

HENPIC 08/22/2019



# Light nuclei production as a probe of QCD phase diagram

*Phys. Lett. B774, 103 (2017)*

*Phys. Lett. B781, 499 (2018)*

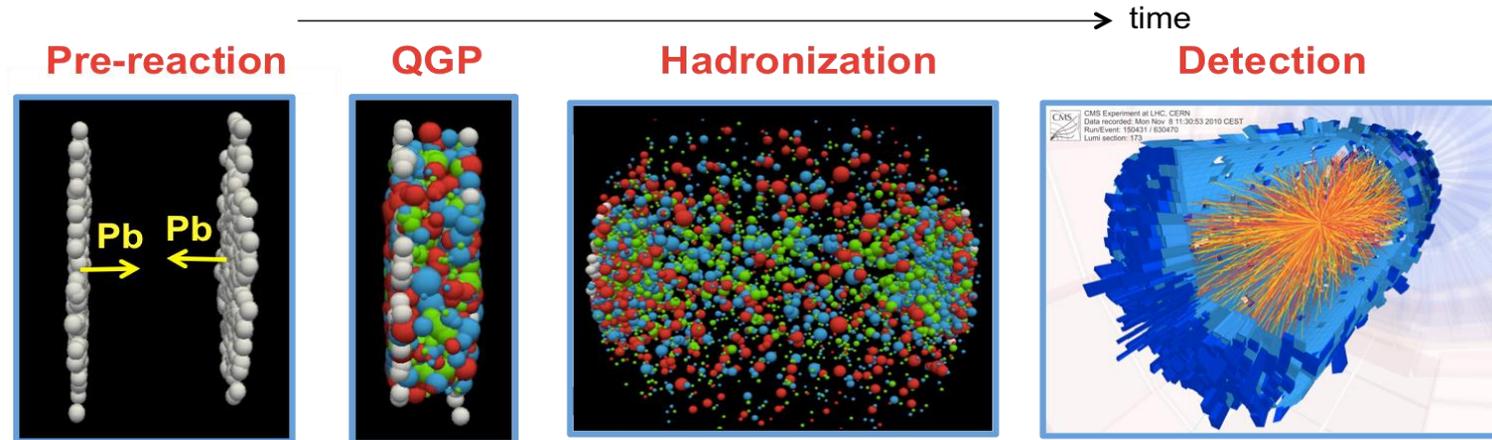
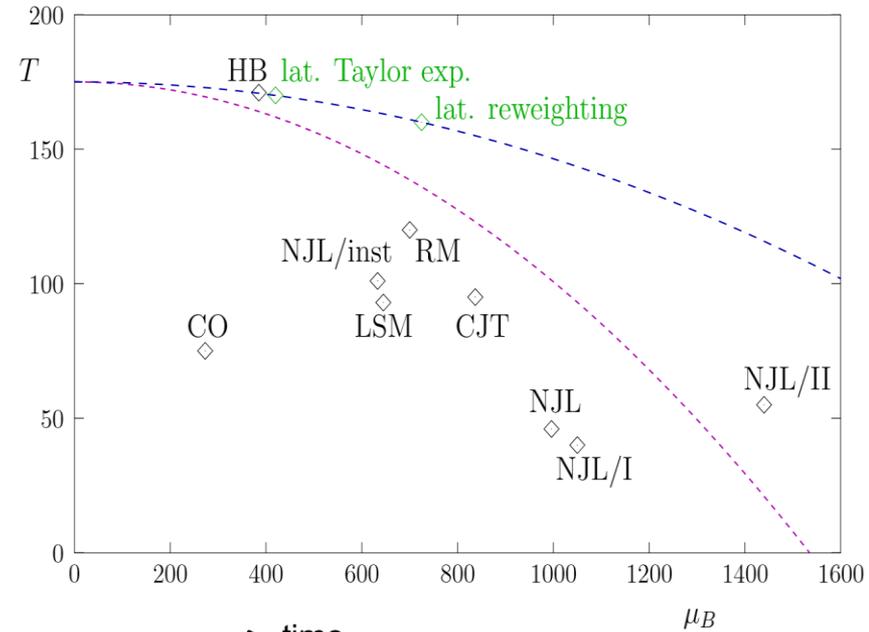
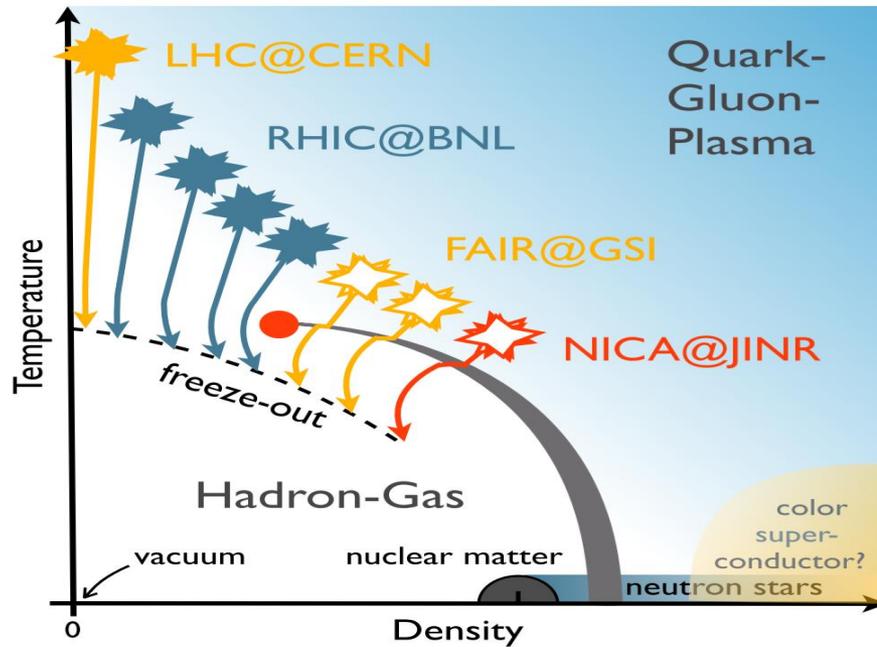
*Phys. Lett. B792, 132 (2019)*

**Kai-Jia Sun (孙开佳)**

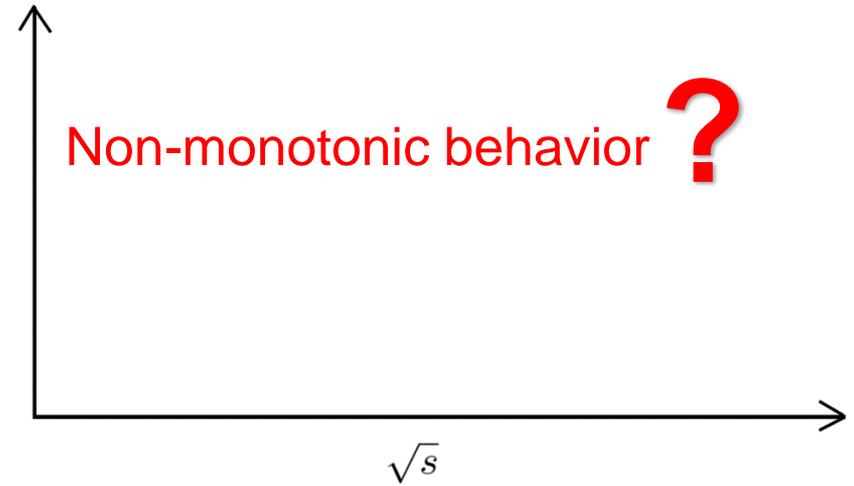
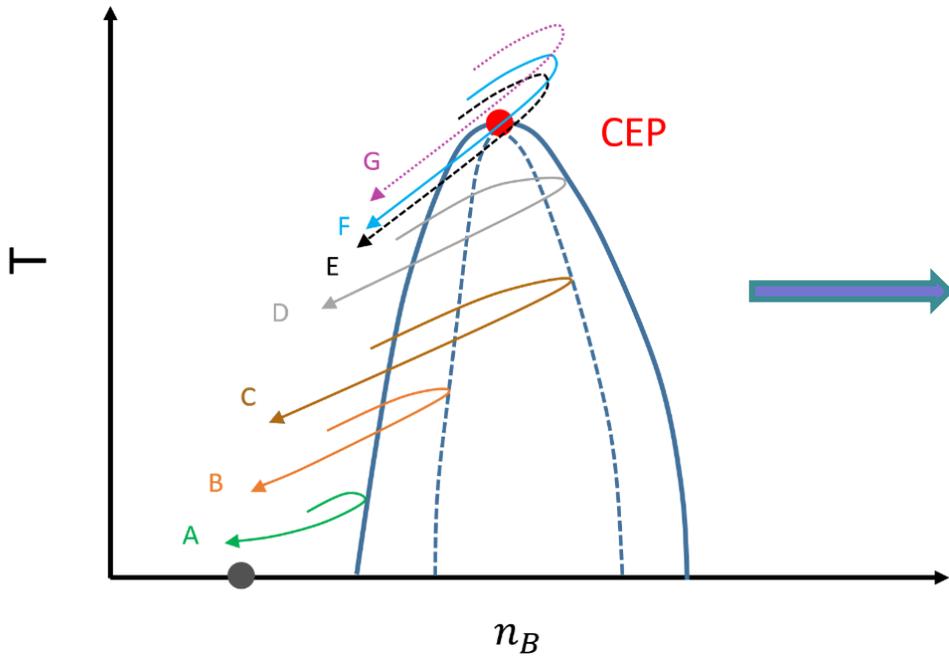
*kjsun@tamu.edu*

