THE CONNECTION BETWEEN SPIN AND THE QCD PHASE TRANSITION

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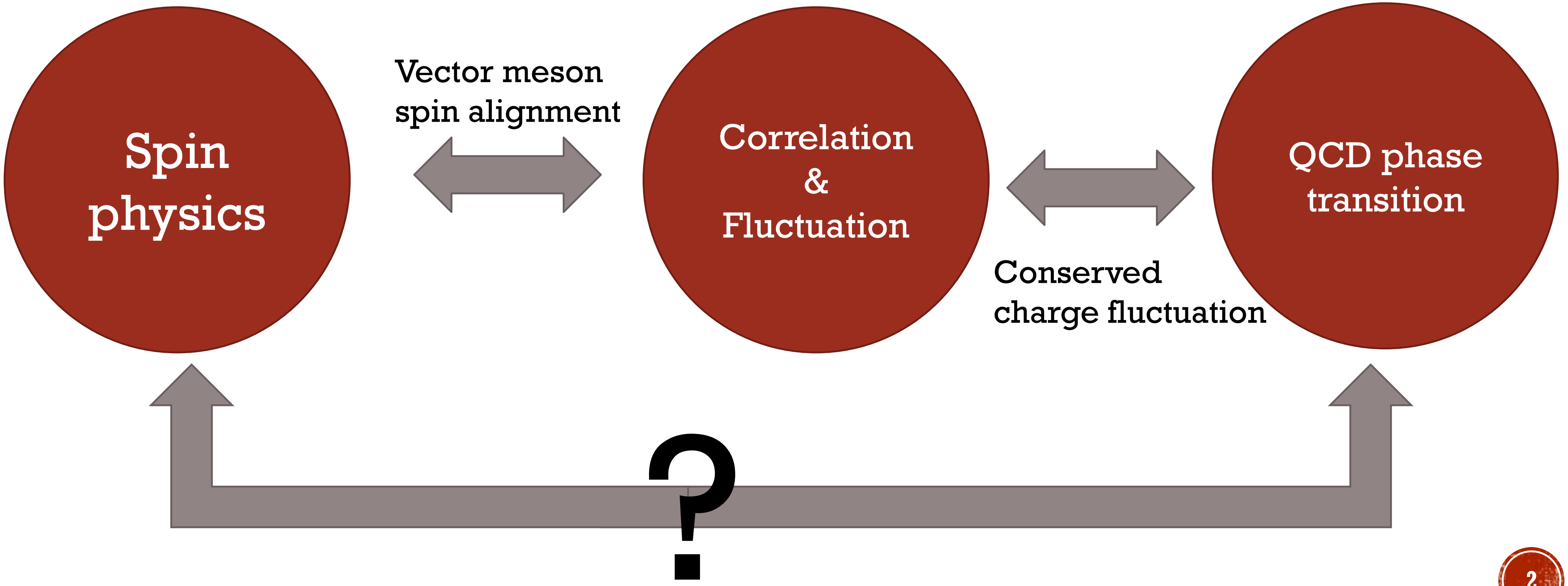
Mainly based on: HLC, Wei-jie Fu, Xu-Guang Huang, Guo-Liang Ma, Phys. Rev. Lett. 135， 032302 (2025)

HENPIC Online Seminar, 21st Aug 2025

OUTLINE


Spin physics
Vector meson spin alignment
Correlation & Fluctuation
Conserved charge fluctuation
QCD phase transition
?

GLOBAL SPIN POLARIZATION

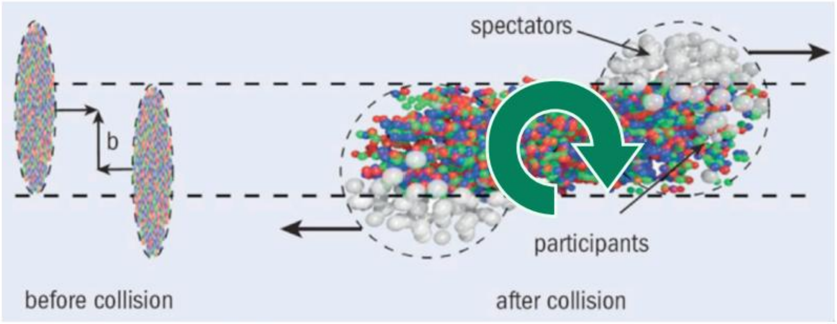


figure: M. Lisa, talk @ "Strangeness in Quark Matter 2016"

- Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

$\boldsymbol{L}_{\text{init}} \sim 10^5 \hbar$ F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

- Quark can be polarized under such a large L
- Coalescence picture : $P_\Lambda = P_s$ Liang, Wang, PRL (2005)
- Estimation given by

Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)

$$\mathbf{P}_\Lambda \simeq \frac{\boldsymbol{\omega}}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}$$

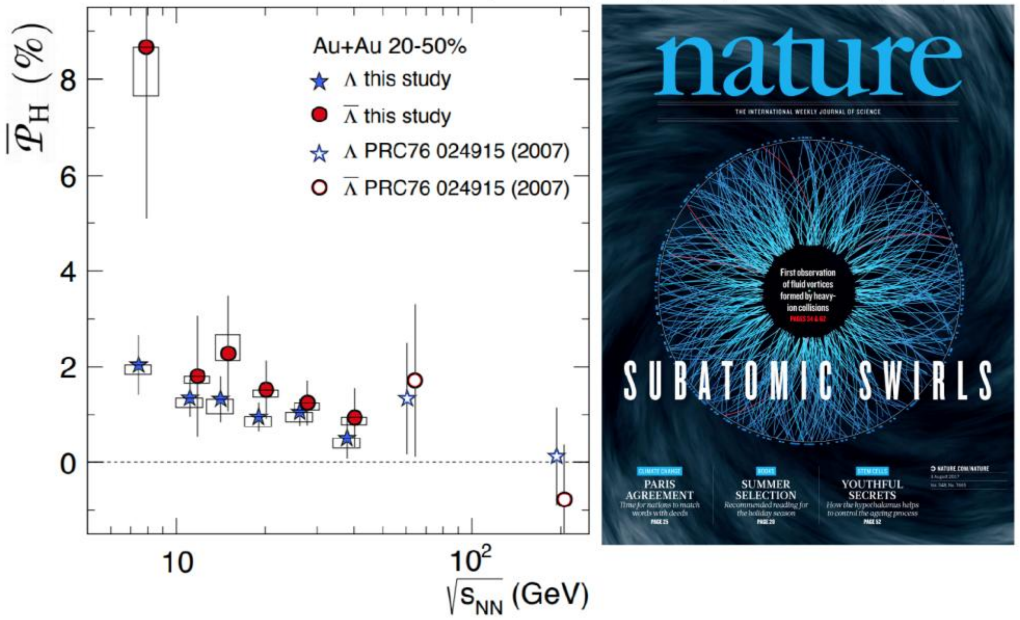
$$\mathbf{P}_{\overline{\Lambda}} \simeq \frac{\boldsymbol{\omega}}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}$$

- angular momenta ⟶ rotation (local/global)

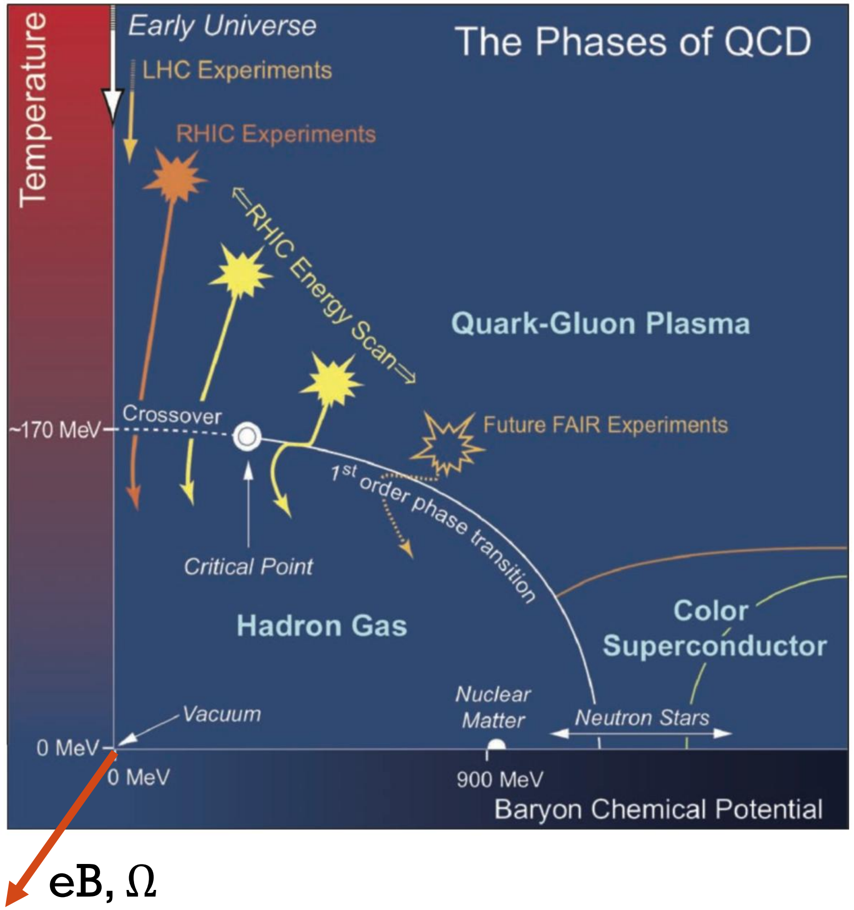
$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

- Rotation related effects attract many attentions

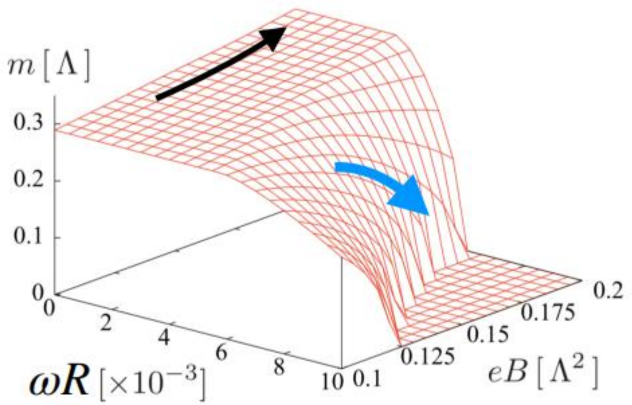
L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

