中国高能核物理网络论坛
HIGH ENERGY NUCLEAR PHYSICS IN CHINA

## HENPIC seminar: 125nd

# 相对论重离子碰撞中的重味强子

## ( Heavy Flavor Hadrons in Heavy Ion Collisions )


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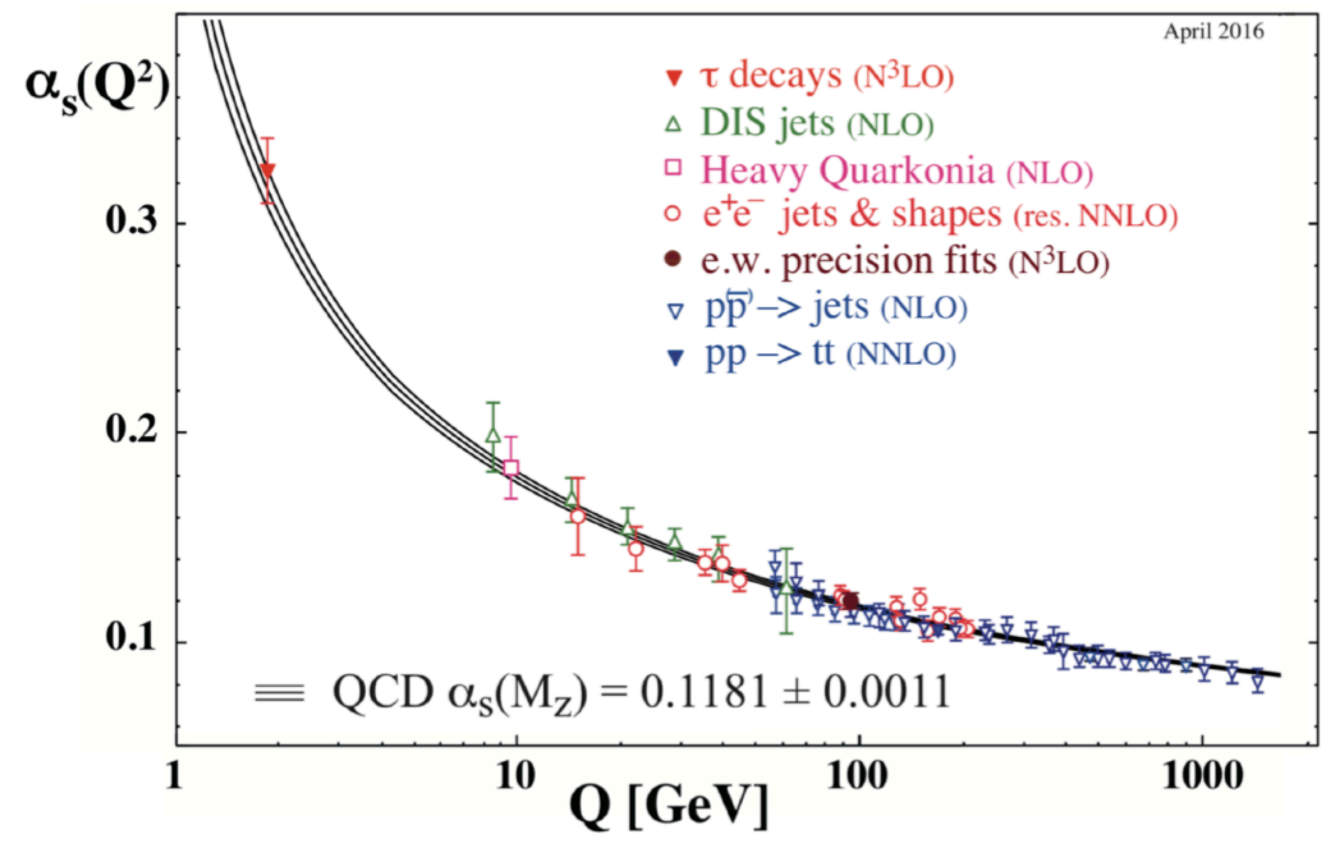
# Outline

- Introduction to heavy-ion collisions and heavy flavor hadrons
- Static properties of heavy flavor hadrons in vacuum and finite-temp.
- Dynamic production of heavy flavor hadrons in heavy-ion collisions
- Using heavy flavor to probe the Hadronization M. and QGP
- Summary and outlook

# Confinement vs. Deconfinement

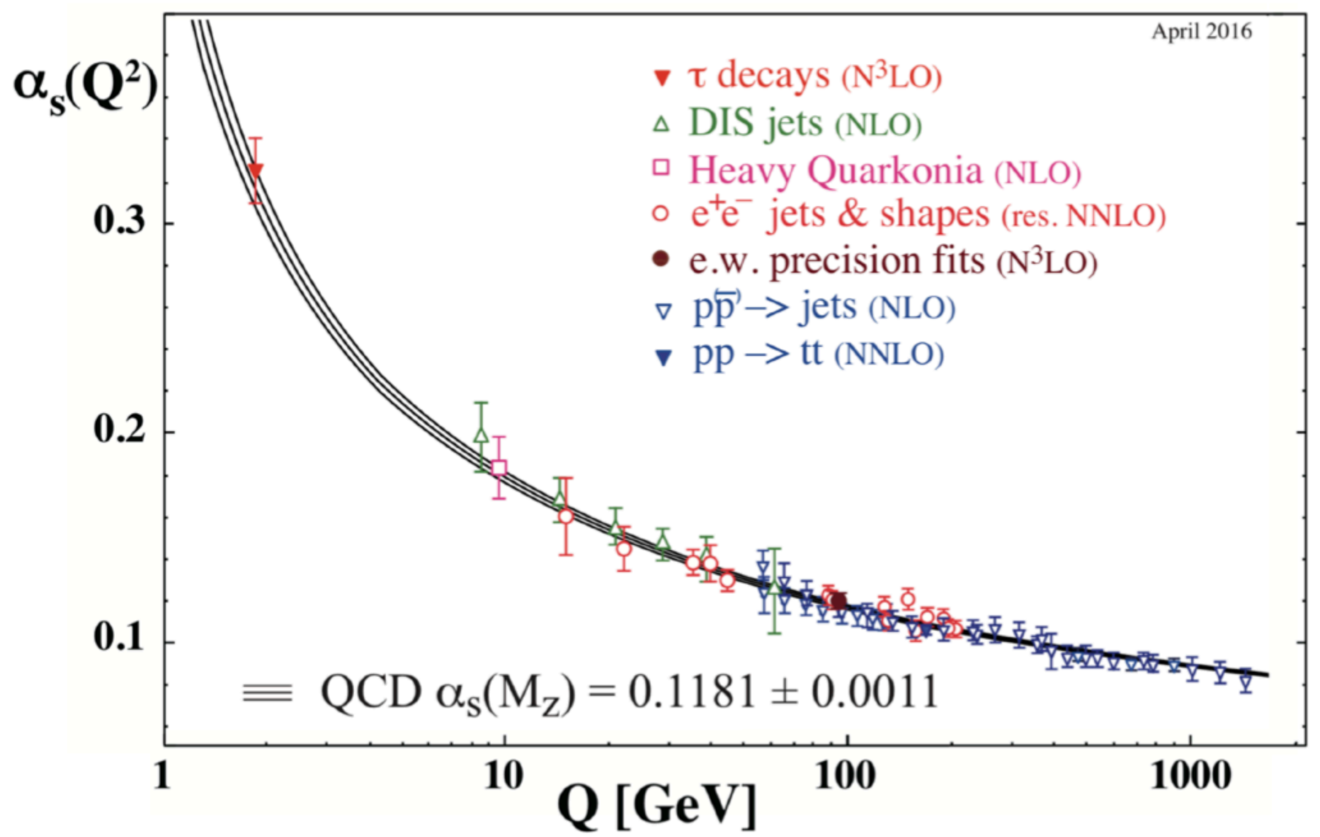



$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2N_f/3)\ln(Q^2/\Lambda^2_{\rm QCD})}.$$