*Cyclotron Institute and Department of Physics and Astronomy Texas A&M University*

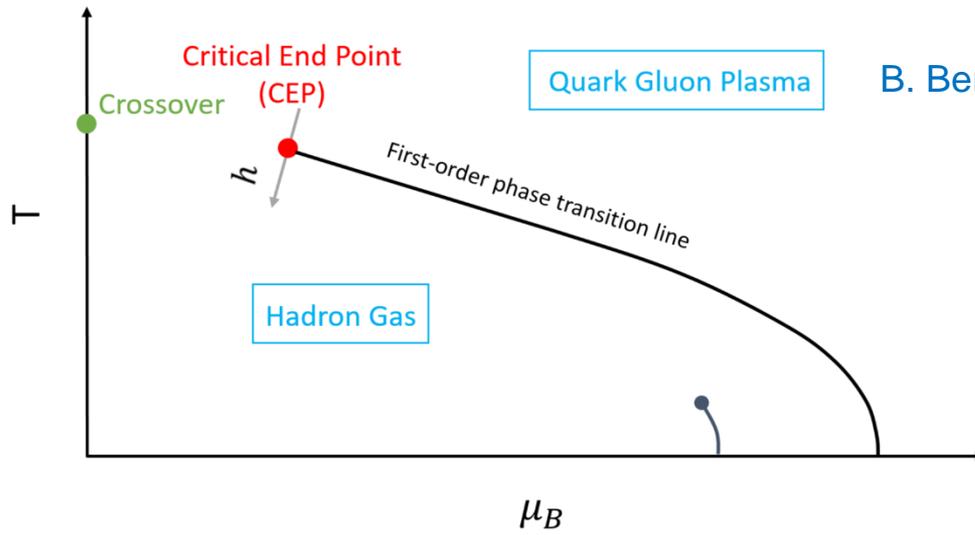
# Where is the critical point in QCD phase diagram?



# What are the observables?



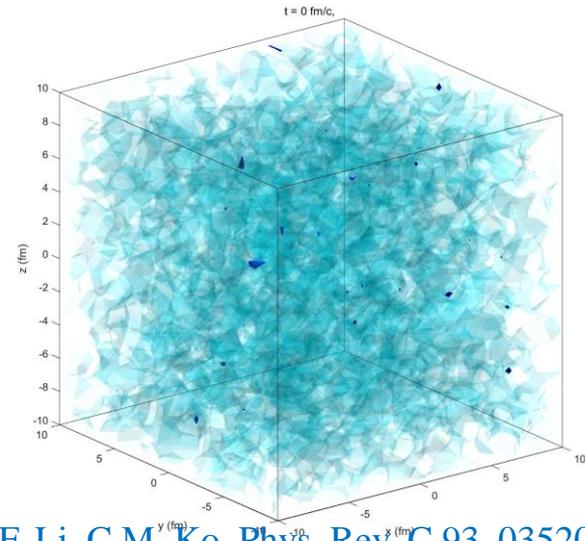
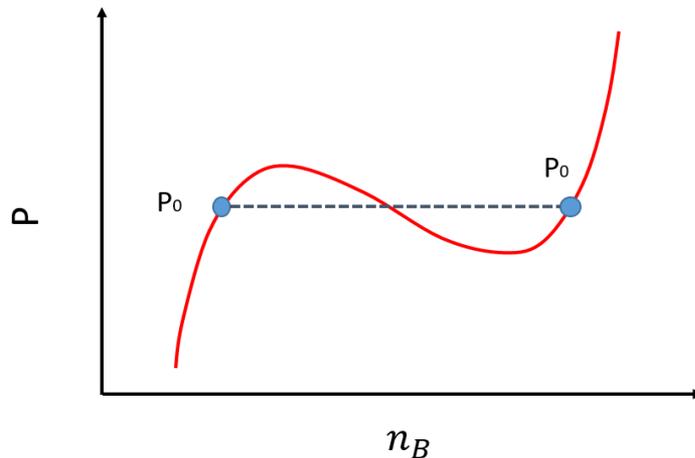
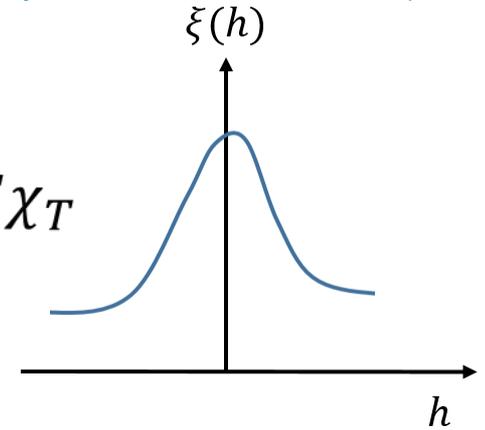
# Enhanced fluctuations during phase transitions



B. Berdnikov, K. Rajagopal, Phys. Rev. D 61 105017, (2000).

$$(\delta Q)^2 = VT\chi_T$$

$$\chi_T \propto \xi^{2-\eta}$$



J. Steinheimer and J. Randrup, Phys. Rev. Lett. 109, 212301 (2012) F. Li, C.M. Ko, Phys. Rev. C 93, 035205 (2016).

# Enhanced fluctuations as a signal of phase transitions

---

Classical ideal gas:

$$\langle (\delta Q_i)^2 \rangle \sim V$$

Cross over:

$$\langle (\delta Q_i)^2 \rangle \sim V$$

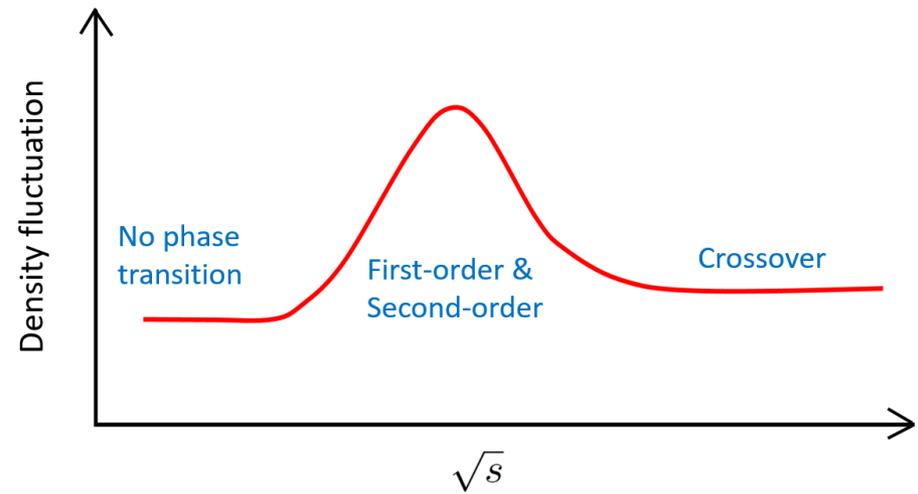
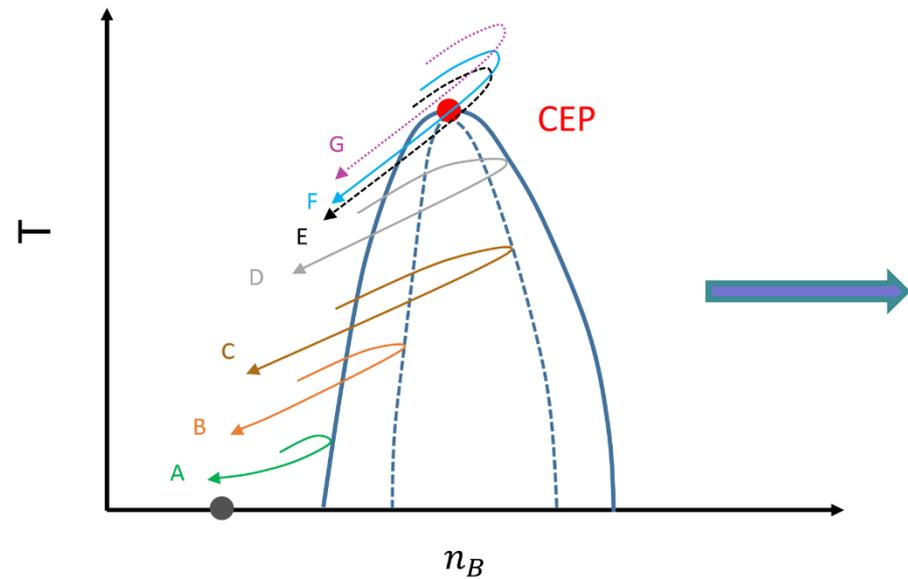
Second-order phase transition:

