Exploring Meson Structure via Lattice QCD

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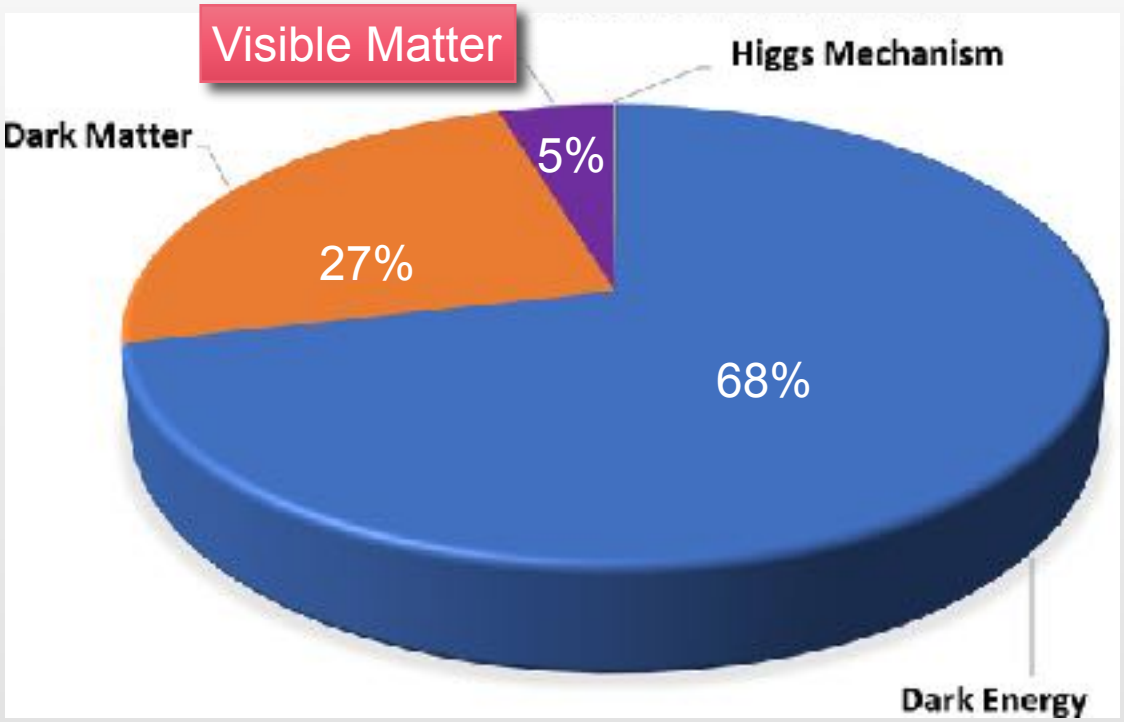
Introduction

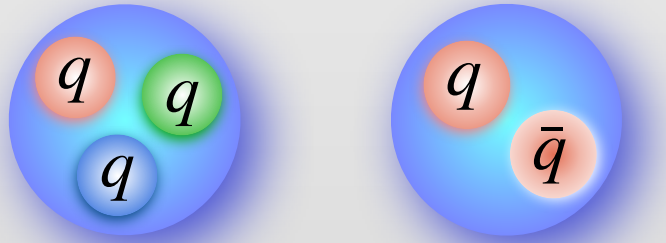
Front. Phys. 16 (2021) 6, 64701



Mass-energy budget of the universe

- Basic structure of visible matter
- Standard Model
- Leptons & **Quarks** & Elementary Bosons
- QCD confinement

Hadrons



Baryon **Meson**

Pion & Kaon:

Goldstone bosons

Mass generation

Hadron Structure

- Naive quark model

only valence quarks

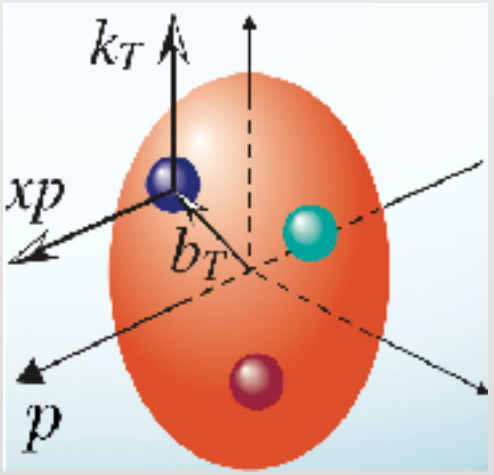
DIS experiments

- Modern quark model

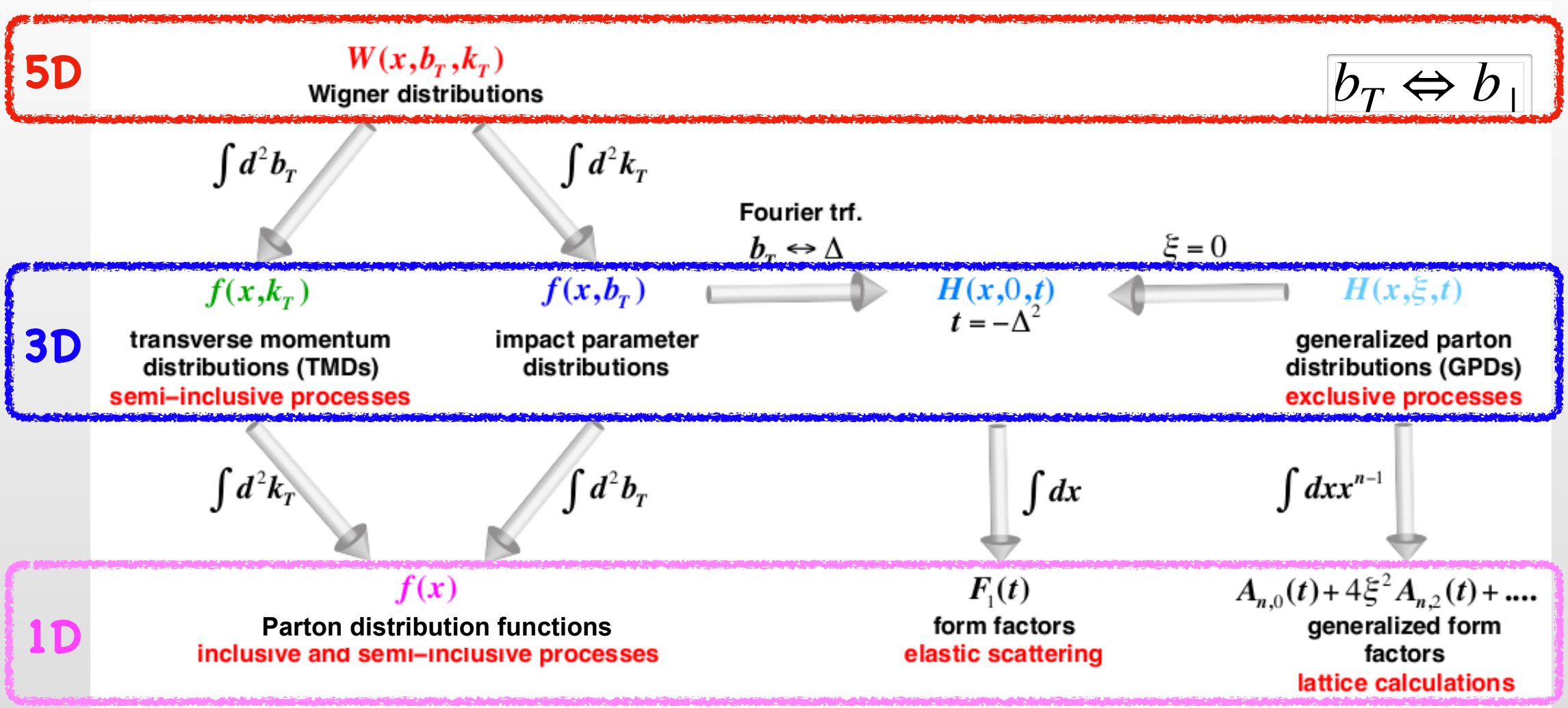
valence quarks +
see quark-antiquark pairs + gluons

Partons

Front. Phys. 16 (2021) 6, 64701

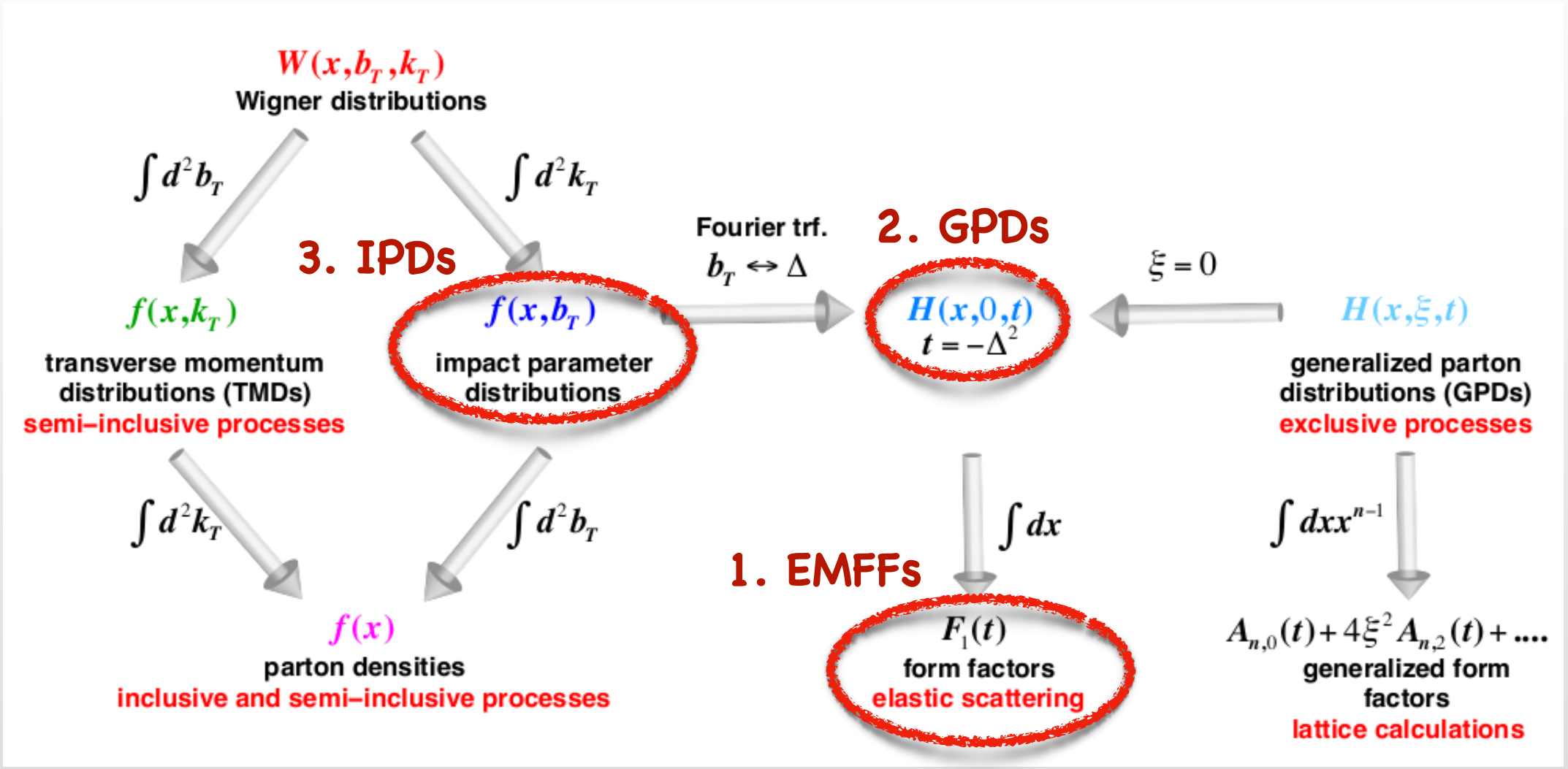


Wigner Distributions



Eur. Phys. J. A 52 (2016) 9, 268

Our topics



Eur. Phys. J. A 52 (2016) 9, 268

From 1D Size to 3D Tomography

1D EMFFs: Global structure

- Low Q^2: How big is the meson?
- High Q^2: Hadronic ⇒ Partonic

  Non-perturbative ⇒ Perturbative

3D GPDs: Internal partons

- Where are the quarks inside?
- 3D impact-parameter distributions

Providing critical benchmarks for the future experimental facilities

Approches

- Experiment: indirect extraction, high cost, limited coverage
- **Lattice QCD: non-perturbative, first principle**
- Perturbative QCD: not reliable at lower energy scales
- Factorization Framework: relies on non-perturbative inputs
- Other methods: model assumptions

