Precise determination of gravitational from factors

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M.-L. Du et al., Deciphering the mechanism of near-threshold J/ψ photoproduction, Eur. Phys. J. C 80 (2020) 1053;
B. Wu, X.-K. Dong, M.-L. Du, FKG, B.-S. Zou, Deciphering the mechanism of J/ψ-nucleon scattering, Fund. Res., in print [arXiv:2410.19526];
X.-H. Cao, FKG, Q.-Z. Li, D.-L. Yao, Precise determination of nucleon gravitational form factors, Nature Commun. 16 (2025) 6979;
X.-H. Cao, FKG, Q.-Z. Li, D.-L. Yao, Gravitational form factors of pions, kaons and nucleons from dispersion relations, Eur. Phys. J. ST, in print [arXiv:2507.05375].

Gravitational form factors

- Electro-magnetic form factors: matrix elements of em current
  - Probes distributions of electric charge, magnetic moment densities, etc.
    - proton is not pointlike, $r_E^p = 0.74(24)$ fm

R. Hofstadter, R. McAllister (1955); R. Hofstadter (1956)

- Gravitational form factors: matrix elements of energy-momentum tensor

$$\left\langle N(p') \left| \hat{T}^{\mu\nu}(0) \right| N(p) \right\rangle = \frac{1}{4m_N} \bar{u}(p') \left[A(t) P^\mu P^\nu + J(t) \left(i P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho \right) + D(t) \left(\Delta^\mu \Delta^\nu - t g^{\mu\nu} \right) \right] u(p)$$

  - Trace anomaly: origin of mass
  - Fundamental properties of the nucleon
    - Energy/mass:

$$m_N = \int d^3 r T^{00}(r), \quad A(0) = 1$$

    - Spin:

$$J^i = \epsilon^{ijk} \int d^3 r r^j T^{0k}(r), \quad J(0) = 1/2$$

    - D-term:

$$D = -\frac{m_N}{2} \int d^3 r \left(r^i r^j - \frac{1}{3} \delta^{ij} \right) T^{ij}(r), \quad D(0): \quad \text{“last global unknown property”}$$

Forces inside hadron, mechanical picture of quark confinement?

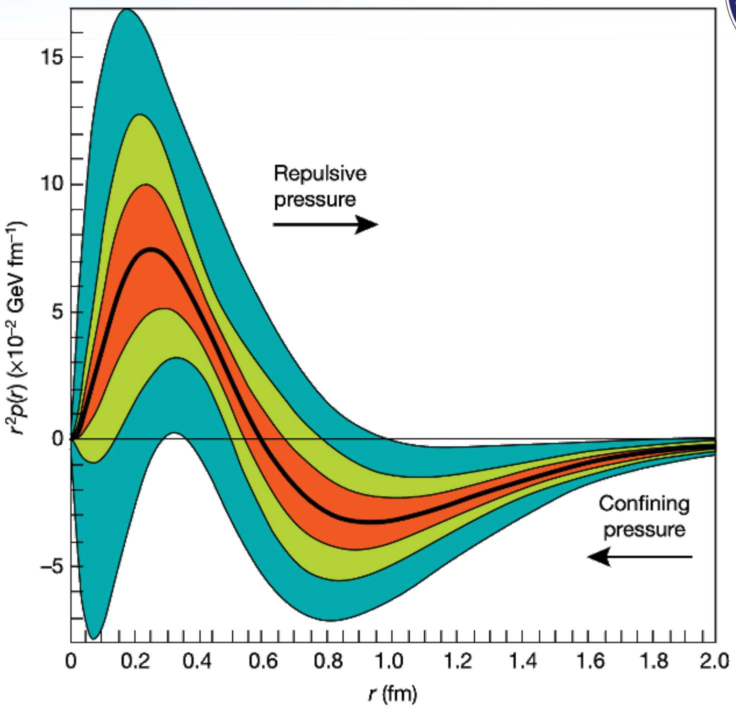
M. Polyakov, PLB 555 (2003) 57;
M. Polyakov, P. Schweitzer, IJMPA 33 (2018) 1830025

Gravitational form factors

- Results from JLab
  - Pressure distribution inside proton using DVCS data

    V. D. Burkert, L. Elouadrhiri, F. X. Girod, Nature 557 (2018) 396

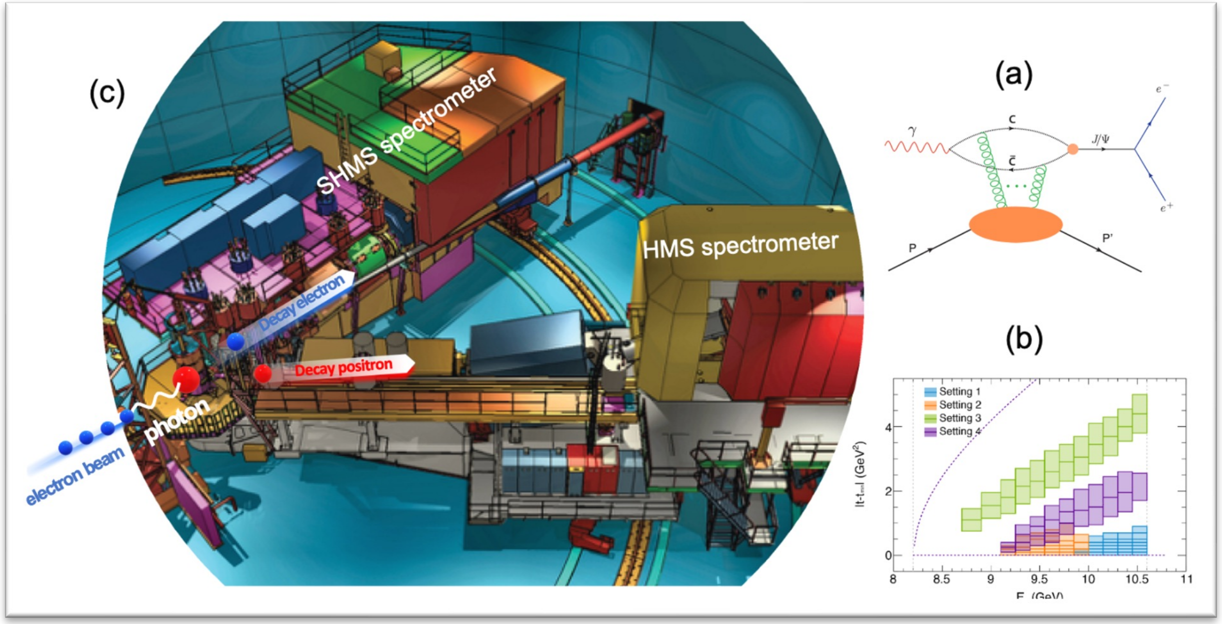
    “We find a strong repulsive pressure near the centre of the proton (up to 0.6 femtometres) and a binding pressure at greater distances. The average peak pressure near the centre is about 10^{35} pascals, which exceeds the pressure estimated for the most densely packed known objects in the Universe, neutron stars.”



  - Nucleon GFFs from near-threshold J/ψ photoproduction with two models   B. Duran et al. [J/ψ-007], Nature 615 (2023) 7954
    - Holographic QCD
    - GPD + vector-meson dominance

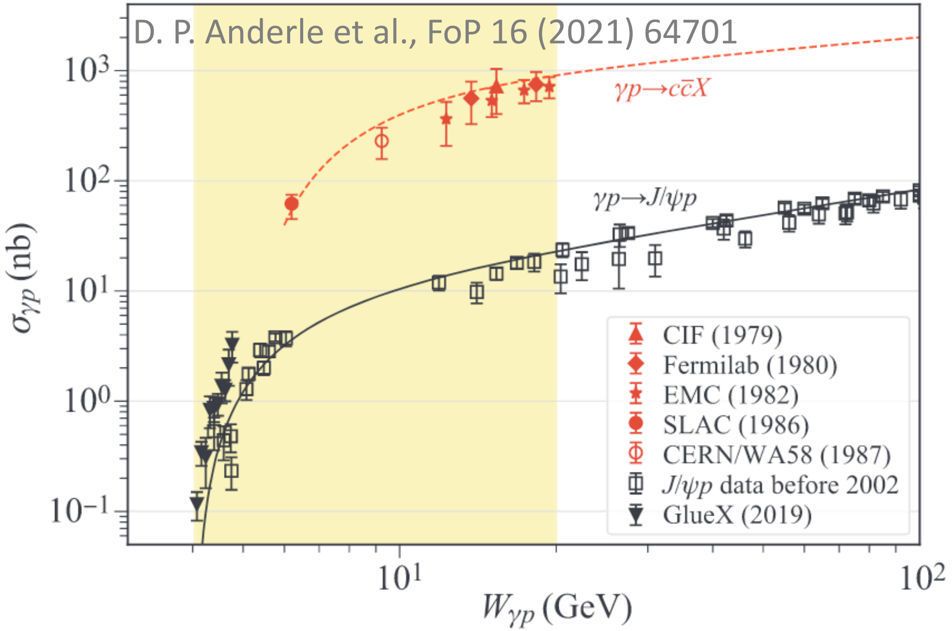
Theoretical approach GFF functional form	$\sqrt{\langle r_m^2 \rangle}$ (fm)	$\sqrt{\langle r_s^2 \rangle}$ (fm)
Holographic QCD Tripole-tripole	0.755±0.035	1.069±0.056
GPD + VMD Tripole-tripole	0.472±0.042	0.695±0.071
Updated GPD analysis:	0.77 ± 0.07	1.20 ± 0.13

Y. Guo et al., PRD 108 (2023) 034003

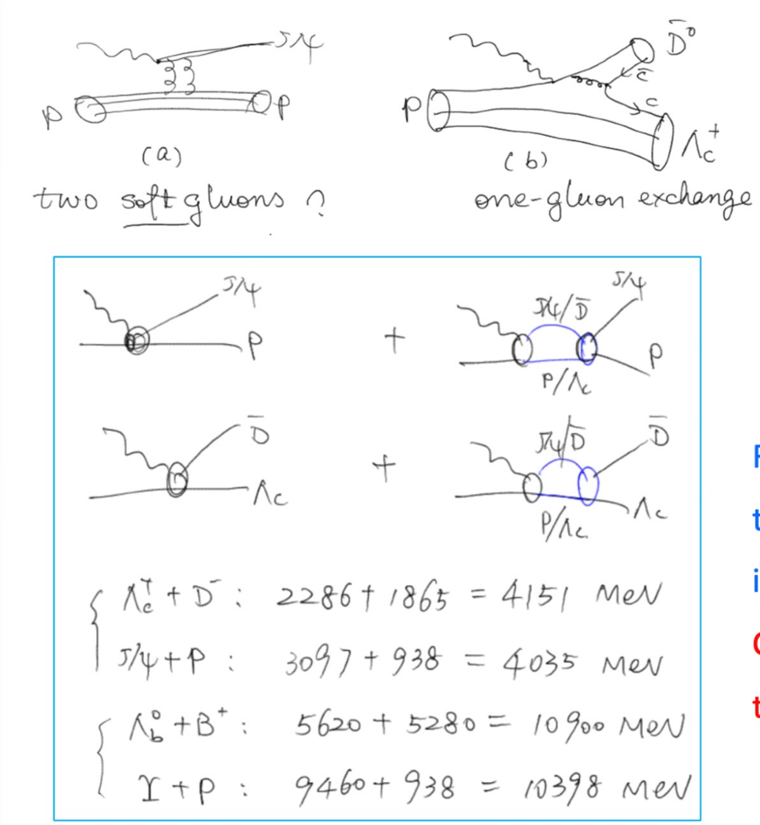


Remarks on J/ψ photoproduction and $J/\psi N$ scattering

- What is the main mechanism for the J/ψ photoproduction in the near-threshold region?
  - ☐ Gluon exchange?
  - ☐ Coupled-channel mechanism?
    - ➢ hinder extraction of gluonic matrix element in the regime
    - ➢ if open-charm channels are produced with larger rates
      - ✧ Thresholds close to $J/\psi N$ threshold
      - ✧ $\sigma_{c\bar{c}}$ inclusive $\gg \sigma_{J/\psi}$



My talk, 2019 EicC discussion @PKU



Naïve estimate:

$\sigma(\Lambda_c\bar{D}) \gg \sigma(J/\psi p)$

For J/psi near-threshold production, the $\Lambda_c\bar{D}$ channel is expected to be important!