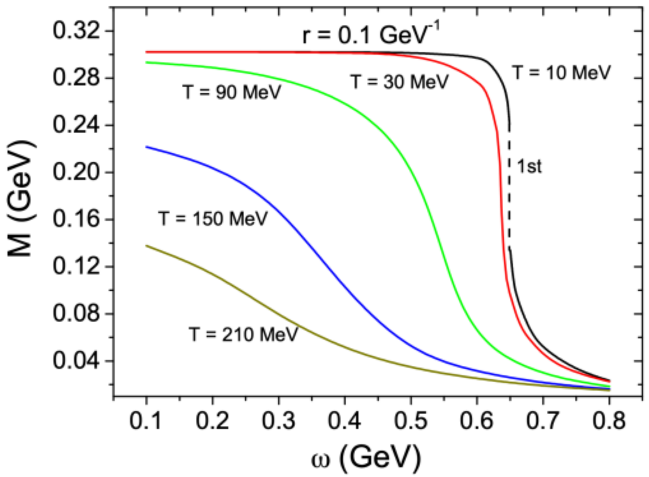


QCD PHASE DIAGRAM UNDER ROTATION



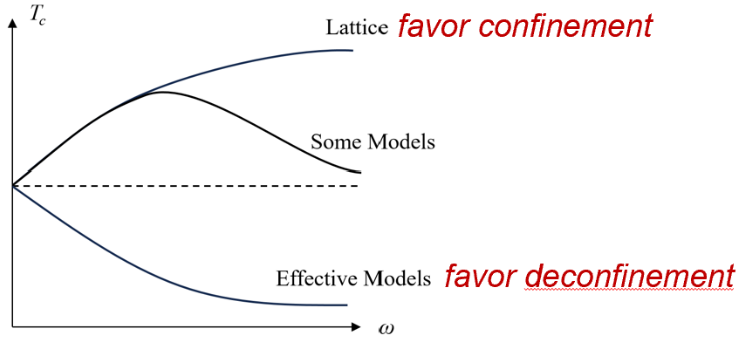
HLC, K. Fukushima, X-G. Huang, K. Mameda, Phys. Rev. D 93, 104052 (2016), 1512.08974





Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016), 1606.03808

- First studied by NJL model (order parameter: quark mass)
- Rotation behaves like chemical potential
- However, lattice studies give opposite result!



P. Zhuang's talk @PHD2024

SPIN ALIGNMENT

- Hyperon polarization only relates to single quark
- Vector meson spin alignment relates to quark pair:
  correlation between quarks will be important
- Coalescence picture : Liang, Wang, PRL (2005)

Spin density matrix element $\rho_{00}^{V} = \frac{1-\langle P_q P_{\bar{q}} \rangle}{3+\langle P_q P_{\bar{q}} \rangle} \approx \frac{1}{3} - \frac{4}{9}\langle P_q P_{\bar{q}} \rangle$

$\rho_{00} \neq 1/3$ indicates spin alignment

- Naïve estimation gives

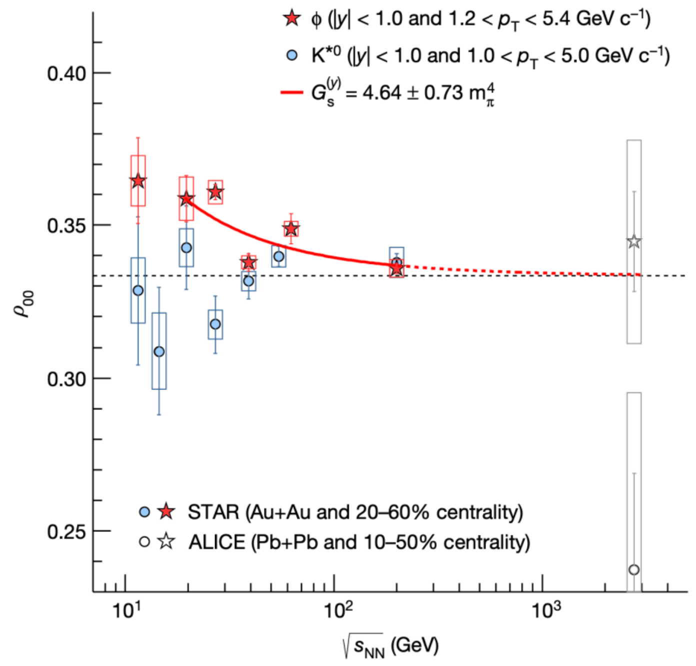
$$\rho_{00} - \frac{1}{3} = -\frac{1}{9}\left(\frac{\omega}{T}\right)^2 \quad \sim 10^{-4}$$

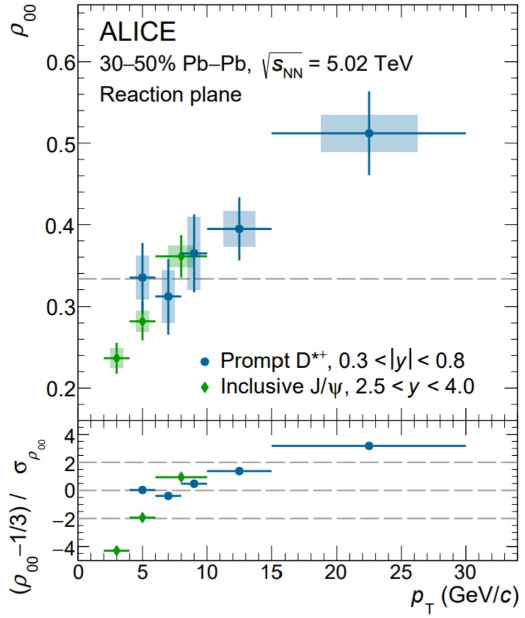
too small

- Further theoretical studies are needed

Correlation and fluctuation are indeed crucial!

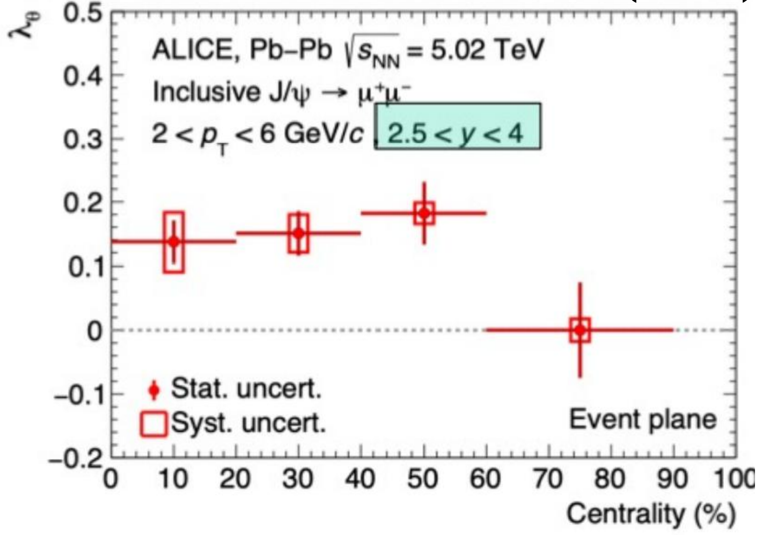
STAR, Nature 614 244 (2023)





ALICE, arXiv: 2504.00714

ALICE, PRL 131 042303 (2023)



$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}}$ $\quad \lambda_\theta > 0, \rightarrow \rho_{00}^{J/\Psi} < 1/3$

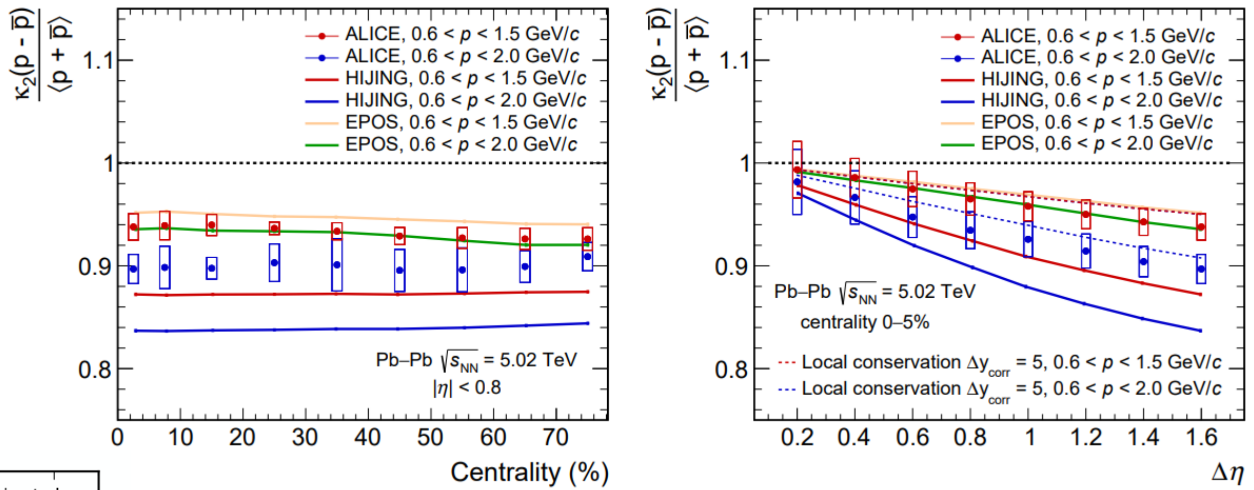
CORRELATION AND FLUCTUATION

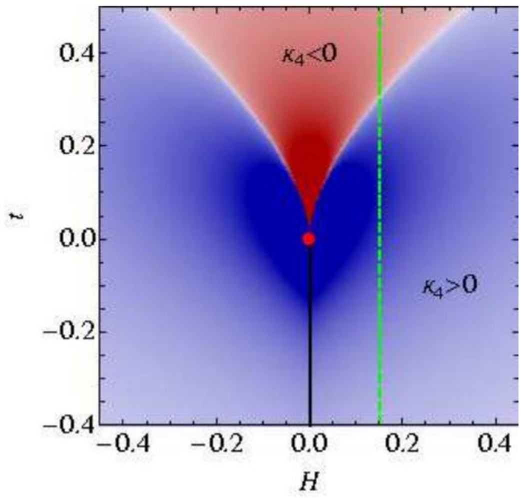
- Baryon number fluctuation

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}$$

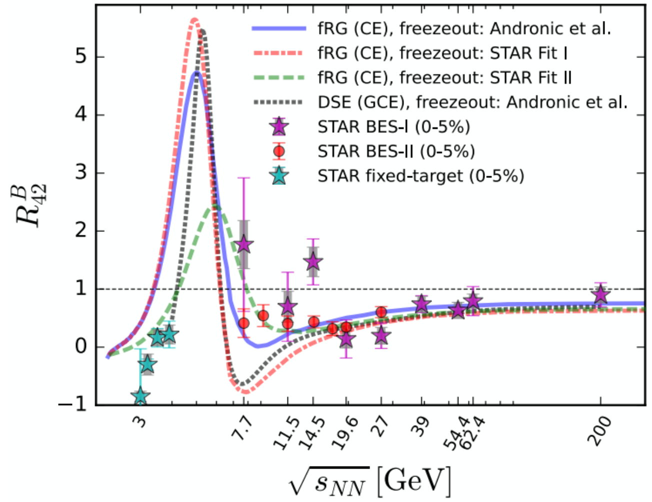
- Quantifying the nature of the phase transition
- At large density: critical endpoint (CEP)

