Anomalous transport from scale symmetry

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(MK, Xu-Guang Huang, Shinya Matsuzaki in preparation)

Outline

- Introduction
  - Anomalous transport
    - from chiral symmetry
    - from scale symmetry
- Our work
  - Anomalous transport based on the dilaton effective theory
- Summary

1. Introduction

QCD Lagrangian

QCD with Nf-flavor light quarks

$$\mathcal{L} = -\frac{1}{2g_s^2}\mathrm{tr}[G_{\mu\nu}^2] + \sum_f \bar{q}_f i\gamma^\mu(\partial_\mu - iG_\mu)q_f - \sum_f m_f\bar{q}q$$

* mass term($-\sum_f m_f\bar{q}q$) can approximately be neglected:

$m_f \ll \Lambda_{\mathrm{QCD}}$ strong coupling scale at which $\alpha_s = g_s^2/(4\pi)^2 = \mathcal{O}(1)$
dynamical mass of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ is generated

⇒ approximately U(Nf)L x U(Nf)R invariant at the classical level

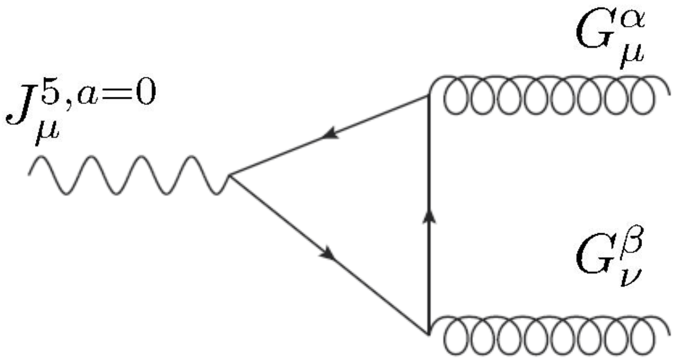
Chiral transform $q_f \to e^{-i\gamma_5 T^a\theta_a}q_f \;(a = 0,\cdots,N_f^2-1)$

Chiral/axial current $J_A^{a\mu} = \bar{q}_f\gamma_\mu\gamma_5 T^a q_f$

$$\partial_\mu J_A^{a\mu} = 0$$

U(1)$_A$ anomaly

U(1)A (axial) symmetry is broken
by gluon config: "U(1)A anomaly"



$$\partial_\mu j_A^\mu = -\frac{g_s^2 N_f}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}[G_{\mu\nu}G_{\rho\sigma}]$$

j_A^μ : the isosinglet axial current

<u>Remarks</u>

- Note: the anomaly form is exact at one-loop order of perturbation.

See, e.g. Adler, arXiv:0405040

- The breaking of U(1)A means the left-right fermion number violation:

$$\frac{d(Q_R - Q_L)}{dt} = -\frac{g_s^2}{16\pi^2}\int d^3x\, G_{\mu\nu}\tilde{G}^{\mu\nu}$$

where L, R number operators $Q_{R,L} = \int d^3x J_0^{R,L}$ $J^\mu_{R,L} = \bar{q}_{R,L}\gamma^\mu q_{R,L}$
(Noether's charges)

where $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$: "pseudo"-gluon field strength
parity(P)-odd

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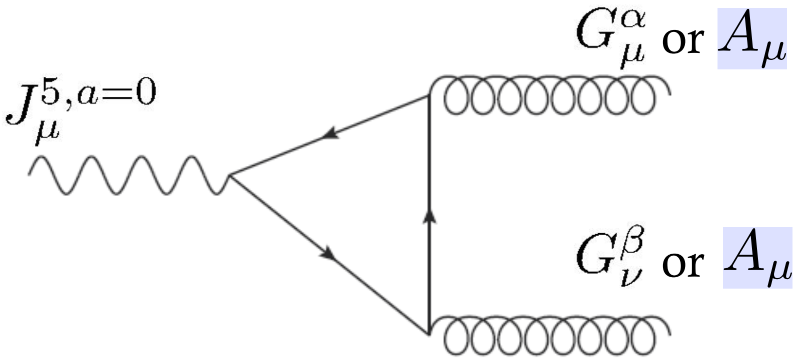

$$\partial_\mu j_A^\mu = -\frac{g_s^2 N_f}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}[G_{\mu\nu}G_{\rho\sigma}]$$

By adding the electromagnetic field…

$$\partial_\mu j_A^\mu = -\frac{g_s^2 N_f}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}[G_{\mu\nu}G_{\rho\sigma}] - \frac{N_c e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\mathrm{tr}[Q^2],$$

The electromagnetic field contributes to U(1)A anomaly.

→ EM fields induce the anomalous transport involving
the nontrivial nature of QCD vacuum.

QCD θ-term

To access the nontrivial nature of QCD vacuum, one adds QCD θ-term to the QCD Lagrangian.

$$\mathcal{L}_\theta = \int d^4x \theta(x) \frac{g_s^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}[G_{\mu\nu} G_{\rho\sigma}]$$

The finiteness of the θ-parameter indicates that the QCD system is put in the CP violation domain.

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In heavy ion collision…

The θ parameter is promoted to the background field θ(x).

Because…

- In heavy ion collision experiments, the hot QCD medium is created at the time scale of O(1–10) fm after the collision.
- Theta parameter depends on the finite temperature.
- θ parameter may get the position-time dependence in the high temperature system.

Nucl. Phys. A 797, 67 (2007). Nucl. Phys. A 803, 227 (2008). Phys. Rev. D 78, 074033 (2008). Phys. Lett. B 710, 230 (2012). Eur. Phys. J. C 73, 2294 (2013).

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By inserting U(1)A anomaly with EM fields…

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$$\mathcal{L}_\theta = \int d^4x\theta(x)\frac{-1}{2N_f}\partial_\mu j_A^\mu + \int d^4x\theta(x)\frac{-N_c e^2}{64\pi^2 N_f}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\mathrm{tr}[Q^2]$$

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QCD Lagrangian gets the parity-odd electromagnetic terms via QCD θ-term along with U(1)A anomaly.

Anomalous transports from the chiral symmetry

QCD Lagrangian has the parity-odd electromagnetic terms.

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$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$V_\mu = \mu_V\delta_{\mu 0} + A_\mu$$

$$\frac{\partial_\mu\theta(x)}{2N_f} = \mu_A\delta_{\mu 0}$$

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$$+\int d^4x\theta(x)\frac{-N_c e^2}{64\pi^2 N_f}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\mathrm{tr}[Q^2]$$

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Electromagnetic field (A_μ)

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Electromagnetic field

μ_V : Baryon chemical potential $\quad \mu_V \neq 0$ ⟺ dense medium is created

$$Q_V = Q_R + Q_L \neq 0$$

μ_A : Chiral chemical potential $\quad \mu_A \neq 0$ ⟺ chiral imbalance medium is created

$$Q_A = Q_R - Q_L \neq 0$$

*Chiral imbalance medium (local parity odd domain) would be realized in heavy ion collisions.