*1. Asymptotic freedom !*

*2. Confinement !*

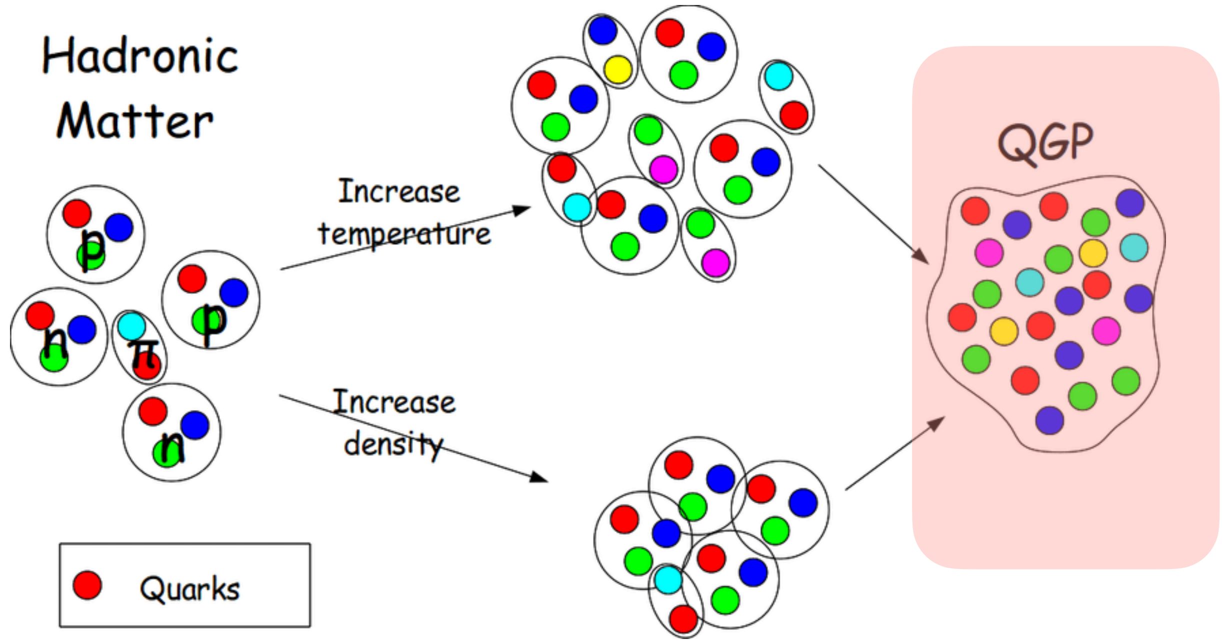
# *Confinement vs. Deconfinement*



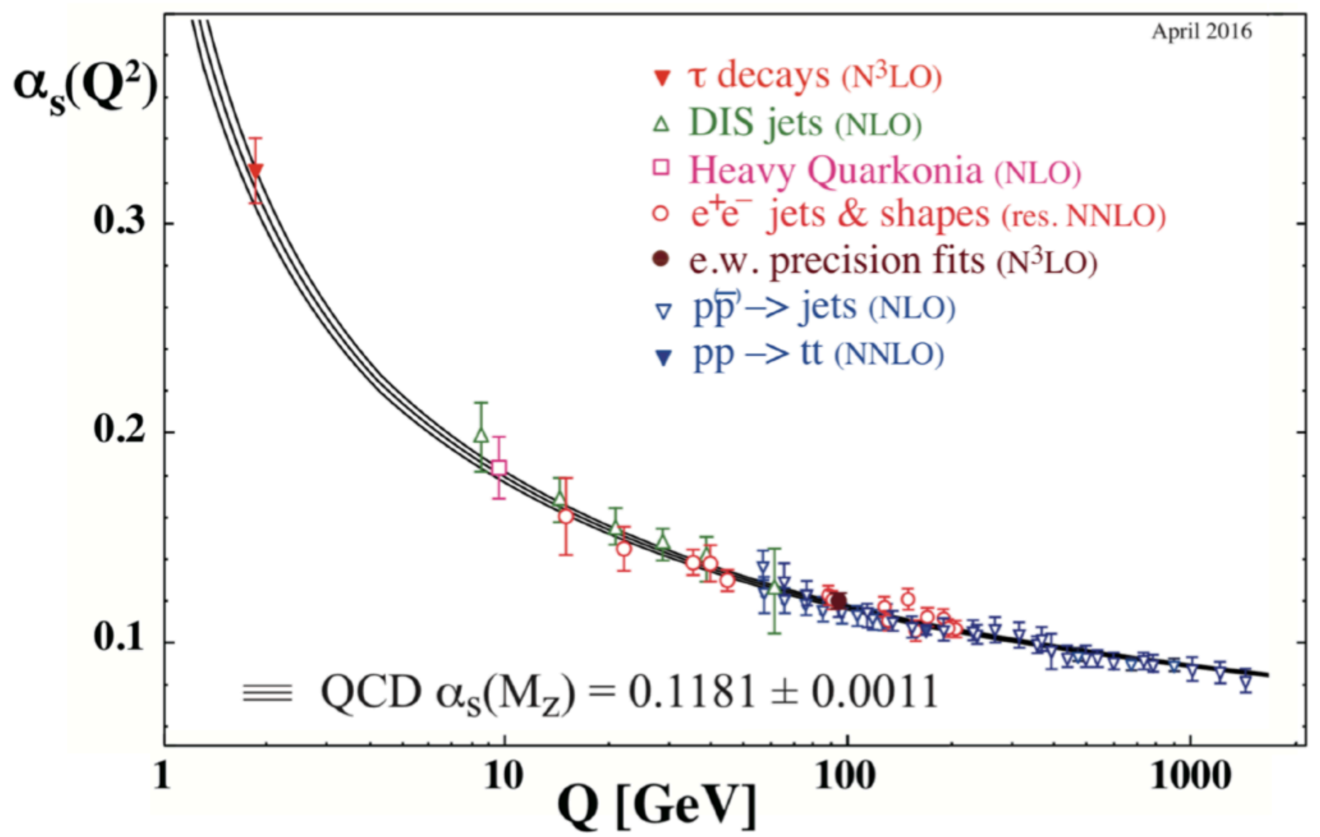

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2N_f/3)\ln(Q^2/\Lambda^2_{QCD})}.$$