  e.g. Dyson-Schwinger equation (DSE), vector meson dominance (VMD) model, …

Current Landscape and Future Reach

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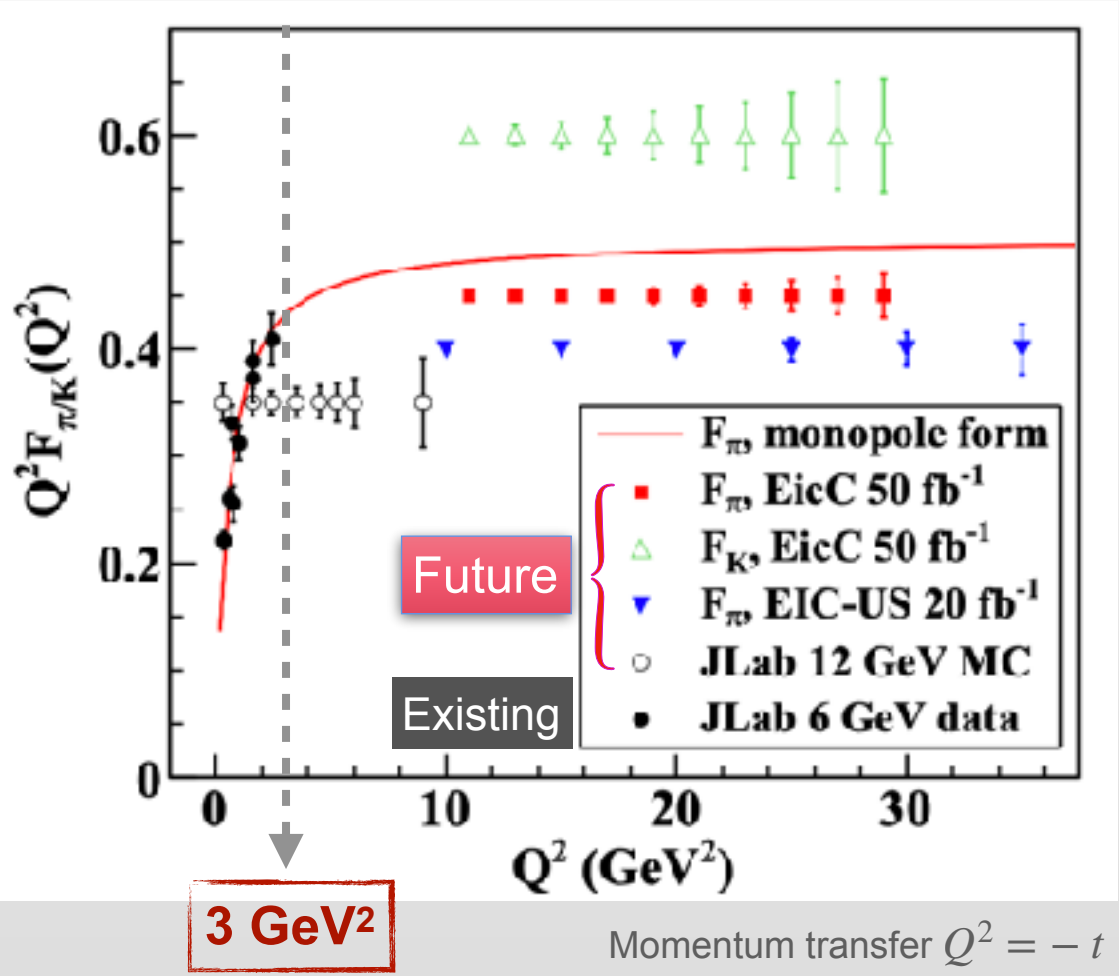


Momentum transfer $Q^2 = -t$

Meson research is very limited

EPJA 48 (2012) 187 JPG 48 (2021) 075106 arXiv: 2102.09222

- Experiment: JLab12, EIC, EicC ...

Gao et al., PRD 96 (2017) 034024

- Effective theory: QCD sum rules, DSE ...

- **Lattice QCD:**
  - State-of-the-art:

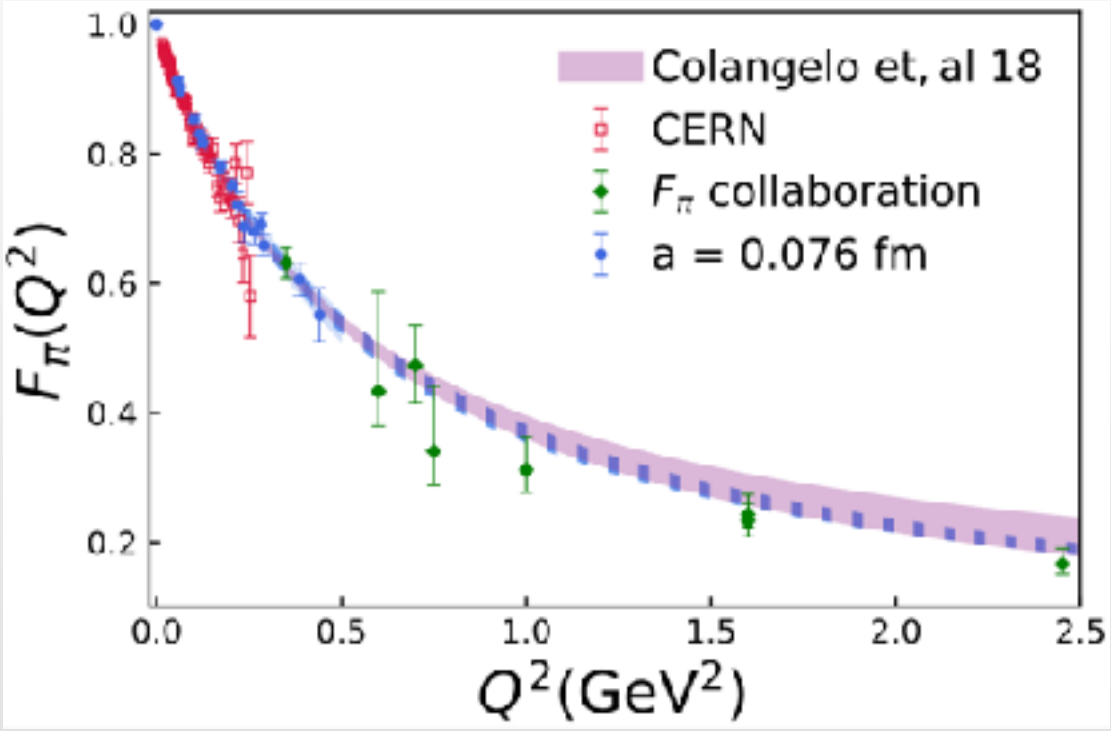
    PRD 96 (2017) 114509 ETMC, PRD 105 (2022) 054502

    $Q^2 \leq 6$ (pion), 3 (kaon) GeV^2
  - This work:

    Q^2 up to 10 (pion), 28 (kaon) GeV^2

The Gap at Higher Q^2

Low Q^2: Good agreement



PRD 104 (2021) 114515

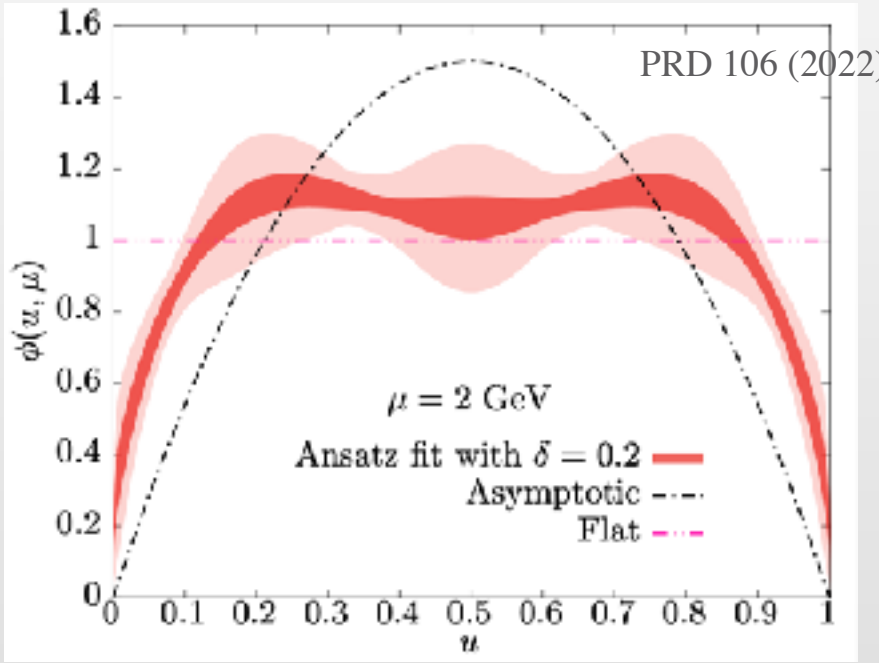
→ **Larger Q^2**

High Q^2: Factorization framework

$$F(Q^2) = \int\int \mathrm{d}x\mathrm{d}y \Phi^*(y,\mu_F^2) T_H(x,y,Q^2,\mu_R^2,\mu_F^2) \Phi(x,\mu_F^2)$$

Hard-process kernel

Distribution amplitude



→ **Test the factorization framework**

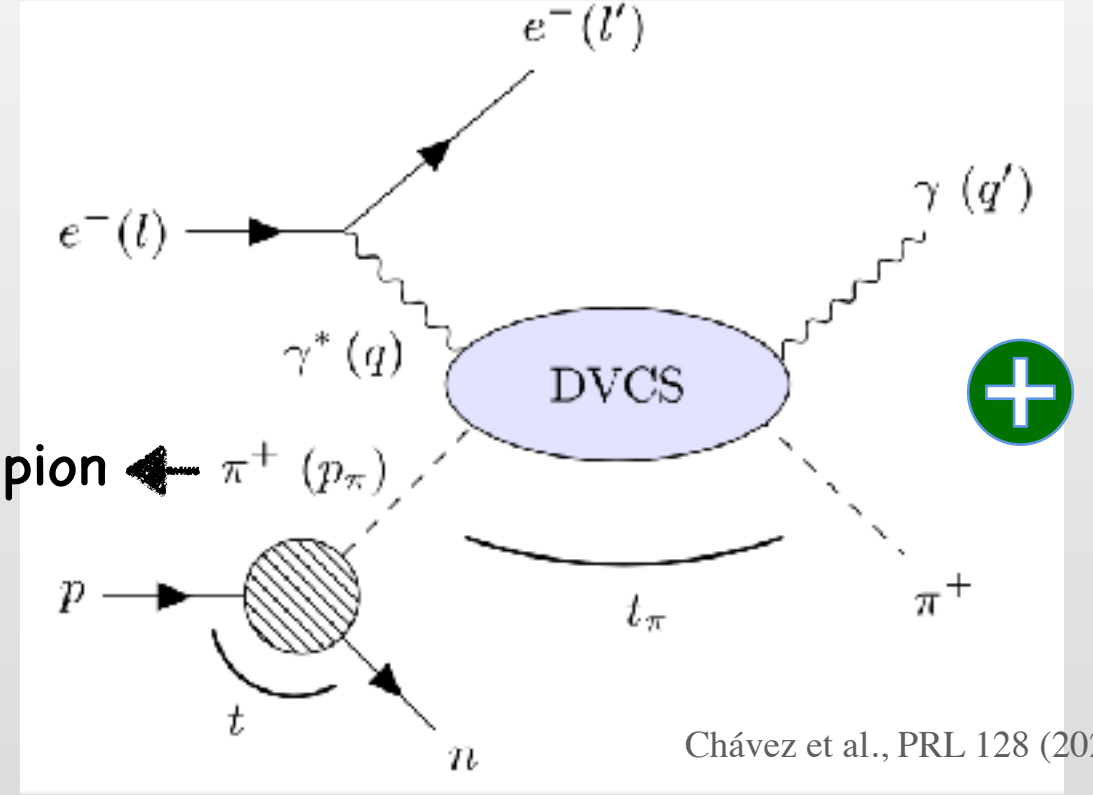
Link

The Challenge of Extracting 3D Structure

1D

- Form Factors (FFs)
- Parton Distribution Functions (PDFs)

→ 3D Generalized Distribution Functions (GPDs)



Produce the pion

⊕ Models

More Difficult

Lattice QCD !

