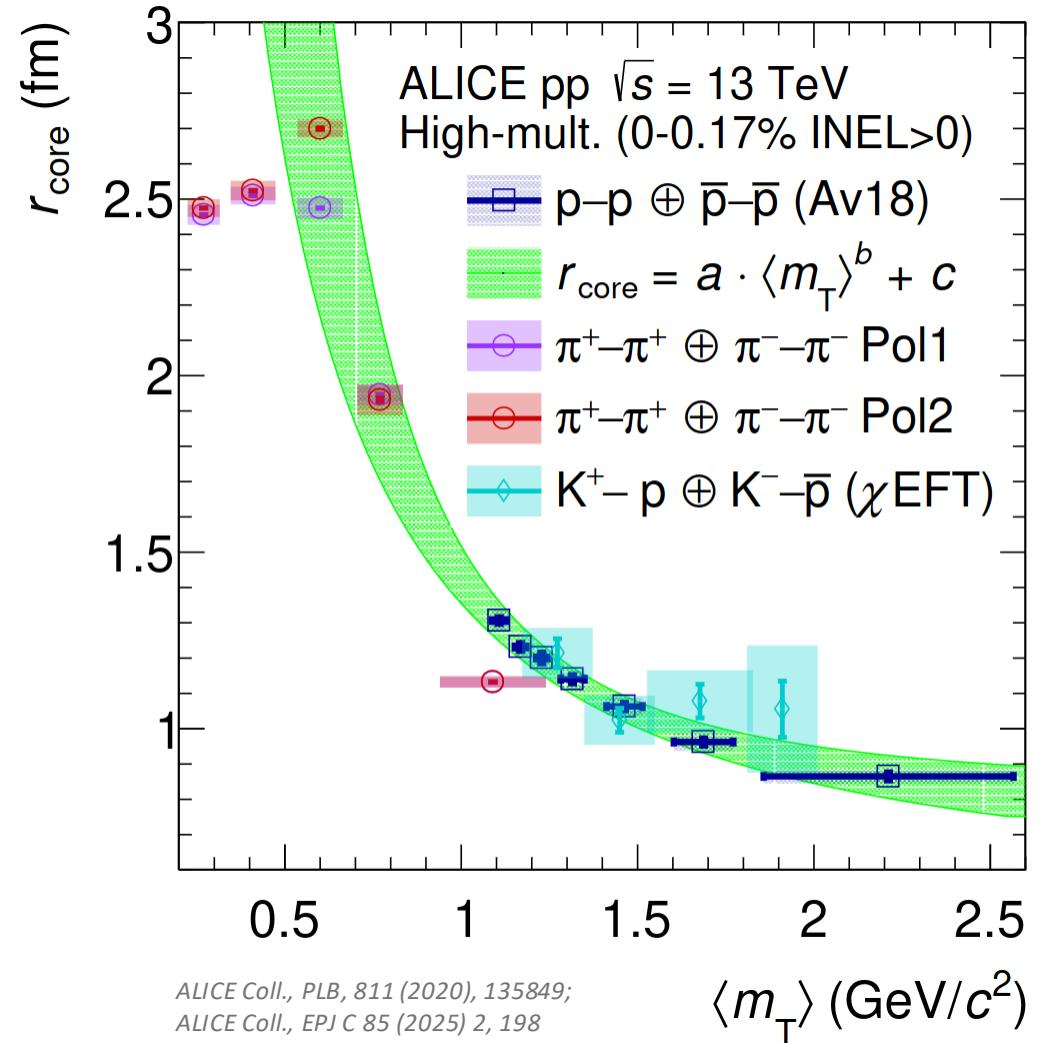
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Effect of short lived  
resonances ( $c\tau \sim 1$  fm)

*ALICE Coll., PLB, 811 (2020), 135849*

- One universal source for all hadrons (cross-check with  $K^+$ -p,  $\pi$ - $\pi$ , p- $\Lambda$ , p- $\pi$ )
- Small particle-emitting source created in pp collisions at the LHC**

*ALICE Coll., PLB, 811 (2020), 135849; ALICE Coll., EPJ C 85 (2025) 2, 198*



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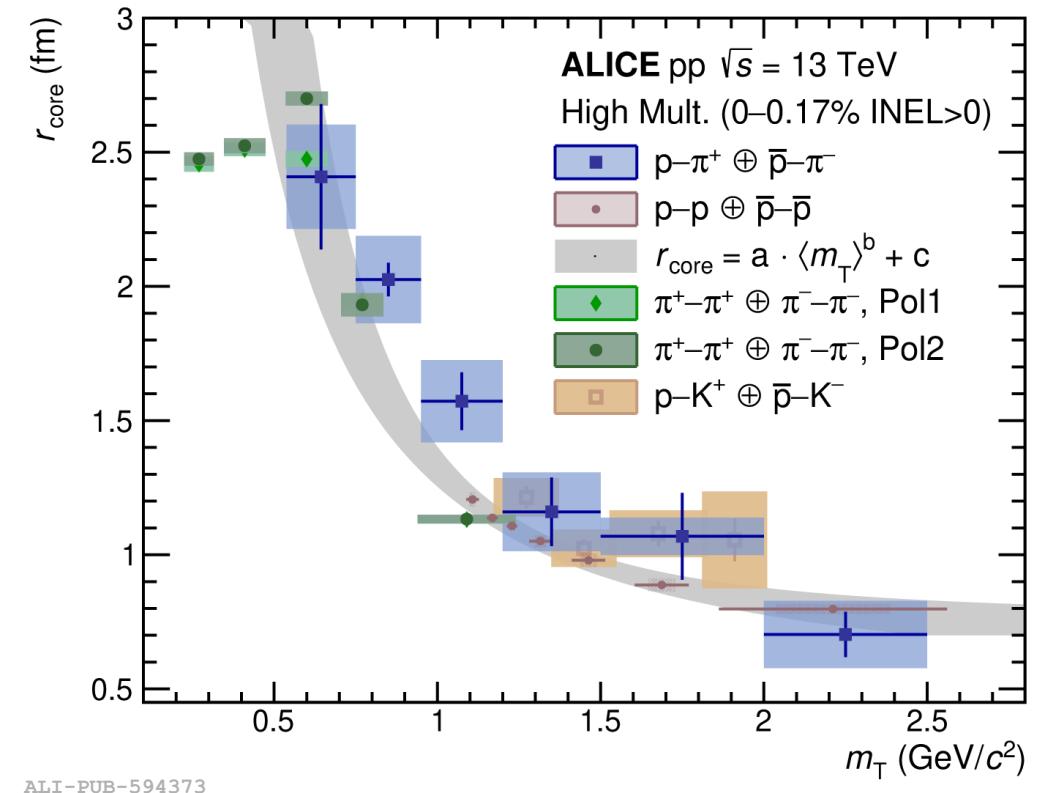
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*ALICE Coll., PLB, 811 (2020), 135849; ALICE Coll., EPJ C 85 (2025) 2, 198;  
ALICE Coll., arXiv:2502.20200 (2025)*



*ALICE Coll., PLB, 811 (2020), 135849;  
ALICE Coll., EPJ C 85 (2025) 2, 198;  
ALICE Coll., arXiv:2502.20200 (2025)*

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measured                                  known interaction

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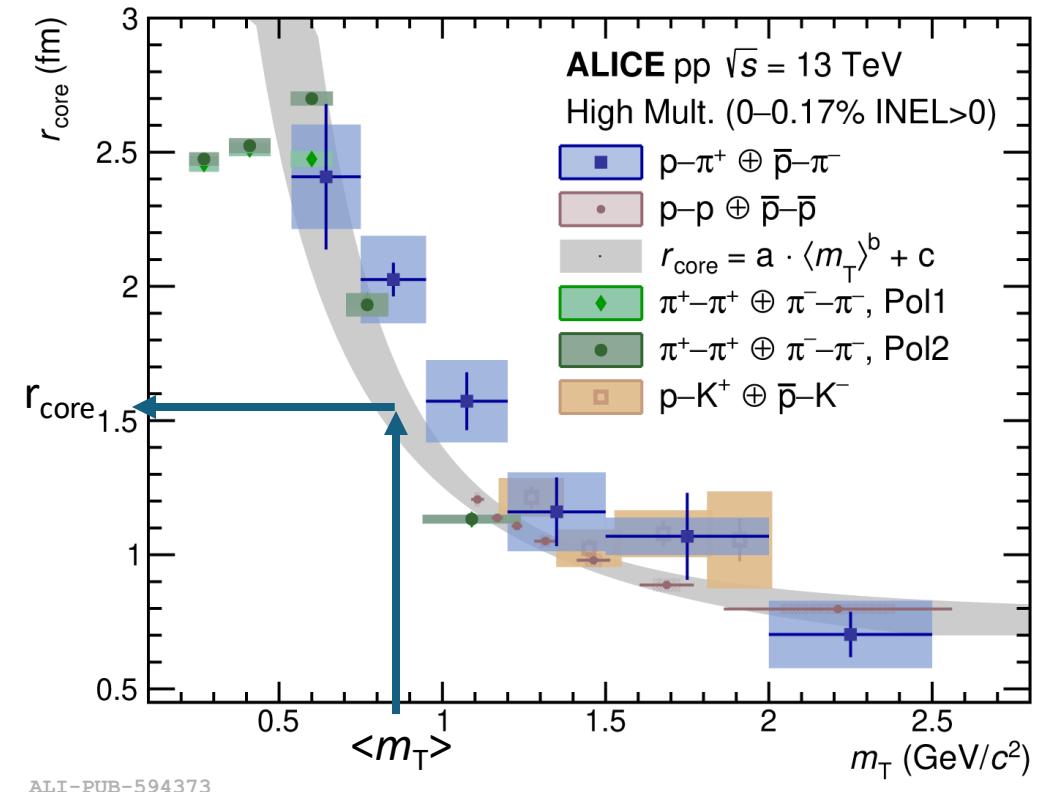
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ALICE Coll., PLB, 811 (2020), 135849

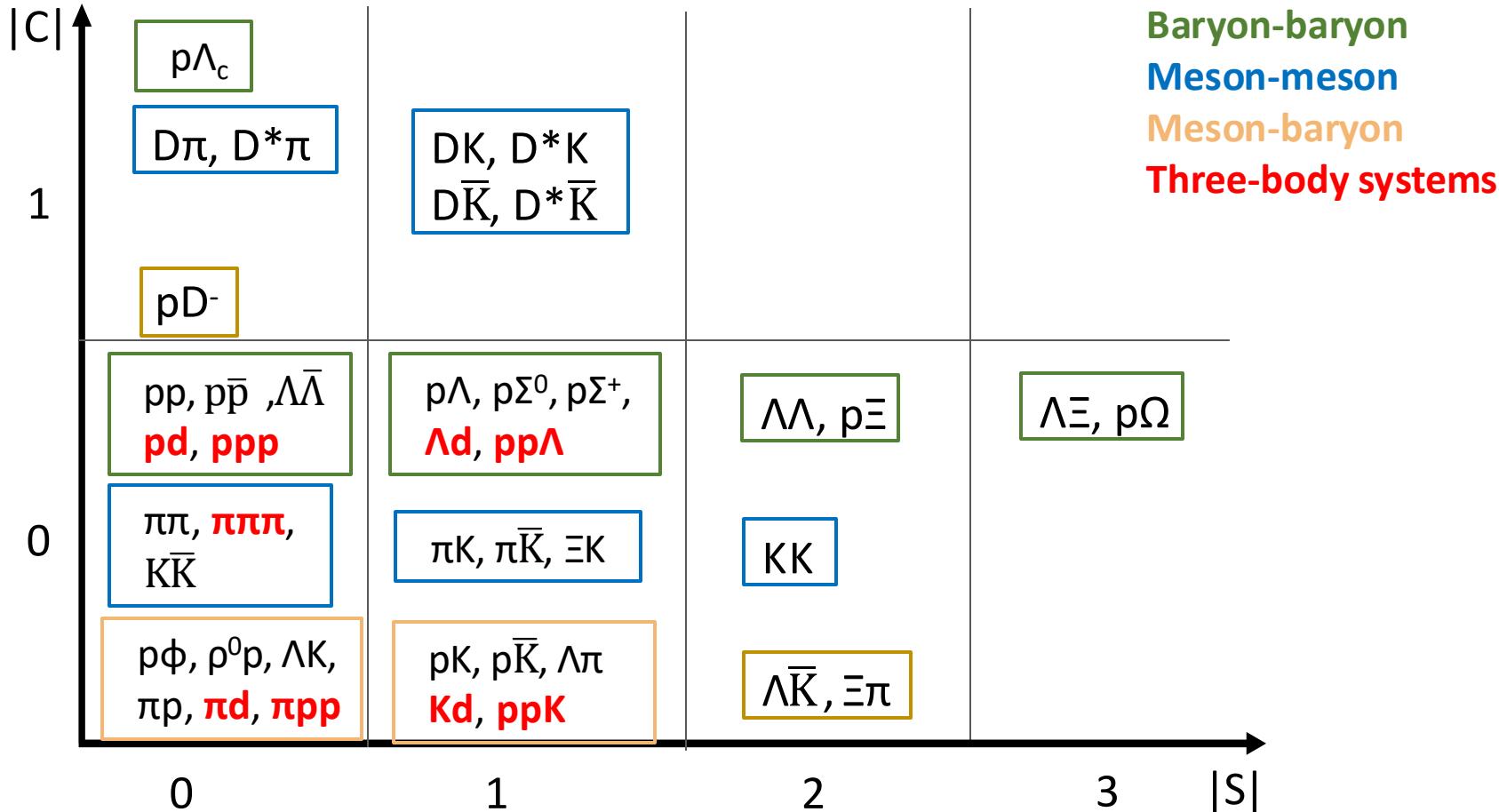
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ALICE Coll., PLB, 811 (2020), 135849; ALICE Coll., EPJ C 85 (2025) 2, 198;  
ALICE Coll., arXiv:2502.20200 (2025)



ALICE Coll., PLB, 811 (2020), 135849;  
ALICE Coll., EPJ C 85 (2025) 2, 198;  
ALICE Coll., arXiv:2502.20200 (2025)

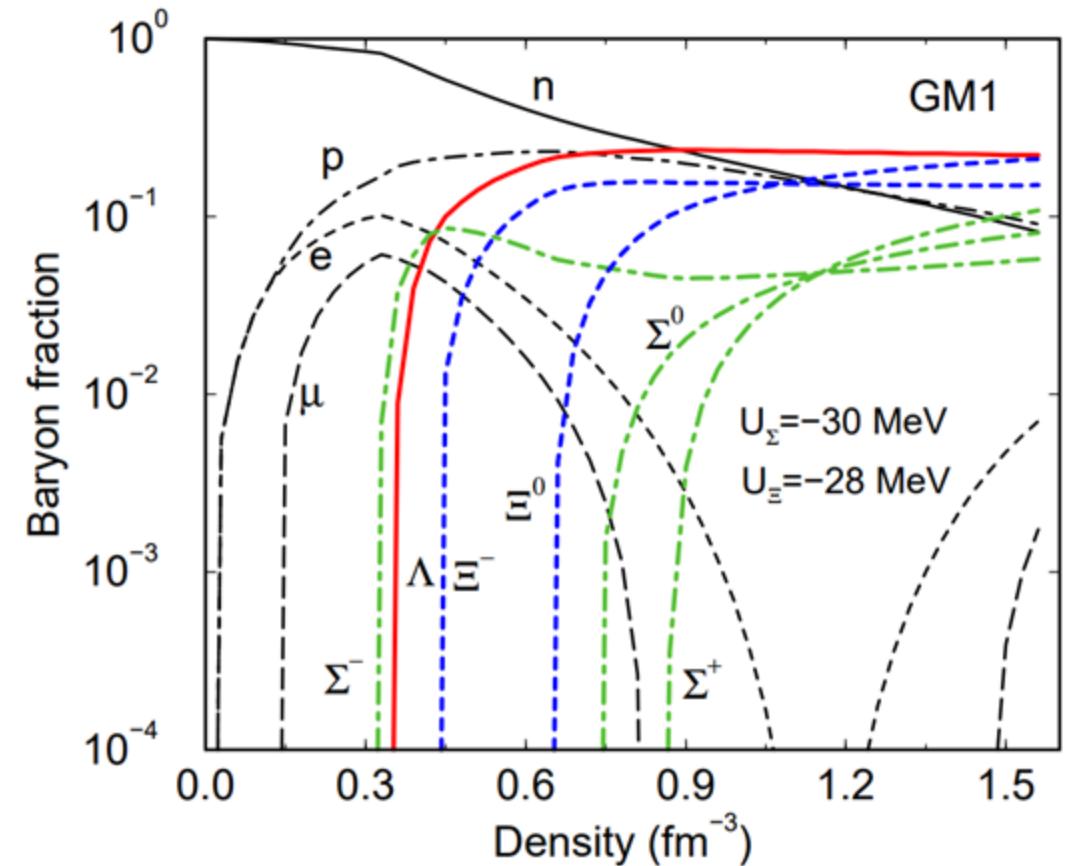
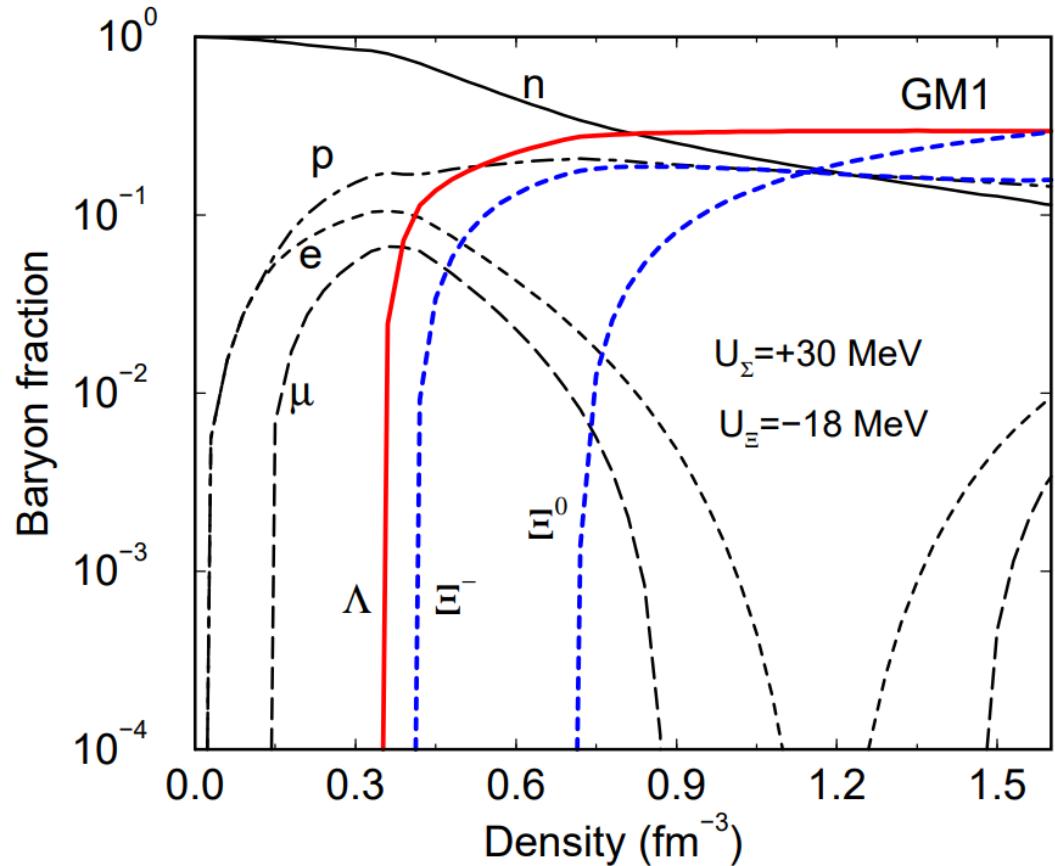
# Femtoscopy measurements at the LHC



