Pushing the Limits of Fluctuating Hydrodynamics

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中国高能核物理网络论坛

HIGH ENERGY NUCLEAR PHYSICS IN CHINA

Content

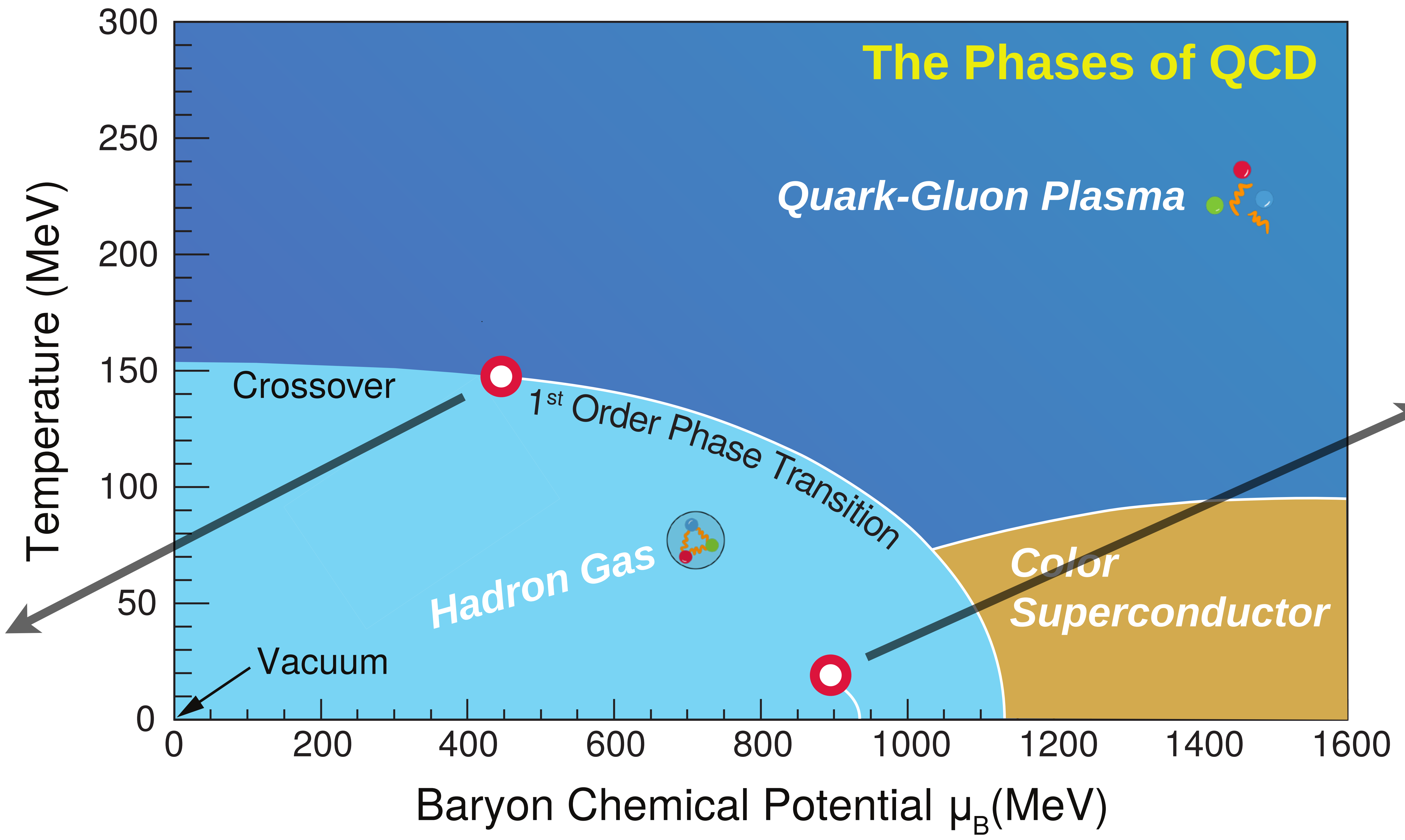
- **Introduction and review**
  - Experiment
  - Theory
- **Pushing fluctuating hydrodynamics to**
  - Full hydrodynamics: SO(3) covariant formalism
  - Far-from-equilibrium regime: U(1) diffusion
- **Recap and outlook**

Introduction and review

Motivation I: understanding QCD phases

- Heavy-ion collisions → QCD phase diagram.

QCD critical point
(hadron-QGP)
TBA



Nuclear critical point
(nuclear liquid-gas)
Pochodzalla et al, 1995

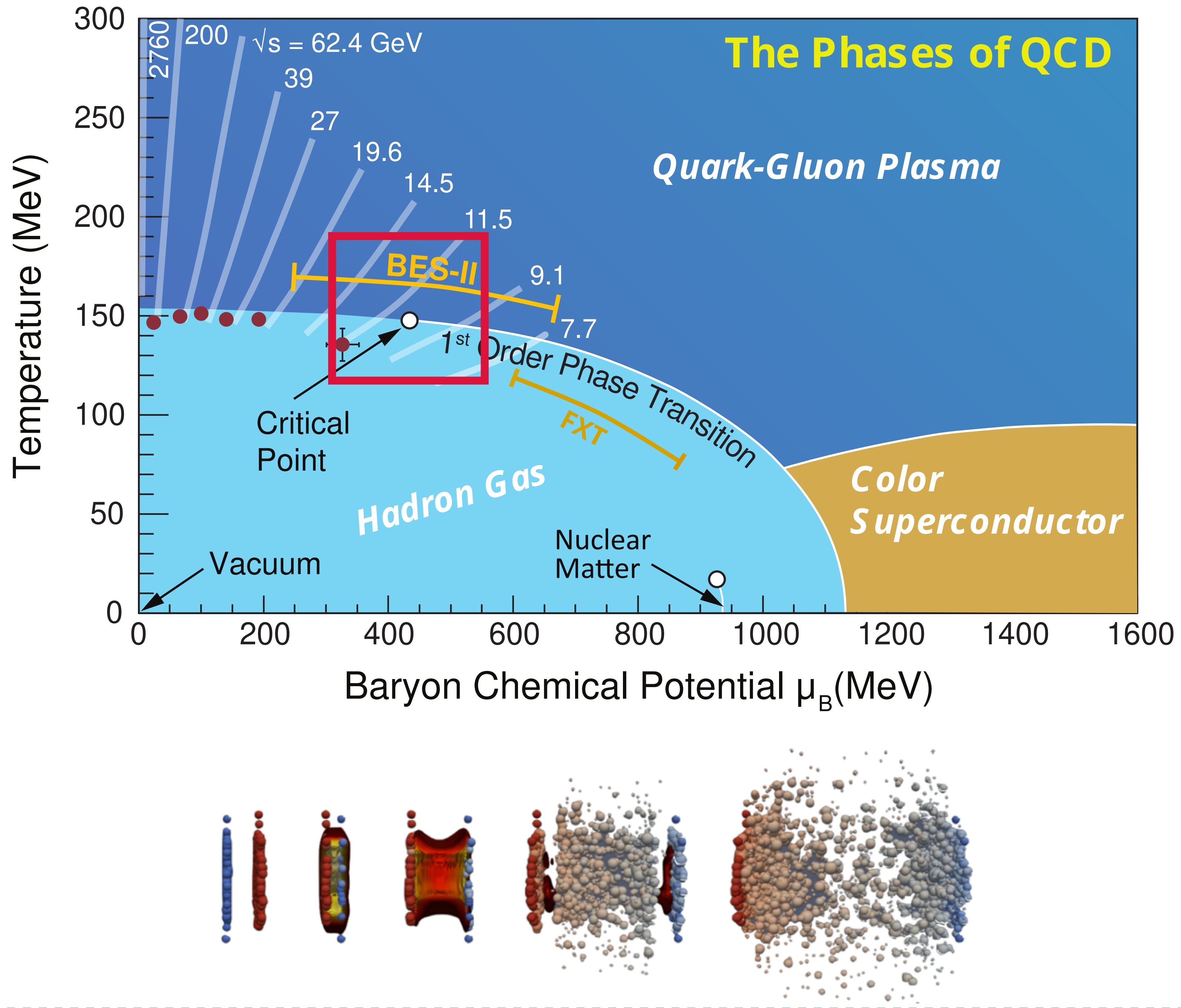
- Understanding the finite temperature and density regime is challenging (sign problem).
- **A simpler task**: search the landmark (critical point) where singularity occurs.

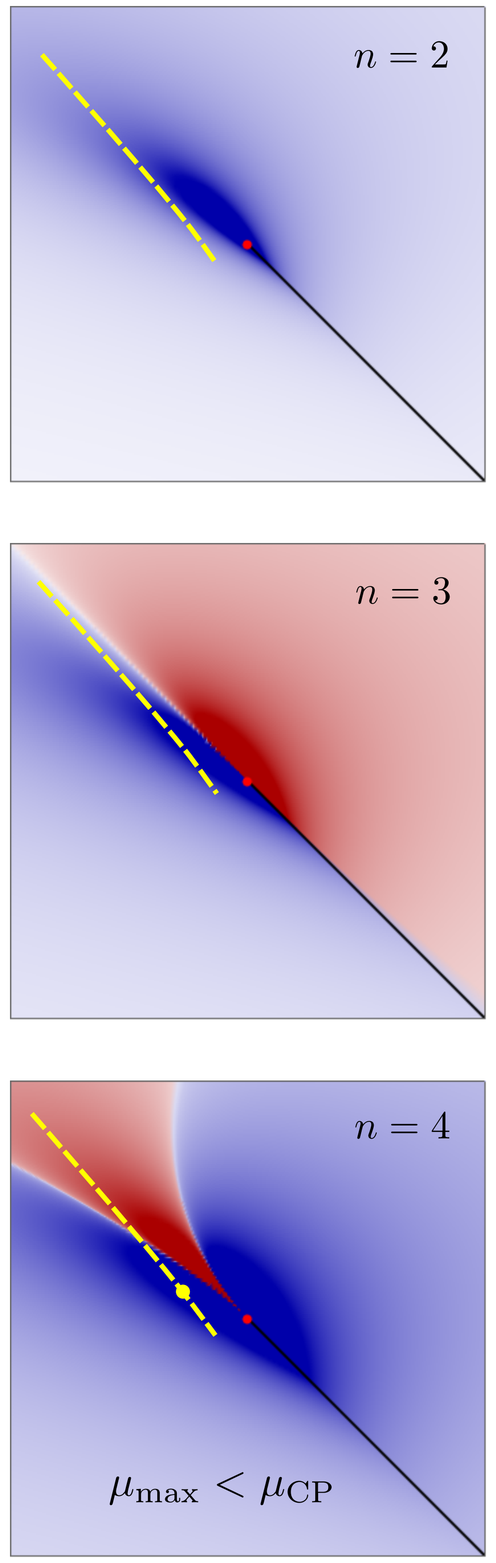
Motivation I: understanding QCD phases

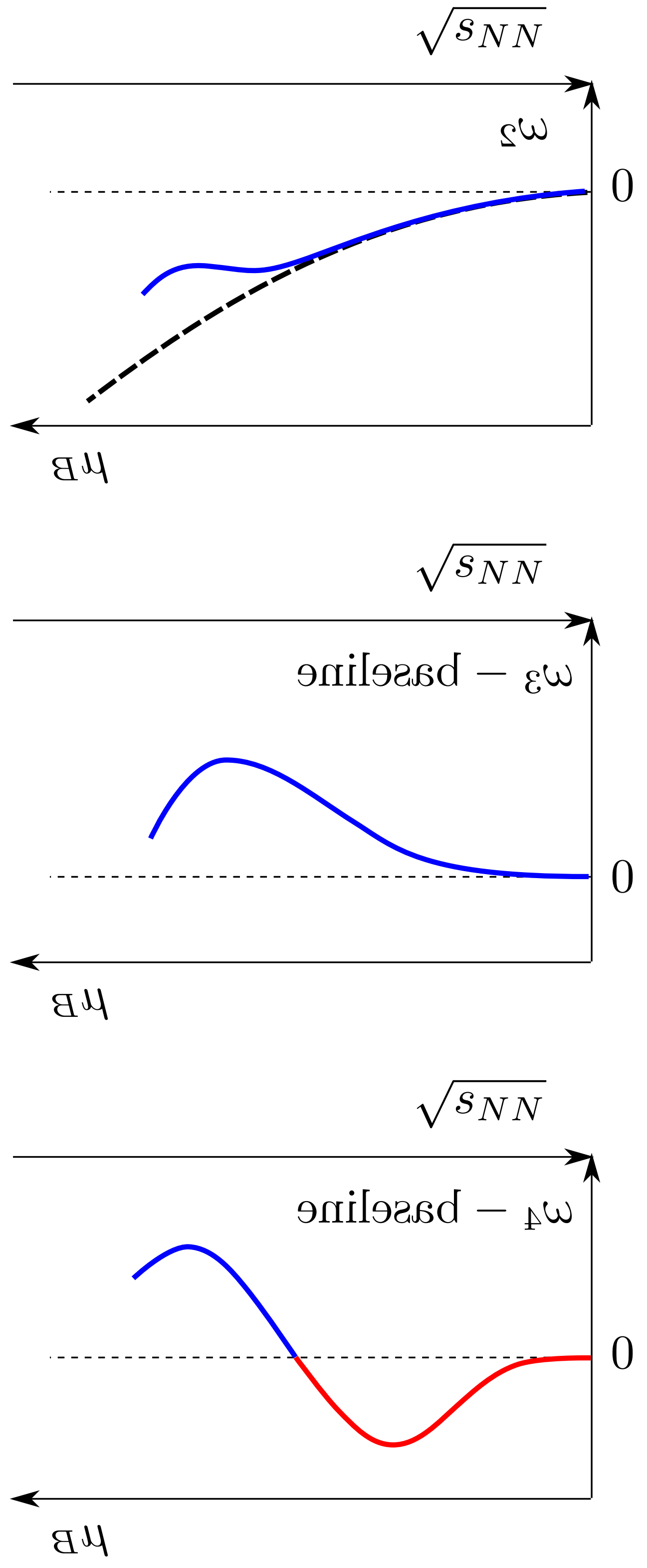
- **RHIC BES**-II data seem to advocate the intriguing hint (non-monotonicity) of QCD critical point based on equilibrium assumption at qualitative level.

STAR, 2504.00817; Stephanov, 2410.02861

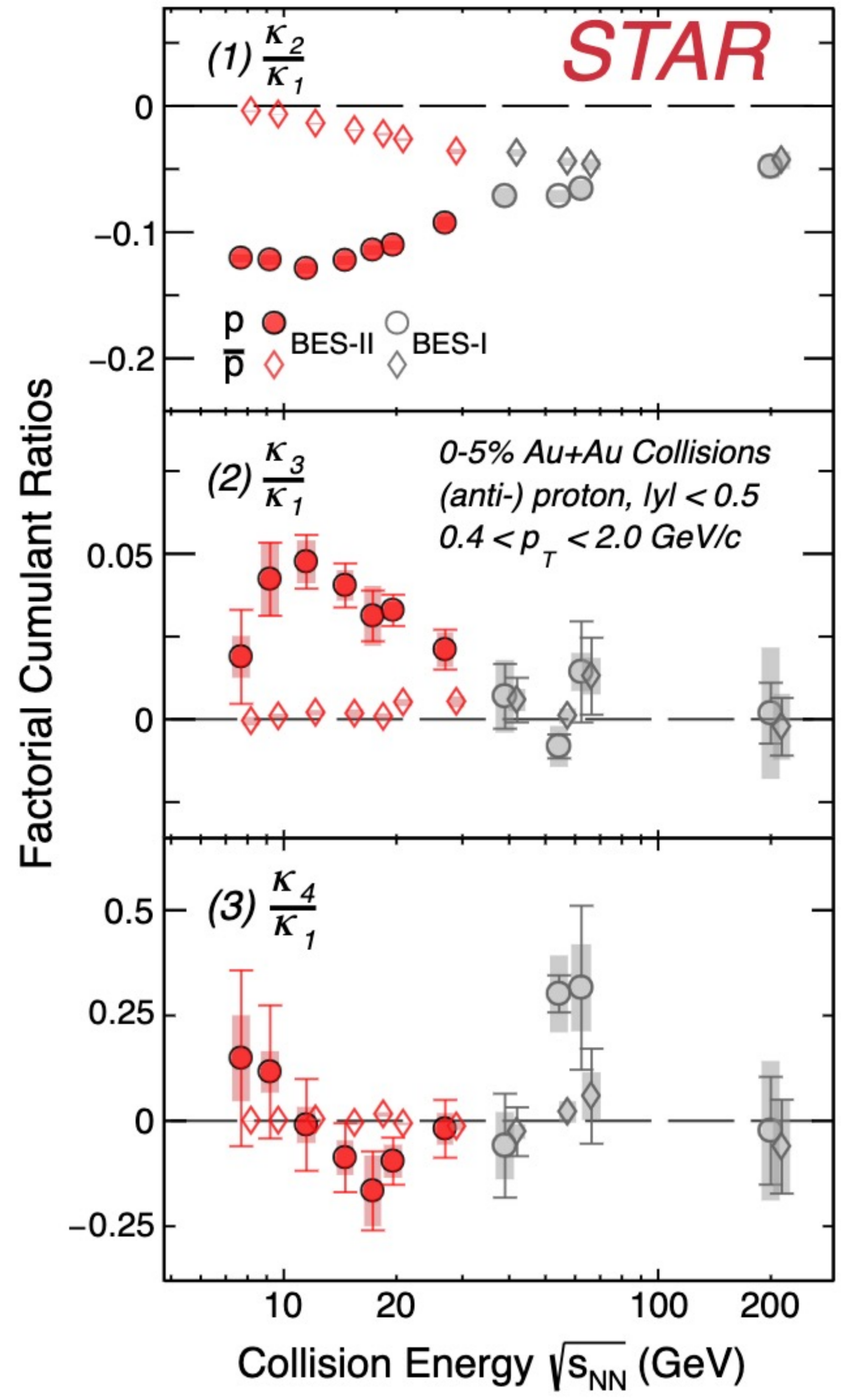
theory (equilibrium)







