Global Analyses of Collinear Fragmentation Functions by the NPC Collaboration

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In collaboration with
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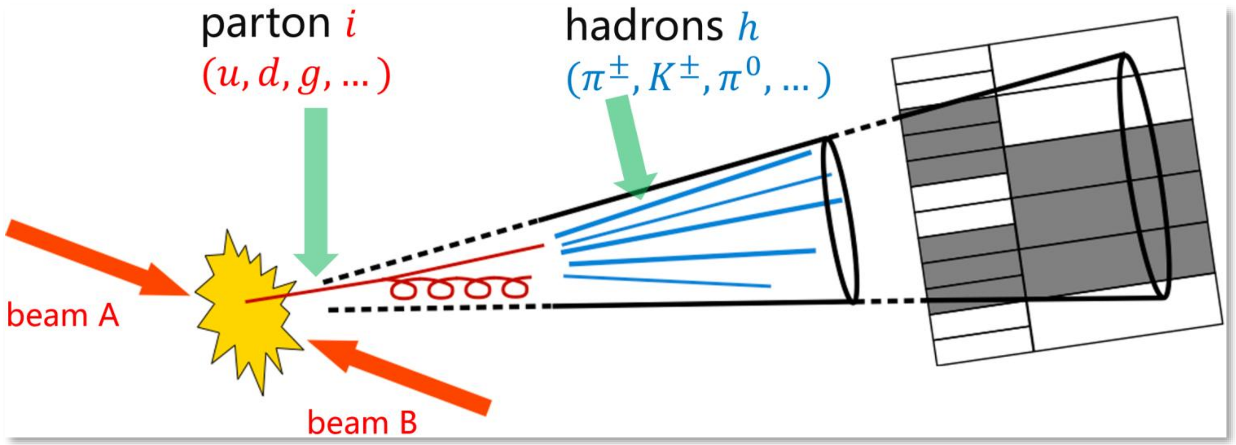
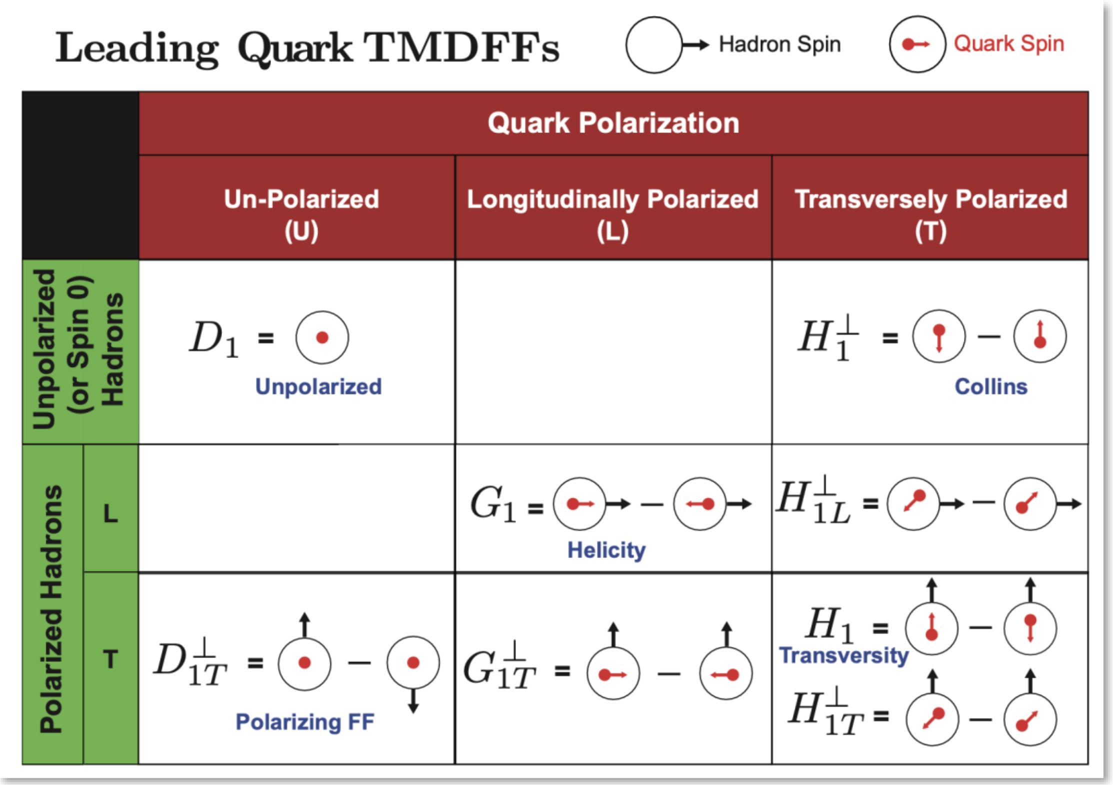
Outline

- Introduction to Fragmentation Functions (FFs)
- NPC global analyses of FFs at NLO
- NPC global analyses of FFs at NNLO + constraining PDFs
- Summary

Fragmentation Functions

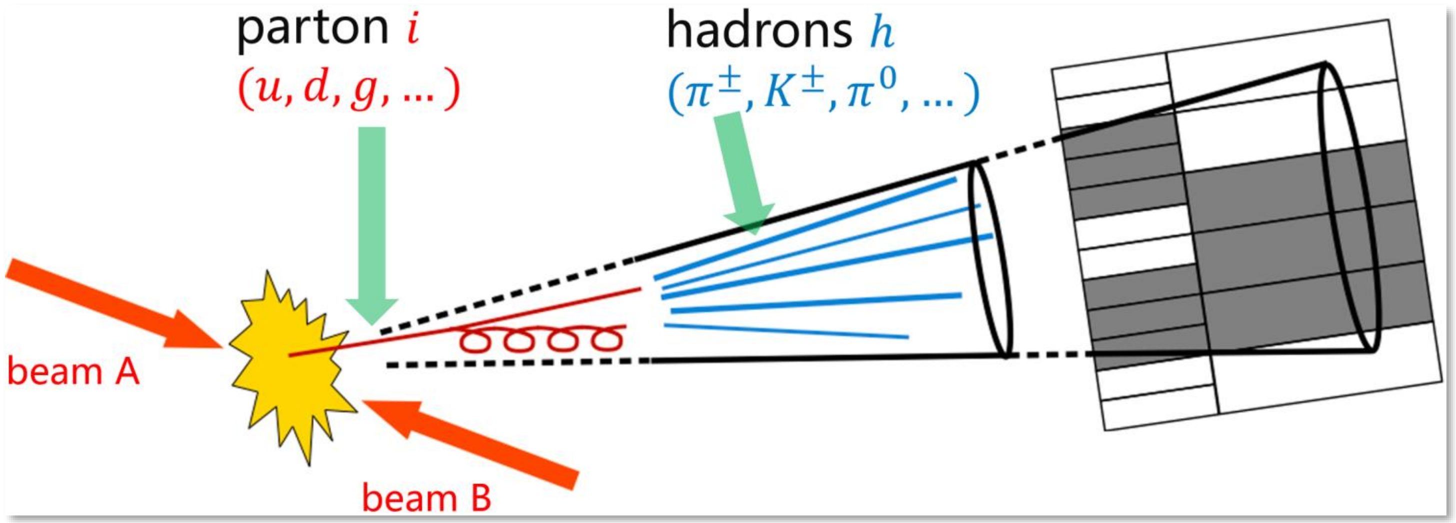




[TMD handbook 2304.03302]

this talk is limited to unpolarized collinear FFs.

Fragmentation Functions in parton model



FF = number density of finding

- a specific hadron *h*
- with momentum fraction z "in" parton i

Bjorken et al., '71
FFs proposed

$$D_{h/i}\left(z = \frac{p_h^+}{p_i^+}\right) \longleftrightarrow f_{i/h}\left(x = \frac{p_i^+}{p_h^+}\right)$$

FFs PDFs

Fragmentation Functions in QCD

1969 — Feynman, '69
Parton model

1971 — Bjorken et al., '71
FFs proposed

1977 — Field, Feynman, '77
First determination

1981 — Collins, Soper, '81
FFs def. in QCD

$$
\begin{aligned}
D_{h/q}(z) \;=\; & \frac{z}{4} \sum\!\!\!\!\!\!\int_X \int \frac{\mathrm{d}\xi^+}{2\pi}\, e^{i P_h^- \xi^+/z}\, \mathrm{Tr}\Big[\langle 0 \,|\, \mathcal{W}(\infty^+, \xi^+)\; \psi_q(\xi^+, 0^-, \vec{0}_T) \,|\, P_h, S_h; X \rangle \\
& \times \langle P_h, S_h; X \,|\, \bar{\psi}_q(0^+, 0^-, \vec{0}_T)\; \mathcal{W}(0^+, \infty^+) \,|\, 0 \rangle \;\gamma^- \Big]
\end{aligned}
$$

Fragmentation Functions in QCD

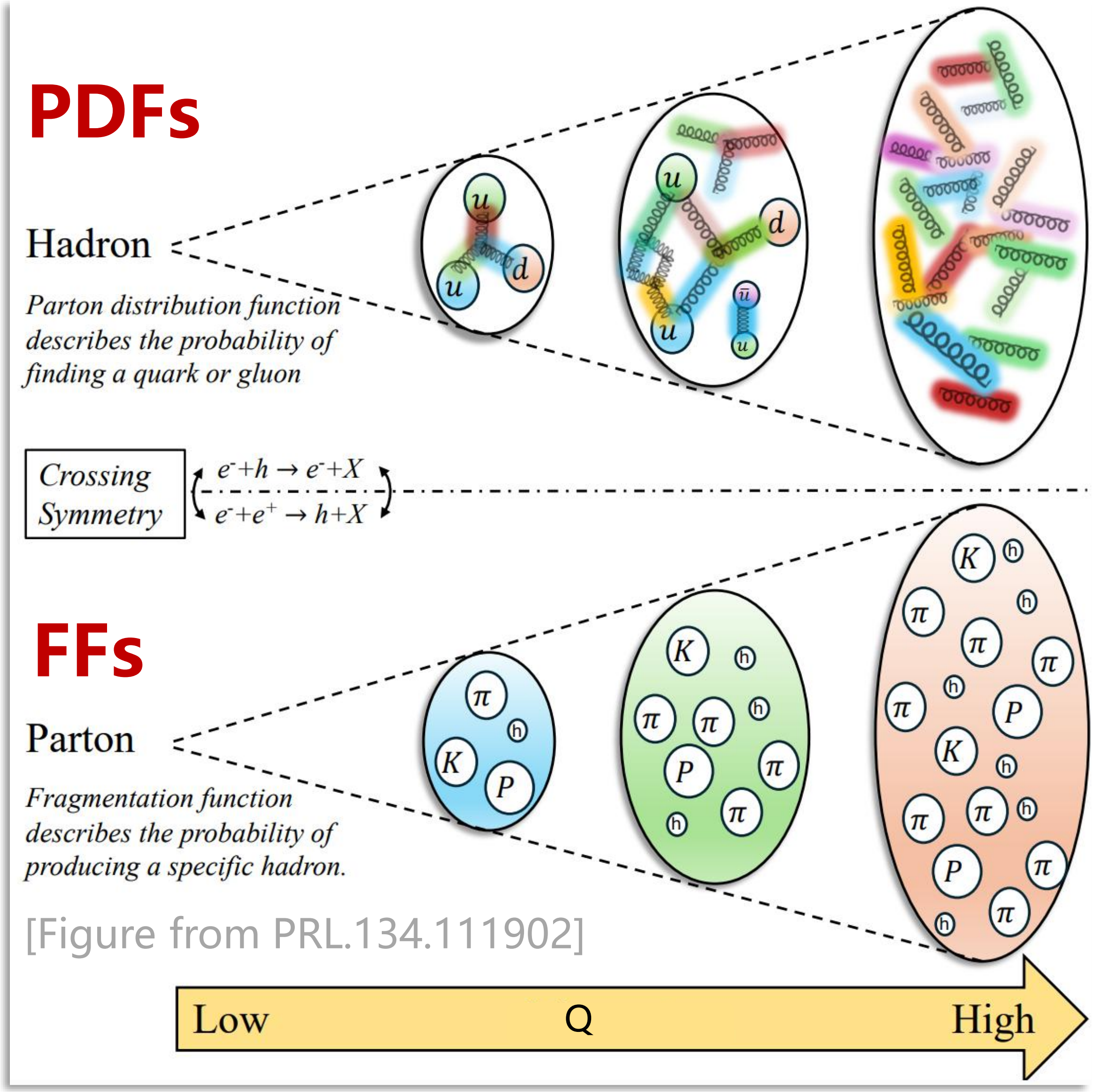
1969	1971	1977	1981
Feynman, '69 Parton model	Bjorken et al., '71 **FFs proposed**	Field, Feynman, '77 First determination	Collins, Soper, '81 **FFs def. in QCD**