*ALICE Collaboration:*  
PRC 99 (2019) 2, 024001  
PLB 797 (2019) 134822  
PRL 123 (2019) 112002  
PRL 124 (2020) 09230  
PLB 805 (2020) 135419  
PLB 811 (2020) 135849  
Nature 588 (2020) 232-238  
PRL 127 (2021), 172301  
PLB 822 (2021), 136708  
PRC 103 (2021) 5, 055201  
PLB 833 (2022), 137272  
PLB 829 (2022), 137060  
PRD 106 (2022), 5, 05201  
PLB 844 (2023) 137223  
EPJA 59 (2023) 145  
EPJC 83 (2023) 4, 340  
PLB 845 (2023) 138145  
EPJA (2023) 59:298  
PRD 110 (2024) 3, 032004  
PRX 14 (2024) 3, 031051  
PLB 856 (2024) 138915  
PRC 109, 024915 (2024)  
EPJC 85 (2025) 2, 198  
arXiv:2502.20200 [nucl-ex]  
arXiv:2504.02333 [nucl-ex]

# Hyperons in neutron stars

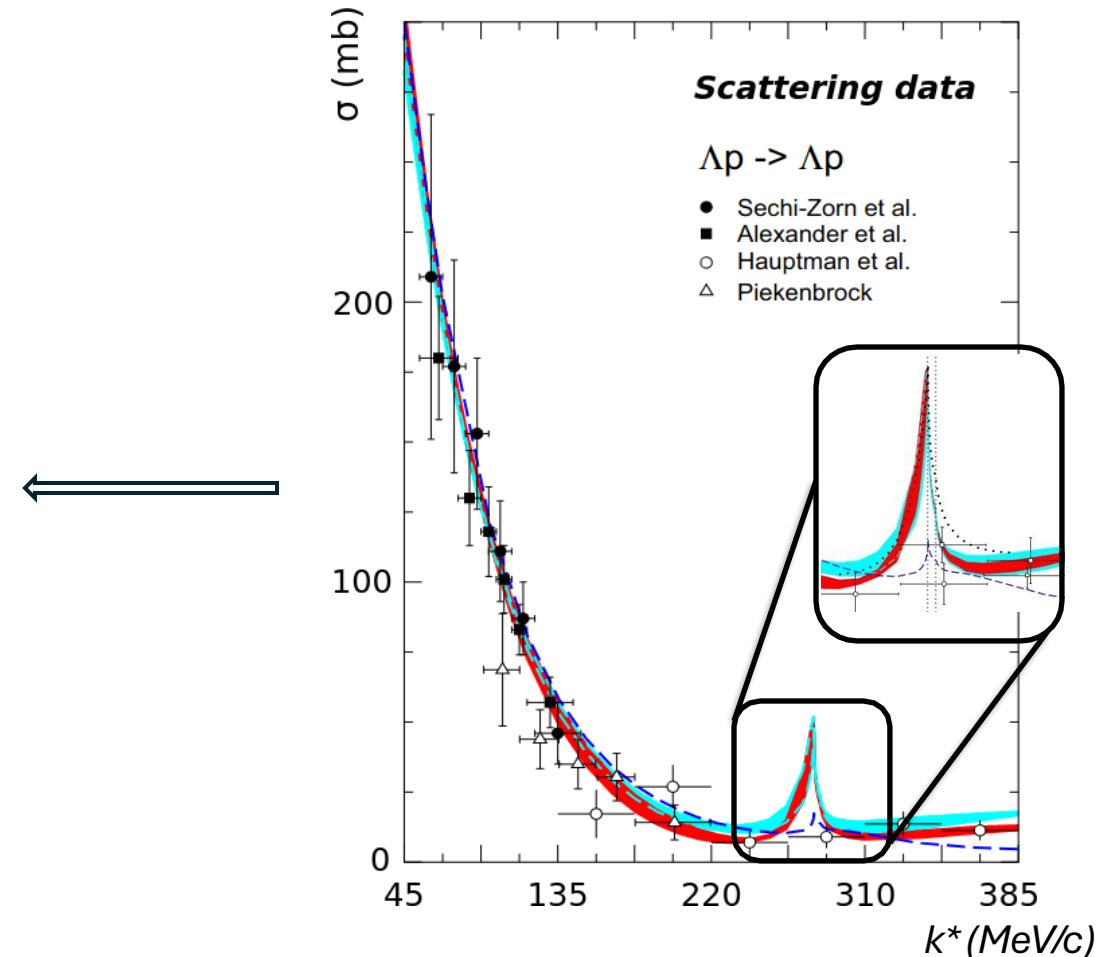
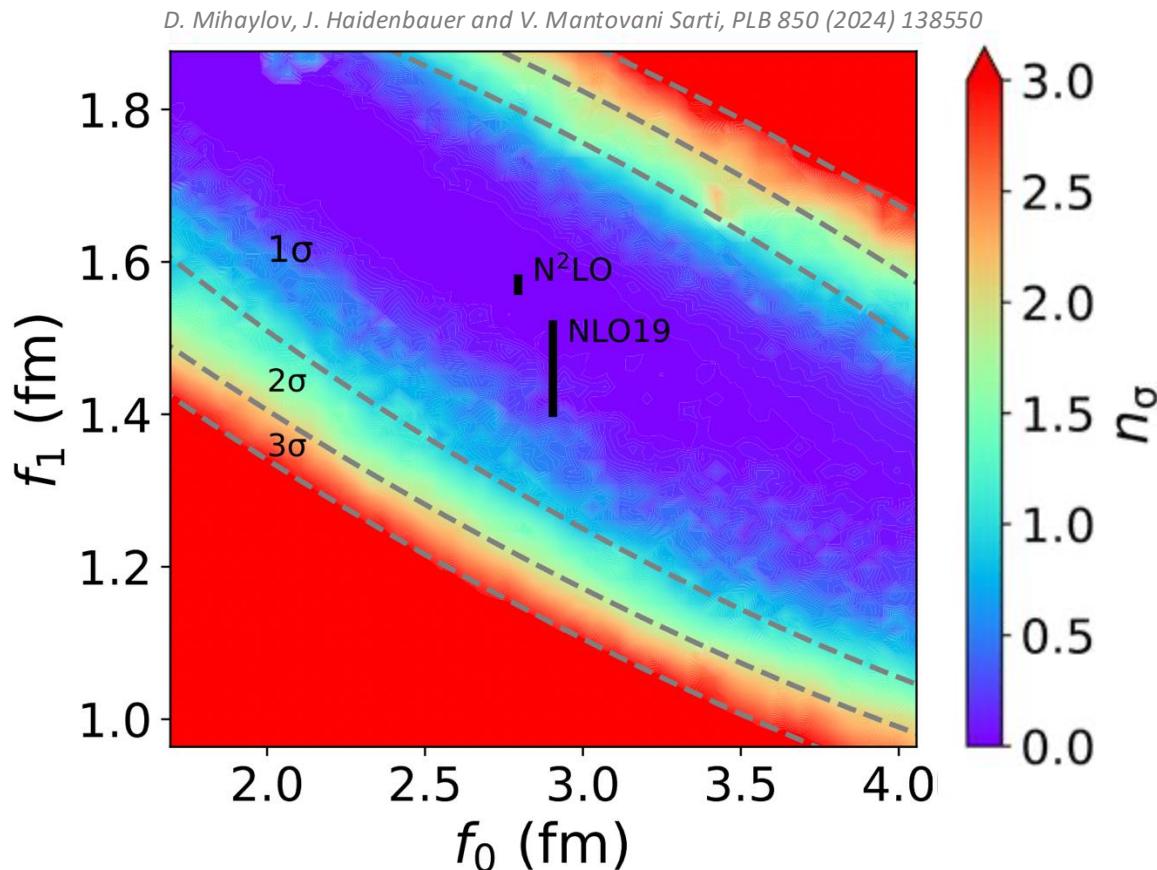
- Exact composition strongly depends on constituent interactions and couplings!



J. Schaffner-Bielich et al NPA 835 (2010)

# The p $\Lambda$ interaction before femtoscopy

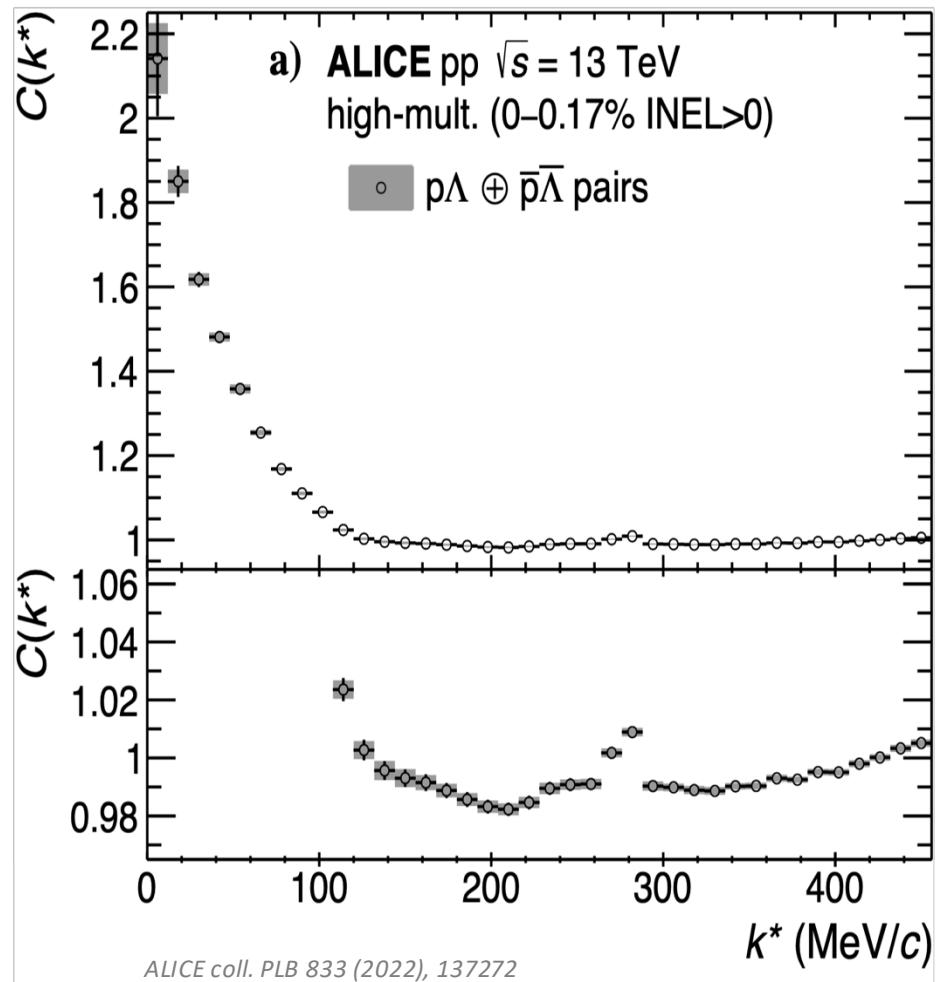
- Spin-0 and Spin-1 scattering length from scattering data
- Agreement with N2LO and NLO19



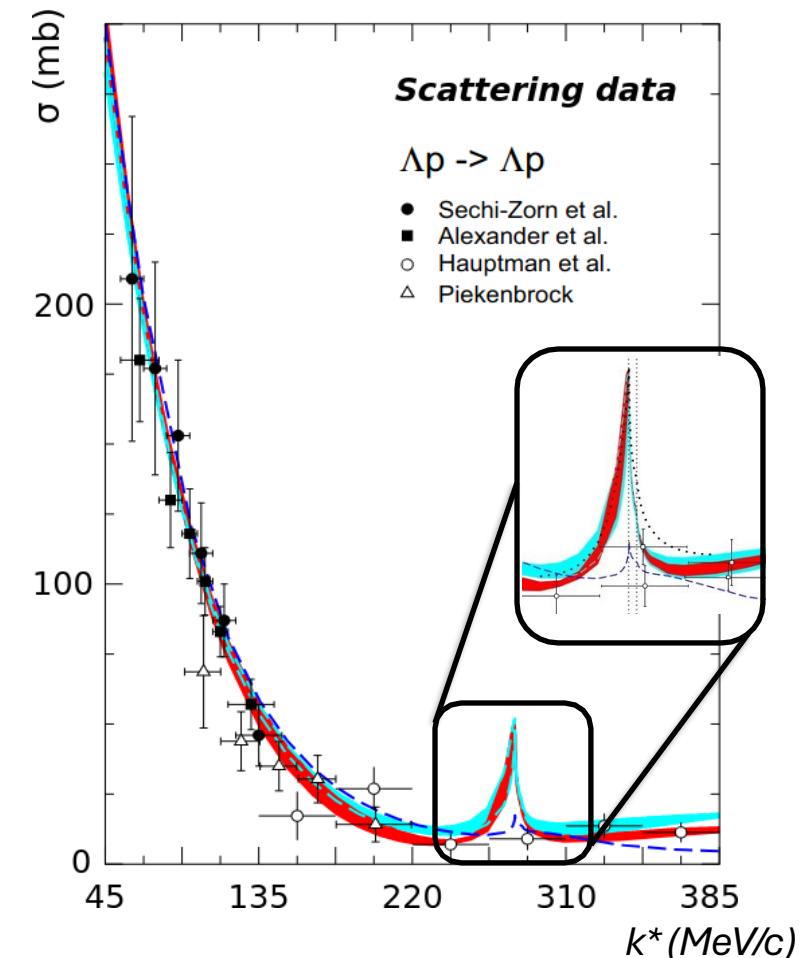
NLO19: J. Haidenbauer, U. Mei $\beta$ ner, EPJA 56 (2020), 3, 91

NLO13: J. Haidenbauer, N. Kaiser et al., NPA 915, 24 (2013)

# The p $\Lambda$ interaction in the femtoscopy era



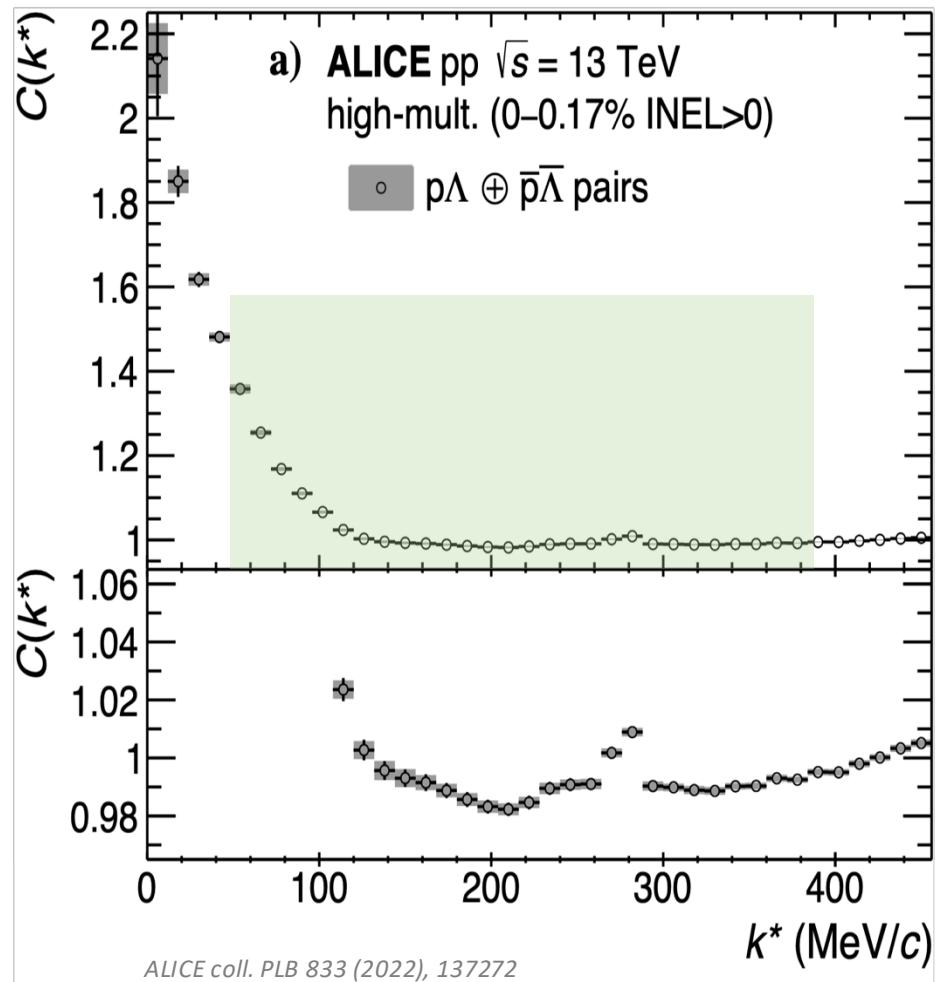
- Measurement down to zero momentum
- Factor 20 improved precision (<1%)
- First experimental evidence of  $\Lambda N$ - $\Sigma N$  opening in 2-body channel