$$\langle (\delta Q_i)^2 \rangle \sim V^{5/3}$$

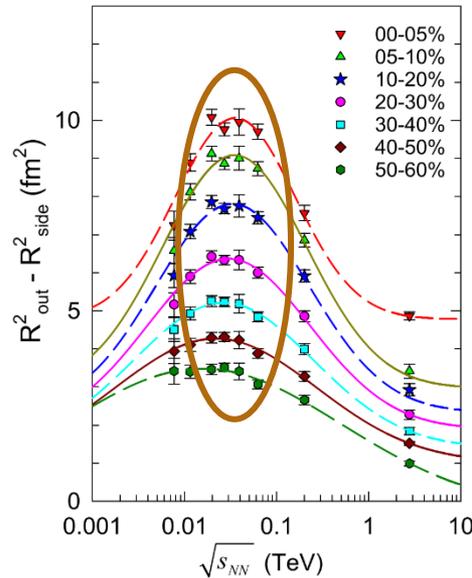
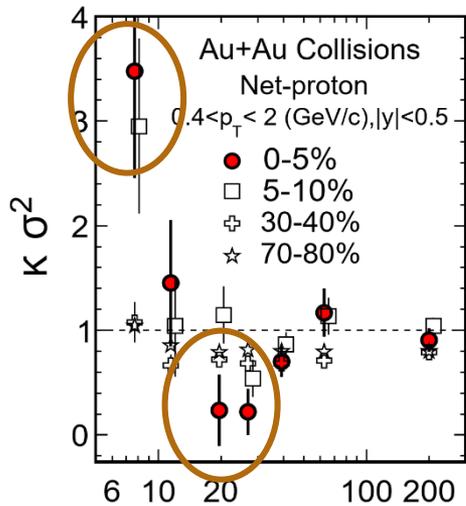
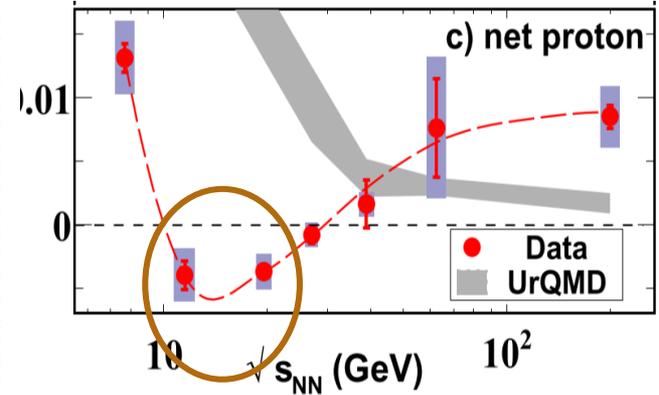
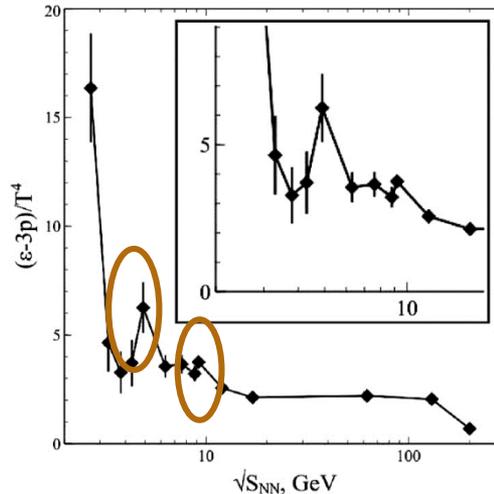
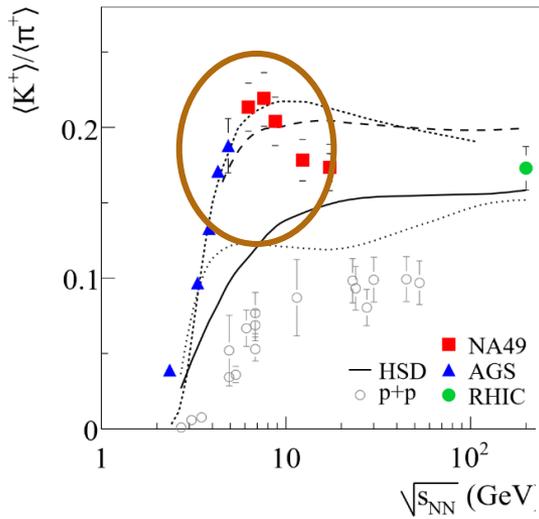
First-order phase transition:

$$\langle (\delta Q_i)^2 \rangle \sim V^2$$

# What are the observables?



# Non-monotonic behaviors



X. F. Luo et al. [STAR Collaboration],  
PoS CPOD2014, 019 (2015).

C. Alt et al, (NA49 Collaboration),  
Phys. Rev. C 77, 024903 (2008).

R. A. Lacey, Phys. Rev. Lett. 114, 142301 (2015).

N. -U. Bastian et al, arXiv: 1608.02851 (2016).

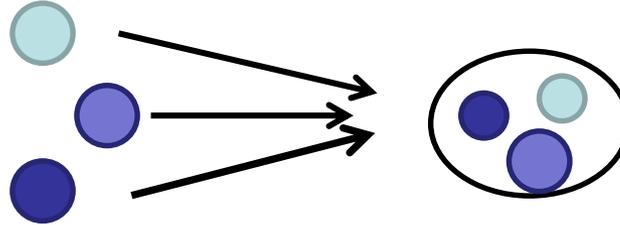
L. Adamczyk et al., arXiv: 1401. 3043 (2014).

K.A. Bugaev et al., arXiv:1709.05419 (2017)

A. Bzdak et al, arXiv:1906. 00936(2019).

# Why light nuclei ?

A simple idea:

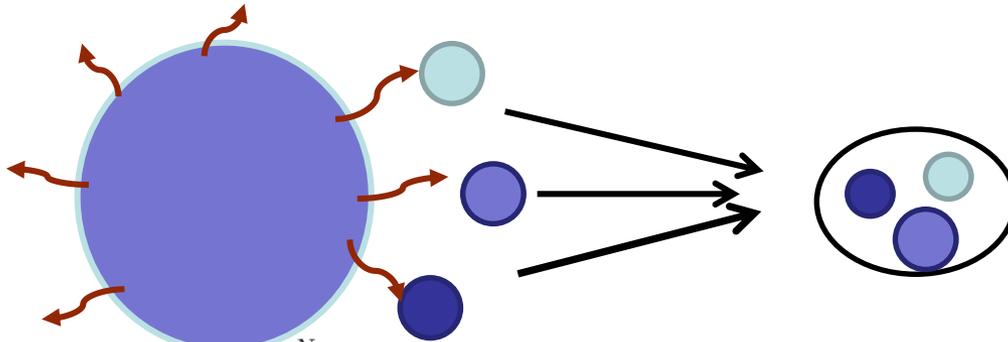


Light nuclei are formed in a restricted volume ( $dx \sim 2$  fm,  $dp \sim 0.1$  GeV) in phase-space.

They can probe **local** nucleon density fluctuations.