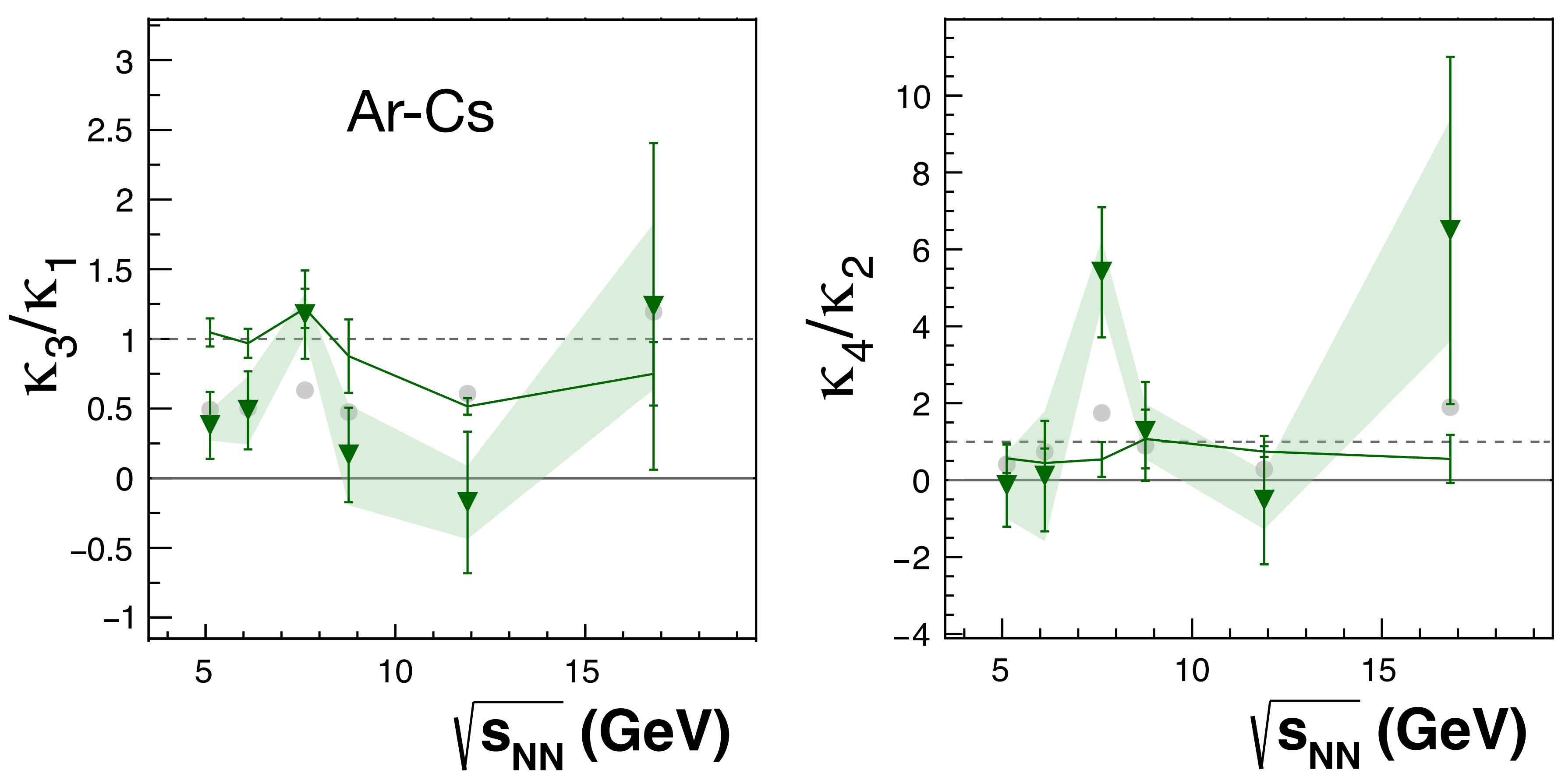
experiment



Motivation I: understanding QCD phases

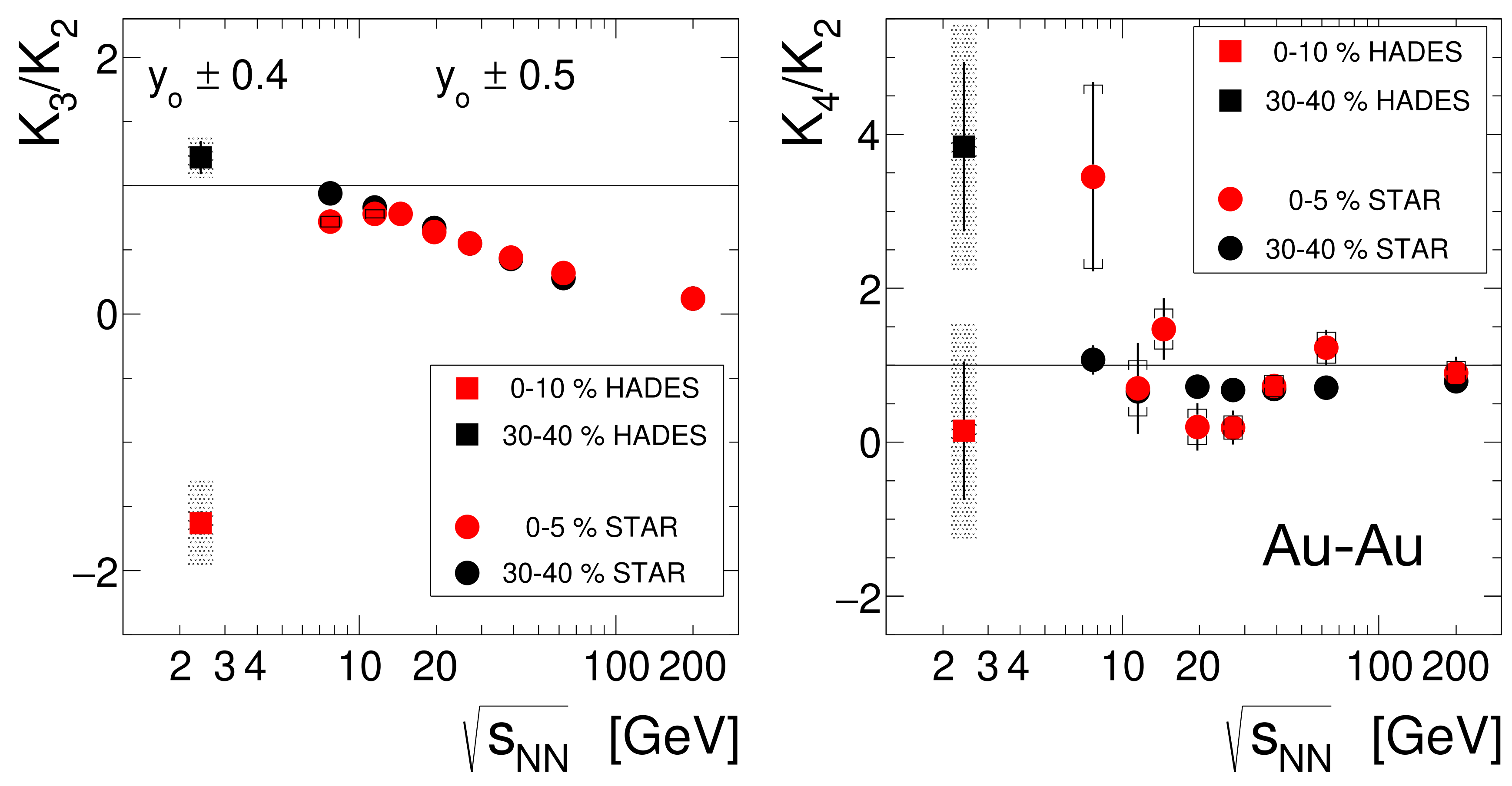
- **NA61/SHINE at CERN SPS**

NA61/SHINE, 2503.22484; CERN 2024 report



- **HADES at GSI**

HADES 2002.08701



Status: "No clear critical point signatures either at SPS/GSI energies" †

Next: "dynamical model calculations including the criticality needed to fully understand the data" †

Freezeout: Karthein et al, 2508.19237; etc

Others: spin correlations; light nuclei production; etc

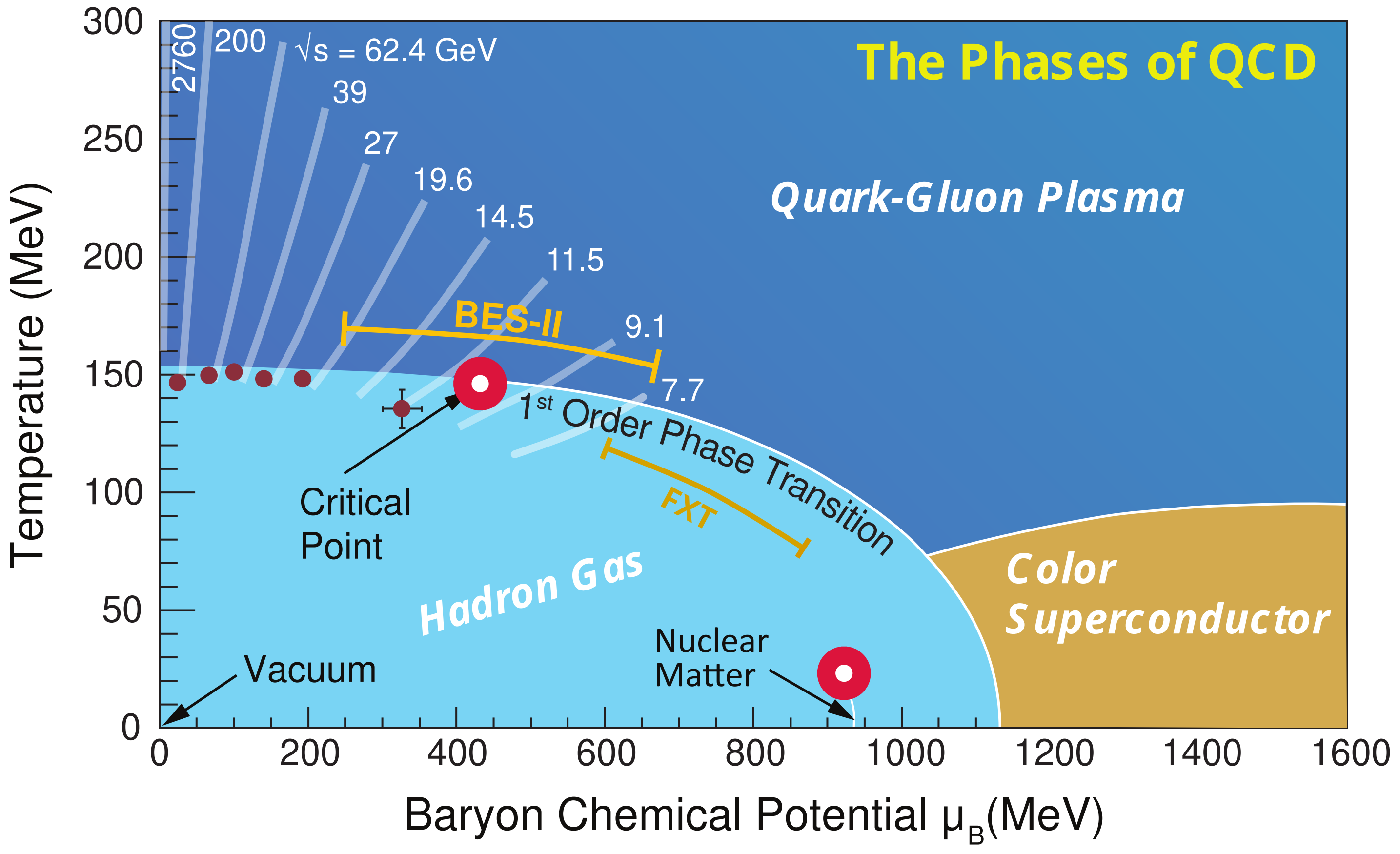
Chen et al, 2410.20704; Shuryak et al, Sun et al, 2020; STAR, 2506.05499, 2209.08058; etc

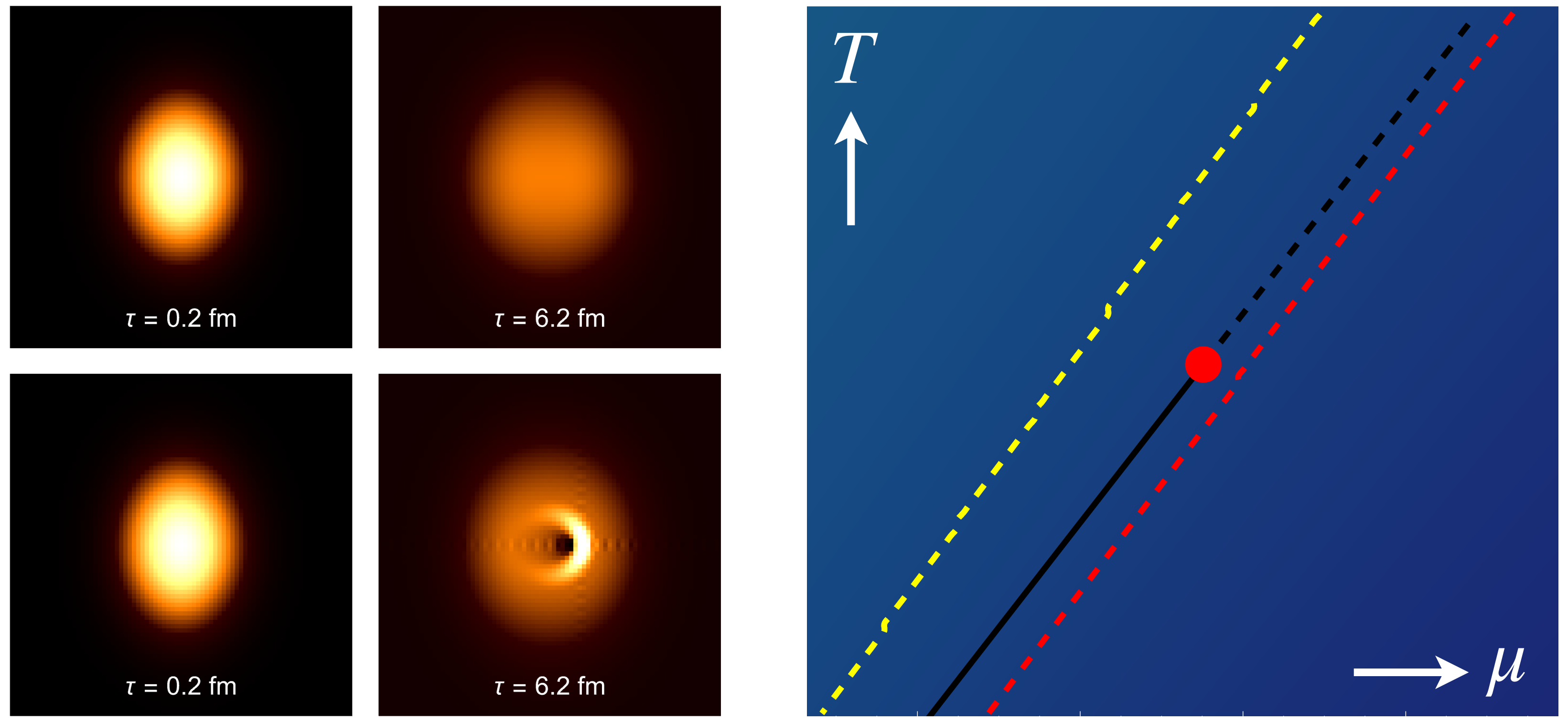
† *Quark Matter 2025 summary*

Motivation I: understanding QCD phases

Theoretical idealization

Experimental realization





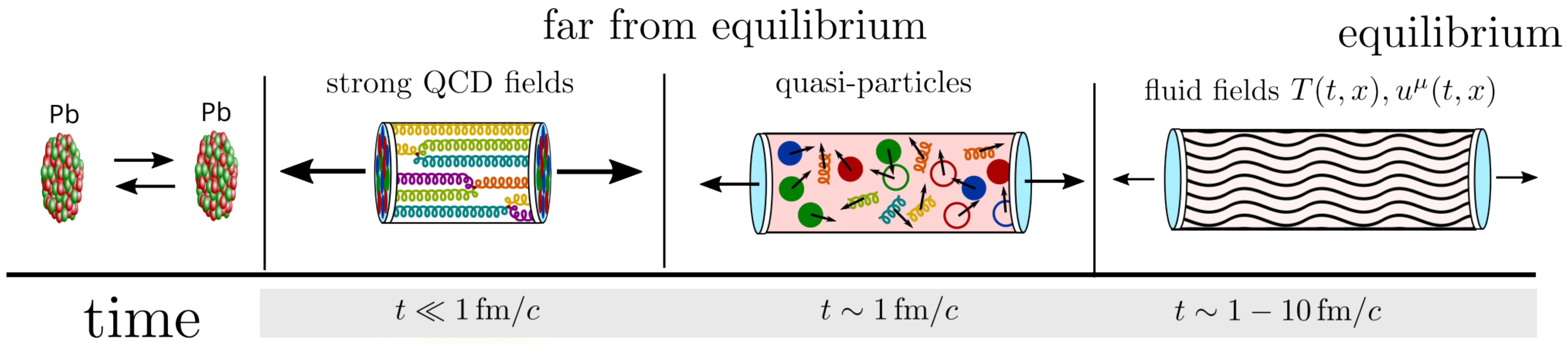
XA et al, 2312.17237, 2503.15719 and WIP

- long time (equilibrium)
- large size (thermodynamic limit)

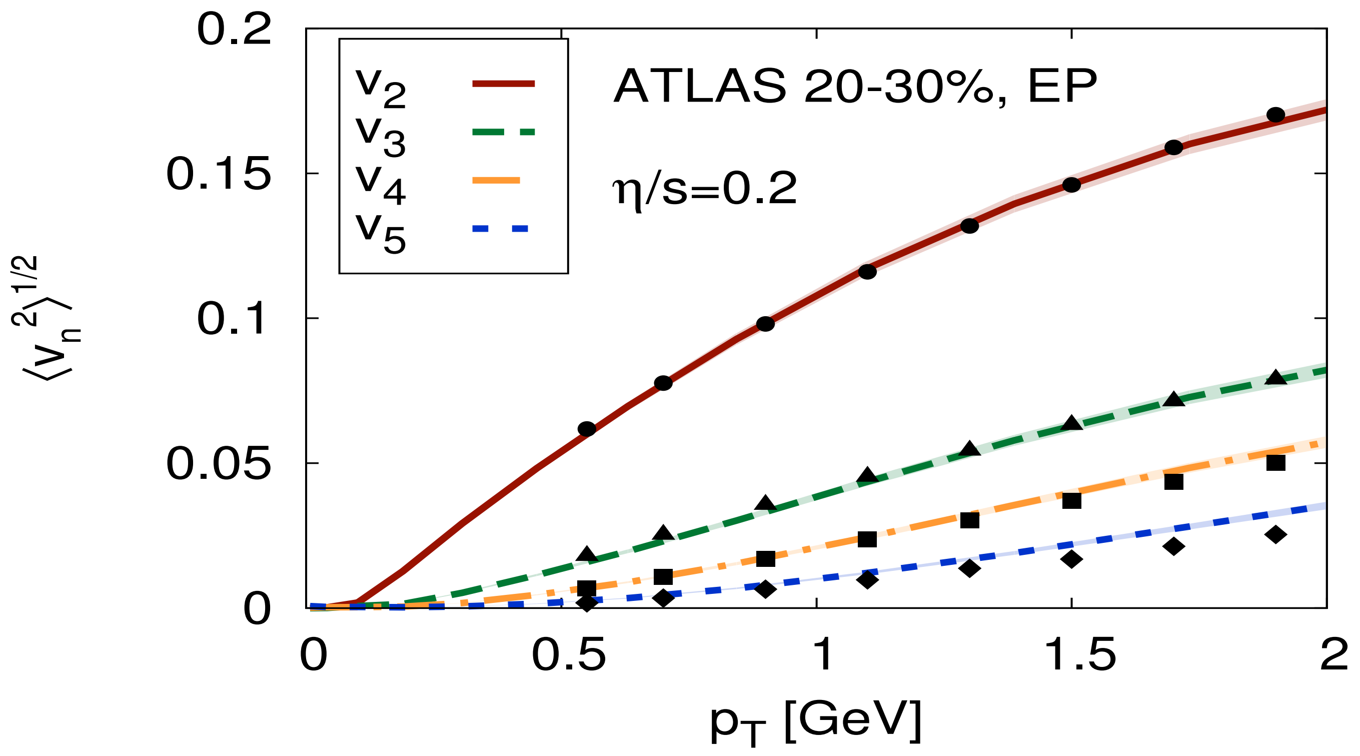
- short time (non-equilibrium)
- small size (non-thermodynamics)

Motivation II: understanding QGP thermalization

- Heavy-ion collisions → fast hydrodynamization/thermalization.

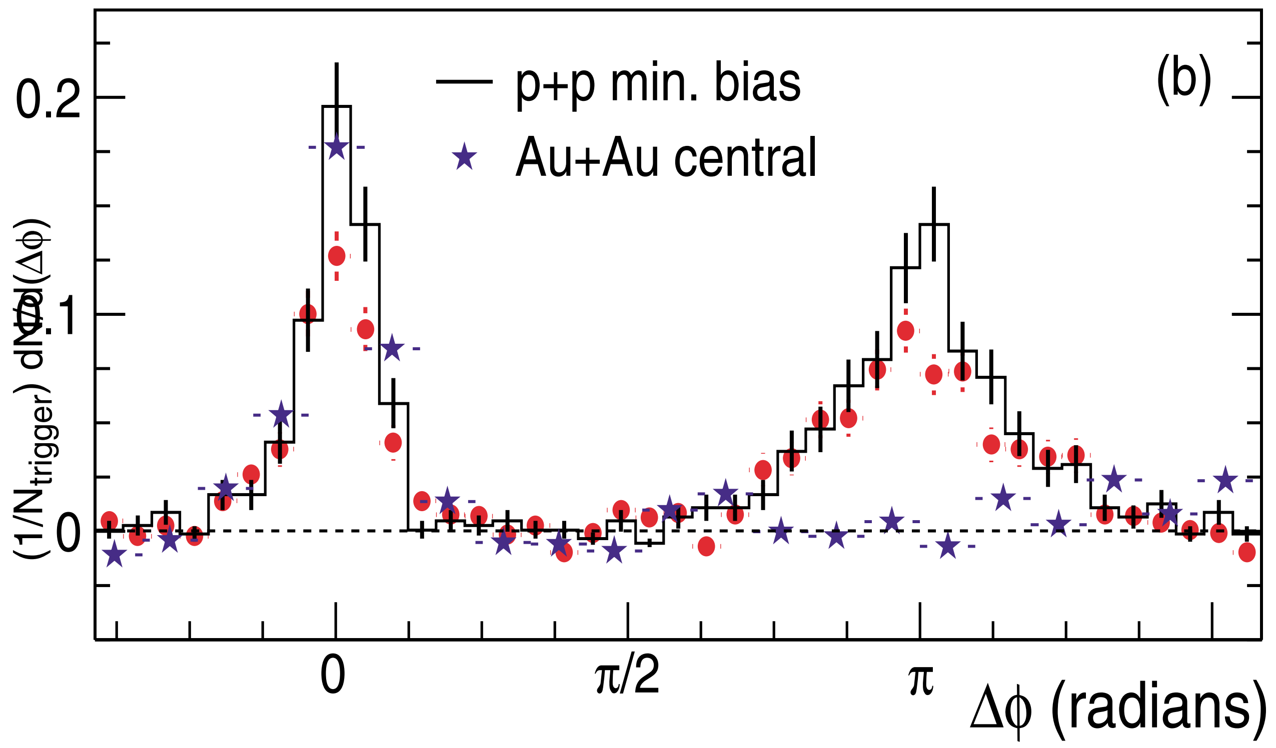


Courtesy of A. Mazeliauskas



Soft probe: flow

Gale et al, 1301.5893



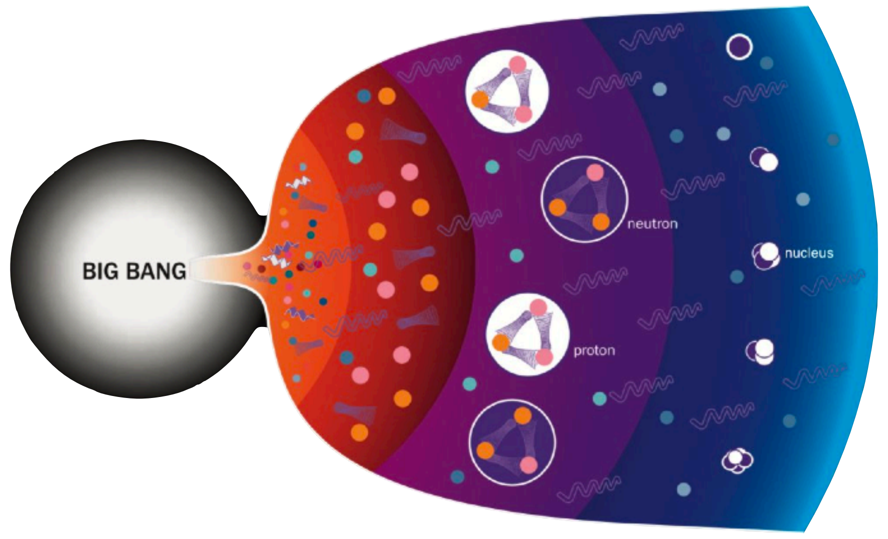
Hard probe: jet etc

STAR, 2003

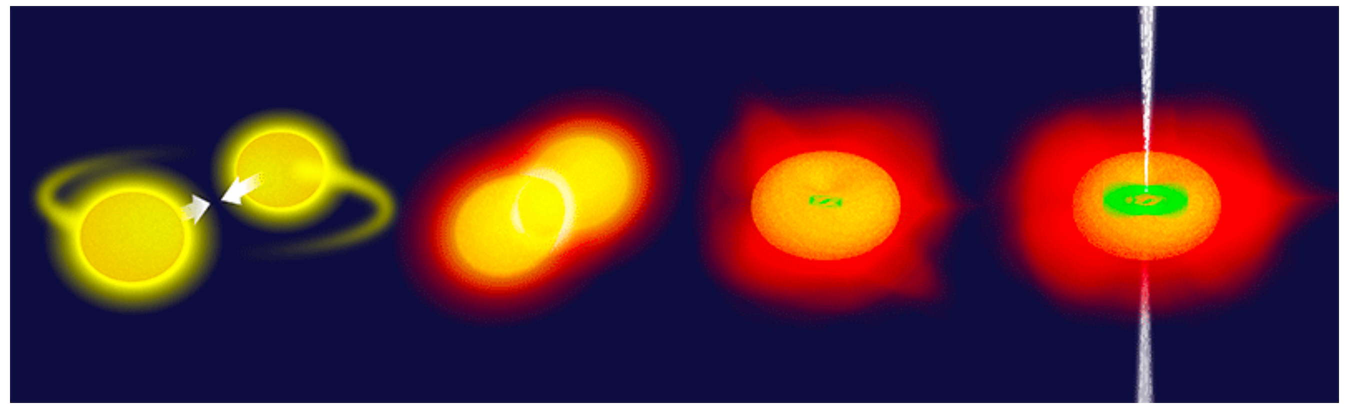
- Understanding the early-time physics is challenging (measurable after freezeout).
- **A simpler task**: find the legacy of non-hydrodynamics.