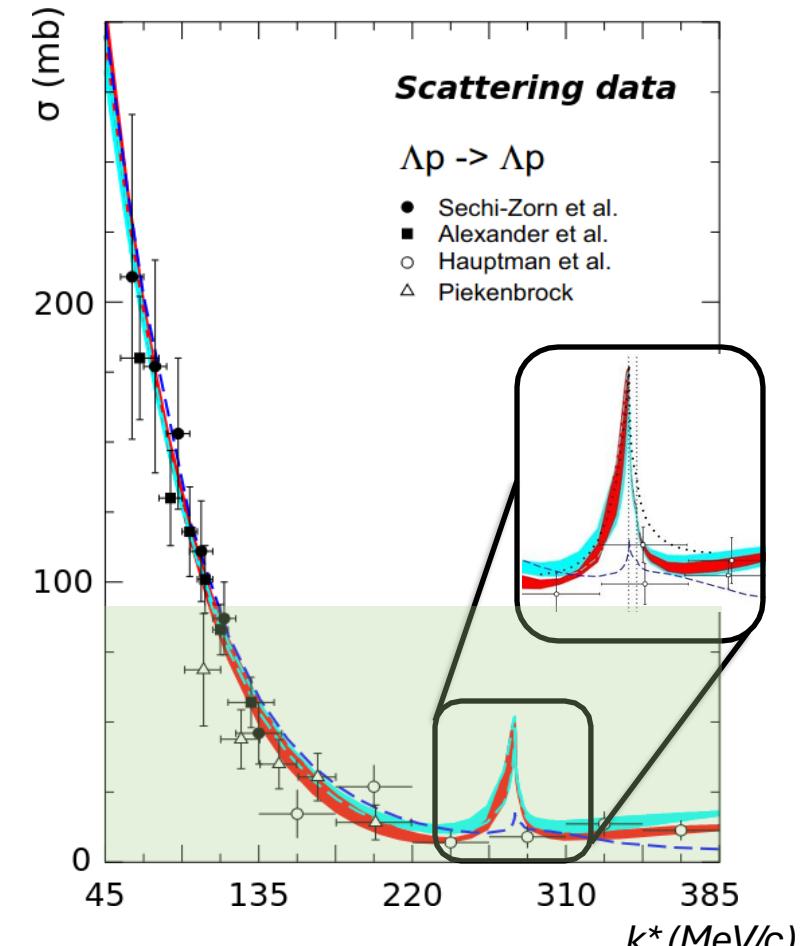
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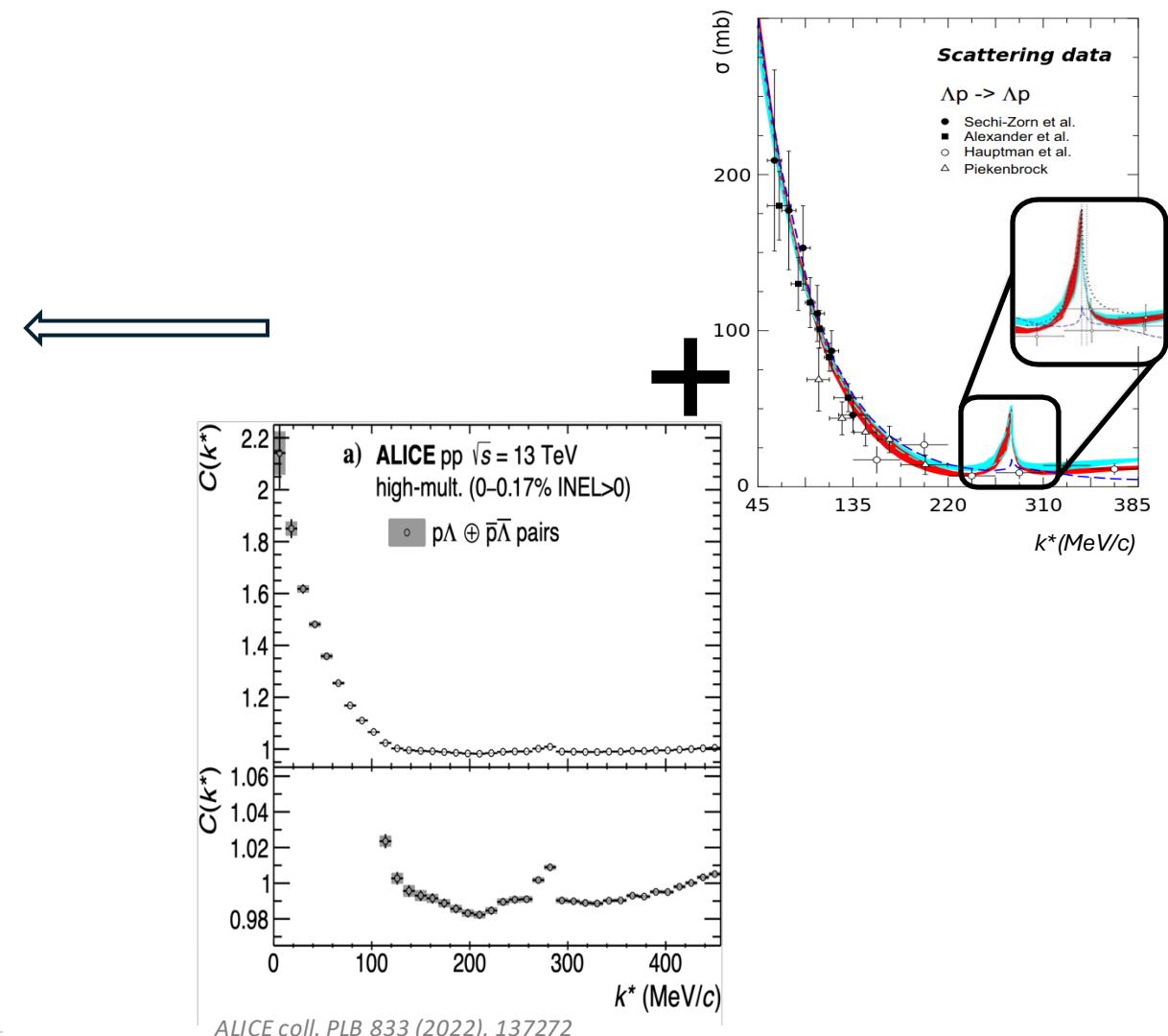
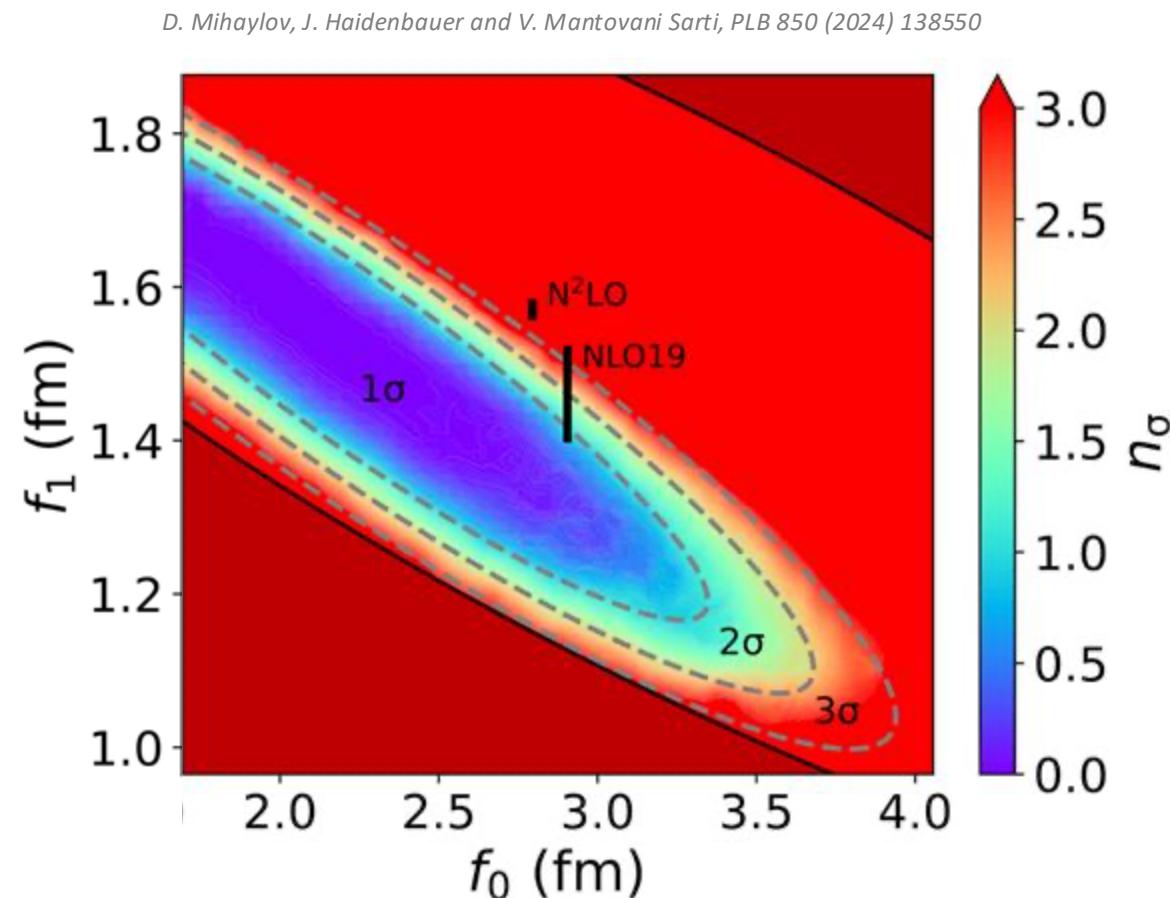


NLO19: J. Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91

NLO13: J. Haidenbauer, N. Kaiser et al., NPA 915, 24 (2013)

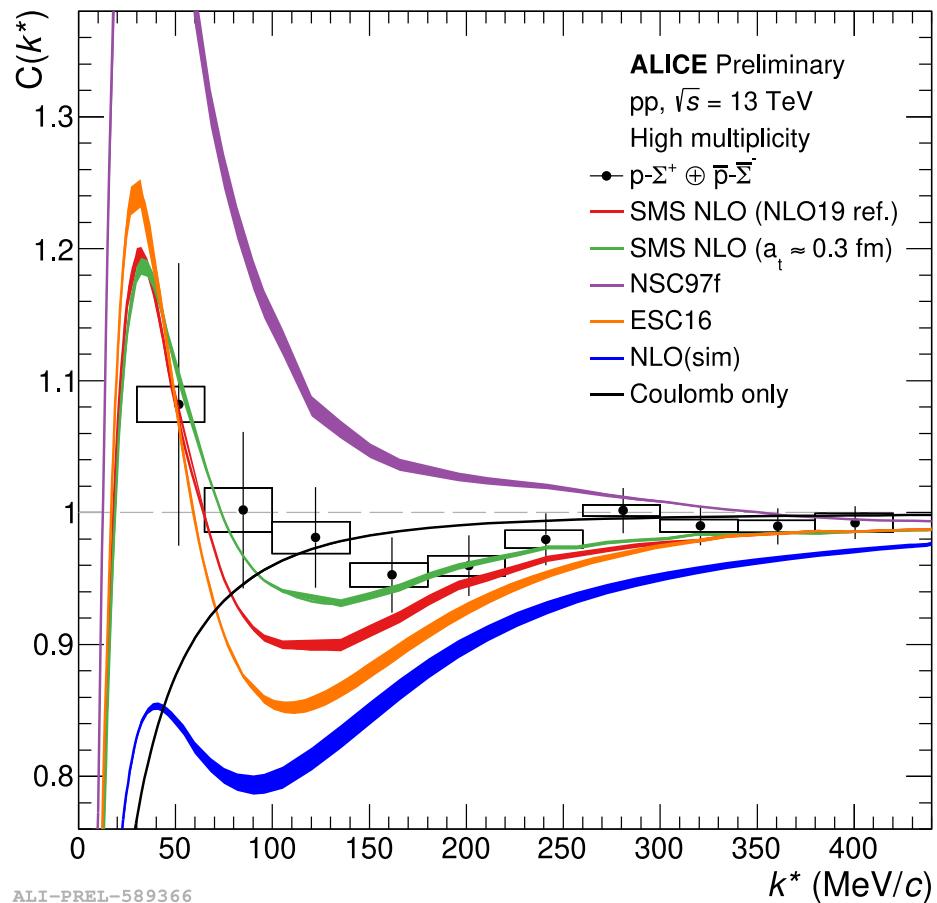
# The p $\Lambda$ interaction in the femtoscopy era

- Combined analysis of femtoscopic and scattering data

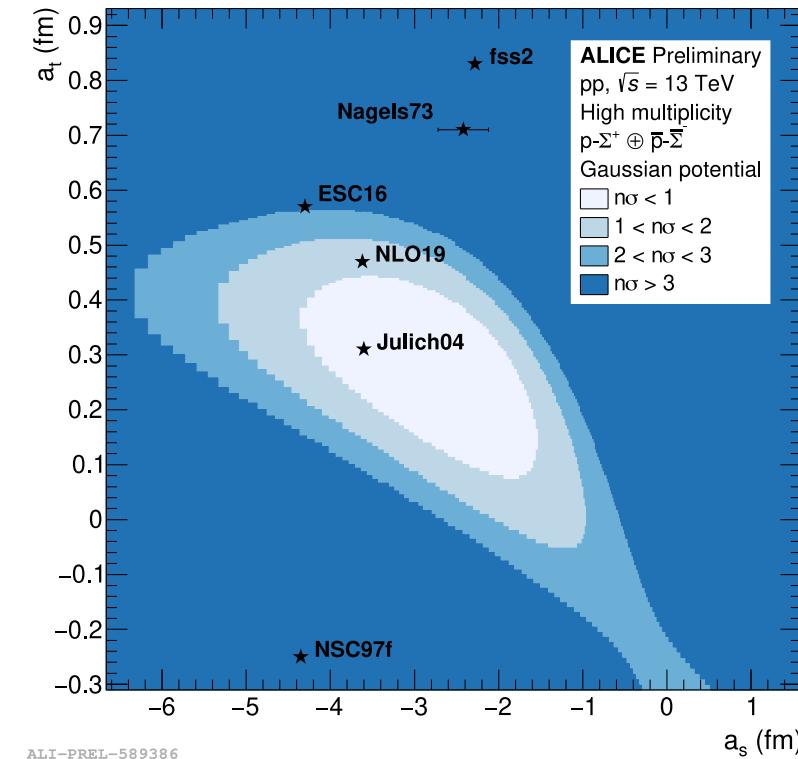


# The $p\Sigma^+$ interaction

- Data sensitive to the triplet channel



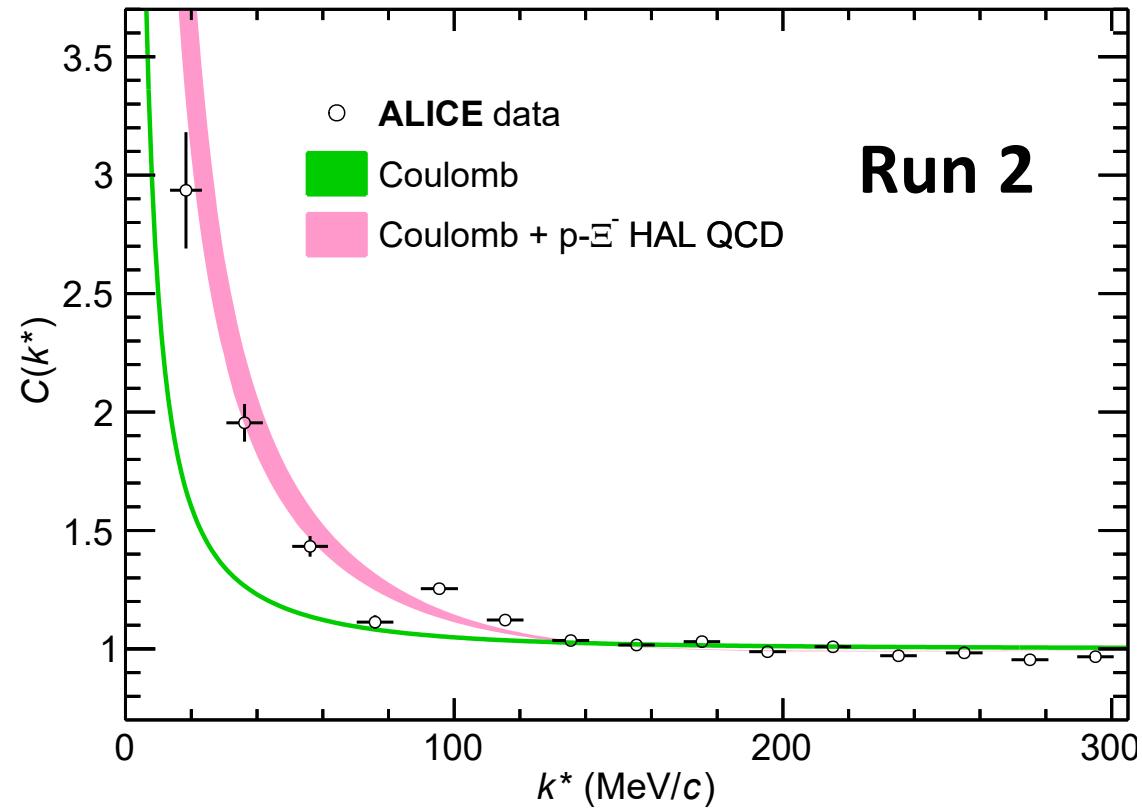
- Fit with Gaussian + Coulomb
- Shallow repulsion in triplet channel



B. Heybeck in collaboration with Y. Kamiya

# The $p\Xi^-$ interaction

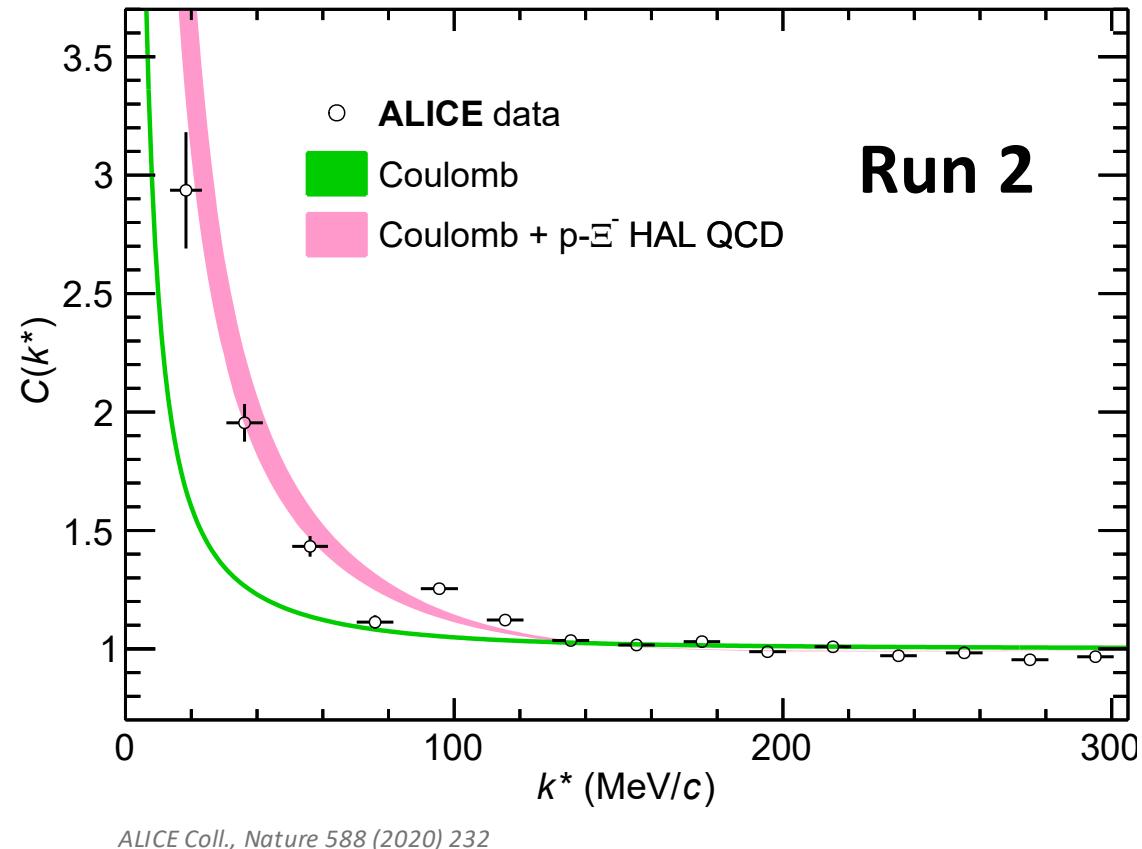
- Evidence of the attractive strong interaction