Specifically, density fluctuations can affect the multiplicities of light nuclei

Coalescence formation:



Takes the internal structure into consideration

Source:  $T, V, N_1, N_2 \dots$

Cluster:  $m_i, s_i; l_i, r_{\text{rms}}(w)$

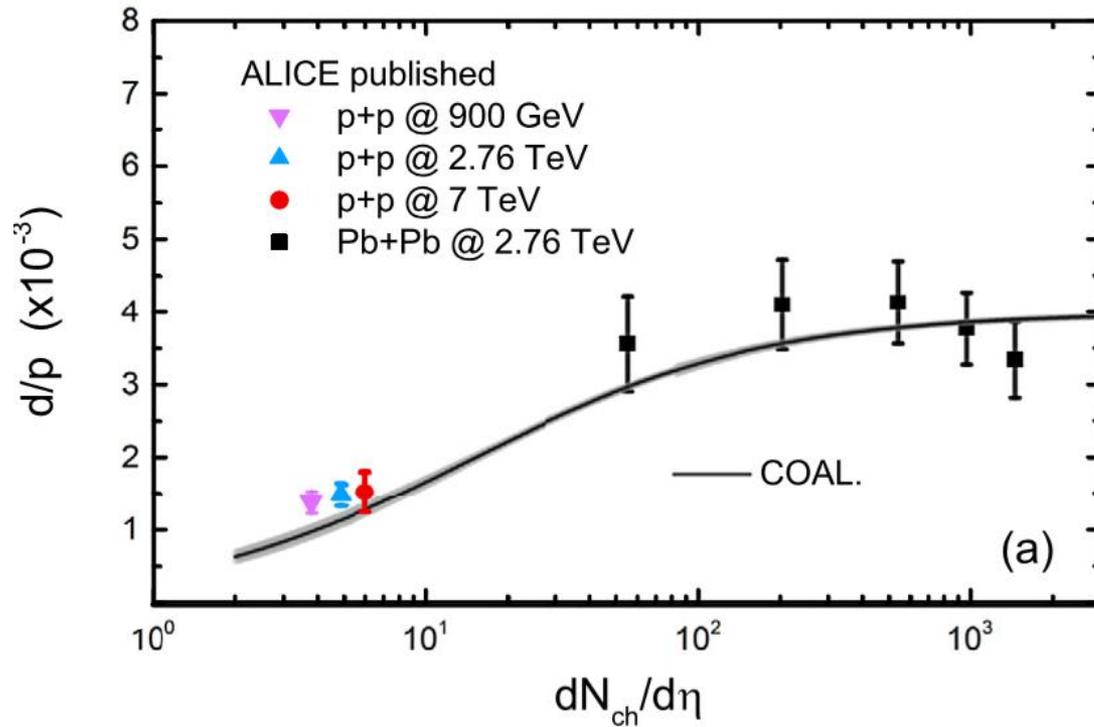
**Wigner function:**

$$\rho^W(\vec{x}, \vec{p}) = \frac{1}{(2\pi\hbar)^3} \int \langle x + y | \hat{\rho} | x - y \rangle e^{-2i\vec{p}\vec{y}/\hbar} d\vec{y}$$

$$N_c = g_c \int \left( \prod_{i=1}^N dN_i \right) \rho_c^W(x_1, \dots, x_N; p_1, \dots, p_N)$$

$$\propto \text{Tr}(\hat{\rho}_i \hat{\rho}_f)$$

# Light nuclei production in collisions of small systems



$$\frac{N_d}{N_p} \approx \frac{4.0 \times 10^{-3}}{\left[1 + \left(\frac{1.6 \text{ fm}}{R}\right)^2\right]^{3/2}}$$

# Recent Studies on Light Nuclei Production

---

Kai-Jia Sun et al., Phys. Lett. B774, 103 (2017)

Kai-Jia Sun et al., Phys. Lett. B781, 499 (2018)

Kai-Jia Sun et al., Phys. Lett. B792, 132 (2019)

Jinhui Chen, Yu-Gang Ma, Aihong Tang, and Zhangbu Xu, Phys. Rept. 760, 1-39 (2018).

Xinyuan Xu and Ralf Rapp, Eur. Phys. J. A55, 68 (2019)

Stanislaw Mrowczynski and Patrycja Slon, arXiv:1904.08320.

Kfir Blum and Masahiro Takimoto, Phys. Rev. C99, 044913 (2019)

Peter Braun-Munzinger and Benjamin Donigus, Nucl. Phys. A987, 144 (2019).

Francesca Bellini and Alexander Philipp Kalweit, Phys. Rev. C99, 054905 (2019).

Edward Shuryak, Juan M. Torres-Rincon, arXiv:1805.0444.

Stanislaw Mrowcznski, Acta Phys. Polon B48, 707 (2017).

Sylwia Bazak and Stanislaw Mrowczynski, Mod. Phys. Lett. A33, 1850142 (2018).

Wenbin Zhao et al., Phys. Rev. C98, 054905 (2018).

Jin Wu, Yufu Lin, Yuanfang Wu and Zhiming Li, arXiv:1901.11193.

Dmytro Oliinychenko, Long-Gang Pang, Hannah Elfner and Volker Koch, arXiv:1812.06225.

K.A. Bugaev et al., arXiv:1812.02509.

Juan M. Torres-Rincon and Edward Shuryak, arXiv:1904.01610.

Ning Yu, Dingwei Zhang and Xiaofeng Luo, 1812.04291.

Yu. B. Ivanov and A.A. Soldatov, arXiv:1703.05040.

# What are the observables?

**Coalescence Model:**

$$N_d = \frac{3}{2^{1/2}} \left( \frac{2\pi}{m_0 T} \right)^{3/2} N_p \langle n \rangle,$$

$$N_{3H} = \frac{3^{3/2}}{4} \left( \frac{2\pi}{m_0 T} \right)^3 N_p \langle n \rangle^2 (1 + \Delta n),$$

$$\Delta n = \frac{\langle (\delta n)^2 \rangle}{\langle n \rangle^2}$$

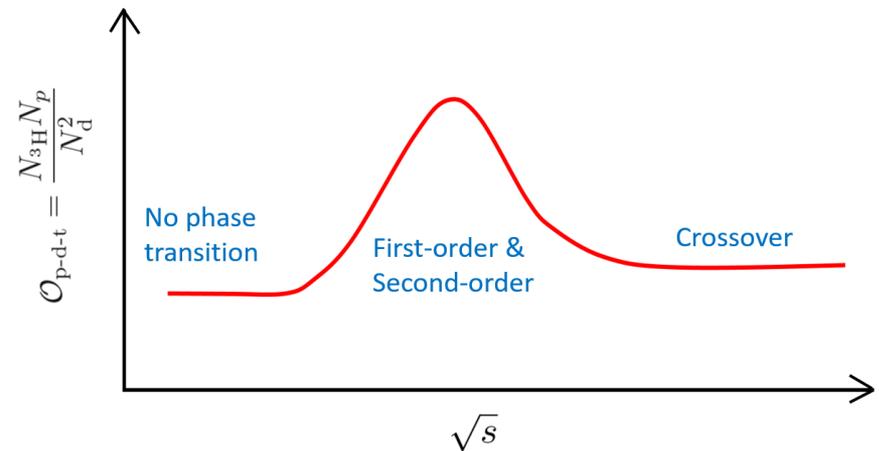
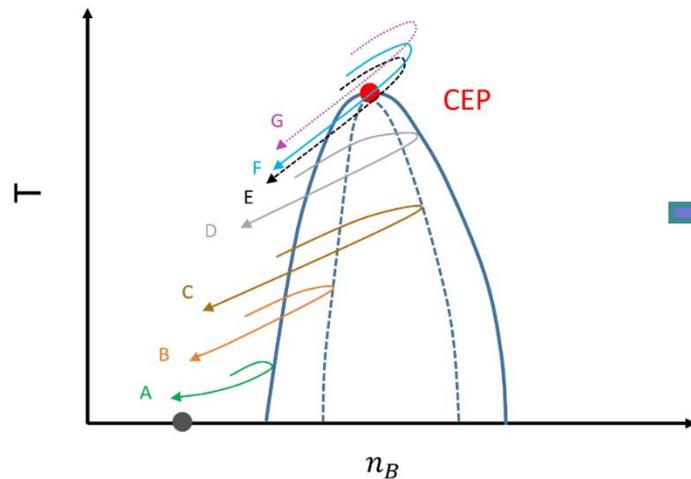
$$\langle n(\vec{r}) \rangle = \frac{1}{V} \int d\vec{r} n(\vec{r})$$

$$\langle (\delta n(\vec{r}))^2 \rangle = \frac{1}{V} \int d\vec{r} (\delta n(\vec{r}))^2$$