$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^-\xi^+/z}\,\mathrm{Tr}\Big[\langle 0|\,\mathcal{W}(\infty^+,\xi^+)\ \psi_q(\xi^+,0^-,\vec{0}_T)\,|P_h,S_h;X\rangle \times \langle P_h,S_h;X\,|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\ \mathcal{W}(0^+,\infty^+)\,|0\rangle\ \gamma^-\Big]$$

$$\boxed{D(z,Q_0)} \longrightarrow \boxed{D(z,Q)}$$

$$\frac{d D_{h/i}(z,Q)}{d\ln^2 Q} = P_{ji}(y)\otimes D_{h/j}\left(\frac{z}{y},Q\right)$$

timelike DGLAP evolution

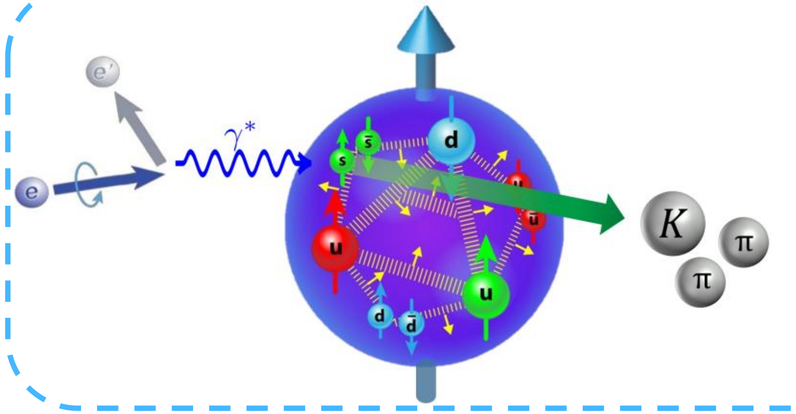


[Figure from PRL.134.111902]

FFs are key ingredients of QCD factorization framework

1969	Feynman, '69 Parton model
1971	Bjorken et al., '71 FFs proposed
1977	Field, Feynman, '77 First determination
1981	Collins, Soper, '81 FF def. in QCD
1989	Collins et al., '89 **Factorization thm.**

- Semi-Inclusive DIS (SIDIS) : e + N -> e + **h** + X

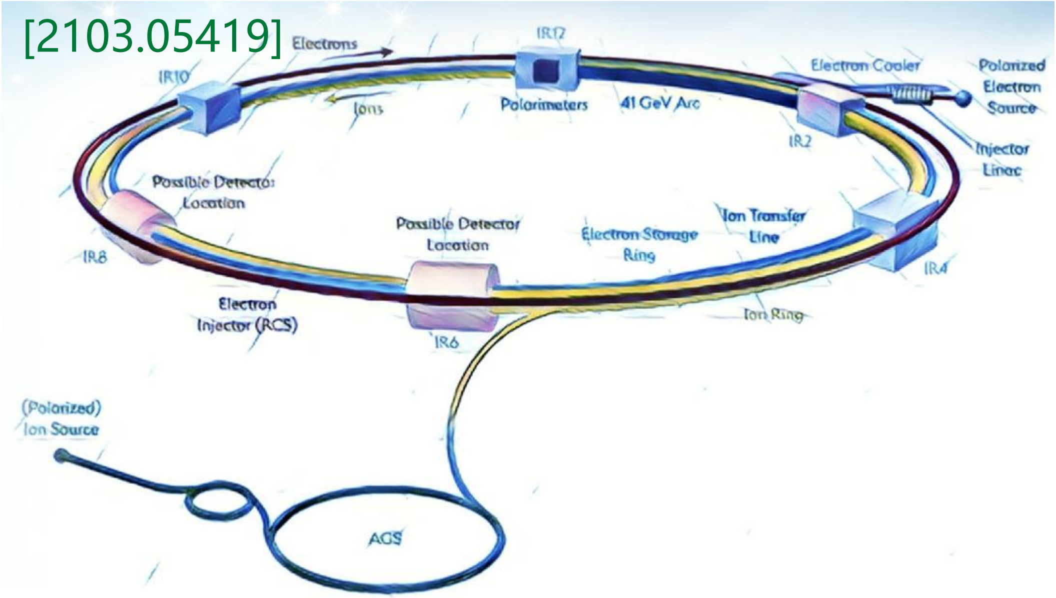


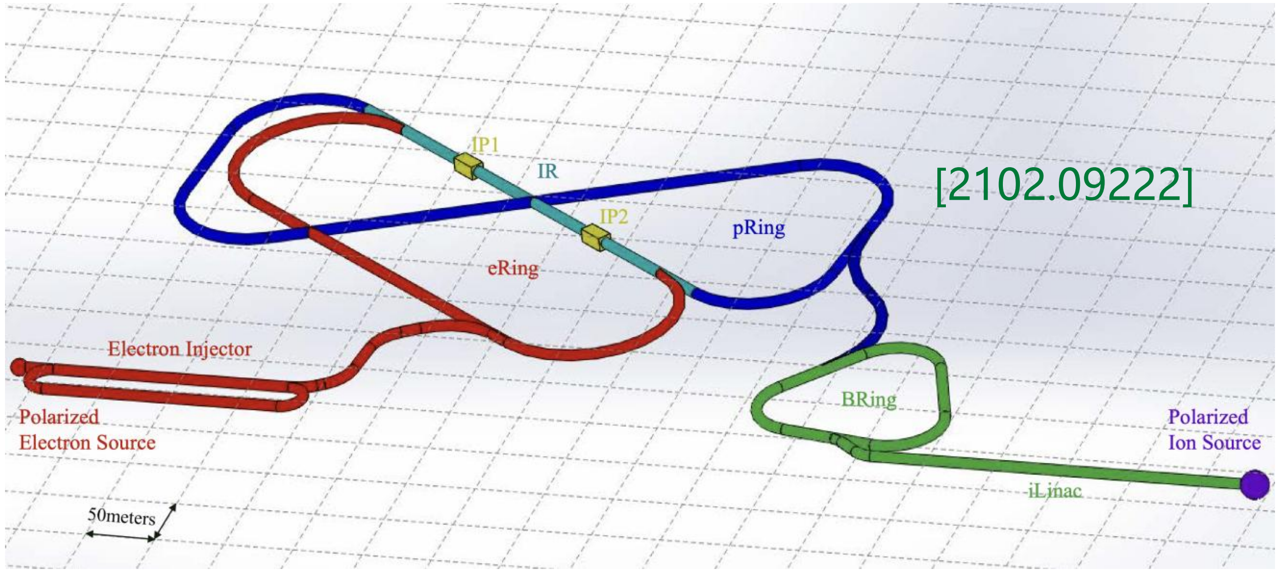
$$\frac{d^3\sigma_h}{dx_B\, dy\, dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x, y, z) \otimes D_{h/j}(z)$$

PDF pQCD **FF**

- $e^+e^- \to h + X$ (SIA) => σ = pQCD ⊗ **FF**
- $pp \to h + X$ => σ = PDF ⊗ PDF ⊗ pQCD ⊗ **FF**

FFs play a key role in the era of high-precision physics





❖ Electron-Ion Collider (EIC)

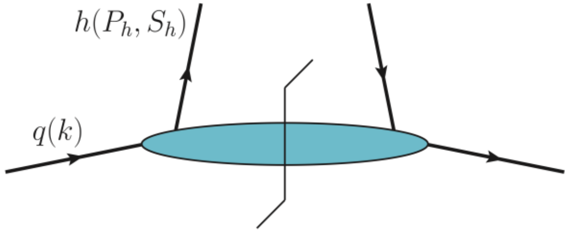
- start operation in the early 2030s
- FFs will be key inputs/outputs

❖ Efforts from China

- **BESIII**
- **STCF**
- **EicC**

FFs are a major physics target and key inputs of future colliders.

Determination of light hadron FFs



$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^-\xi^+/z} \mathrm{Tr}\Big[\langle 0 \,|\, \mathcal{W}(\infty^+, \xi^+)\; \psi_q(\xi^+, 0^-, \vec{0}_T) \,|\, P_h, S_h; X\rangle \times \langle P_h, S_h; X \,|\, \bar{\psi}_q(0^+, 0^-, \vec{0}_T)\; \mathcal{W}(0^+, \infty^+) \,|\, 0\rangle\; \gamma^-\Big]$$

❖ Lattice QCD: many PDFs studies have been done, but no FFs study yet.

❖ Quantum computers

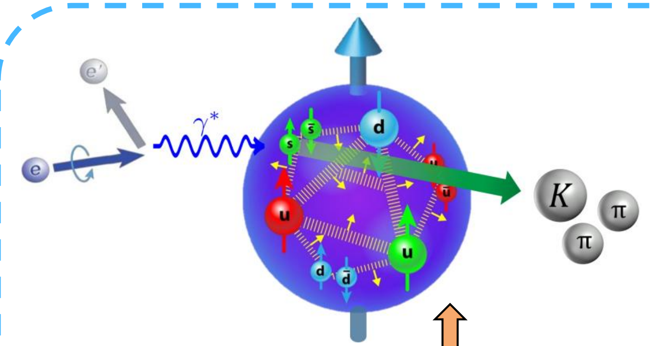
[Li, Xing, Zhang, 2406.05683]
[Galvez-Viruet et al. 2510.18869]

❖ Models

[Jia, Mo, Xiong, 2310.17640]
[Xing, Bian, Cui, Roberts 2504.08142]
…

Determination of FFs: global data fits

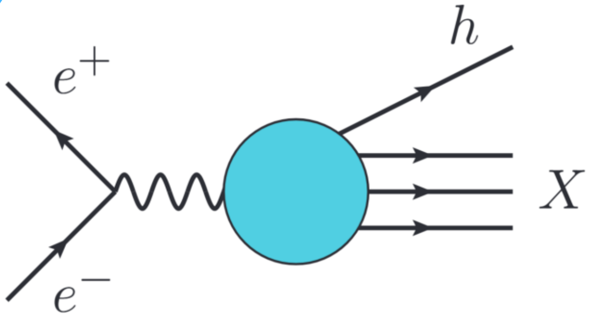
❖ **e p collision:**



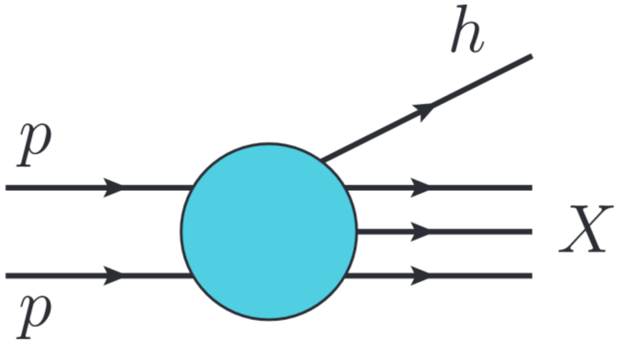
$$= \quad f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x, y, z) \otimes D_{h/j}(z)$$

measurement   input   pQCD   to be determined.   Universal !

❖ **ee and pp**



σ = pQCD ⊗ **FF**



σ = PDF ⊗ PDF ⊗ pQCD ⊗ **FF**

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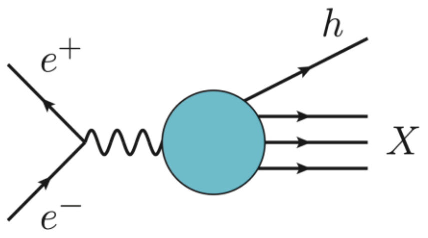
- **Global fits of FFs at NLO**
- Global fits of FFs at NNLO
- Summary

NPC= Non-Perturbative Physics Collaboration

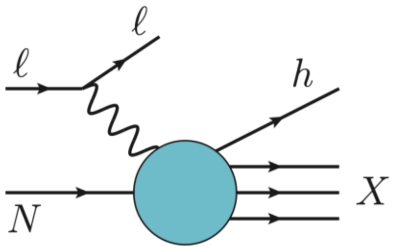
collaboration	NNFF	JAM	DSS+	BDSSV	MAP	**NPC**
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	$\pi^\pm$,$K^\pm$,p	$\pi^\pm$,$K^\pm$	$\pi^\pm$,$K^\pm$,p	$\pi^\pm$	$\pi^\pm$,$K^\pm$	$\pi^\pm$,$K^\pm$,p
FFs (neutral h)			η			K^0,η,Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

Only some of the recent global analyses are shown here.