Chávez et al., PRL 128 (2022) 202501

CONTENTS

Lattice QCD

- Discretizing spacetime: $N_s^3 \times N_t$
- From first principles ⇔ From Lagrangian
  - Lagrangian $\mathscr{L}_{\mathrm{QCD}}$ → action S →

$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}A \mathscr{D}\bar{\psi} \mathscr{D}\psi \, O \, e^{-S}$$

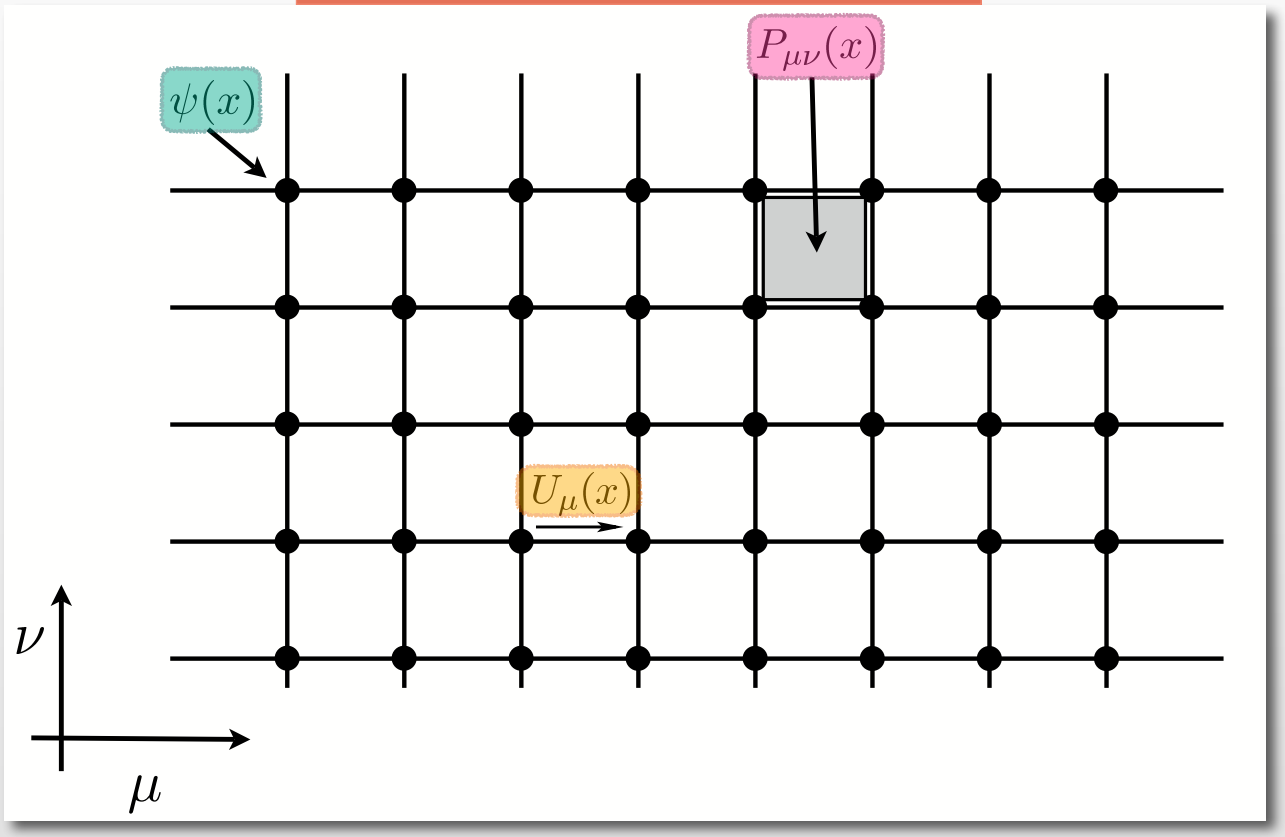
Gluon fields | Quark fields

- Gauge invariance: $A \rightarrow U \rightarrow P$

Link variable $U_\mu(x) = \exp\left(ig_s a A_\mu(x)\right)$

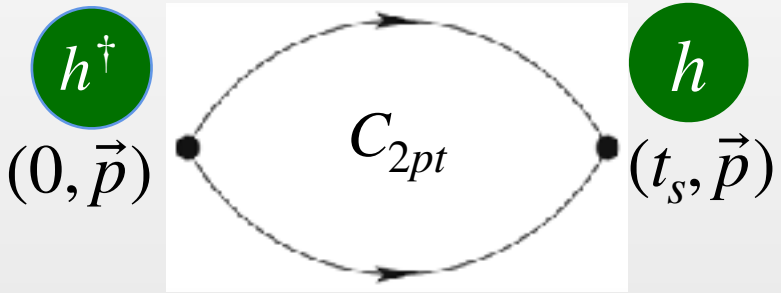
Plaquette $P_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)$

Simplified 2D lattice



Lattice QCD

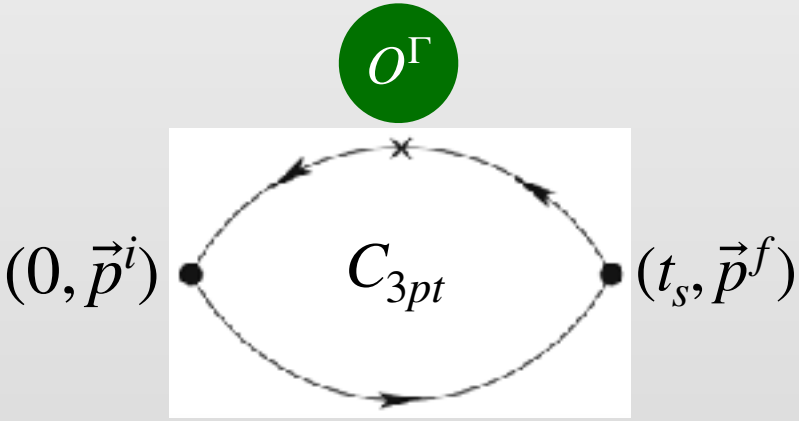
$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}A \mathscr{D}\bar{\psi} \mathscr{D}\psi \, O \, e^{-S}$$



$$C_{2\mathrm{pt}}(\mathbf{P}, t_s) = \langle h(\mathbf{P}, t_s) h^\dagger(\mathbf{P}, 0) \rangle$$

$$O^\Gamma(\boldsymbol{\Delta}, \tau, z) = \sum_y e^{-i\boldsymbol{\Delta}\cdot\mathbf{y}} \bar{\psi}(\mathbf{y}, \tau) \Gamma \mathscr{W}(z) \psi(\mathbf{y} + z, \tau)$$

$$\mathbf{P}^f = \mathbf{P}^i + \boldsymbol{\Delta}$$



$$C^\Gamma_{3\mathrm{pt}}(\mathbf{P}^f, t_s, \boldsymbol{\Delta}, \tau, z) = \langle h(\mathbf{P}^f, t_s) O^\Gamma(\boldsymbol{\Delta}, \tau, z) h^\dagger(\mathbf{P}^i, 0) \rangle$$

Lattice QCD

$$C_{2\mathrm{pt}}(t_s) = \sum_{k=0}^{N_{\mathrm{state}}-1} A_k \left[e^{-E_k t_s} + e^{-E_k (aN_t - t_s)} \right]$$

Fit with $N_{\mathrm{state}} = 2$ → $E_0, E_1;\ A_0, A_1$

$$C^{\Gamma}_{3\mathrm{pt}}(\mathbf{P}^f, t_s, \boldsymbol{\Delta}, \tau, z) = \sum_{n,k=0}^{N_{\mathrm{state}}-1} \langle 0 | \hat{h}(\mathbf{P}^f, t_s) | n \rangle \langle k | \hat{h}^{\dagger}(\mathbf{P}^i, 0) | 0 \rangle e^{-(\Delta E^i_k - \Delta E^f_n)\tau} e^{-\Delta E^f_n t_s} \langle n | \hat{O}^{\Gamma}(\boldsymbol{\Delta}, \tau, z) | k \rangle$$

A_n, A_k

Can be canceled by C_{2pt}

$n = k = 0$ → M^{B}

→ $R^{fi} \sim C_{3pt} \,/\, C_{2pt} \xrightarrow{t_s \to \infty}$ Bare matrix element $M^{\mathrm{B}} \xrightarrow[\text{local}]{z=0}$ Bare form factor F^{B}

Lattice data (Euclidean space) ⇒ Physical observables (Minkowski space) ?

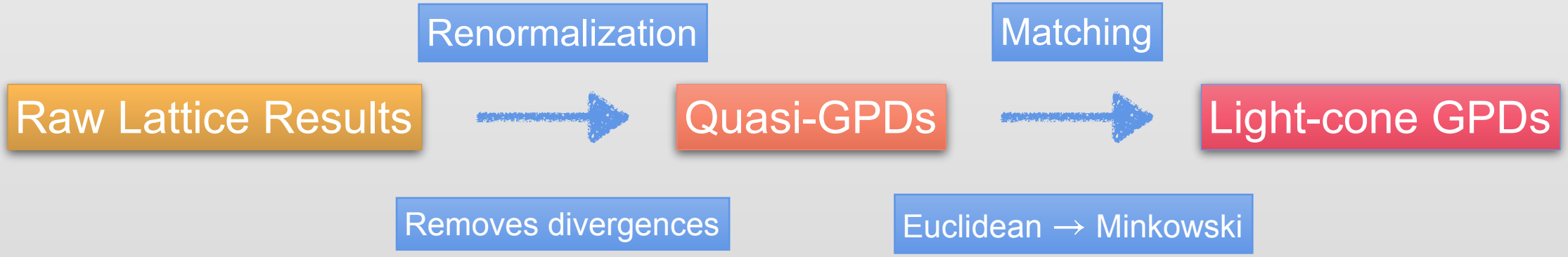
From Lattice to Physical World

Local Operator

- EMFFs
- Wilson line length: $z = 0$
- No anomalous dimension
- Divided by $1/Z_V = F^B(0)$

Non-local Operator

- GPDs
- Quarks separated by a Wilson line
- Suffer from various divergences
- Renormalization + Matching



CONTENTS

EMFFs: Lattice Setup

- Lattice size: $N_s^3 \times N_t = 64^3 \times 64$
- Lattice spacing: $a = 0.076$ & 0.04 **fm for kaon,** $a = 0.076$ **fm for pion**
- Sea quark: Highly Improved Staggered Quark (HISQ) action