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Implement the linear response of the vector current to get the anomalous transport from the chiral anomaly.

$$\langle j^\mu(x)\rangle = i\frac{\delta \ln Z_{QCD}}{\delta A_\mu(x)}$$

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Chiral magnetic effect

$$\langle \vec{j}_V\rangle = \frac{\mu_A}{2\pi^2}e\vec{B}$$

Nucl. Phys. A 803, 227 (2008)

Phys. Rev. D 78, 074033 (2008)

U(1)A anomaly along with QCD θ-term induces the EM current (anomalous transport).

Anomalous transports from the chiral symmetry

chiral magnetic effect

$$\langle \vec{j}_V \rangle = \frac{\mu_A}{2\pi^2} e\vec{B}$$

QCD theta-vacuum

$$\mathcal{L}_\theta = \int d^4x\, \theta(x) \frac{g_s^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}[G_{\mu\nu} G_{\rho\sigma}]$$

*The axial anomalous current is also induced by the chiral anomaly.

The chiral separation effect: $\langle \vec{j}_A \rangle = \dfrac{\mu_V}{2\pi^2} e\vec{B}$

Phys. Rev. D 70, 074018 (2004)
Phys. Rev. D 72, 045011 (2005)

*In the rotating system, the anomalous transports are induced.

Anomalous transports from the chiral symmetry

chiral magnetic effect

$$\langle \vec{j}_V \rangle = \frac{\mu_A}{2\pi^2} e\vec{B}$$

Anomalous transport is closely tied with the vacuum structure of QCD (QCD θ-vacuum).

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*In the rotating system, the anomalous transports are induced.

Anomalous transports would be generated in a strong magnetic field region relevant to the heavy ion collision.

Anomalous transports would be a probe to reveal a nontrivial aspect of the QCD phase (vacuum) structure.

This topic is under intensive investigations from theoretical as well as from experimental sides

Anomalous transports from the chiral symmetry

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Anomalous transports would be generated in a strong magnetic field region relevant to the heavy ion collision.

Anomalous transports would be a probe to reveal a nontrivial aspect of the QCD phase (vacuum) structure.

→ Other symmetry also induces the anomalous transports.

Scale symmetry

As a simple case, let's consider massless-QED (not QCD) to obtain the anomalous transport from scale symmetry.

- Scale invariant at the classical level

$$\langle T^{\mu}{}_{\mu} \rangle = \partial_\mu j^\mu_D = 0$$

where j^μ_D is the dilatation current.

Scale symmetry

As a simple case, let's consider massless-QED (not QCD) to obtain the anomalous transport from scale symmetry.

- Scale invariant at the classical level

$$\langle T^{\mu}{}_{\mu} \rangle = \partial_\mu j^\mu_D = 0$$

where j^μ_D is the dilatation current.

- Scale symmetry is broken at the quantum level.

$$\langle T^{\mu}{}_{\mu} \rangle = \partial_\mu j^\mu_D = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$$

e : QED coupling constant

$\beta(e)$: Beta function of QED

The electromagnetic field contributes to the scale anomaly.

Anomalous transport from scale symmetry

Trace anomaly (scale anomaly) $\langle T^{\mu}{}_{\mu} \rangle = \partial_\mu j^\mu_D = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$

In the curved spacetime, the trace (scale) anomaly induces the anomalous current.

→ Putting QED on a conformally-flat spacetime: $\gamma_{\mu\nu}(x) = e^{2\tau(x)} \eta_{\mu\nu}$

$\tau(x)$: local scale factor

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Action is changed in the background of the rescaled metric.

$$S \to S_\tau = S + \int d^4x \, \tau(x) T^{\mu}{}_{\mu}(x) + \mathcal{O}(\tau^2)$$

In the curved spacetime, QED action gets the nonzero energy momentum term.

Anomalous transport from scale symmetry

QED action in the curved spacetime

$$S \to S_\tau = S + \int d^4x\, \tau(x) T^\mu{}_\mu(x) + \mathcal{O}(\tau^2)$$

By using the linear response of the vector current…

$$\langle j^\mu(x) \rangle = i \frac{\delta \ln Z_{QED}}{\delta A_\mu(x)}$$

Scale electric/magnetic effect

Phys. Rev. Lett. 117, no. 14, 141601 (2016)

$$\langle \vec{j}_V \rangle = -\frac{2\beta(e)}{e} \frac{\partial \tau(x)}{\partial t} \vec{E}(x) + \frac{2\beta(e)}{e} \vec{\nabla}\tau(x) \times \vec{B}(x),$$

- Scale anomaly also induces the anomalous EM current (anomalous transport).
- This current may presumably be realized in solid state materials, such as strained graphene or elastically deformed Weyl/Dirac semimetals.

Anomalous transport

- Chiral anomaly with QCD θ-term

Chiral magnetic effect

$$\langle \vec{j}_V \rangle = \frac{\mu_A}{2\pi^2} e\vec{B}$$

Anomalous transport is closely tied with the vacuum structure of QCD.

- Scale anomaly (QED) in the curved spacetime

Scale electric/magnetic effect

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QCD has the scale anomaly (which would be closely tied with the QCD phase structure)
$\longrightarrow$ QCD scale anomaly would induce anomalous transports.

2. Our work

Anomalous transport from the scale symmetry in QCD

- In the curved QED, the scale anomaly induces the anomalous transports.

$$\langle T^{\mu}{}_{\mu} \rangle = \partial_{\mu} j^{\mu}_{D} = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \quad \langle \vec{j}_V \rangle = -\frac{2\beta(e)}{e} \frac{\partial \tau(x)}{\partial t} \vec{E}(x) + \frac{2\beta(e)}{e} \vec{\nabla} \tau(x) \times \vec{B}(x),$$

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- In QCD+QED, the scale anomaly has quark mass terms and a gluon part.

$$\partial_\mu j^\mu_D = (1+\gamma_m) \sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G^2_{\mu\nu} + \frac{\beta_F(e)}{2e} F^2_{\mu\nu}$$

g_s : QCD coupling constant

$\beta_F(g_s)$: Beta function of QCD

? ? ?

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$\Rightarrow$? ? ? $\Leftarrow$

Anomalous transport from the scale symmetry would be a new probe to reveal a nontrivial aspect of the QCD phase structure.

Anomalous transport from the scale symmetry in QCD

- In the curved QED, the scale anomaly induces the anomalous transports.

$$\langle T^{\mu}{}_{\mu} \rangle = \partial_\mu j_D^\mu = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \quad \langle \vec{j}_V \rangle = -\frac{2\beta(e)}{e} \frac{\partial \tau(x)}{\partial t} \vec{E}(x) + \frac{2\beta(e)}{e} \vec{\nabla} \tau(x) \times \vec{B}(x),$$

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$$\partial_\mu j_D^\mu = (1+\gamma_m) \sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G_{\mu\nu}^2 + \frac{\beta_F(e)}{2e} F_{\mu\nu}^2$$

$\Rightarrow$? ? ?