*1. Asymptotic freedom !*

*2. Confinement !*



# Confinement vs. Deconfinement

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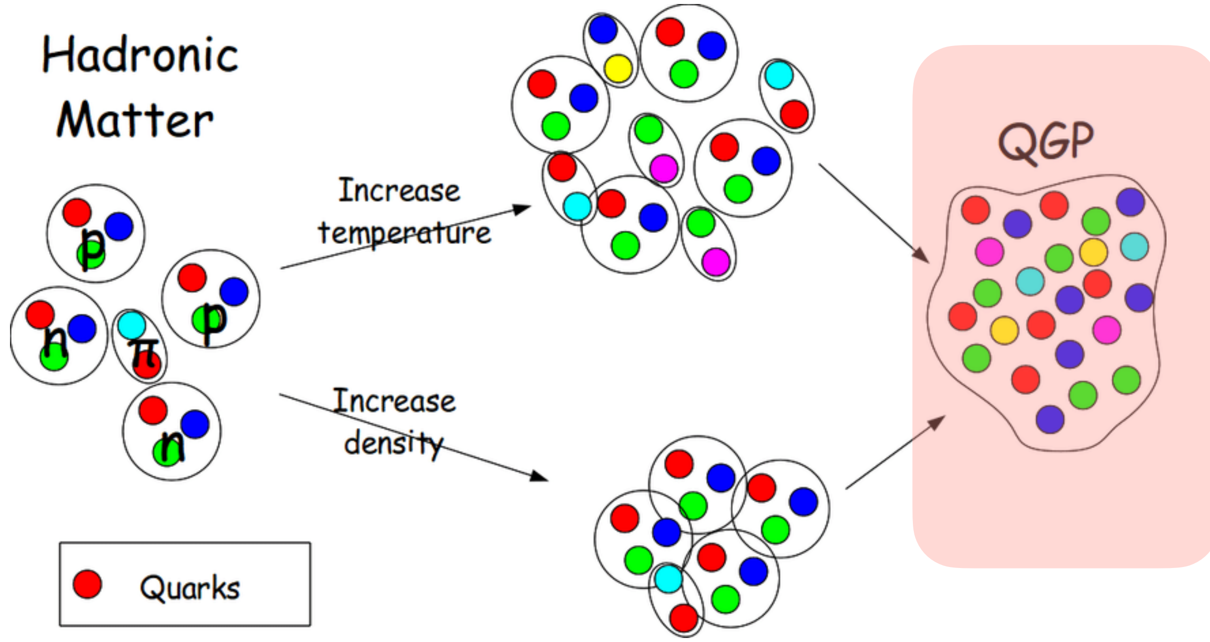
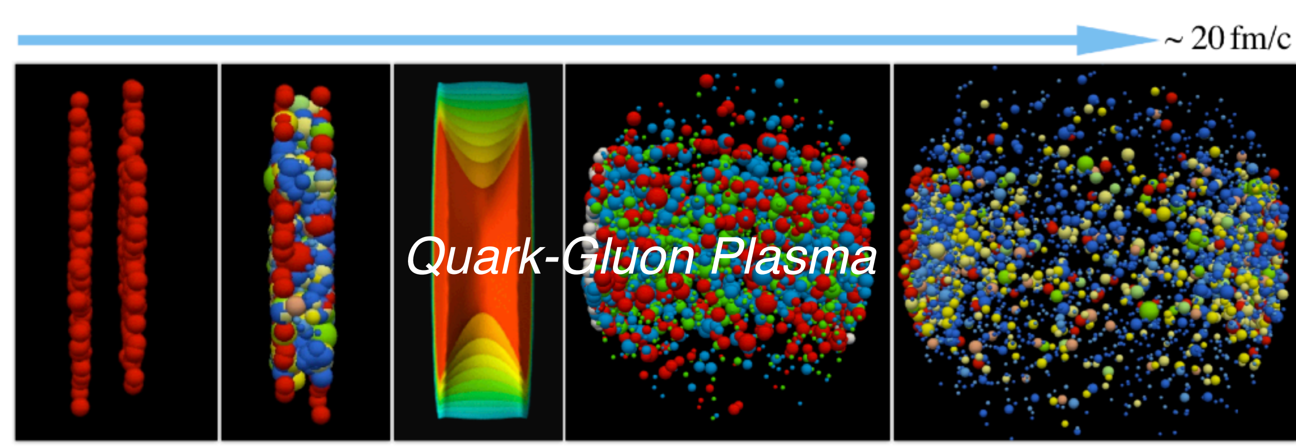