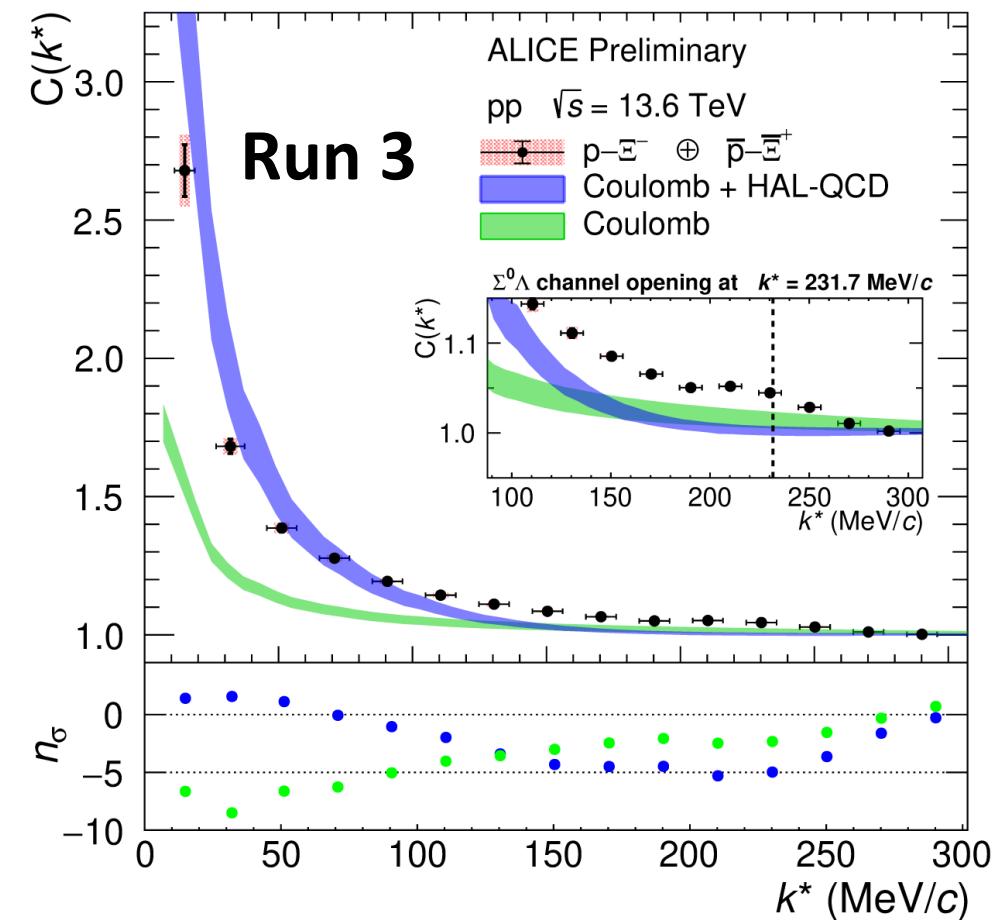
ALICE Coll., Nature 588 (2020) 232

# The $p\Xi^-$ interaction

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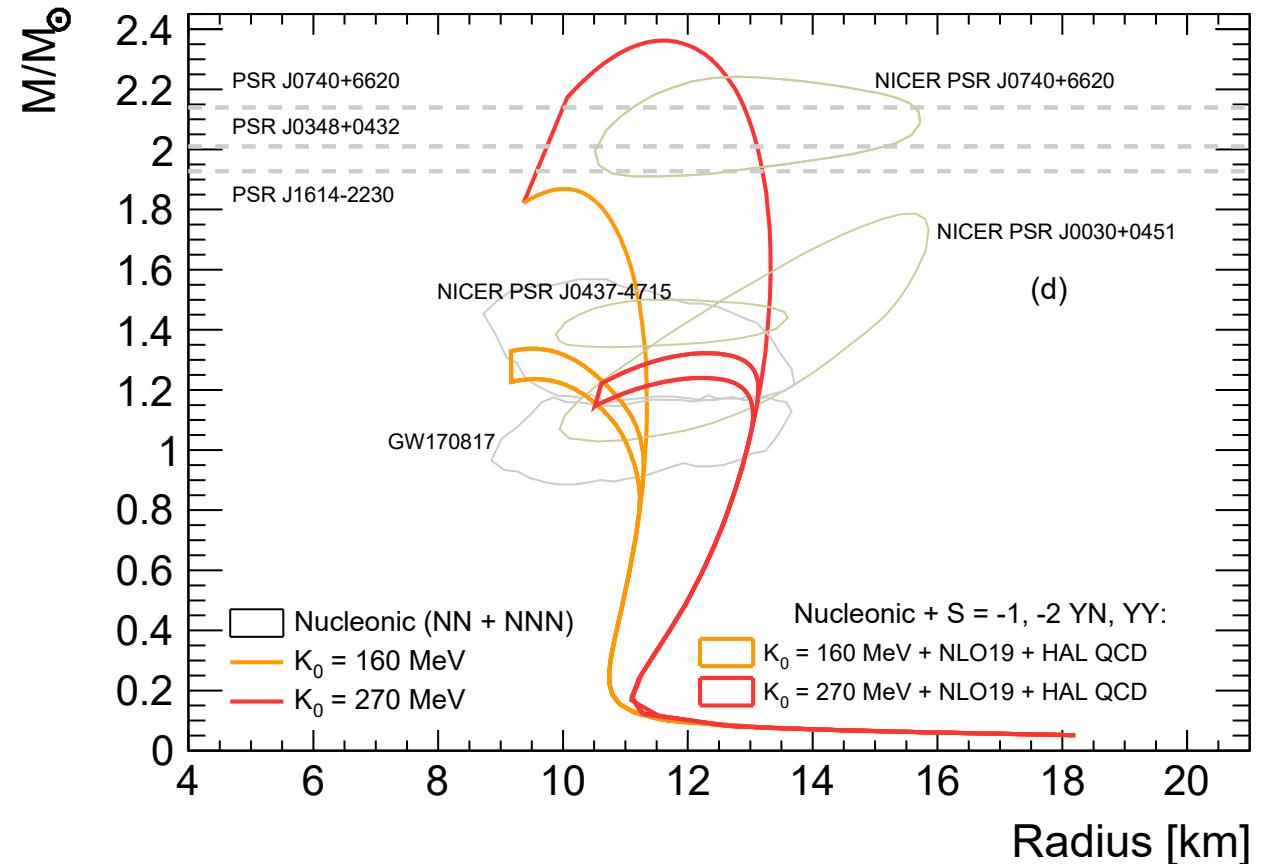
- Opening of the  $p\Xi \rightarrow \Lambda\Sigma^0$  channel



# Towards a realistic equation of state of neutron stars

- *State-of-the-art interactions* for NN, NNN, YN (S=-1 and S=-2) and YY fail to reproduce observed heavy neutron stars

I. Vidaña, V. Mantovani Sarti, J. Haidenbauer, D. Mihaylov, L. Fabbietti, arXiv: 2412.12729  
(2024)

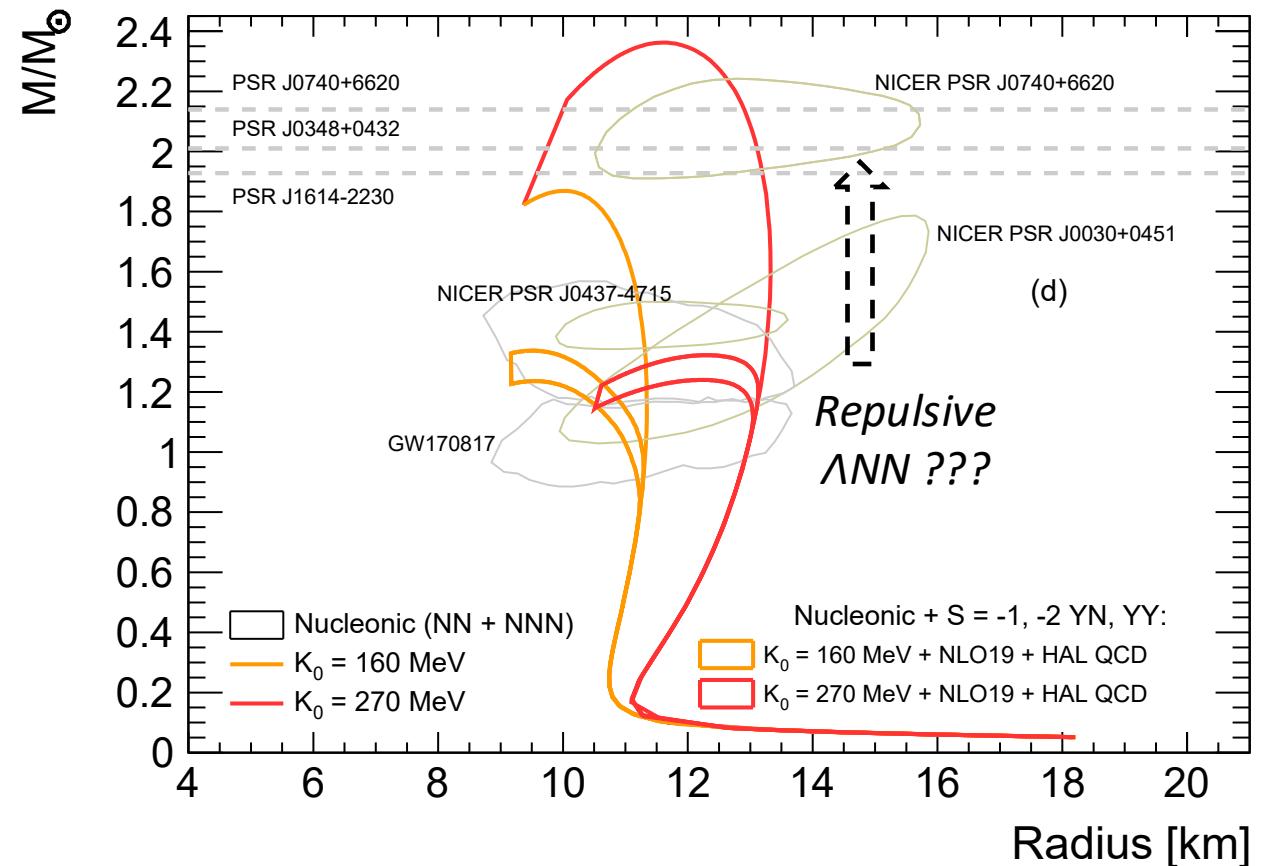


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(2024)

- Next step → inclusion of three body interaction involving hyperons



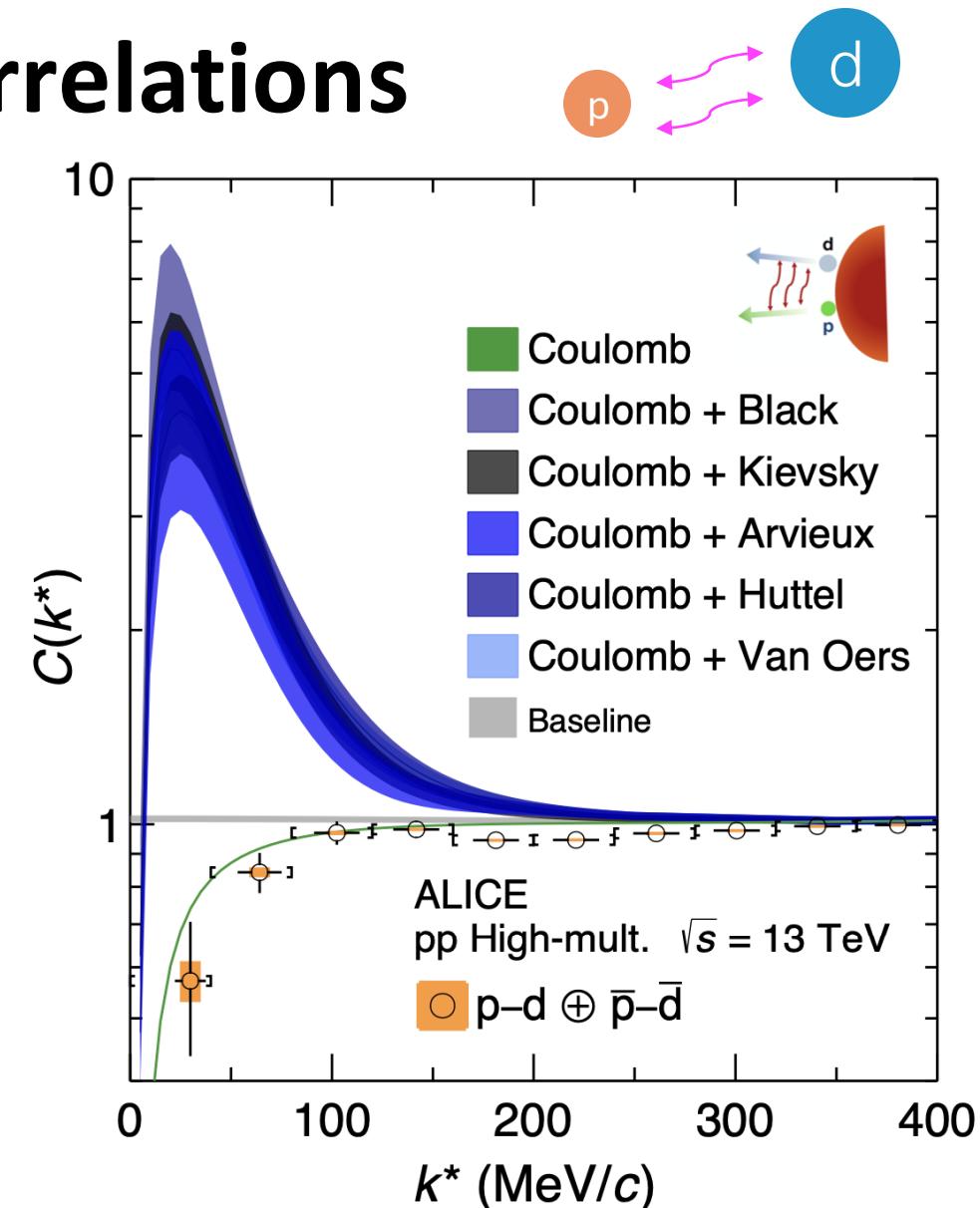
# NNN using proton-deuteron correlations

- Point-like particle models anchored to scattering experiments

W. T. H. Van Oers et al., NPA 561 (1967);  
J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);  
A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);

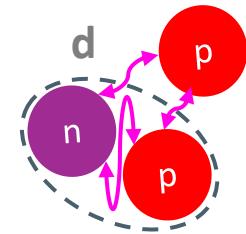
- Coulomb + strong interaction using Lednický model  
Lednický, R. Phys. Part. Nuclei 40, 307–352 (2009)
- Only s-wave interaction
- Source radius evaluated using the universal  $m_T$  scaling

**Point-like particle description doesn't work for p-d**



ALICE Coll. Phys. Rev. X 14, 031051 (2024)

# NNN using proton-deuteron correlations



- The p-d **correlation function**, assuming that p-p-n forms p-d

$$C_{pd}(k) = \frac{1}{A_d} \frac{1}{6} \sum_{m_1, m_2} \int d^3r_1 d^3r_2 d^3r_3 S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_1, m_2}|^2$$

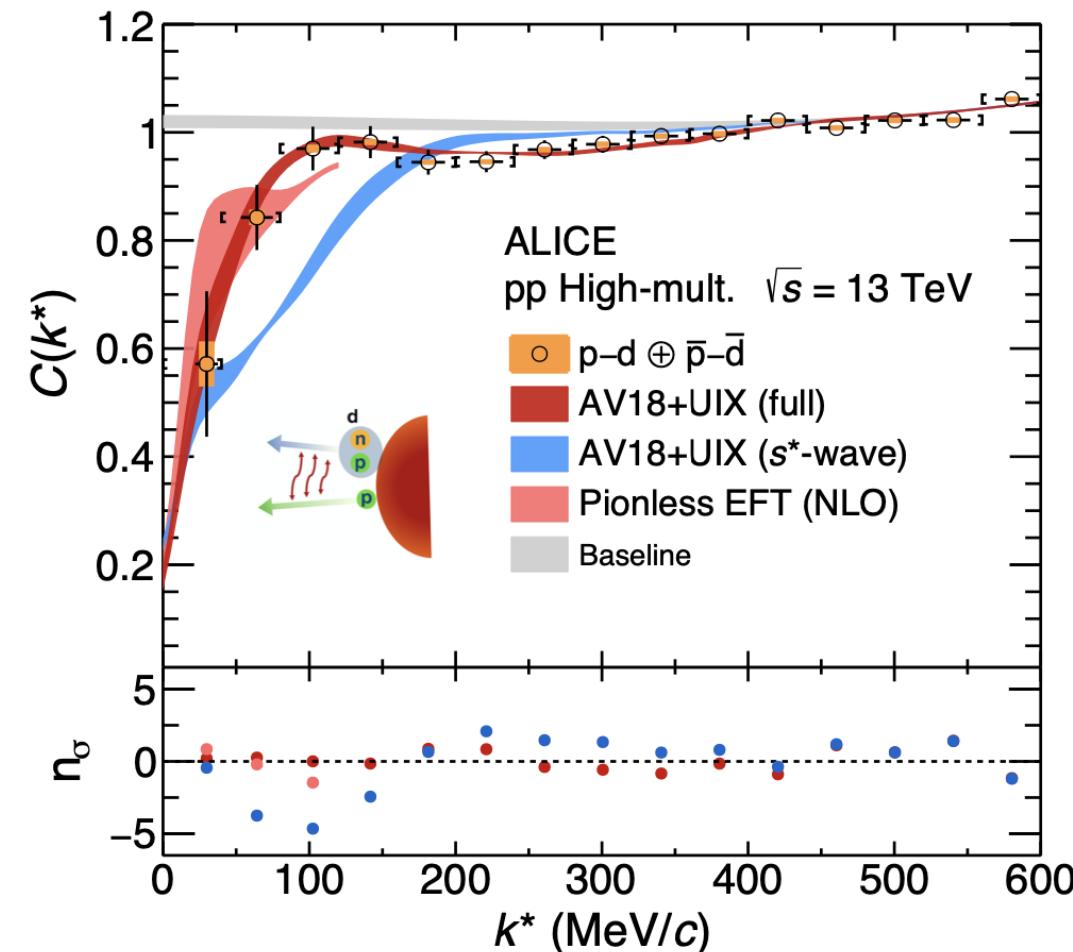
where  $S_1(r)$  is a single-particle Gaussian source and  $A_d$  is the formation probability of a deuteron