Motivation III: ubiquity and universality

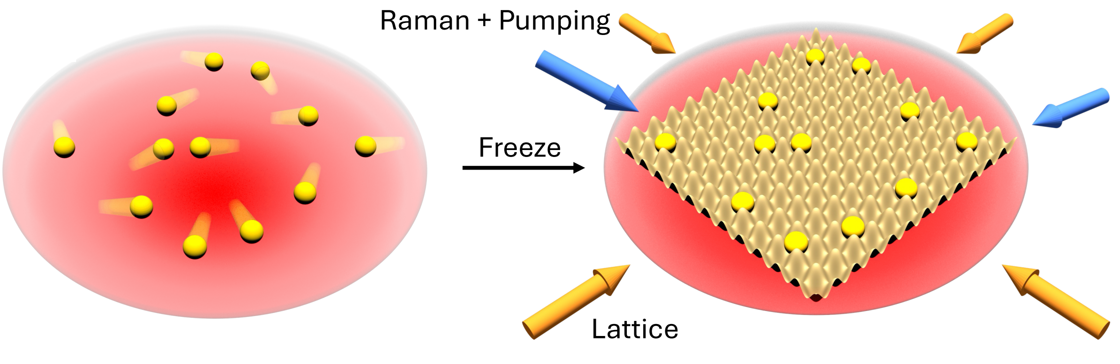
- Fluctuations/hydrodynamics describe systems across different scales: from **big bang** to **small bang**.



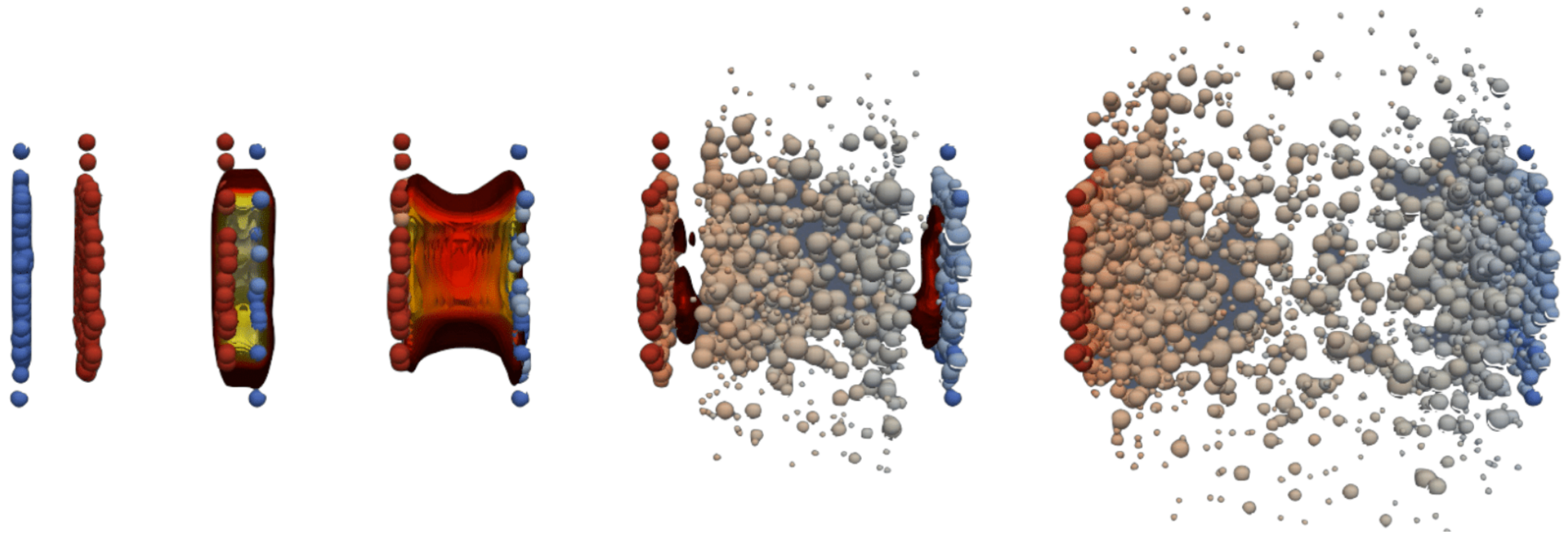
History of Universe



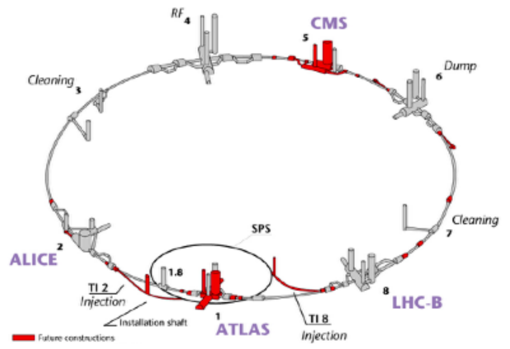
Neutron star merger (Rezzolla et al)



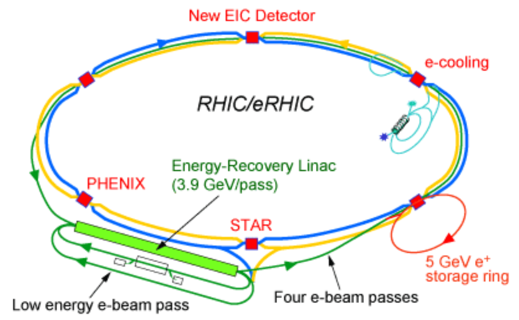
Quantum gases (Yao et al)



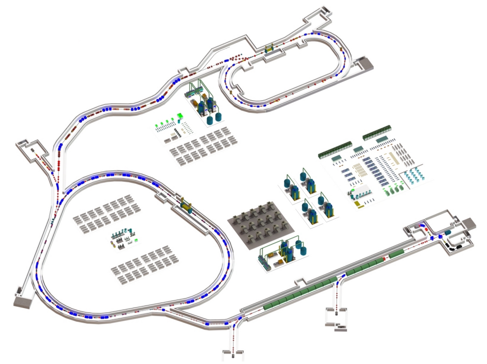
History of a heavy-ion collision (HIC)



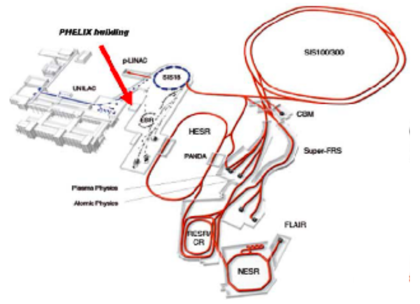
LHC (EU)



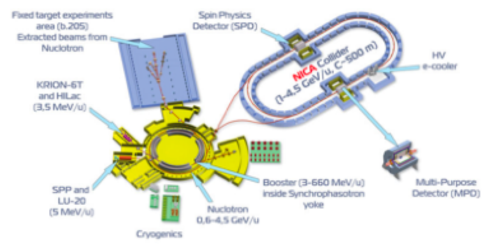
RHIC (US)



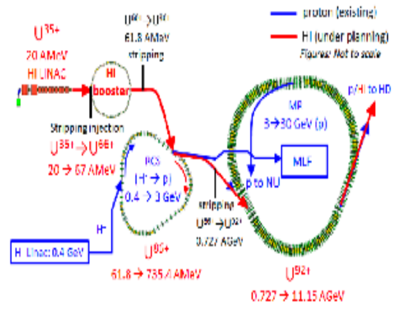
HIAF (CN)



GSI (DE)



NICA (RU)



J-PARC (JP)

Effective theories and scale separation

- Hydrodynamics + fluctuation & noise (source):

Top-down:
e.g., Schwinger-Keldysh

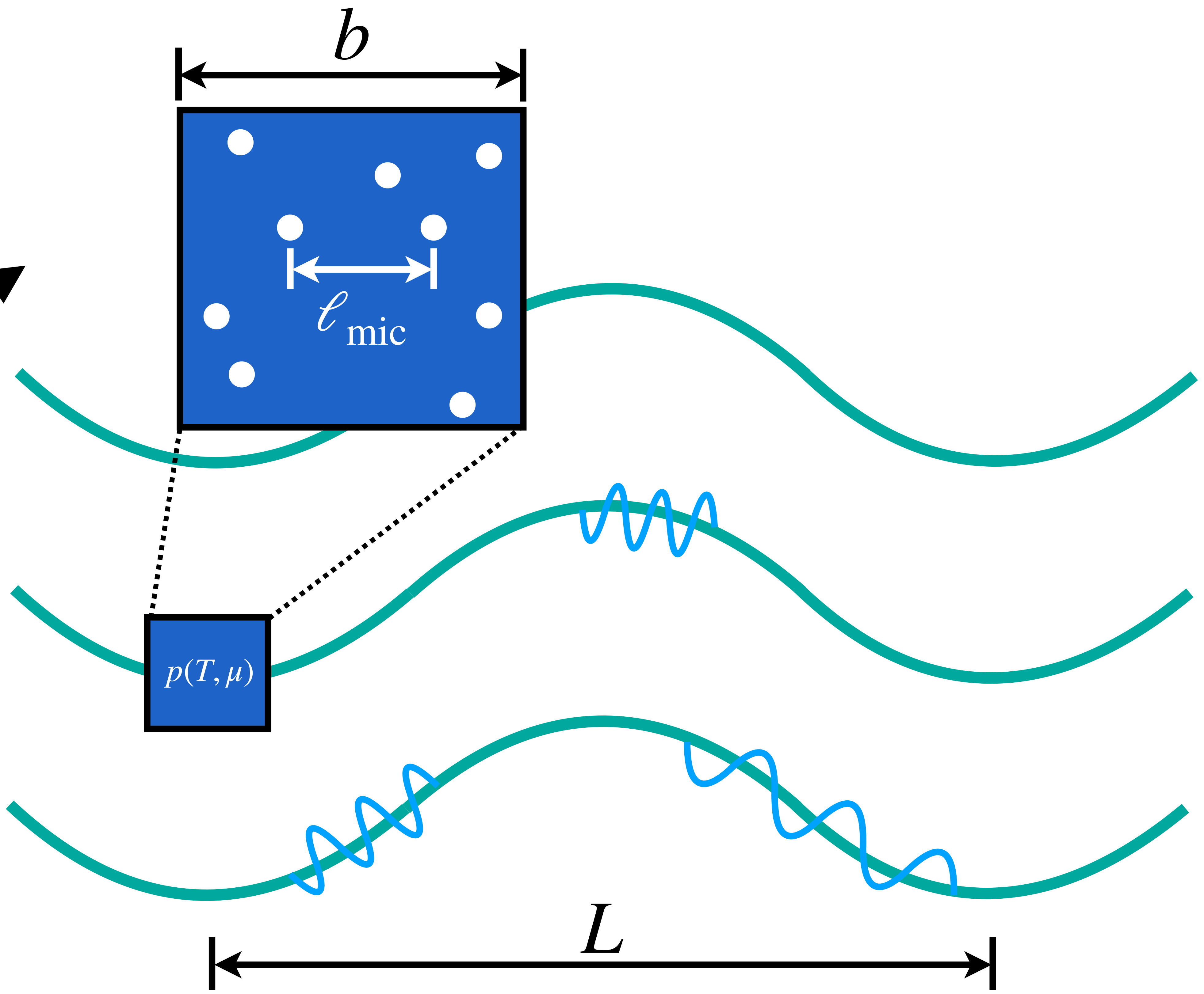
UV

integrating out
UV DOFs

emergence of:
short-range
stochastic noises
&
long-range
conserved quantities

IR

Bottom-up:
e.g., Langevin / Fokker-Planck



The scale hierarchy ensures the applicability of:

1) thermodynamics ($\ell_{\mathrm{mic}} \ll b$);

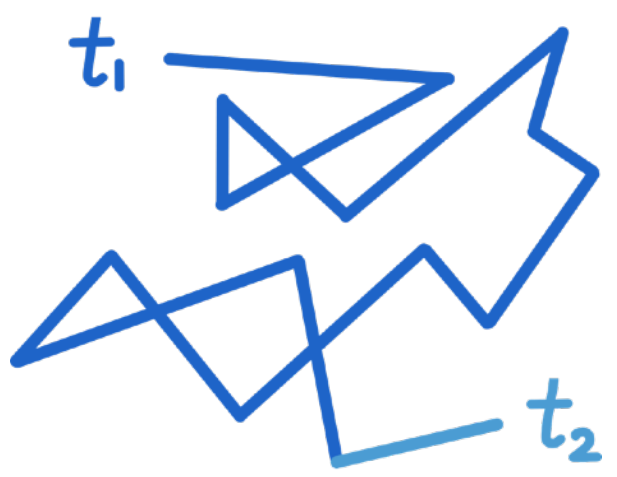
2) hydrodynamics ($\ell_{\mathrm{mic}} \ll L$).

Theoretical/numerical approaches

- Different approaches are complementary and related

Stochastic Navier-Stokes

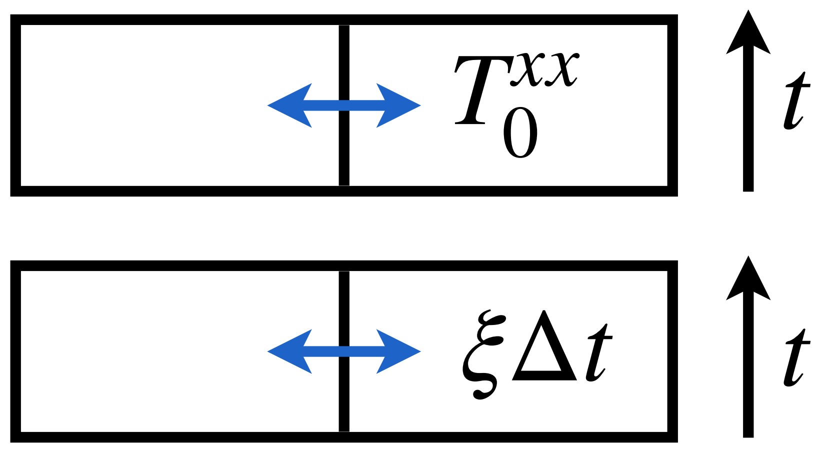
$$\dot\psi_i = F_i + \xi_i,$$
$$\langle \xi_i(t)\xi_j(t')\rangle = M_{ij}\delta_{tt'}$$



infinite/multiplicative noise; unstable & acausal

Metropolis algorithm

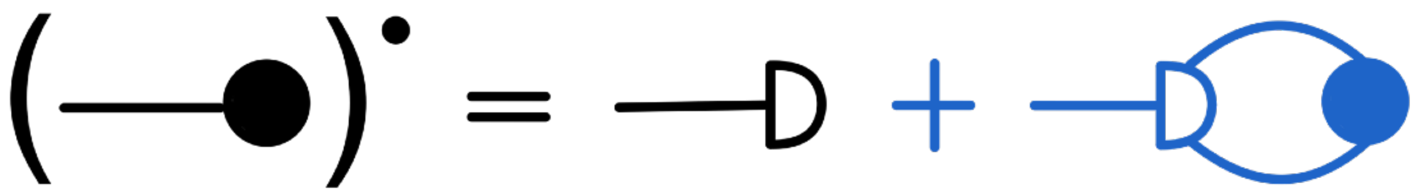
$$\dot\psi_i = -M_{ij}H_{,j} + \xi_i,$$
$$P[\psi] = e^{-H[\psi]}$$



stable; need renormalized quantities

Deterministic equations

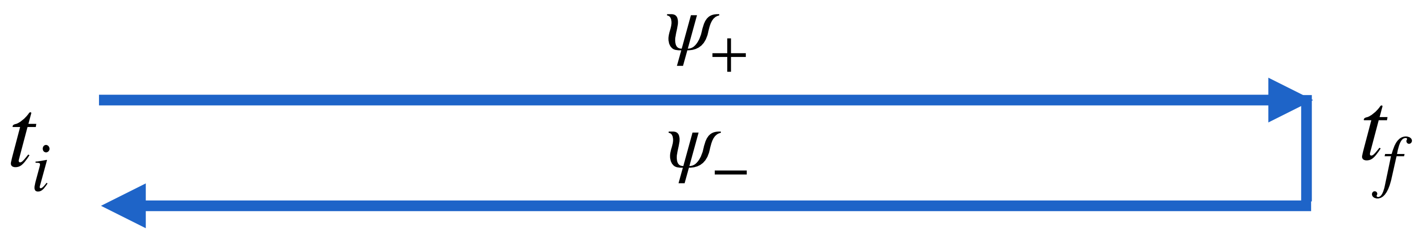
$$\dot\psi_i = M_{ij}(S_{,j} + S_{,jk\ell}G_{k\ell}) + \ldots$$



analytic; so far hydro regime

Schwinger-Keldysh formalism

$$\mathbb{L}(\psi_r, \psi_a) = E_i(\psi_r)\psi_i^a + i\psi_i^a M_{ij}^{-1}\psi_j^a$$



field-theoretical, top-down

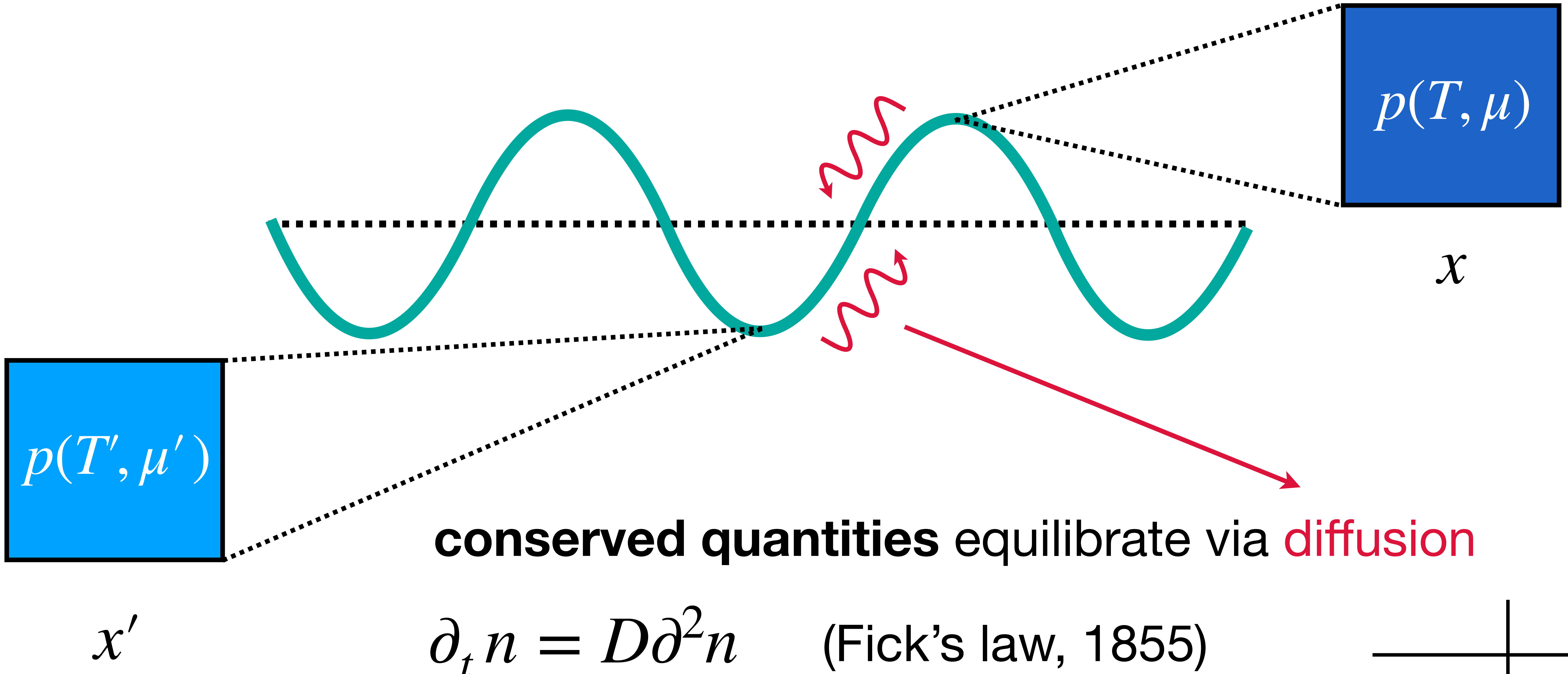
Full hydrodynamics:
SO(3) covariant formalism

Goals and constraints

- Goals in the deterministic method:
  - Deal with stochastic process as if one knows nothing about randomness (i.e., like how one deals with non-fluctuating theories).
  - Deal with relativity/covariance as if one knows nothing about relativity (i.e., like how one deals with non-relativistic theories in the lab).
  - Deal with multi-point correlations as if one knows nothing about multiple points (i.e., like how one deals with one-point functions).
- Bottom-up symmetry constraints:
  - The systems must be **SO(1,3)**/Lorentz covariant;
  - The systems in any local rest frame must be **SO(3)** covariant;
  - The systems must satisfy fluctuation-dissipation relations (KMS **Z(2)** invariant).