ALICE, PLB 844 (2023) 137545



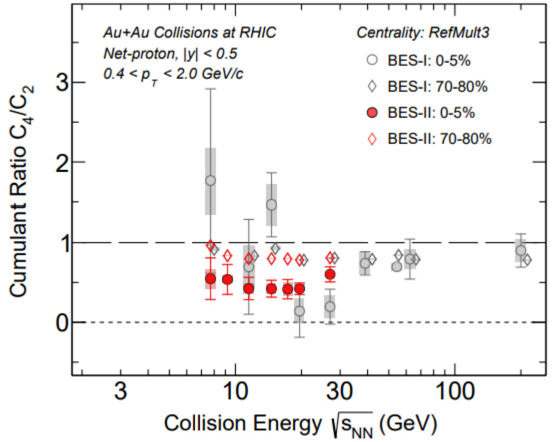


M. Stephanov, PRL 107 (2011) 052301



From W. Fu's HENPIC seminar

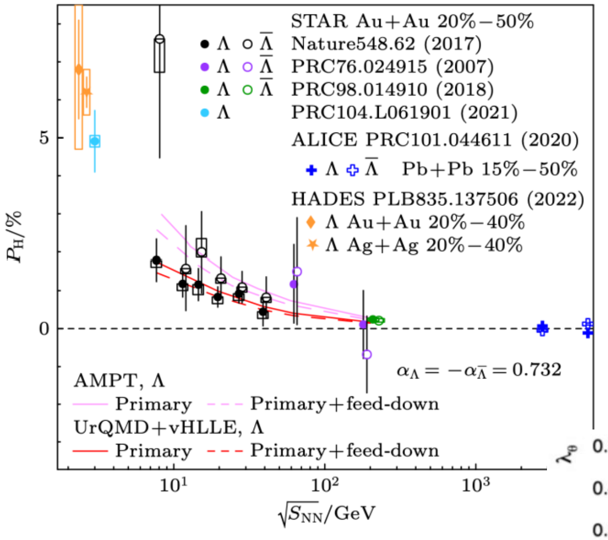
STAR Collaboration, arXiv:2504.00817

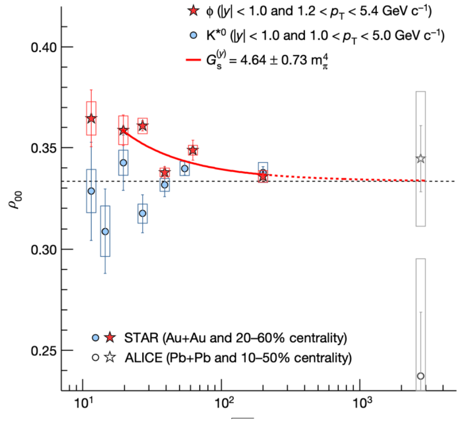


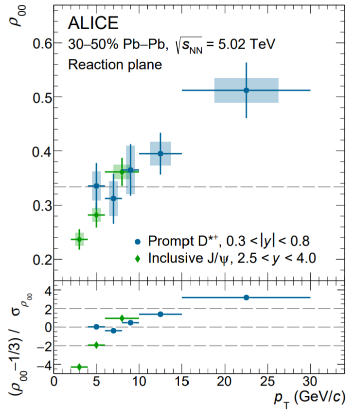
MOTIVATION

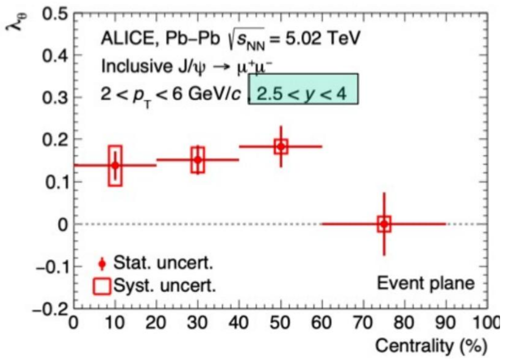
- Almost studied separately

Spin polarization/alignment

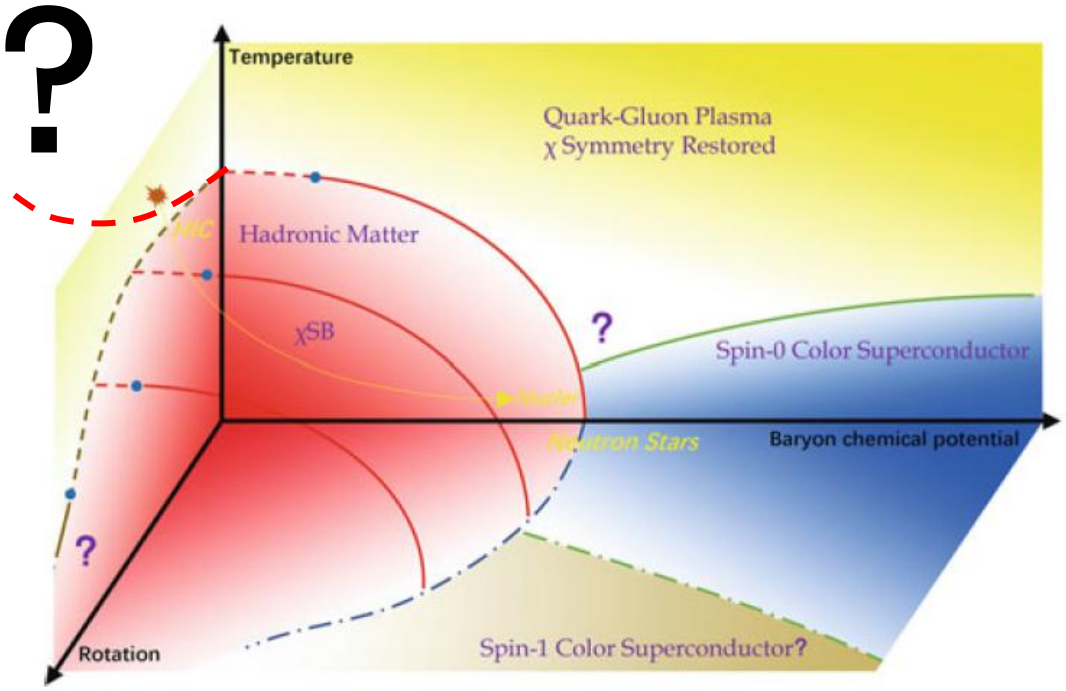








Phase transition

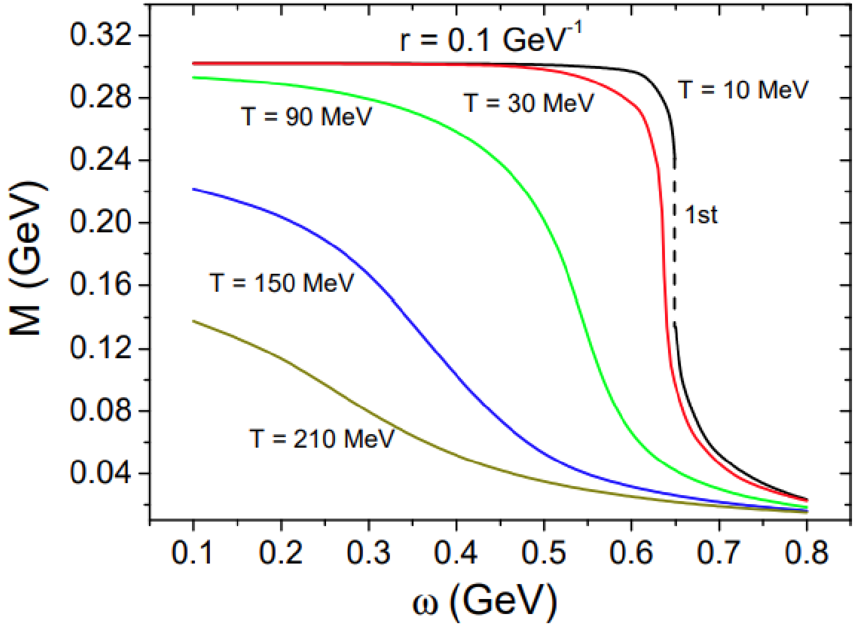


Q: Are these two aspects related?