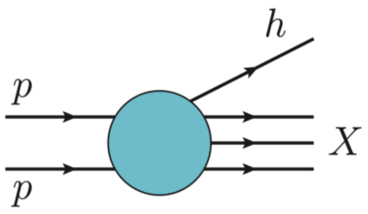
NPC FFs analyses incorporate various types of data



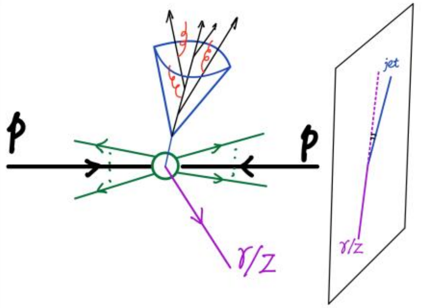
ee(SIA)



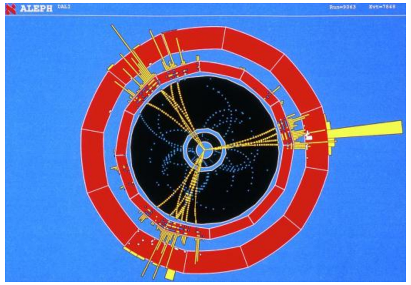
ep(SIDIS)



pp



pp



ee

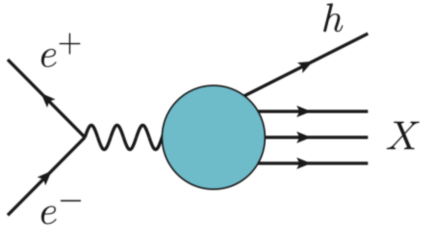
single-inclusive hadron production

hadron-in-jet measurements

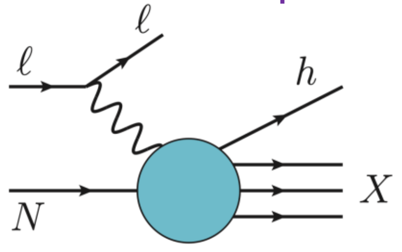
Figures from: A. Metz, A. Vossen,1607.02521; ALEPH Collaboration

NPC FFs analyses incorporate various types of data

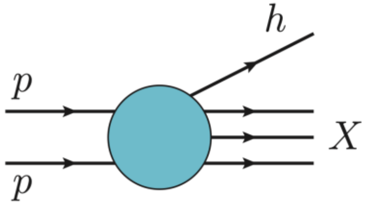
single-inclusive hadron production



ee(SIA)



ep(SIDIS)

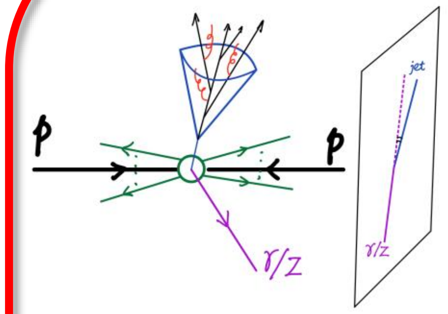


pp

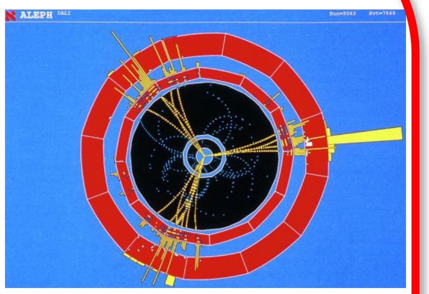
$$\frac{\mathrm{d}\sigma_{pp\to h+X}}{\mathrm{d}\,p_{T,h}} = \int f(x_1)f(x_2)\hat{\sigma}(p_{T,h},x_1,x_2,z)D(z,Q)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}z$$

z dependence not directly probed

hadron-in-jet measurements



pp



ee

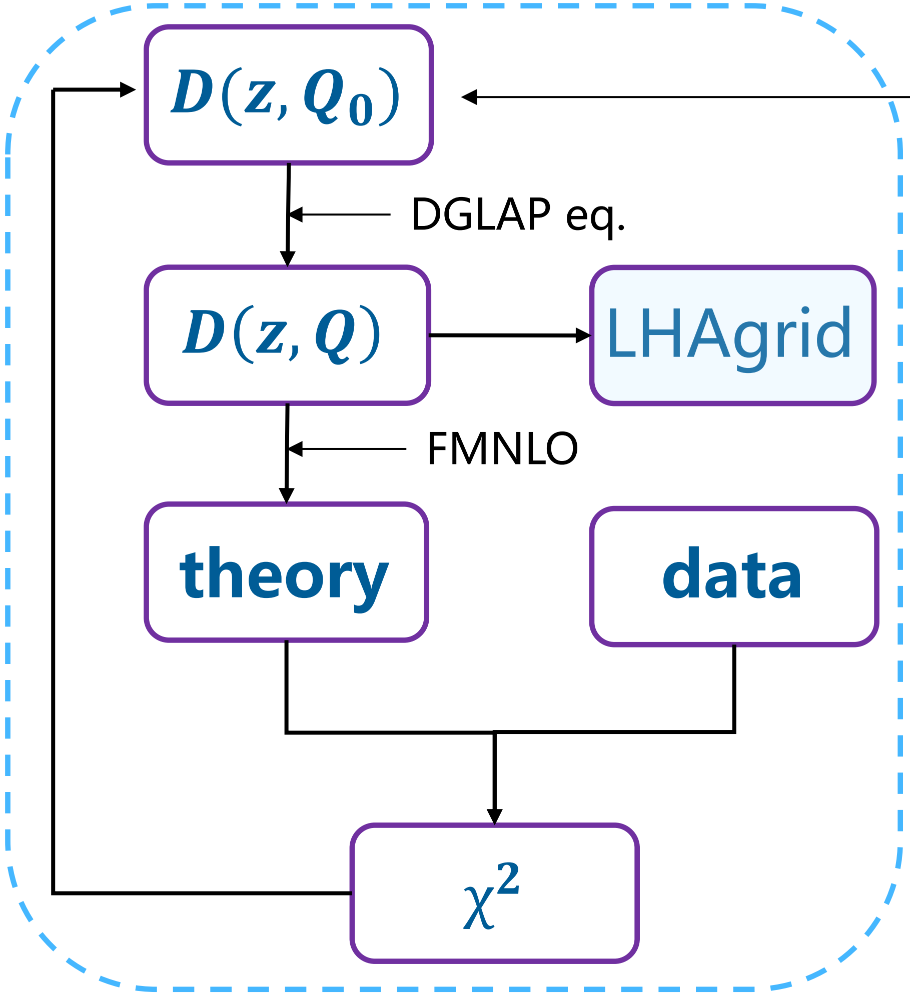
$$\frac{p_{T,h}}{p_{T,j}} \xrightarrow{\text{LO}} z$$

direct probe of z dependence

➢ All theoretical predictions calculated with **FMNLO.**

[Liu, **XS**, Zhou, Gao, 2305.14620 (JHEP)]

Fit framework



Parameterization at starting scale Q_0=4.0 GeV

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to-π^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$\bar{d} \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F				4

parton-to-K^+	favored	α	β	a_0	a_1	a_2	d.o.f.
u	Y					x	4
$\bar{s} \simeq u$	Y	-	-		-	x	1
$\bar{u}=d=\bar{d}=s$	N					x	4
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

parton-to-p	favored	α	β	a_0	a_1	a_2	d.o.f.
$u = 2d$	Y					x	4
$\bar{u} = \bar{d} = s = \bar{s}$	N				x	x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F			x	3

simultaneous fit of $\pi^\pm, K^\pm, p/\bar{p}$ FFs

Good agreement between theory and data

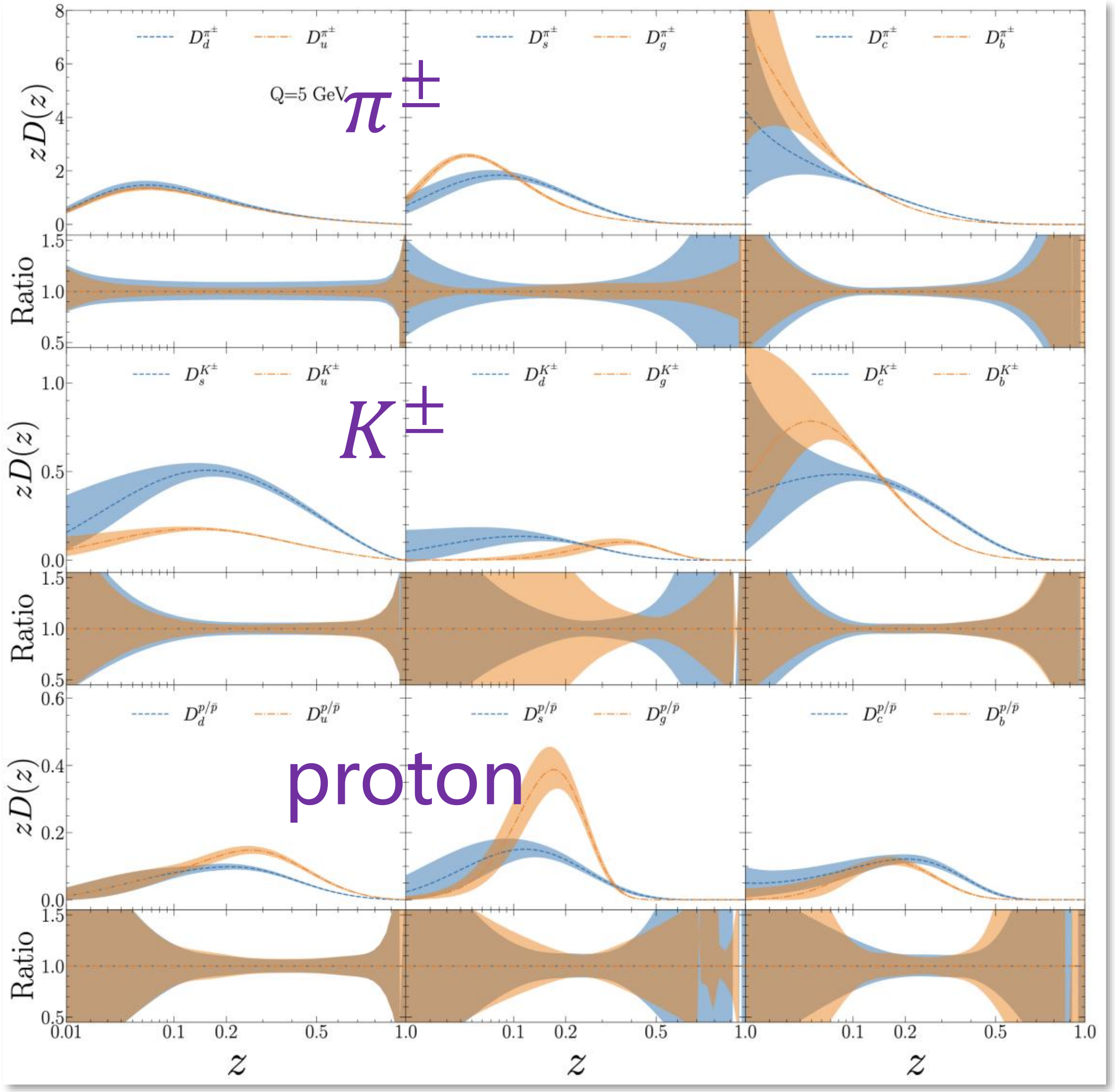
	Experiments	N_{pt}	χ^2	χ^2/N_{pt}
h in jet	ATLAS jets †	446	350.8	0.79
	ATLAS $Z/\gamma+\text{jet}$ †	15	31.8	2.12
	CMS $Z/\gamma+\text{jet}$ †	15	17.3	1.15
	LHCb $Z+\text{jet}$	20	30.6	1.53
pp	ALICE inc. hadron	147	150.6	1.02
	STAR inc. hadron	60	42.2	0.70
	pp sum	703	623.3	0.89
ee	TASSO	8	7.0	0.88
	TPC	12	11.6	0.97
	OPAL	20	16.3	0.81
	OPAL (202 GeV) †	17	24.2	1.42
	ALEPH	42	31.4	0.75
	DELPHI	78	36.4	0.47
	DELPHI (189 GeV)	9	15.3	1.70
	SLD	198	211.6	1.07
	SIA sum	384	353.8	0.92
ep	H1 †	16	12.5	0.78
	H1 (asy.) †	14	12.2	0.87
	ZEUS †	32	65.5	2.05
	COMPASS (06l)	124	107.3	0.87
	COMPASS (16p)	97	56.8	0.59
	SIDIS sum	283	254.4	0.90
	Global total	1370	1231.5	0.90