Hydrodynamics

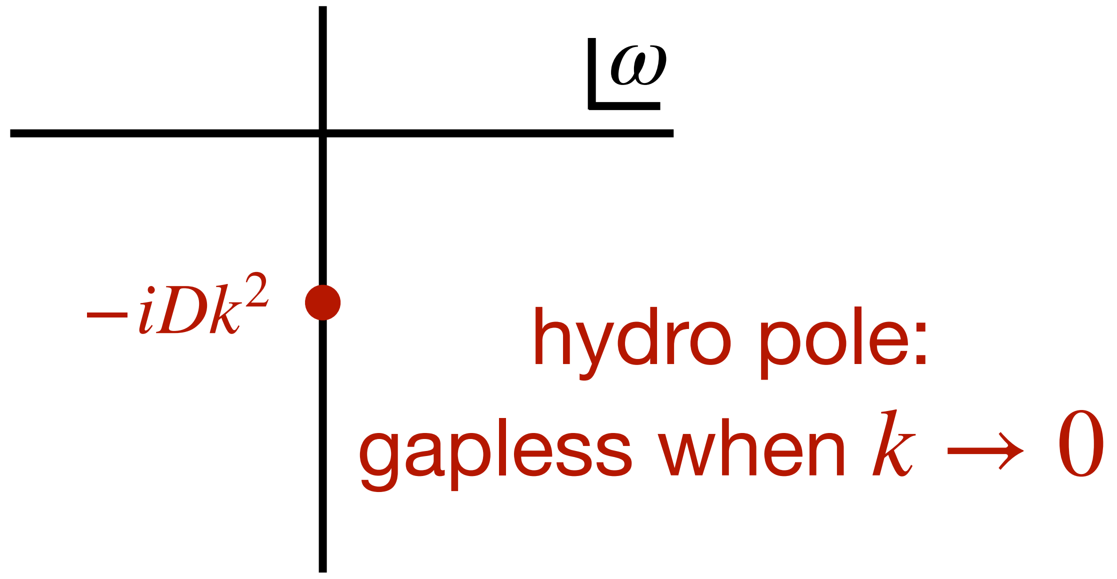
- Hydrodynamics deals with a collection of thermodynamical yet **inhomogeneous** systems.



conserved quantities equilibrate via diffusion

$$\partial_t n = D\partial^2 n \qquad \text{(Fick's law, 1855)}$$

$$G_R(\omega, k) \sim \frac{1}{-i\omega + Dk^2}$$



Hydrodynamic frames

- There are infinitely many equilibrium proxies for a non-equilibrium state, choosing a proxy amounts to specifying a hydrodynamic frame.

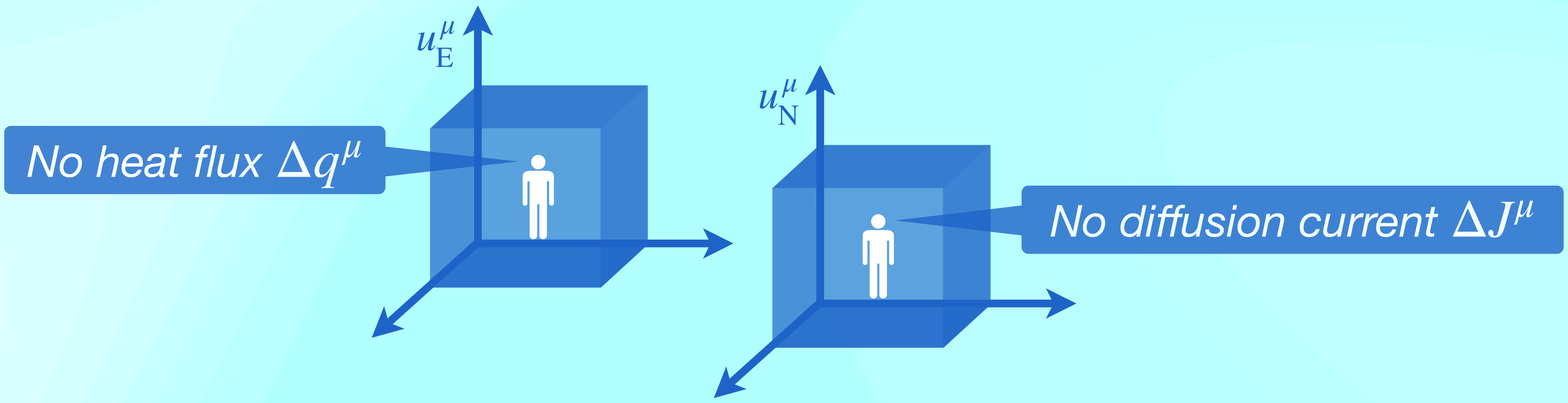
ΔX : Non-equilibrium quantities

Energy: $\varepsilon + \Delta\varepsilon = T^{\mu\nu}u_\mu u_\nu$

Charge: $n + \Delta n = -J^\mu u_\mu$

Velocity: $T^{\mu\nu}u^{\mathrm{E}}_\nu = -\varepsilon u^\mu_{\mathrm{E}} + \cancel{\Delta q}^\mu$ $\qquad$ $J^\mu = n u^\mu_{\mathrm{N}} + \cancel{\Delta J}^\mu$

Landau frame (E-frame) $\qquad$ Eckart frame (N-frame)



Stochastic (Landau) frames

$$\langle \breve{u}_\mu \rangle \neq u_\mu$$

Averaged frame

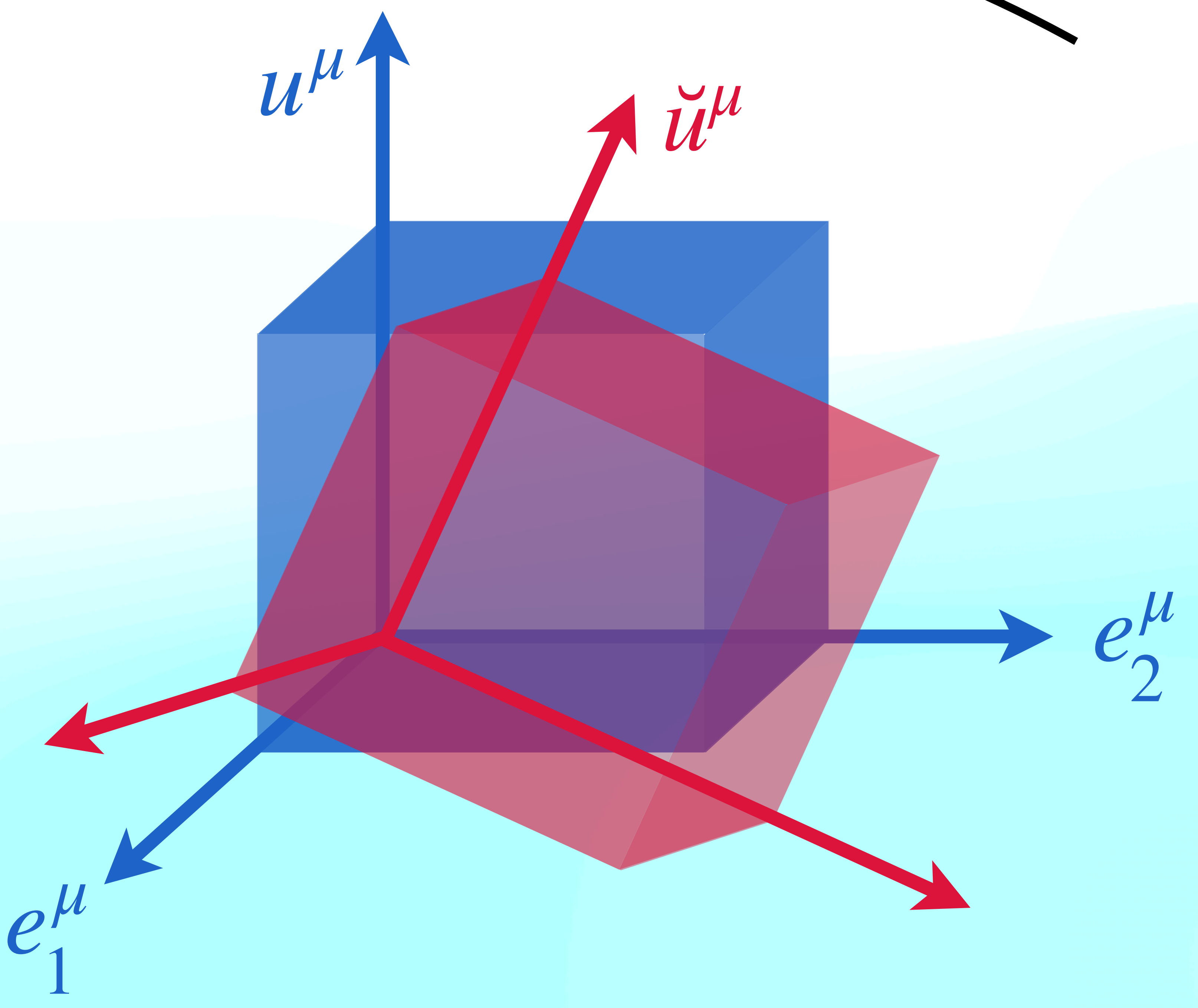
$$\langle \breve{T}^{\mu\nu} \rangle u_\nu = -\,\varepsilon u^\mu$$

$$\langle \breve{J}^{\mu} \rangle u_\mu = -\,n$$

Stochastic frames (comoving)

$$\breve{T}^{\mu\nu} \breve{u}_\nu = -\,\breve{\varepsilon} \breve{u}^\mu$$

$$\breve{J}^{\mu} \breve{u}_\mu = -\,\breve{n}$$



$$\langle \tilde{\pi}^a \rangle = 0$$

$$\langle \tilde{\varepsilon} \rangle = \varepsilon$$

$$\langle \tilde{n} \rangle = n$$

$$\breve{T}^{\mu\nu} u_\nu = -\tilde{\varepsilon} u^\mu + \tilde{\pi}^a e_a^\mu$$

$$\breve{J}^{\mu} u_\mu = -\tilde{n}$$

Primary variables are unconstrained & conserved:

$$\tilde{\psi} = (\tilde{\pi}, \tilde{\varepsilon}, \tilde{n})$$

XA, Basar and Stephanov, to appear

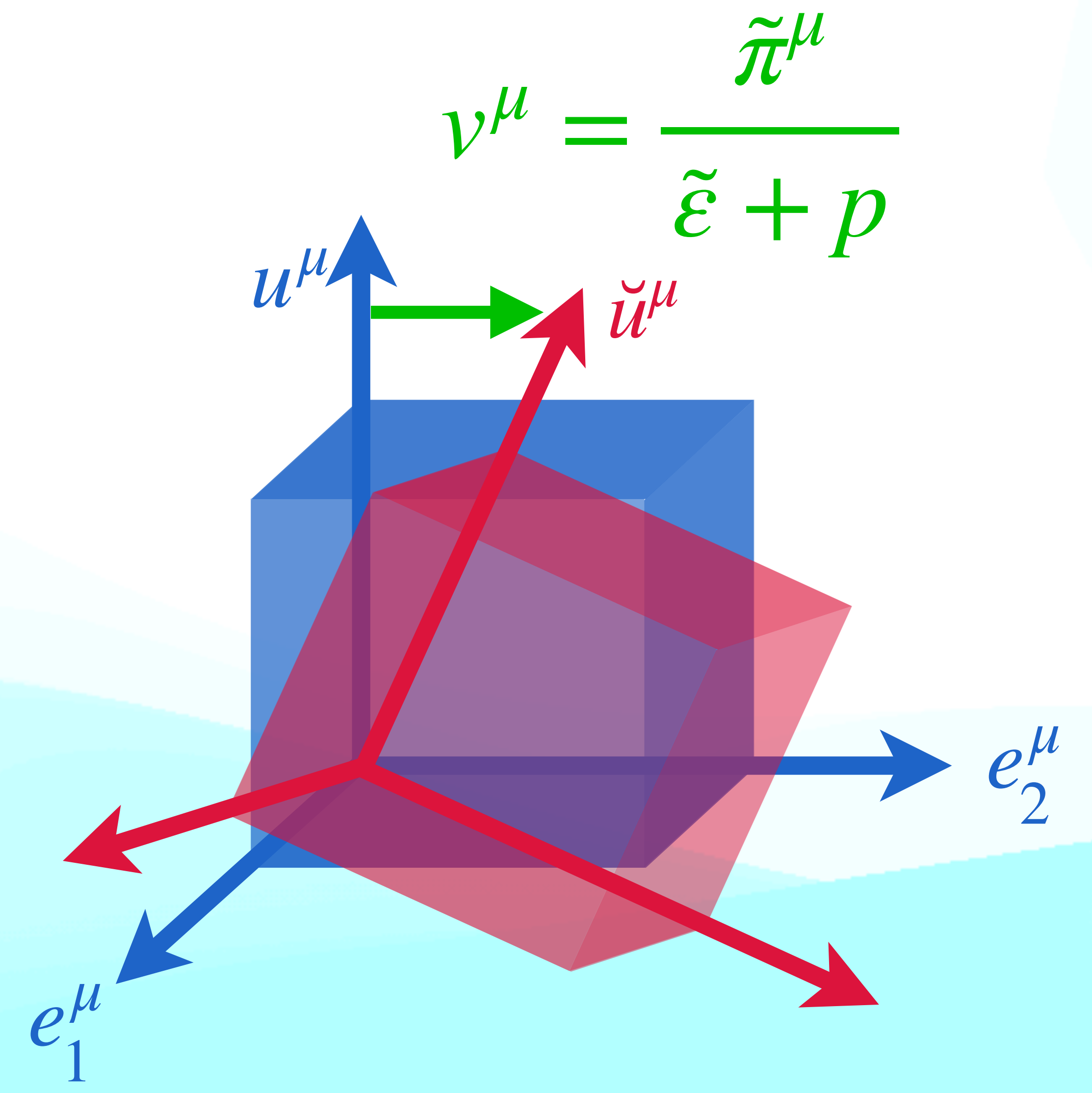
Thermodynamics in different frames

- Thermodynamic laws:
  - **1st law** for $\tilde{\psi} = (\tilde{\pi}, \tilde{\varepsilon}, \tilde{n})$:

$$\boxed{\tilde{T} d\tilde{s} = -v \cdot d\tilde{\pi} + d\tilde{\varepsilon} - \tilde{\mu} d\tilde{n}}$$

  - **2nd law** for $S^\mu = \tilde{s}(u^\mu + v^\mu) + \Delta S^\mu$:

$$\boxed{\partial_\mu S^\mu > 0}$$



- Thermodynamic relations:

$$c_s^2 = p_{,\breve{\varepsilon}} + \frac{\breve{n}}{\breve{\varepsilon} + p} p_{,\breve{n}}$$

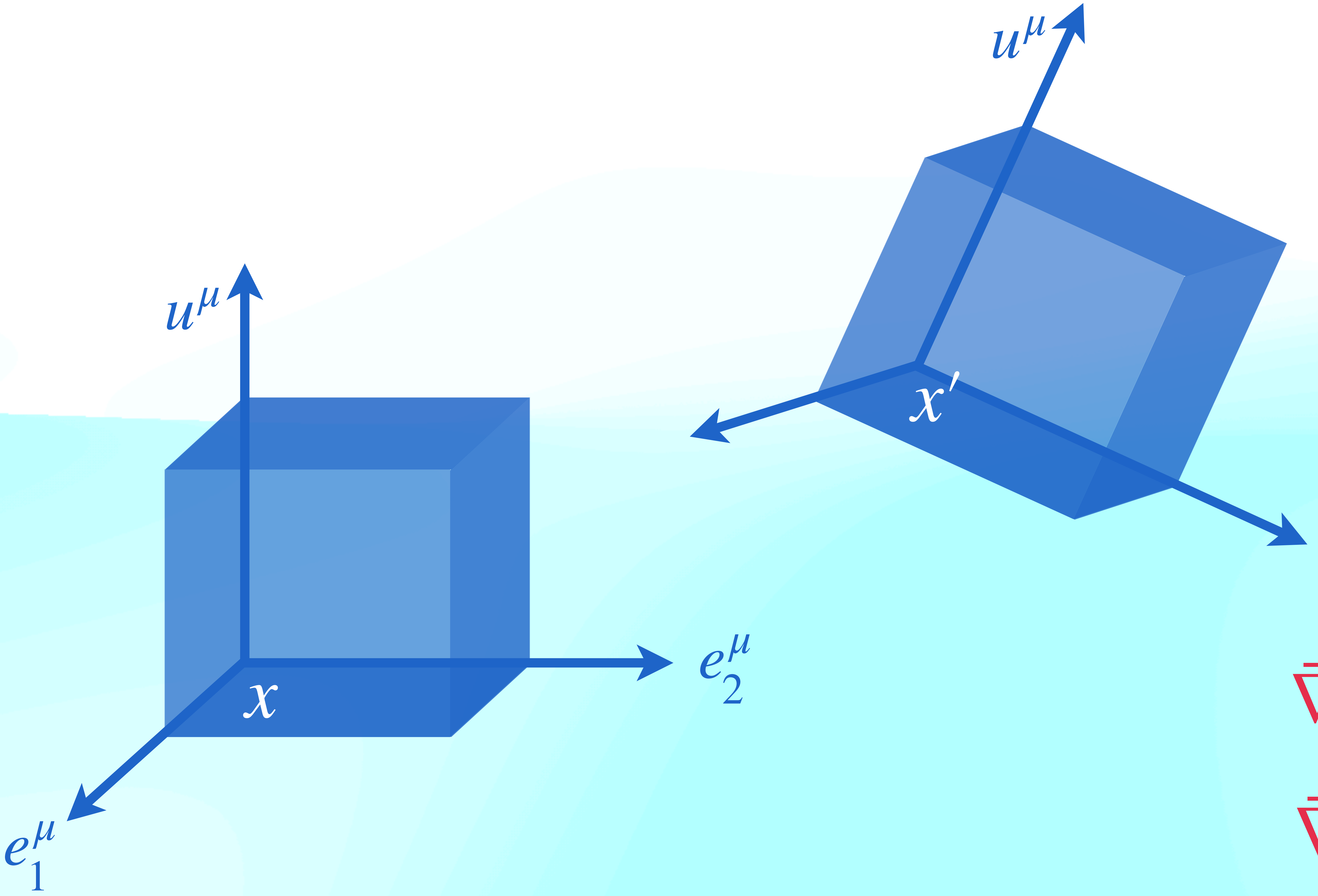
averaged frame
(hydro simulation)

$$p_{,\tilde{\varepsilon}} = \frac{p_{,\breve{\varepsilon}} + v^2 c_s^2}{1 - v^2 c_s^2}$$

stochastic frame
(Lattice QCD input)

Confluent transport

- More convenient to work with fluid mechanics **independent** of its kinematics.