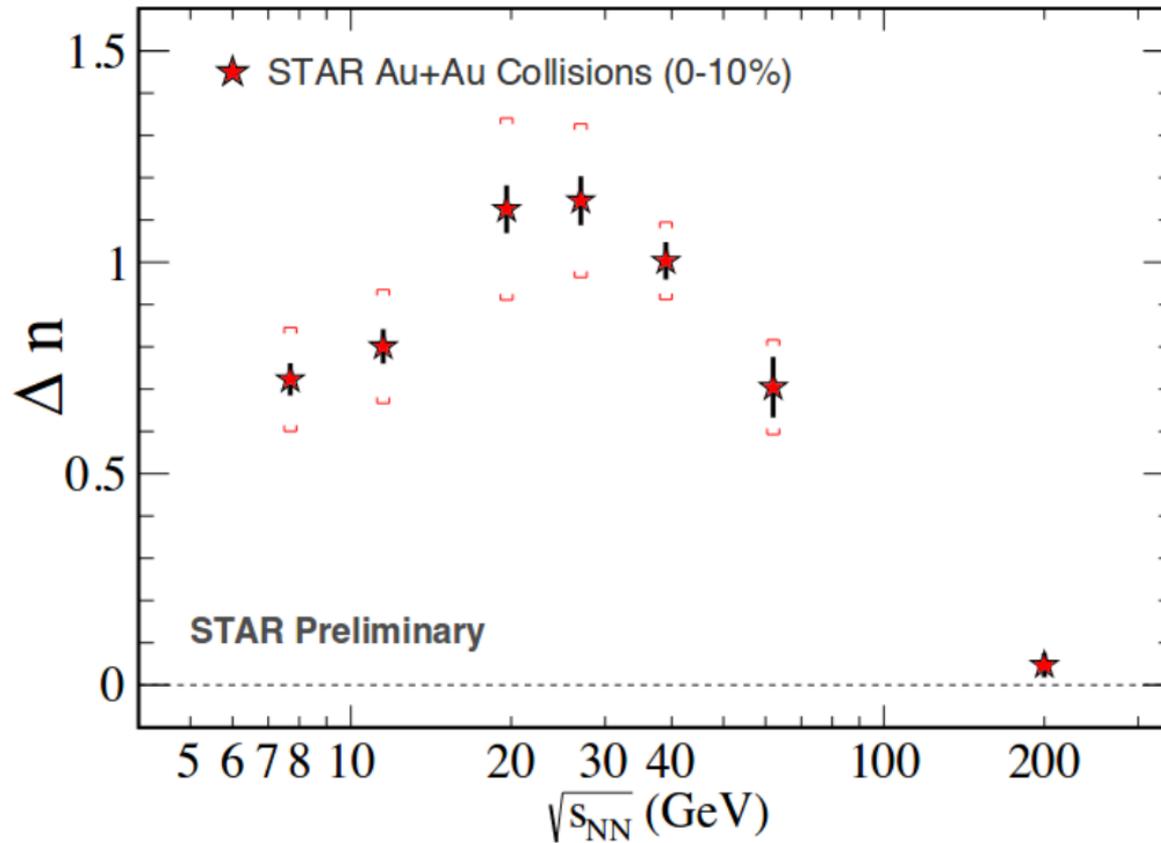
$$\mathcal{O}_{p-d-t} = \frac{N_{3H} N_p}{N_d^2} = g(1 + \Delta n)$$

$$g = 4/9 \times (3/4)^{3/2} \approx 0.29$$

Kai-Jia Sun et al., Phys. Lett. B774, 103 (2017)

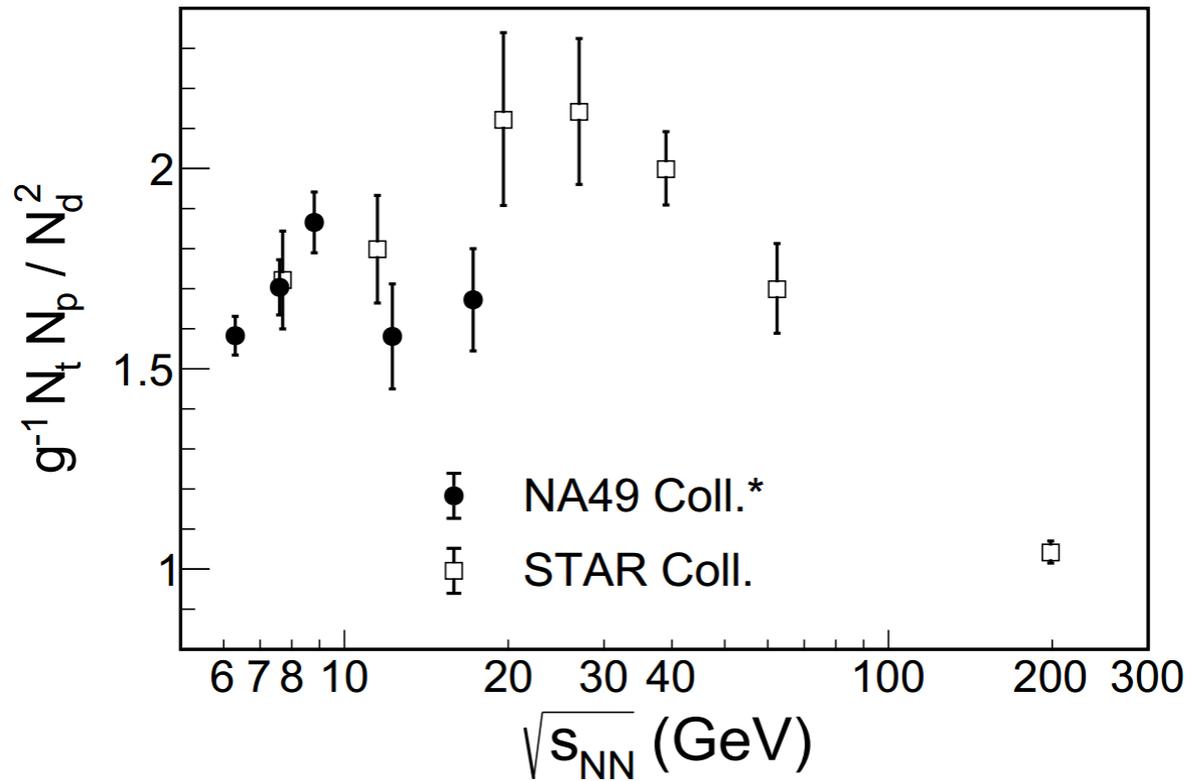


# Experimental Results at RHIC

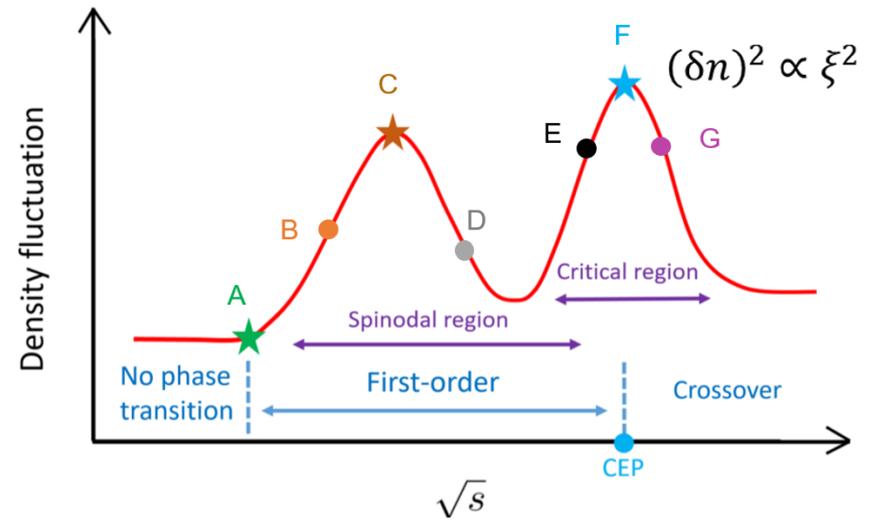
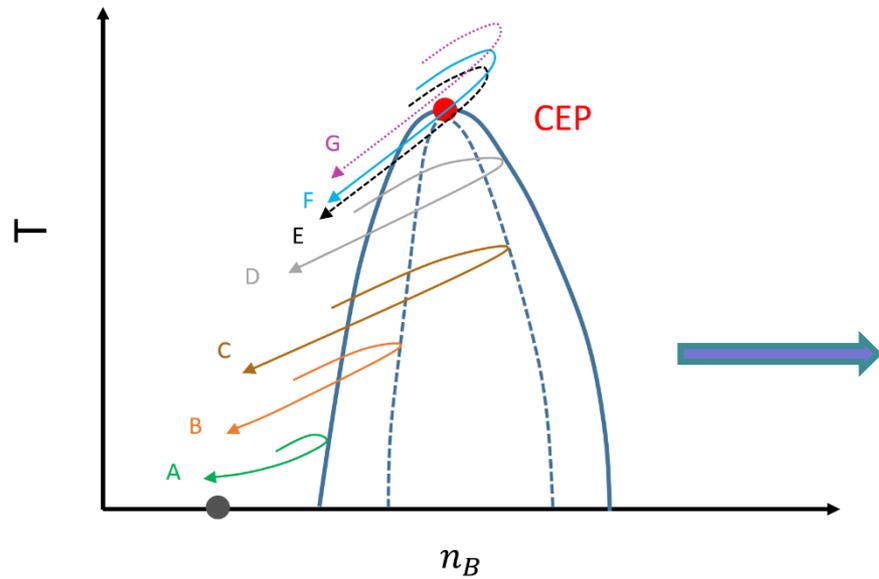