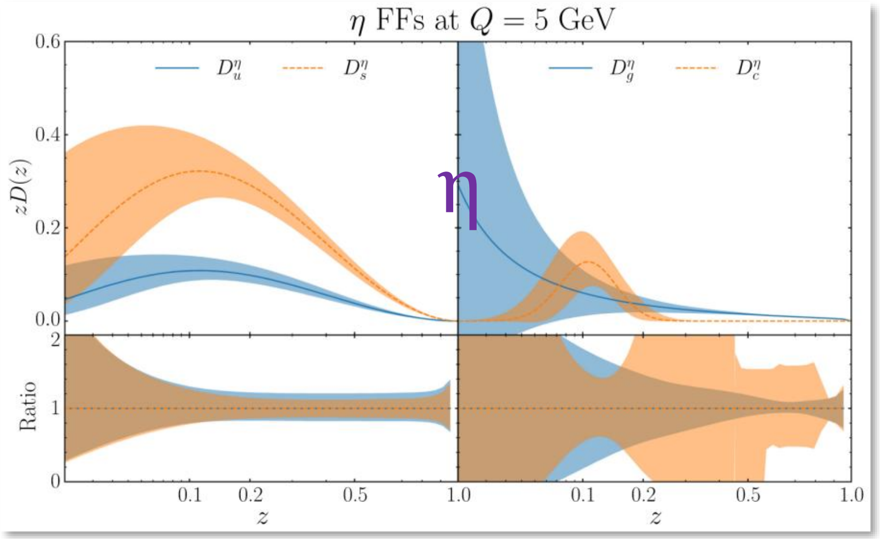
NPC23 $\pi^{\pm}, K^{\pm}, p$ FFs fit

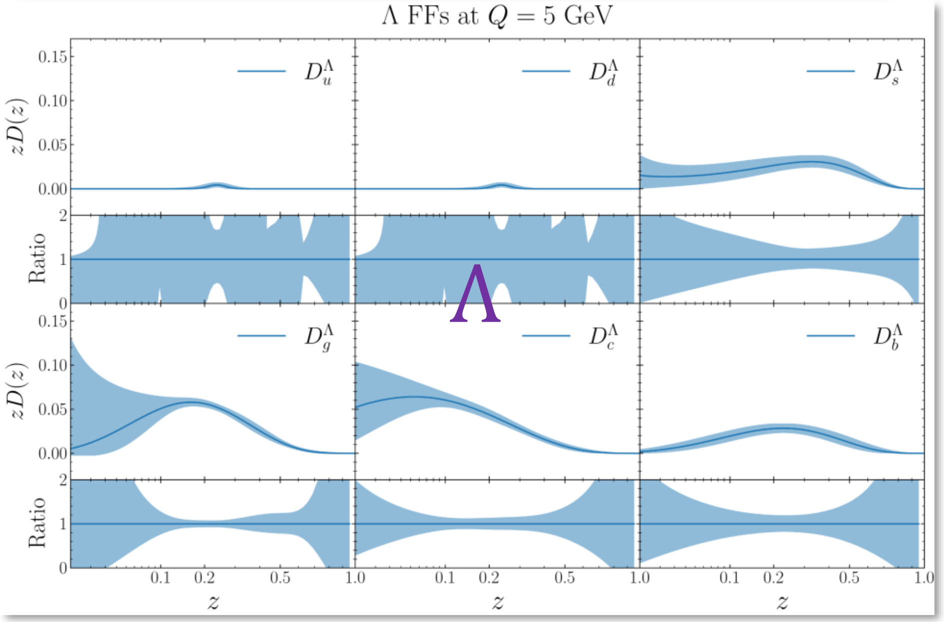
collaboration	year	$\sqrt{s}$[GeV]	χ^2	N_{pt}	χ^2/N_{pt}
TASSO	1985	14	5.65	9	0.63
TASSO	1985	22	5.87	6	0.98
TASSO	1985	34	16.03	13	1.23
TASSO	1990	14.8	12.56	9	1.40
TASSO	1990	21.5	3.78	6	0.63
TASSO	1990	34.5	17.51	13	1.35
TASSO	1990	35	14.76	13	1.14
TASSO	1990	42.6	33.60	13	2.58
TPC	1984	29	2.75	8	0.34
MARK II	1985	29	12.65	17	0.74
HRS	1987	29	33.16	12	2.76
CELLO	1990	35	2.71	9	0.30
TOPAZ	1995	58	0.29	4	0.07
OPAL	1991	91.2	7.75	7	1.11
OPAL	1995	91.2	13.63	16	0.85
OPAL	2000	91.2	8.62	16	0.54
ALEPH	1998	91.2	6.39	16	0.40
ALEPH	2000	91.2	12.72	14	0.91
ALEPH jet 1	2000	91.2	14.91	12	1.24
ALEPH jet 2	2000	91.2	8.21	13	0.63
ALEPH jet 3	2000	91.2	8.55	11	0.78
DELPHI	1995	91.2	7.55	13	0.58
SLD	1999	91.2	7.39	9	0.82
SLD c-tagged	1999	91.2	17.44	9	1.94
SLD b-tagged	1999	91.2	11.12	9	1.24
SIA sum			285.60	277	1.03
ZEUS $Q^2 \in 160, 640\,\text{GeV}^2$	2012	318	4.41	5	0.88
ZEUS $Q^2 \in 640, 2560\,\text{GeV}^2$	2012	318	3.26	5	0.65
ZEUS $Q^2 \in 2560, 10240\,\text{GeV}^2$	2012	318	2.74	2	1.37
SIDIS sum			10.41	12	0.87
ALICE $N^{13\,\text{TeV}}_{K^0_S}/N^{7\,\text{TeV}}_{K^0_S}$	2021	13000 & 7000	2.88	10	0.29
ALICE $N_{K^0_S}/N_{\pi^{\pm}}$	2021	13000	5.79	15	0.39
pp sum			8.67	25	0.35
total sum			**304.68**	**314**	**0.97**

NPC23 K^0 FFs fit

The NPC FF sets at NLO





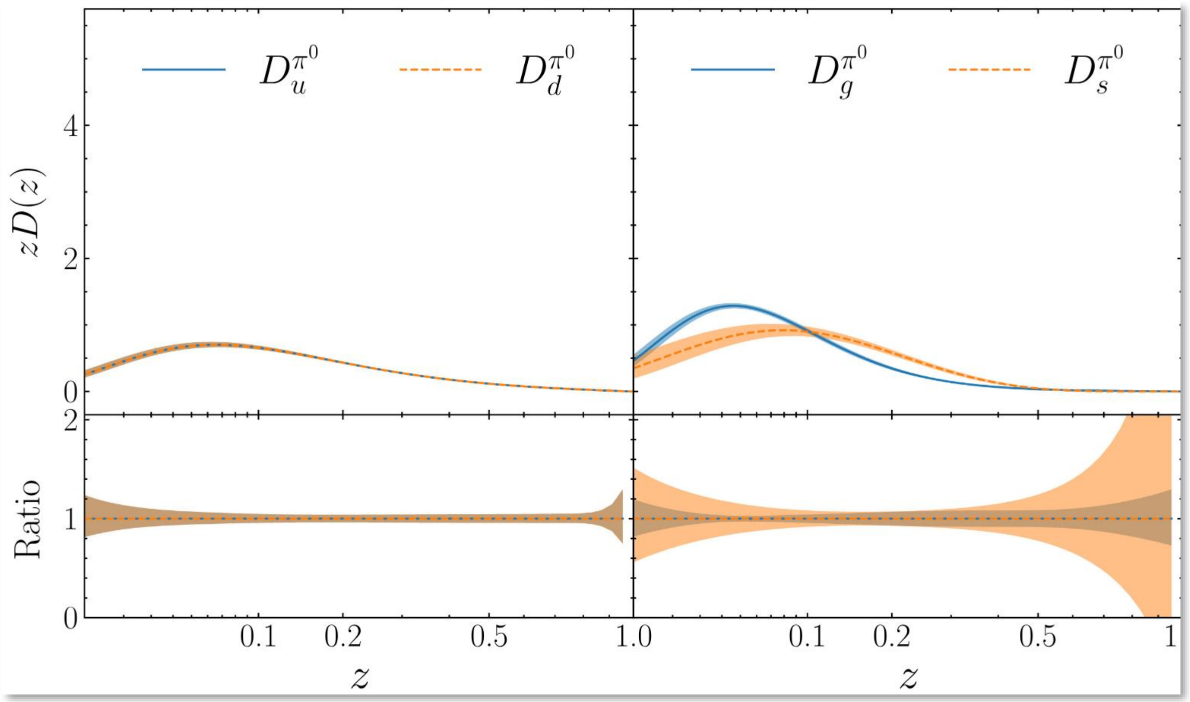


Gao, Liu, **XS**, Xing, Zhao, *PRL 132, 261903*, '24

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045*, '25

Example: NPC23 gluon→π^0 FF

The gluon FF is well constrained:

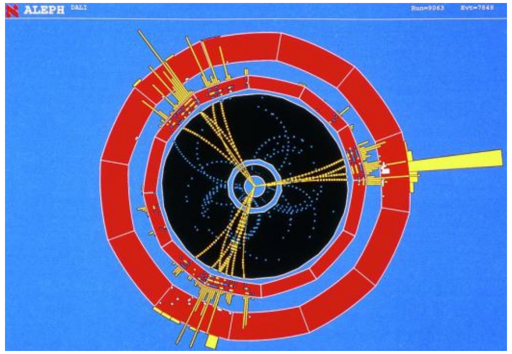


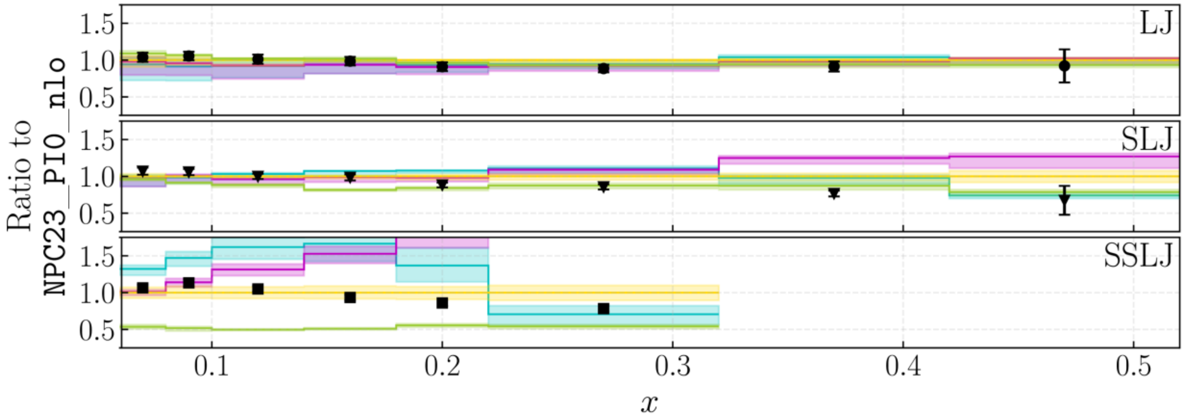
Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24

[Bonino et al. 2602.12344 (PRL)]

Theory predictions v.s. ALEPH 3-jet exclusive events

$$e^+e^- \rightarrow \text{jet} + \text{jet} + \text{jet}(\pi^0)$$
$$e^+e^- \rightarrow q + \bar{q} + g$$





Combination of all types of hadron production data leads to **good constraints** on FFs.

Both charged and neutral hadron FFs are determined

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p
FFs (neutral h)			η			K^0,η,Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

- FFs determination at NLO from Non-perturbative Physics Collaboration (NPC)
  - NPC23 FFs to light **charged** hadrons:

    Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
    Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019,* '24 (Editors' suggestion)

  - NPC23 FFs to light **neutral** hadrons:

    Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045, '25*

Test sum rule using neutral + charged hadron FFs

parton i $\longrightarrow$ hadrons $h = \pi^{\pm}, \pi^{0}, K^{\pm}, K^{0}, \cdots$

The **momentum sum rule**:

$$\lim_{z_{\min}\to 0} \sum_{h} \underbrace{\int_{z_{\min}}^{1} \left[z D_{h/i}(z) \right] \mathrm{d}z}_{\text{total momentum fraction carried by } h} = 1$$

Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045, '25*

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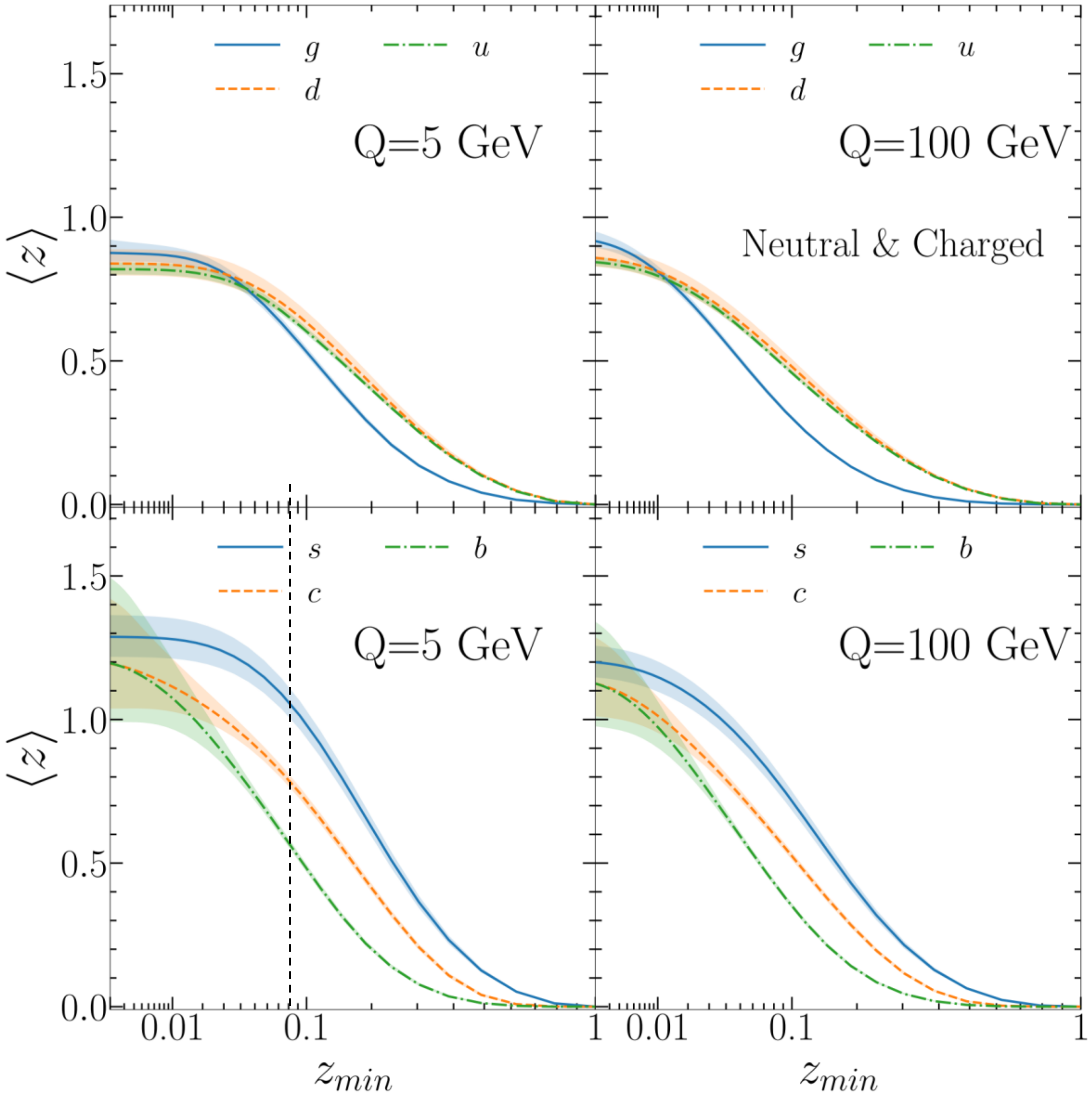

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SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ K^0,η,Λ	$\pi^{\pm}$,$K^{\pm}$
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

Only some of the recent global analyses are shown here.

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025]

Motivations to NNLO FF analyses

Higher-order corrections are essential for analyzing low-energy data

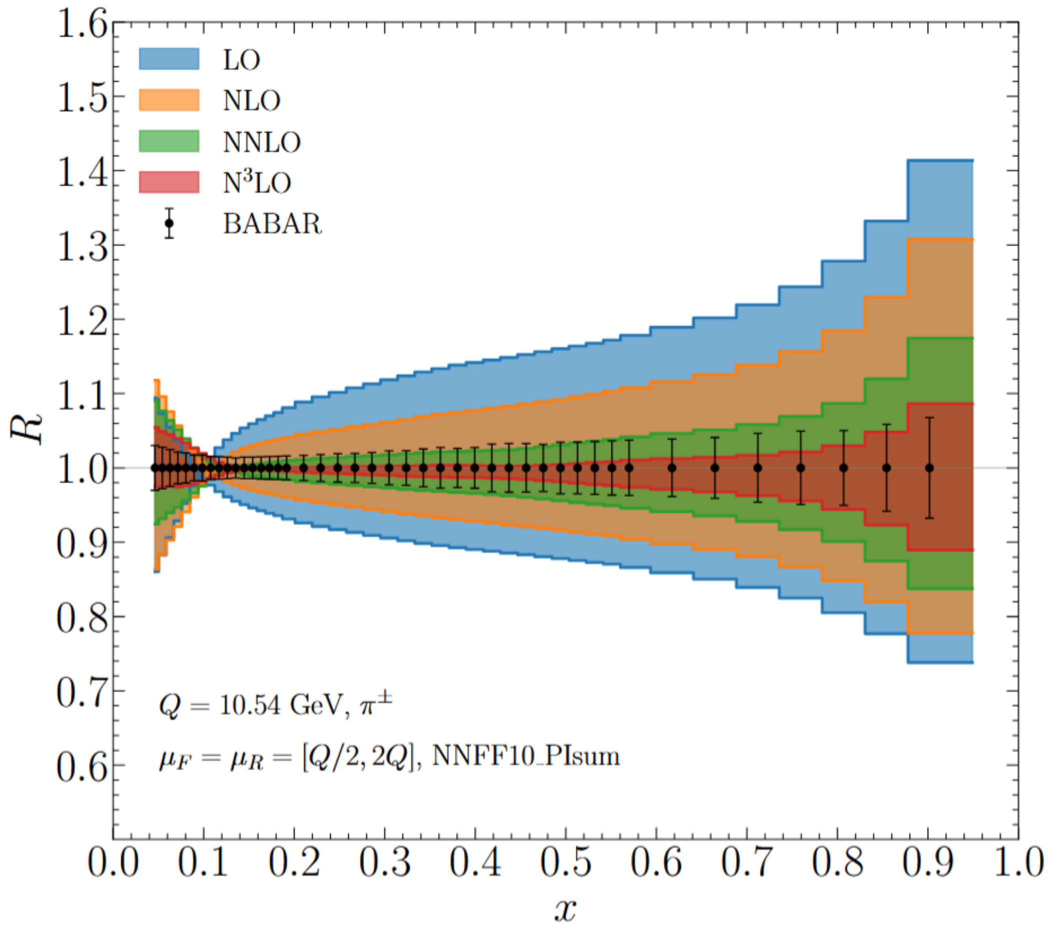
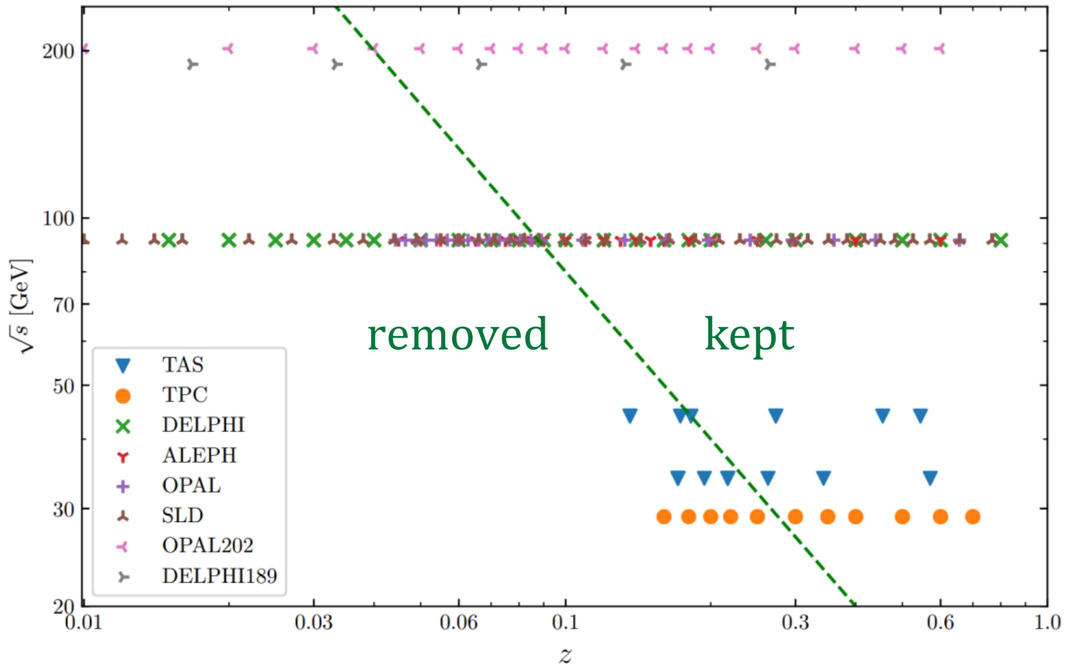


Figure from [He, Xing, Yang, Zhu, PRL.135.101901(2025)]



low-energy data have been excluded in NPC23 **NLO** analysis

Motivations to NNLO FF analyses

Higher-order corrections are essential for analyzing low-energy data

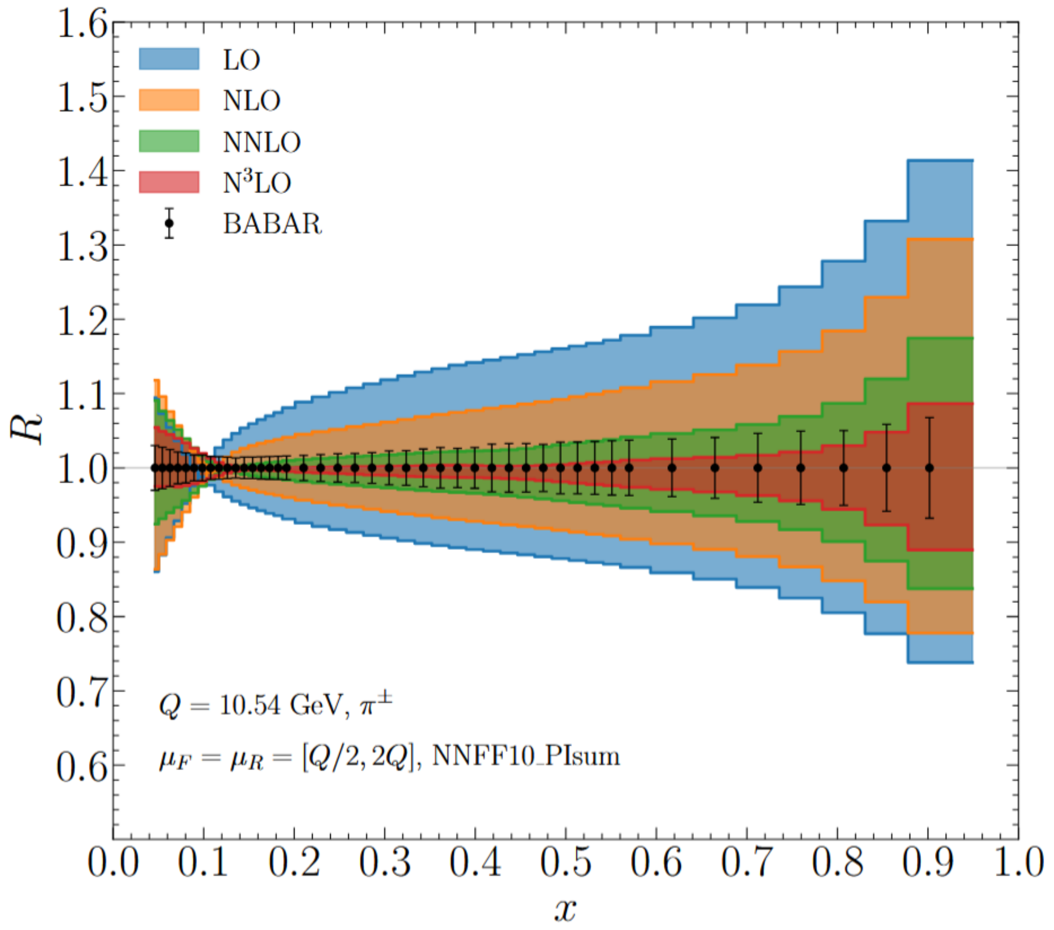
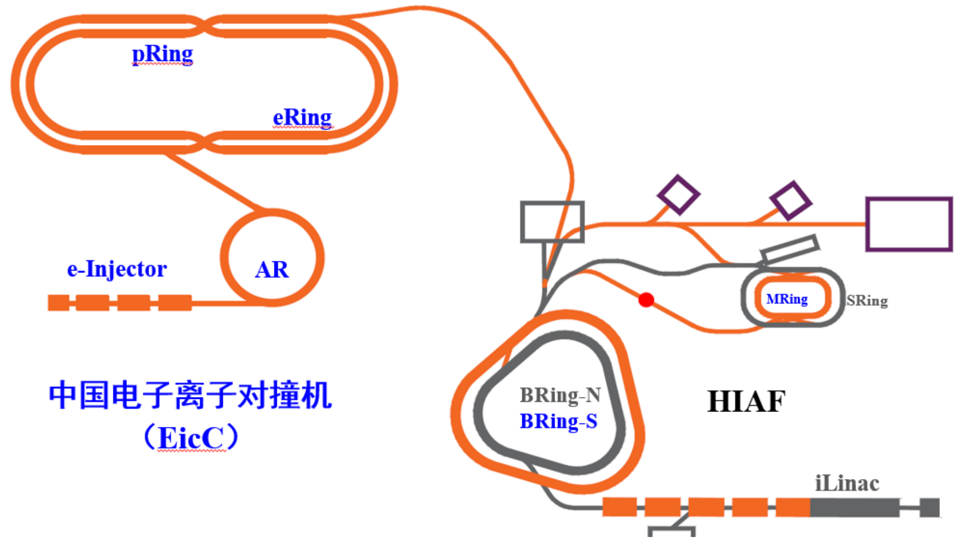