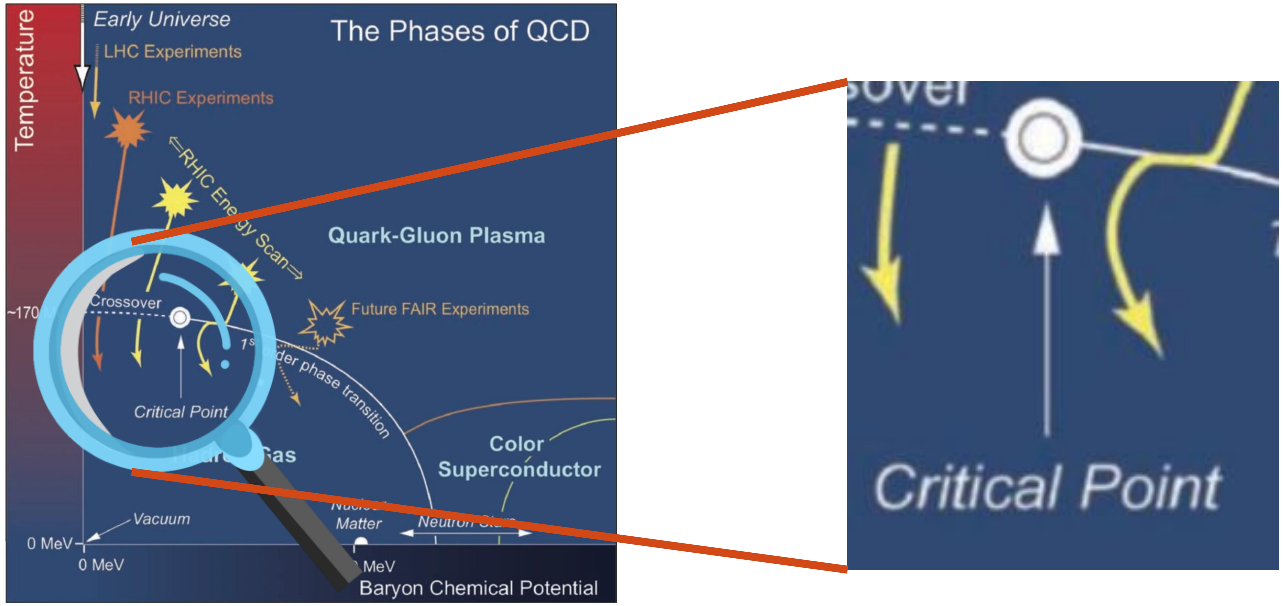
Minghua Wei , Mei Huang, Chin.Phys.C 47 (2023) 10, 104105
Fei Sun, Jingdong Shao, Rui Wen, Kun Xu, Mei Huang, PhysRevD.109.116017
Sushant K. Singh, Jan-e Alam, Eur. Phys. J. C 83, 585 (2023)

NAÏVE ESTIMATIONS

Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016),



- Spin polarization $\sim\Omega$
- Spin alignment $\sim\Omega^2$
- Chiral condensate does not change much at small Ω
- The effect seems not large enough for measurement
- However



!!

Spin fluctuation? Just like baryon number?

QUALITATIVE STUDY: NJL MODEL

Rotating spacetime metric

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Nambu–Jona-Lasinio model is a low energy effective model of QCD

$$\mathcal{L}_{NJL} = \bar{\psi} i\gamma^\mu \nabla_\mu \psi - m_0 \bar{\psi}\psi + \mu_B \bar{\psi}\gamma^0\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma^5\vec{\tau}\psi)^2]$$

- Vierbein formulism

$$e^t_0 = e^x_1 = e^y_2 = e^z_3 = 1, \qquad e^x_0 = y\Omega, \qquad e^y_0 = -x\Omega,$$

- Hamiltonian for fermion

$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}.$$

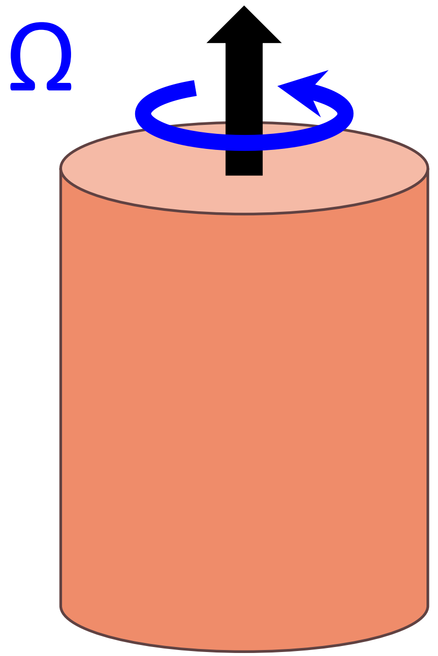
- Similar as finite density

L. Landau and E. Lifshitz, Statistical Physics, Part 1

Rotation: $E \rightarrow E + (L_z + S_z)\Omega$

Finite density: $E \rightarrow E - \mu$

Rotation behaves as an effective chemical potential



QUALITATIVE STUDY: NJL MODEL

- General thermodynamic potential under rotation

$$V_{eff}(r) = \frac{(m-m_0)^2}{4G} - N_c N_f \sum_l \int_0^\Lambda \frac{p_t \mathrm{d}p_t \mathrm{d}p_z}{(2\pi)^2} [\varepsilon_p + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p-\mu-\Omega j)}) + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p+\mu+\Omega j)})](J_l^2(p_t r) + J_{l+1}^2(p_t r)).$$

- Away from the center, contribution from orbital angular momentum is dominant
- Since we are interested in spin, we first focus on the physics near the center (r=0)

$$\begin{aligned} V_{eff}^0(\Omega,\mu) = & \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p \\ & + N_c N_f \int_0^\infty \frac{\mathrm{d}^3 p}{(2\pi)^2} [\underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})}_{q,\uparrow} + \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})}_{q,\downarrow} \\ & + \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})}_{\bar{q},\uparrow} + \underbrace{T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})}_{\bar{q},\downarrow}]. \end{aligned}$$

- We can get information about average spin from this expression

QUALITATIVE STUDY: NJL MODEL

- Thermodynamic contribution (without critical fluctuation)

$$V^0_{eff}(\Omega,\mu)=\frac{(m-m_0)^2}{4G}-N_cN_f\int_0^{\Lambda}\frac{\mathrm{d}^3p}{(2\pi)^2}2\varepsilon_p$$

$q,\uparrow$ $\quad$ $q,\downarrow$

$$+N_cN_f\int_0^{\infty}\frac{\mathrm{d}^3p}{(2\pi)^2}[T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T})+T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})$$

$$+T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T})+T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})].$$

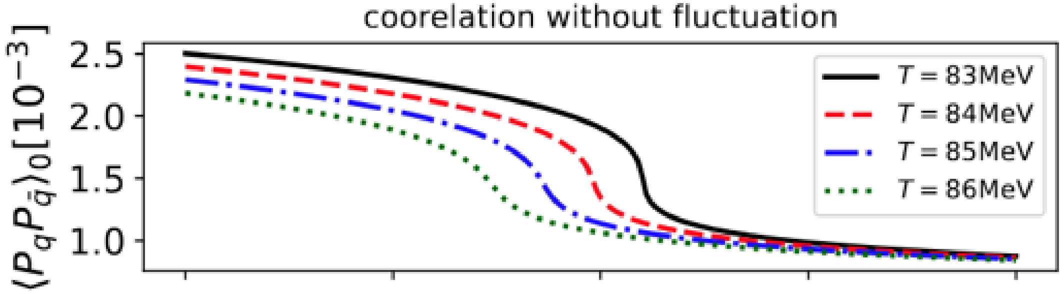
$\bar{q},\uparrow$ $\quad$ $\bar{q},\downarrow$

0 (Gap Eq)

$$\langle S\rangle=-\frac{V}{T}\frac{\mathrm{d}V^0_{eff}}{\mathrm{d}(\frac{\Omega}{T})}=-\frac{V}{T}\frac{\partial V^0_{eff}}{\partial(\frac{\Omega}{T})}-\frac{V}{T}\frac{\partial V^0_{eff}}{\partial m}\frac{\partial m}{\partial(\frac{\Omega}{T})}.$$

$$\langle P_qP_{\bar{q}}\rangle_0=\frac{\int\mathrm{d}^3p(f_q^{\uparrow}-f_q^{\downarrow})(f_{\bar{q}}^{\uparrow}-f_{\bar{q}}^{\downarrow})}{\int\mathrm{d}^3p(f_q^{\uparrow}+f_q^{\downarrow})(f_{\bar{q}}^{\uparrow}+f_{\bar{q}}^{\downarrow})}.$$

$$f_q^{\uparrow/\downarrow}=\frac{1}{\mathrm{e}^{\beta(\epsilon_p-\mu\mp\frac{\Omega}{2})}+1},\quad f_{\bar{q}}^{\uparrow/\downarrow}=\frac{1}{\mathrm{e}^{\beta(\epsilon_p+\mu\mp\frac{\Omega}{2})}+1}.$$



However, what we want to see is the fluctuation related to phase transition

QUALITATIVE STUDY: NJL MODEL

- To get correlation, further techniques are needed
- Introducing rotation and chemical potential only act on quark or antiquark

$$V_{\rm eff}(\Omega_q^s, \Omega_{\bar q}^s, \Omega, \mu_q, \mu_{\bar q}, \mu; r)$$
$$= \frac{[m(r)-m_0]^2}{4G} - N_c N_f \int_0^\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^\infty \frac{\mathrm{d}^3 p}{(2\pi)^3} J_l^2(p_t r) \Big[T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\Omega l-\mu_q)/T})$$
$$+ T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\Omega l-\mu_q)/T}) + T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega_{\bar q}^s/2+\Omega l-\mu_{\bar q})/T}) + T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega_{\bar q}^s/2+\Omega l-\mu_{\bar q})/T}) \Big]$$

  - Then by taking derivative, we can get correlation of quark/antiquark spin and particle number

$$\langle S_q S_{\bar q}\rangle - \langle S_q\rangle\langle S_{\bar q}\rangle = \frac{\partial^2 V_{eff}}{\partial\Omega_q^s \partial\Omega_{\bar q}^s}\Big|_{\Omega_q^s=\Omega_{\bar q}^s=\Omega} \qquad \langle N_q N_{\bar q}\rangle - \langle N_q\rangle\langle N_{\bar q}\rangle = \frac{\partial^2 V_{eff}}{\partial\mu_q \partial\mu_{\bar q}}\Big|_{\mu_q=\mu_{\bar q}=0}$$