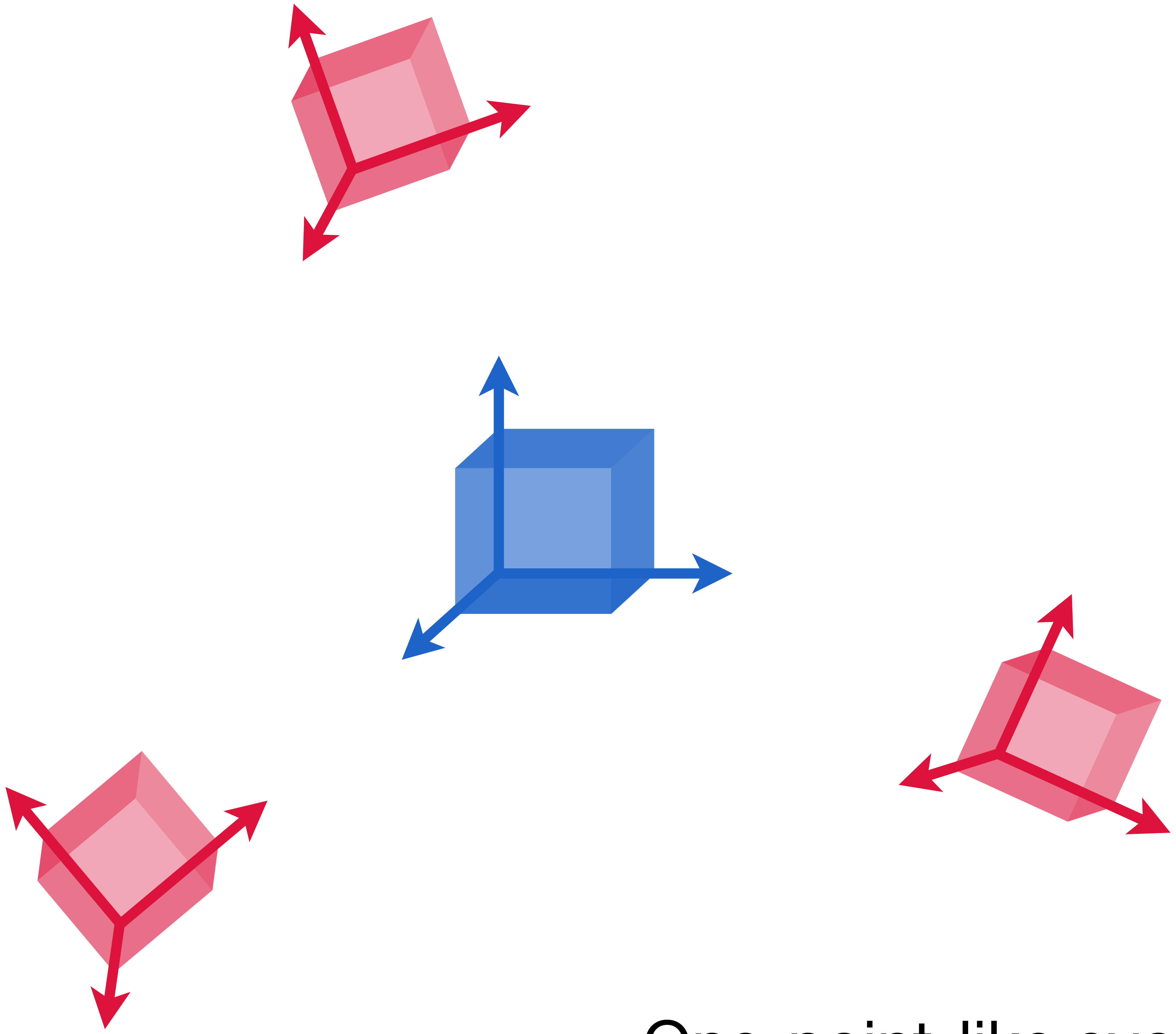
$$\partial_\mu u^\nu \neq 0$$

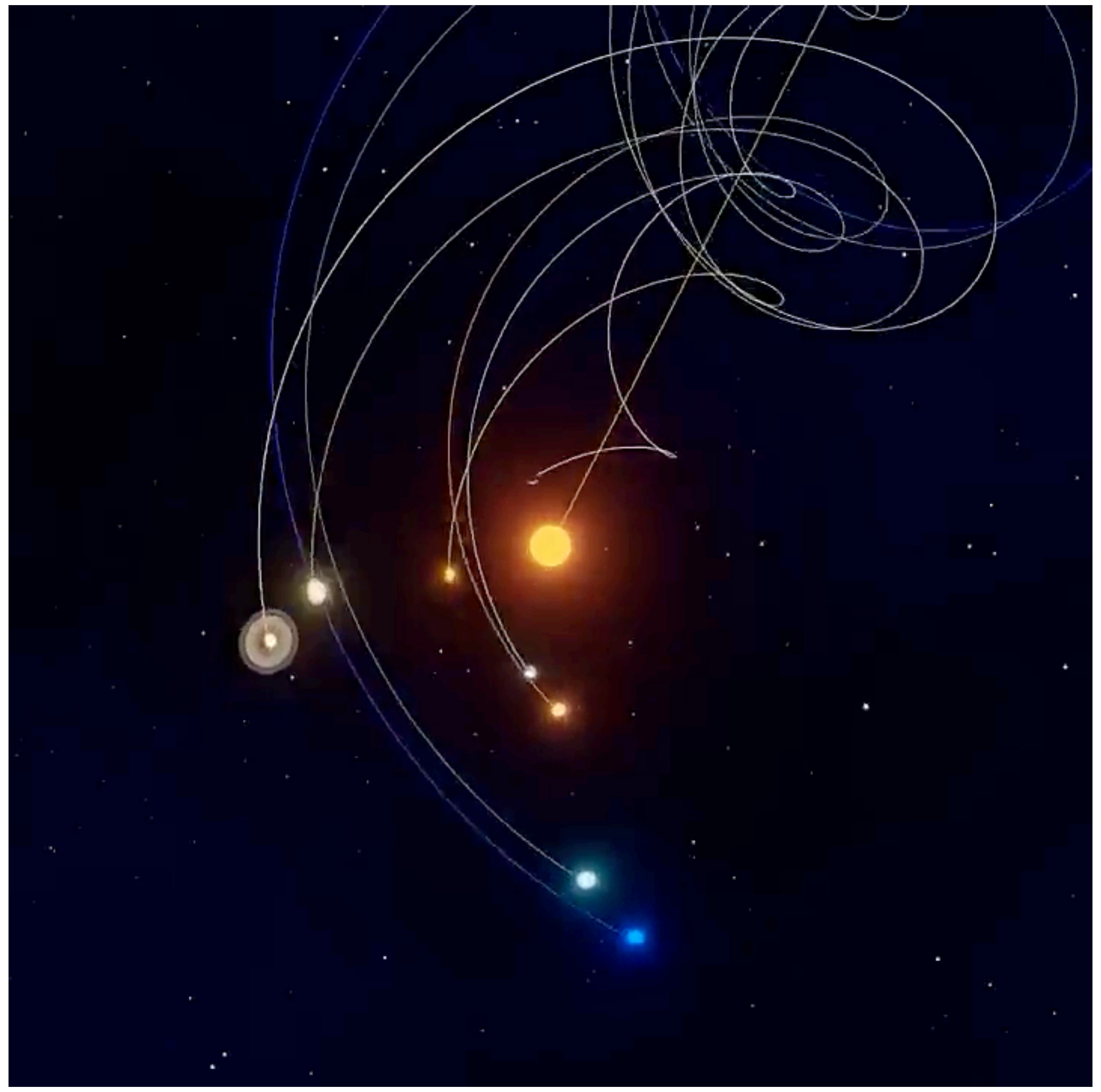
$$\partial_\mu e_a^\nu \neq 0$$

$$\bar{\nabla}_\mu u^\nu = \partial_\mu u^\nu + \Gamma^\nu_{\mu\lambda} u^\lambda = 0$$

$$\bar{\nabla}_\mu e_a^\nu = \partial_\mu e_a^\nu + \Gamma^\nu_{\mu\lambda} e_a^\lambda - \Gamma^b_{\mu a} e_b^\nu = 0$$

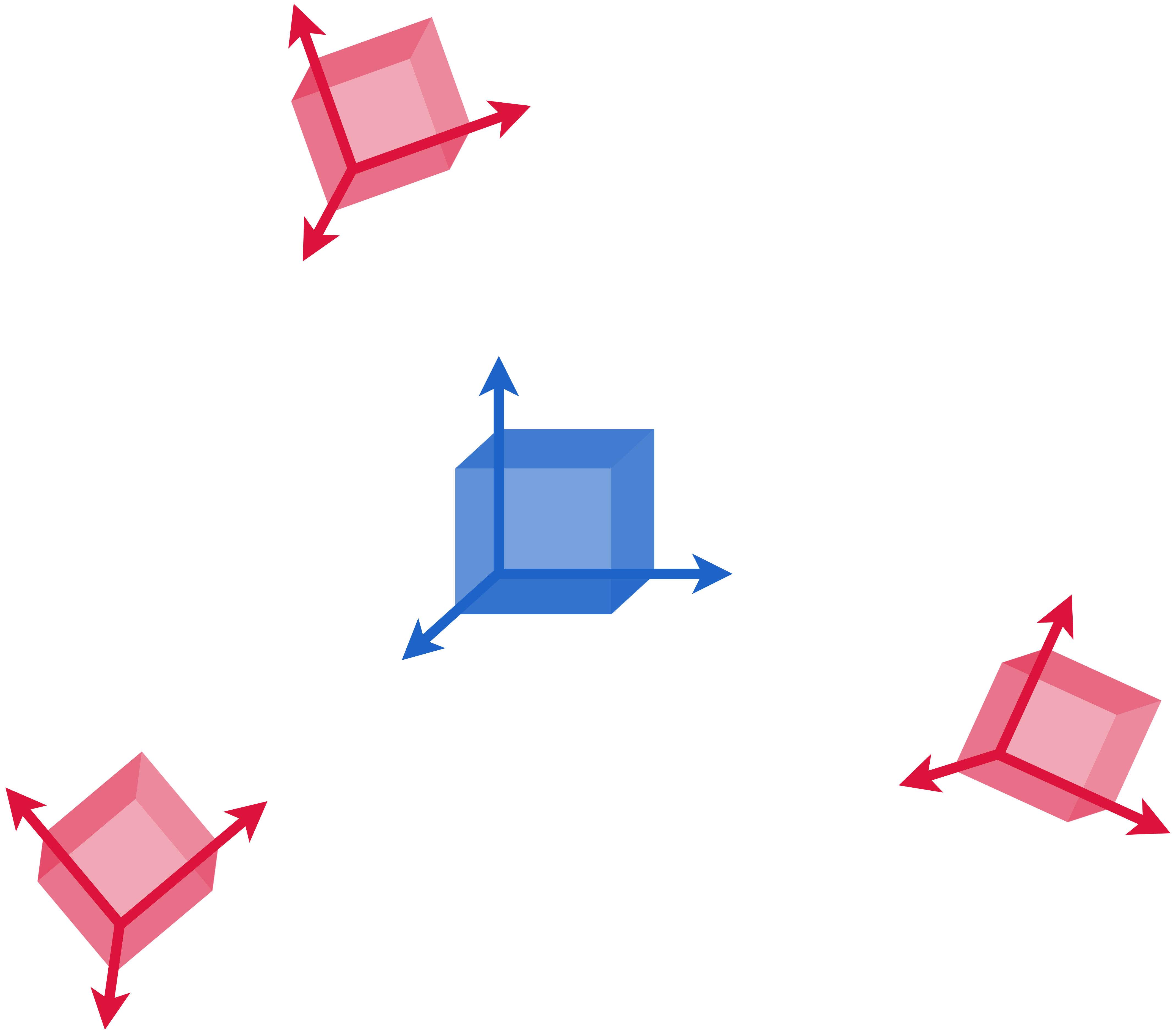
SO(3) covariant formalism



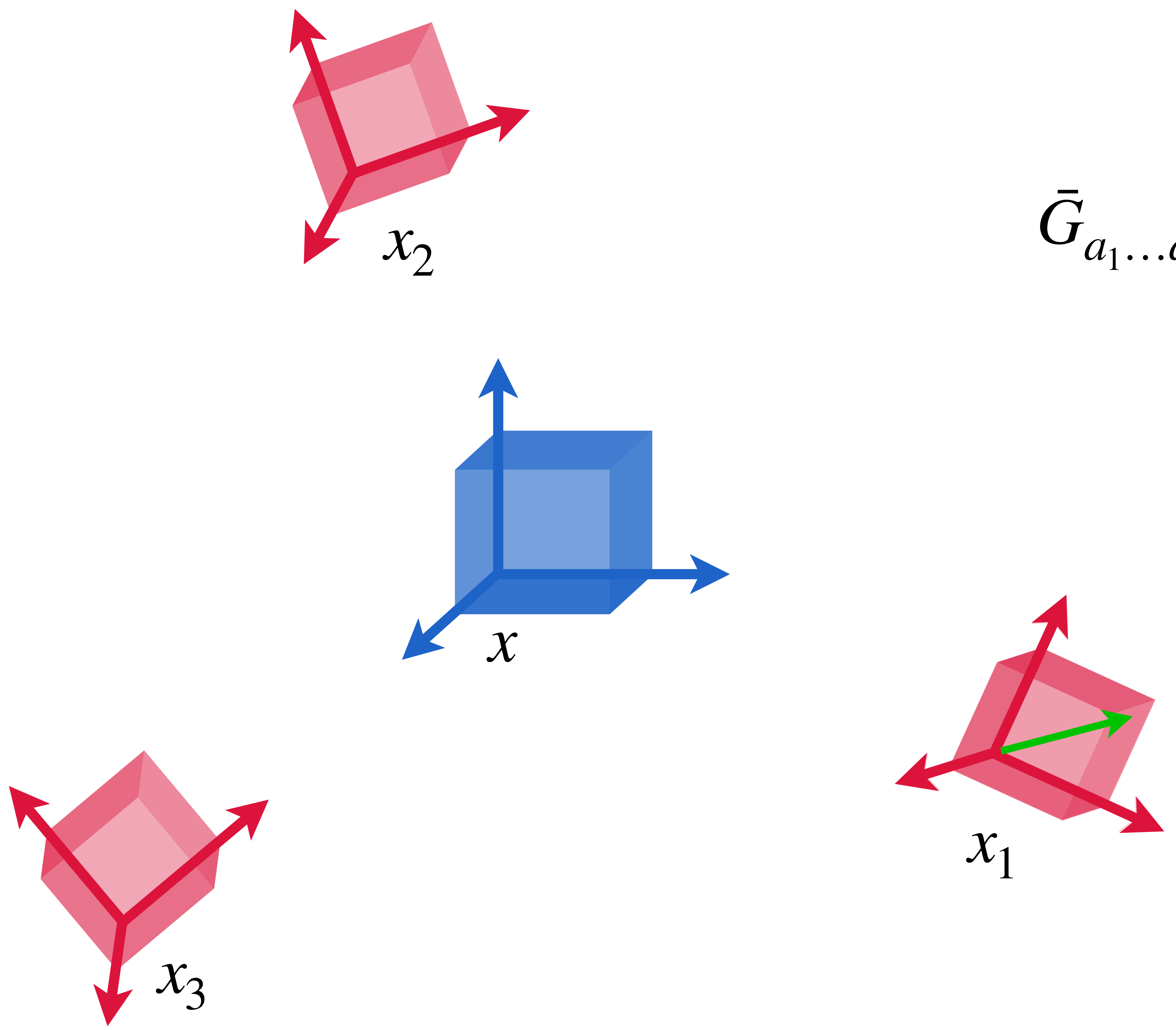


One-point-like evolution

SO(3) covariant formalism



SO(3) covariant formalism



$$\bar{G}_{a_1 \dots a_N}(x_1, \dots, x_N) \equiv \left\langle \prod_{i=1}^{N} \left[R(x, x_i) \phi(x_i) \right]_{a_i} \right\rangle$$

$$R(x, x')_a{}^b \equiv \underbrace{e(x)_a}_{\text{rotation}} \cdot \underbrace{\Lambda(x, x')}_{\text{boost}} \cdot \underbrace{e(x')^b}_{\text{projection}}$$

In terms of $\bar{\nabla}$ and $\bar{G}$, equations are manifestly SO(3) covariant

Generic equation of motion: a priori

- Assuming the **most generic** form of the **full nonlinear** equations for **any** $\tilde{\psi}_a$:

$$\partial_\mu \breve{T}^{\mu\nu} = 0, \quad \partial_\mu \breve{J}^\mu = 0$$

$$\downarrow$$

$$u \cdot \bar{\nabla} \tilde{\psi}_a = B_a + A^{\mu}{}_a{}^b \bar{\nabla} \tilde{\psi}_b \qquad \text{(Ideal/Hamiltonian)}$$

$$+ D^{\mu\nu}{}_a{}^b \bar{\nabla}_\mu \bar{\nabla}_\nu \tilde{\psi}_b + \tilde{D}^{\nu\mu}{}_a{}^{bc} (\bar{\nabla}_\mu \tilde{\psi}_b)(\bar{\nabla}_\nu \tilde{\psi}_c) + \dots \qquad \text{(Dissipation)}$$

$$+ C^{\mu}_{\hat{c}a} (\bar{\nabla}_\mu \eta^{\hat{c}}) + \tilde{C}^{\mu}{}_{\hat{c}a}{}^{b} \eta^{\hat{c}} (\bar{\nabla}_\mu \tilde{\psi}_b) + \dots \qquad \text{(Noise)}$$

$$\langle \eta^{\hat{c}}(x) \eta^{\hat{d}}(x') \rangle = 2 \delta^{\hat{c}\hat{d}} \delta^{(4)}(x - x')$$

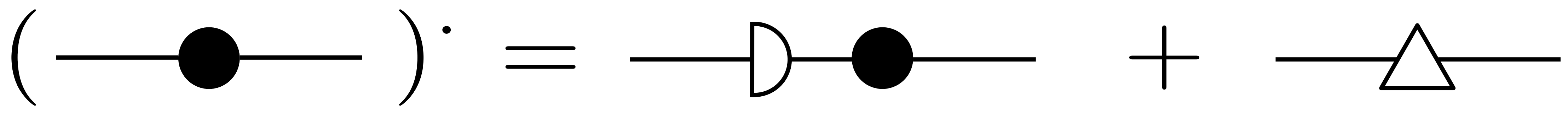
We do not need to know anything about operators $ABCD$!

SO(3) covariant confluent equations

- Equations for $W_{a_1\ldots a_N}(q_1,\ldots,q_N) = \mathrm{FT}\left\{\bar{G}_{a_1\ldots a_N}(x_1,\ldots,x_N)\right\}$ in **full hydrodynamics**:

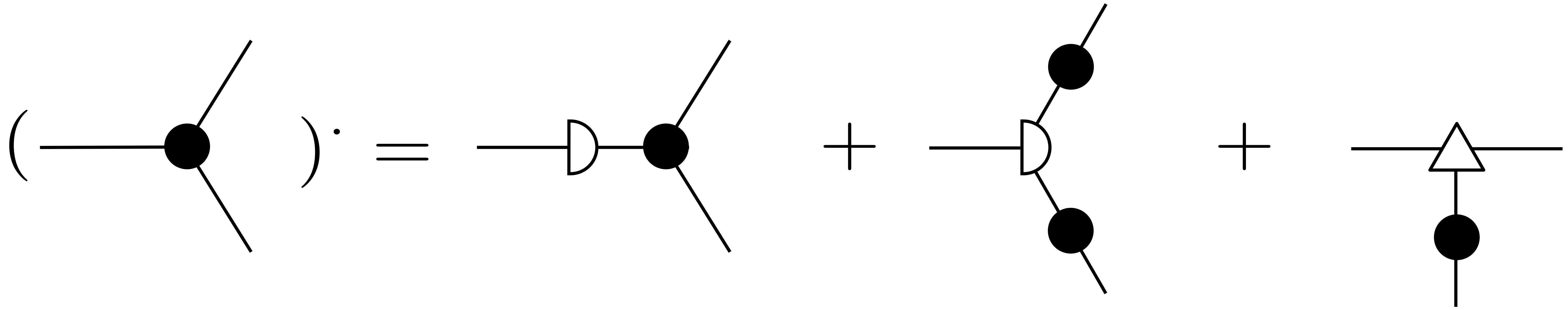
$$u\cdot\bar{\nabla}\,W^{(q_1,q_2)}_{a_1a_2} = 2\left[\Gamma_{a_1}^{\;b\,(q_1,-q_1)}W^{(q_1,q_2)}_{ba_2} + \Xi^{(q_1,q_2)}_{a_1a_2}\right]_{\mathrm{Perm}(12)}$$

Γ, Ξ depend on $ABCD$



Consistent with fluctuation-dissipation relation *regardless* of the explicit form of $ABCD$: a very *nontrivial* check!

$$u\cdot\bar{\nabla}\,W^{(q_1,q_2,q_3)}_{a_1a_2a_3} = 3\left[\Gamma_{a_1}^{\;b\,(q_1,-q_1)}W^{(q_1,q_2,q_3)}_{ba_2a_3} + \Gamma_{a_1}^{\;bc\,(q_1,q_2,q_3)}\left(W^{(-q_2,q_2)}_{ba_2}, W^{(-q_3,q_3)}_{ca_3}\right) + \Xi_{a_1a_2}^{\;\;\;\;b}W^{(-q_3,q_3)}_{ba_3}\right]_{\mathrm{Perm}(123)}$$