Figure from [He, Xing, Yang, Zhu, PRL.135.101901(2025)]

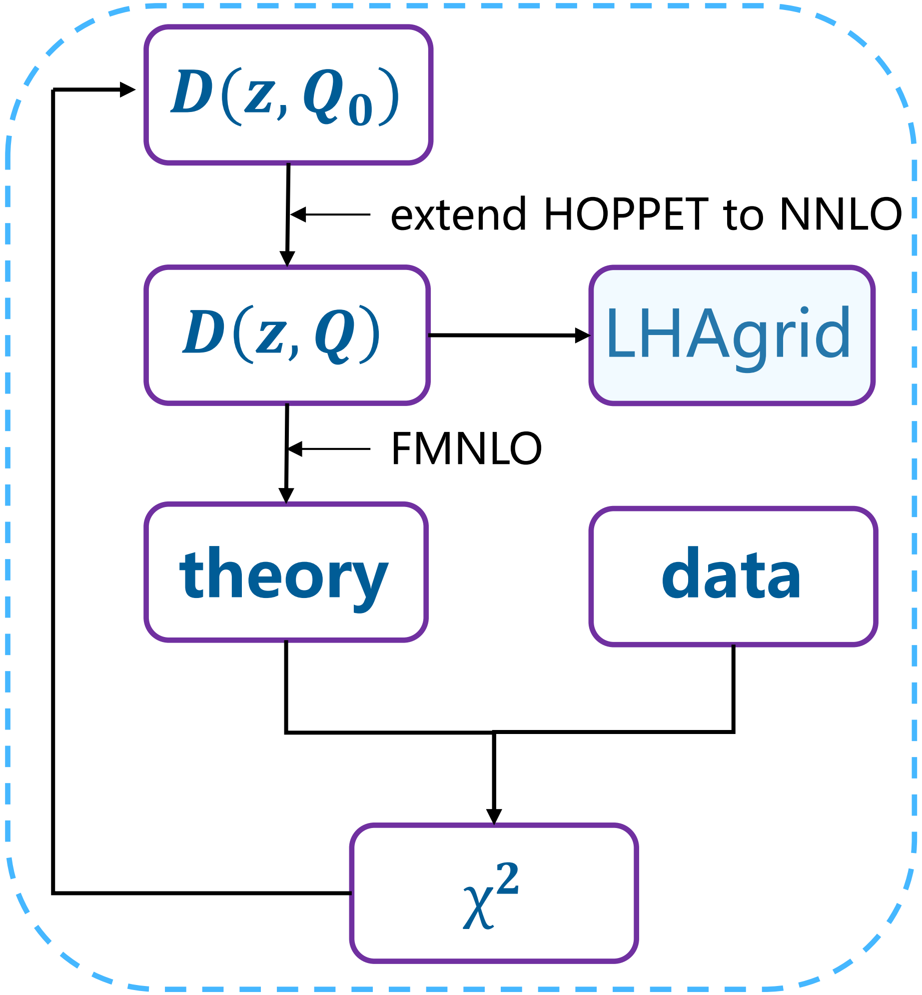
EIC/EicC require high-precision FFs as input



	HERA	EIC	EicC
峰值亮度 ($10^{33}cm^{-2}s^{-1}$)	0.05	10	6
积分亮度 (fb^{-1})	0.21	100	**150**
电子极化率	60%	70%	**80%**
质子极化率	无	70%	70%

Figure&Table from Jiancheng Yang's talk at EICC2026

The framework of NNLO analyses



- FFs at starting scale $Q_0 = 1.4$ GeV

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

+charge/isospin symmetries

- FFs at arbitrary energy scale Q

3-loop timelike splitting functions

[Mitov, Moch, Vogt, Almasy]

[Chen, Yang, Zhu, Zhu, '20]

- SIA/SIDIS coefficient functions at NNLO

[Bonino+, '24], [Goyal+, '24]

The **first** global FF fit (ee+SIDIS) at full NNLO accuracy

The datasets

- SIA(e^+e^-) data used in the fit:

exp.	$\sqrt{s}$/GeV	lum.(n_Z)	year	final states	hadrons
DELPHI	189	157.7 pb^{-1}	2002	inc. had.	$\pi^{\pm}$,$K^{\pm}$
OPAL	m_Z	780 000	1994	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
ALEPH	m_Z	520 000	1995	$Z \to q\bar{q}$	$\pi^{\pm}$, $K^{\pm}$
DELPHI	m_Z	1 400 000	1998	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
				$Z \to b\bar{b}$	$\pi^{\pm}$,$K^{\pm}$
SLD	m_Z	400 000	2004	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
				$Z \to b\bar{b}$	$\pi^{\pm}$, $K^{\pm}$
				$Z \to c\bar{c}$	$\pi^{\pm}$,$K^{\pm}$
TASSO	44	34 pb^{-1}	1989	inc. had.	$\pi^{\pm}$, π^0
TASSO	34	77 pb^{-1}	1989	inc. had.	$\pi^{\pm}$,$K^{\pm}$
TPC/2γ	29	70 pb^{-1}	1988	inc. had.	$\pi^{\pm}$,$K^{\pm}$
Belle	10.52	68 fb^{-1}	2013	inc. had.	$\pi^{\pm}$, $K^{\pm}$
BaBar	10.54	0.91 fb^{-1}	2013	inc. had.	$\pi^{\pm}$,$K^{\pm}$
BESIII	2.0-3.671	253 pb^{-1}	2025	inc. had.	$\pi^{\pm}$, $K^{\pm}$

unique c.m. energy

[BESIII, **PRL**135, 151901, 2025]

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TASSO	34	77 pb^{-1}	1989	inc. had.	$\pi^\pm, K^\pm$
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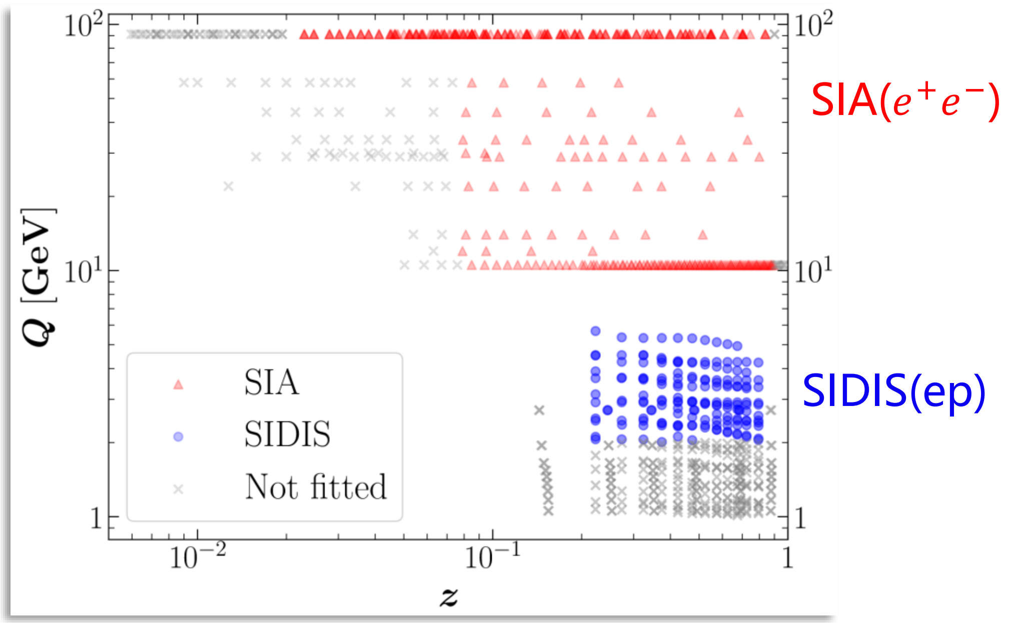
BESIII	2.0-3.671	253 pb^{-1}	2025	inc. had.	$\pi^\pm, K^\pm$

[BESIII, **PRL**135, 151901, 2025]

➢ **separated** kinematic region of e^+e^- and ep data



Datasets used by MAP group

Khalek,Bertone,Nocera PRD.104.034007, '21

➢ This work include both ee and ep data at Q~3GeV

The **first** test on **universality of FFs** at Q~3 GeV (using **ee +ep**)

The results

① Our FFs describe **both ee and ep** data well

	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}$[GeV]	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

⇧

energy cut of the identified hadron

② Test of **leading-twist** factorization

- $E_h \gtrsim 0.8$ GeV
- $Q \gtrsim 3$ GeV (e^+e^-)

③ This work: $\chi^2_{\text{full NNLO}} < \chi^2_{\text{NLO}}$

Previous studies: $\chi^2_{\text{appr.NNLO}} > \chi^2_{\text{NLO}}$

[PRL.129.012002, PLB.2022.137456]

The results

① Our FFs describe **both ee and ep** data well

$E_{h,\min}$[GeV]	BESIII $N_{\rm pt}$	BESIII $\chi^2/N_{\rm pt}$	COMPASS $N_{\rm pt}$	COMPASS $\chi^2/N_{\rm pt}$	B-factories $N_{\rm pt}$	B-factories $\chi^2/N_{\rm pt}$	HE-SIA $N_{\rm pt}$	HE-SIA $\chi^2/N_{\rm pt}$	global $N_{\rm pt}$	global χ^2	global $\chi^2/N_{\rm pt}$
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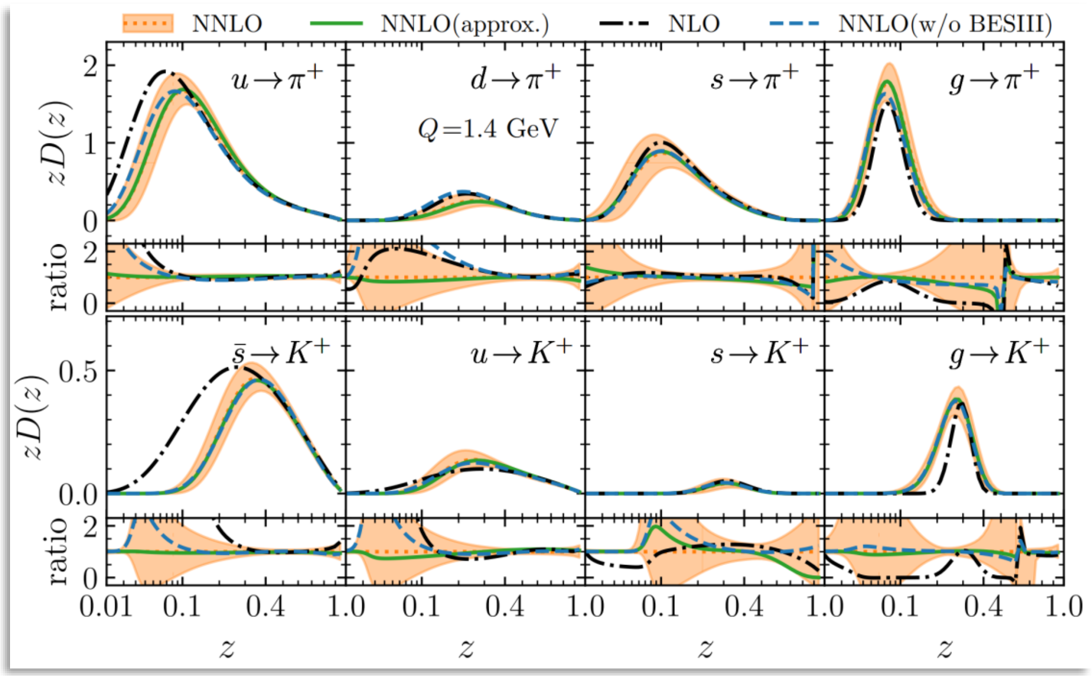
[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]



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Previous studies: $\chi^2_{\rm appr.NNLO} > \chi^2_{\rm NLO}$

[PRL.129.012002, PLB.2022.137456]

LHAgrids of our FFs have been submitted to the LHAPDF repository.