- *Relativistic Heavy-ion Collisions*

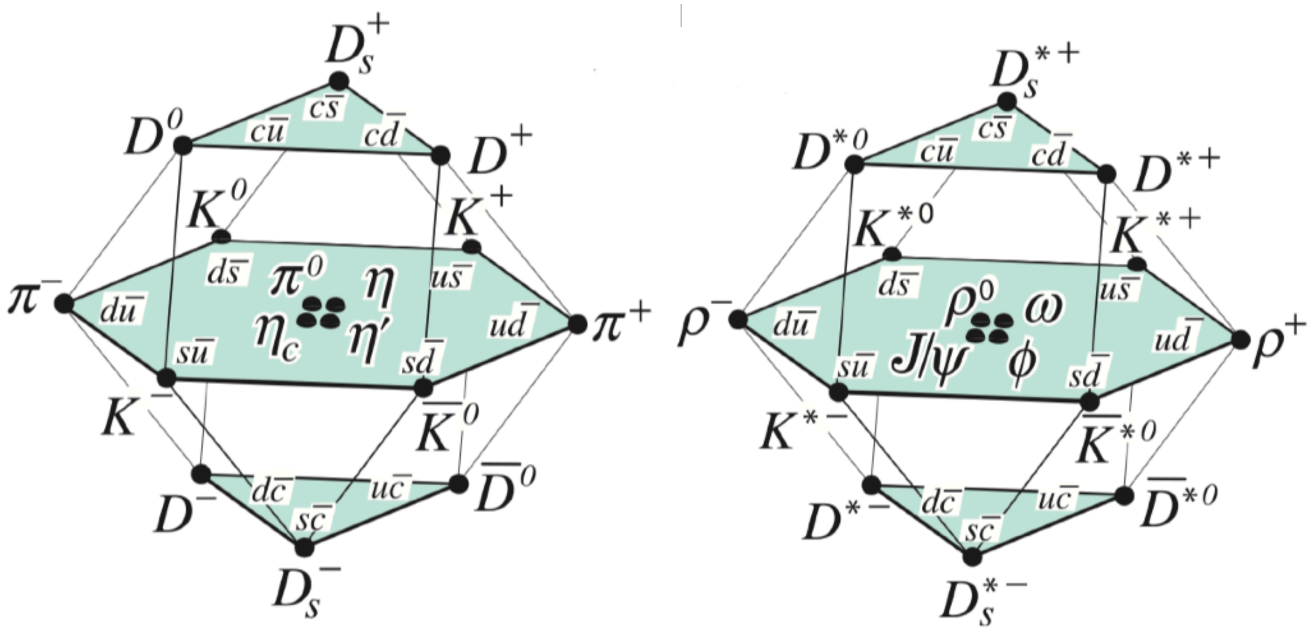



- *Massive neutron stars*

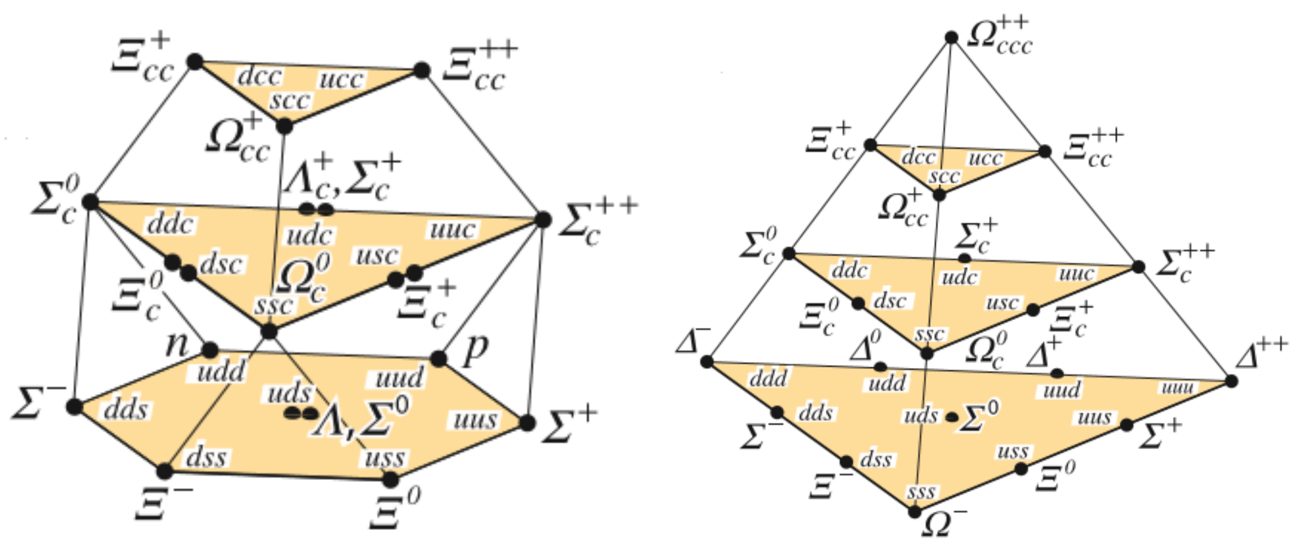
***Annala, E., Gorda, T., Kurkela, A. et al. Nat. Phys. (2020).***

# Heavy flavor hadrons

*The hadron which contains at least one heavy quark (charm or bottom).*




$$4 \otimes \bar{4} = 1 \oplus 15$$




$$4 \otimes 4 \otimes 4 = 20 \oplus 20' \oplus 20' \oplus \bar{4}$$

http://pdg.web.cern.ch/pdg/

## Charmonium

| $n^{2s+1}\ell_J$ | $J^{PC}$ | I $= 0$<br>$c\bar{c}$ | I $= \frac{1}{2}$<br>$c\bar{u}$, $c\bar{d}$;<br>$\bar{c}u$, $\bar{c}d$ | I $= 0$<br>$c\bar{s}$;<br>$\bar{c}s$ |
|---|---|---|---|---|
| $1\,^1S_0$ | $0^{-+}$ | $\eta_c(1S)$ | $D$ | $D_s^{\pm}$ |
| $1\,^3S_1$ | $1^{--}$ | $J/\psi(1S)$ | $D^*$ | $D_s^{*\pm}$ |
| $1\,^3P_0$ | $0^{++}$ | $\chi_{c0}(1P)$ | $D_0^*(2300)$ | $D_{s0}^*(2317)^{\pm\dagger}$ |
| $1\,^3P_1$ | $1^{++}$ | $\chi_{c1}(1P)$ | $D_1(2430)$ | $D_{s1}(2460)^{\pm\dagger}$ |
| $1\,^1P_1$ | $1^{+-}$ | $h_c(1P)$ | $D_1(2420)$ | $D_{s1}(2536)^{\pm}$ |
| $1\,^3P_2$ | $2^{++}$ | $\chi_{c2}(1P)$ | $D_2^*(2460)$ | $D_{s2}^*(2573)$ |
| $2\,^1S_0$ | $0^{-+}$ | $\eta_c(2S)$ | | |
| $2\,^3S_1$ | $1^{--}$ | $\psi(2S)$ | | $D_{s1}^*(2700)^{\pm\ddagger}$ |
| $1\,^3D_1$ | $1^{--}$ | $\psi(3770)$ | | $D_{s1}^*(2860)^{\pm\ddagger}$ |
| $1\,^3D_2$ | $2^{--}$ | $\psi_2(3823)$ | | |
| $2\,^3P_J$ | $0,1^{++}$ | $\chi_{c0}(3860)$ | | |
| | $2^{++}$ | $\chi_{c2}(3930)$ | | |
| $3\,^3S_1$ | $1^{--}$ | $\psi(4040)$ | | |
| $2\,^3D_1$ | $1^{--}$ | $\psi(4160)$ | | |
| $4\,^3S_1$ | $1^{--}$ | $\psi(4415)$ | | |
| $1\,^3D_3$ | $3^{--}$ | | $D_3^*(2750)$ | $D_{s3}^*(2860)^{\pm}$ |

## Bottomonium

| $n^{2s+1}\ell_J$ | $J^{PC}$ | I $= 0$<br>$b\bar{b}$ | I $= \frac{1}{2}$<br>$b\bar{u}$, $b\bar{d}$;<br>$\bar{b}u$, $\bar{b}d$ | I $= 0$<br>$b\bar{s}$;<br>$\bar{b}s$ | I $= 0$<br>$b\bar{c}$;<br>$\bar{b}c$ |
|---|---|---|---|---|---|
| $1\,^1S_0$ | $0^{-+}$ | $\eta_b(1S)$ | $B$ | $B_s^0$ | $B_c^{\pm}$ |
| $1\,^3S_1$ | $1^{--}$ | $\Upsilon(1S)$ | $B^*$ | $B_s^*$ | |
| $1\,^3P_0$ | $0^{++}$ | $\chi_{b0}(1P)$ | | | |
| $1\,^3P_1$ | $1^{++}$ | $\chi_{b1}(1P)$ | | | |
| $1\,^1P_1$ | $1^{+-}$ | $h_b(1P)$ | $B_1(5721)$ | $B_{s1}(5830)^0$ | |
| $1\,^3P_2$ | $2^{++}$ | $\chi_{b2}(1P)$ | $B_2^*(5747)$ | $B_{s2}^*(5840)^0$ | |
| $2\,^1S_0$ | $0^{-+}$ | $\eta_b(2S)$ | | | $B_c(2S)^{\pm}$ |
| $2\,^3S_1$ | $1^{--}$ | $\Upsilon(2S)$ | | | $B_c^*(2S)^{\pm}$ |
| $1\,^3D_2$ | $2^{--}$ | $\Upsilon_2(1D)$ | | | |
| $2\,^3P_J$ | $0,1,2^{++}$ | $\chi_{b0,1,2}(2P)$ | | | |
| $2\,^1P_1$ | $1^{+-}$ | $h_b(2P)$ | | | |
| $3\,^3S_1$ | $1^{--}$ | $\Upsilon(3S)$ | | | |
| $3\,^3P_J$ | $0,1,2^{++}$ | $\chi_{b1,2}(3P)$ | | | |
| $4\,^3S_1$ | $1^{--}$ | $\Upsilon(4S)$ | | | |

# Heavy flavor exotic hadrons

*M. Gell-Mann indicated: "Baryons can now be constructed from quarks by using the combinations $(qqq)$, $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc"*

***M. Gell-Mann. Phys. Lett. 8 (1964) 214–215.***

- *Open heavy-flavor hadrons* $D_{s0}^*(2317)$ , $D_{s1}(2460)$, $D_{s1}^*(2860)$
- *Charmonium-like XYZ states.*

***Belle Collaboration, Phys. Rev. Lett., 2003, 91:022001.***

*2003, Belle, Find tetraquark candidate, named X(3872), which is the first member of the charmonium-like states family.*