Consequence: a cusp at the $\Lambda_c\bar{D}$ threshold

Remarks on J/ψ photoproduction and $J/\psi N$ scattering

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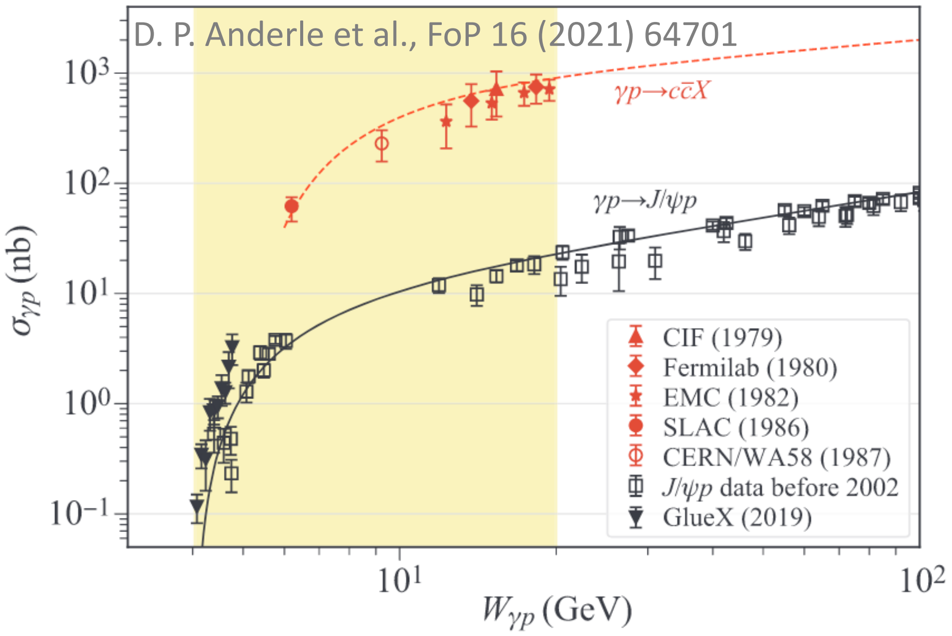
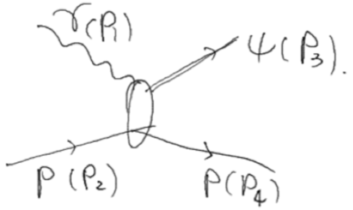


My talk, 2019 EicC discussion @PKU

t at threshold production:



For J/ψ: $t|_{th} \simeq -2.2$ GeV^2. For J/ψ: $m_c v_c \sim 1$ GeV.

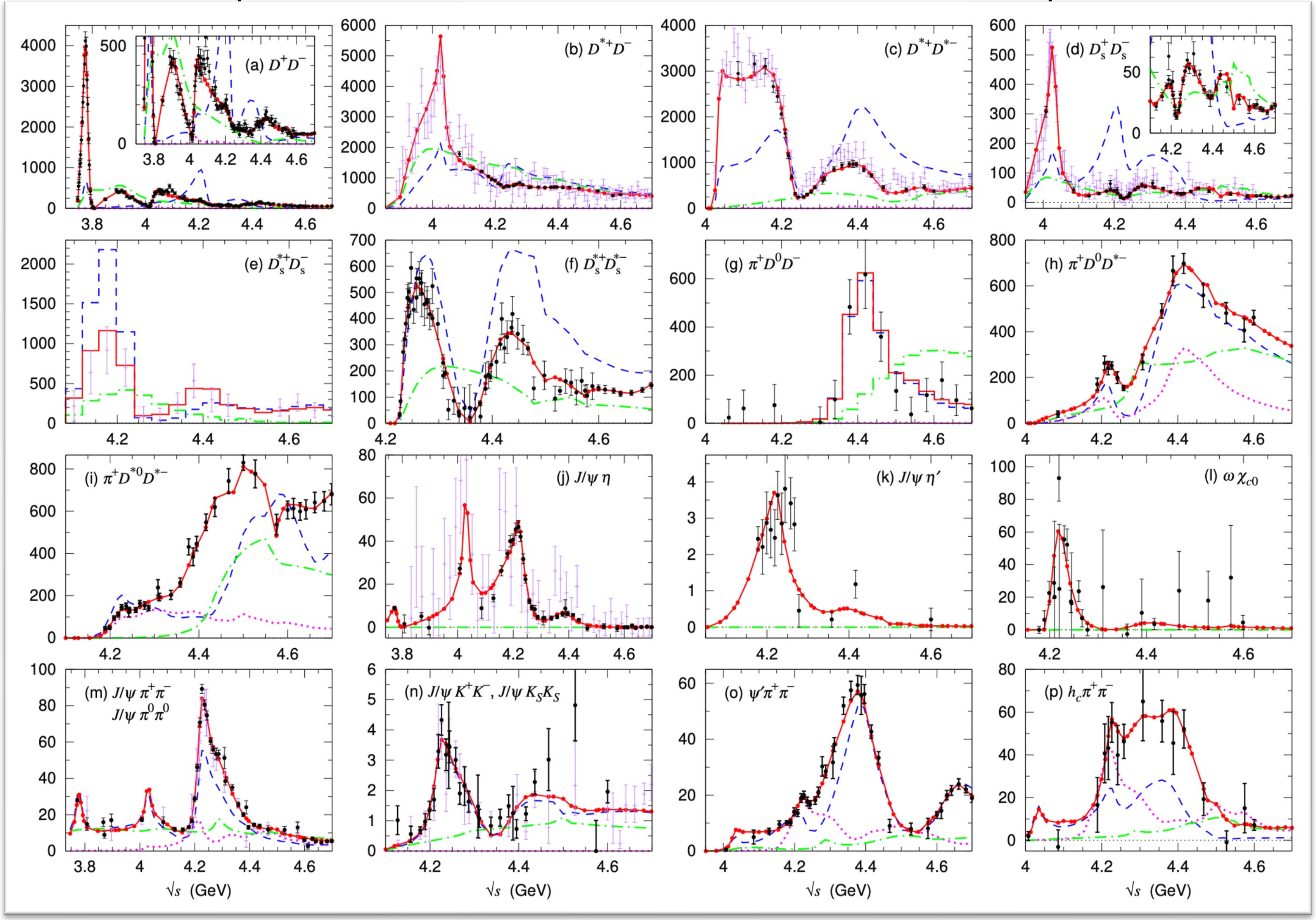
For Υ: $t|_{th} \simeq -8.1$ GeV^2. For Υ: $m_b v_b \sim 1.5$ GeV.

Thus, we have $|t|_{th}| > m_Q v_Q$. multipole expansion not working!!

The threshold region is far away from t=0.

Remarks on J/ψ photoproduction and $J/\psi N$ scattering

- Production rates of open-charm v.s. charmonium channels in electron-positron collisions (units: nb)

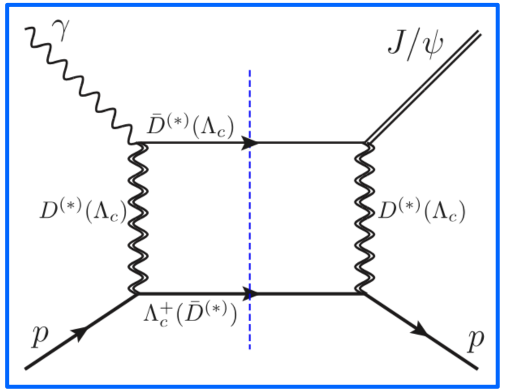


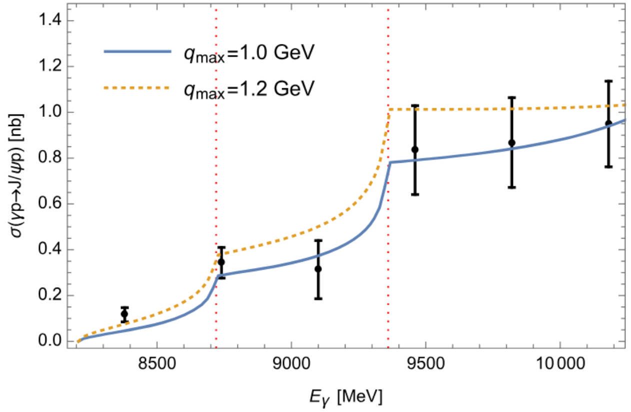
Plot taken from S.X. Nakamura et al., PRD 112 (2025) 054027

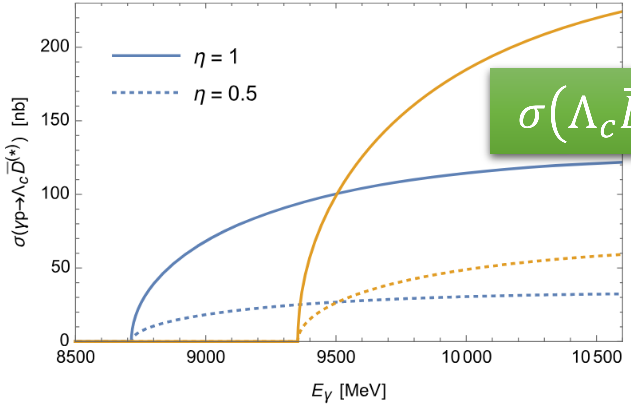
Remarks on J/ψ photoproduction and $J/\psi N$ scattering

- Coupled-channel mechanism M.-L. Du et al., EPJC 80 (2020) 1053
  - Unitarity: $J/\psi p \to J/\psi p$ enters coupled-channel scattering ($J/\psi p$, $\Lambda_c \bar{D}^{(*)}$, $\Sigma_c^{(*)} \bar{D}^{(*)}$) w/o VMD, but cannot be singled out
  - Simple model estimate: consider $\Lambda_c \bar{D}^{(*)}$ channels
    - ➢ Charmed meson/baryon exchanges at 1-loop

Measure the open-charm production!





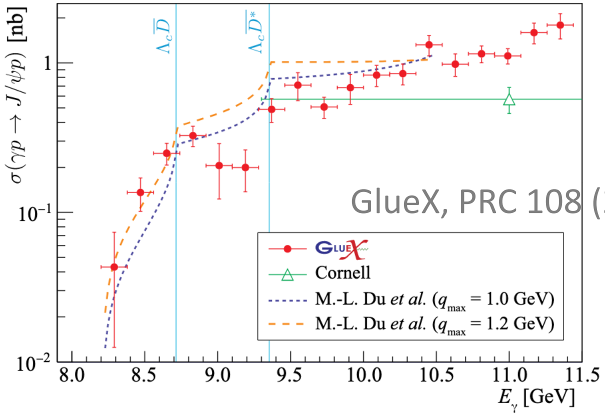


$\sigma(\Lambda_c \bar{D}^{(*)}) \gg \sigma(J/\psi p)$

- ➢ Threshold cusps (1-loop result is not robust)
  - ✧ Needs to be analyzed with unitarity

    For such analyses, see, JPAC, PRD 108 (2023) 054018; X. Zhang, EPJC 85 (2025) 1120
  - ✧ Nontrivial cusp only for attractive interaction

    X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001



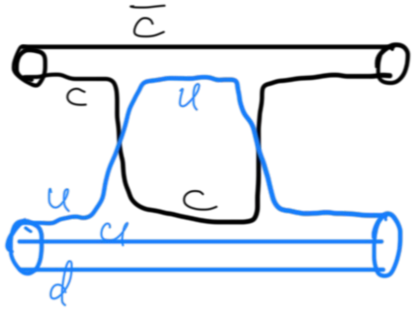
GlueX, PRC 108 (2023) 025201

Remarks on J/ψ photoproduction and $J/\psi N$ scattering

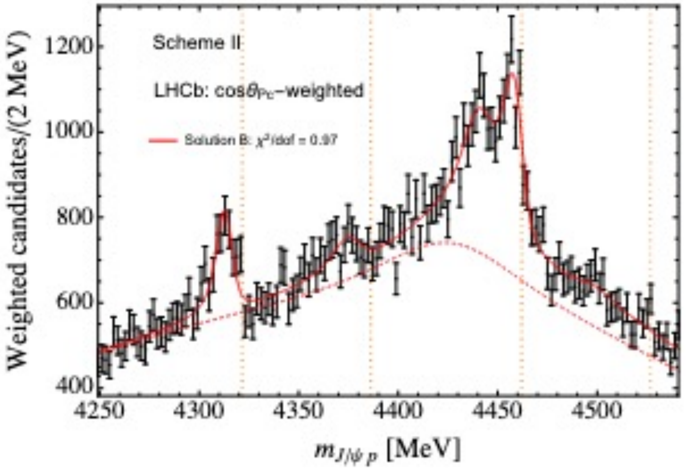
- $J/\psi N$ scattering length mechanisms