To reveal the anomalous transport, we employ the dilaton effective theory.

Effective theory in the scale symmetry

- Scale anomaly in QCD

$$\partial_\mu j_D^\mu = (1+\gamma_m)\sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G_{\mu\nu}^2$$

Dilatation current

- Scale anomaly in effective field theory

$$j_D^\mu = f_\phi \partial^\mu \phi + \cdots$$

ϕ is the NG boson field of the scale symmetry. (dilaton field)

Effective theory in the scale symmetry

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ϕ is the NG boson field of the scale symmetry. (dilaton field)

Dilaton is massless: $\langle 0|\partial_\mu j_D^\mu|\phi(p)\rangle = 0$

- Conformal compensator

$$\chi = f_\phi e^{\phi/f_\chi} \qquad \chi(x) \to e^\sigma \chi(x)$$

Dilaton Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi$$

*Action is invariant under the scale transformation

Effective theory in the scale symmetry

- Scale anomaly in QCD

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Scale anomaly supplies the dilaton mass.

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Scale anomaly supplies the dilaton mass.

- Scale anomaly in effective field theory

$$j_D^\mu = f_\phi \partial^\mu \phi + \cdots$$

ϕ is the NG boson field of the scale symmetry. (dilaton field)

Dilaton is massive: $\langle 0|\partial_\mu j_D^\mu|\phi(p)\rangle = -f_\phi m_\phi^2 e^{-ip\cdot x}$

Lagrangian gets a dilaton mass term.

- Conformal compensator

$$\chi = f_\phi e^{\phi/f_\chi} \qquad \chi(x) \to e^\sigma \chi(x)$$

Dilaton Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{f_\phi^2 m_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4\left(\log\frac{\chi}{f_\phi} - \frac{1}{4}\right)$$

*Action is invariant under the scale transformation

Dilaton effective theory in electromagnetic field (Our work)

- Scale anomaly in QCD

$$\partial_\mu j^\mu_D = (1+\gamma_m)\sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G^2_{\mu\nu}$$

Scale anomaly can be described by the dilaton field with mass.

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right]$$

- Dilaton potential term (mass term) characterizes the QCD scale anomaly.

Dilaton effective theory in electromagnetic field (Our work)

- Scale anomaly in QCD+QED

$$\partial_\mu j_D^\mu = (1+\gamma_m)\sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G_{\mu\nu}^2 + \frac{\beta_F(e)}{2e} F_{\mu\nu}^2$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e} F_{\mu\nu}^2$$

- Dilaton potential term (mass term) characterizes the QCD scale anomaly.

- Introduce an additional term having EM fields by matching QCD+QED with the dilaton effective theory.

Dilaton effective theory in electromagnetic field (Our work)

- Scale anomaly in QCD+QED

$$\partial_\mu j^\mu_D = (1+\gamma_m)\sum m_q \bar{q}q + \frac{\beta_F(g_s)}{2g_s}G^2_{\mu\nu} + \frac{\beta_F(e)}{2e}F^2_{\mu\nu}$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4\left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e}F^2_{\mu\nu}$$

- Dilaton potential term (mass term) characterizes the QCD scale anomaly.

- Introduce an additional term having EM fields by matching QCD+QED with the dilaton effective theory.

→ In the effective theory, dilaton field directly couples with EM fields.

Dilaton effective theory in electromagnetic field (Our work)

- Scale anomaly in QCD+QED

$$\partial_\mu j_D^\mu = (1+\gamma_m)\sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G_{\mu\nu}^2 + \frac{\beta_F(e)}{2e} F_{\mu\nu}^2$$

Dilaton is a mediator between QCD and QED in the scale anomaly.

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e} F_{\mu\nu}^2$$

- Dilaton potential term (mass term) characterizes the QCD scale anomaly.

- Introduce an additional term having EM fields by matching QCD+QED with the dilaton effective theory.

→ In the effective theory, dilaton field directly couples with EM fields.
→ It is expected that a novel aspects of QCD scale anomaly would be extracted from the additional EM term.

Anomalous transport in dilaton effective theory

- Scale anomaly in QCD+QED induces the anomalous transport

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e}F_{\mu\nu}^2$$

Implement the linear response

$$\langle j^\mu(x)\rangle = i\frac{\delta \ln Z_{\rm eff}}{\delta A_\mu(x)}$$

Anomalous transport in the dilaton effective theory

$$\langle j^\mu(x)\rangle = -\frac{2\beta(e)}{e}F^{\mu\nu}\left[\partial_\nu \log\left(\frac{\chi_0}{f_\phi}\right)\right]$$

Anomalous transport in dilaton effective theory

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Anomalous transport in the dilaton effective theory

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$$\langle j^\mu(x)\rangle = i\frac{\delta \ln Z_{\text{eff}}}{\delta A_\mu(x)}$$

$$\langle j^\mu(x)\rangle = -\frac{2\beta(e)}{e}F^{\mu\nu}\left[\partial_\nu \log\left(\frac{\chi_0}{f_\phi}\right)\right]$$

$$\langle \chi \rangle = \chi_0$$

Expectation value of dilaton field is determined from the stationary point of the dilaton potential.

In the vacuum of the dilaton potential, the anomalous current shows up.

Anomalous transport in scale effective theory

- Scale anomaly in QCD+QED induces the anomalous transport

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log \frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e}F_{\mu\nu}^2$$

Anomalous transport in the dilaton effective theory

$$\langle j^\mu(x)\rangle = -\frac{2\beta(e)}{e}F^{\mu\nu}\left[\partial_\nu \log\left(\frac{\chi_0}{f_\phi}\right)\right]$$

- In QCD, the scale anomaly can induce the anomalous current even in the flat spacetime.

*Anomalous transport in a curved QED

$$\langle j^\mu(x)\rangle_{\rm scale} = -\frac{2\beta(e)}{e}F^{\mu\nu}(x)\partial_\nu \tau(x)$$

Anomalous transport in scale effective theory

- Scale anomaly in QCD+QED induces the anomalous transport

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e}F_{\mu\nu}^2$$

Anomalous transport in the dilaton effective theory

$$\langle j^\mu(x)\rangle = -\frac{2\beta(e)}{e}F^{\mu\nu}\left[\partial_\nu \log\left(\frac{\chi_0}{f_\phi}\right)\right]$$

- If the dilaton condensation is constant, the anomalous current vanishes.

$\chi_0 = \text{const}$ ⇒ $j^\mu = 0$

- To obtain the nonzero anomalous current, a dilaton should get a inhomogeneous condensation (condensation has position-time dependence).