Dingwei Zhang, poster at Quark Matter 2018

# Experimental Results



# Possible Double-Peak Structure?

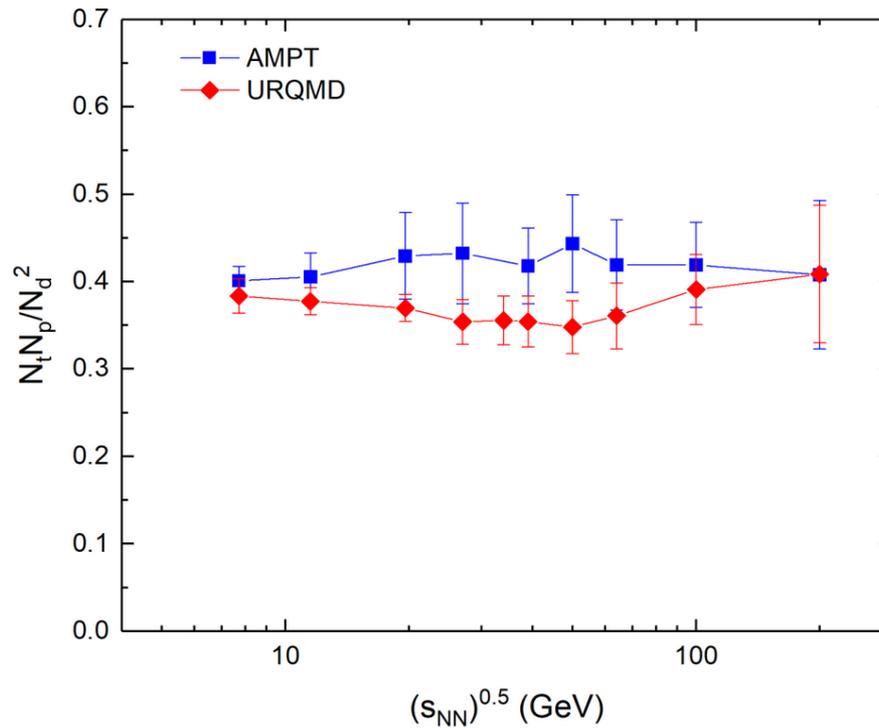


Kai-Jia Sun et al., Phys. Lett. B781, 499 (2018)

# Work in Progress

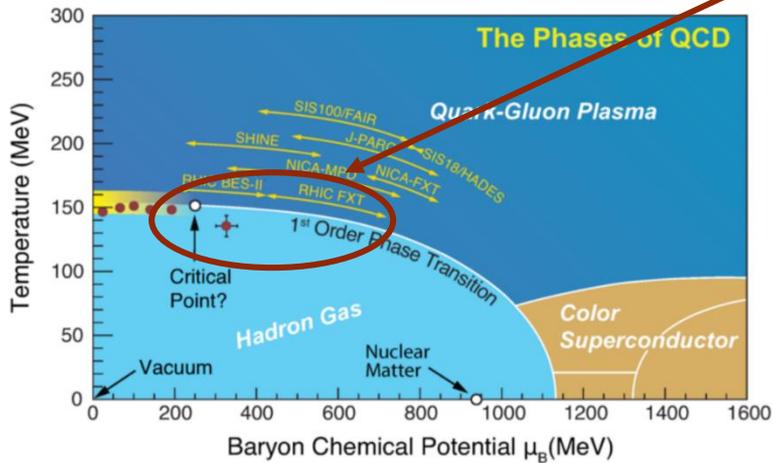
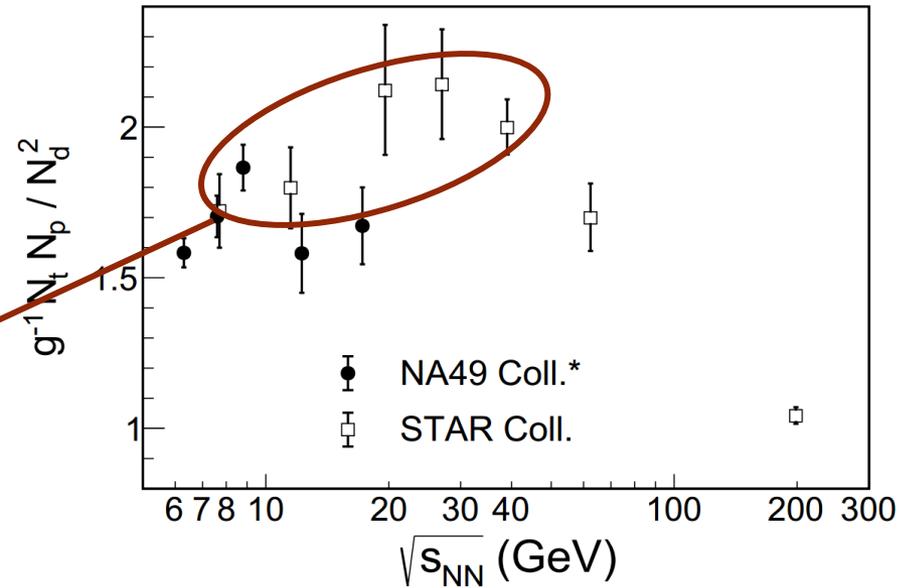
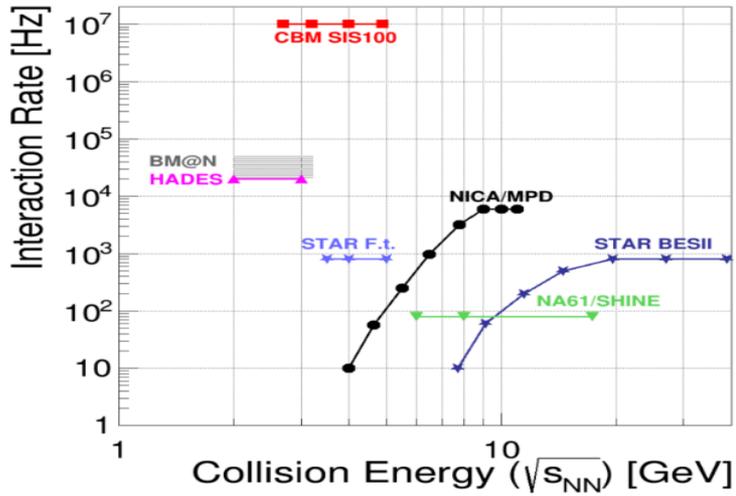
## Microscopic Calculations

### Yield ratio from AMPT and URQMD



Implementing phase transitions in transport codes

# Summary



# *Thank You !*

**Collaborators:** Prof. Che Ming Ko<sup>1</sup>  
Prof. Lie-Wen Chen<sup>2</sup>  
Prof. Zhangbu Xu<sup>3</sup>  
Dr. Jie Pu<sup>2</sup>

1. Cyclotron Institute and Department of Physics and Astronomy Texas A&M University.
2. School of Physics and Astronomy, and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, China.
3. Brookhaven National Laboratory; School of Physics & Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University.

# Backup

---

# Particle coalescence production

Deuteron:

$$N_d = g_d \int d^3\mathbf{x}_1 \int d^3\mathbf{k}_1 \int d^3\mathbf{x}_2 \int d^3\mathbf{k}_2 f_n(\mathbf{x}_1, \mathbf{k}_1) f_p(\mathbf{x}_2, \mathbf{k}_2) W_d(\mathbf{x}_1 - \mathbf{x}_2, (\mathbf{k}_1 - \mathbf{k}_2)/2),$$

Gaussian approximation:

$$f(\mathbf{x}, \mathbf{k}) = \frac{2\xi}{(2\pi)^3} e^{-\frac{k^2}{2mT}}$$

$$W_d(\mathbf{x}, \mathbf{k}) = 8 e^{-\frac{x^2}{\sigma^2}} e^{-\sigma^2 k^2}$$

$$\mathbf{X} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \quad \mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2,$$

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2, \quad \mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2},$$

$$N_d = \frac{32g_d\xi_1\xi_2}{(2\pi)^6} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\frac{x^2}{\sigma^2}} \int d^3\mathbf{K} e^{-\frac{K^2}{4mT}} \int d^3\mathbf{k} e^{-k^2(\sigma^2 + \frac{1}{mT})}$$

$$= \frac{32g_d\xi_1\xi_2}{(2\pi)^6} V (\pi\sigma^2)^{3/2} (4\pi mT)^{3/2} \left(\frac{\pi}{\sigma^2 + \frac{1}{mT}}\right)^{3/2} \xrightarrow{mT \gg 1/\sigma^2} N_d \approx \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} \frac{N_n N_p}{V}$$

$$= \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} \frac{1}{\left(1 + \frac{1}{mT\sigma^2}\right)^{3/2}} \frac{N_n N_p}{V}.$$