B. Wu et al., Fund. Res., in print (2025) [arXiv:2410.19526]

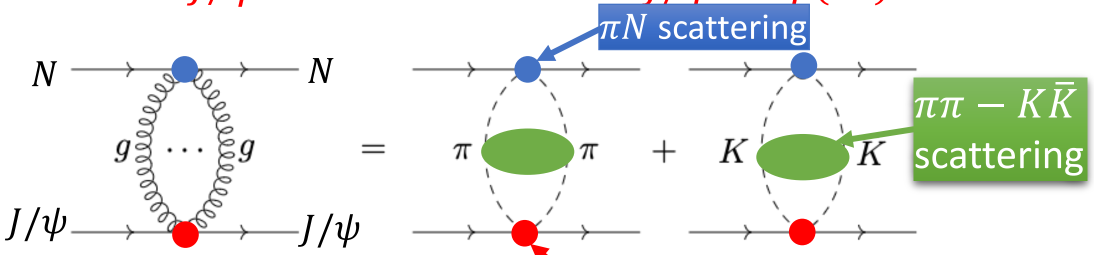
- Open-charm coupled channels $(J/\psi N - \Lambda_c \bar{D}^{(*)}/\Sigma_c^{(*)}\bar{D}^{(*)} - J/\psi N)$
  - Based on solution of coupled-channel LSE fitted to P_c data (direct $J/\psi N$ scattering neglected, only via coupled channels)
  - Result: $\mathcal{O}(-(0.1 \ldots 10)\times 10^{-3}\ \text{fm})$

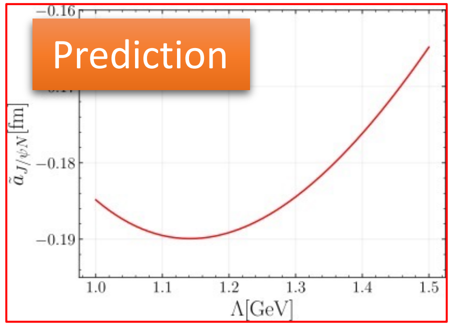


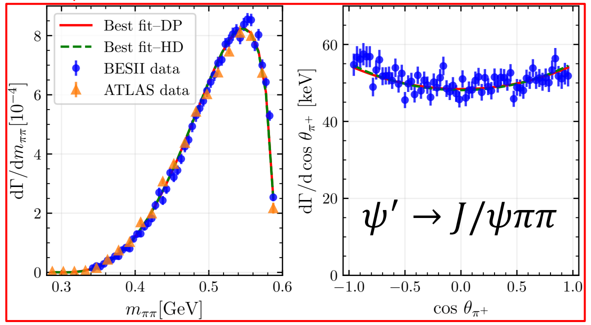
M.-L. Du et al., PRL 124 (2020) 072001; JHEP 08 (2021) 157



- Soft-gluon exchange
  - Based on dispersion relation
  - Result: $a_{J/\psi N} \lesssim -0.16$ fm, $a_{J/\psi N}\, a_{\psi(2S)N} \geq (-0.15\ \text{fm})^2$







- consistent with later lattice QCD result

Y. Lyu et al. [HALQCD], PLB 860 (2024) 139178

$$a_{J/\psi N\,(S=3/2)} = -0.30^{+0.02+0.00}_{-0.02-0.02}\ \text{fm}$$

Low-energy $J/\psi N$ scattering dominated by gluonic exchange

Pion and nucleon GFFs

X.-H. Cao, FKG, Q.-Z. Li, D.-L. Yao, Precise determination of nucleon gravitational form factors, Nature Commun. 16 (2025) 6979;
X.-H. Cao, FKG, Q.-Z. Li, D.-L. Yao, Gravitational form factors of pions, kaons and nucleons from dispersion relations, Eur. Phys. J. ST, in print [arXiv:2507.05375].

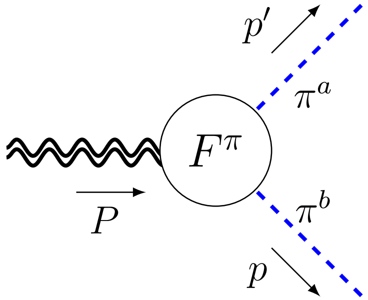
- Definitions
  - Gravitational form factors (GFFs) for spin-0 particles, e.g., for pion:

$$\left\langle \pi^a(p') \left| \hat{T}^{\mu\nu}(0) \right| \pi^b(p) \right\rangle = \frac{\delta^{ab}}{2} \left[A^\pi(t) P^\mu P^\nu + D^\pi(t) \left(\Delta^\mu \Delta^\nu - t g^{\mu\nu} \right) \right]$$

$$P^\mu = p'^\mu + p^\mu, \Delta^\mu = p'^\mu - p^\mu$$

Crossing, for constructing dispersion relations

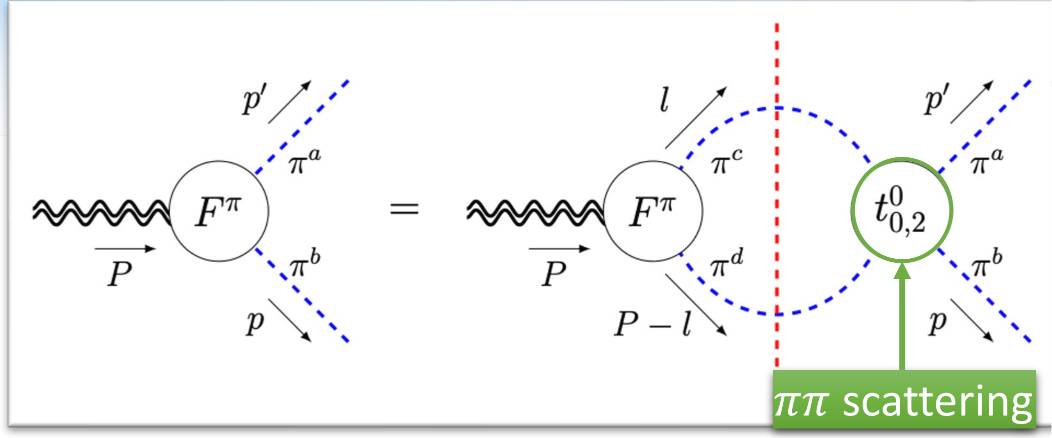
$$\left\langle \pi^a(p') \pi^b(p) \left| \hat{T}^{\mu\nu}(0) \right| 0 \right\rangle = \frac{\delta^{ab}}{2} \left[A^\pi(t) \Delta^\mu \Delta^\nu + D^\pi(t) \left(P^\mu P^\nu - t g^{\mu\nu} \right) \right]$$



  - Nucleon GFFs

$$\left\langle N(p') \bar{N}(p) \left| \hat{T}^{\mu\nu}(0) \right| 0 \right\rangle = \frac{1}{4 m_N} \bar{u}(p') \left[\hat{A}(t) \Delta^\mu \Delta^\nu + \hat{J}(t) \left(i \Delta^{\{\mu} \sigma^{\nu\}\rho} P_\rho \right) + \hat{D}(t) \left(P^\mu P^\nu - t g^{\mu\nu} \right) \right] u(p)$$

Unitarity relation for the pion GFFs



- Unitarity ⇒ discontinuity (imaginary part) of the pion GFFs

$$\mathrm{Disc}\left\langle \pi^a(p')\pi^b(p)\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle$$

$$=\frac{\delta^{ab}}{2}\left[\mathrm{Disc}\,A^\pi(t)\Delta^\mu\Delta^\nu+\mathrm{Disc}\,D^\pi(t)\left(P^\mu P^\nu-tg^{\mu\nu}\right)\right]$$

$$=\frac{1}{2}\frac{i}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int \mathrm{d}\Omega_l\left\langle \pi^a(p')\pi^b(p)\right|\left.\pi^c(l)\pi^d(P-l)\right\rangle\left\langle\pi^c(l)\pi^d(P-l)\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle^*$$

$$=\frac{1}{2}\frac{i}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int \mathrm{d}\Omega_l\frac{\delta^{ab}}{2}\left[3A(t,s,u)+A(s,t,u)+A(u,s,t)\right]$$
$$\times\left[\left(A^\pi(t)\right)^*(2l-P)^\mu(2l-P)^\nu+\left(D^\pi(t)\right)^*\left(P^\mu P^\nu-tg^{\mu\nu}\right)\right]$$

$$=\frac{1}{2}\frac{i}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int \mathrm{d}\Omega_l\frac{\delta^{ab}}{2}A^{I=0}(t,s,u)\left[\left(A^\pi(t)\right)^*\underbrace{\boxed{(2l-P)^\mu(2l-P)^\nu}}_{\text{S-, D-waves}}+\left(D^\pi(t)\right)^*\underbrace{\boxed{\left(P^\mu P^\nu-tg^{\mu\nu}\right)}}_{\text{S-wave}}\right]$$

Mandelstam variables: $t=P^2, s=(p'-l)^2, u=(p-l)^2$; only two independent, $A^{I=0}(t,s)\equiv A^{I=0}(t,s,u)$

Lorentz structures: $\int \mathrm{d}\Omega_l A^{I=0}(t,s)(2l-P)^\mu(2l-P)^\nu=A_1\Delta^\mu\Delta^\nu+A_2\left(P^\mu P^\nu-tg^{\mu\nu}\right)$

Contract with $\Delta_\mu\Delta_\nu$ and $g_{\mu\nu}$ ⇒ A_1, A_2

Unitarity relation for the pion GFFs

$A_1, A_2 \Rightarrow$ combinations of isoscalar S-, D-wave $\pi\pi$ amplitudes $t_0^0(t), t_2^0(t)$: $A^I(t,s) = 32\pi \sum_J (2J+1) P_J(\cos\theta) t_J^I(t)$

$$\int \mathrm{d}\Omega_l A^{I=0}(t,s)(2l-P)^\mu (2l-P)^\nu = 128\pi^2 t_2^0(t) \Delta^\mu \Delta^\nu + \frac{512\pi^2}{3t}\left[t_0^0(t) - t_2^0(t)\right]\left(P^\mu P^\nu - t g^{\mu\nu}\right)$$

- Discontinuity of A^π and D^π

$$\mathrm{Disc}\left\langle \pi^a(p')\pi^b(p) \middle| \hat{T}^{\mu\nu}(0) \middle| 0 \right\rangle = \frac{\delta^{ab}}{2}\left[\mathrm{Disc}\, A^\pi(t) \Delta^\mu \Delta^\nu + \mathrm{Disc}\, D^\pi(t)\left(P^\mu P^\nu - t g^{\mu\nu}\right)\right]$$

$$= 2i \frac{2p_\pi}{\sqrt{t}} \frac{\delta^{ab}}{2}\left[\left(A^\pi(t)\right)^* \left(\frac{4}{3t} p_\pi^2 \left(t_0^0(t) - t_2^0(t)\right)\left(P^\mu P^\nu - t g^{\mu\nu}\right) + t_2^0(t) \Delta^\mu \Delta^\nu\right) + \left(D^\pi(t)\right)^* t_0^0(t)\left(P^\mu P^\nu - t g^{\mu\nu}\right)\right]$$

$$\mathrm{Im}\, A^\pi(t) = \frac{2p_\pi}{\sqrt{t}}\left(t_2^0(t)\right)^* A^\pi(t)\ ,$$

$$\mathrm{Im}\, D^\pi(t) = \frac{2p_\pi}{\sqrt{t}}\left[\frac{4}{3}\frac{p_\pi^2}{t}\left(t_0^0(t) - t_2^0(t)\right)^* A^\pi(t) + \left(t_0^0(t)\right)^* D^\pi(t)\right] \Rightarrow \mathrm{Im}\, \Theta^\pi(t) = \frac{2p_\pi}{\sqrt{t}}\left(t_0^0(t)\right)^* \Theta^\pi(t)$$

- Decomposition into $J^{PC} = 0^{++}, 2^{++}$ matrix elements (conserved separately):