Inhomogeneous dilaton condensation

Consider a time-dependent dilaton condensation in constant EM fields: $\chi_0(t)$

$$V_{\text{eff}} = -\frac{1}{2}\partial_t\chi_0\partial_t\chi_0 + \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi_0}{f_\phi}\right)^4\left[\log\frac{\chi_0}{f_\phi} - \frac{1}{4}\right] - \log\left(\frac{\chi_0}{f_\phi}\right)\frac{\beta(e)}{2e}F_{\mu\nu}^2$$

- Add the kinetic term to get a time-dependent dilaton condensation

Inhomogeneous dilaton condensation

Consider a time-dependent dilaton condensation in constant EM fields: $\chi_0(t)$

$$V_{\rm eff} = -\frac{1}{2}\partial_t\chi_0\partial_t\chi_0 + \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi_0}{f_\phi}\right)^4\left[\log\frac{\chi_0}{f_\phi} - \frac{1}{4}\right] - \log\left(\frac{\chi_0}{f_\phi}\right)\frac{\beta(e)}{2e}F_{\mu\nu}^2$$

- Stationary condition (equation of motion)

$$\partial_t^2\chi_0 + \frac{m_\phi^2}{f_\phi^2}\chi_0^3\log\frac{\chi_0}{f_\phi} - \frac{1}{\chi_0}\frac{\beta(e)}{2e}F_{\mu\nu}^2 = 0.$$

Impose initial conditions to solve this equation.

$$\chi_0(t=0) = f_\phi \qquad \left.\frac{\partial\chi_0(t)}{\partial t}\right|_{t=0} = 0$$

(Dilaton condensation is rest at t=0.)

→ consider the “static” potential at t=0.

To give a numerical solution, we identify the dilaton as $f_0(500)$.

$$m_\phi = 441[\mathrm{MeV}], \quad f_\phi = 100[\mathrm{MeV}]$$

R. J. Crewther and L. C. Tunstall
Phys. Rev. D 91, no. 3, 034016 (2015)

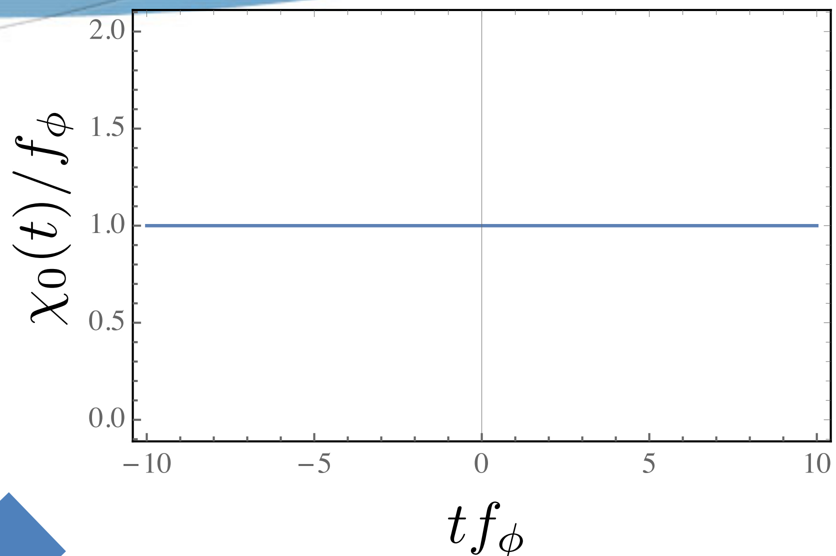
$$\text{1-loop}: \beta(e) = \frac{e^3}{12\pi^2}$$

Inhomogeneous dilaton condensation

Time-dependent dilaton condensation

- EM fields induces the inhomogeneous dilaton condensation which is time-dependent oscillating solutions.