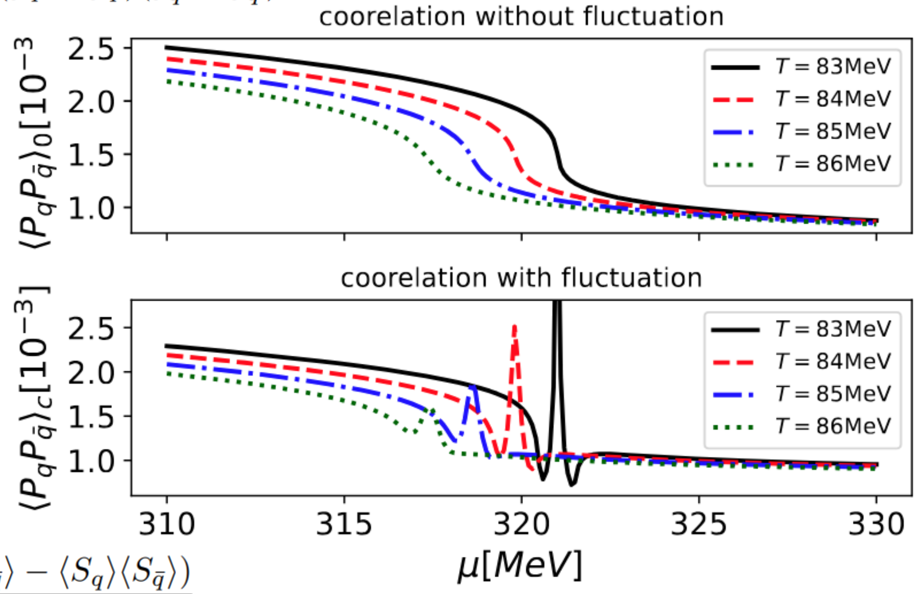
  - Then we can define the spin correlation of quark-antiquark as

$$\langle P_q P_{\bar q}\rangle_c = \frac{4(\langle S_q S_{\bar q}\rangle - \langle S_q\rangle\langle S_{\bar q}\rangle)}{\langle N_q N_{\bar q}\rangle - \langle N_q\rangle\langle N_{\bar q}\rangle}$$

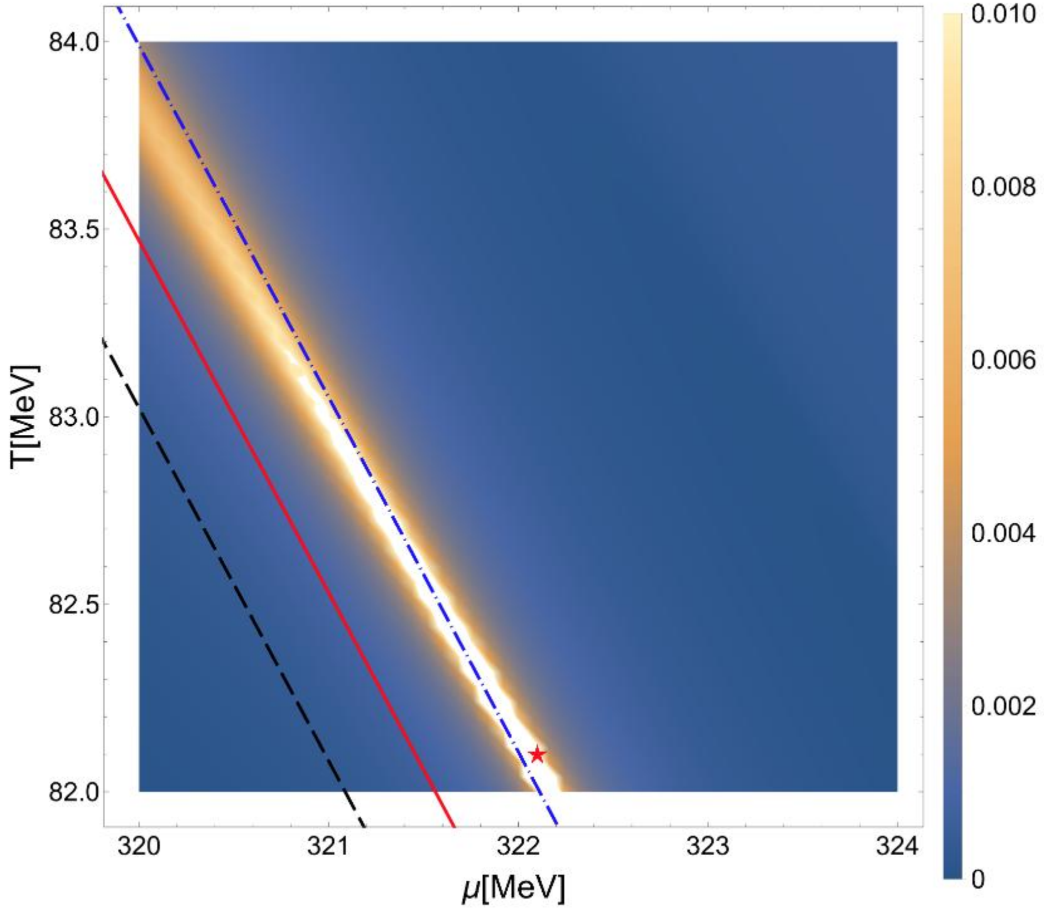
SPIN CORRELATION ENHANCED BY CEP!

- First we consider r=0
- Comparison with the case w/o fluctuation

$$\langle P_q P_{\bar{q}} \rangle_0 = \frac{\int \mathrm{d}^3 p (f_q^{\uparrow} - f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} - f_{\bar{q}}^{\downarrow})}{\int \mathrm{d}^3 p (f_q^{\uparrow} + f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} + f_{\bar{q}}^{\downarrow})}.$$



$$\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}$$



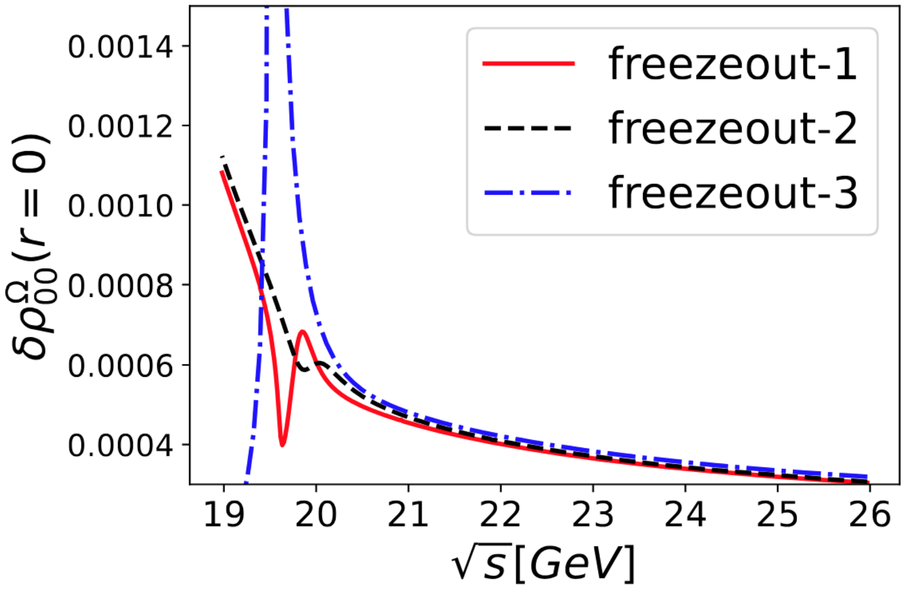
VECTOR MESON SPIN ALIGNMENT

- Along imaginary freezeout lines

$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta\rho_{00}^{\Omega}.$$

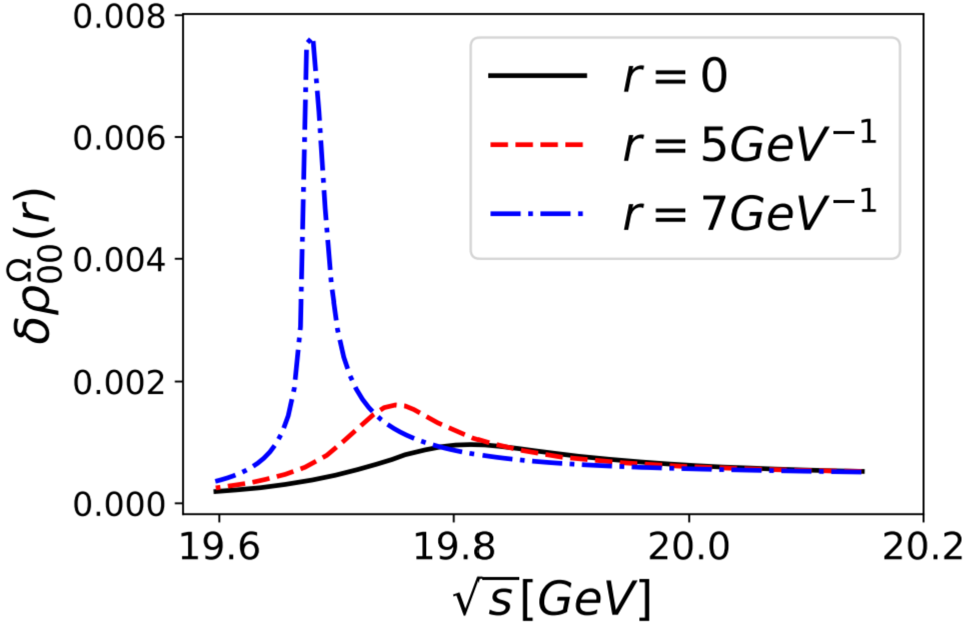
Contribution from critical fluctuation

At r=0



$$V_{\text{eff}}(\Omega_q^s, \Omega_{\bar{q}}^s, \Omega, \mu_q, \mu_{\bar{q}}, \mu; r)$$
$$= \frac{[m(r) - m_0]^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^3} 2\varepsilon_p - \sum_{l=-\infty}^{\infty} N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^3} J_l^2(p_t r) \Big[T \ln(1 + \mathrm{e}^{-(\varepsilon_p - \mu - \Omega_q^s/2 - \Omega l - \mu_q)/T})$$
$$+ T \ln(1 + \mathrm{e}^{-(\varepsilon_p - \mu + \Omega_q^s/2 - \Omega l - \mu_q)/T}) + T \ln(1 + \mathrm{e}^{-(\varepsilon_p + \mu - \Omega_{\bar{q}}^s/2 + \Omega l - \mu_{\bar{q}})/T}) + T \ln(1 + \mathrm{e}^{-(\varepsilon_p + \mu + \Omega_{\bar{q}}^s/2 + \Omega l - \mu_{\bar{q}})/T}) \Big]$$