K. Raman (1971)

$$\left\langle \pi^a(p')\pi^b(p) \middle| \hat{T}^{\mu\nu}(0) \middle| 0 \right\rangle = \delta^{ab}\left\{\frac{1}{3}\left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right)\Theta^\pi(t) + \left[\Delta^\mu \Delta^\nu + \frac{\Delta^2}{3t}\left(P^\mu P^\nu - t g^{\mu\nu}\right)\right] A^\pi(t)\right\}$$

$$\text{trace part: } \left\langle \pi^a(p')\pi^b(p) \middle| \hat{T}^\mu_{\ \mu}(0) \middle| 0 \right\rangle = \delta^{ab}\Theta^\pi(t), \quad \Theta^\pi(t) = -\frac{1}{2}\left(4p_\pi^2 A^\pi(t) + 3t D^\pi(t)\right)$$

$\pi\pi$-$K\bar{K}$ coupled channels

- $\pi\pi$ phase shifts known precisely from Roy(-like) equation analyses — Bern group; Madrid-Krakow group
- Generalization to coupled channels: isoscalar, scalar $\pi\pi$-$K\bar{K}$; $f_0(500)$, $f_0(980)$ mesons
  - Unitarity relation for $\Theta^\pi(t)$ ⇒ matrix relation for coupled channels (both pion and kaon trace GFFs):

$$\mathrm{Im}\,\Theta^\pi(t) = \frac{2p_\pi}{\sqrt{t}}\left(t_0^0(t)\right)^*\Theta^\pi(t)$$

$$\mathrm{Im}\,\mathbf{\Theta}(t) = [\mathbf{T}_0^0(t)]^*\,\mathbf{\Sigma}_0^0(t)\,\mathbf{\Theta}(t), \quad \mathbf{\Theta}(t) = \begin{pmatrix} \Theta^\pi(t) \\ \frac{2}{\sqrt{3}}\Theta^K(t) \end{pmatrix}$$

phase-space factor

$$\mathbf{\Sigma}_0^0(t) \equiv \mathrm{diag}(\sigma_\pi\theta(t-t_\pi), \sigma_K\theta(t-t_K))$$

$$\text{with } \sigma_i(t) \equiv \sqrt{1-4m_i^2/t}\ (i=\pi, K)$$

$\pi\pi$-$K\bar{K}$ T-matrix

$$\mathbf{T}_0^0(t) = \begin{pmatrix} \frac{\eta_0^0(t)e^{2i\delta_0^0(t)}-1}{2i\sigma_\pi} & |g_0^0(t)|e^{i\Psi_0^0(t)} \\ |g_0^0(t)|e^{i\Psi_0^0(t)} & \frac{\eta_0^0(t)e^{2i(\Psi_0^0(t)-\delta_0^0(t))}-1}{2i\sigma_K} \end{pmatrix}$$

$$\eta_0^0(t) = \sqrt{1-4\sigma_\pi\sigma_K|g_0^0(t)|^2\theta\left(t-t_K\right)}$$

J. Donoghue, J. Gasser, H. Leutwyler, NPB 343 (1991) 341

Muskhelishvili-Omnès representation

- Single-channel: Watson's theorem ⇒ phase of FF = scattering phase shift   K.M. Watson (1952)

$$\text{disc}\, A^\pi(t) = 2iA^\pi(t)\theta(t-t_\pi)\sin\delta_2^0(t)e^{-i\delta_2^0(t)}$$

$$|t_2^0|\, e^{i\phi_2^0} = \frac{\eta_2^0 e^{2i\delta_2^0}-1}{2i\sigma_\pi}$$

- ☐ Omnès solution

$$A^\pi(t) = P_2^\pi(t)\,\Omega_2^0(t)\ , \quad \Omega_2^0(t) \equiv \exp\left\{\frac{t}{\pi}\int_{t_\pi}^{\infty}\frac{dt'}{t'}\frac{\phi_2^0(t')}{t'-t}\right\}$$

δ_2^0 replaced by the phase of $\pi\pi$ partial wave ϕ_2^0 to account for inelasticity
M. Hoferichter et al., Phys. Rept. 625 (2016) 1

- ➢ δ_2^0, η_2^0 up to $E_0 \simeq 2$ GeV from latest dispersive analysis   P. Bydžovský et al., PRD 94 (2016) 116013
- ➢ Beyond matching point E_0,   B. Moussallam, EPJC 14 (2000) 111

$$\delta_2^0(t) = \pi + \left(\delta_2^0\left(E_0^2\right)-\pi\right)\frac{2}{1+(\sqrt{t}/E_0)^3}$$

$$\eta_2^0(t) = 1 + \left(\eta_2^0\left(E_0^2\right)-\pi\right)\frac{2}{1+(\sqrt{t}/E_0)^3}$$

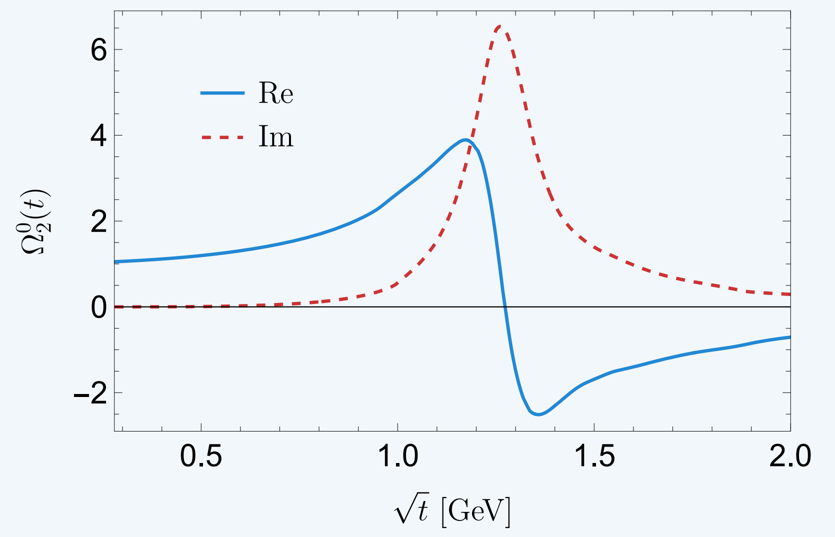
- ➢ Polynomial: $P_2^\pi(t) = 1+\alpha t$ matched to NLO ChPT

$$A^{\pi,K}(t) = 1 - \frac{2L_{12}^r}{F_\pi^2}t$$

J. Donoghue, H. Leutwyler, ZPC 52 (1991) 343

tensor-meson dominance estimate: $L_{12}^r = -\frac{F_\pi^2}{2m_{f_2}^2}$

$$P_2^\pi(t) = 1 + \left(\frac{1}{m_{f_2}^2} - \dot{\Omega}_2^0(0)\right)t \simeq 1-(0.01\ \text{GeV}^{-2})t$$



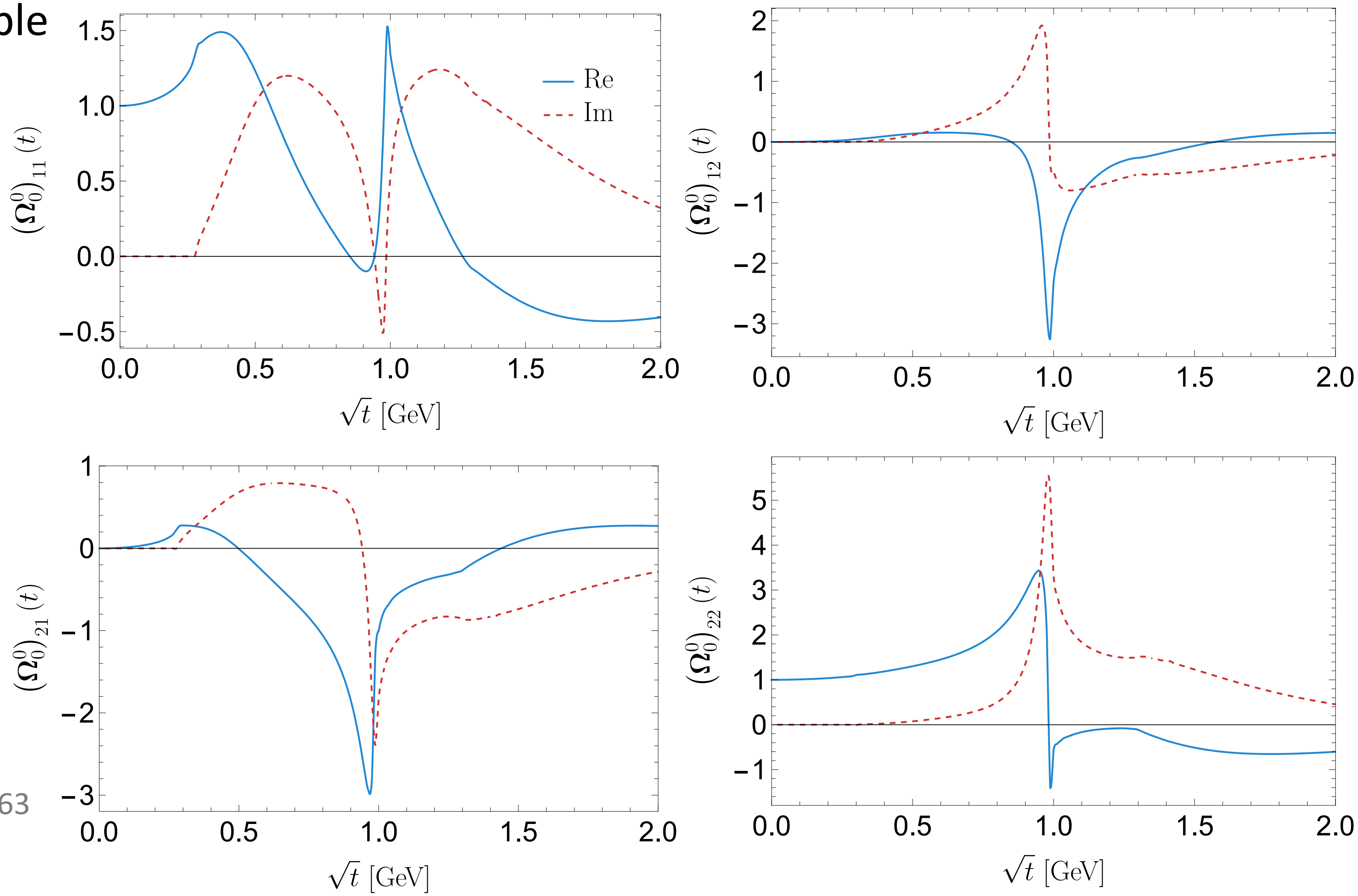
PDG average: $m_{f_2} = (1275.4 \pm 0.8)$ MeV   PDG2024
We take $m_{f_2} = (1275 \pm 20)$ MeV for a conservative error estimate

Meson dominance picture for GFFs, W. Broniowski, E. Ruiz Arriola, PRD 112 (2025) 054028

Muskhelishvili-Omnès representation

- Coupled-channel: solution known as the Muskhelishvili-Omnès (MO) representation
  - The above can be generalized to $\pi\pi$-$K\bar{K}$ coupled channels (matching point: $\sim$ 1.3 GeV)
  - Take isoscalar scalar $\pi\pi$-$K\bar{K}$ as example

$$\boldsymbol{\Omega}_0^0(t) = \frac{1}{\pi}\int_{t_\pi}^{\infty} \frac{\mathrm{d}t'}{t'-t}[\mathbf{T}_0^0(t)]^*\,\boldsymbol{\Sigma}_0^0(t)\,\boldsymbol{\Omega}_0^0(t')$$



$\pi\pi$ phase shifts: Roy equation

I. Caprini et al. (2012);

$\pi\pi \to K\bar{K}$: Roy-Steiner equation

P. Büttiker et al. (2004);

M. Hoferichter et al., JHEP 06 (2012) 063

Muskhelishvili-Omnès representation

- Pion and kaon trace GFFs:

$$[\boldsymbol{\Theta}(t)]^T = [\boldsymbol{P}_0(t)]^T\, \boldsymbol{\Omega}_0^0(t) \ , \quad \boldsymbol{P}_0(t) = \begin{pmatrix} 2m_\pi^2 + \beta_\pi t \\ \frac{2}{\sqrt{3}}\left(2m_K^2 + \beta_K t\right) \end{pmatrix}$$

$$\beta_\pi = \dot{\Theta}^\pi(0) - 2m_\pi^2 \left(\dot{\boldsymbol{\Omega}}_0^0\right)_{11}(0) - \frac{4m_K^2}{\sqrt{3}} \left(\dot{\boldsymbol{\Omega}}_0^0\right)_{12}(0) \ ,$$

$$\beta_K = \dot{\Theta}^K(0) - \sqrt{3}m_\pi^2 \left(\dot{\boldsymbol{\Omega}}_0^0\right)_{21}(0) - 2m_K^2 \left(\dot{\boldsymbol{\Omega}}_0^0\right)_{22}(0)$$