A general formula (COAL-SH) with boost invariance:

$$\frac{dN_c}{dy} \approx g_{\text{rel}} g_{\text{size}} g_c \mu_0^{\frac{3}{2}} \left[ \prod_{i=1}^N \frac{dN_i}{m_i^{\frac{3}{2}}} \right] \times \mathbf{K. J. Sun and L. W. Chen, arXiv:1701.01935 (2017).}$$

$$\prod_{i=1}^{N-1} \frac{\left(\frac{4\pi}{w}\right)^{\frac{3}{2}}}{V \left(\frac{2T}{w}\right)^{\frac{1}{2}} \left(1 + \frac{2T}{w}\right)} \left(\frac{2T}{w} + 1\right)^{l_i} G\left(l_i, \left(\frac{2T}{w}\right)^{\frac{1}{2}}\right).$$

# Without density fluctuation

---

Coalescence model:

$$\begin{aligned} N_d &= \frac{3}{2^{1/2}} \left( \frac{2\pi}{m T} \right)^{3/2} \frac{N_p N_n}{V}, & \sim \int d\vec{r} n(\vec{r}) n_p(\vec{r}) \\ N_{3H} &= \frac{3^{3/2}}{4} \left( \frac{2\pi}{m T} \right)^3 \frac{N_p N_n^2}{V^2}. & \sim \int d\vec{r} n(\vec{r})^2 n_p(\vec{r}) \end{aligned}$$

# In vicinity of density fluctuation

Density fluctuation over space:

$$n(\vec{r}) = \frac{1}{V} \int n(\vec{r}') d\vec{r}' + \delta n(\vec{r}) = \langle n \rangle + \delta n(\vec{r})$$

When:  $\delta n \neq 0$

Neutron:  $n(\vec{r}) = \langle n \rangle + \delta n(\vec{r}),$

Proton:  $n_p(\vec{r}) = \langle n_p \rangle + \delta n_p(\vec{r}),$

$$\langle \delta n \rangle = 0, \langle \delta n_p \rangle = 0,$$

$$N_n = \int d\vec{r} n = V \langle n \rangle, N_p = \int d\vec{r} n_p = V \langle n_p \rangle.$$

Approximately:

$$\begin{aligned} N_d &= \frac{3}{2^{1/2}} \left( \frac{2\pi}{m T} \right)^{3/2} \int d\vec{r} n(\vec{r}) n_p(\vec{r}) \\ &= \frac{3}{2^{1/2}} \left( \frac{2\pi}{m T} \right)^{3/2} \int d\vec{r} (\langle n \rangle + \delta n(\vec{r})) (\langle n_p \rangle + \delta n_p(\vec{r})) \quad \text{Cross terms vanish} \\ &= \frac{3}{2^{1/2}} \left( \frac{2\pi}{m T} \right)^{3/2} \int d\vec{r} (\langle n \rangle \langle n_p \rangle + \delta n(\vec{r}) \delta n_p(\vec{r})) \\ &= \frac{3}{2^{1/2}} \left( \frac{2\pi}{m T} \right)^{3/2} (N_p \langle n \rangle + \int d\vec{r} \delta n(\vec{r}) \delta n_p(\vec{r})). \end{aligned}$$

# Density correlation and fluctuation

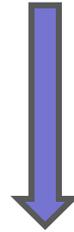
$$N_d = \frac{3}{2^{1/2}} \left( \frac{2\pi}{m T} \right)^{3/2} N_p \langle n \rangle (1 + \alpha \Delta n),$$

$$N_{3H} = \frac{3^{3/2}}{4} \left( \frac{2\pi}{m T} \right)^3 N_p \langle n \rangle^2 (1 + (1 + 2\alpha) \Delta n)$$

$$C_{np} = \langle \delta n \delta n_p \rangle / (\langle n \rangle \langle n_p \rangle) = \alpha \Delta n$$

$$\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$$

$$\Delta n_I = \frac{\langle (\delta n - \delta n_p)^2 \rangle}{(\langle n \rangle + \langle n_p \rangle)^2}$$



$$C_{np} \approx g_{p-d} R_{np} V_{ph} \mathcal{O}_{p-d} - 1,$$

$$\Delta n \approx g_{p-d-t} (1 + C_{np})^2 \mathcal{O}_{p-d-t} - 2C_{np} - 1,$$

$$\langle n(\vec{r}) \rangle = \frac{1}{V} \int d\vec{r} n(\vec{r})$$

$$\langle (\delta n(\vec{r}))^2 \rangle = \frac{1}{V} \int d\vec{r} (\delta n(\vec{r}))^2$$

$$g_{p-d} = \frac{2^{1/2}}{3(2\pi)^3} \approx 0.0019, \quad g_{p-d-t} = 9/4 \times (4/3)^{3/2} \approx 3.5, \quad R_{np} = N_p / N_n = \langle n_p \rangle / \langle n \rangle$$

$$\mathcal{O}_{p-d} = N_d / N_p^2, \quad \mathcal{O}_{p-d-t} = N_p N_{3H} / N_d^2, \quad R_{np} = (\pi^+ / \pi^-)^{1/2}$$

Can be well determined from experiments

$$V_{ph} = (2\pi m T)^{3/2} V \quad ?$$

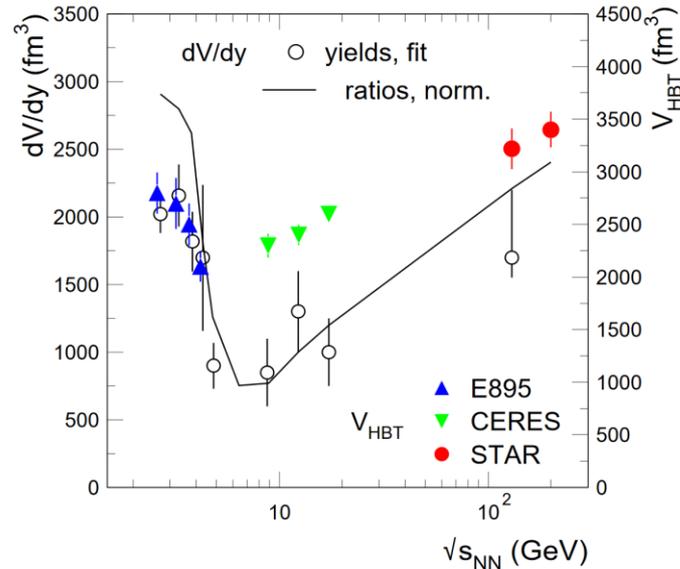
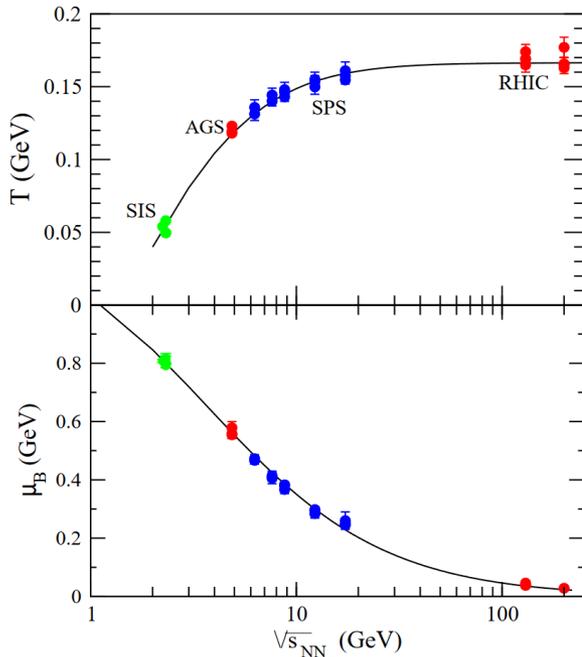
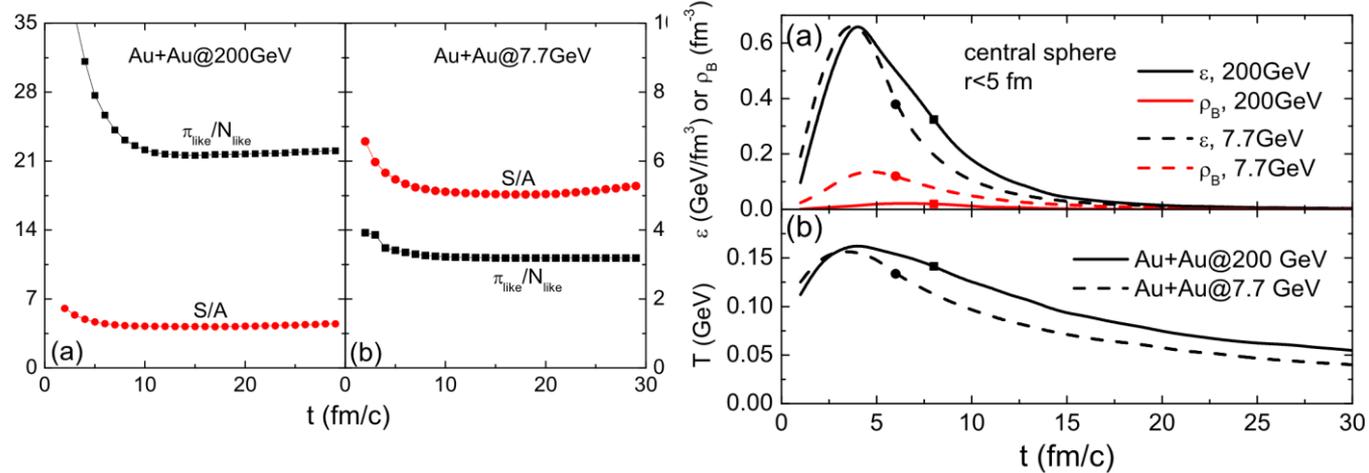
# Relating kinetic and chemical FO