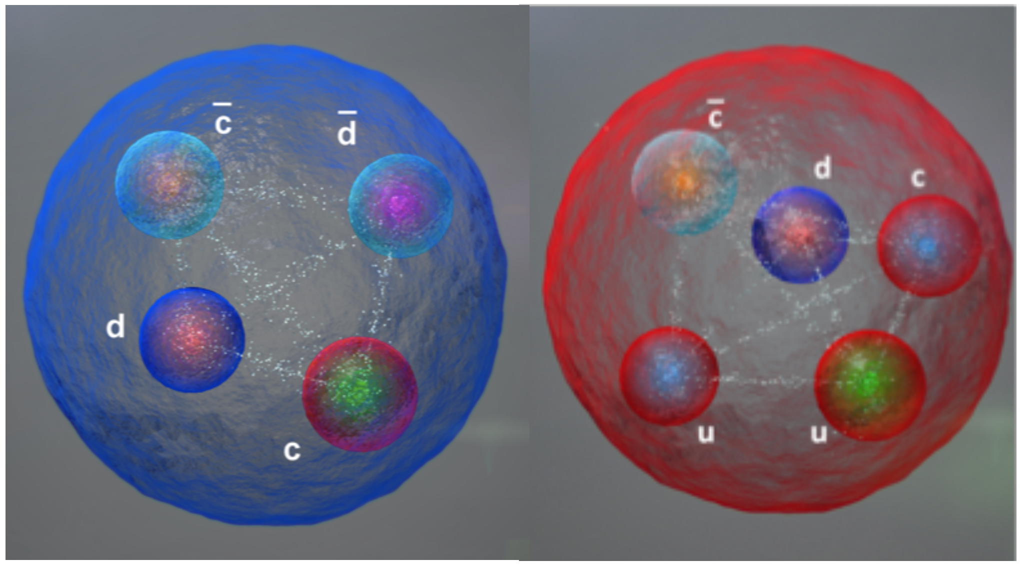
*2004-2020, Find so many charmonium-like states, named X, Y, Z.*

- *Heavy flavor pentaquark states.*

***LHCb Collaboration, Phys. Rev. Lett., 2015, 115:072001.***

*2015, LHCb, Find pentaquark-charmonium-like states, Pc*

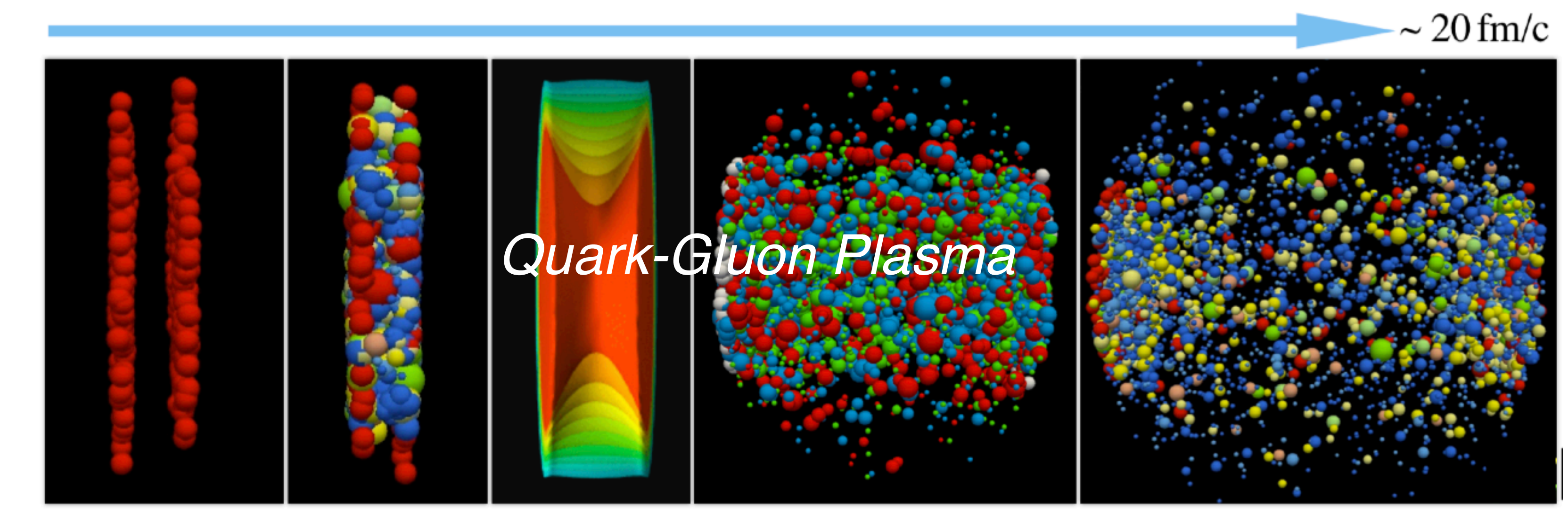
| A [1–5] | B [6–10] | C [11, 12] | D [13–15] | E [16–20] |
|---|---|---|---|---|
| $X(3872)$ | $Y(4260)$ | $X(3940)$ | $X(3915)$ | $Z_b(10610)$ |
| $Y(3940)$ | $Y(4008)$ | $X(4160)$ | $X(4350)$ | $Z_b(10650)$ |
| $Z^+(4430)$ | $Y(4360)$ | – | $Z(3930)$ | $Z_c(3900)$ |
| $Z^+(4051)$ | $Y(4660)$ | – | – | $Z_c(4025)$ |
| $Z^+(4248)$ | $Y(4630)$ | – | – | $Z_c(4020)$ |
| $Y(4140)$ | – | – | – | $Z_c(3885)$ |
| $Y(4274)$ | – | – | – | – |




***lhcb-public.web.cern.ch***

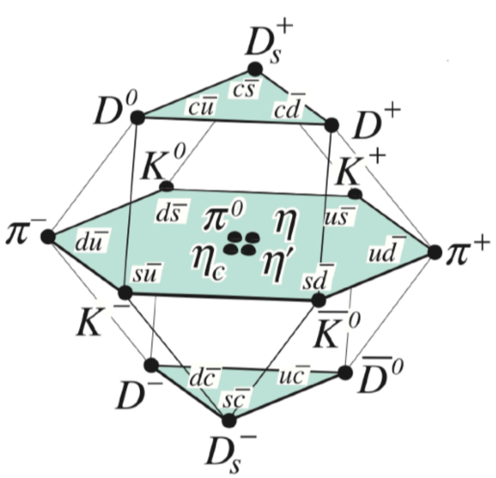
# Question:

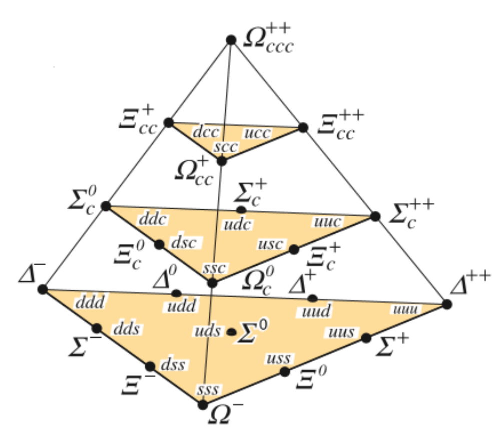
*Heavy flavor hadrons are produced not only in elementary particle collisions(p+p, e+e-,…) but also in relativistic heavy-ion collisions(A+A).*