  Valence quark: Wilson-Clover action

  ⇒ **at the physical point**

- State-of-the-art:

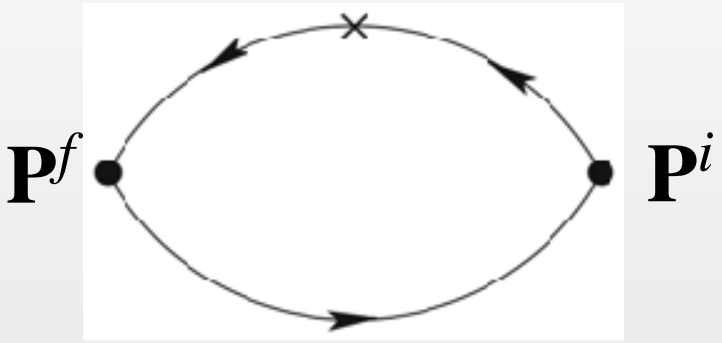
  $Q^2 \leq 6$ (pion), 3 (kaon) GeV

- **This work:**

  Q^2 **up to 10 (pion), 28 (kaon) GeV^2**

EMFFs: Techniques to High Q^2

Previous studies:

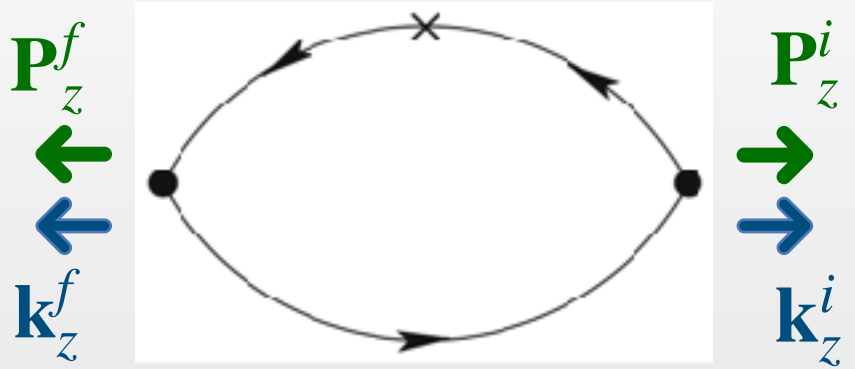


- Fix $\mathbf{P}_z$, only vary $\mathbf{P}_\perp$
- Fix $\mathbf{P}^f$, only vary $\mathbf{P}^i$

↓

Limited Q^2

This work:



- Vary $\mathbf{P}^i$ & $\mathbf{P}^f$ ($\mathbf{P}_\perp$ & $\mathbf{P}_z$)
- Momentum smearing: $\zeta = \dfrac{\mathbf{k}_z^i}{\mathbf{p}_z^i} = \dfrac{\mathbf{k}_z^f}{\mathbf{p}_z^f}$

↓

Higher Q^2

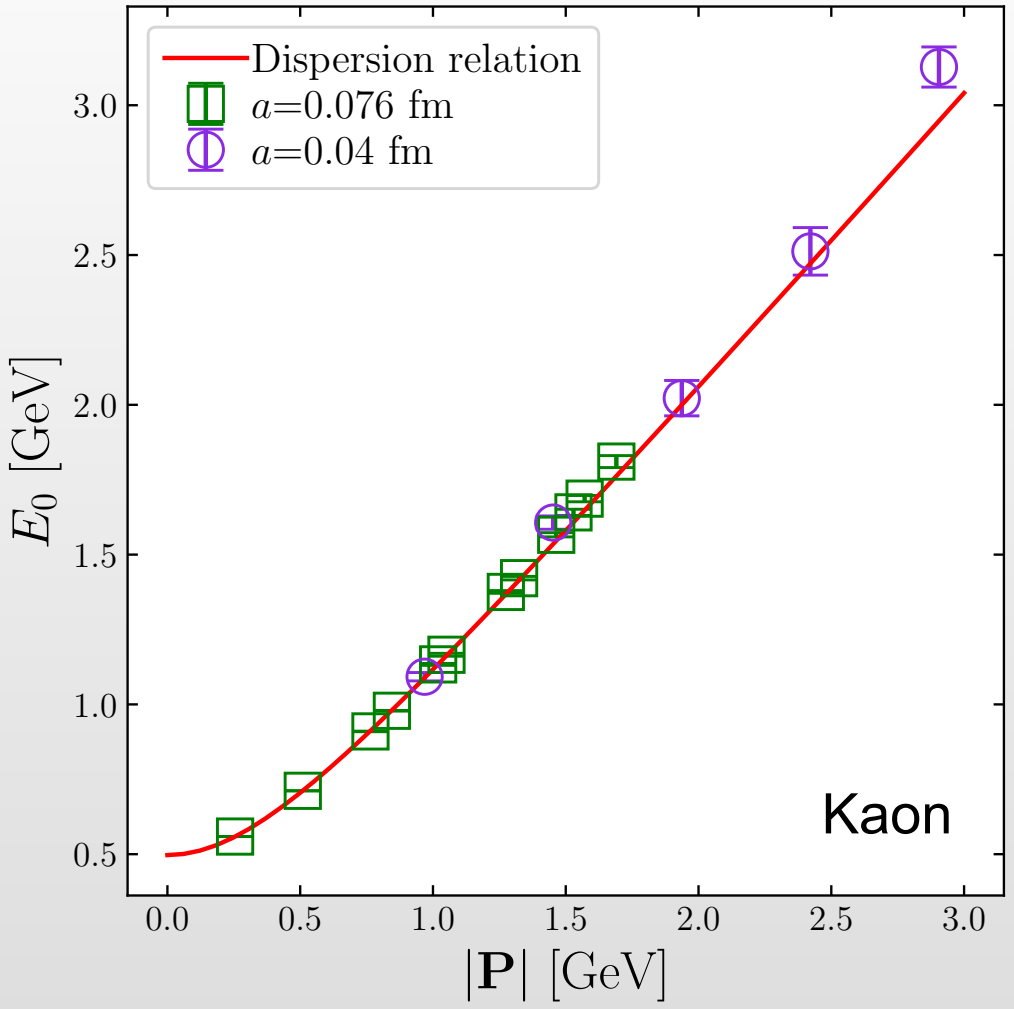
EMFFs: C_{2pt}

- Extract E & A from C_{2pt}

$$C_{2\mathrm{pt}}(t_s) = \sum_{k=0}^{N_{\mathrm{state}}-1} A_k \left[e^{-E_k t_s} + e^{-E_k (aN_t - t_s)} \right]$$