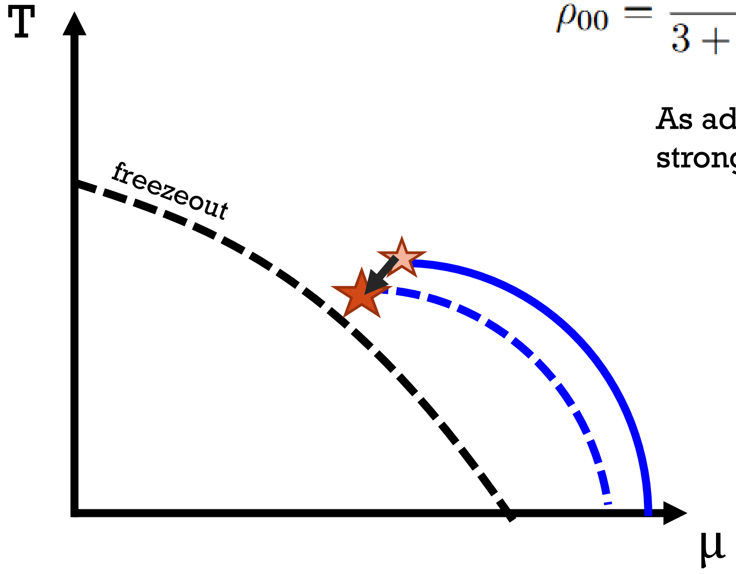
Orbit angular momentum contribution to Freezeout – 2



PHI MESON SPIN ALIGNMENT

- The CEP is shifted closer to freezeout line by orbit angular momentum contribution
- The peak structure is enhanced

A SCHEMATIC FIGURE

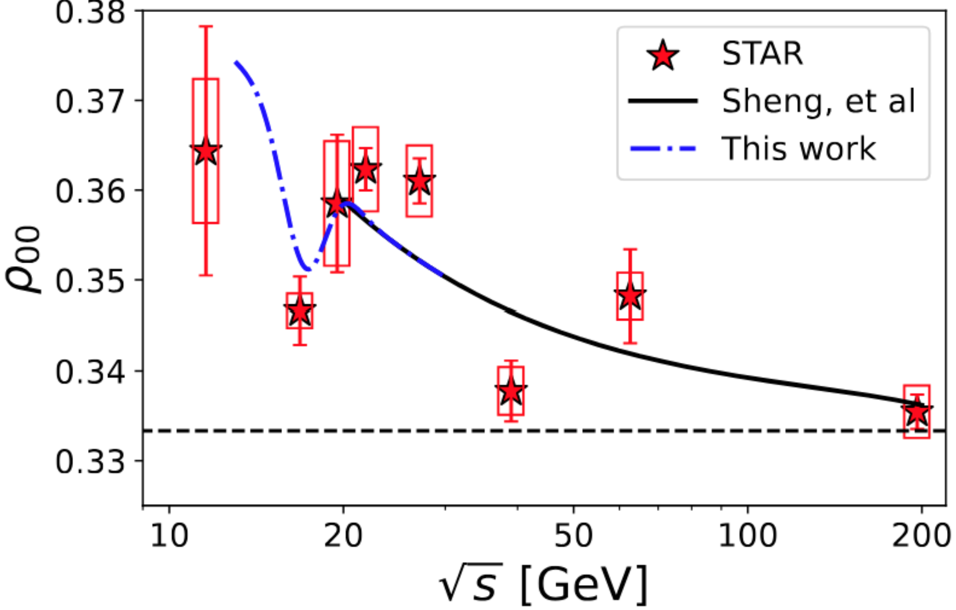


$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta \rho_{00}^{\Omega}.$$

As additional contribution (to strong force field contribution)

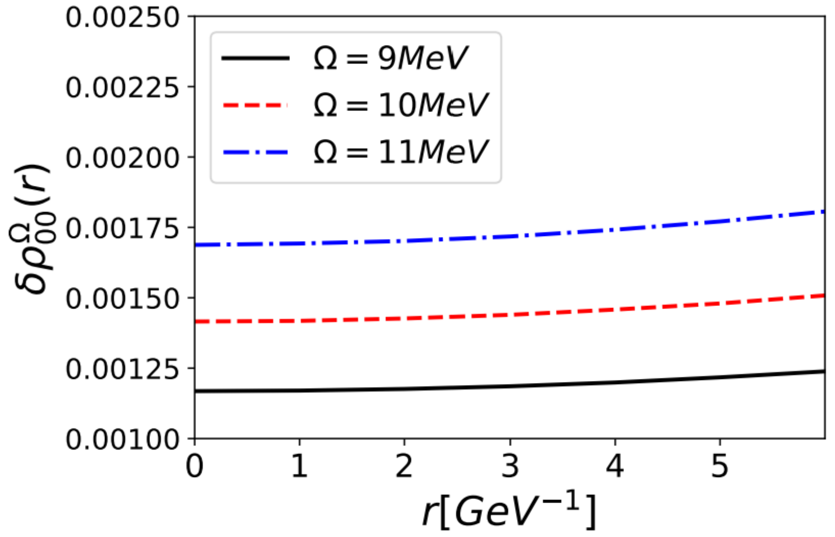
G. Wilks' talk@SQM2024

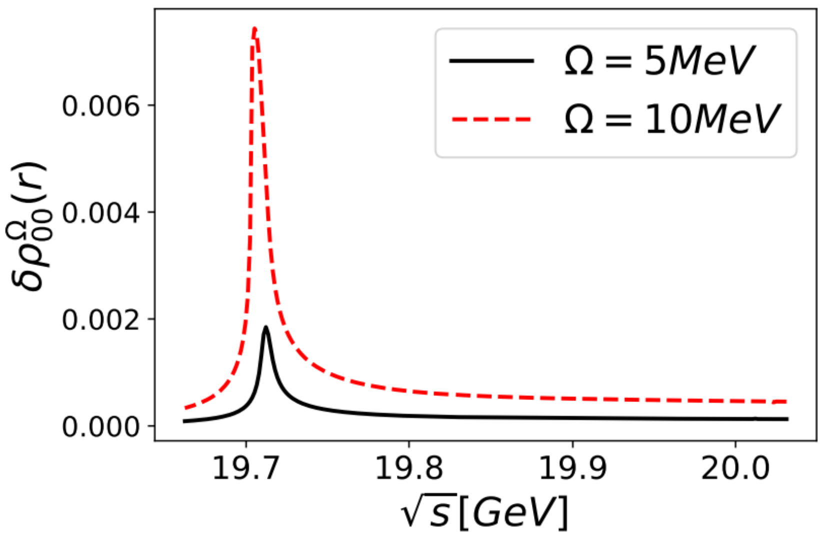
X.-L. Sheng, et al, Phys. Rev. Lett. 131, 042304 (2023)



ANGULAR VELOCITY DEPENDENCE

- Far from CEP the angular velocity dependence $\sim\Omega^2$
- r dependence is minor: our approximation works well
- The peak is sensitive to angular velocity: CEP is shifted close to freezeout line

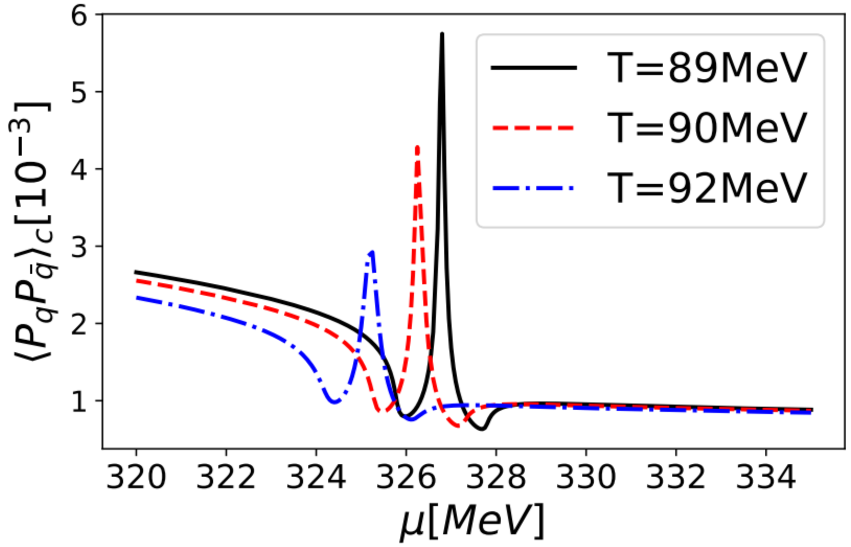




PNJL MODEL

Λ [MeV]	m_0 [MeV]	$G_{\rm PNJL}\Lambda^2$	N_f	a_0	a_1	a_3	b_3	T_0[MeV]
651	5.5	2.135	2	3.51	-2.47	15.2	-1.75	210

$$
\begin{aligned}
&V_{\rm PNJL}(\Omega_q^s, \Omega_{\bar q}^s, \mu_q, \mu_{\bar q}; r=0)\\
=&\frac{[m-m_0]^2}{4G_{\rm PNJL}} - 2N_f\int_0^\Lambda \frac{{\rm d}^3p}{(2\pi)^3}3\varepsilon_p\\
&-N_f\int_0^\infty \frac{{\rm d}^3p}{(2\pi)^3}\Big[T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T}+3\bar\Phi{\rm e}^{-2(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T}+{\rm e}^{-3(\varepsilon_p-\mu-\Omega_q^s/2-\mu_q)/T})\\
&+T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T}+3\bar\Phi{\rm e}^{-2(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T}+{\rm e}^{-3(\varepsilon_p-\mu+\Omega_q^s/2-\mu_q)/T})\\
&+T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p+\mu-\Omega_{\bar q}^s/2-\mu_{\bar q})/T}+3\bar\Phi{\rm e}^{-2(\varepsilon_p+\mu-\Omega_{\bar q}^s/2-\mu_{\bar q})/T}+{\rm e}^{-3(\varepsilon_p+\mu-\Omega_{\bar q}^s/2-\mu_{\bar q})/T})\\
&+T\ln(1+3\Phi{\rm e}^{-(\varepsilon_p+\mu+\Omega_{\bar q}^s/2-\mu_{\bar q})/T}+3\bar\Phi{\rm e}^{-2(\varepsilon_p+\mu+\Omega_{\bar q}^s/2-\mu_{\bar q})/T}+{\rm e}^{-3(\varepsilon_p+\mu+\Omega_{\bar q}^s/2-\mu_{\bar q})/T})\Big]\\
&+T^4\Big\{-\frac{1}{2}\Big[a_0+a_1(\frac{T_0}{T})+a_2(\frac{T_0}{T})^2\Big]\bar\Phi\Phi+b_3(\frac{T_0}{T})^3\ln\Big[1-6\bar\Phi\Phi+4(\bar\Phi^3+\Phi^3)-3(\bar\Phi\Phi)^2\Big]\Big\},
\end{aligned}
$$



- Quantitatively agree with NJL model results
- Here we do not modify Polyakov potential, since we only care about quark spin