*The emerges of QGP would affect the production of heavy flavor hadrons or not?*

*Can we use heavy flavor hadrons to probe each stage of heavy-ion collisions ?*



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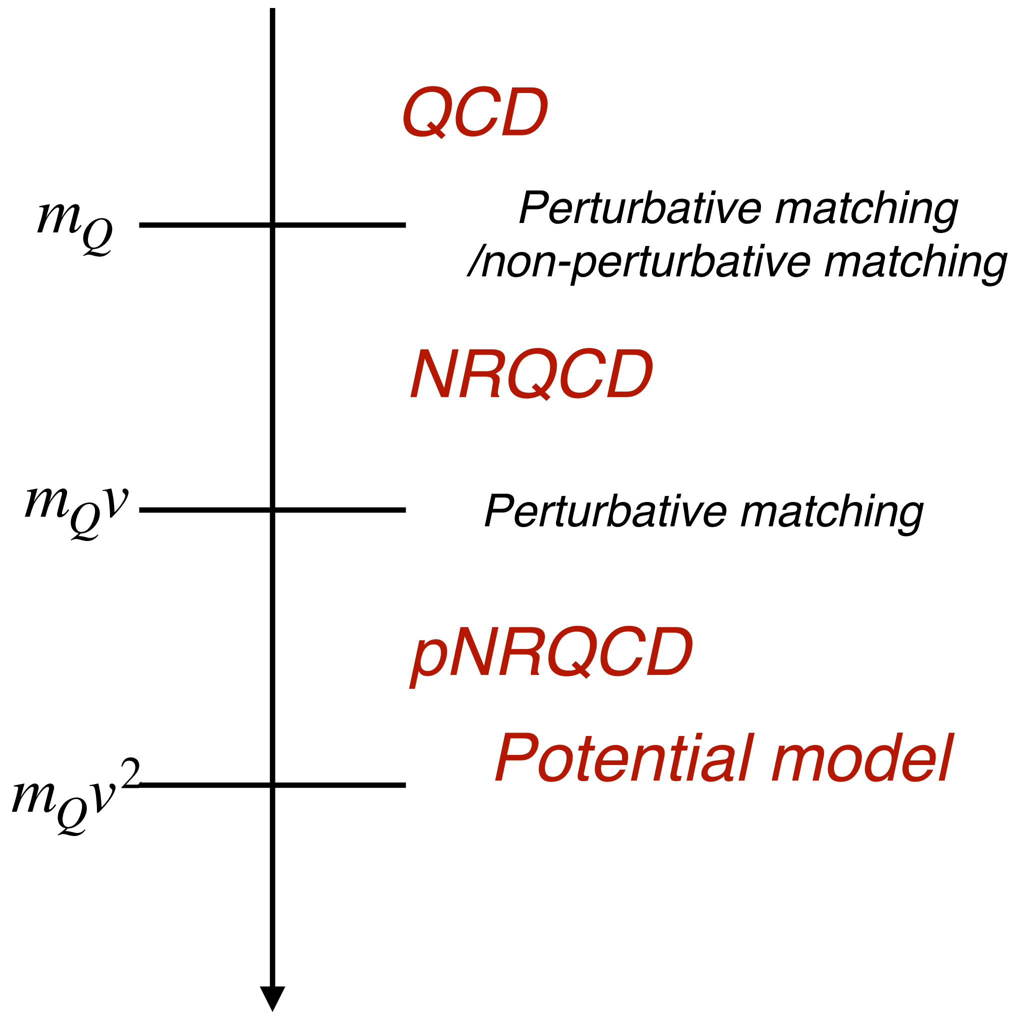
*JX Zhao and Pengfei Zhuang, Few Body Syst. 58, 100(2017).*

*JX Zhao, Hang He and Pengfei Zhuang, Phys. Lett. B771,349(2017).*

*Shuzhe Shi, JX Zhao, and Pengfei Zhuang, Chin.Phys.C 44 (2020) 8, 084101.*

# Potential model

Separation of scales: $m_Q \gg m_Q v \gg m_Q v^2$

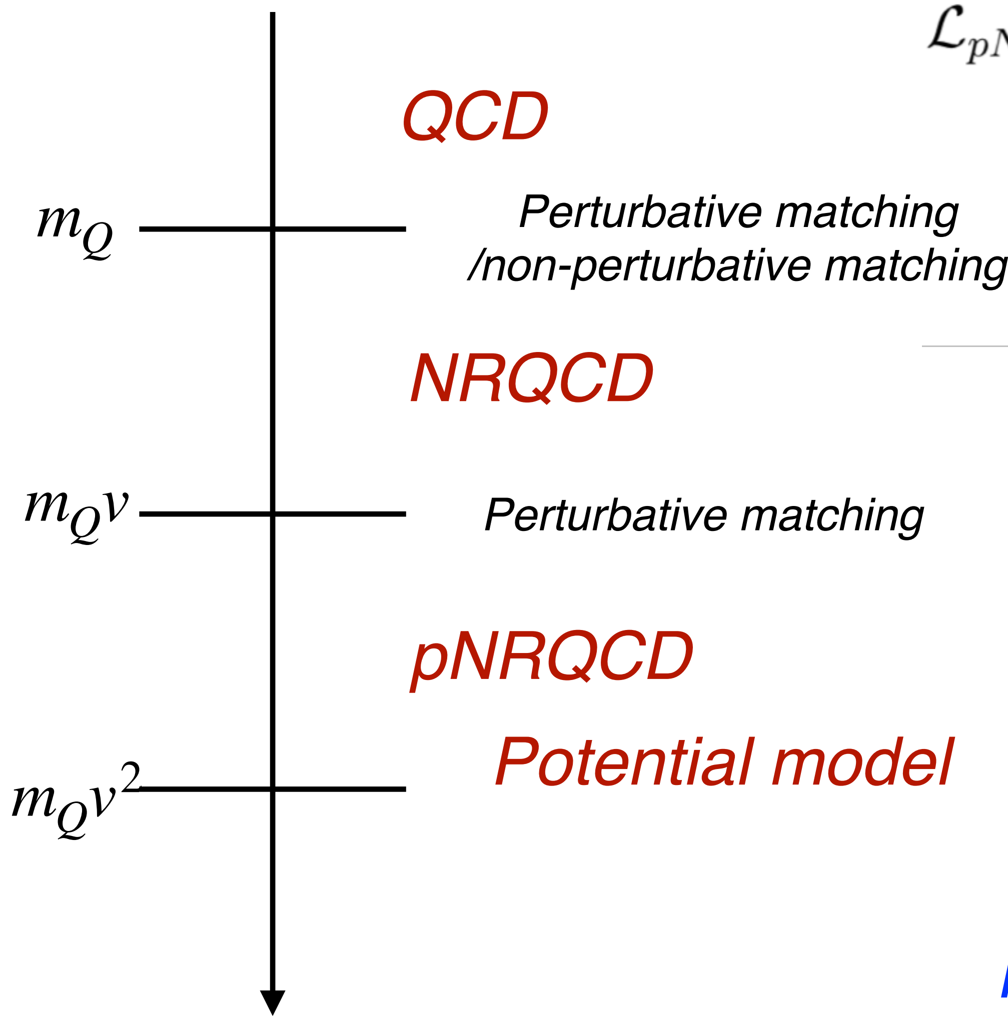



"Top - down"

*From Xiaojun Yao's slides.*

# Potential model

*Separation of scales:* $m_Q \gg m_Q v \gg m_Q v^2$




$$
\begin{aligned}
\mathcal{L}_{pNRQCD} &= \int d^3r \mathrm{Tr}\Big[S^\dagger(i\partial_0 - H_S)S + O^\dagger(i\partial_0 - H_O)O\Big] \\
&+ V_A(r)\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}S + S^\dagger \mathbf{r}\cdot g\mathbf{E}O] \\
&+ \frac{V_B(r)}{2}\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E}] + \mathcal{L}'_g + \mathcal{L}'_l
\end{aligned}
$$