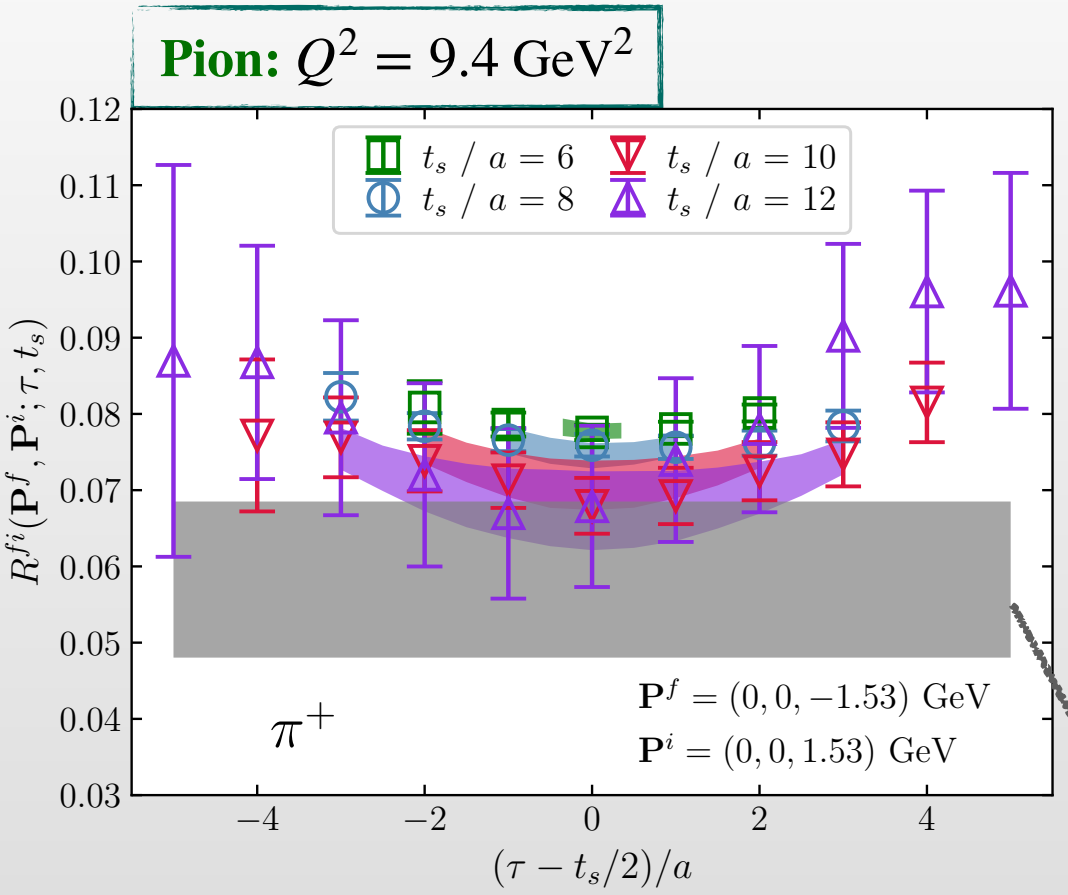
Fit with $N_{\mathrm{state}} = 2$

E_0, E_1; A_0, A_1

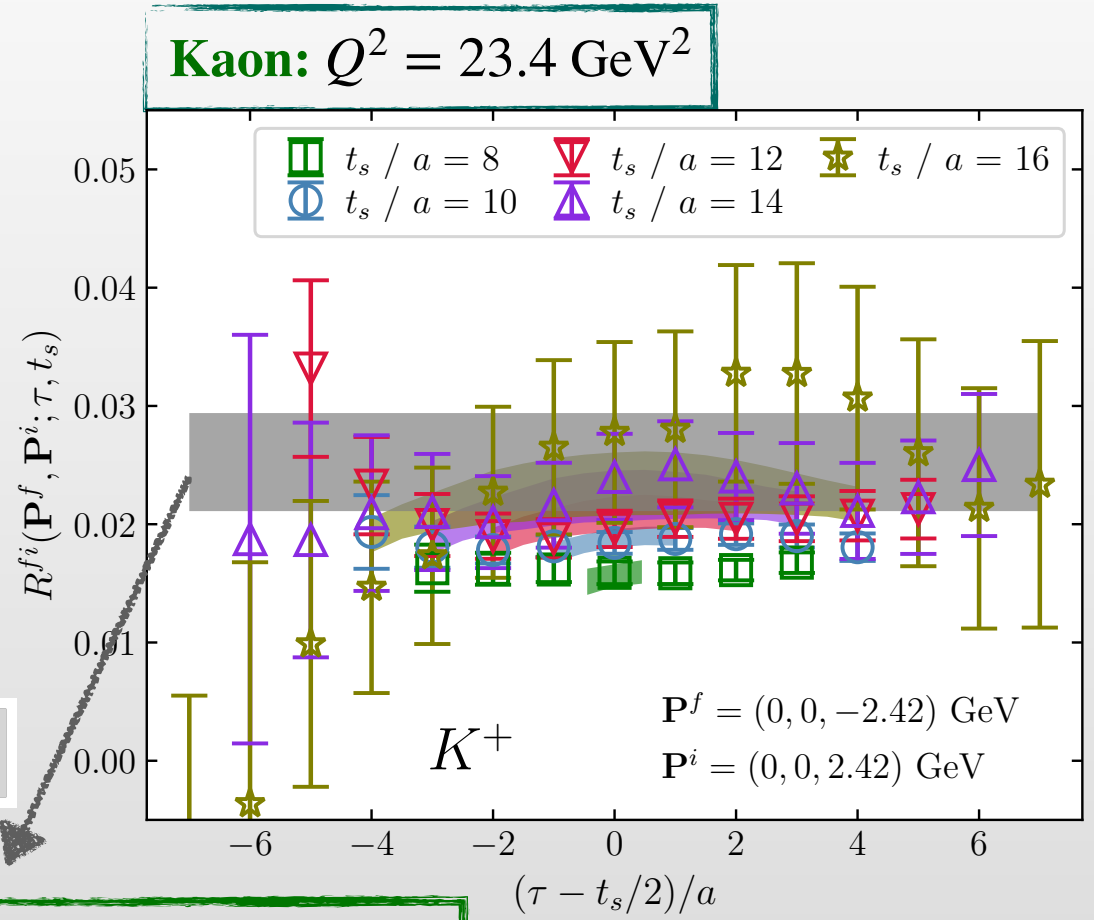


EMFFs: Bare FF F^B

Extract F^{B} from $R^{fi} \sim C_{3\mathrm{pt}}/C_{2\mathrm{pt}}$



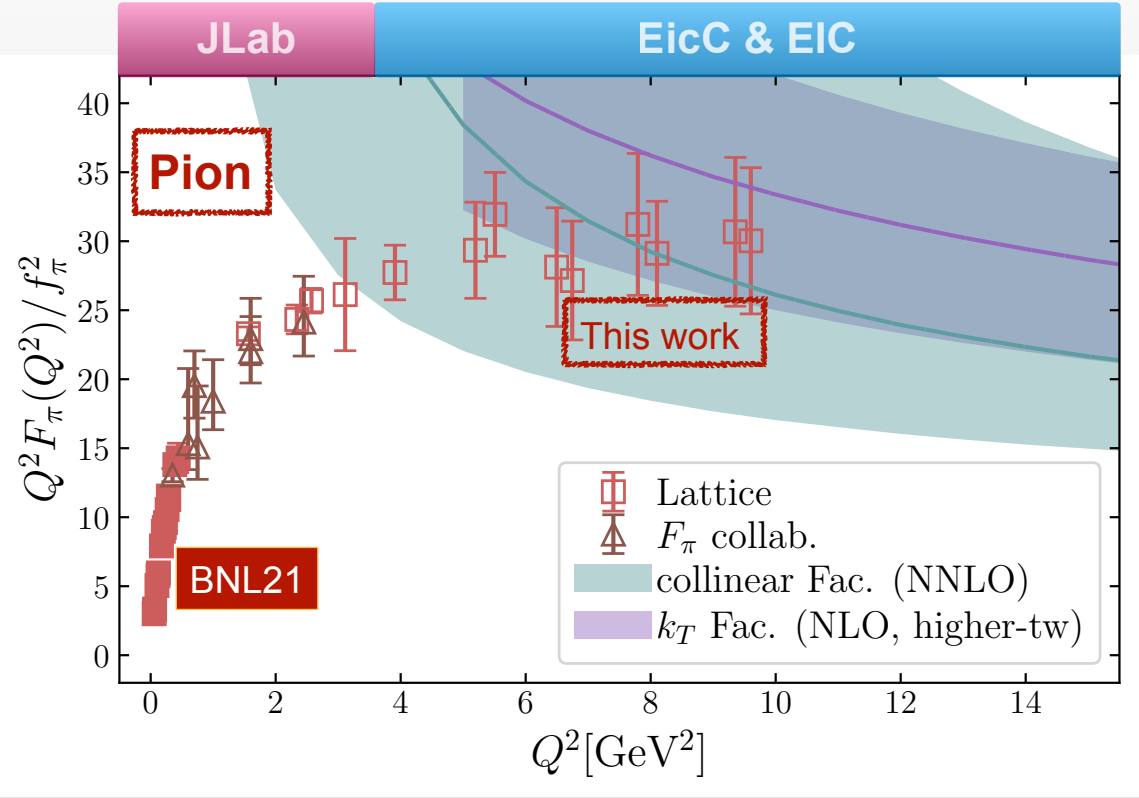
$t_s \to \infty$

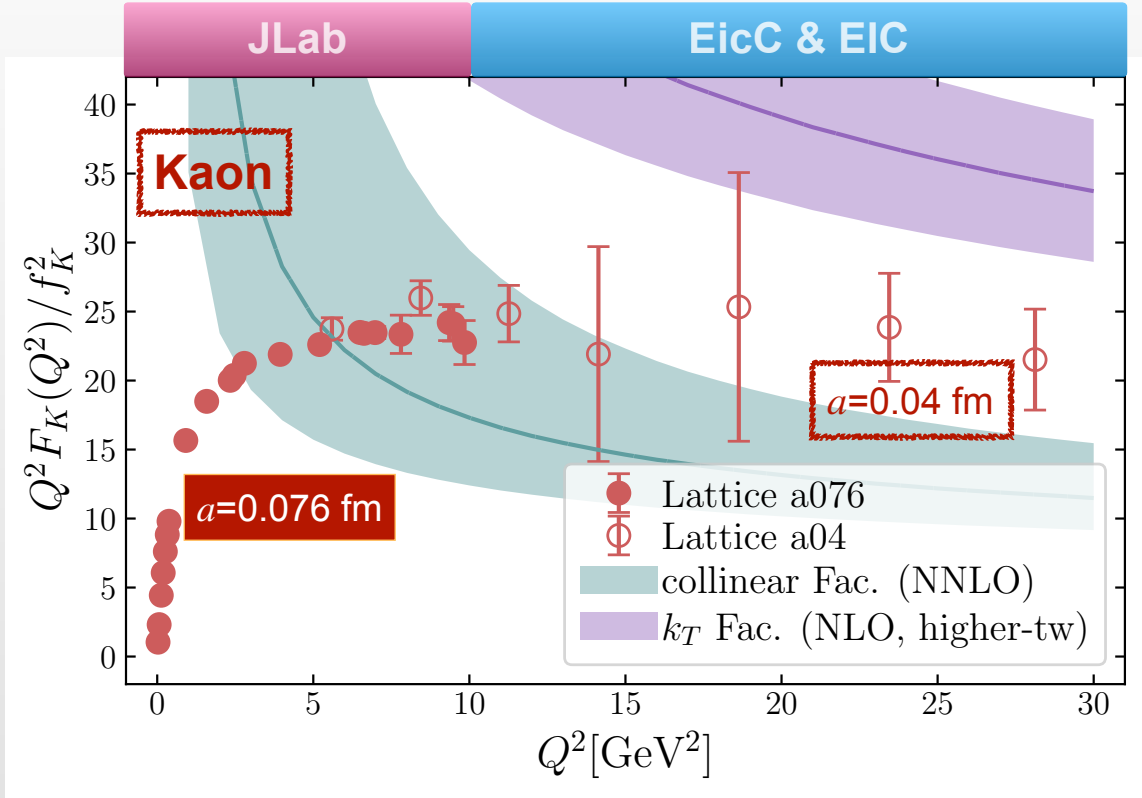


Bare Form factor $F^{\mathrm{B}} \times Z_V^{-1} = F(Q^2)$

EMFFs

$$Q^2F(Q^2)/f^2 \xrightarrow{Q^2\to\infty} \text{Constant}$$





Hard-process kernel

$$F(Q^2) = \int\int \mathrm{d}x\mathrm{d}y \Phi^*(y,\mu_F^2) T_H(x,y,Q^2,\mu_R^2,\mu_F^2) \Phi(x,\mu_F^2)$$

Distribution amplitude

Lattice (BNL21): Gao et al., PRD 104 (2021) 114515

F^π collaboration: Huber et al., PRC 78 (2008) 045203

k_T factorization: Cheng, PRD 100 (2019) 013007

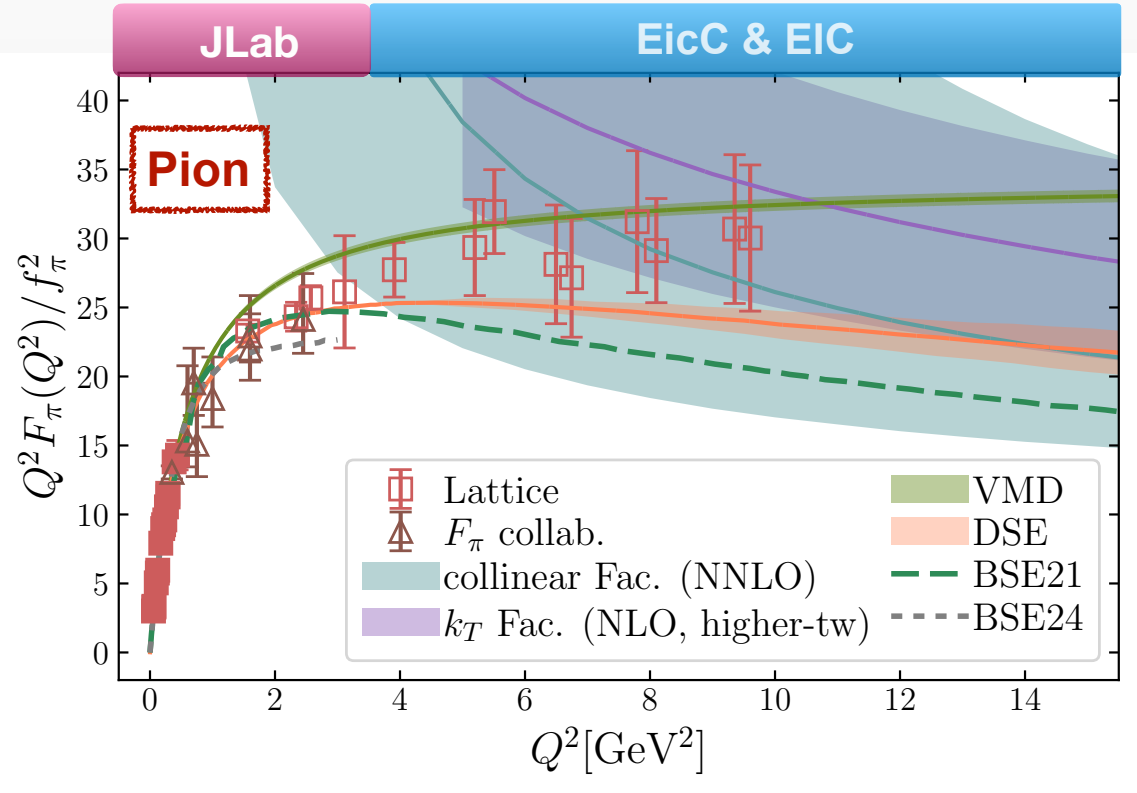
pion: Chai et al., EPJC 83 (2023) 556;