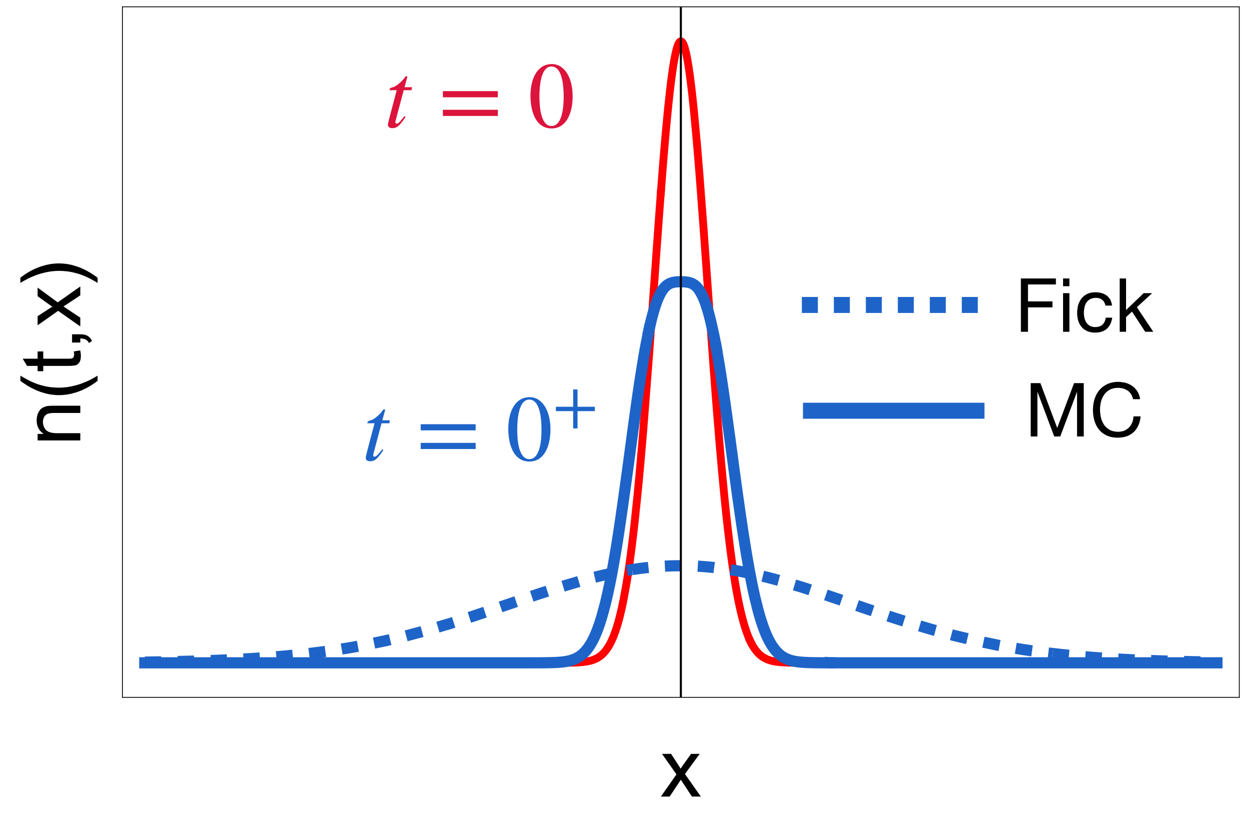


XA, Basar and Stephanov, to appear

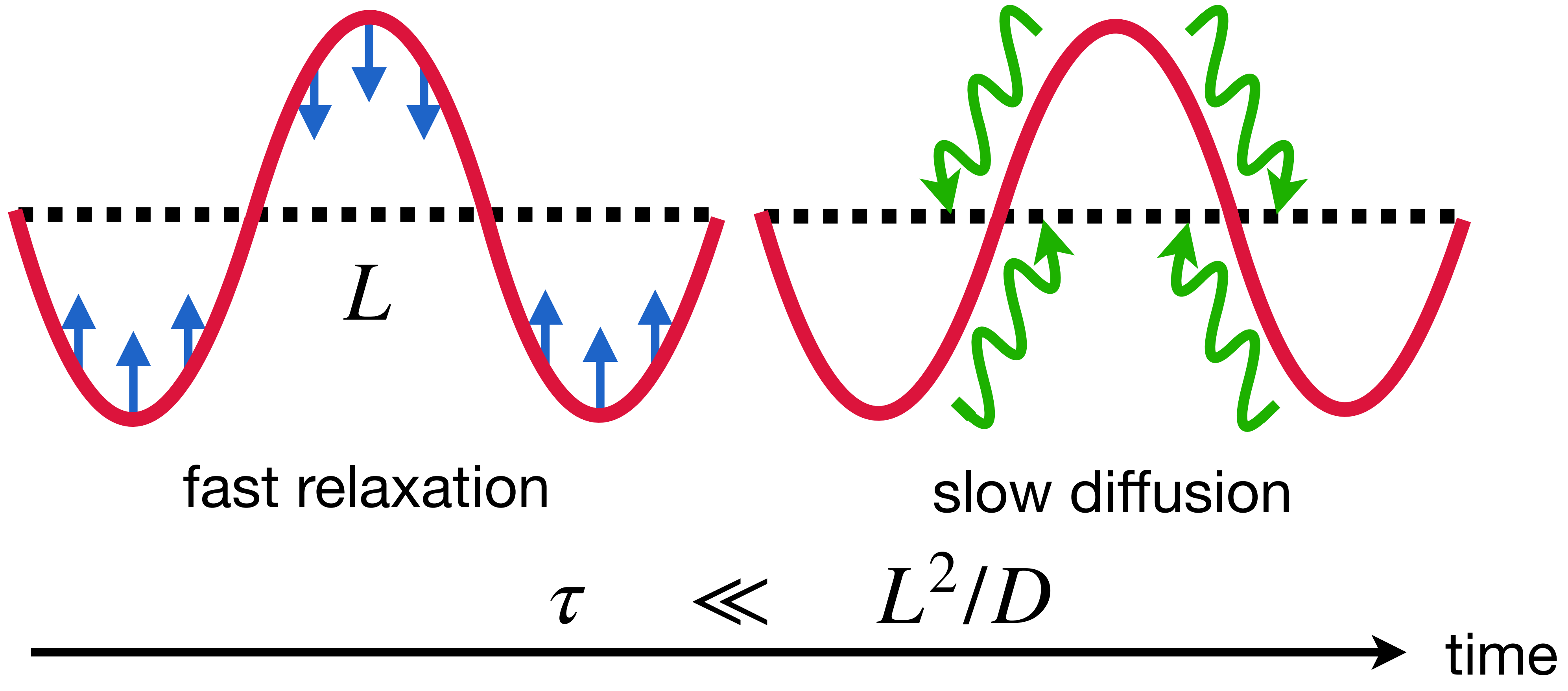
Read for use before freezeout

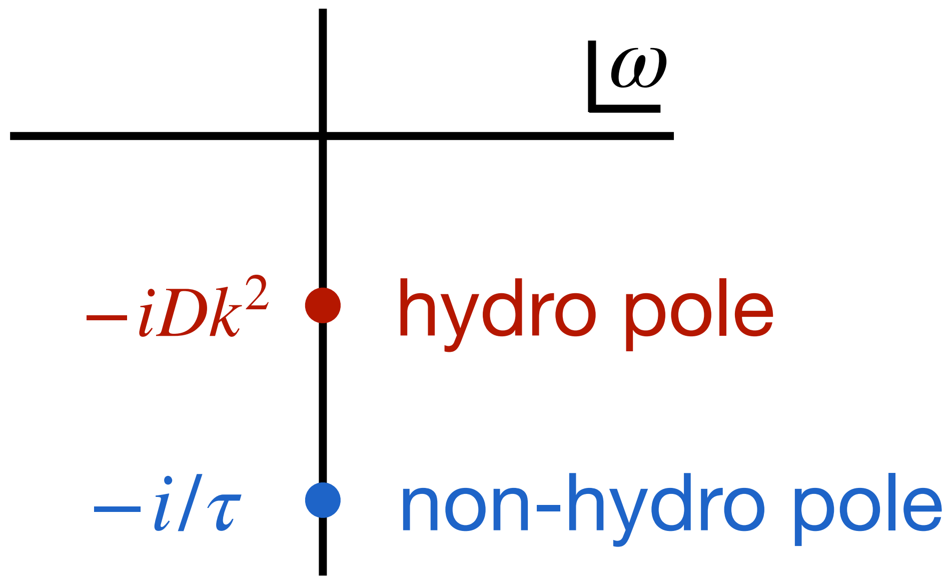
Far-from-equilibrium regime:
U(1) diffusion

Maxwell-Cattaneo diffusion



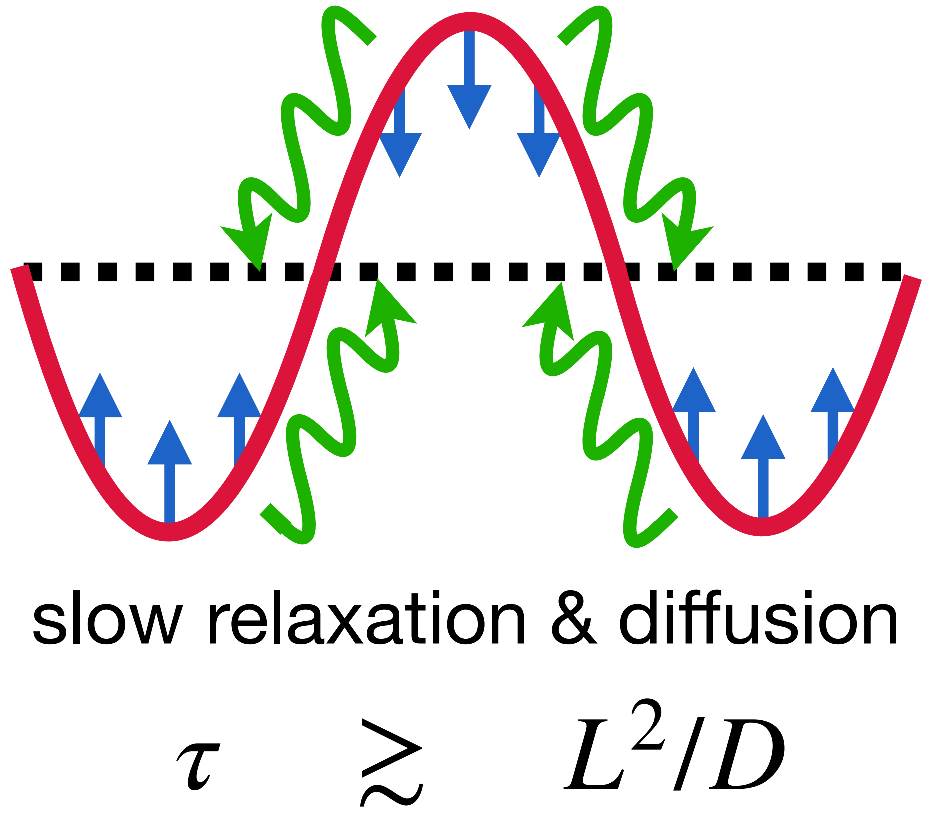
- Fick diffusion (gapped modes are transient)

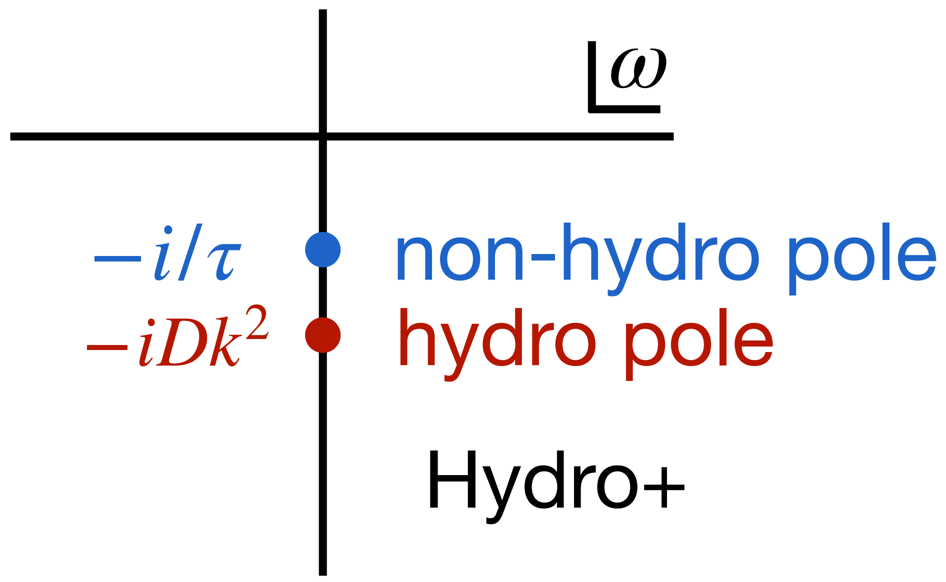




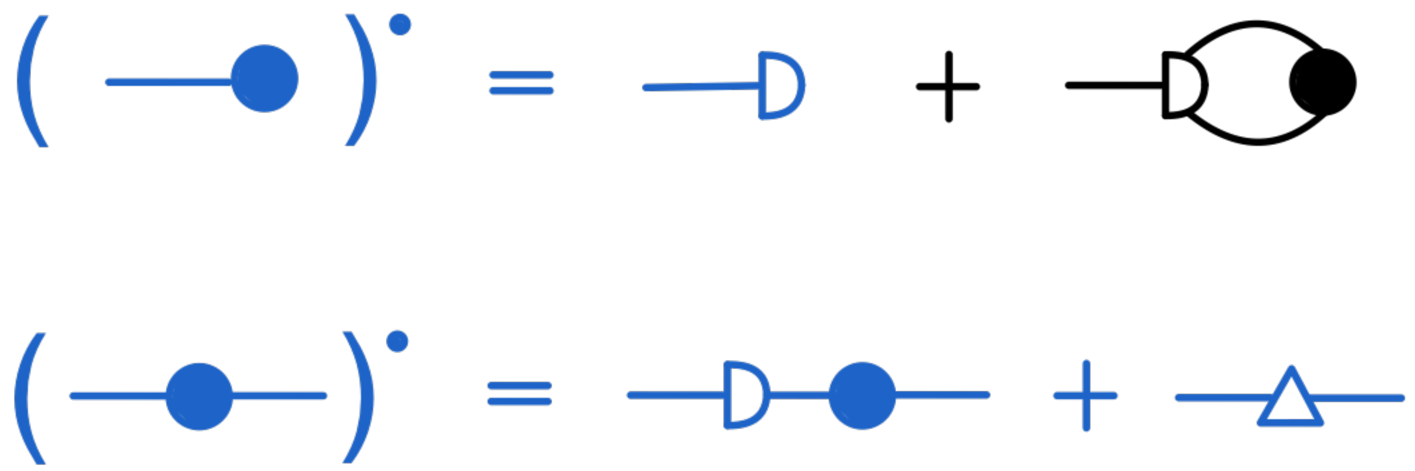
$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0 \qquad \vec{J} = -D\vec{\partial} n$$

- Maxwell-Cattaneo diffusion (e.g., critical slowing down $\tau\partial_t \gg 1$)





$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0 \qquad \tau\partial_t \vec{J} = -(\vec{J} + D\vec{\partial} n)$$



Maxwell-Cattaneo diffusion near critical point

- Evolution of dynamic variable n and $g \equiv \dot{n} = -\vec{\partial} \cdot \vec{J}$

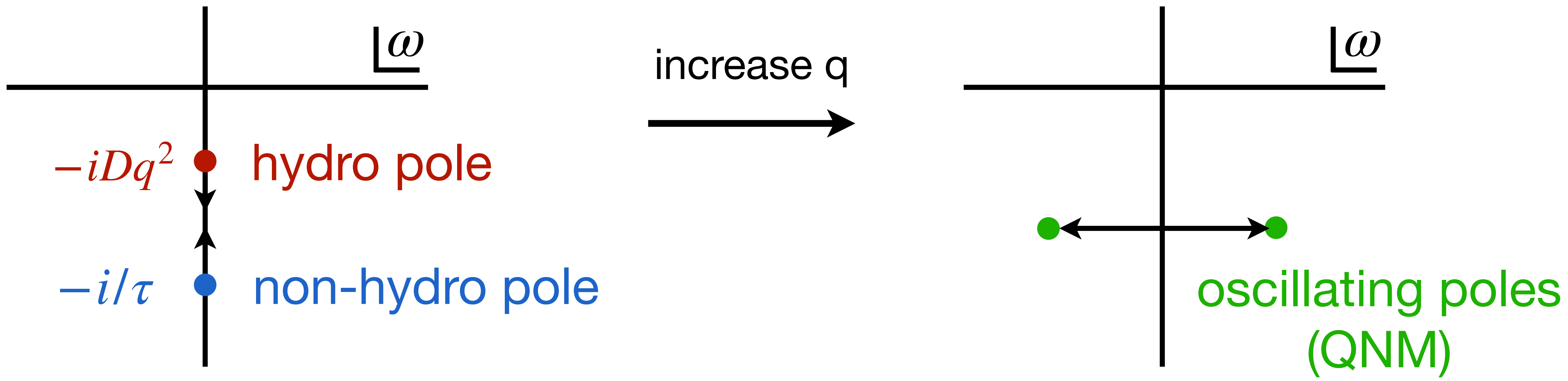
a toy model for full hydrodynamics

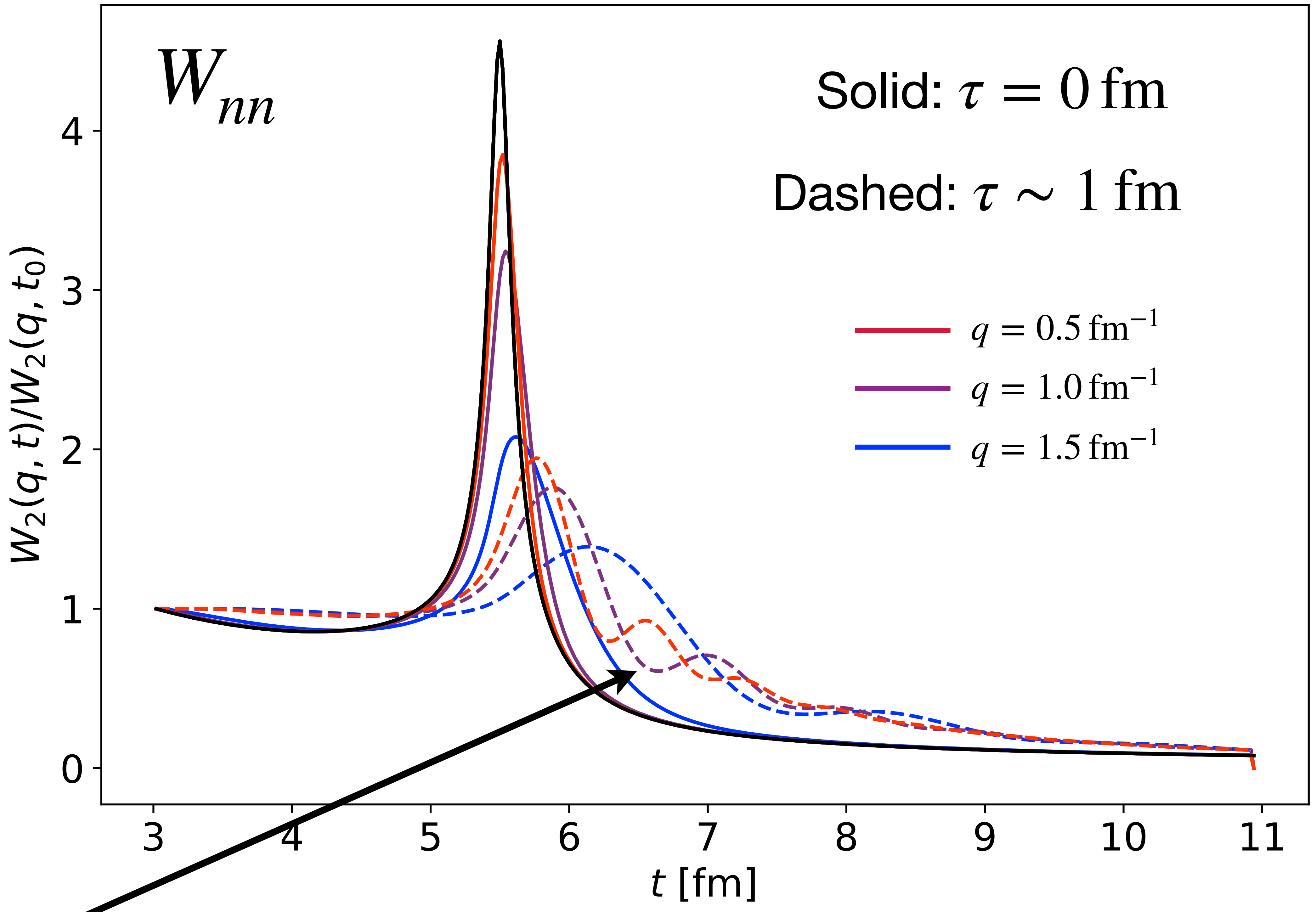
$$W_{nn}(q) \equiv \langle n(q)n(-q)\rangle \qquad W_{ng}(q) \equiv \langle n(q)g(-q)\rangle \qquad W_{gg}(q) \equiv \langle g(q)g(-q)\rangle$$

$$\tau\partial_t W_{nn}(q) = 2\tau W_{ng}(q)\,|_{\text{perm}}$$

$$\tau\partial_t W_{ng}(q) = -W_{ng}(q) - Dq^2 W_{nn}(q) + \tau W_{gg}(q)$$

$$\tau\partial_t W_{gg}(q) = -2[W_{gg}(q) + Dq^2 W_{ng}(q) - \lambda q^2/\tau]_{\text{perm}}$$

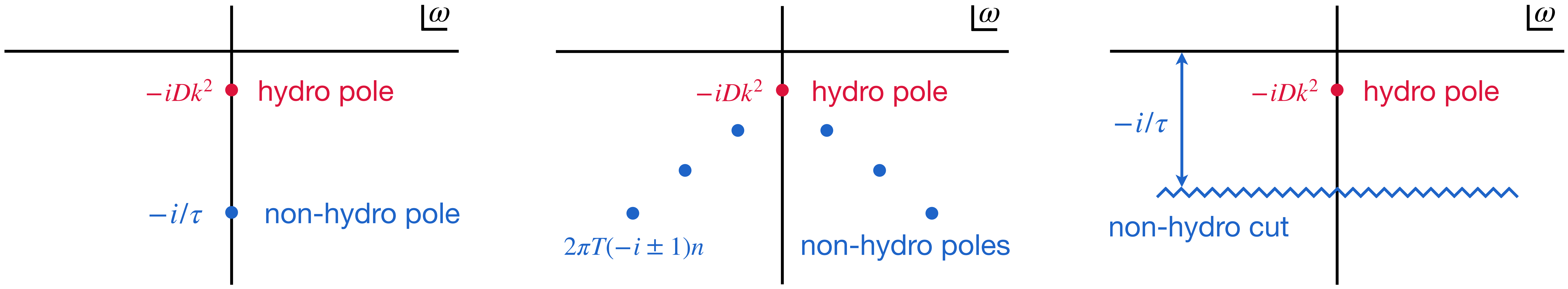




Abbasi, XA and Wu, WIP

Non-universal non-hydro modes

- Analytic structures of retarded **Green functions** differ beyond hydro regime.