- Matching to NLO ChPT J. Donoghue, H. Leutwyler, ZPC 52 (1991) 343

$$\dot{\Theta}^\pi(0) = 1 - 4L_{12}^r \frac{m_\pi^2}{F_\pi^2} - 24\left(L_{11}^r - L_{13}^r\right)\frac{m_\pi^2}{F_\pi^2} - \frac{3}{2}\frac{m_\pi^2}{F_\pi^2} I_\pi + \frac{m_\pi^2}{2F_\pi^2} I_\eta = 0.98(2) \ ,$$

$$\dot{\Theta}^K(0) = 1 - 4L_{12}^r \frac{m_K^2}{F_\pi^2} - 24\left(L_{11}^r - L_{13}^r\right)\frac{m_K^2}{F_\pi^2} - \frac{m_K^2}{F_\pi^2} I_\eta = 0.94(14)$$

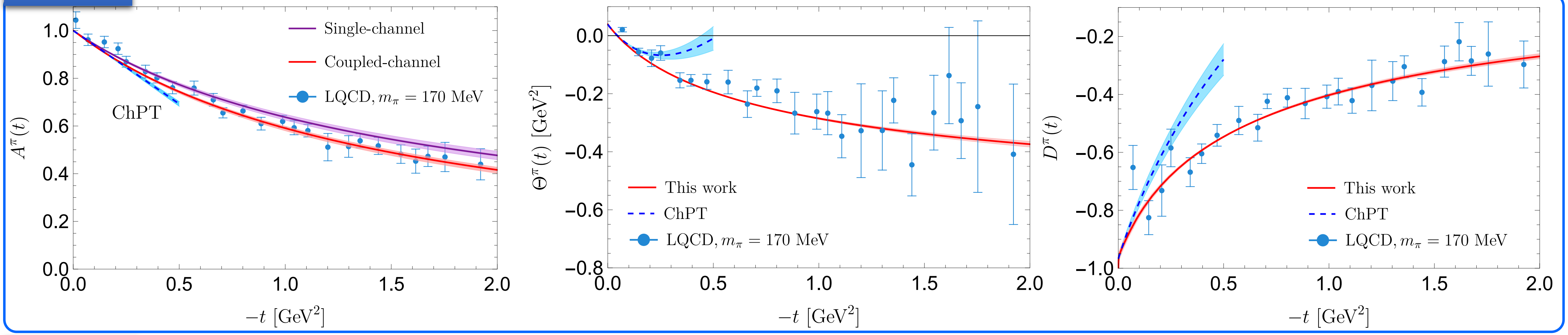
Chiral logs: $I_i = \frac{1}{48\pi^2}\left(\ln\left(\frac{\mu^2}{m_i^2}\right) - 1\right)$

- Similar coupled-channel analysis for D-wave $\pi\pi$-$K\bar{K}$ $\Rightarrow$ coupled-channel results for A^π, A^K and D^π, D^K

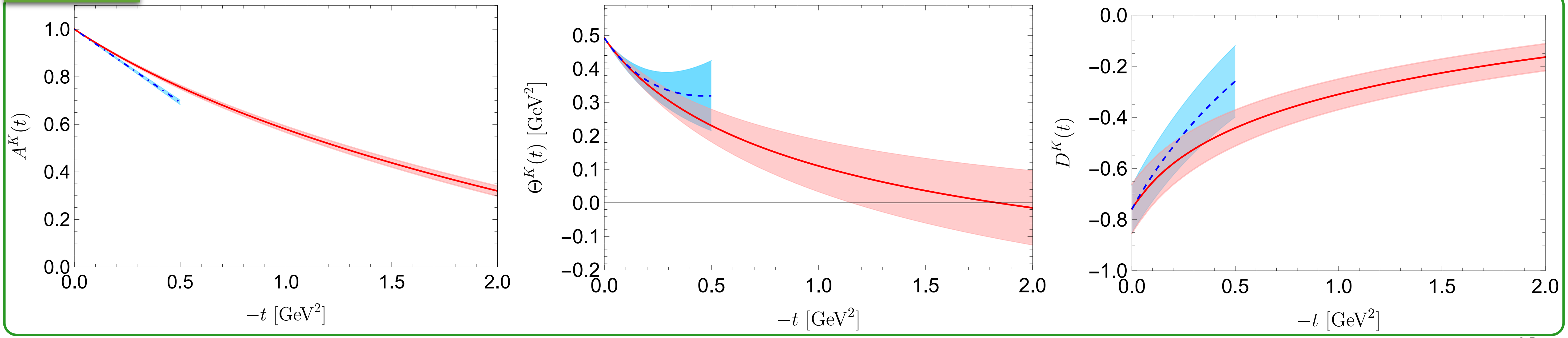
Pion and kaon GFFs **Prediction, NOT fit**

➢ Pion

LQCD ($m_\pi = 170$ MeV): D.C. Hackett et al., PRL 132 (2024) 251904

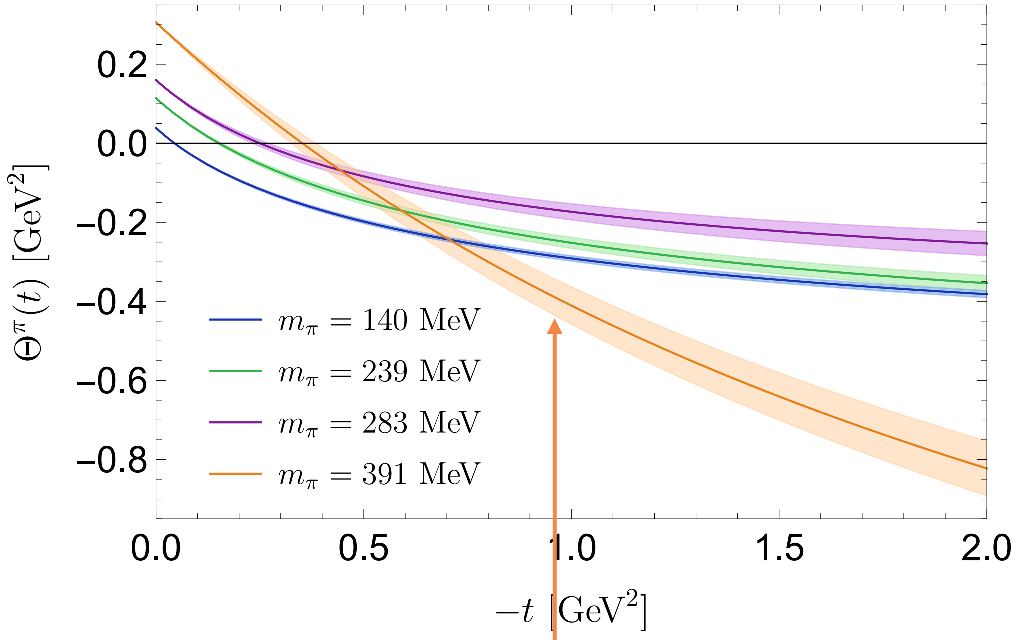


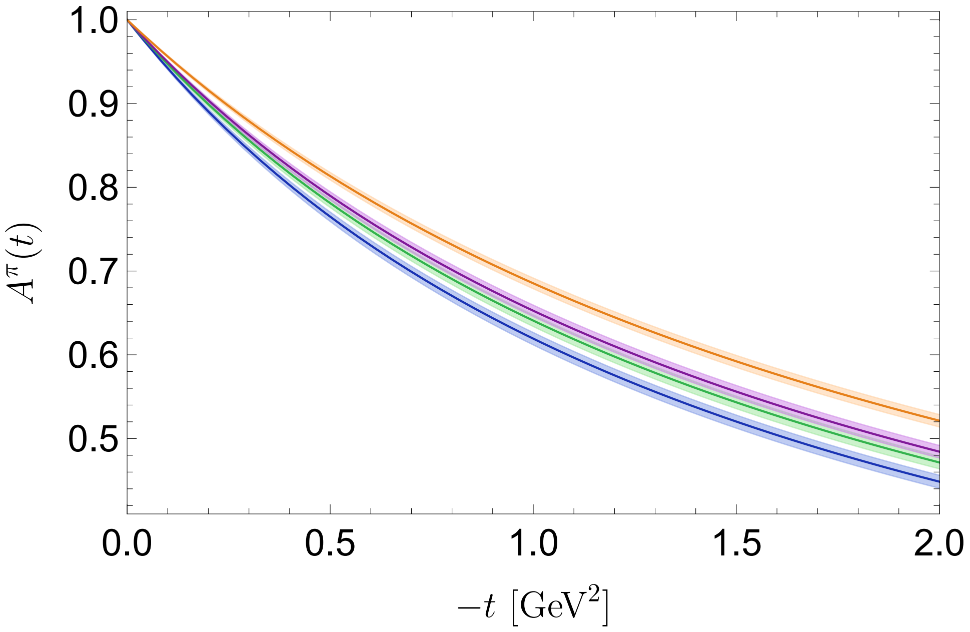
➢ Kaon



Pion GFFs: pion mass dependence

- Using the $\pi\pi$ scattering phase shifts at unphysical pion masses (239 MeV, 283 MeV, 391 MeV) obtained from Roy equation analyses X.-H. Cao et al., PRD 108 (2023) 034009; A. Rodas et al., PRD 109 (2024) 034513





Fast change before $m_\pi = 391$ MeV:

- σ meson becomes a $\pi\pi$ bound state from lattice HadSpec, PRL 118 (2017) 022002

- Singularity in quark-mass dependence at such critical point:

FKG, C. Hanhart, F. Llanes-Estrada, U.-G. Meißner, PLB 703 (2011) 510;
FKG, U.-G. Meißner, PRL 109 (2012) 062001

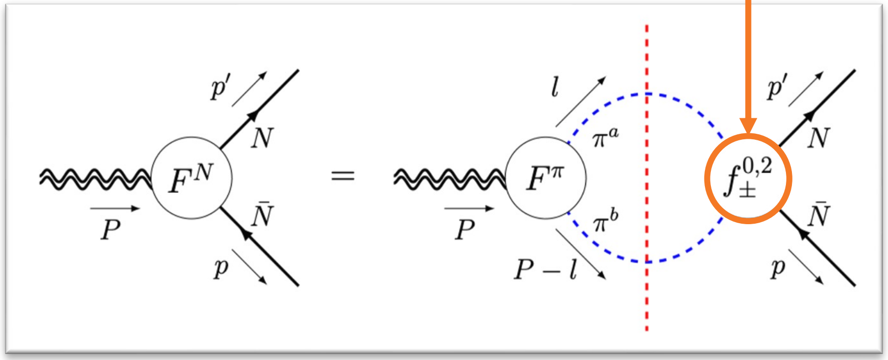
Unitarity relation for nucleon GFFs

- Discontinuity:

$$\mathrm{Disc}\left\langle N(p')\bar{N}(p)\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle$$

$$\propto \sum_n \langle N(p')\bar{N}(p)|\, n\rangle \left\langle n\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle^* \delta^4(p+p'-p_n)$$

$\pi N, \pi\pi/K\bar{K} \to N\bar{N}$ scattering



- In the region $t \in (4M_\pi^2, 16M_\pi^2)$, only $\pi\pi$ intermediate state

$$\mathrm{Disc}\left\langle N(p')\bar{N}(p)\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle$$

$$= \frac{1}{4m_N}\bar{u}(p')\left[\mathrm{Disc}\,\hat{A}(t)\Delta^\mu\Delta^\nu + \mathrm{Disc}\,\hat{J}(t)\left(i\Delta^{\{\mu}\sigma^{\nu\}\rho}P_\rho\right) + \mathrm{Disc}\,\hat{D}(t)\left(P^\mu P^\nu - t g^{\mu\nu}\right)\right]v(p)$$

$$= \frac{1}{2}\frac{i}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int \mathrm{d}\Omega_l \langle N(p')\bar{N}(p)|\,\pi^a(l)\pi^b(P-l)\rangle \left\langle \pi^a(l)\pi^b(P-l)\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle^*$$

$$= \frac{1}{2}\frac{i}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int \mathrm{d}\Omega_l\, \bar{u}(p')\Bigg[\underbrace{\delta^{ab}\mathbb{1}\left(A^+ + \frac{(\not{P}-2\not{l})}{2}B^+\right)}_{\text{Isospin-even}} + \underbrace{i\epsilon_{bac}\tau^c\left(A^- + \frac{(\not{P}-2\not{l})}{2}B^-\right)}_{\text{Isospin-odd}}\Bigg]v(p)$$

$$\times \frac{\delta^{ab}}{2}\left[(A^\pi(t))^*(2l-P)^\mu(2l-P)^\nu + (D^\pi(t))^*\left(P^\mu P^\nu - t g^{\mu\nu}\right)\right]$$

$$= \frac{1}{2}\frac{i}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int \mathrm{d}\Omega_l\, \bar{u}(p')\frac{3}{2}\left(A^+ + \frac{(\not{P}-2\not{l})}{2}B^+\right)v(p)\left[(A^\pi(t))^*(2l-P)^\mu(2l-P)^\nu + (D^\pi(t))^*\left(P^\mu P^\nu - t g^{\mu\nu}\right)\right]$$