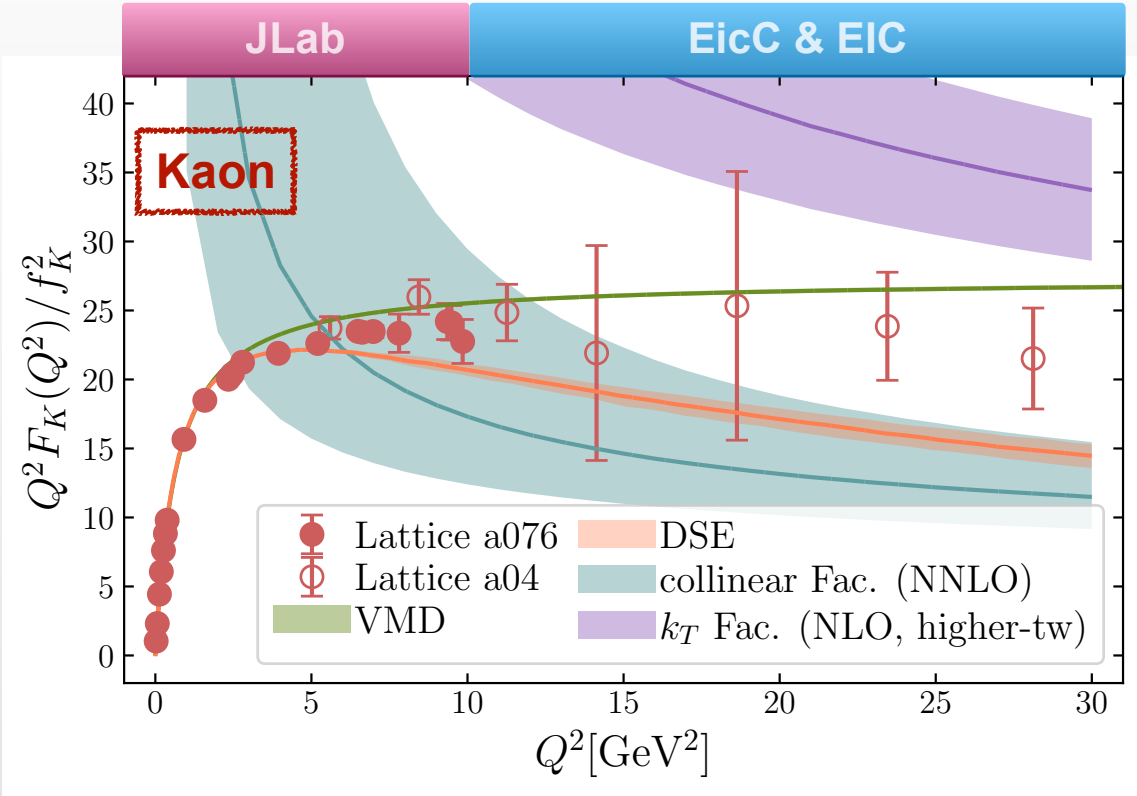
kaon: in preparation -> Chai and Cheng, JHEP 06 (2025) 229

T_H in the collinear factorization: Chen et al., PRL 132 (2024) 201901

DA in the collinear factorization: Cloët et al., arXiv:2407.00206

EMFFs





- **VMD**: Vector Meson Dominance
- **DSE**: Dyson-Schwinger Equation
- **BSE**: Bethe-Salpeter Equation

Lattice (BNL21): Gao et al., PRD 104 (2021) 114515
F^π collaboration: Huber et al., PRC 78 (2008) 045203
k_T factorization: Cheng, PRD 100 (2019) 013007
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T_H in the collinear factorization: Chen et al., PRL 132 (2024) 201901
DA in the collinear factorization: Cloët et al., arXiv:2407.00206
DSE (Dyson-Schwinger equation): Yao et al., PLB 855 (2024) 138823
BSE21 (Bethe-Salpeter equation): Ydrefors et al., PLB 820 (2021) 136494
BSE24: Jia and Cloët, arXiv:2402.00285

CONTENTS

GPDs: Lattice Setup & Techniques

- $N_s^3 \times N_t = 64^3 \times 64, \qquad a = 0.04$ fm
- HISQ action + Wilson-Clover action $\Rightarrow m_\pi^{\text{val}} = 0.3$ GeV
- Using momentum smearing to enhance the signal
- Momentum transfer Q^2: 0 ~ 1.7 GeV^2
- Zero skewness $\xi = 0\ (\Delta^z = 0)$
- Frame-independent approach: save computational cost

Bhattacharya et al., PRD 106 (2022) 114512

GPDs: Frame-independent Approach

- Lorentz-invariant amplitudes A_i's (frame-independent)

$$M^\mu(P^m u, z^\mu, \Delta^\mu) = \bar{P}^\mu A_1 + m^2 z^\mu A_2 + \Delta^\mu A_3, \qquad \bar{P}^\mu = (p_f^\mu + p_i^\mu)/2, \quad \Delta^\mu = p_f^\mu - p_i^\mu \,.$$

- Lorentz-invariant quasi-GPD $\tilde{H}_{\rm LI}$

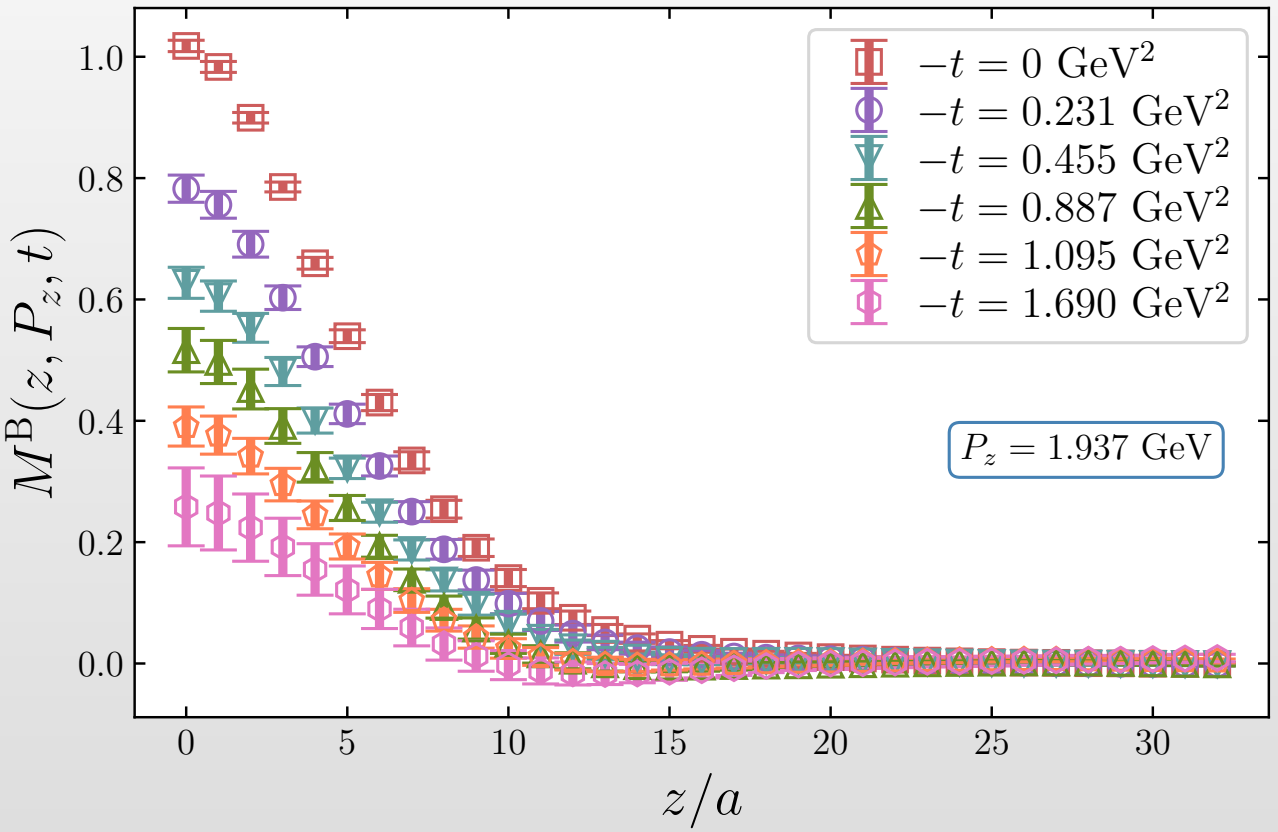
$$\tilde{H}_{\rm LI}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1 + \frac{z\cdot\Delta}{z\cdot\bar{P}} A_3$$

$$A_3(z\cdot\Delta = 0) = 0 \quad \Rightarrow \quad \tilde{H}_{\rm LI} = A_1 = M^\mu/\bar{P}^\mu \equiv M^B$$

Bare Matrix Elements

GPDs: Bare Matrix Elements

- Largest momentum: $P_z = 1.937$ GeV



$$\left.\begin{array}{l} z \uparrow \\ -t \uparrow \end{array}\right\} M^B \downarrow$$

Renormalization

Fourier Transform

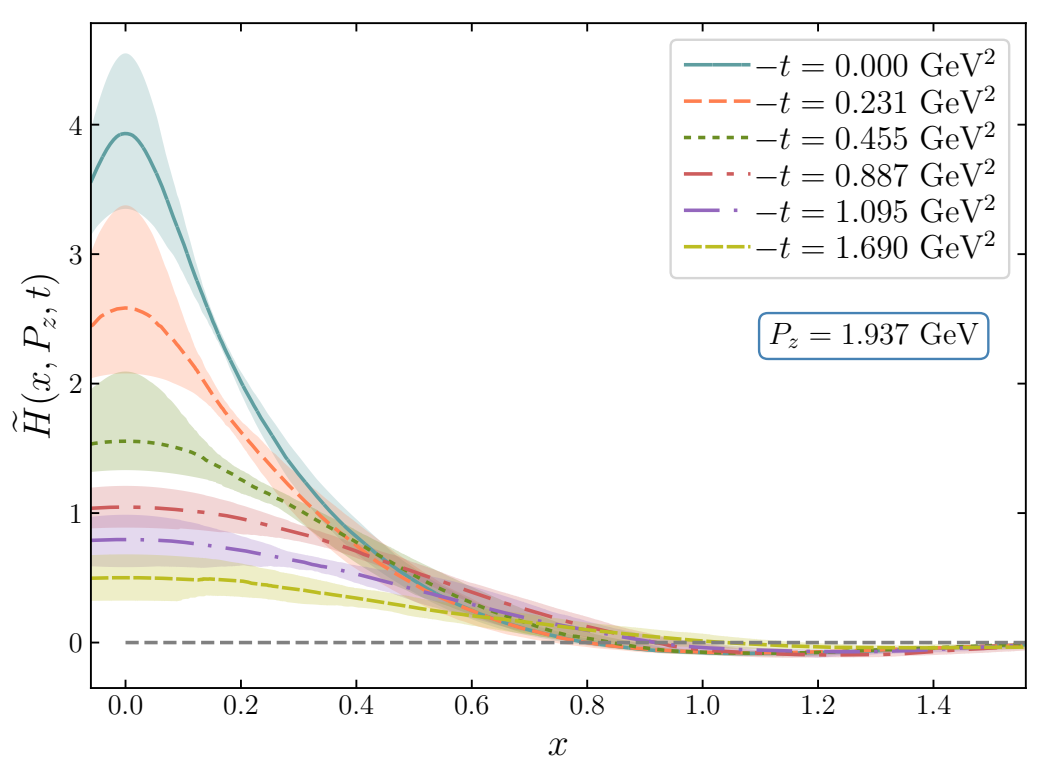
Quasi-GPDs

$\tilde{H}(x, P_z, t)$

GPDs: Quasi-GPDs $\tilde{H}$

- Renormalization:
  - ☆ RI/MOM, ratio schemes (short distance)
  - ★ **Hybrid scheme (short & long distance)**

Ji et al., NPB 964 (2021) 115311



$$M^{\mathrm{R}}(z, z_s; P_z, t) = \begin{cases} |z| \le |z_S| : \dfrac{M^{\mathrm{B}}(z, P_z, t)}{M^{\mathrm{B}}(z, 0, 0)}, \text{ Ratio scheme} \\ |z| > |z_S| : \dfrac{M^{\mathrm{B}}(z, P_z, t)}{M^{\mathrm{B}}(z_S, 0, 0)} e^{(\delta m + \bar{m}_0)|z - z_S|}. \end{cases}$$

- Fourier Transform:

$$\int \frac{\mathrm{d}\lambda}{\pi} e^{ix\lambda} M^R(z, P_z, t) \Rightarrow \tilde{H}(x, P_z, t)$$