Maxwell-Cattaneo | Holography | RTA kinetic

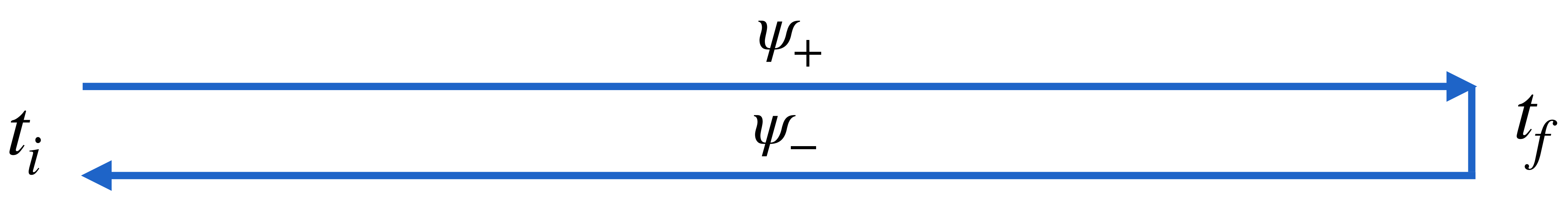
Hydro modes: governed by manifest symmetries associated with conservation laws.

Non-hydro modes: governed by emergent hidden symmetries. XA, Brants, Heller and Yin, 2511.11555

Constraint and symmetries in Keldysh formalism

- Starting from UV/bare action to IR/effective action (top-down)

$$\text{UV} \quad \boxed{I_0[\psi, \chi]} \xrightarrow{\text{Integrate out } \chi} \boxed{I[\psi]} \quad \text{IR}$$



Schwinger, 1961; Keldysh, 1965

Martin et al, 1973

Glorioso et al, 1805.09331

$$\psi_r = \frac{1}{2}(\psi_+ + \psi_-) \qquad \psi_a = \psi_+ - \psi_-$$

- Constraints:

Unitarity: $I^*[\psi_r, \psi_a] = -I[\psi_r, -\psi_a]$, (Reflectivity) $\quad I[\psi_r, \psi_a = 0] = 0$, (Normalizability) $\quad \mathrm{Im}\, I[\psi_r, \psi_a] \geq 0$ (Positivity)

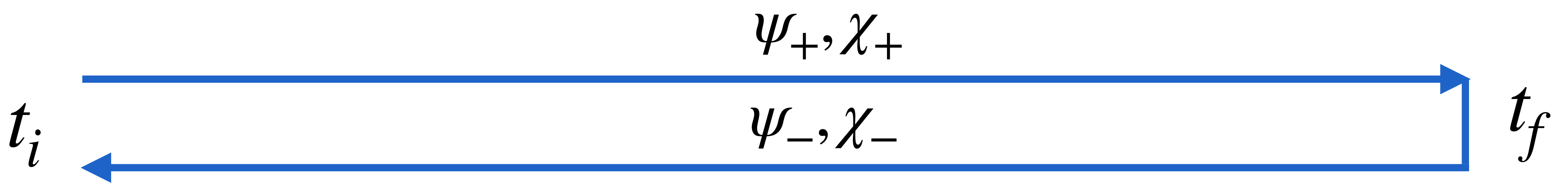
Kubo-Martin-Schwinger **Z(2) symmetry**:

$$I[\psi_r, \psi_a] = I[\tilde{\psi}_r, \tilde{\psi}_a] \quad \Longrightarrow \quad \tilde{\psi}_r(-t) = \psi_r(t), \quad \tilde{\psi}_a(-t) = \psi_a(t) + i\beta\dot{\psi}_r(t)$$

Constraint and symmetries in Keldysh formalism

- Starting from IR/effective action to UV/bare action (bottom-up)

$$\text{UV} \quad \boxed{I_0[\psi,\chi]} \xleftarrow{\text{Integrate in } \chi} \boxed{I[\psi]} \quad \text{IR}$$



Jain et al, 2309.00511

Mullins et al, 2309.00512

$$\psi_r = \frac{1}{2}(\psi_+ + \psi_-) \qquad \psi_a = \psi_+ - \psi_-$$

- Constraints:

Unitarity: $I^*[\chi_r, \psi_a] = -I[\chi_r, -\chi_a]$, $I[\chi_r, \psi_a = 0] = 0$, $\mathrm{Im}\, I[\chi_r, \chi_a] \geq 0$

(Reflectivity) (Normalizability) (Positivity)

Kubo-Martin-Schwinger **Z(2) symmetry**:

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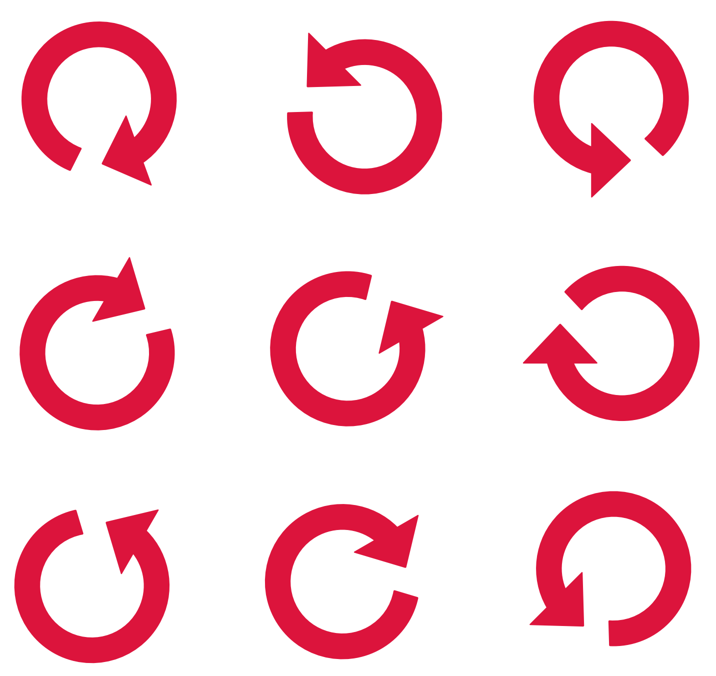
Explicit U(1) and emergent shift

- The action is invariant under
  - *Explicit* **U(1) gauge symmetry**

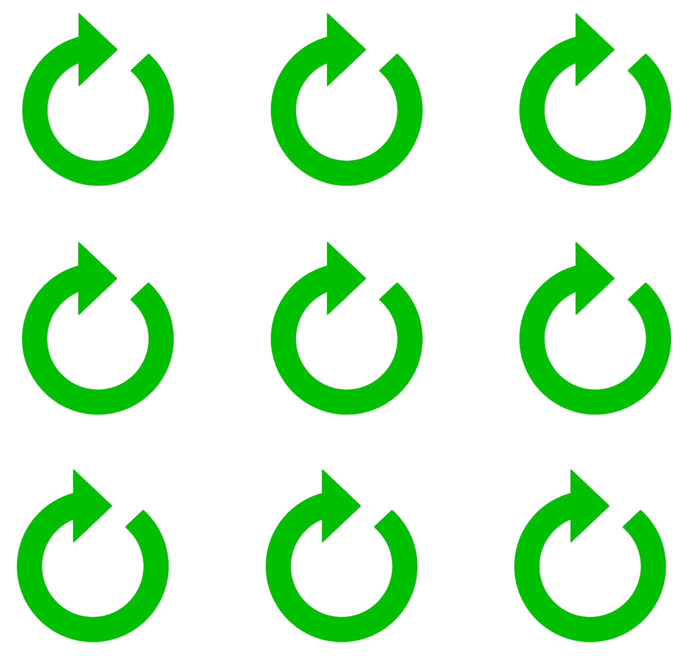
$$\psi \to \psi + \theta(x), \quad A_\mu \to A_\mu - \partial_\mu \theta(x)$$

Gauge-invariant block: $\mathbb{A}_\mu = A_\mu + \partial_\mu \psi$

  - *Emergent* **shift symmetry** in symmetry unbroken phase:



unbroken (disordered) phase



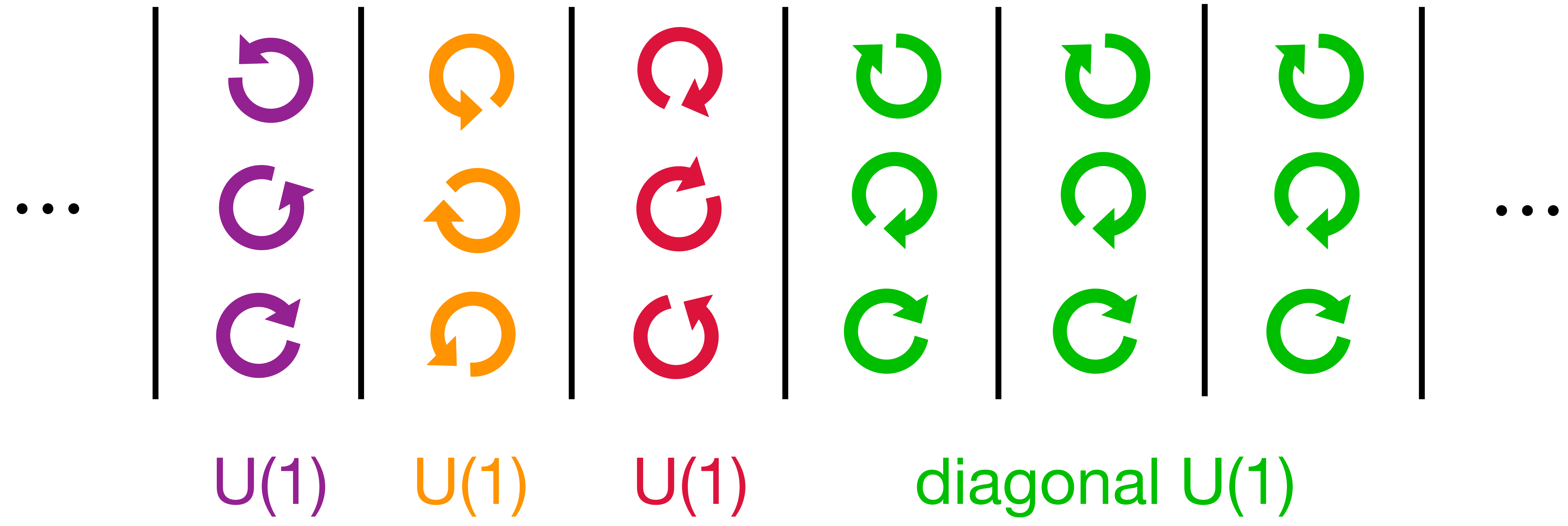
broken (ordered) phase

$$\psi_r \to \psi_r + \lambda(\mathbf{x}), \quad \mathbb{A}_\mu \to \mathbb{A}_\mu + \partial_\mu \lambda(\mathbf{x})$$

$$\psi_r \to \psi_r + \lambda$$

Multiple U(1)s

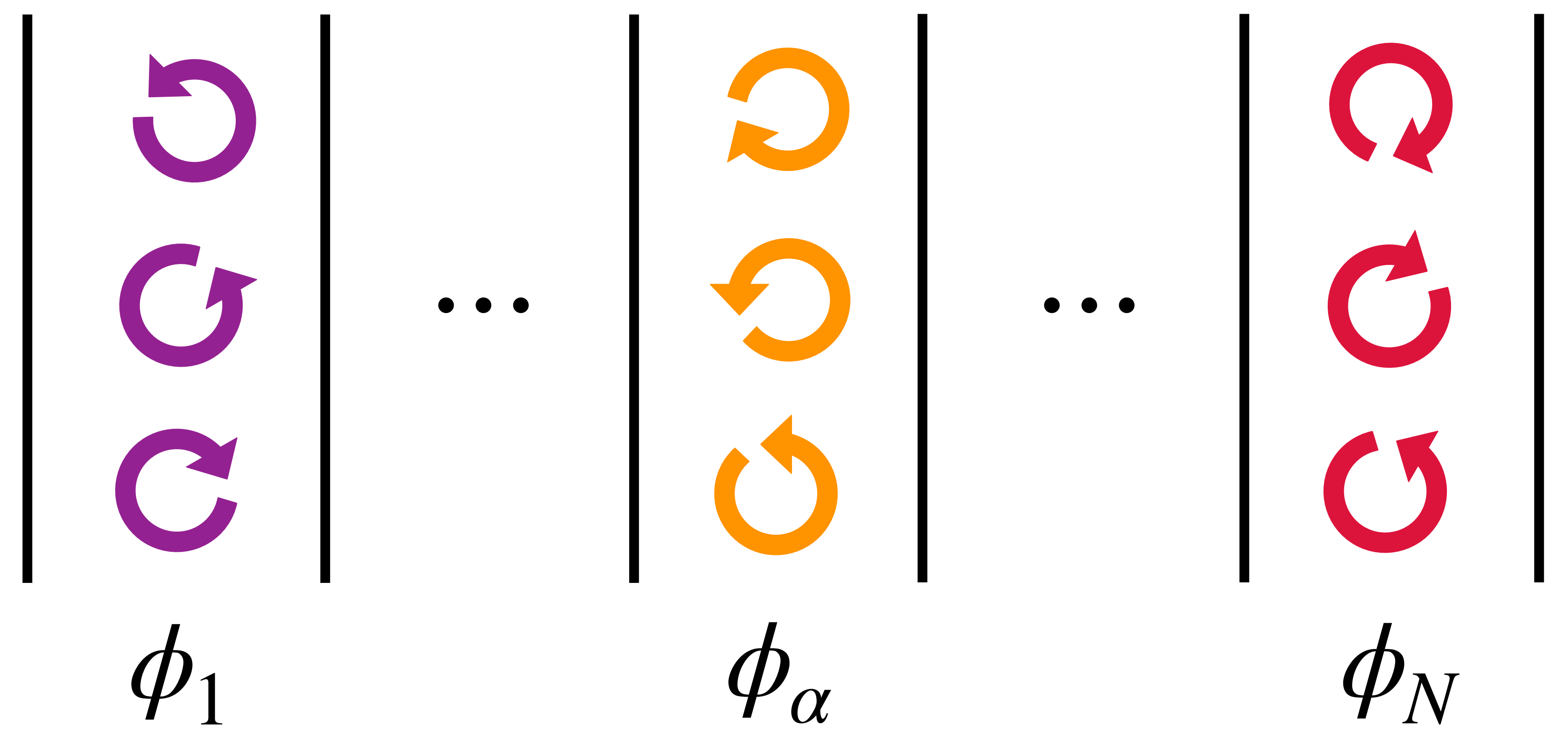
- Generalization to a collection of systems with multiple U(1)s:

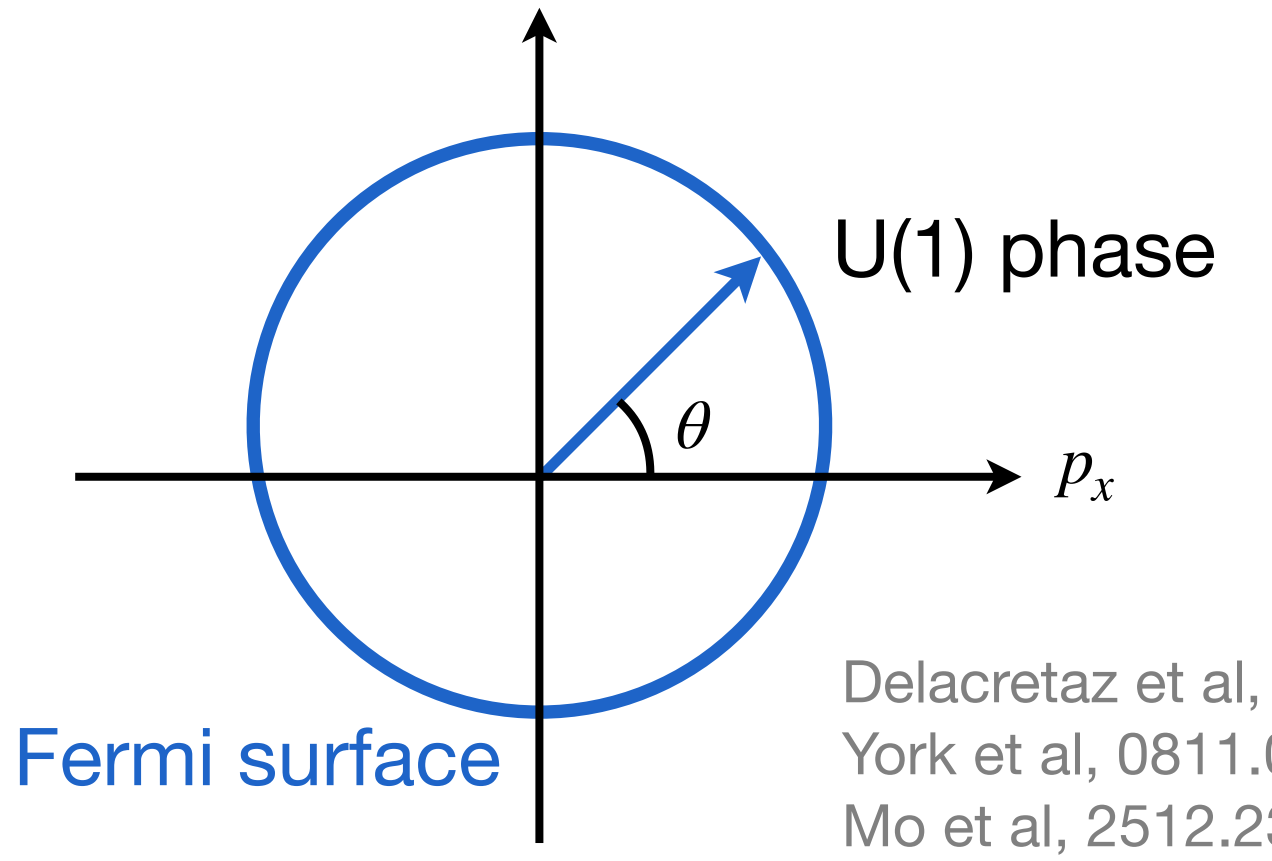


- Scenarios:
  - all U(1)s are **unbroken**
  - all U(1)s are **broken** into a diagonal U(1)
  - **mixture** of the two above

Scenario I: Unbroken U(1) and kinetic-like theory

- All global U(1)s are unbroken





Delacretaz et al, 2203.05004
York et al, 0811.0729
Mo et al, 2512.23960

$$\delta f = \chi_0 (\partial_t \phi^r)$$

$$(\partial_t + \mathbf{v} \cdot \partial)\, \delta f + F \cdot \mathbf{v} = 0$$

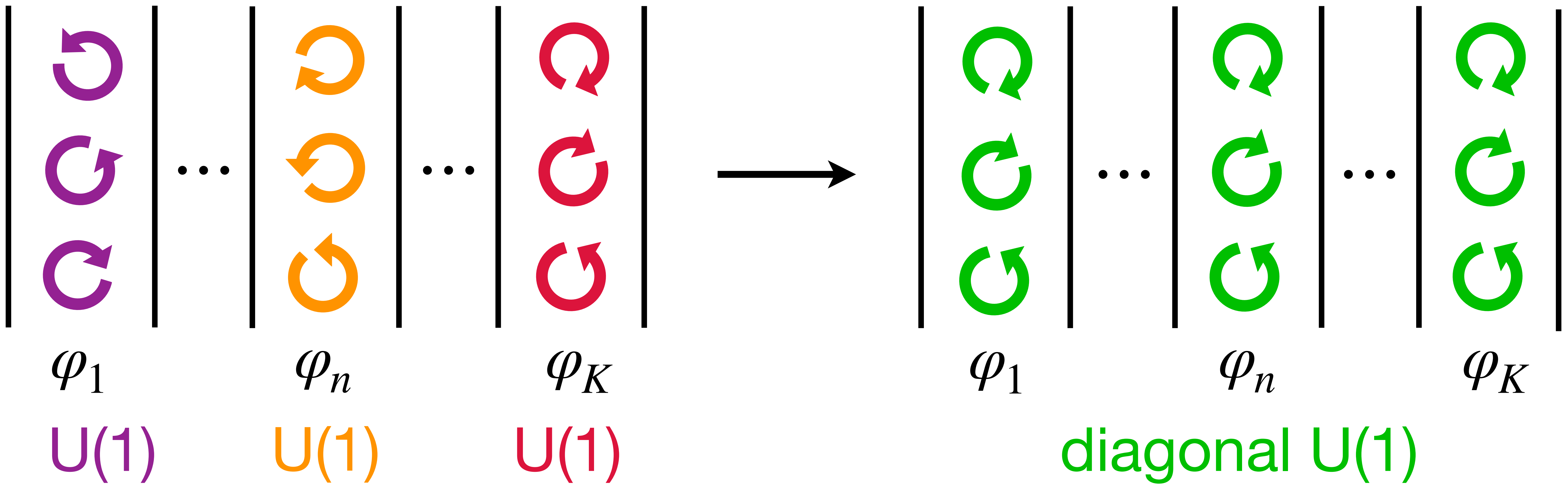
Action for ϕ_α is **invariant** under **shift**