Singlet field S; Octet field O.

$$
\begin{aligned}
H_S &= \{c_1^s(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^s(r)\frac{\mathbf{P}^2}{2M} + V_S^{(0)} + \frac{V_S^{(1)}}{m_Q} + \frac{V_S^{(2)}}{m_Q^2}, \\
H_O &= \{c_1^o(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^o(r)\frac{\mathbf{P}^2}{2M} + V_O^{(0)} + \frac{V_O^{(1)}}{m_Q} + \frac{V_O^{(2)}}{m_Q^2}.
\end{aligned}
$$

Potential model *(Schroedinger eq. & Dirac eq.)*

*"Top - down"*

$$
H_{S,O} = \frac{(i\nabla)^2}{m_Q} + V_{S,O}^{(0)}
$$

*From Xiaojun Yao's slides.*

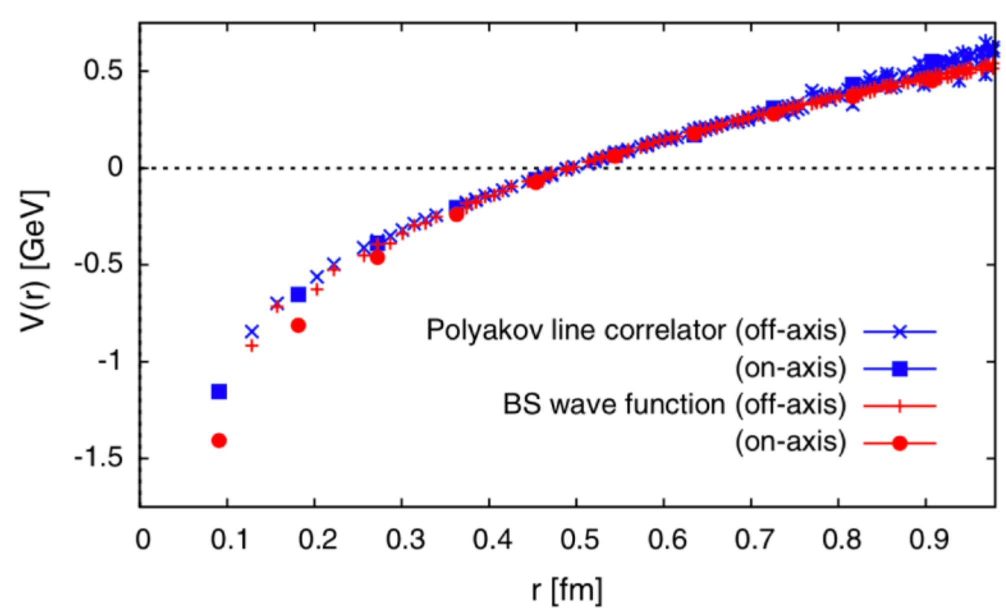
# Potential

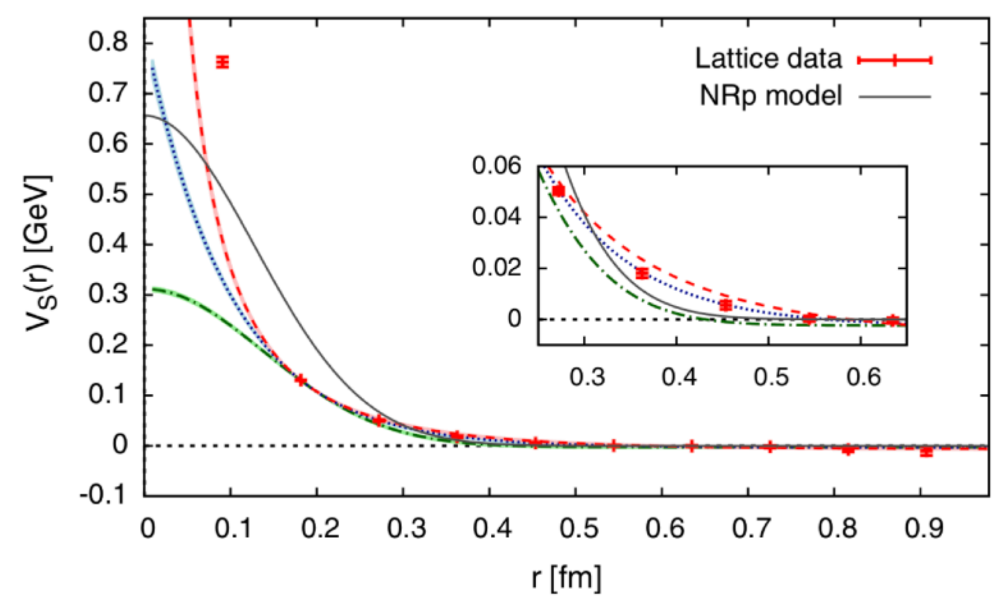
*The interaction potential can be calculated by the perturbation expansion of the above effective model or lattice QCD*

*One gluon exchange (OGE) plus a phenomenological linear confinement interaction*

$$V_{ij}(|\mathbf{r}_{ij}|) = -\frac{1}{4}\lambda_i^a \cdot \lambda_j^a \left(V_{ij}^c(|\mathbf{r}_{ij}|) + V_{ij}^{ss}(|\mathbf{r}_{ij}|)\mathbf{s}_i \cdot \mathbf{s}_j\right)$$

$\lambda_i^a (a = 1,...,8)$ *SU(3) Gell-Mann matrices*




***T. Kawanai, S. Sasaki, Phys. Rev. D 85 (2012) 091503.***

$$V_{ij}^c(|\mathbf{r}_{ij}|) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma|\mathbf{r}_{ij}|,$$ *Cornell potential*

$$V_{ij}^{ss}(|\mathbf{r}_{ij}|) = \beta e^{-\gamma|\mathbf{r}_{ij}|}.$$ *Spin-spin or chromomagnetic interaction*

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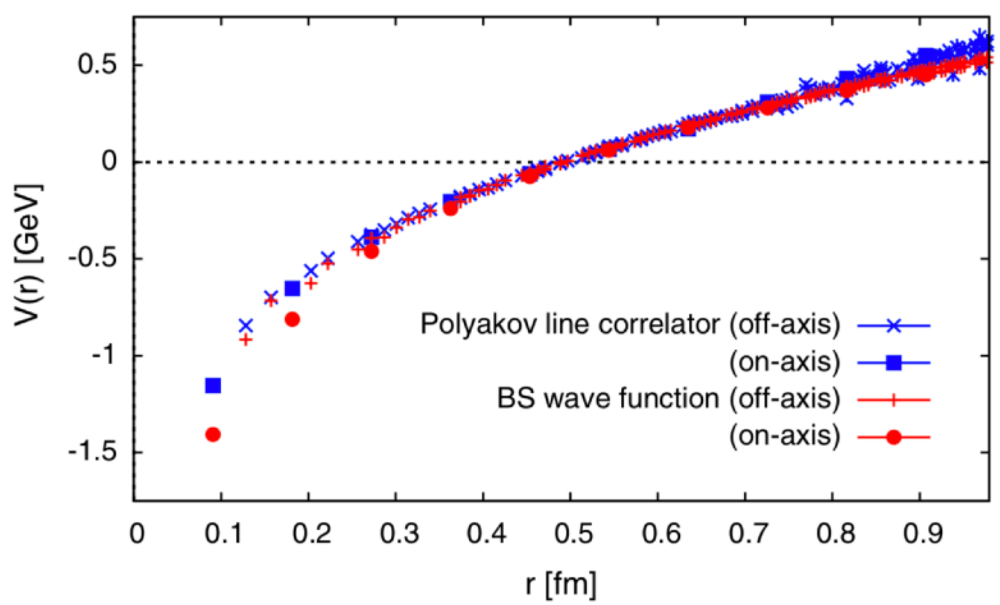