GPDs: Matching

- Large Momentum Effective Theory (LaMET) framework

LC GPDs

NNLO+LRR+RGR

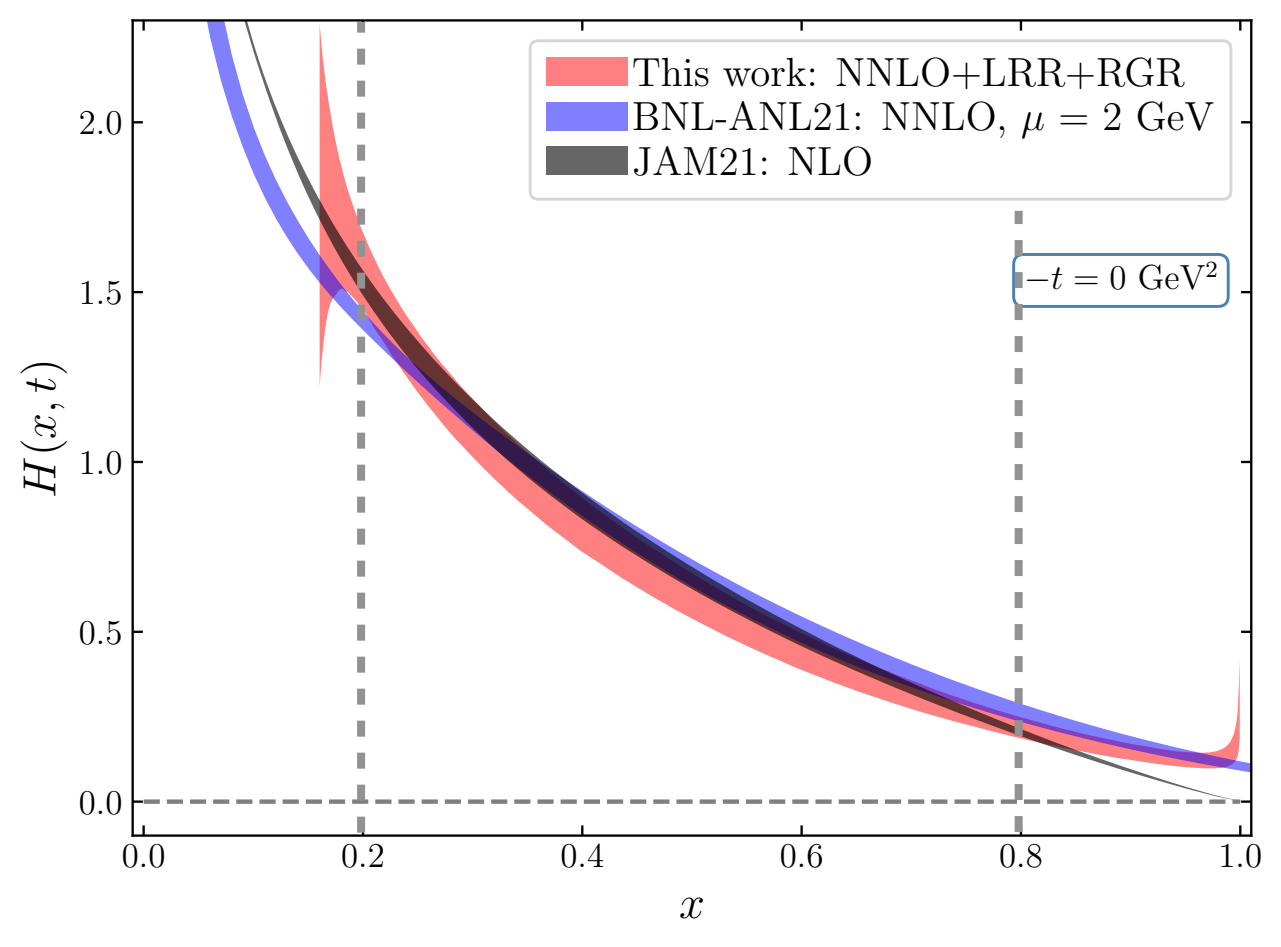
quasi-GPDs

$$H(x,t) = \int \frac{\mathrm{d}k}{|k|} \int \frac{\mathrm{d}y}{|y|} C_{\mathrm{evo}}^{-1}\left(\frac{x}{k}, \frac{\mu}{\mu_0}\right) C^{-1}\left(\frac{k}{y}, \frac{\mu_0}{yP_z}, |y|\lambda_S\right) \tilde{H}(y, P_z, t, z_S, \mu_0) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(xP_z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{[(1-x)P_z]^2}\right)$$

- Next-to-next-to-leading order (NNLO) correction → More precise matching
- Leading-renormalon resummation (LRR) → Improve convergence of perturbation series
- Renormalization group resummation (RGR) → Improves reliability of matching at small x

Physical scale: $\mu_0 = 2\kappa e^{-\gamma_E}/|z|, \kappa = [1, \sqrt{2}, 2]$ → Factorization scale: $\mu = 2$ GeV.

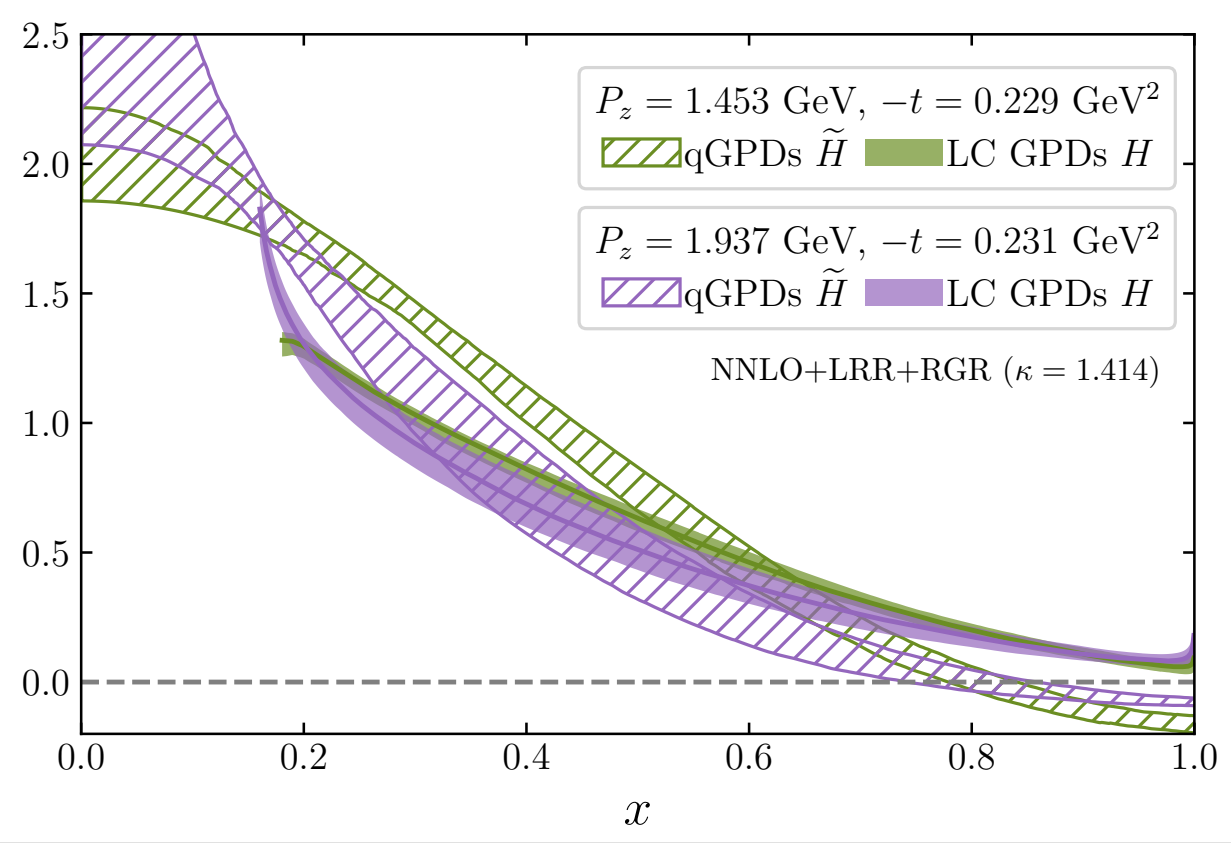
GPDs: Light-cone (LC) PDFs



- **GPDs at $-t = 0$ GeV^2 → PDFs**
- $0.2 < x < 0.8$: comparable with JAM21
- Both ends of x:

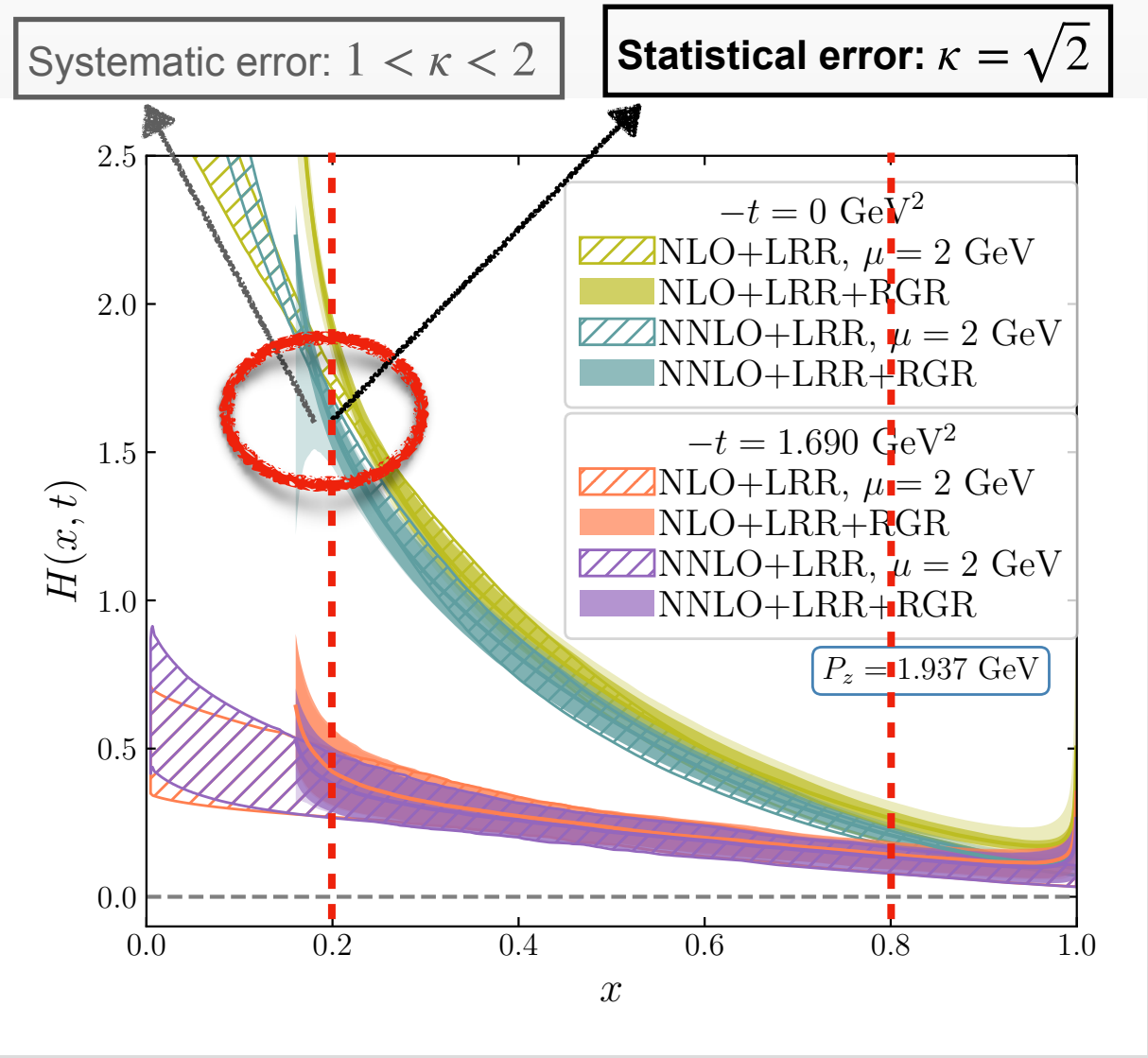
  LRR + RGR → different with BNL-ANL21

GPDs: Matching effect on P_z-dependence, q V.S. LC



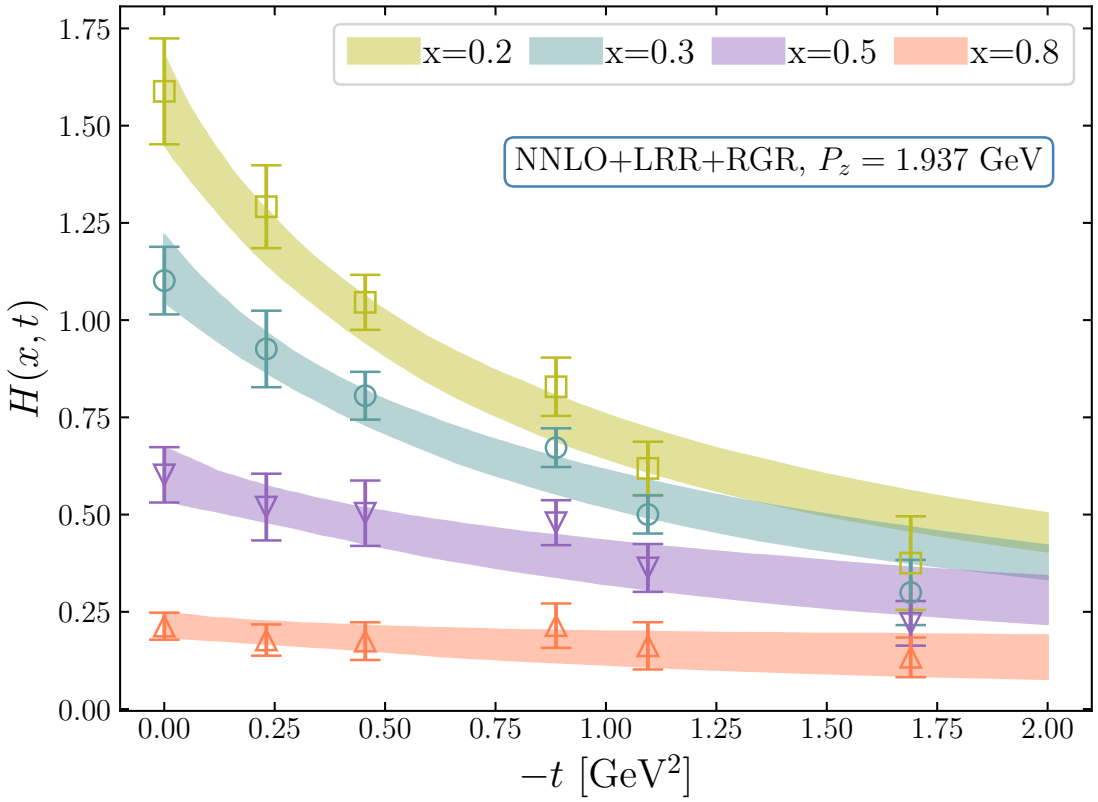
- Perturbation matching:
  - Same data set, difference → effect of perturbation matching
  - Different data sets → Reduce the P_z-dependence

LC GPDs: Scale & Order dependence



- Scale dependence
  - More significant at small and large x
  - Smaller at larger $-t$
- Good convergence: NLO & NNLO
- RGR:
  - reliable x: 0.2 - 0.8
  - small x: perturbation theory breaks down
  - large x: threshold logarithms in the future

LC GPDs: t-, x-dependence & IPDs



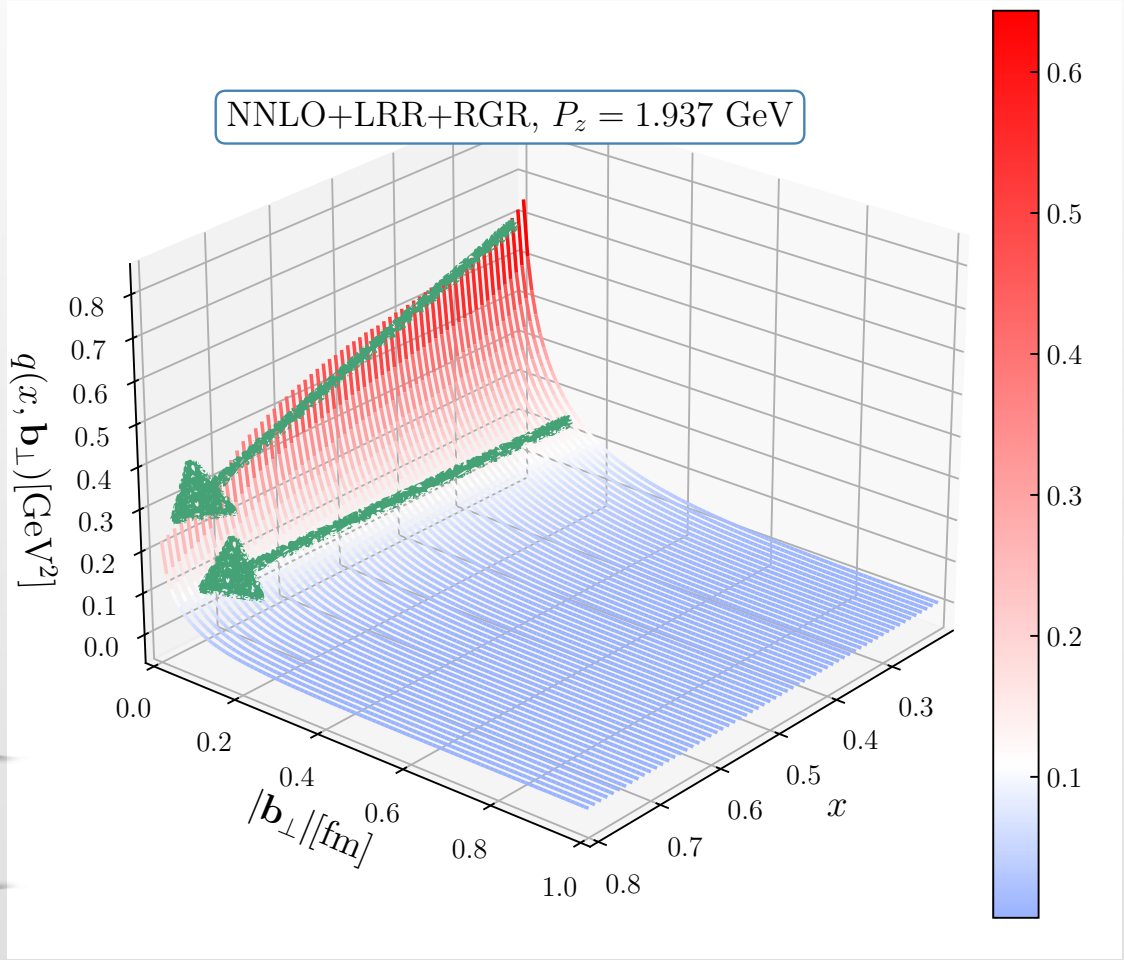
- Monopole model: $H(x,\,t) = \dfrac{H(x,\,0)}{1 - t/M^2(x)}$ — PDFs
- t-dependence becomes milder as x increases

LC GPDs → IPDs

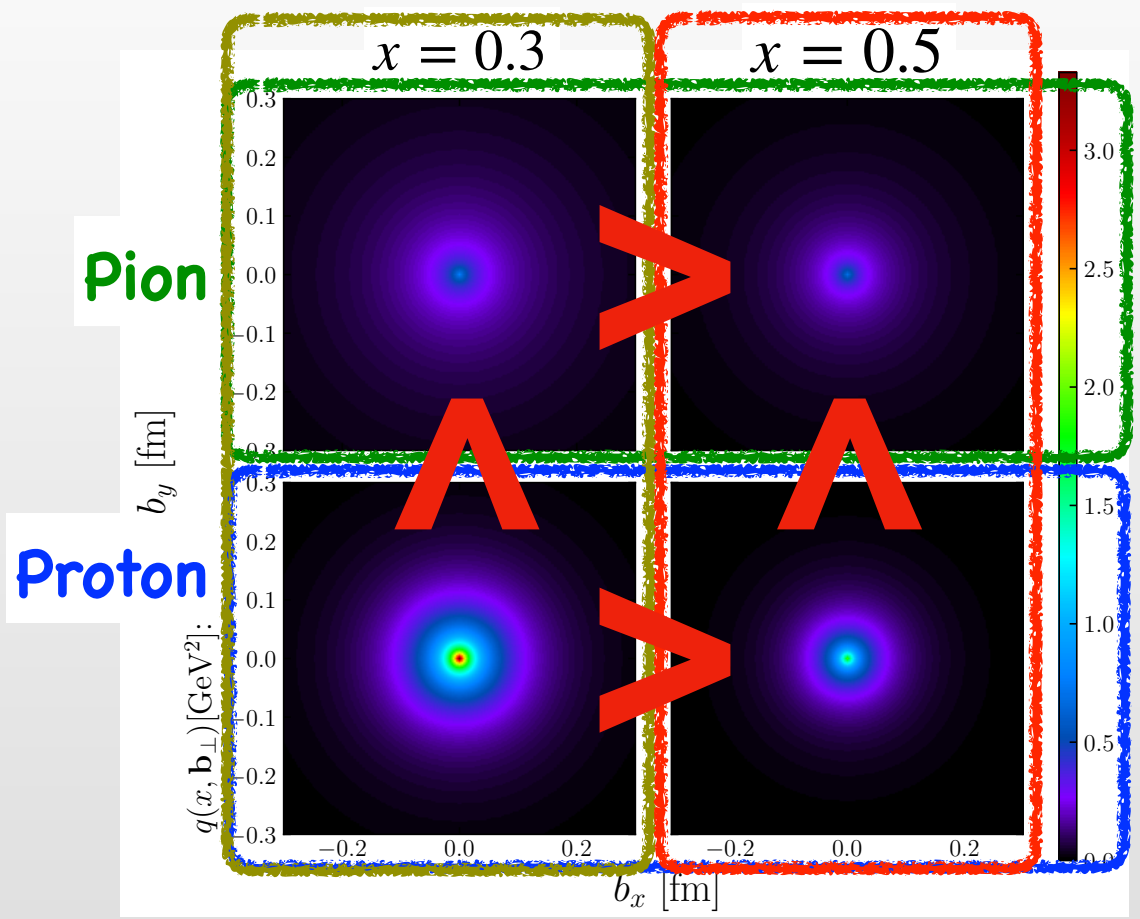
$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H(x, \xi = 0, \boldsymbol{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

- $x \uparrow$
  - Probability $\downarrow$
  - Distributions on $b_\perp$ become narrow

Quarks with higher x are more concentrated in the transverse space



IPDs: Pion v.s. Proton



- LC Proton GPDs [Cichy et al., arXiv:2304.14970]
- Distributions are more concentrated at larger x in the transverse space
- Quark distributions in the proton are broader than those of the pion

Conclusions

- **Pion and kaon EMFFs at the physical point**
  - Q^2 up to 10 and 28 GeV^2 for the pion and kaon
  - Consistent with experimental results and collinear factorization predictions
  - Serve as benchmark QCD predictions for model-based studies and the future experimental measurements
- **Pion LC GPD at zero skewness**
  - Improved perturbation matching with NNLO + LRR + RGR
  - P_z-, t-, x-, scale and order dependence of the LC GPDs & IPDs
  - Provide the three-dimensional image of the pion structure

Thanks for your attantion !