https://www.lhapdf.org/pdfsets.html

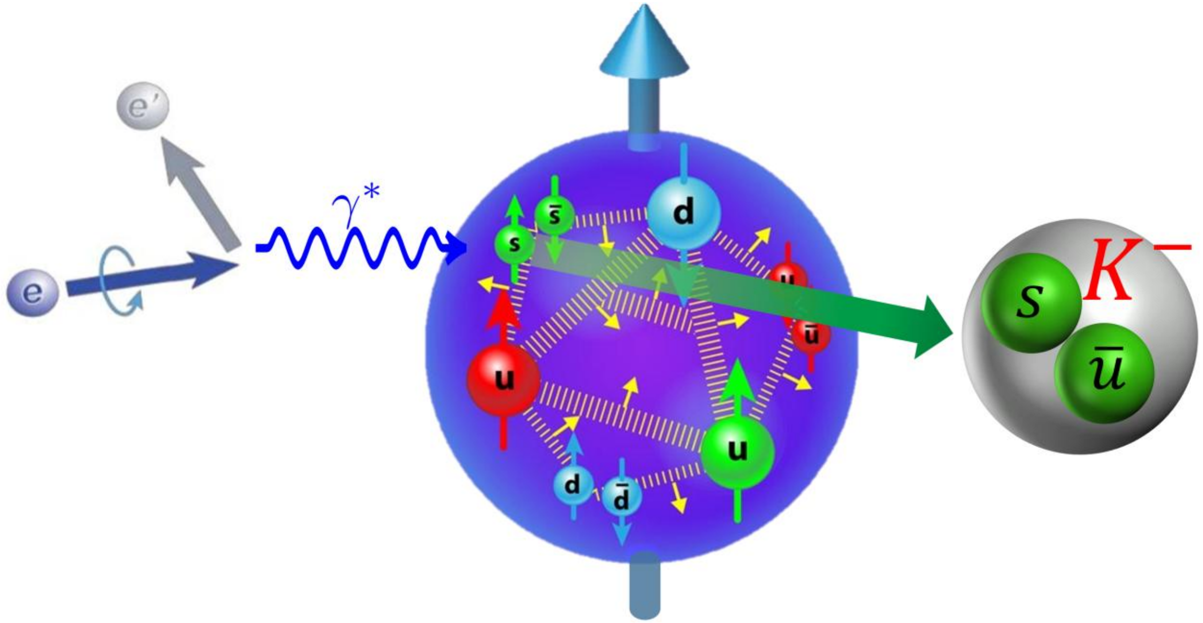
Application of our framework

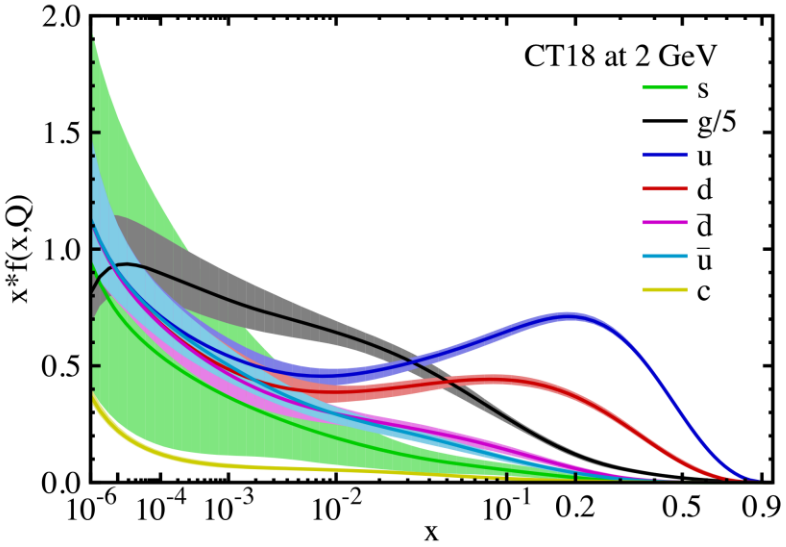
Application: constraining proton PDFs at NNLO

SIDIS may also constrain PDFs

$$\frac{d^3\sigma_h}{dx_B dy dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z)$$

unpolarized PDF **FF**





strangeness asymmetry:

$$r_a = \frac{s - \bar{s}}{s + \bar{s}}$$

Application: constraining proton PDFs at NNLO

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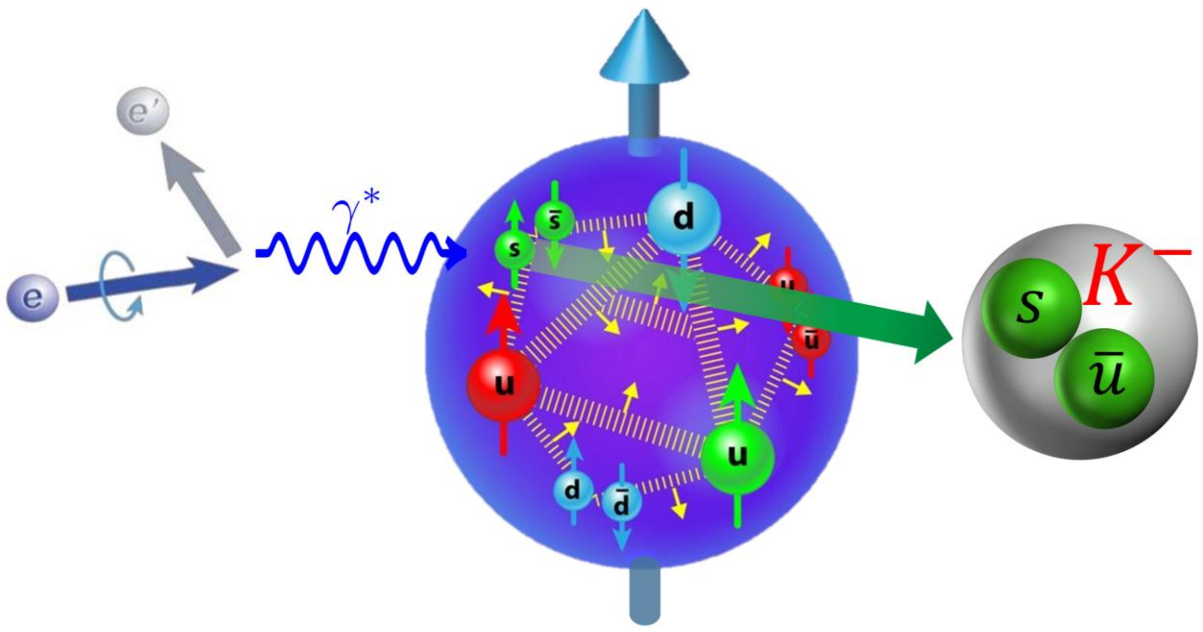
$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x_B\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z)$$

unpolarized PDF FF



- LO xsec of SIDIS off an isoscalar target (COMPASS)

$$\frac{\mathrm{d}\sigma^{K^+}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} - \frac{\mathrm{d}\sigma^{K^-}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z}$$

$$\sim 2\Big(u_v(x) + d_v(x)\Big)\Big(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)\Big)$$

$$+\Big(s(x) - \bar{s}(x)\Big)\Big(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z)\Big) + \cdots$$

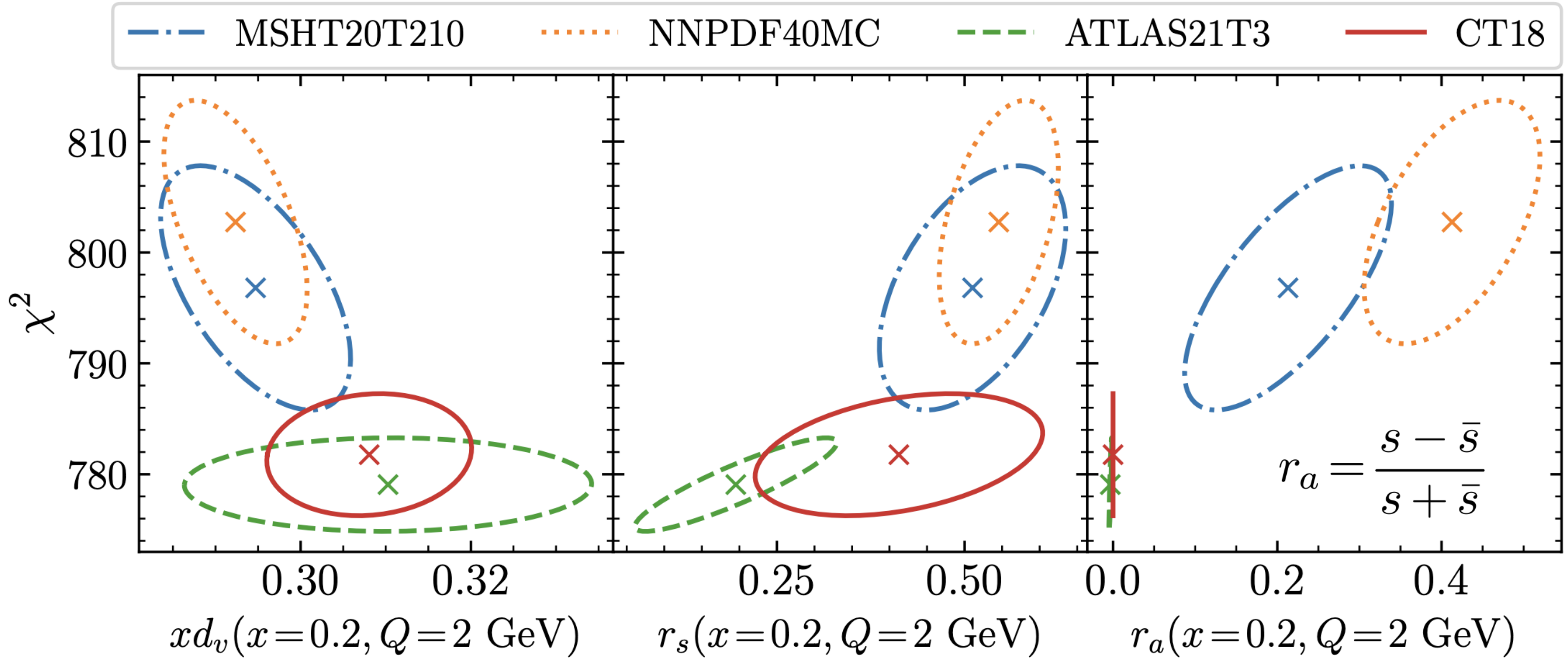
PDF FF

is sensitive to **strangeness asymmetry**

$$r_a = \frac{s - \bar{s}}{s + \bar{s}}$$

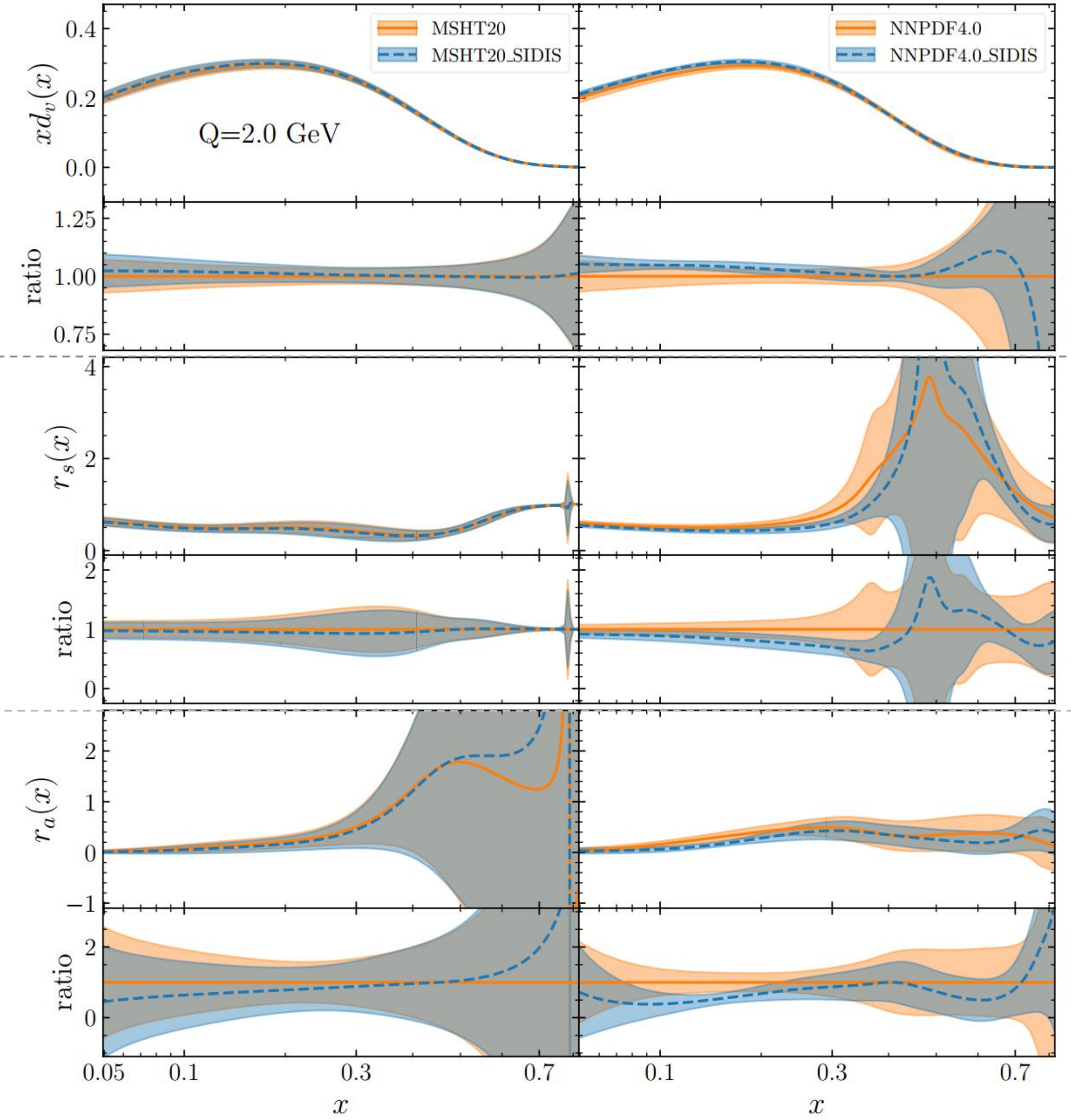
Application: constraining proton PDFs at NNLO