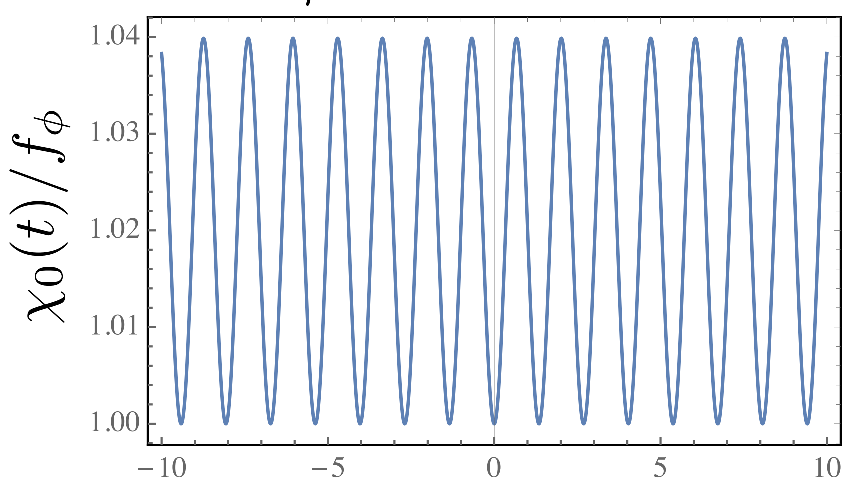
$F_{\mu\nu}^2 = 0$



Homogeneous condensation without EM fields

Inhomogeneous condensation in EM field

$F_{\mu\nu}^2 = 10^{10}[\mathrm{MeV}^4]$



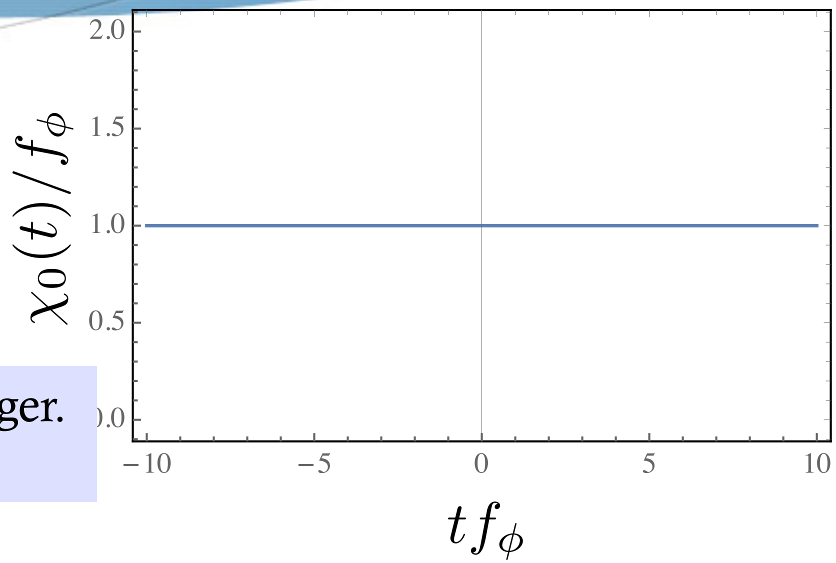
Inhomogeneous dilaton condensation

Time-dependent dilaton condensation

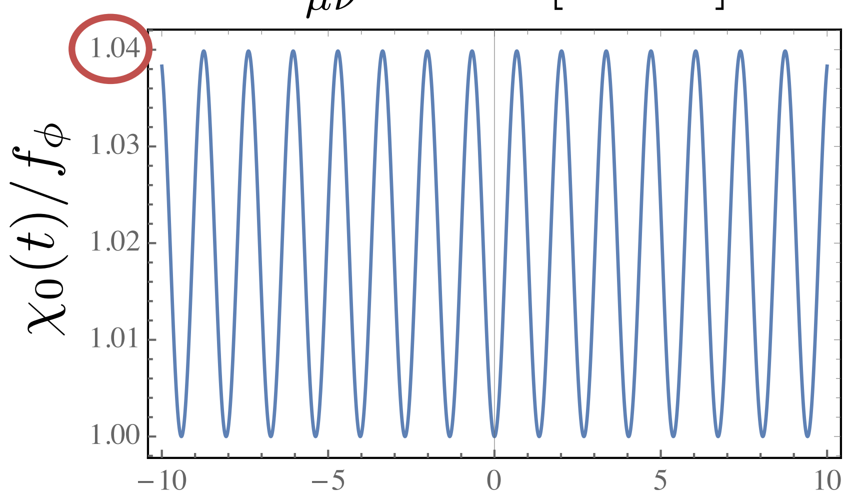
- EM fields induces the inhomogeneous dilaton condensation which is time-dependent oscillating solutions.

- As EM fields increase, the amplitude becomes larger. Furthermore, the frequency is more rapid.

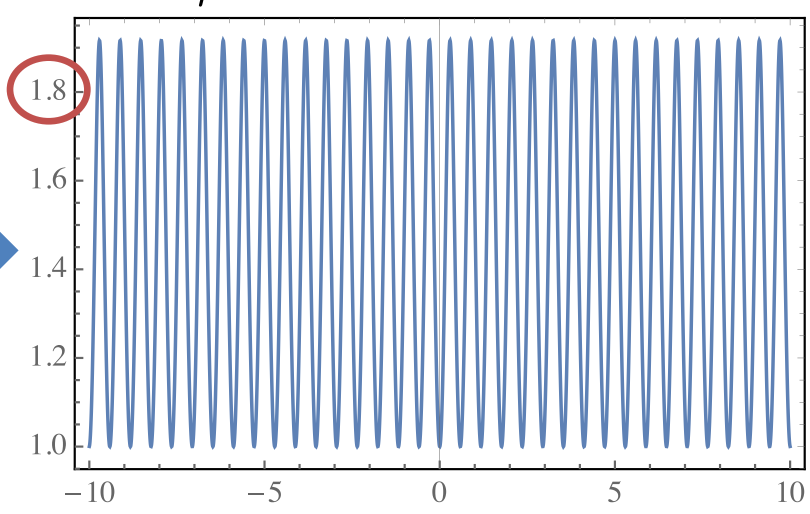
$F_{\mu\nu}^2 = 0$



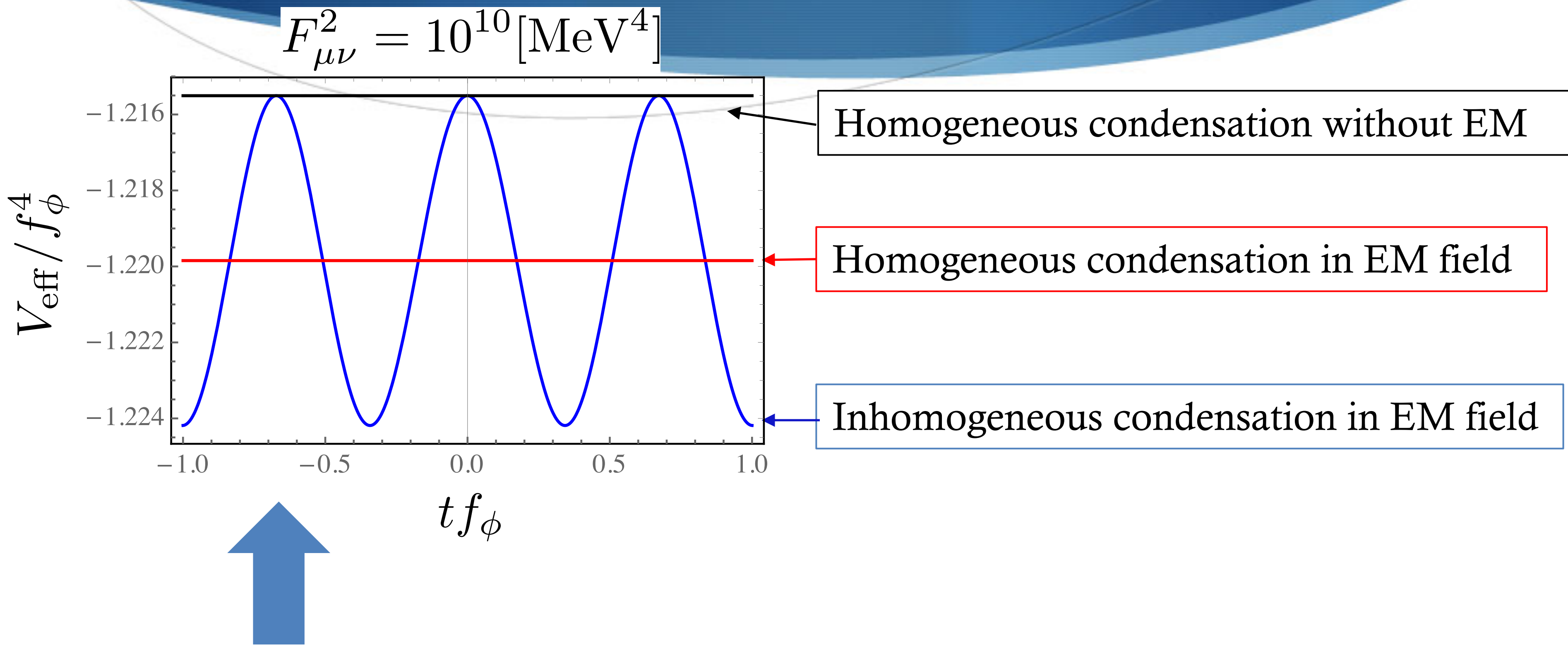
$F_{\mu\nu}^2 = 10^{10}[\mathrm{MeV}^4]$