- The **three-body wavefunction** of the p-d System

$$\Psi_{m_2, m_1}(x, y) = \Psi_{m_2, m_1}^{free} + \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} \left( 1m_2 \frac{1}{2}m_1 \left| SJ_z \right\rangle \langle L0SJ_z | JJ_z \right) \tilde{\Psi}_{LSJJ_z}$$

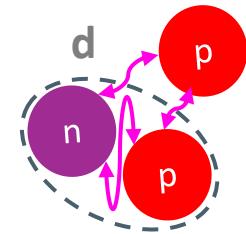
Asymptotic solution      Three-body dynamics

- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics**

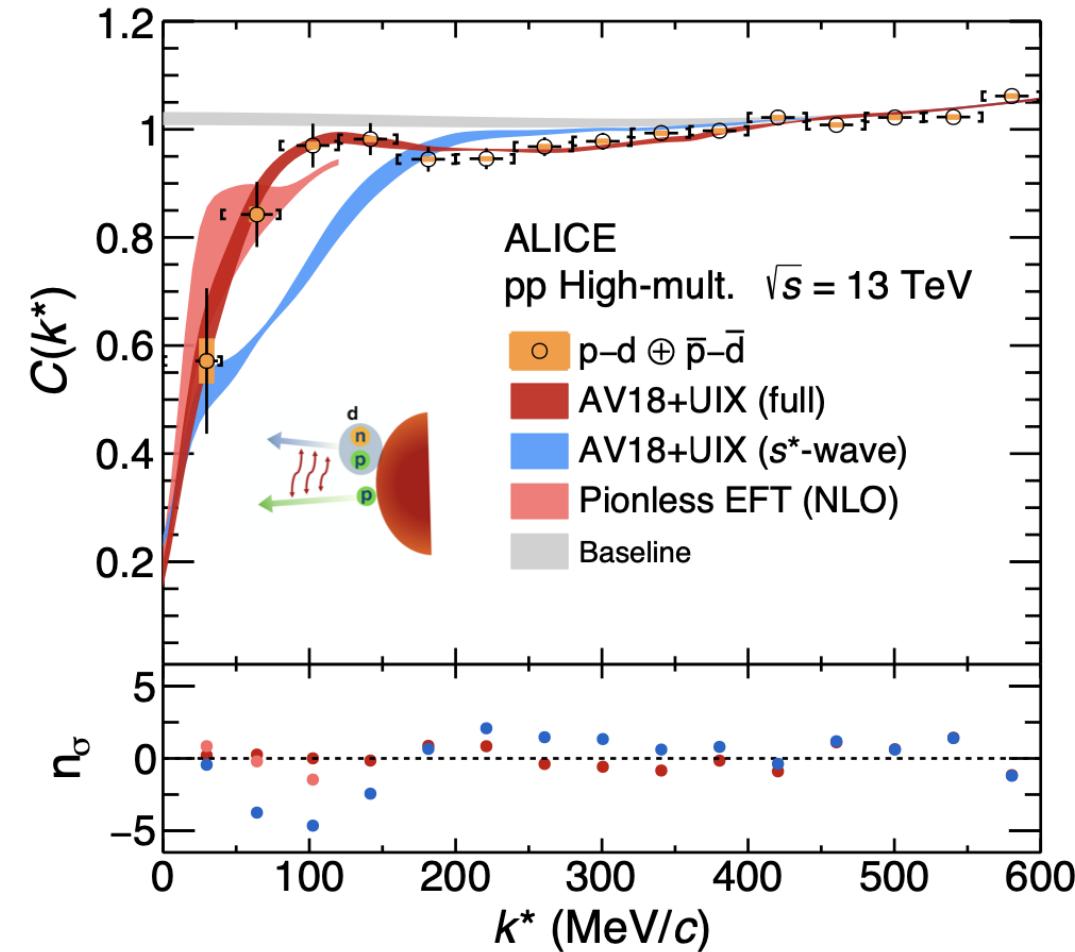


ALICE Coll. Phys. Rev. X 14, 031051 (2024)  
M. Viviani et al, Phys. Rev. C 108 (2023) 6, 064002

# NNN using proton-deuteron correlations

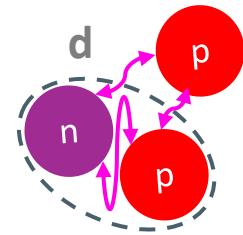


- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

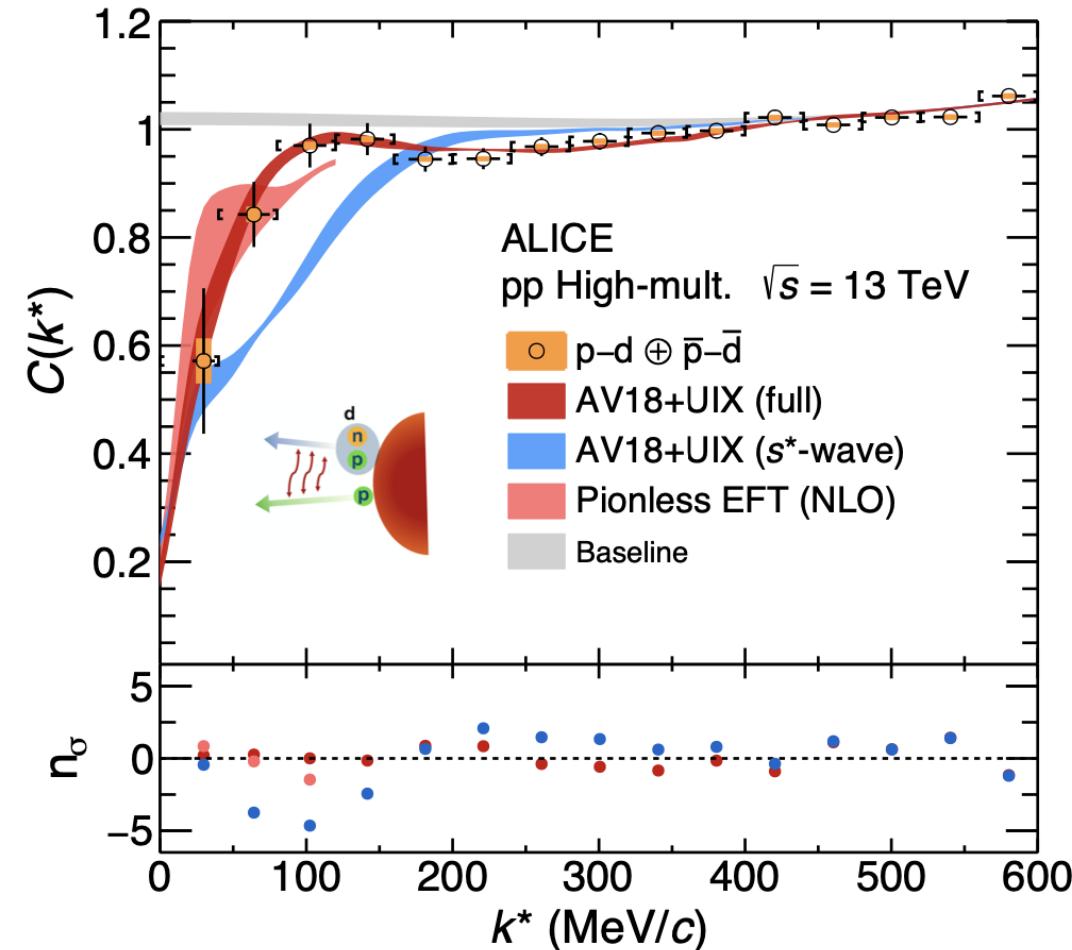
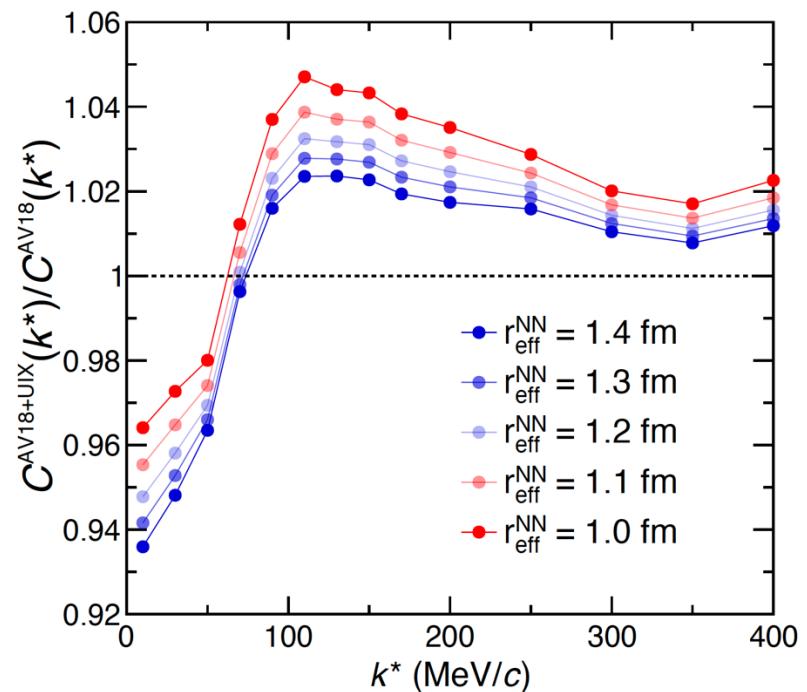


ALICE Coll. Phys. Rev. X 14, 031051 (2024)  
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# NNN using proton-deuteron correlations

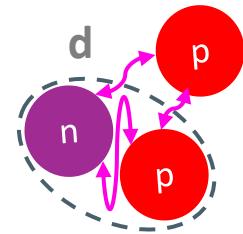


- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics
- Sensitivity to three-body forces up to 5%

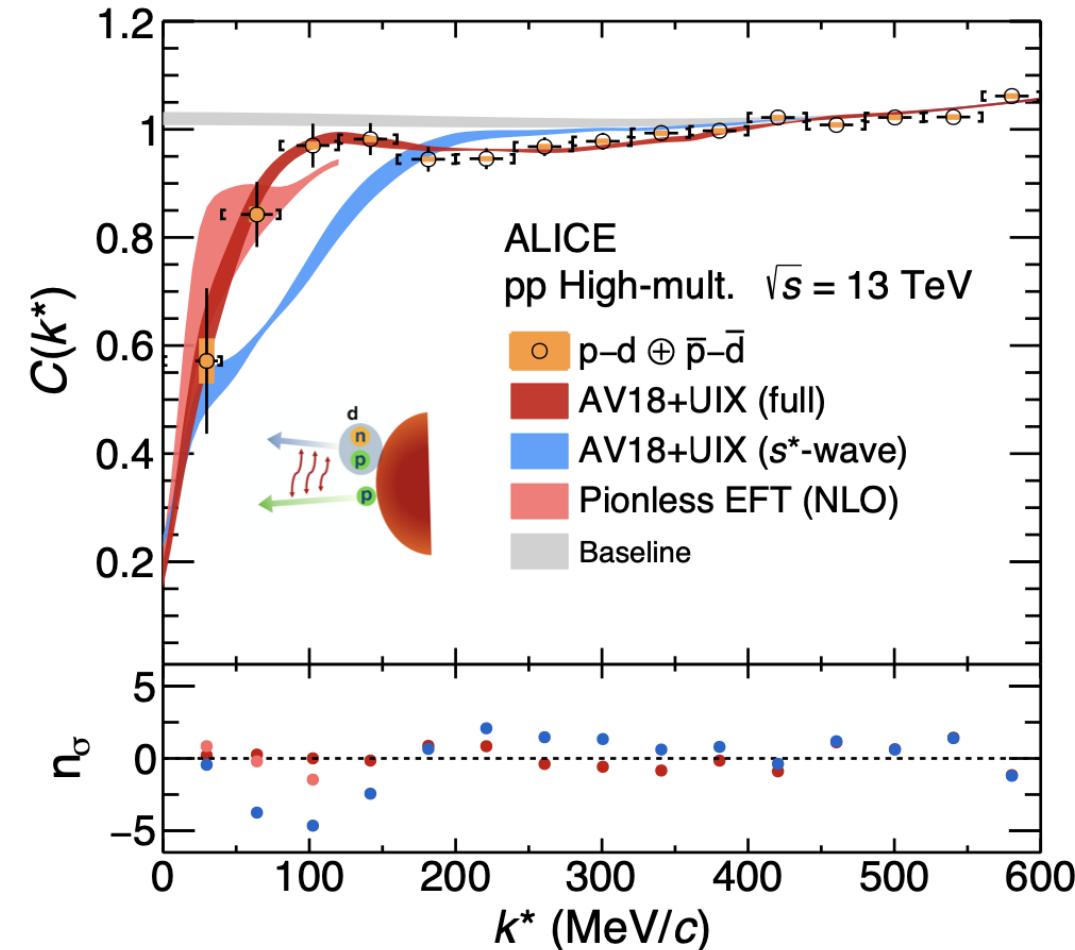
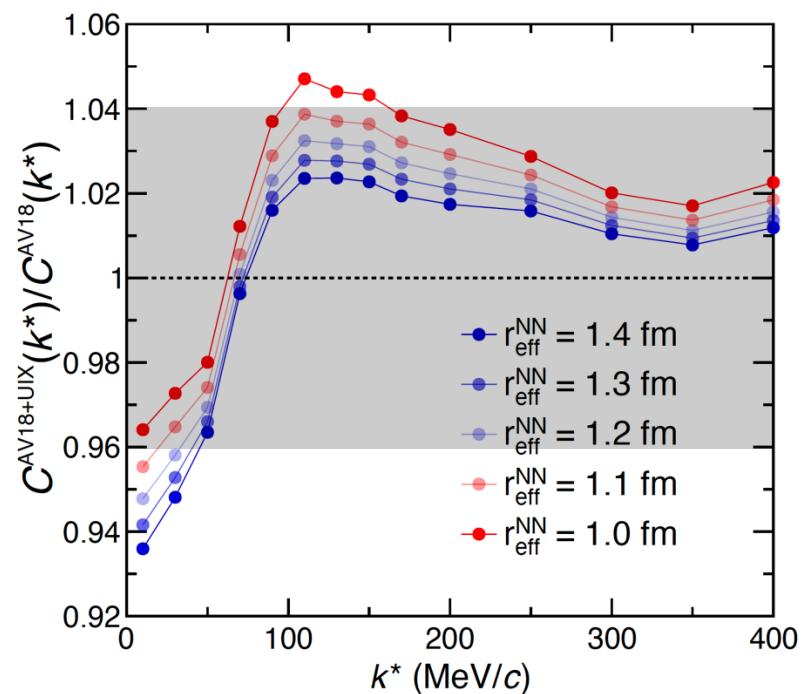


ALICE Coll. Phys. Rev. X 14, 031051 (2024)  
M. Viviani et al, Phys. Rev. C 108 (2023) 6, 064002

# NNN using proton-deuteron correlations

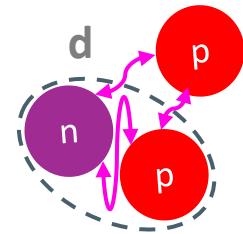


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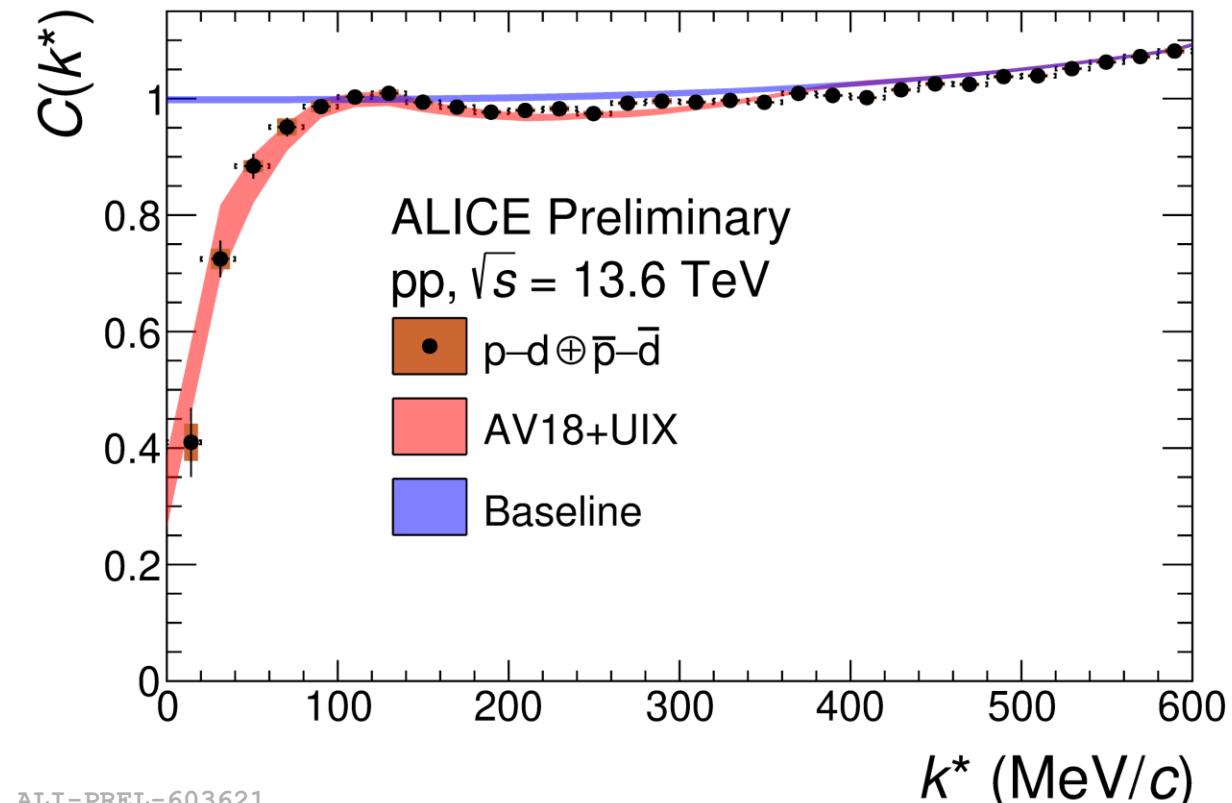
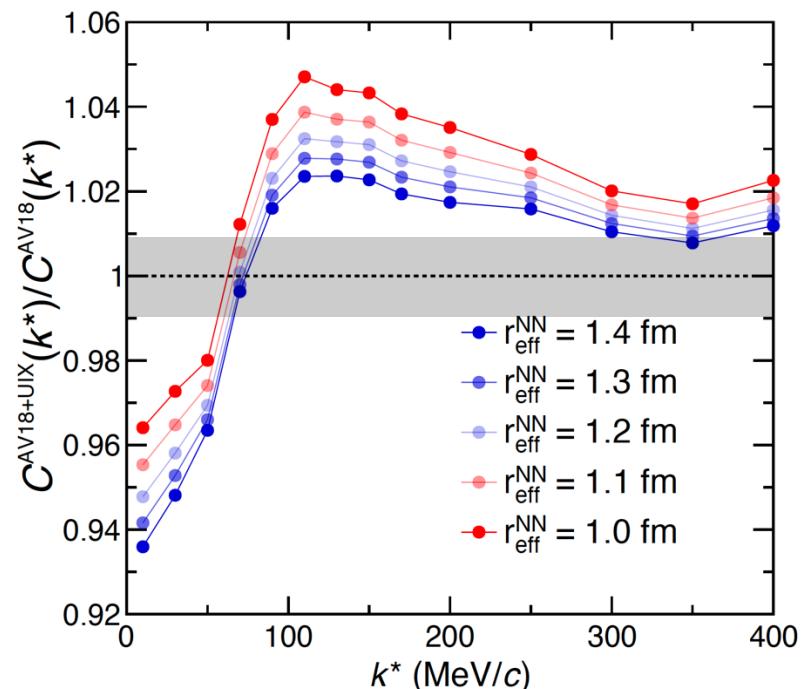