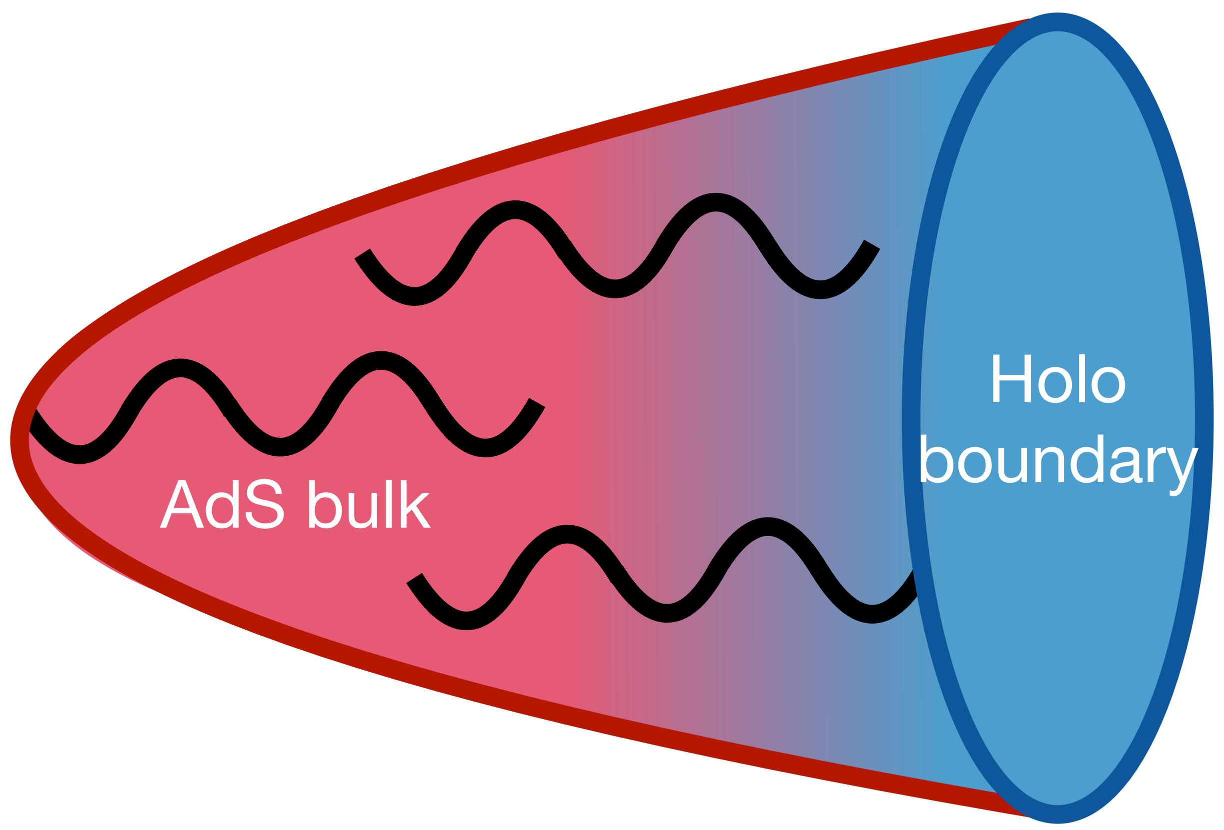
$$I_{\mathrm{U}}[\{\phi^r_\alpha, \phi^a_\alpha\}] = I_{\mathrm{U}}[\{\phi^r_\alpha + \lambda_\alpha(\mathbf{x}), \phi^a_\alpha\}]$$

$$I_{\mathrm{U}}[\{\phi_\alpha\}] = \sum_{\alpha=1}^{N} \int_x \chi_0 \Big\{ (\partial_t \phi^r) \partial_t \phi^a + \frac{1}{2} \left[(\mathbf{v} \cdot \partial) \phi^r \partial_t \phi^a + (r \leftrightarrow a) \right] \Big\}_\alpha$$

Scenario II: Broken into diagonal U(1) and holo-like theory

- All (local) U(1)s are broken into a diagonal U(1)





Holographic liquid with hidden local symmetry

Son and Stephanov, 0304182

Nickel and Son, 1009.3094

Action for $\mathbb{A}_n = A_n + \partial\varphi_n$ is **invariant** under (diagonal) **shift**

$$I_{\rm B}[\{\mathbb{A}_n^r, \mathbb{A}_n^a\}] = I_{\rm B}\{\mathbb{A}_n^r + \partial\lambda(x), \mathbb{A}_n^a\}]$$

$$I_{\rm B}[\{\mathbb{A}_n\}] = \sum_{n=1}^{K} \int_x \left(f^2\, \Delta\mathbb{A}_t^r\, \Delta\mathbb{A}_t^a - g^2 \Delta\mathbb{A}_i^r \Delta\mathbb{A}_i^a + \epsilon \mathbf{E}^r \cdot \mathbf{E}^a - \kappa \mathbf{B}^r \cdot \mathbf{B}^a + \sigma \mathbb{A}^a \cdot \mathbf{E}^r\right)_n$$

where $\Delta\mathbb{A}_n = \mathbb{A}_{n+1} - \mathbb{A}_n$ is a shift-invariant block with nearest-neighbor coupling

Scenario II: Broken into diagonal U(1) and holo-like theory

- When $K = 1$ (one layer), identifying

$$n \leftrightarrow f^2 \Delta \mathbb{A}_0, \qquad J_i \leftrightarrow -g^2 \Delta \mathbb{A}_i$$

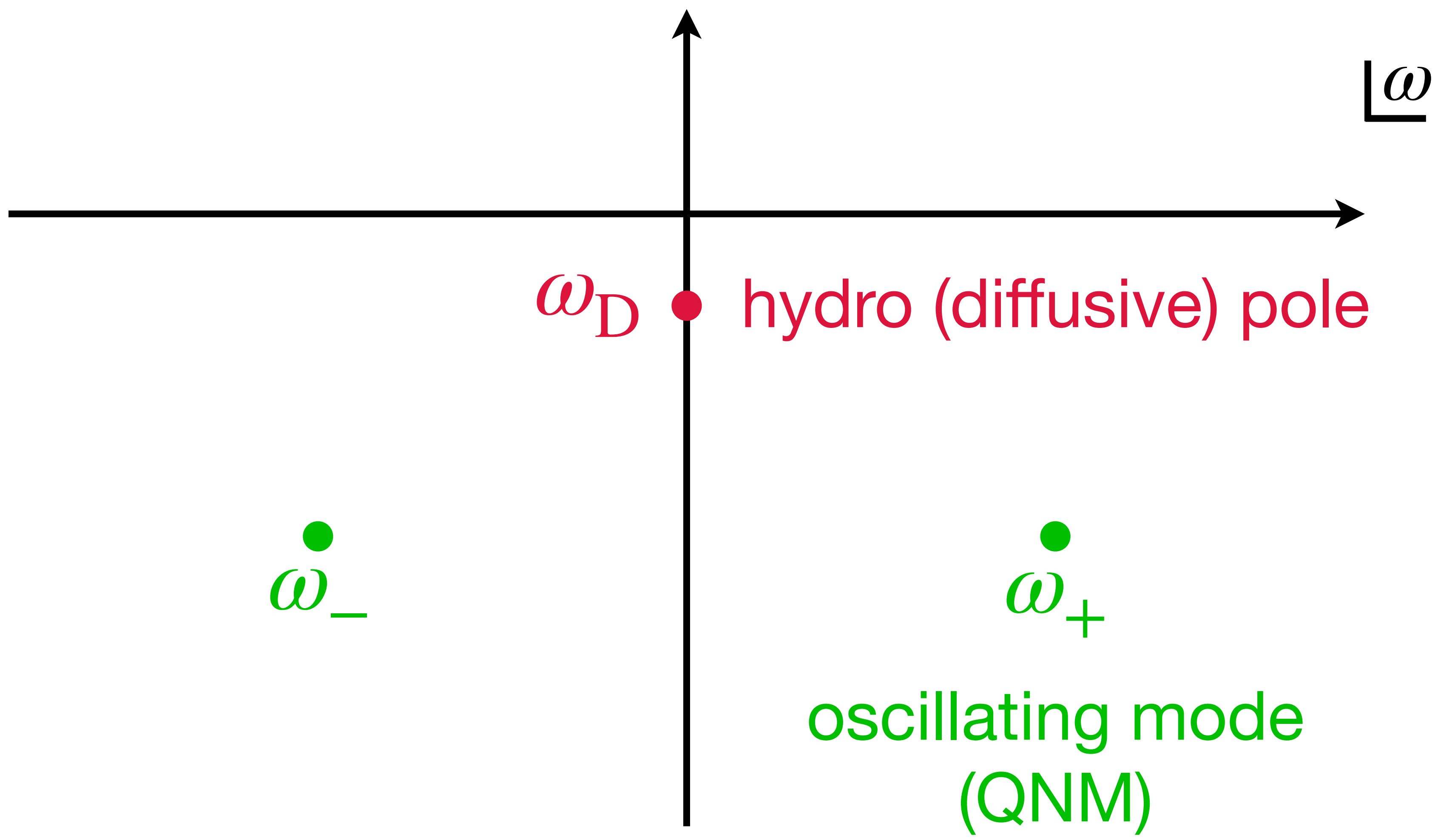
$$\chi \equiv \frac{\lambda}{D} \leftrightarrow f^2, \qquad \frac{\lambda}{\tau} \leftrightarrow g^2,$$

the model is identical to Maxwell-Cattaneo:

$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0, \qquad \tau \partial_t \vec{J} = -(\vec{J} + D \vec{\partial} n)$$

NB: the non-hydrodynamic (gapped) mode results from mass acquired by Higgs mechanism

Maxwell 1867, Cattaneo 1948
Ahn et al, 2506.00926



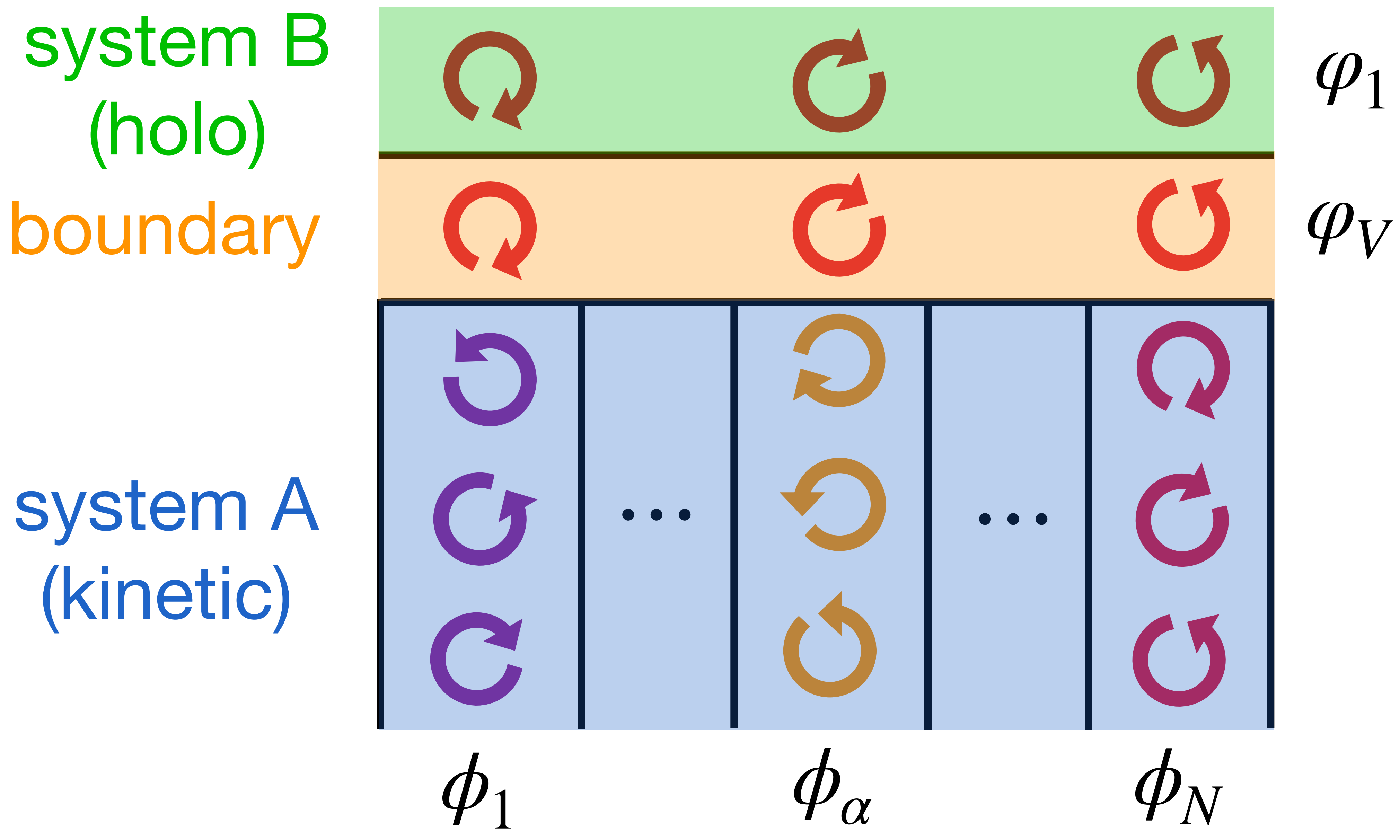
- When $K \to \infty$ (continuous limit), the model is identical to holography via dimensional deconstruction. Arkani-Hamed et al, 0104005

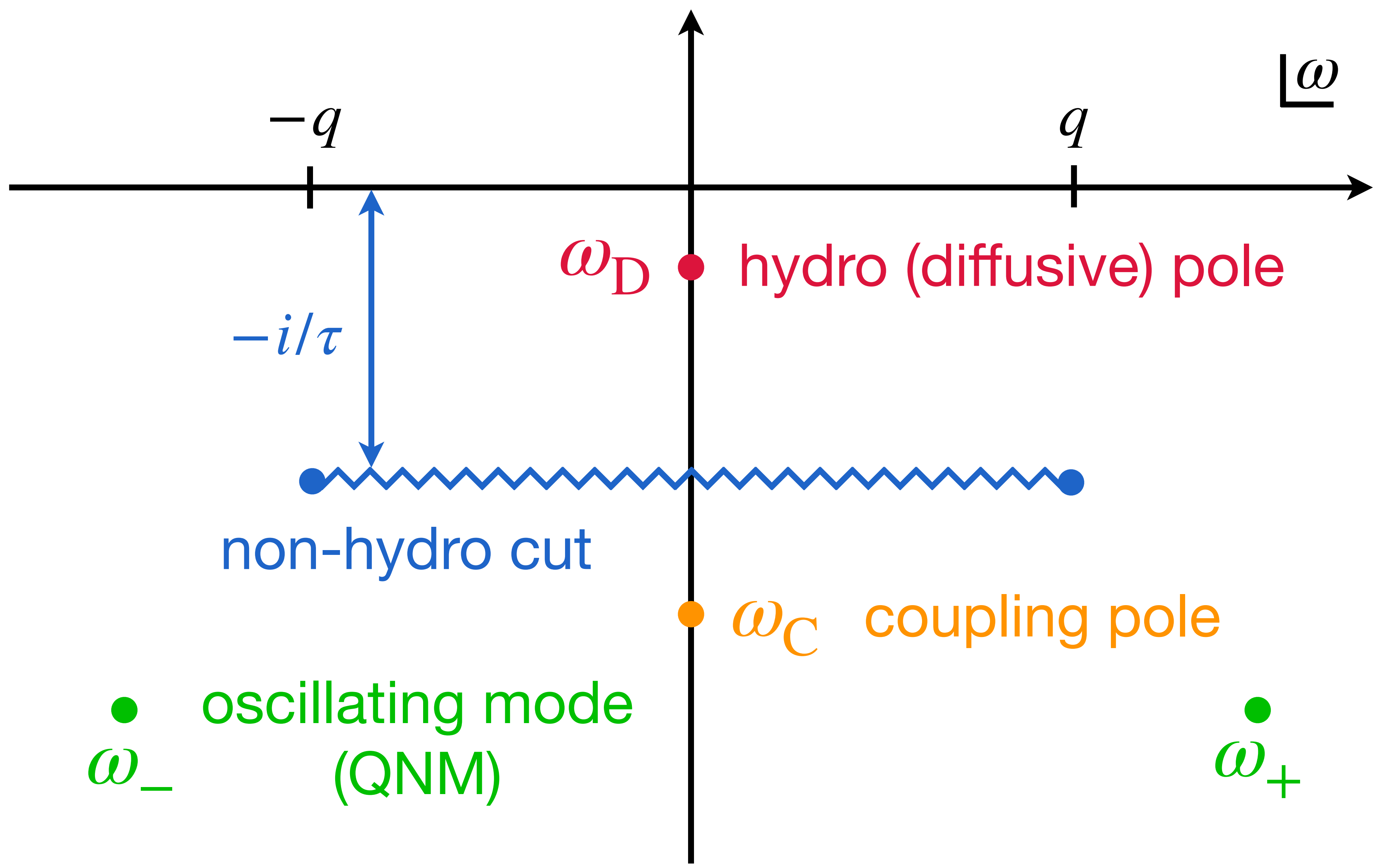
Scenario III: Mixture

- The analytic structure of the simplest mixture model

$$I_{(a)} = I_{\mathrm{U}}[\{\phi^r_\alpha, \phi^a_\alpha\}] + I_{\mathrm{B}}[\{\mathbb{A}^r_n, \mathbb{A}^a_n\}] + \Delta I[\phi_\alpha, \varphi_V]$$

$$\Delta I = -\frac{\chi_{\mathrm{U}}}{\tau_{\mathrm{R}}} \sum_\alpha \int_x (\partial_t \phi_\alpha - \partial_t \varphi_V)(\phi^a_\alpha - \varphi^a_V)$$



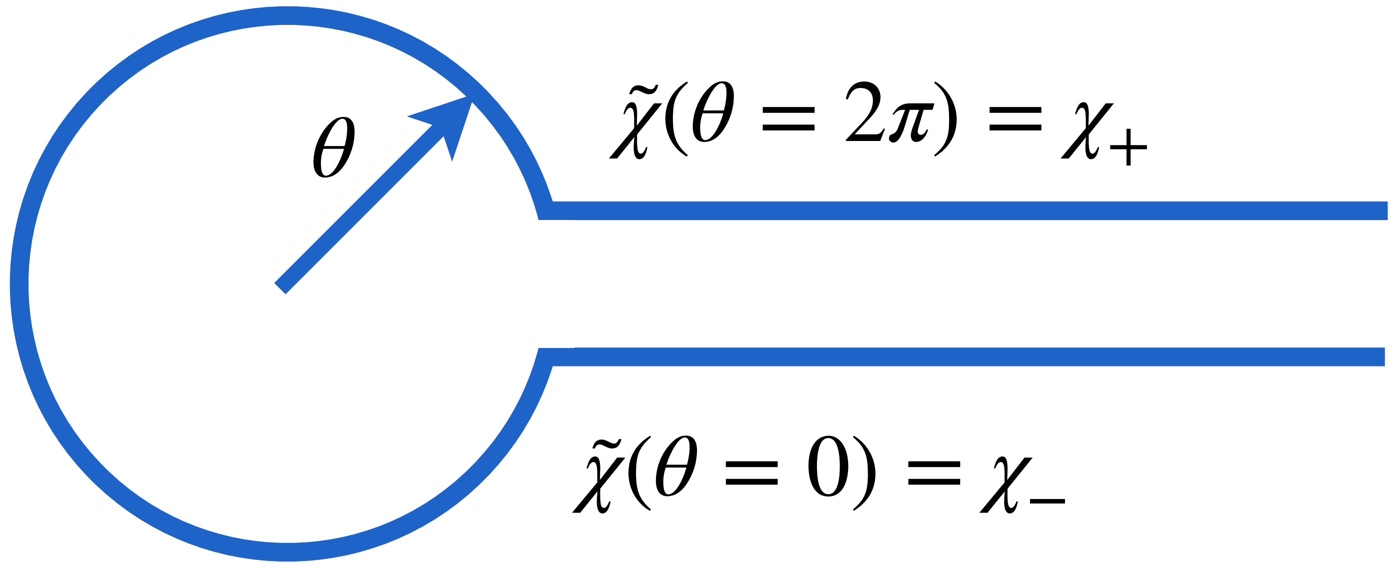


Fluctuations and shift symmetry

- “Integrated in” action **invariant** under **shift**

$$\tilde{I}[\tilde{\chi}(\theta, x)] = \tilde{I}[\tilde{\chi}(\theta, x) + \lambda(x)]$$

$$\tilde{I}[\tilde{\chi}] = \int_x \int_0^{2\pi} d\theta \, \frac{\sigma}{2} \left[i(\partial_\theta \tilde{\chi})^2 - \frac{\beta}{2\pi} (\partial_\theta \tilde{\chi}) \, \partial_t \tilde{\chi} \right] \qquad \text{(local)}$$



$\downarrow$ integrate out $\tilde{\chi}$

$$I_{\mathrm{FD}} = \int_{\mathbf{x},\omega} \omega \, \sigma \, \chi^a(-\omega) \left[i\chi^r(\omega) + \coth\left(\frac{\beta\omega}{2}\right) \chi^a(\omega) \right] \qquad \text{(non-local)}$$

The power of **shift symmetry**: fluctuation-dissipation relation in the far-from-equilibrium regime is reproduced very *nontrivially*!

Recap

- Fluctuation equations for full hydrodynamics are ready for use now.
- The non-universal non-hydro behavior are governed by symmetries.

Outlook

- “Dynamical modeling” of the fluctuation equations with QCD critical point, compare to experimental data after freezeout.
- Construct EFT with different degrees of freedom (e.g., energy-momentum, spin, isospin) and symmetries (e.g., diffeomorphism, SU(2)); Understand the universality in far-from-equilibrium dynamics.

Thank You