F. Sun, et al., Phys.Rev.D, 109 (2024) 11, 116017

G. Cao, Phys.Rev.D, 109 (2024) 1, 014001

Ji-peng Lv, et al., Phys.Rev.D 109 (2024) 11, 114003

OTHER CORRELATIONS

$$\delta\rho_{00}(\phi) \approx -\frac{4}{9}\frac{\langle\delta N_s \delta N_{\bar{s}}\rangle}{N_s N_{\bar{s}}} = -\frac{32}{9c^2}\frac{T}{V}\frac{N_c^2 G_A}{\rho_s^2}L^2$$

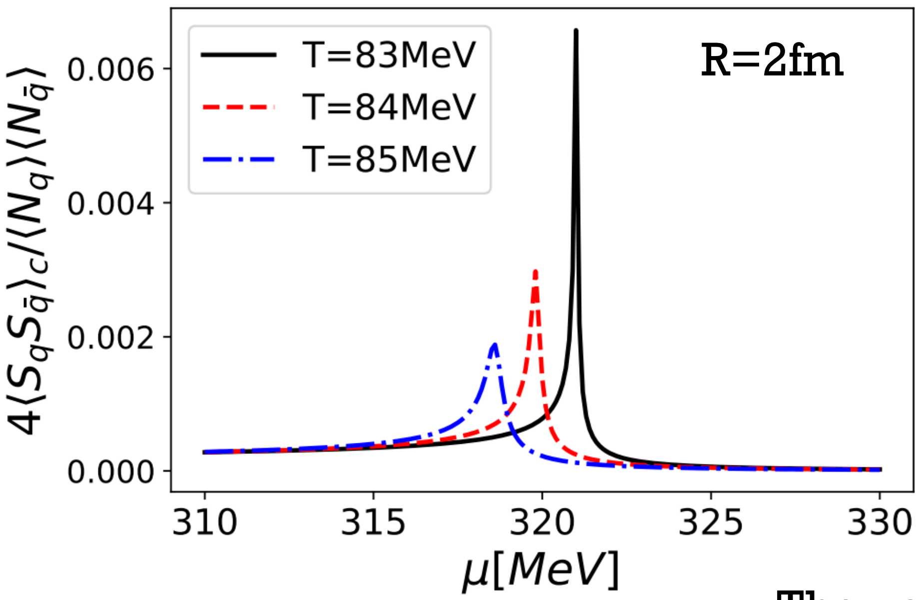
Kun Xu, Mei Huang, Phys.Rev.D 110 (2024) 9, 094034

$$c_{ij} = \frac{f_{\uparrow\uparrow} + f_{\downarrow\downarrow} - f_{\downarrow\uparrow} - f_{\uparrow\downarrow}}{f_{\uparrow\uparrow} + f_{\downarrow\downarrow} + f_{\downarrow\uparrow} + f_{\uparrow\downarrow}},$$

The same?

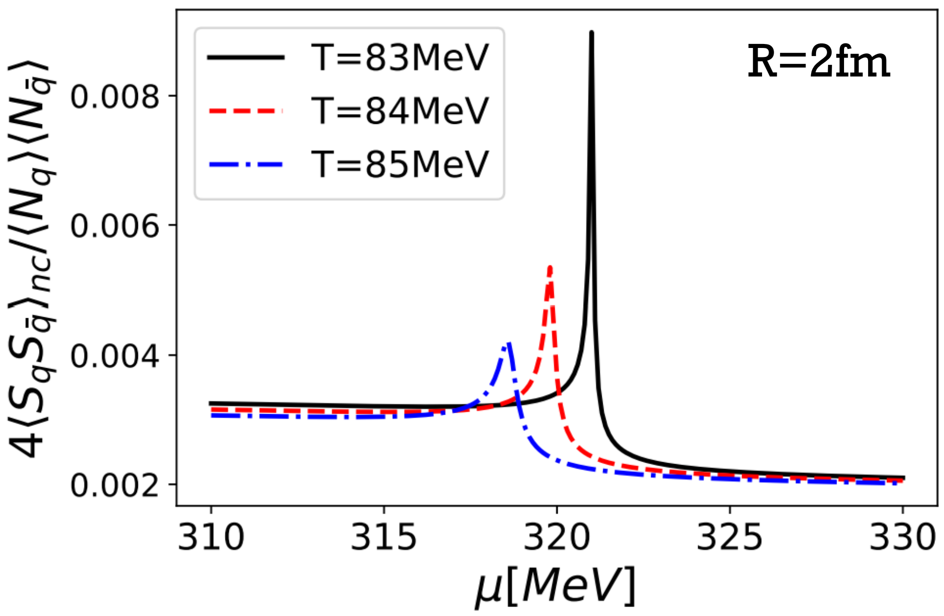
If we take into account critical behavior

$$\frac{4(\langle S_q S_{\bar{q}}\rangle - \langle S_q\rangle\langle S_{\bar{q}}\rangle)}{\langle N_q\rangle\langle N_{\bar{q}}\rangle}$$



Non-connected correlation

$$\frac{4\langle S_q S_{\bar{q}}\rangle}{\langle N_q\rangle\langle N_{\bar{q}}\rangle}$$



The volume dependence is annoying

OTHER CORRELATIONS

More reasonable than our original definition?

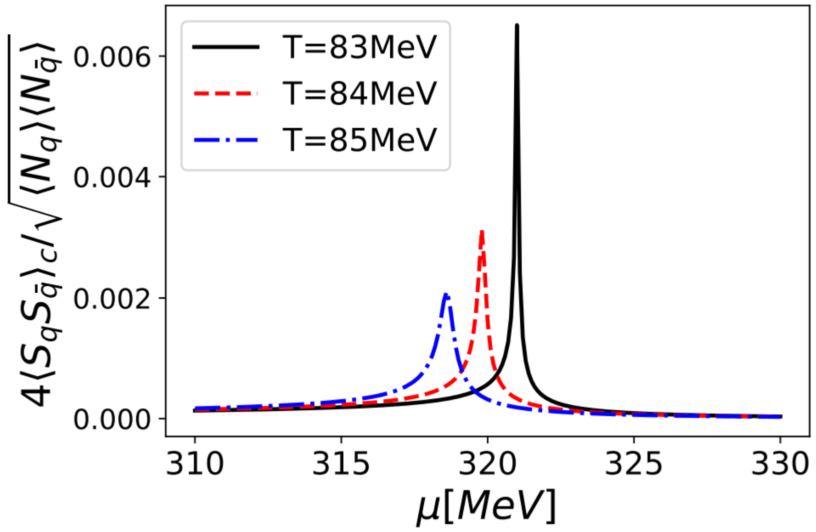
$$\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle}$$

- Inspired by baryon number fluctuation C2/C1

$$\langle P_q P_{\bar{q}} \rangle = \frac{4(\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle)}{\sqrt{\langle N_q \rangle \langle N_{\bar{q}} \rangle}} = \frac{4C^S_{2,q\bar{q}}}{C^N_{1,q\bar{q}}}$$

With

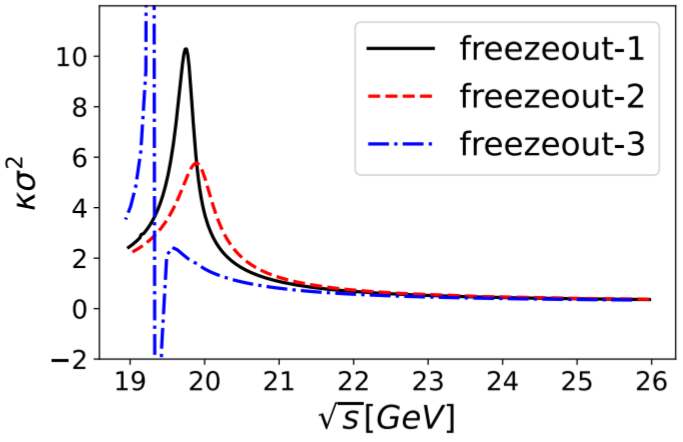
$$C^S_{2,q\bar{q}} = VT \frac{\partial^2 p}{\partial \omega_q \omega_{\bar{q}}}, \qquad C^N_{1,q\bar{q}} = \sqrt{\langle N_q \rangle \langle N_{\bar{q}} \rangle} = V\left(\frac{\partial p}{\partial \mu_q} \frac{\partial p}{\partial \mu_{\bar{q}}}\right)^{1/2}$$



Kurtosis of quark spin fluctuation

more sensitive to CEP

$$\kappa\sigma^2 = \frac{C_4^S}{C_2^S} \qquad C_n^S = VT^{n-1} \frac{\partial^n p}{\partial \omega^n}$$



FURTHER IMPROVEMENTS

- Contribution from spin correlation of other direction $\langle P_q^x P_{\bar{q}}^x \rangle$ and $\langle P_q^y P_{\bar{q}}^y \rangle$
- Momentum dependence of the correlation
- Finite size & inhomogeneity

If we assume m(r)=m, and integrate over the whole system

$$\int r\mathrm{d}r \langle S_q \rangle = \int r\mathrm{d}r \langle S_{\bar{q}} \rangle = 0 \quad \text{and} \quad \int r\mathrm{d}r \langle S_q S_{\bar{q}} \rangle = 0$$

Inhomogeneity might be important

- Other definitions of correlation are possible
- Gluon spin polarization

GLUON SPIN

- Up to now, we only focus on the quark sector
- Proton spin crisis: contribution from sea quark or gluon field
- Gluon has spin one: more sensitive to rotation in principle
- It is interesting to see how rotation affects gluon field

Shi Chen, Kenji Fukushima, Yusuke Shimada, Phys.Rev.Lett. 129 (2022) 24, 242002
Victor V. Braguta, Maxim N. Chernodub, Ilya E. Kudrov, Artem A. Roenko, Dmitrii A. Sychev, Phys.Rev.D 110 (2024) 1, 014511
Yin Jiang, Phys.Lett.B 853 (2024) 138655
Guojun Huang, Shile Chen, Yin Jiang, Jiaxing Zhao, Pengfei Zhuang, Phys.Lett.B 862 (2025) 139274
Kenji Fukushima, Yusuke Shimada, Phys.Lett.B 868 (2025) 139716
Sheng Wang, Jun-Xia Chen, Defu Hou, Hai-Cang Ren, arXiv: 2505.15487

POLARIZED GLUON FIELD: SAVVIDY VACUUM

- QCD vacuum can have a nonzero chromomagnetic condensate G.K. Savvidy, Phys. Lett. B, 71:133, 1977
- Nielsen-Olesen Instability N.K Nielsen and P. Olesen., Nucl. Phys. B, 144(2-3):376–396, 1978.
- Asymptotic freedom can be understood in term by vacuum polarizability

$$E_{\mathrm{vac,QCD}} = -\tfrac{1}{2}VH^2\frac{(33-2N_F)g^2}{48\pi^2}\log\frac{\Lambda^2}{|gH|}$$

QCD Beta function

$$\beta(g) = -\frac{(33-2N_F)g^3}{48\pi^2}.$$

Q: What will happen if we add rotation to the system?