$F_{\mu\nu}^2 = 10^{12}[\mathrm{MeV}^4]$



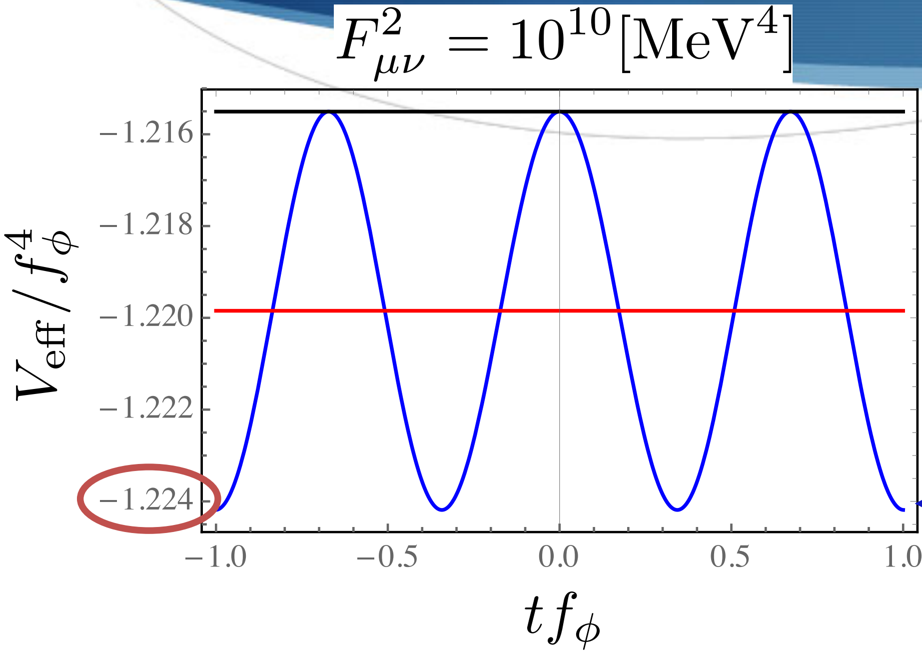
Dilaton potential in electromagnetic field



Time-dependent potential at the stationary condition: $V_{\rm eff}\left[\chi_0(t)\right]$

- The dilaton potential at the stationary point gets to oscillate by a constant EM field.

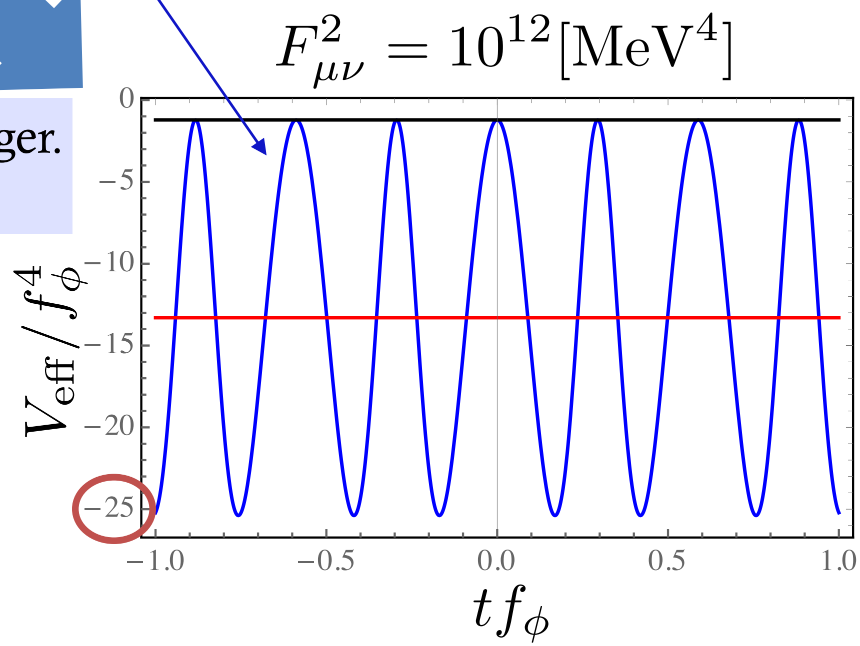
Dilaton potential in electromagnetic field



As EM fields increase…

Inhomogeneous condensation in EM field

- Amplitude of the dilaton potential becomes larger.
- Frequency is more rapid.



Anomalous current in inhomogeneous condensate

Anomalous current in the time-dependent dilaton condensation

$$\langle j^i(x)\rangle = -E^i \left[\partial_t \log\left(\frac{\chi_0(t)}{f_\phi}\right)\right]\frac{2\beta(e)}{e}$$

Electric field is oriented to the x-direction.

$$E^x = \frac{\sqrt{F_{\mu\nu}^2}}{2}$$

$$\langle j^{i=x}(x)\rangle = -E^{i=x} \left[\partial_t \log\left(\frac{\chi_0(t)}{f_\phi}\right)\right]\frac{2\beta(e)}{e}$$

Anomalous current in inhomogeneous condensate

Anomalous current in the time-dependent dilaton condensation

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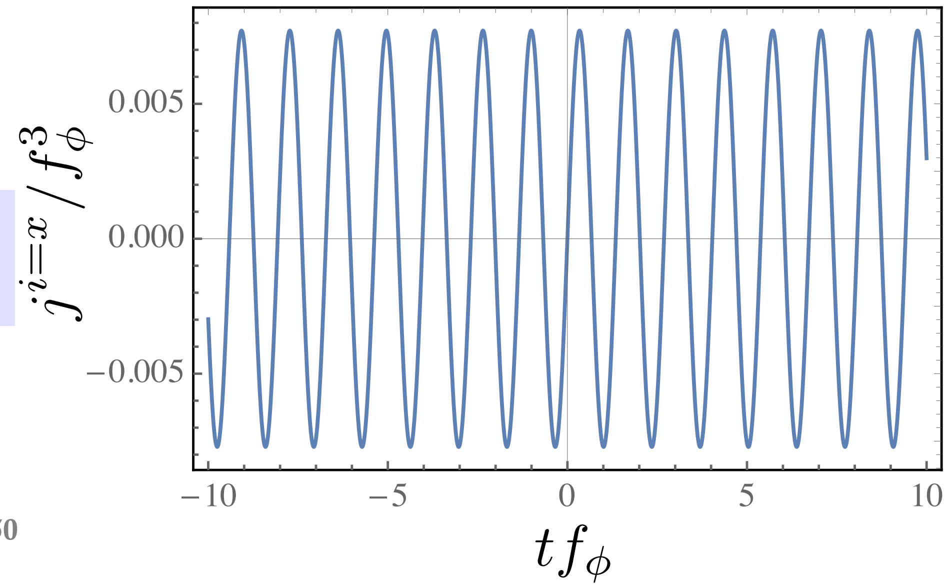
- Anomalous current is along a electric field.
- Anomalous current oscillates.