$A^\pm, B^\pm$: Lorentz invariant πN scattering amplitudes G. Höhler (1983)

$\pi\pi \to N\bar{N}$ amplitudes

- Partial-wave amplitudes for $\pi\pi \to N\bar{N}$

W. Frazer, J. Fulco (1960); G. Höhler (1983)

$$A^I(t,s) = -\frac{8\pi}{p_N^2}\sum_{J=0}^{\infty}\left(J+\frac{1}{2}\right)(p_\pi p_N)^J\left\{P_J(\cos\theta)f_+^J(t) - \frac{m_N\cos\theta}{\sqrt{J(J+1)}}P_J'(\cos\theta)f_-^J(t)\right\},$$

$$B^I(t,s) = 8\pi\sum_J \frac{J+\frac{1}{2}}{\sqrt{J(J+1)}}(p_\pi p_N)^{J-1}P_J'(\cos\theta)\,f_-^J(t)$$

I =+/− for even/odd J;
$f_\pm^J$: $\pi\pi \to N\bar{N}$ partial-wave amp. with
+/− for parallel/antiparallel $N\bar{N}$ helicities

- Discontinuity of the nucleon GFFs

$$\mathrm{Im}\,A^s(t) = \frac{3p_\pi^5}{\sqrt{6t}}\left[f_-^2(t) + \sqrt{\frac{3}{2}}\frac{m_N}{p_N^2}\left(m_N\sqrt{\frac{2}{3}}f_-^2(t) - f_+^2(t)\right)\right]^* A^\pi(t)$$

$$\mathrm{Im}\,J^s(t) = \frac{3p_\pi^5}{2\sqrt{6t}}\left(f_-^2(t)\right)^* A^\pi(t),$$

$$\Rightarrow \mathrm{Im}\,\Theta^s(t) = -\frac{3p_\pi}{4p_N^2\sqrt{t}}\left(f_+^0(t)\right)^*\Theta^\pi(t)$$

$$\mathrm{Im}\,D^s(t) = -\frac{3m_N p_\pi}{2p_N^2\sqrt{t}}\left[\frac{4p_\pi^2}{3t}\left(\left(f_+^0(t)\right)^* - (p_\pi p_N)^2\left(f_+^2(t)\right)^*\right)A^\pi(t) + \left(f_+^0(t)\right)^* D^\pi(t)\right]$$

coupled-channel

$$\mathrm{Im}\,\Theta^s(t) = -\frac{3}{4p_N^2\sqrt{t}}\left[p_\pi\left(f_+^0(t)\right)^*\Theta^\pi(t)\theta(t-t_\pi) + \frac{4}{3}p_K\left(h_+^0(t)\right)^*\Theta^K(t)\theta(t-t_K)\right]$$

$K\bar{K} \to N\bar{N}$ amplitude

- Decomposition into $J^{PC} = 0^{++}, 2^{++}$ matrix elements

$$\left\langle N(p')\bar{N}(p)\left|\hat{T}^{\mu\nu}(0)\right|0\right\rangle = \bar{u}(p')\left(T_S^{\mu\nu} + T_T^{\mu\nu}\right)v(p)$$

$$T_S^{\mu\nu} = \frac{1}{3}\left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right)\Theta^s(t)\,, \qquad \Theta^s(t) = \frac{1}{4m_N}\left[-4p_N^2A^s(t) + 2tJ^s(t) - 3tD^s(t)\right]$$

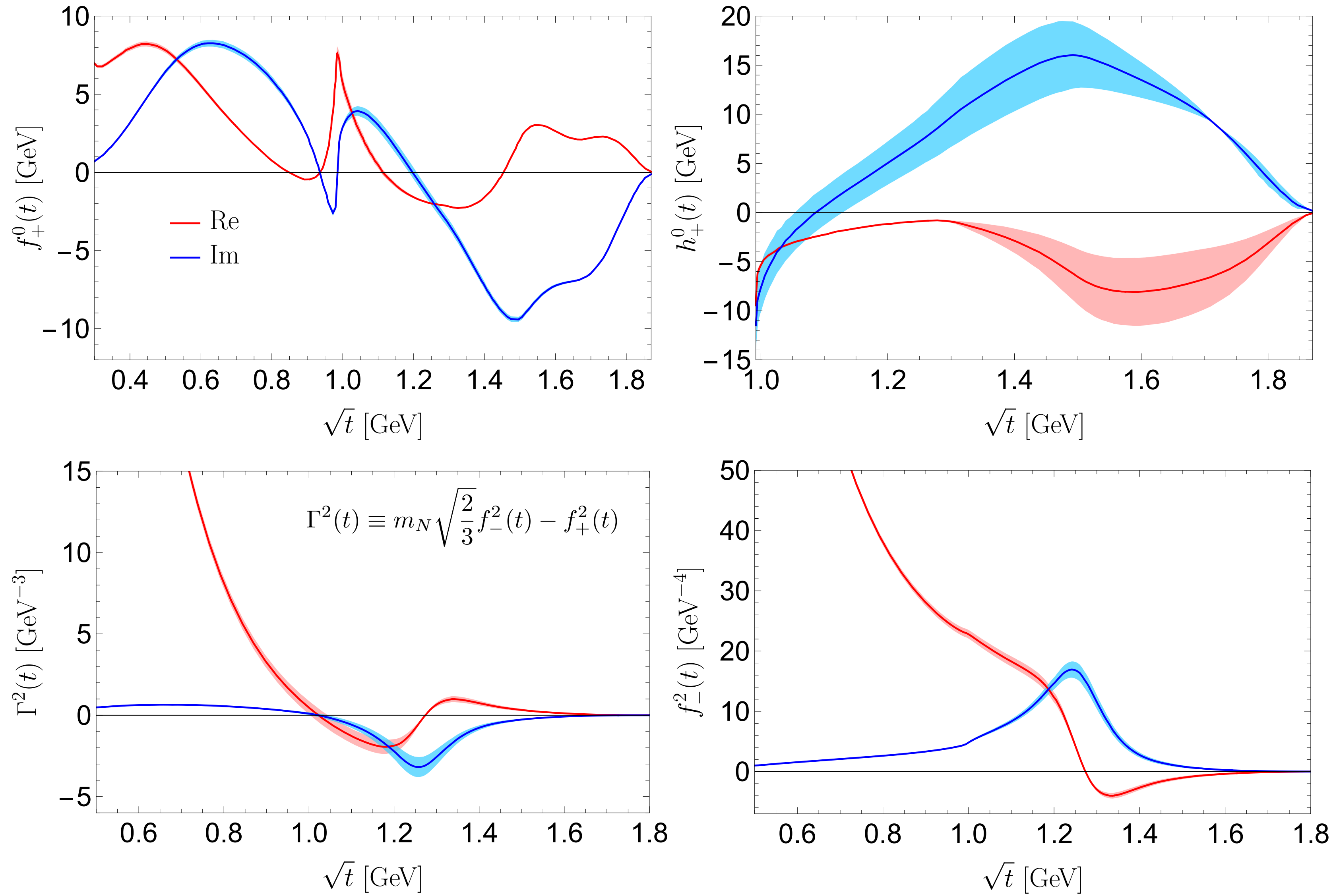
$$T_T^{\mu\nu} = \frac{1}{4m_N}\left[\Delta^\mu\Delta^\nu + \frac{\Delta^2}{3t}\left(P^\mu P^\nu - tg^{\mu\nu}\right)\right]A^s(t) + \left[i\Delta^{\{\mu}\sigma^{\nu\}\rho}P_\rho + \frac{2i\sigma^{\rho\kappa}\Delta_\rho P_\kappa}{3t}\left(P^\mu P^\nu - tg^{\mu\nu}\right)\right]J^s(t)$$

$\pi\pi/K\bar{K} \to N\bar{N}$ S-wave amplitudes

G.E. Hite, F. Steiner (1973)

- Inputs: $\pi\pi/K\bar{K} \to N\bar{N}$ S-wave amplitudes $f_\pm^{0,2}, h_+^0$ from Roy-Steiner equation analyses

M. Hoferichter et al., Phys. Rept. 625 (2016) 1; PLB 853 (2024) 138698; X.-H. Cao et al., JHEP 12 (2022) 073



Nucleon GFFs

G.E. Hite, F. Steiner (1973)

- Inputs: $\pi\pi/K\bar{K} \to N\bar{N}$ S-wave amplitudes $f_{\pm}^{0,2}, h_{+}^{0}$ from Roy-Steiner equation analyses

M. Hoferichter et al., Phys. Rept. 625 (2016) 1; PLB 853 (2024) 138698; X.-H. Cao et al., JHEP 12 (2022) 073

- Dispersive relations for the nucleon GFFs

$$(A, J, \Theta)(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}(A, J, \Theta)(t')}{t'-t}$$

  - Normalization ⇒ sum rules

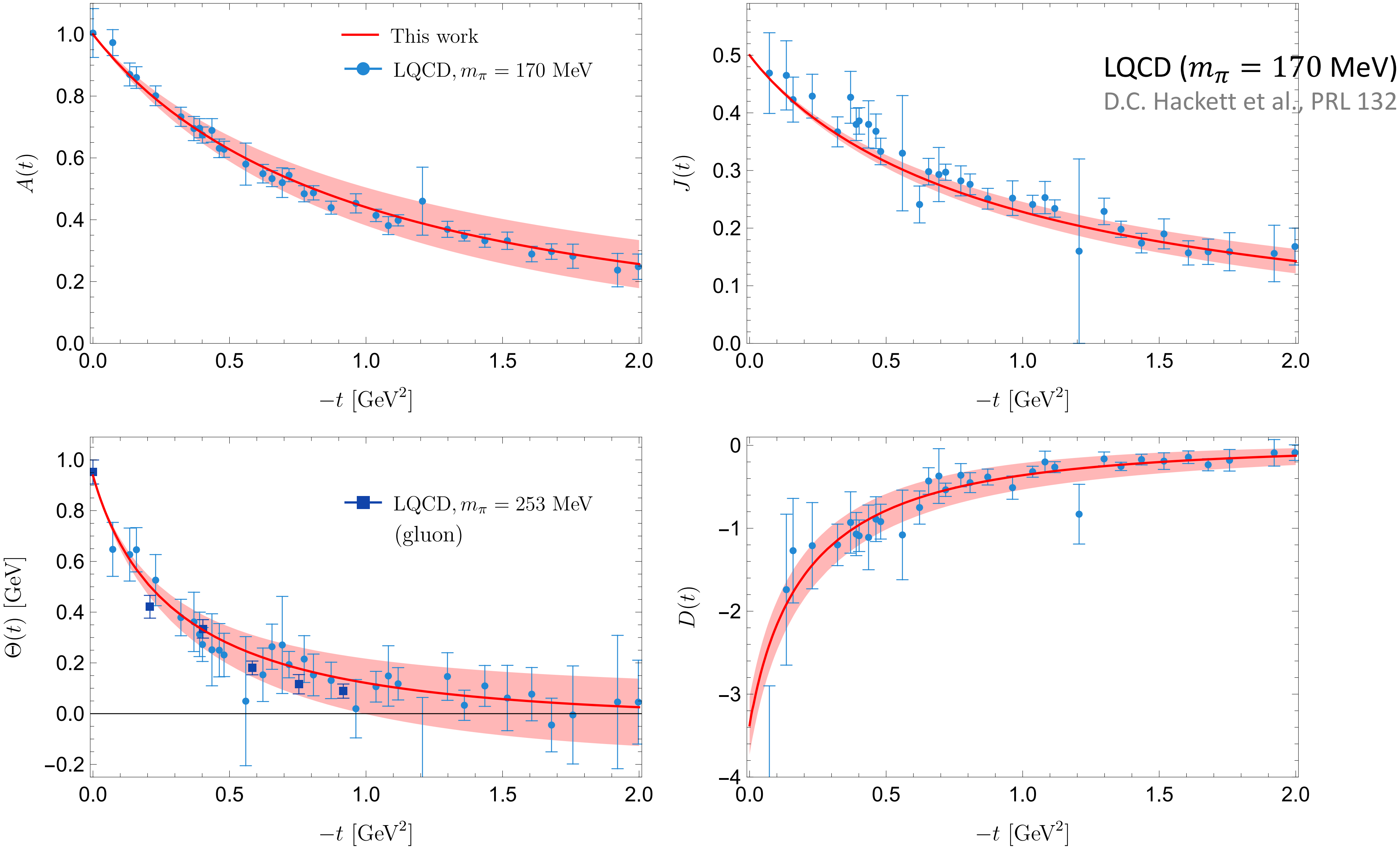
$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}(A, J, \Theta)(t')}{t'} = \left(1, \frac{1}{2}, m_N\right)$$