- Correlation between χ^2 and PDFs



- We generate **modified PDFs** which reflect the impact of SIDIS data
  - Reweighting of the NNPDF4.0 PDF set
  - Profiling of the MSHT20 PDF set

Application: PDF sets before and after reweighting/profiling



[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

$$d_v := d(x,Q) - \bar{d}(x,Q)$$

$$r_s := \frac{s(x,Q)+\bar{s}(x,Q)}{\bar{u}(x,Q)+\bar{d}(x,Q)}$$

$$r_a := \frac{s(x,Q)-\bar{s}(x,Q)}{s(x,Q)+\bar{s}(x,Q)}$$

Summary

- FFs are key inputs for calculations of hadron production rate from first principles.
- **Precise and comprehensive** FF sets at NLO.
- **First** global (SIA+SIDIS) FF determination at full NNLO accuracy.
- Based on our NNLO framework, we can assess the impact of SIDIS data on NNLO **PDFs**.

NPC FF sets		
SIA (ee) SIDIS (ep) pp incl. hadron hadron in jet	✓ ✓ ✓ ✓	✓ ✓
hadrons	$\pi^{\pm}, K^{\pm}, p$ K^0, η, Λ	$\pi^{\pm}, K^{\pm}$
pQCD order	NLO	NNLO
$Q_{\min}$	4GeV	1.3GeV

NPC FF sets are available at https://www.lhapdf.org/pdfsets.html

NLO charged hadron:
Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903*, 2024
Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019*, 2024

NLO neutral hadron:
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045*, 2025

NNLO:
Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025

Summary

- FFs are key inputs for calculations of hadron production rate from first principles.
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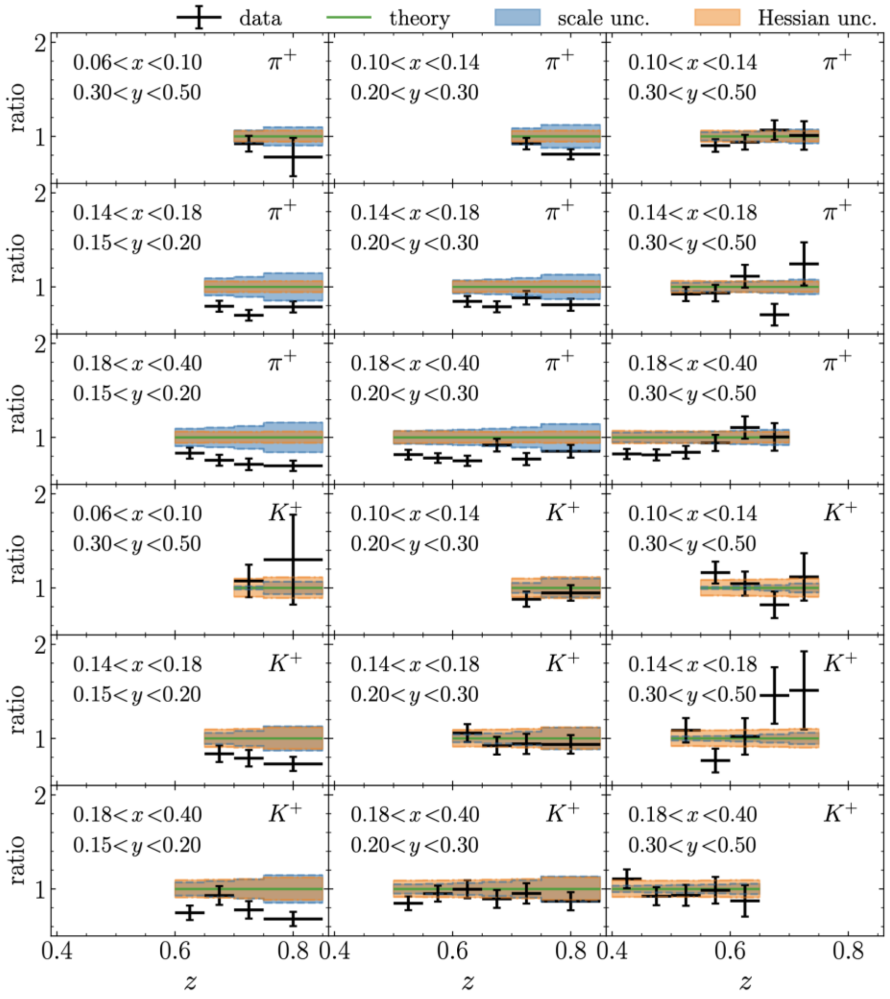
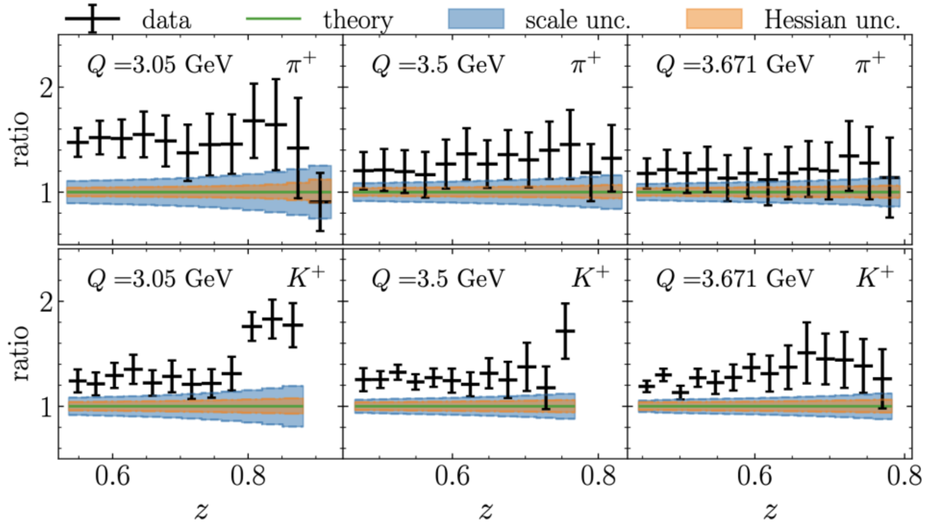
Thank you!

Backup

Theory v.s. data for COMPASS06 (SIDIS: 2~5GeV)

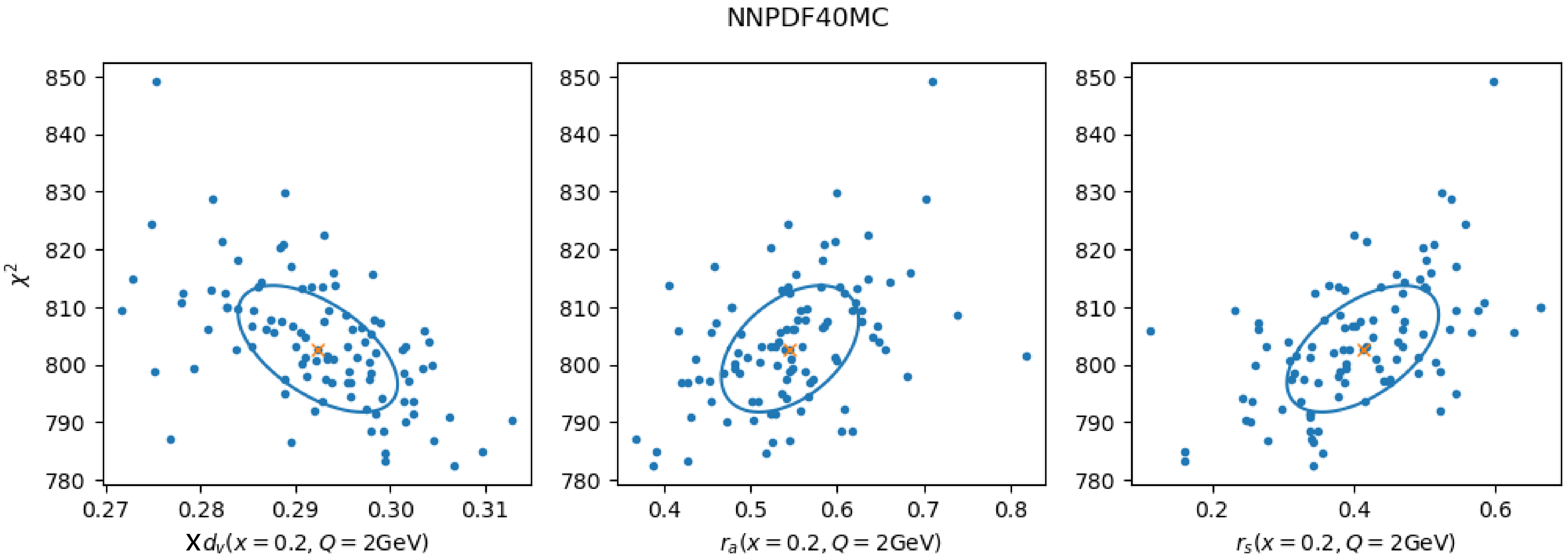


Theory v.s. data for BESIII (SIA: ~3GeV)



- Kinematic cuts in our analyses:
  - $Q > 3$ GeV (SIA)
  - $Q > 2$ GeV(SIDIS)
  - $z > 0.01, E_h > 0.8$ GeV

Backup



NNPDF40MC
850
840
830
820
810
800
790
780
χ^2
0.27
0.28
0.29
0.30
0.31
$\mathrm{X}d_v(x=0.2, Q=2\mathrm{GeV})$
0.4
0.5
0.6
0.7
0.8
$r_a(x=0.2, Q=2\mathrm{GeV})$
0.2
0.4
0.6
$r_s(x=0.2, Q=2\mathrm{GeV})$

lhapdf.org/pdfsets.html

2070200	NPC23_KAp_nlo	(tarball)	(info file)	127
2070400	NPC23_PRp_nlo	(tarball)	(info file)	127
2070600	NPC23_PIm_nlo	(tarball)	(info file)	127
2070800	NPC23_KAm_nlo	(tarball)	(info file)	127
2071000	NPC23_PRm_nlo	(tarball)	(info file)	127
2071200	NPC23_PIsum_nlo	(tarball)	(info file)	127
2071400	NPC23_KAsum_nlo	(tarball)	(info file)	127
2071600	NPC23_PRsum_nlo	(tarball)	(info file)	127
2071800	NPC23_CHHAp_nlo	(tarball)	(info file)	127
2072000	NPC23_CHHAm_nlo	(tarball)	(info file)	127
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2072800	NPC23_lowQ_PIsum_nlo	(tarball)	(info file)	109
2073000	NPC23_lowQ_KAp_nlo	(tarball)	(info file)	109
2073200	NPC23_lowQ_KAm_nlo	(tarball)	(info file)	109
2073400	NPC23_lowQ_KAsum_nlo	(tarball)	(info file)	109
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2073800	NPC23_lowQ_K0s_nlo	(tarball)	(info file)	109
2074000	NPC23_PIp_nnlo	(tarball)	(info file)	109
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2075000	NPC23_KAsum_nnlo	(tarball)	(info file)	109
2075200	NPC23_PI0_nnlo	(tarball)	(info file)	109
2075400	NPC23_K0s_nnlo	(tarball)	(info file)	109

NPC collaboration gathering on July 19th 2025

in neighborhood of Huizhou city (host of EICc)

Jun Gao, ChongYang Liu, Meng Yang Li, **XiaoMin Shen**, **HongXi Xing**, **YuXiang Zhao**, Bin Zhou, YiYu Zhou
Shanghai JiaoTong Univ., South China Normal Univ., Institute of Modern Physics, CAS

NPC=Non-perturbative Physics Collaboration

Slide from Jun Gao