Asymptotic freedom as a spin effect

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Fysisk Institut, Odense University, Odense, Denmark
(Received 25 August 1980; accepted 26 November 1980)

It is shown how both the qualitative and the quantitative features of the asymptotic freedom of quantum chromodynamics can be understood in a rather intuitive way. The starting point is the spin of the gluon, which because of the gluon self-coupling makes the vacuum behave like a paramagnetic substance. Combining this result with Lorentz invariance, we conclude that the vacuum exhibits dielectric antiscreening and hence asymptotic freedom. The calculational techniques are with some minor modifications those of the Landau theory on the diamagnetic properties of a free-electron gas.

SU(2) YANG-MILLS THEORY UNDER ROTATION

- For simplicity, I focus on imaginary rotation here
- The nonzero background gauge potential $\bar{A}^3_\mu = (\phi, \frac{1}{2}Hy, -\frac{1}{2}Hx, 0)$

$$V(r) = \frac{1}{2}H^2 + \sum_n \sum_{l=-\lambda}^{N-\lambda} \sum_{\lambda\in\mathbb{N}} \sum_{s=\pm1} \int \frac{dk_z}{2\pi} \ln[(\omega_n - \Omega_I(\mathrm{sgn}(gH)l - s) + g\phi)^2 + |gH|(2\lambda+1+2s) + k_z^2] \frac{|gH|}{2\pi} \Phi_l^2(\lambda, \tfrac{1}{2}|gH|r^2)$$

Tachyonic mode when $\lambda = 0, \quad s = -1$

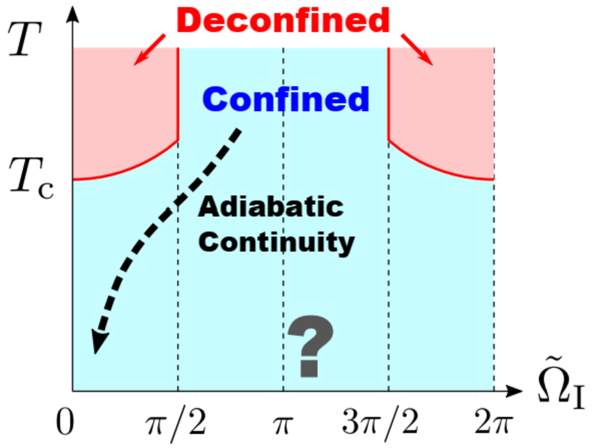
- At the center (r=0),

$$V_R = \frac{11g^2H^2}{48\pi^2}\ln(\frac{gH}{\mu_0^2}) - \frac{(gH)^{\frac{3}{2}}}{\pi^2\beta}\sum_{n=1}^{\infty}\frac{1}{n}[K_1(n\beta\sqrt{gH}) - \frac{\pi}{2}Y_1(n\beta\sqrt{gH})]\cos n(\tilde{\phi} - \tilde{\Omega}_I)$$

$$- 2\frac{(gH)^{\frac{3}{2}}}{\pi^2\beta}\sum_{n=1}^{\infty}\sum_{\lambda=0}^{\infty}\frac{1}{n}\sqrt{2\lambda+3}K_1(n\beta\sqrt{gH(2\lambda+3)})\cos n\tilde{\phi}\cos n\tilde{\Omega}_I$$

$$V_I = -\frac{(gH)^2}{8\pi} - \frac{(gH)^{\frac{3}{2}}}{2\pi^2\beta}\sum_{n=1}\frac{1}{n}J_1(n\beta\sqrt{gH})\cos n(\tilde{\phi} - \tilde{\Omega}_I)$$

Shi Chen, Kenji Fukushima, Yusuke Shimada,
Phys.Rev.Lett. 129 (2022) 24, 242002



GLUON CONDENSATE

- Here we first ignore the instability
- Minimize the real part of effective potential at high temperature

$$\frac{\partial V_R}{\partial (gH)} = \frac{\partial V_R}{\partial (g\phi)} = 0$$

- For small imaginary rotation, chromomagnetic condensate increases

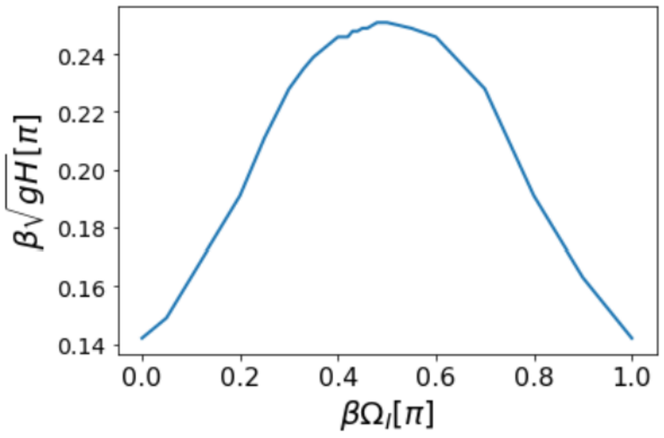


FIG. 3. Chromomagnetic condensate as a function of imaginary angular velocity at $T = 10\mu_0$.

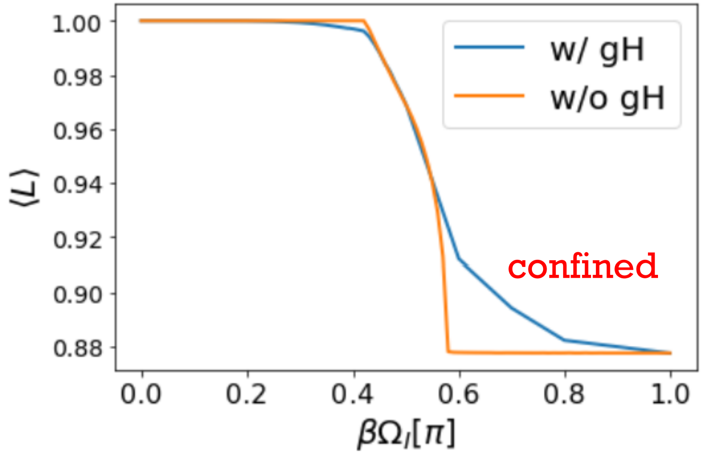


FIG. 6. Ployakov loop at $T = 10\mu_0$.

EFFECTIVE COUPLING CONSTANT

- For small imaginary rotation, we can expand the effective potential (where $b=\beta\sqrt{gH}$ and $\tilde{\Omega}_I=\beta\Omega$)

$$V_R=[-\frac{11}{24\pi^2\beta^4}(\ln\frac{\beta\mu_0}{4\pi}-\gamma)+\frac{7+4C_1}{32\pi^2\beta^4}-\frac{11}{96\pi^4\beta^4}\zeta(3)\tilde{\Omega}_I^2]b^4-\frac{C_2}{2\pi\beta^4}b^3-[\frac{11}{24\pi\beta^4}-\frac{C_3}{2\pi\beta^4}]\tilde{\Omega}_I^2 b$$

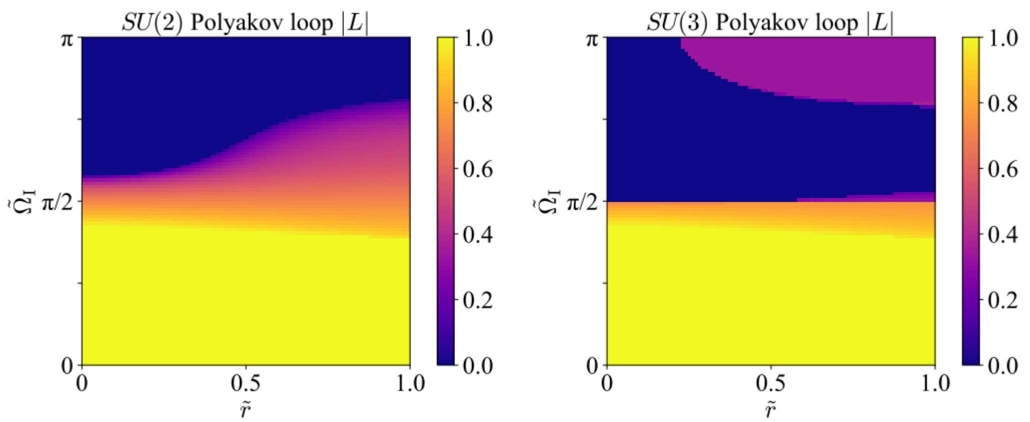
- We can extract effective coupling constant for VR

$$g_{eff}^2(T,\tilde{\Omega}_I)=\frac{1}{-\frac{11}{12\pi^2}(\ln\frac{\beta\mu_0}{4\pi}-\gamma)+\frac{7+4C_1}{16\pi^2}-\frac{11}{48\pi^4}\zeta(3)\tilde{\Omega}_I^2}$$

- The coupling increase with imaginary rotation: Tc increases with imaginary rotation
- Analytic continuation: Tc decreases with real rotation

In consistent with previous model studies

FUTURE DIRECTION



- Including Quark degree of freedom
- Inhomogeneity Shi Chen, et al., Phys.Lett.B 859 (2024) 139107
- Two-loop contribution
- Higgs mechanism

} might stabilize the system

- Whether this polarized gluon field relates to some observables?
- For example, proton spin

SUMMARY & OUTLOOK

- Interaction is important to when we study quark spin correlation
- Quark-antiquark spin correlation has a peak near the CEP
- Critical fluctuation near CEP can lead to non-monotonic behavior of spin alignment & Hyperon-anti-Hyperon correlation
- Spin alignment & Hyperon-anti-Hyperon correlation can serve as signatures for CEP
- Connection between spin and phase transition is an interesting direction
- Contribution from gluons should be considered
- Spin alignment of other vector mesons
- More realistic and detailed studies in future

THANK YOU FOR ATTENTION!