  - Introduce S-wave (0^{++}) and D-wave (2^{++}) poles to the spectral functions: $\pi c_{S,D} m_{S,D}^2 \delta\left(t - m_{S,D}^2\right)$ to satisfy the sum rules
    - 0^{++}: $m_S \in (1.5, 1.8)$ GeV to cover $f_0(1500)$ and $f_0(1710)$
    - 2^{++}: $m_D \in (1.5, 2.2)$ GeV to cover $f_2(1565), f_2(1950)$ and $f_2(2010)$

error estimate

M.A. Belushkin et al., PRC 75 (2007) 035202; M. Hoferichter et al., EPJA 52 (2016) 331; J.M. Alarcón, C. Weiss, PLB 784 (2018) 373

Nucleon GFFs: results **Prediction, NOT fit**



LQCD ($m_\pi = 170$ MeV):
D.C. Hackett et al., PRL 132 (2024) 251904

LQCD ($m_\pi = 253$ MeV), gluon part only:
B. Wang et al. [χQCD], PRD 109 (2024) 094504

Nucleon GFFs: results

- D-term: $D \equiv D(0)$
- Various radii in the Breit frame
  - From the trace FF (scalar radius):

$$\langle r_\Theta^2 \rangle = \frac{6\dot{\Theta}(0)}{m_N} = 6\dot{A}(0) - \frac{9D}{2m_N^2}$$

  - Radius of the energy density (mass radius):

$$\langle r_{\rm Mass}^2 \rangle = 6\dot{A}(0) - \frac{3D}{2m_N^2}$$

  - Mechanical radius: M. Polyakov, PLB 555 (2003) 57; M. Polyakov, P. Schweitzer, IJMPA 33 (2018) 183005; C. Lorcé et al., EPJC 79 (2019) 89

$$\langle r_{\rm Mech}^2 \rangle = \frac{6D}{\int_{-\infty}^{0} \mathrm{d}t\ D(t)}$$

  - Radius of the density $J(t) + \frac{2}{3} t \frac{\mathrm{d}J(t)}{\mathrm{d}t}$:

$$\langle r_J^2 \rangle = 20 J'(0)$$

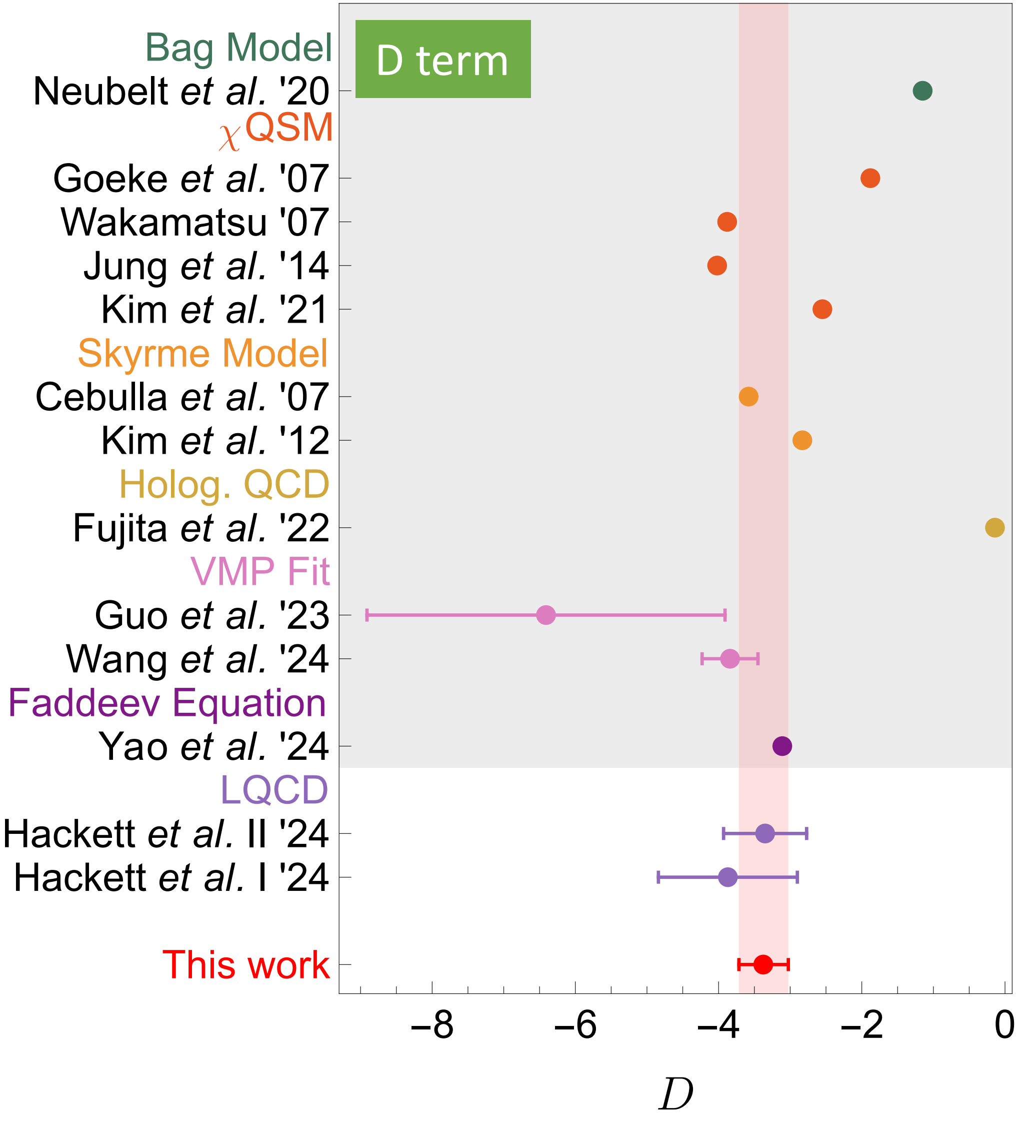
M. Polyakov, PLB 555 (2003) 57; C. Lorcé et al., PLB 776 (2018) 38

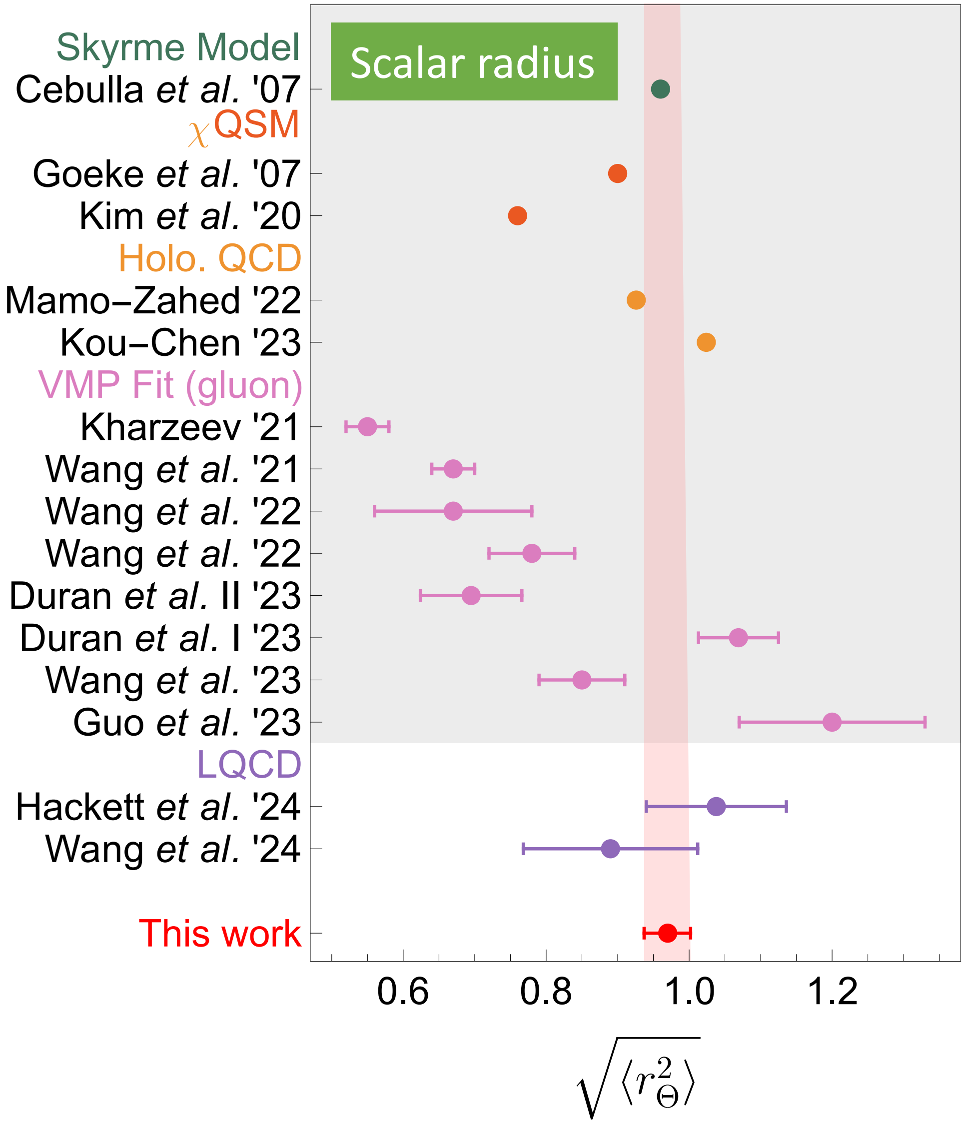
Quantity	Result	Error budget
D-term	$-3.38^{+0.34}_{-0.35}$	$+(0.18)_{\rm ChPT}(0.12)_{\rm pwa}(0.26)_{\rm eff}$
		$-(0.16)_{\rm ChPT}(0.12)_{\rm pwa}(0.29)_{\rm eff}$
$\sqrt{\langle r_\Theta^2 \rangle}$ [fm]	$0.97^{+0.03}_{-0.03}$	$+(0.01)_{\rm ChPT}(0.01)_{\rm pwa}(0.03)_{\rm eff}$
		$-(0.02)_{\rm ChPT}(0.01)_{\rm pwa}(0.02)_{\rm eff}$
$\sqrt{\langle r_{\rm Mass}^2 \rangle}$ [fm]	$0.70^{+0.03}_{-0.04}$	$+(0.02)_{\rm ChPT}(0.01)_{\rm pwa}(0.02)_{\rm eff}$
		$-(0.02)_{\rm ChPT}(0.01)_{\rm pwa}(0.03)_{\rm eff}$
$\sqrt{\langle r_{\rm Mech}^2 \rangle}$ [fm]	$0.72^{+0.09}_{-0.08}$	$+(0.02)_{\rm ChPT}(0.00)_{\rm pwa}(0.09)_{\rm eff}$
		$-(0.03)_{\rm ChPT}(0.01)_{\rm pwa}(0.07)_{\rm eff}$
$\sqrt{\langle r_J^2 \rangle}$ [fm]	$0.70^{+0.02}_{-0.02}$	$+(0.01)_{\rm ChPT}(0.01)_{\rm pwa}(0.01)_{\rm eff}$
		$-(0.01)_{\rm ChPT}(0.00)_{\rm pwa}(0.02)_{\rm eff}$