$$F_{\mu\nu}^2 = 10^{10}[\mathrm{MeV}^4]$$



Anomalous current in inhomogeneous condensate

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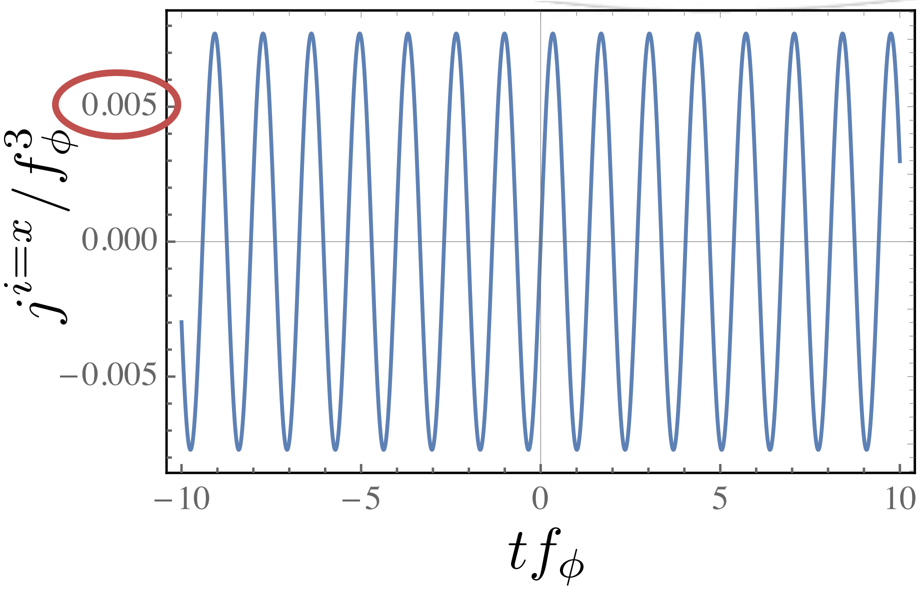
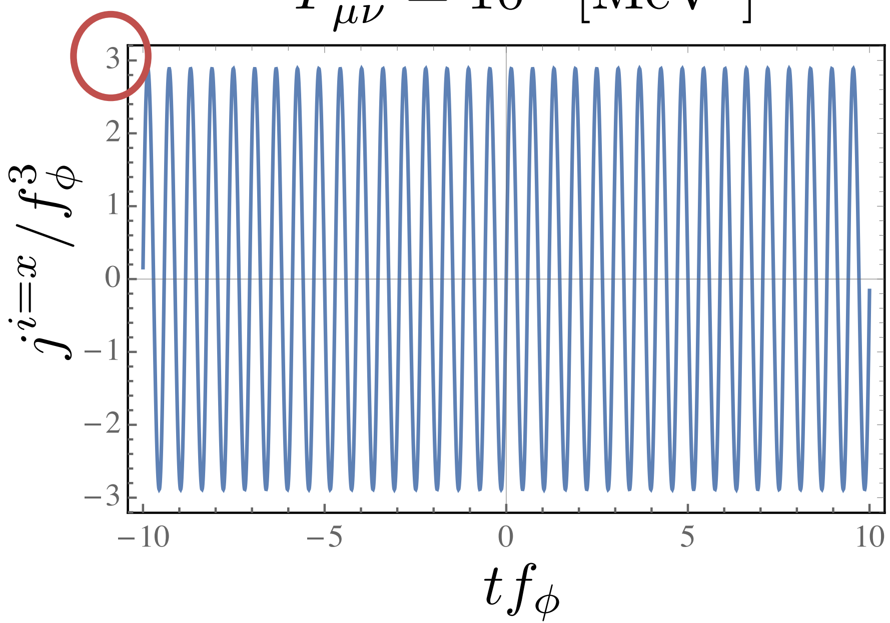


As an electric field increases…

$F_{\mu\nu}^2 = 10^{12}[\mathrm{MeV}^4]$



- Amplitudes becomes larger.
- Frequency is more rapid.

Summary

To reveal the anomalous transport from the scale symmetry in QCD (hadron), we employ the dilaton effective theory.

- In the scale anomaly of QCD+QED, dilaton acts as a mediator between QCD and QED.

$$\partial_\mu j^\mu_D = (1+\gamma_m)\sum m_q \bar{q} q + \frac{\beta_F(g_s)}{2g_s} G^2_{\mu\nu} + \frac{\beta_F(e)}{2e} F^2_{\mu\nu}$$

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4 \left[\log\frac{\chi}{f_\phi} - \frac{1}{4}\right] + \log\left(\frac{\chi}{f_\phi}\right)\frac{\beta(e)}{2e} F^2_{\mu\nu}$$

- We find the anomalous current involving the feature of the QCD scale anomaly.

$$\langle j^\mu(x)\rangle = -\frac{2\beta(e)}{e} F^{\mu\nu}\left[\partial_\nu \log\left(\frac{\chi_0}{f_\phi}\right)\right]$$

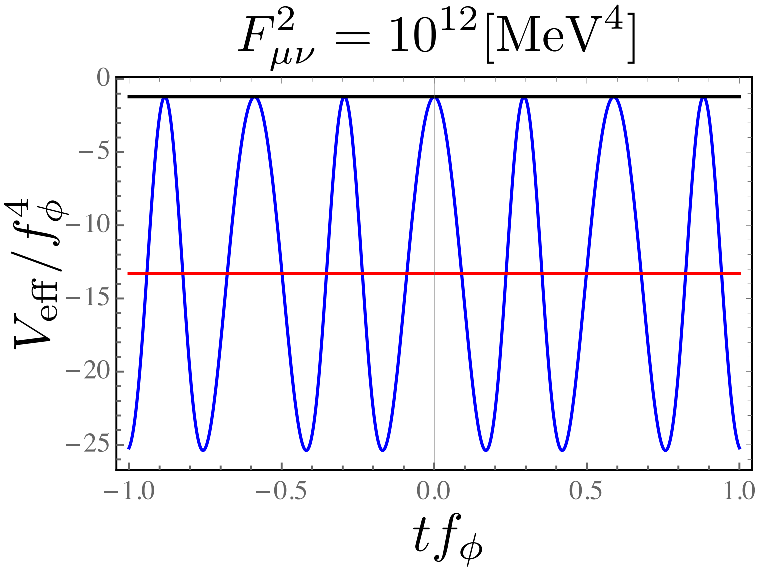
Summary

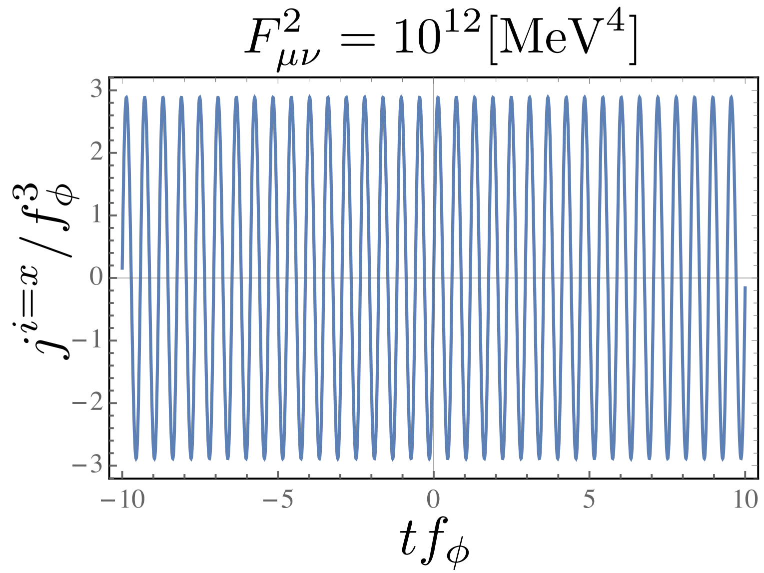
To reveal the anomalous transport from the scale symmetry in QCD (hadron), we employ the dilaton effective theory.