ALICE Coll. Phys. Rev. X 14, 031051 (2024)  
M. Viviani et al, Phys. Rev. C 108 (2023) 6, 064002

# NNN using proton-deuteron correlations

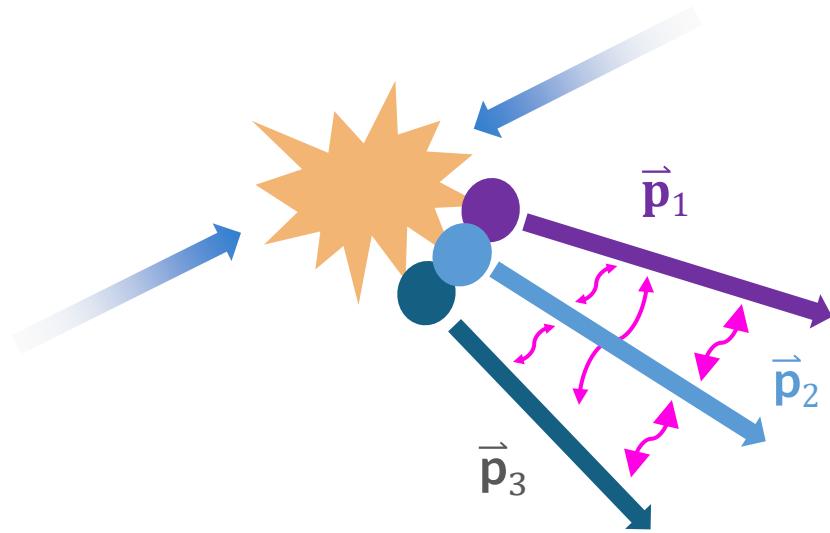


- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Results from Run 3 are promising!
- In Run 3 expected uncertainty of 1%



M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

# Three-body femtoscopy in pp collisions



Correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

Three-body scattering wave function

Hyper-momentum:

$$Q_3 = 2\sqrt{k_{12}^2 + k_{23}^2 + k_{31}^2}$$

R. Del Grande et al. EPJC 82 (2022) 244  
ALICE Coll., EPJA 59, 145 (2023)

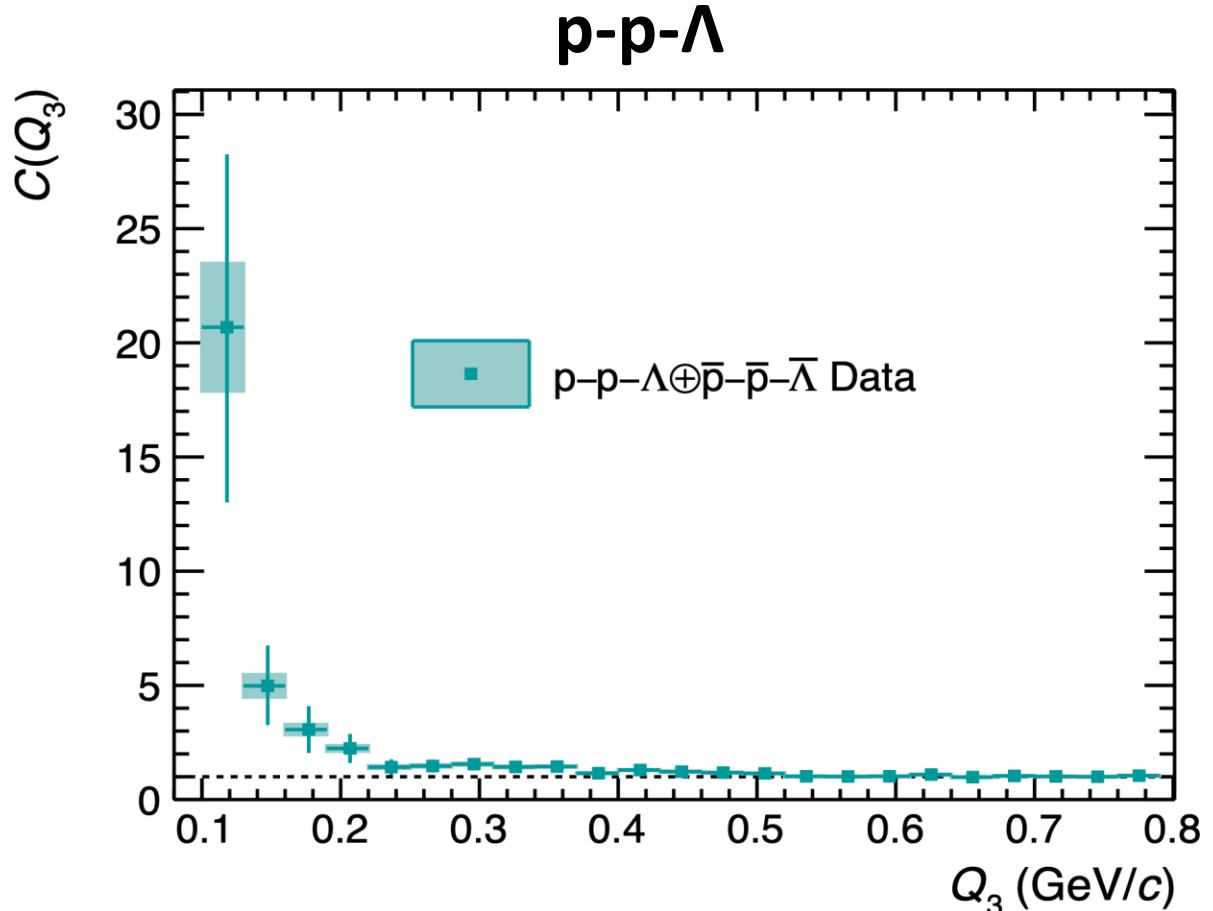
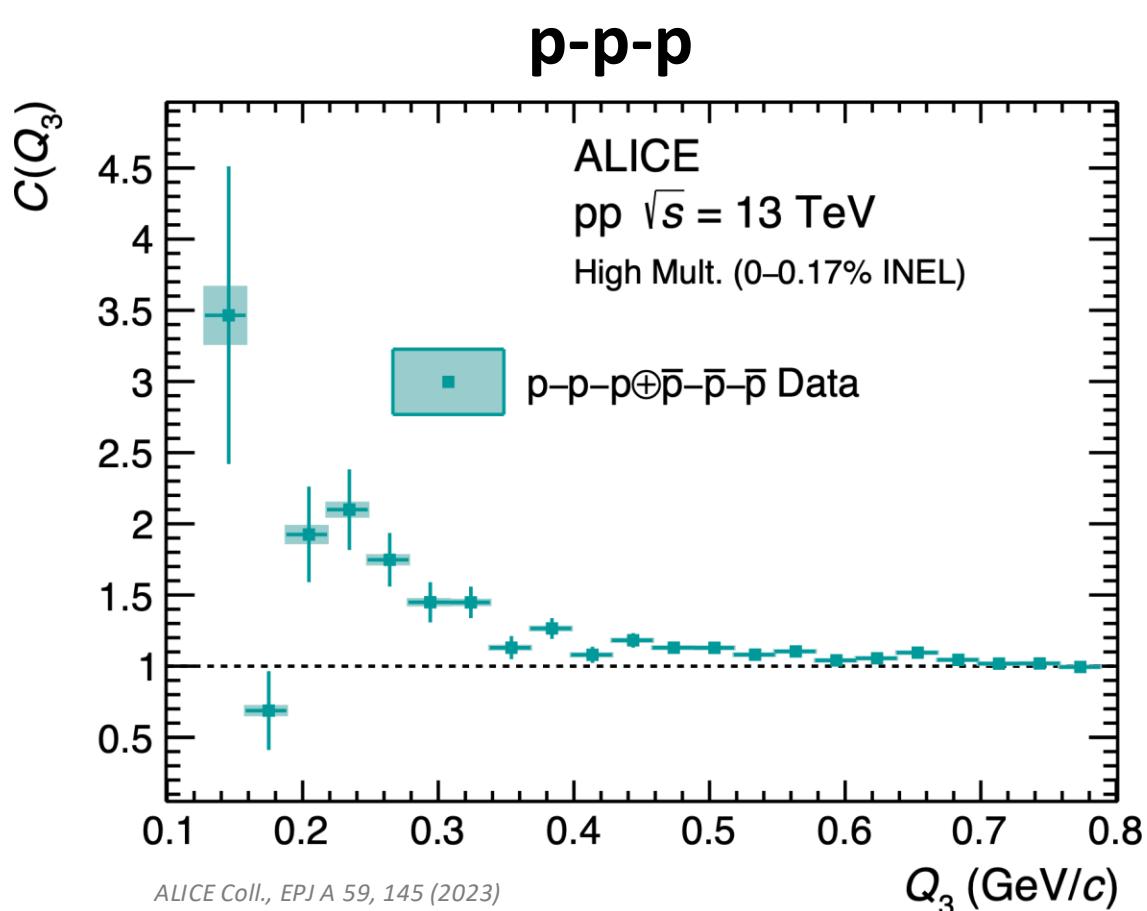
Hyper-radius:

$$\rho = 2\sqrt{r_{12}^2 + r_{23}^2 + r_{31}^2}$$

L. E. Marcucci et al., Front. in Phys. 8, 69 (2020).

# Extension to three-particle system

- First measurement of the free scattering of three hadrons
- Deviation from unity in p-p-p and p-p- $\Lambda$  correlation functions



# p-p-p correlation function

- First ever full three-body correlation function calculations

$$C(Q_3) = \int \rho^5 d\rho S(\rho, \rho_0) | \Psi(\rho, Q_3) |^2$$

three-proton wave function  
hyperradius

- Wave function in hyperspherical harmonics

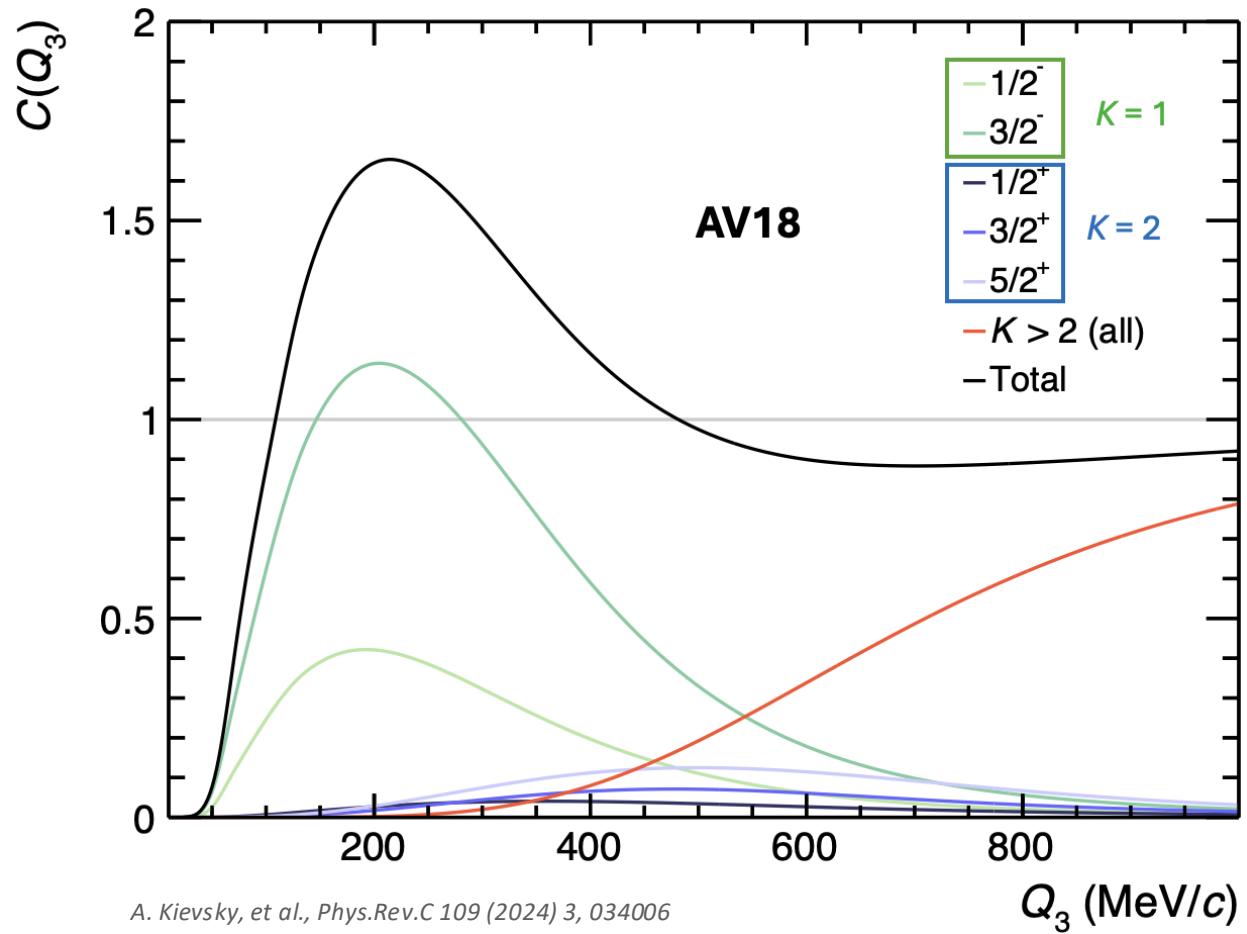
$$\Psi(\rho, Q_3) = \sum_K R_K(\rho) Y_K(\Omega)$$

- Interactions:

- pp strong interaction (AV18)
- Coulomb
- No three-body forces

A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

- p-p-p correlation function:  
superposition of many partial waves

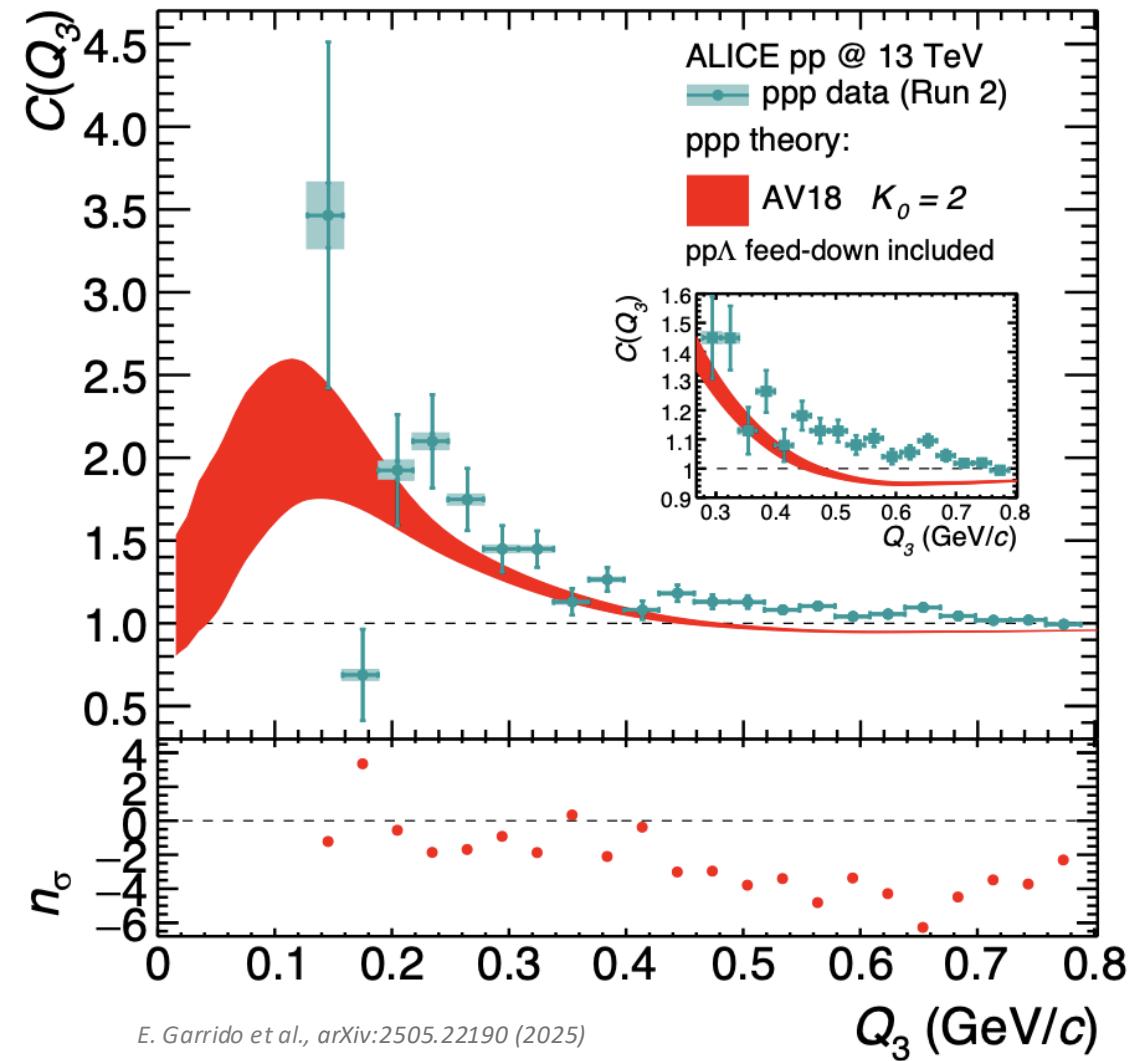


A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

# Comparison Run-2 data

Comparison with the ALICE Run-2 measurement:

- calculations can describe the shape observed in the data
- tension at large values of  $Q_3$

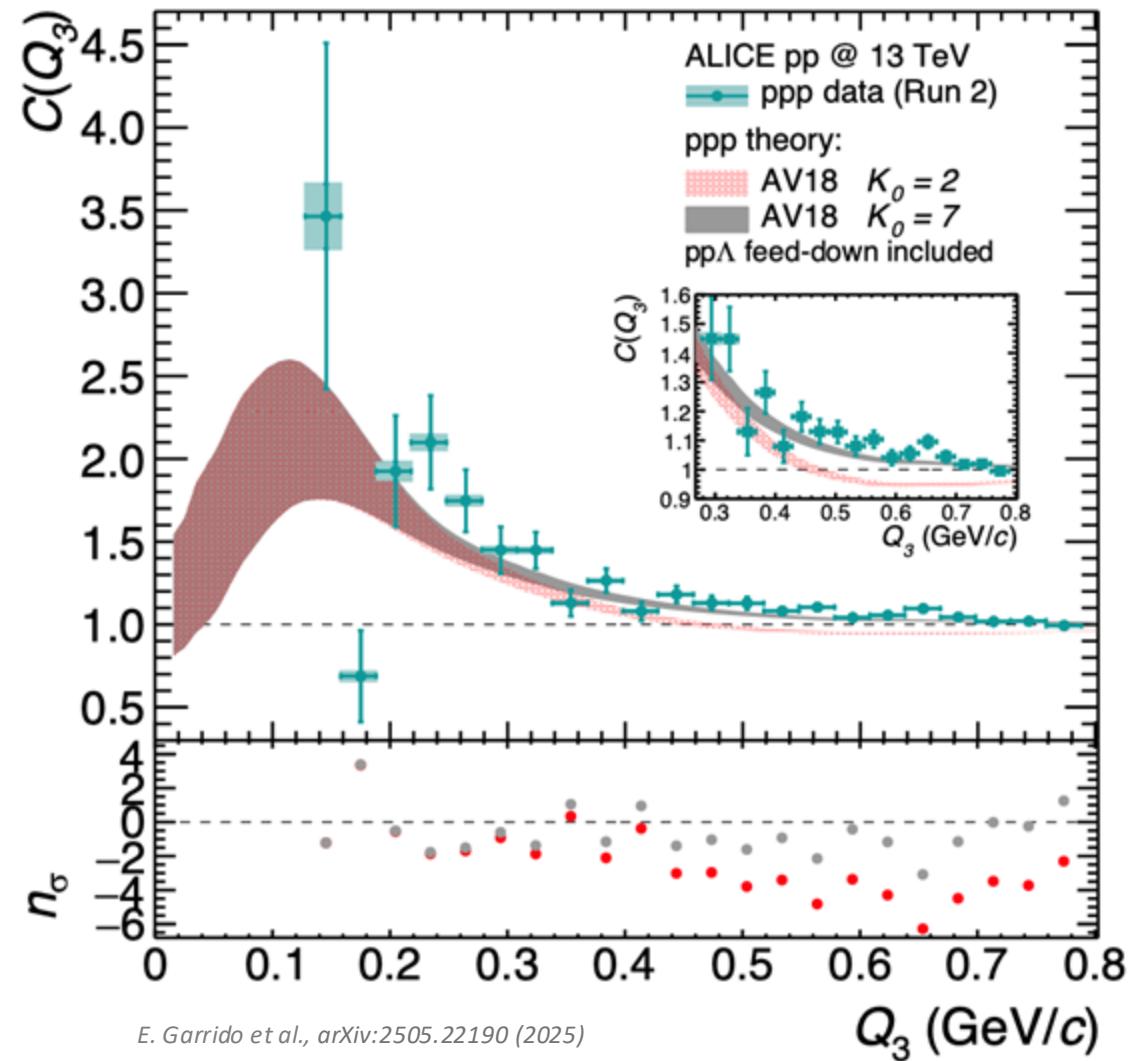


# Comparison Run-2 data

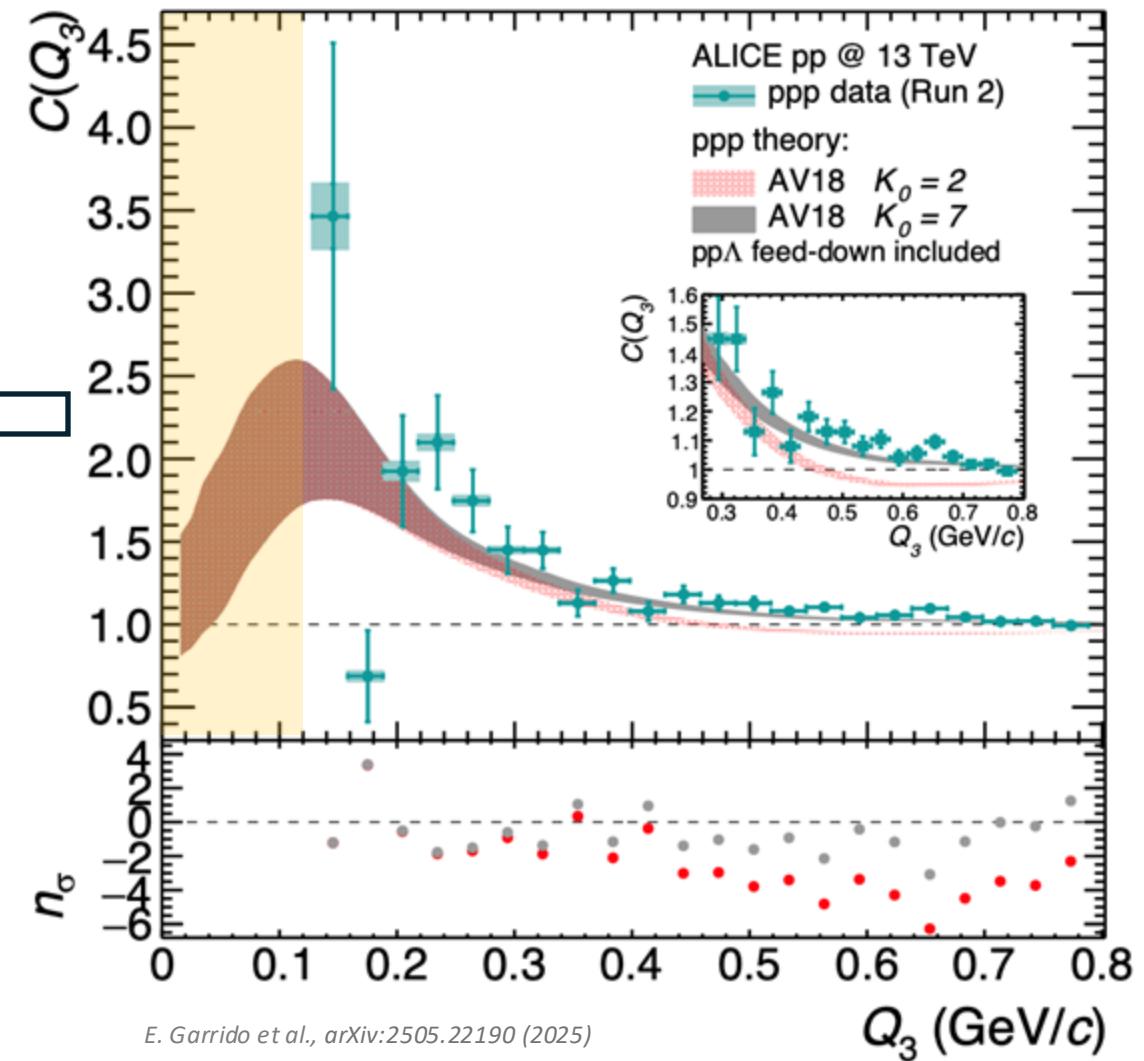
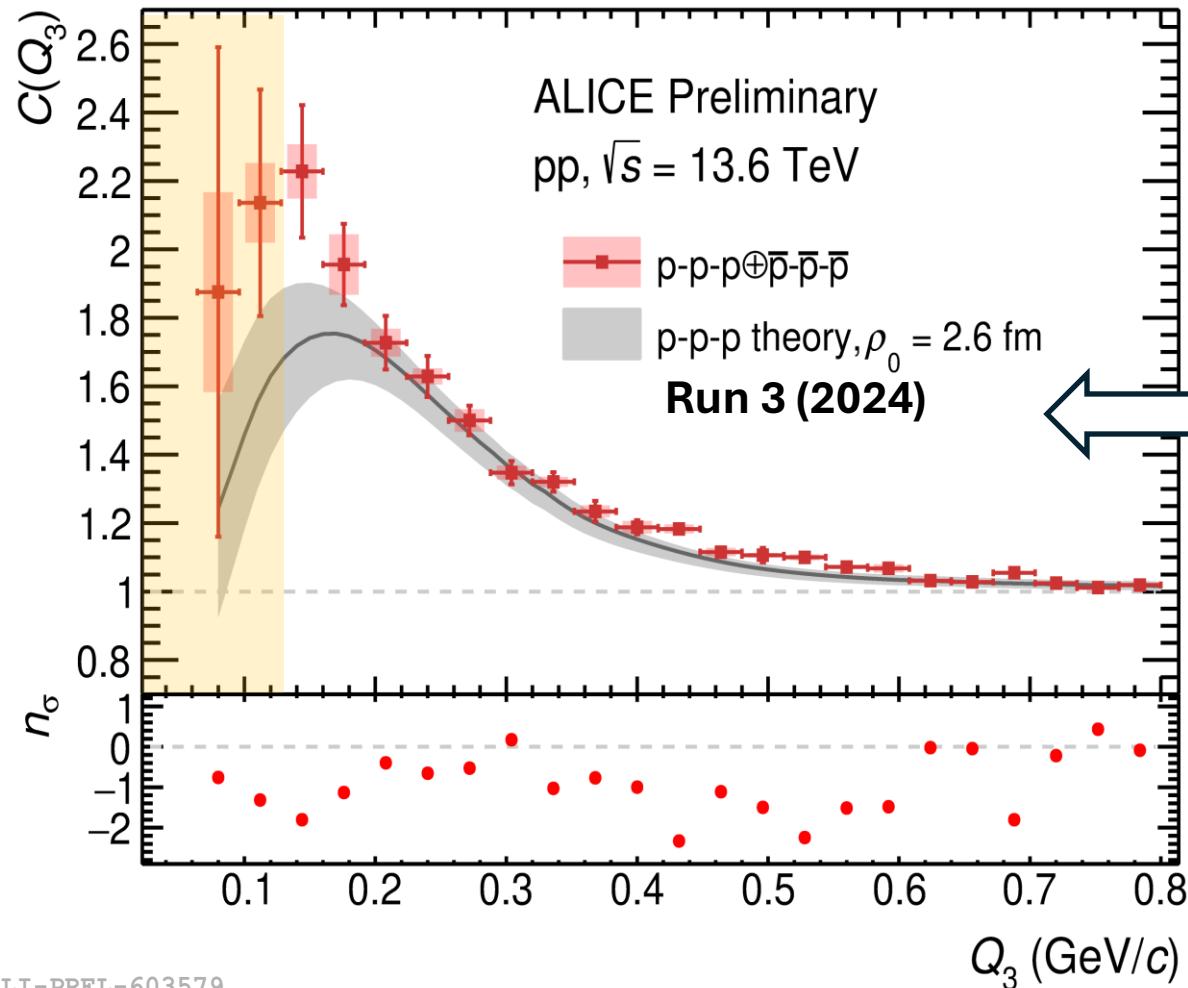
Comparison with the ALICE Run-2 measurement:

- calculations can describe the shape observed in the data
- interaction at higher partial waves ( $K \leq 7$ ) must be included in the calculations
- missing data in the low-energy region

Negligible three-body interactions in p-p-p.



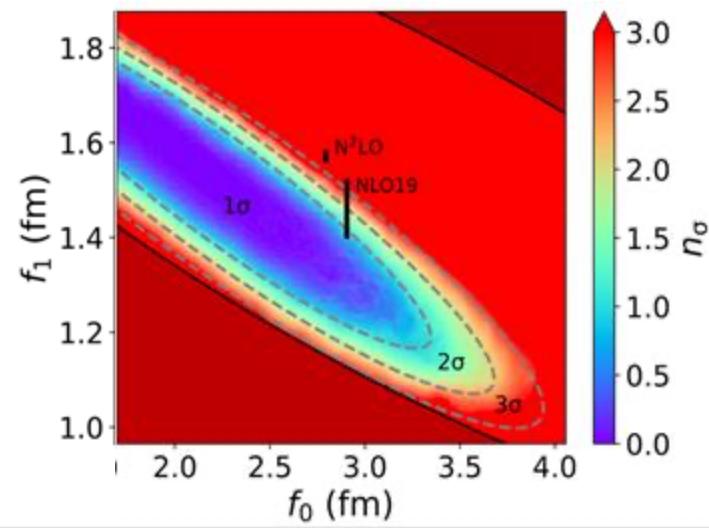
# p-p-p correlation function in Run-3



# p-p- $\Lambda$ correlation function

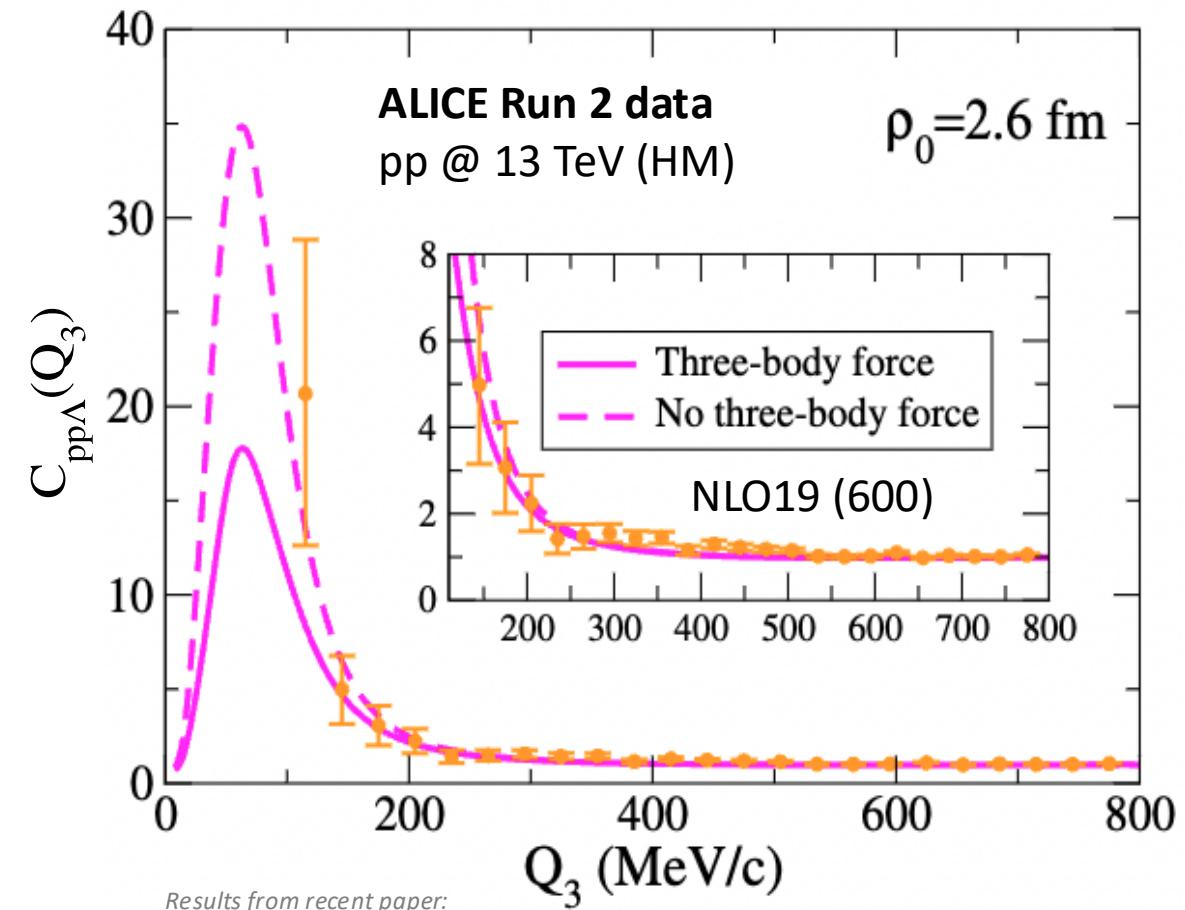
- NLO19 (600) is used to describe the p $\Lambda$  interaction