- ChPT: NLO ChPT inputs
- pwa: $\pi\pi/K\bar{K} \to N\bar{N}$
- eff: effective poles m_S, m_D

Nucleon GFFs: results

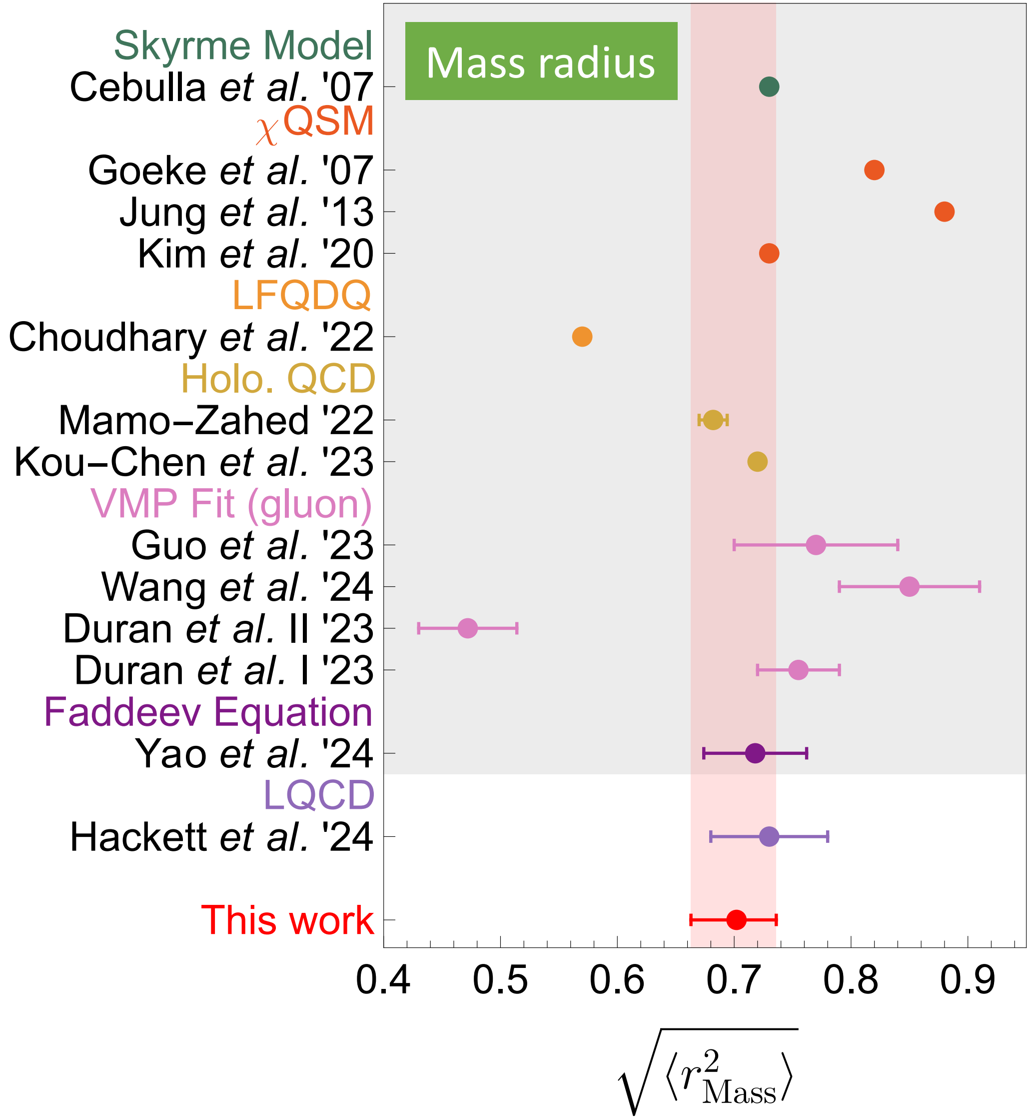
- Comparison with other results

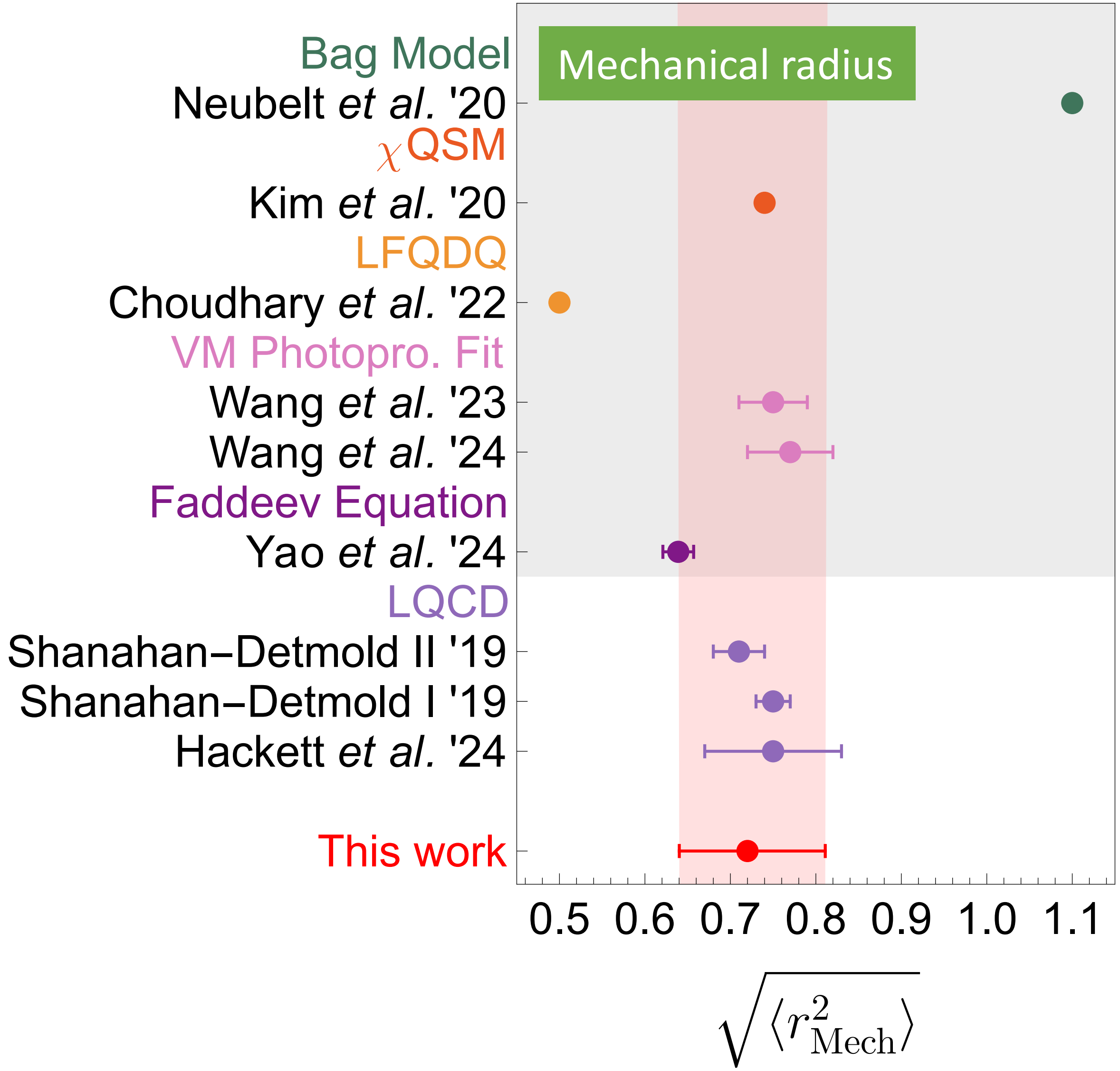




Nucleon GFFs: results

- Comparison with other results



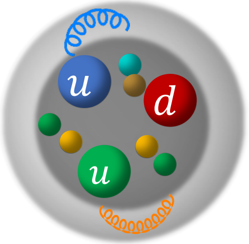


Summary and outlook

- The pion, kaon and nucleon GFFs are precisely determined using dispersive method with inputs:
  - Coupled-channel $\pi\pi$-$K\bar{K}$, $\pi\pi/K\bar{K} \to N\bar{N}$ amplitudes
  - Low energy: NLO ChPT with LECs estimated using resonance saturation, improvable with lattice calculations
  - High energy: highly excited meson resonances

- $D^N = -3.38^{+0.34}_{-0.35}$, $\underbrace{\sqrt{\langle r_\Theta^2 \rangle} = 0.97^{+0.03}_{-0.03}\ \mathrm{fm}}_{\text{May be regarded as a confinement radius}} > \underbrace{\sqrt{\langle r_{E,p}^2 \rangle} \simeq 0.84\ \mathrm{fm}}_{\text{proton charge radius}} > \sqrt{\langle r_{\mathrm{Mass}}^2 \rangle} = 0.70^{+0.03}_{-0.04}\ \mathrm{fm}$

  Bag radius in MIT bag model X. Ji, Front. Phys. (Beijing) 16 (2021) 64601



  - ➢ Gluons distributed over a larger region than (anti-)quarks
  - ➢ Foundation for further analyses, interpretations, etc.

- More results to come. Outlook:
  - Pion mass dependence
  - Extension to hyperons

Thank you for your attention!

πN amplitudes

- Amplitudes for $\pi^a(q) + N(p) \to \pi^a(q') + N(p')$

$$T_{\pi N}^{a'a}(s,t,u) = \chi_{N'}^{\dagger}\left\{\delta_{a'a}T^{+}(s,t,u) + \frac{1}{2}\left[\tau_{a'},\tau_a\right]T^{-}(s,t,u)\right\}\chi_N$$ with isospinors $\chi_N, \chi_{N'}$

- Lorentz and isospin decompositions:

$$T^{\pm}(s,t,u) = \bar{u}^{(s')}(p')\left\{A^{\pm}(s,t,u) + \frac{1}{2}\left(\not{q}' + \not{q}\right)B^{\pm}(s,t,u)\right\}u^{(s)}(p)$$

$$A^{I=1/2} = A^{+} + 2A^{-}, \quad A^{I=3/2} = A^{+} - A^{-}$$

Spatial density profiles

- Consider various densities:

M. Polyakov, PLB 555 (2003) 57; M. Polyakov, P. Schweitzer, IJMPA 33 (2018) 183005;
C. Lorcé et al., EPJC 79 (2019) 89; C. Lorcé et al., PLB 776 (2018) 38;
X. Ji, Front. Phys. (Beijing) 16 (2021) 64601; D.E. Kharzeev, PRD 104 (2021) 054015; …

$$\rho_\Theta(r) = m_N \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \left[A(t) - \frac{t}{4m_N^2}\left[A(t) - 2J(t) + 3D(t)\right]\right] = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \Theta(t),$$

$$\rho_{\text{Ener.}}(r) = m_N \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \left[A(t) - \frac{t}{4m_N^2}\left[A(t) - 2J(t) + D(t)\right]\right] = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \left[\Theta(t) + \frac{t}{2m_N} D(t)\right],$$

$$\rho_J(r) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \left[J(t) + \frac{2}{3} t \frac{\mathrm{d}}{\mathrm{d}t} J(t)\right],$$

$$p_r(r) = m_N \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \left[-\frac{1}{r^2}\frac{1}{\sqrt{t m_N^2}}\frac{\mathrm{d}}{\mathrm{d}t} t^{\frac{3}{2}} D(t)\right] \equiv p(r) + \frac{2}{3}s(r), \qquad p_t(r) \equiv p(r) - \frac{1}{3}s(r)$$

$$p(r) = \frac{1}{6m_N}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r} r^2 \frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r), \quad s(r) = -\frac{1}{4m_N} r \frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r) \qquad \tilde{D}(r) \equiv \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} D(t)$$

Zero-average-momentum frame (ZAMF) E. Epelbaum et al., PRL 129 (2022) 012001; J.Y. Panteleeva et al., EPJC 83 (2023) 617; …

$$\rho_{\text{Ener}}^{\text{ZAMF}}(r) = \frac{m_N}{4\pi r}\int_0^\infty \mathrm{d}\Delta\, \Delta \sin(\Delta r) \int_{-1}^{1} \mathrm{d}\alpha\, A\left[(\alpha^2 - 1)\Delta^2\right]$$

$$\rho_{\text{Ener}}^{\text{naive}}(r) = \lim_{m_N\to\infty} \rho_{\text{Ener.}}(r) = m_N \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} A(t)$$

Spatial density profiles