- Anomalous current involving the feature of the QCD scale anomaly.

$$\langle j^\mu(x)\rangle = -\frac{2\beta(e)}{e}F^{\mu\nu}\left[\partial_\nu \log\left(\frac{\chi_0}{f_\phi}\right)\right]$$

- Dilaton has inhomogenous dilaton condensations in EM fields.
  $\rightarrow$ dilaton potential gets to oscillate by constant EM fields.
  $\rightarrow$ Anomalous transport shows up (in the stationary point of the dilaton potential).





Summary

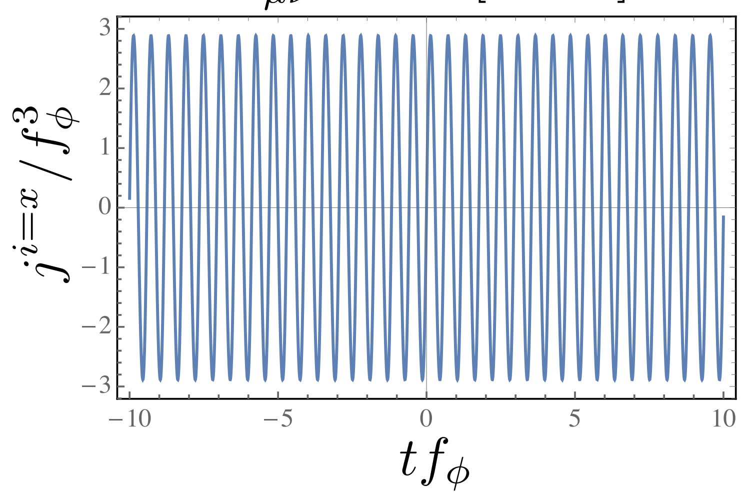
- time-dependent dilaton condensation in constant EM fields

$$V_{\text{eff}} = -\frac{1}{2}\partial_t\chi_0\partial_t\chi_0 + \frac{m_\phi^2 f_\phi^2}{4}\left(\frac{\chi_0}{f_\phi}\right)^4\left[\log\frac{\chi_0}{f_\phi} - \frac{1}{4}\right] - \log\left(\frac{\chi_0}{f_\phi}\right)\frac{\beta(e)}{2e}F_{\mu\nu}^2$$

Anomalous transport

$$\langle j^i(x)\rangle = -E^i\left[\partial_t \log\left(\frac{\chi_0(t)}{f_\phi}\right)\right]\frac{2\beta(e)}{e}$$

$$F_{\mu\nu}^2 = 10^{12}[\text{MeV}^4]$$



Would also induce photon productions

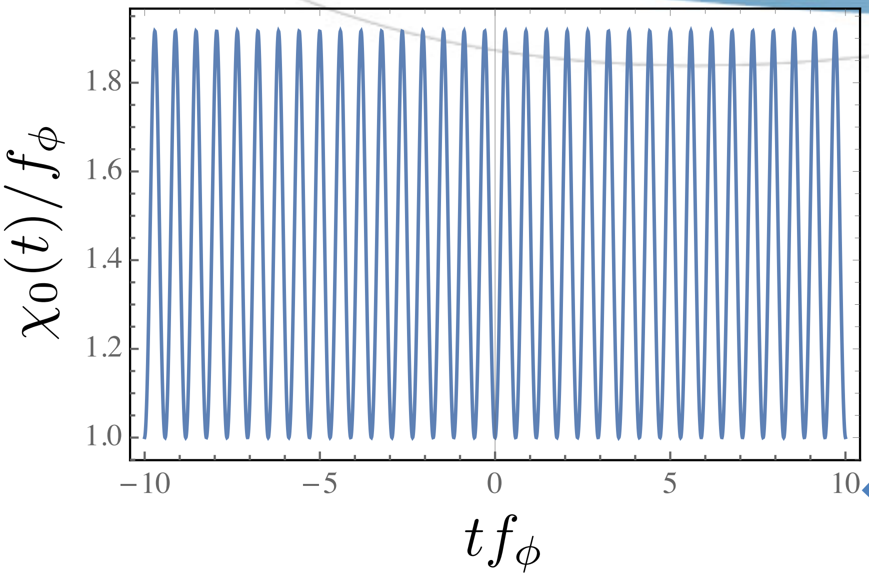
$$\langle \epsilon^{(i)}(\vec{p})\epsilon^{(j)}(\vec{q})|\Omega\rangle$$

- Our anomalous current would be closely correlated with photon productions.
- The photon emission would be a new probe to reveal the novel feature of QCD.

Thank you

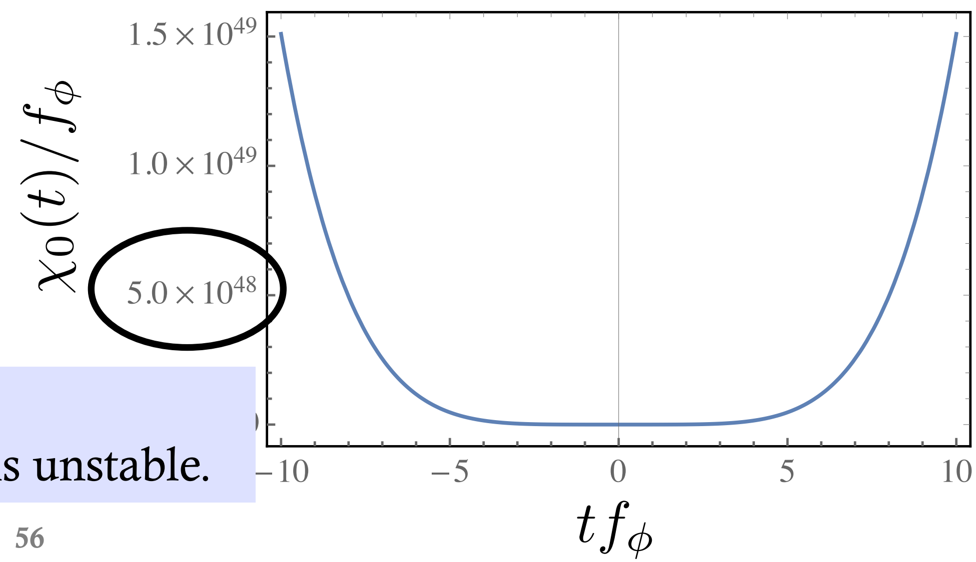
Inhomogeneous dilaton condensation

$F_{\mu\nu}^2 = 10^{12}[\mathrm{MeV}^4]$



Move onto extremely strong EM field region

$F_{\mu\nu}^2 = 10^{32}[\mathrm{MeV}^4]$



$\chi_0(t)$ is too large.

→ dilaton potential is not stabilized.

- In extremely strong EM field regins,
  the inhomogeneous dilaton condensation is unstable.