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550



- Three-body force constrained to the hypertriton binding energy

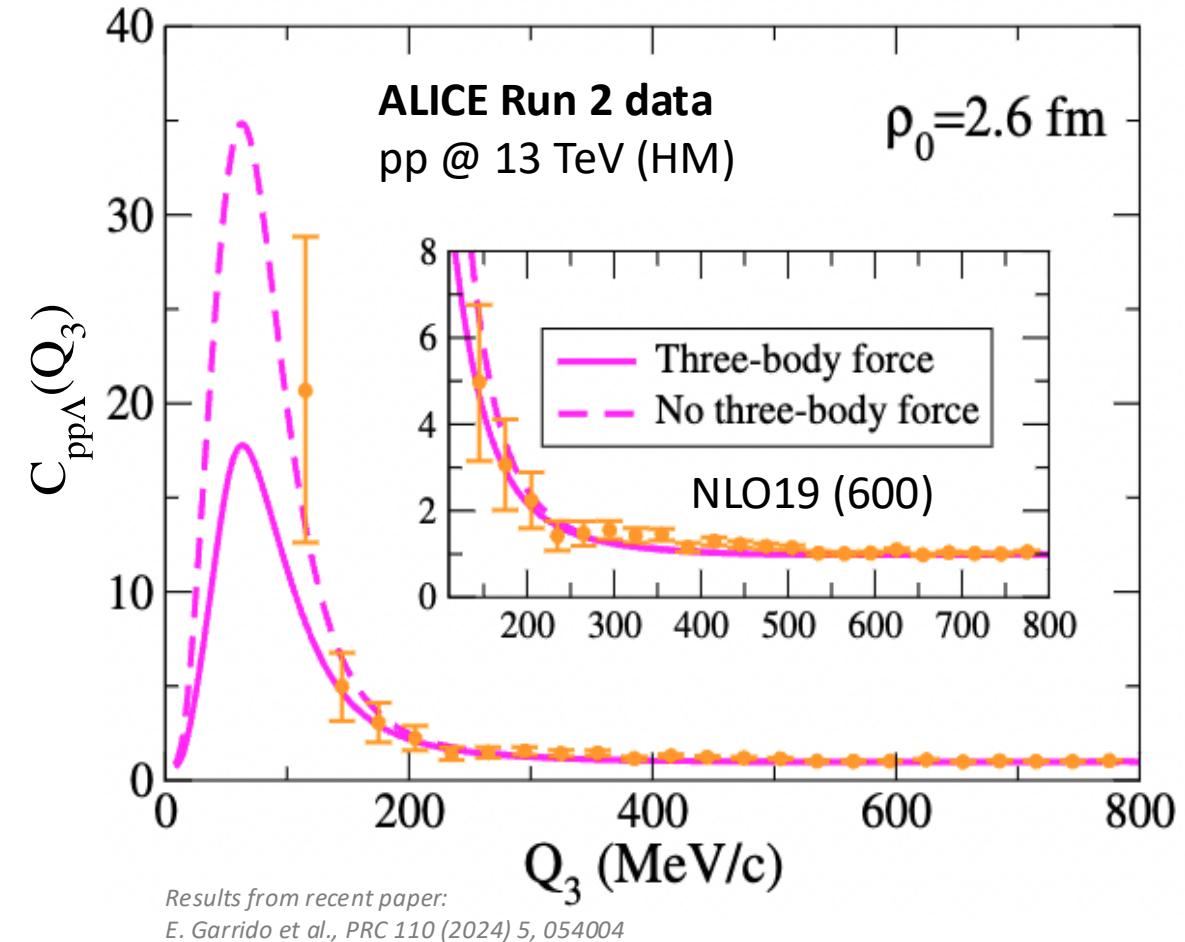
cutoff (MeV)	NLO19				N2LO			Exp.
	500	550	600	650	500	550	600	
$B(^3\Lambda H)$ (MeV)	2.792	2.839	2.904	3.255	2.819	2.799	2.878	2.39



Results from recent paper:  
E. Garrido et al., PRC 110 (2024) 5, 054004

# p-p- $\Lambda$ correlation function

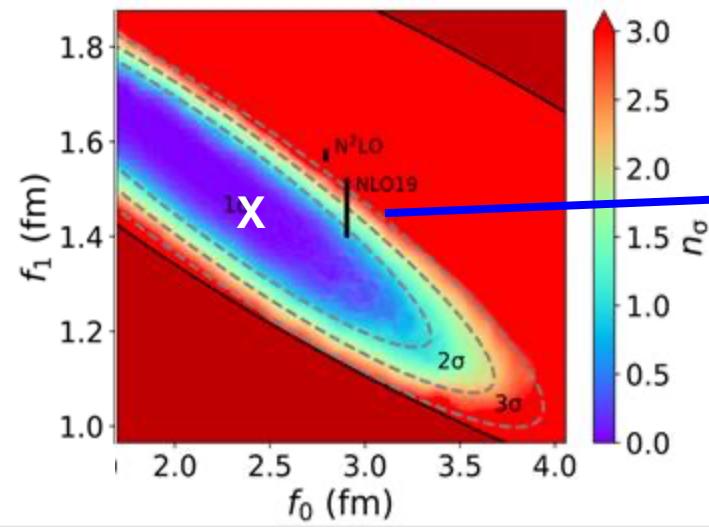
- NLO19 (600) is used to describe the p $\Lambda$  interaction  
*D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550*
- Three-body force constrained to the hypertriton binding energy
- 40% effect of three-body interactions
- Run-2 data: one data point in the region of the maximum



# p-p- $\Lambda$ correlation function

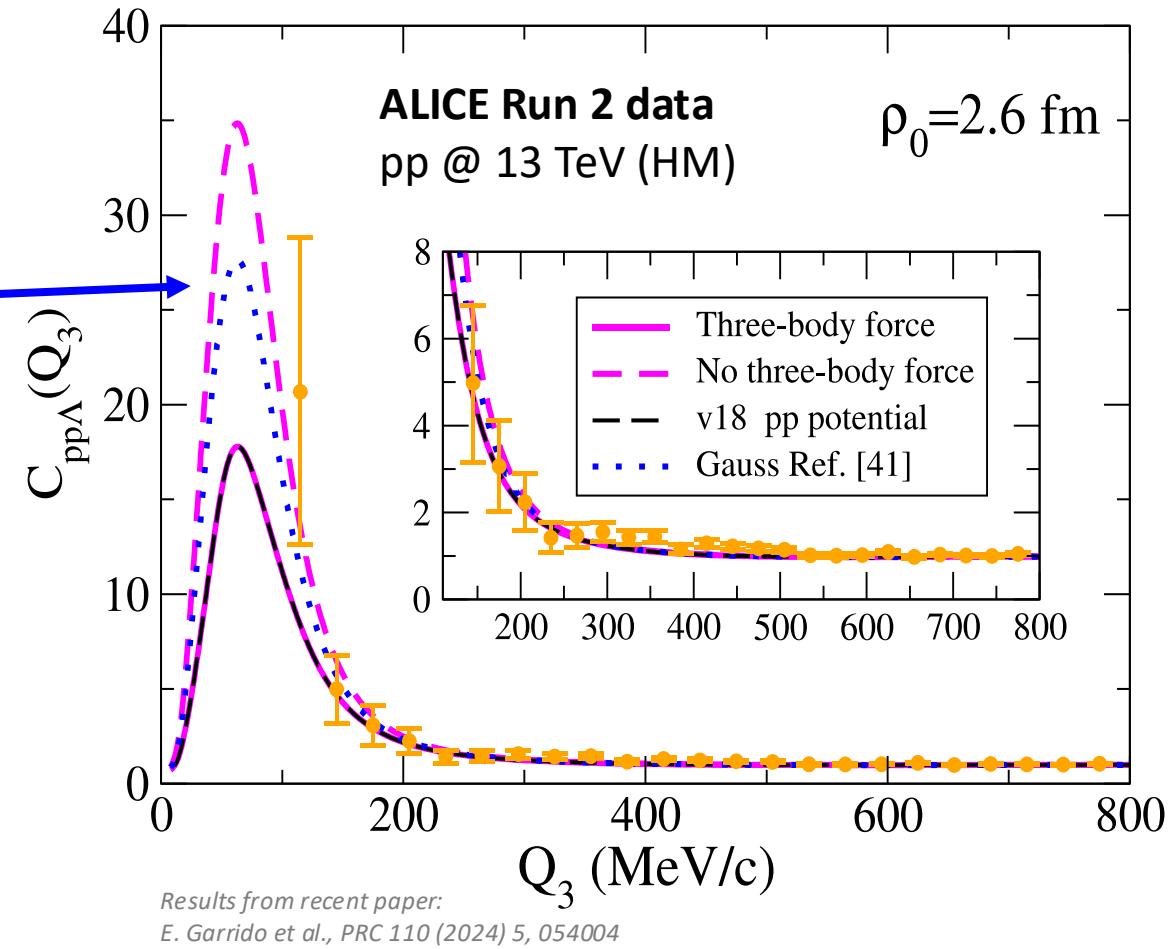
- p $\Lambda$  interaction from best fit of femto+scattering

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

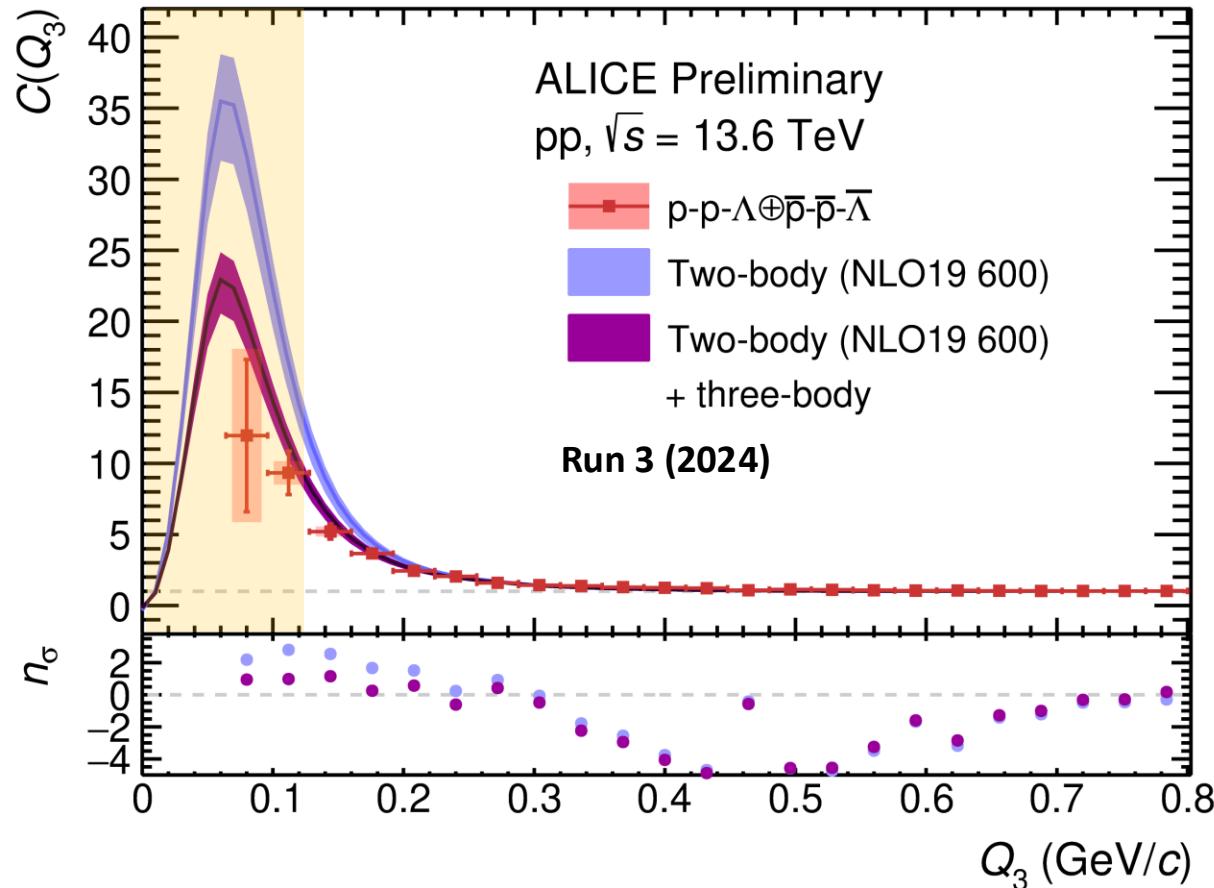


- Hypertriton binding energy well described without the need of a three-body force

	femto+scatt.	Exp.
$B(^3_\Lambda H)$ (MeV)	2.41	2.39

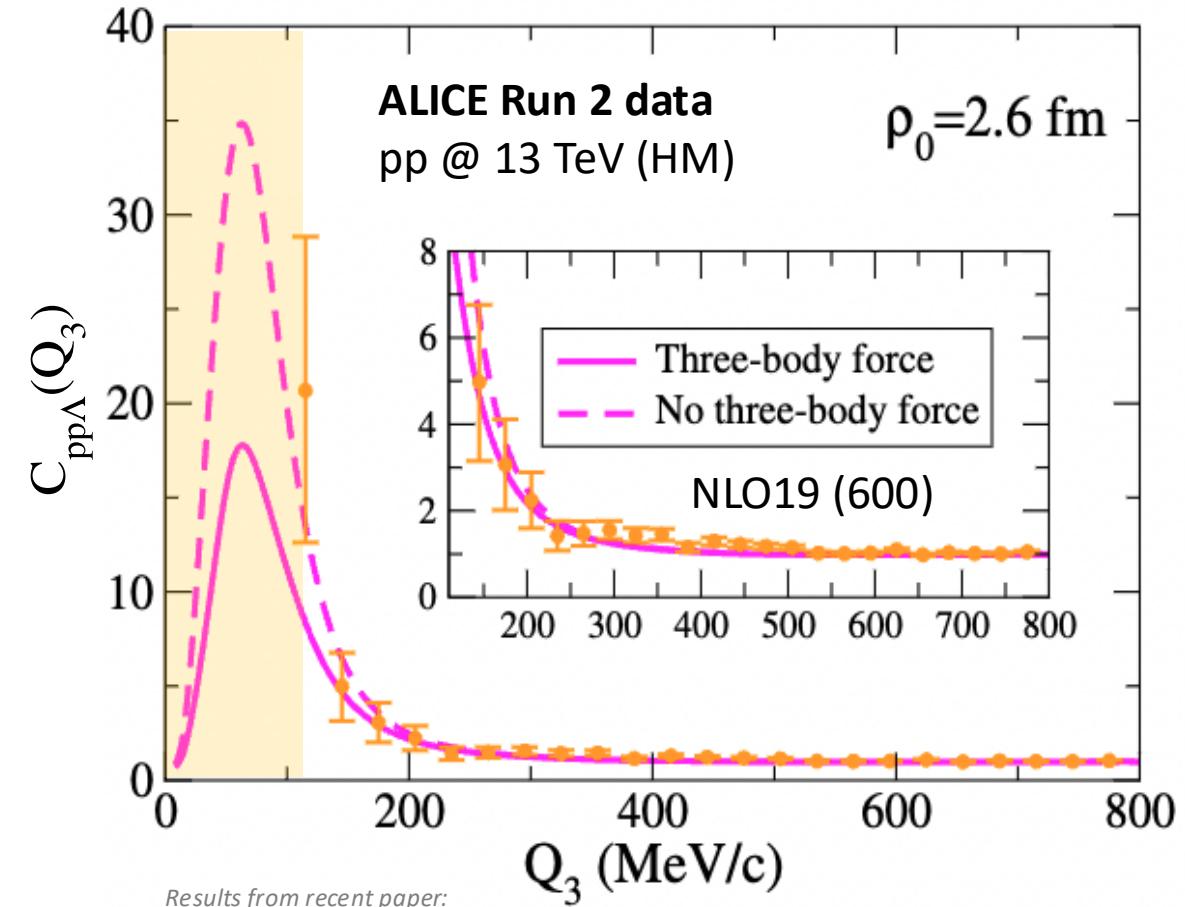


# p-p- $\Lambda$ correlation function



ALI-PREL-603530

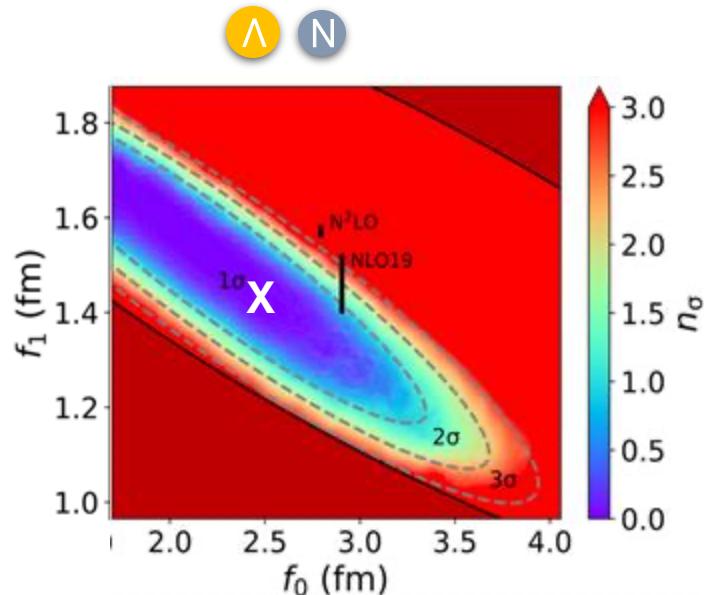
By the end of Run 3: 100 times larger statistical triplets sample expected compared to Run 2 due to developed software triggers!



Results from recent paper:  
E. Garrido et al., PRC 110 (2024) 5, 054004

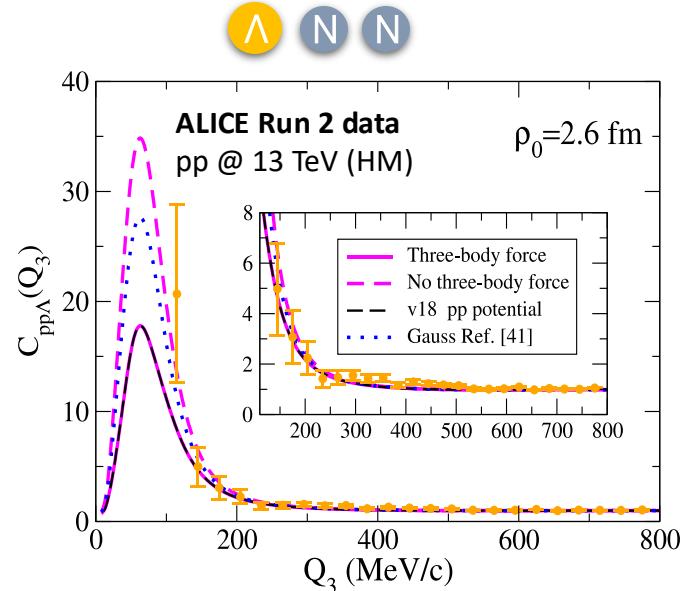
# The future of hadron physics

- 1) Two-body forces from femtoscopy + scattering data



D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

- 2) Three-body femtoscopy to constrain three-body forces

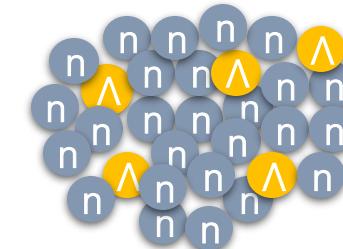


Results from recent paper:  
E. Garrido et al., Phys. Rev. C 110 (2024) 5, 054004

- 3) Four-body systems to construct realistic potentials



- 4) Heavier nuclei
- 5) Neutron stars



# Conclusions and Outlook

- Exciting results from femtoscopy
  - Important experimental input to understand the many facets of QCD in strange sector
    - Most precise p- $\Lambda$  data at low momenta
    - First extraction of the p- $\Lambda$  scattering parameters using femtoscopy and scattering data
    - Shallow repulsion in the p- $\Sigma^+$  interaction in the triplet channel
    - Evidence of the attractive p- $\Xi^-$  strong interaction
    - Opening of the p- $\Xi^- \rightarrow \Lambda\Sigma^0$  channel
    - First measurements of three-particle correlation functions
      - NNN interaction: up to 5% in p-d and negligible in the p-p-p measurement
      - NN $\Lambda$  interaction: 40% effect in the correlation function
- On-going Run 3 and future Run 4
  - Access to precise data on three-particle interactions
  - Sensitivity to the effect of three-body forces in the correlation functions