$$V_{\text{ph}} = (2\pi mT)^{3/2} V$$

$$S/N = 5/2 + \ln(V_{\text{ph}}/N)$$

$$T^{3/2} V = \lambda T_{\text{ch}}^{3/2} V_{\text{ch}}$$

$$\lambda = 1.6$$



J. Xu and C. M. Ko,  
arXiv: 1704.04934 (2017)

J. Cleymans et al.,  
Phys. Rev. C 73, 034905 (2006).

A. Andronic et al.,  
B. Nucl. Phys. A 772, 167 (2006).

# Results

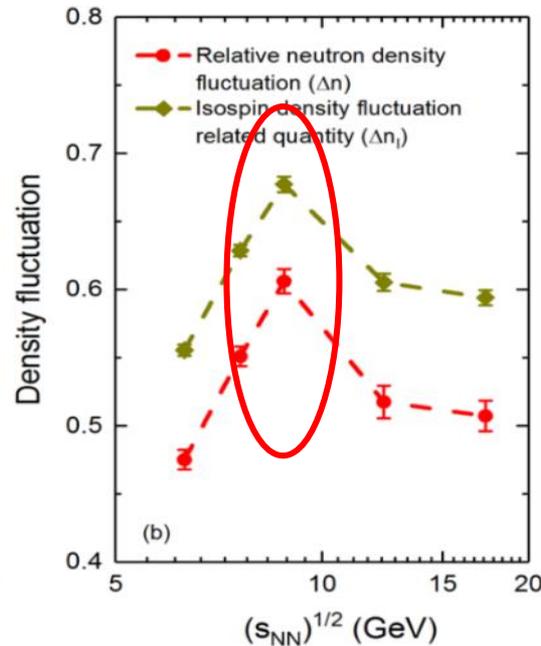
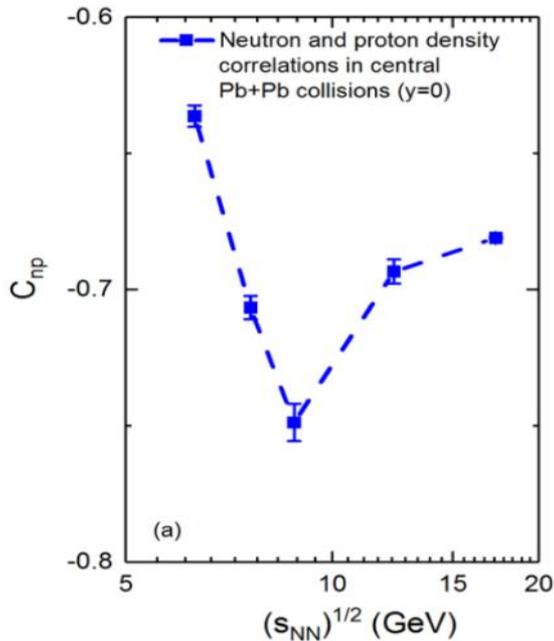
$$C_{np} \approx g_{p-d} R_{np} V_{ph} \mathcal{O}_{p-d} - 1,$$

$$\Delta n \approx g_{p-d-t} (1 + C_{np})^2 \mathcal{O}_{p-d-t} - 2C_{np} - 1,$$

$$\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$$

$$C_{np} = \langle \delta n \delta n_p \rangle / (\langle n \rangle \langle n_p \rangle) = \alpha \Delta n$$

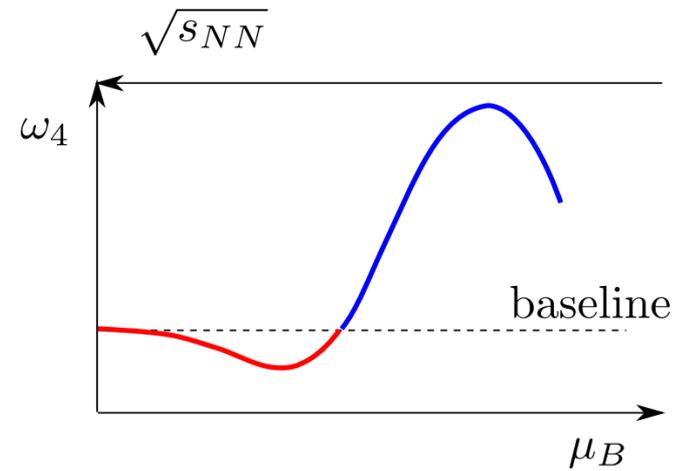
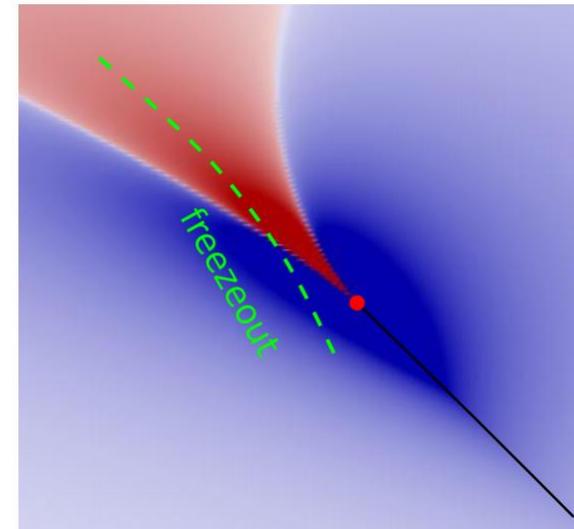
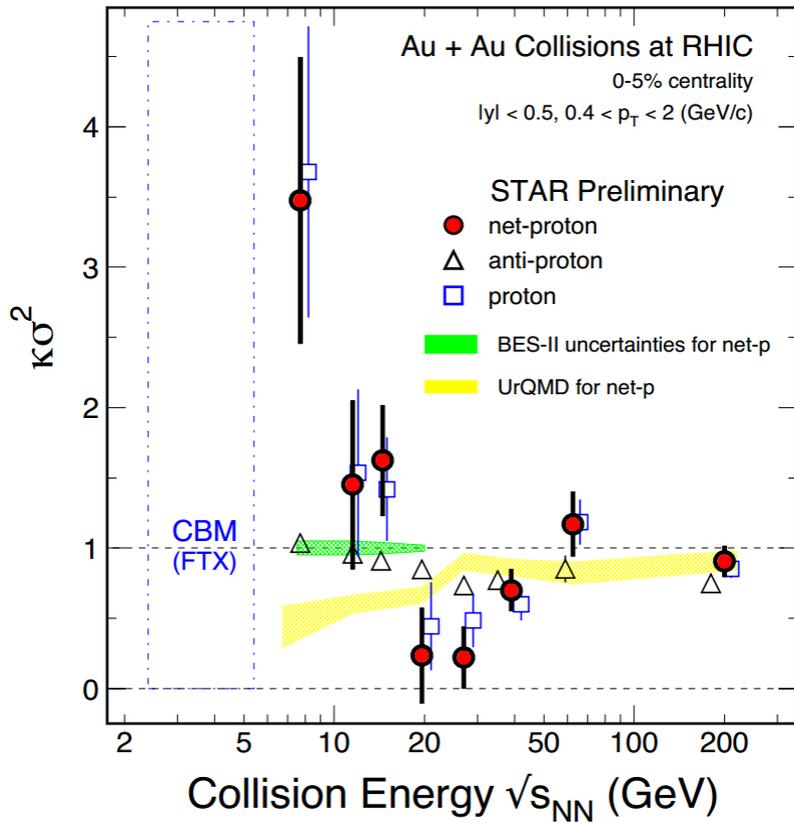
$$\Delta n_I = \frac{\langle (\delta n - \delta n_p)^2 \rangle}{(\langle n \rangle + \langle n_p \rangle)^2}$$



*Driven by the same underlined physics, namely, enhancement of correlation length in the second-order phase transition*

**T. Anticic et al. (NA49 Collaboration),  
Phys. Rev. C 94, 044906 (2016).**

# Another Scenario



A. Bzdak et al, arXiv:1906.00936(2019).