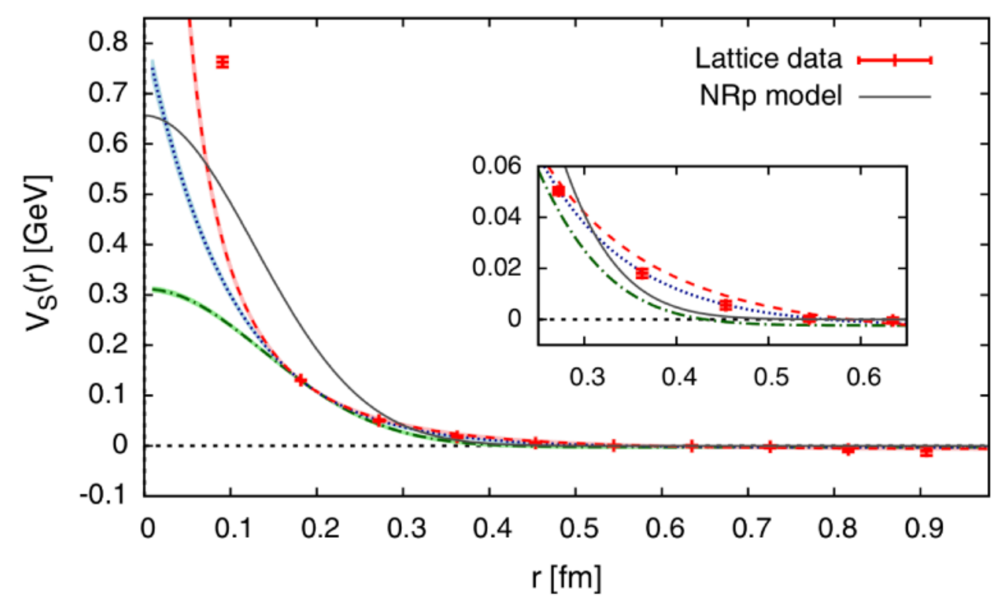


*T. Kawanai, S. Sasaki, Phys. Rev. D 85 (2012) 091503.*

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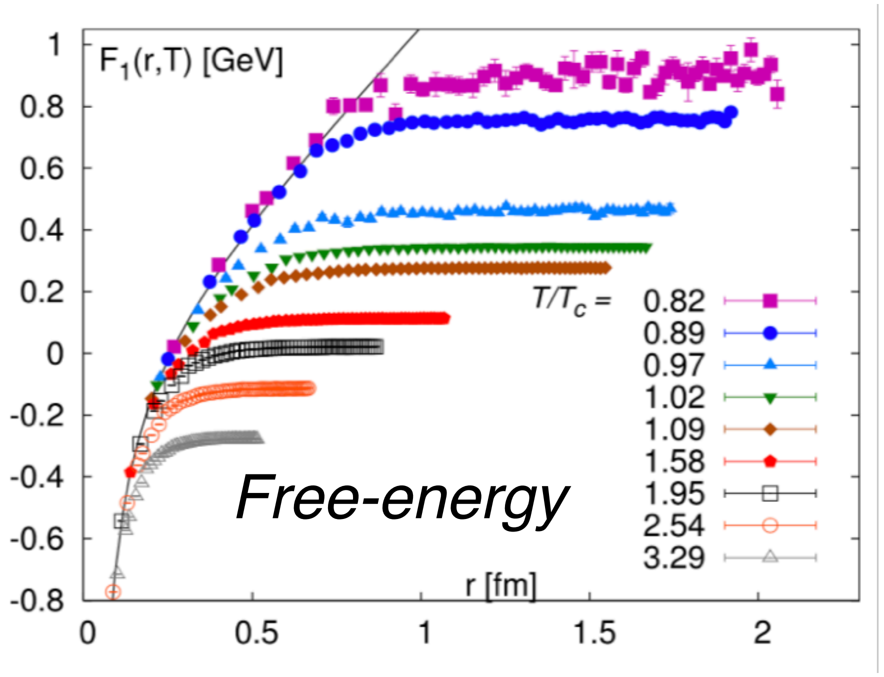
$$V_{ij}^{ss}(|\mathbf{r}_{ij}|) = \beta e^{-\gamma|\mathbf{r}_{ij}|}.$$ *Spin-spin or chromomagnetic interaction*

- *Mesons* $V_{Q\bar{Q}} = \frac{4}{3}(V_{ij}^c(r) + V_{ij}^{ss}\mathbf{s}_i \cdot \mathbf{s}_j)$
- *Baryons* $V_{QQ} = \frac{2}{3}(V_{ij}^c(r) + V_{ij}^{ss}\mathbf{s}_i \cdot \mathbf{s}_j)$
- *Tetraquark*

*JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319*

# Potential

## Finite-Temperature




*P. Petreczky, J. Phys. G 37 (2010) 094009.*

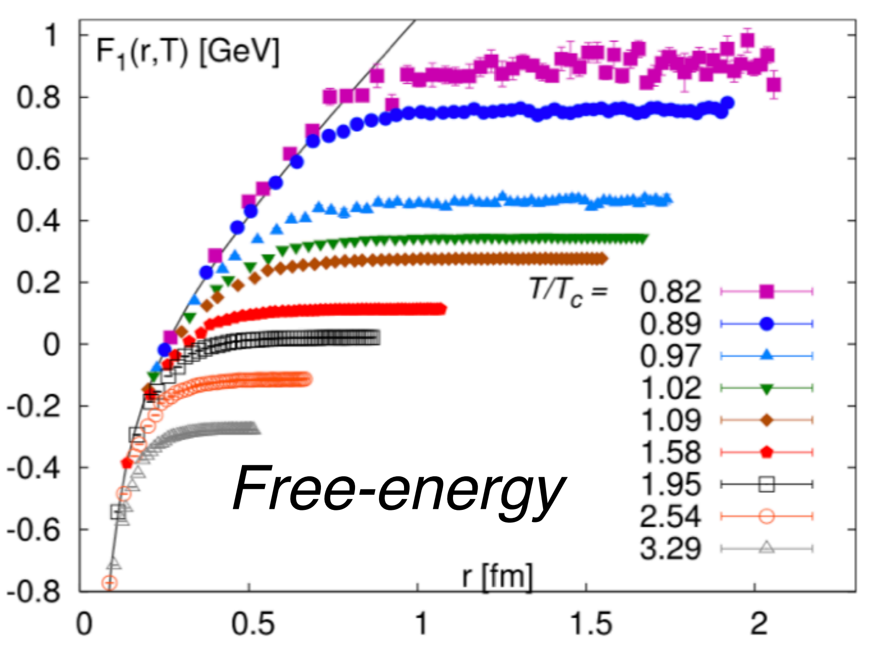
$$F_{Q\bar{Q}}(r,T) = \frac{\sigma}{m_D}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{m_D r}}{2^{3/4}\Gamma(3/4)}K_{1/4}(m_D^2 r^2)\right] - \alpha\left[m_D + \frac{e^{-m_D r}}{r}\right]$$

$$U = F + TS.$$

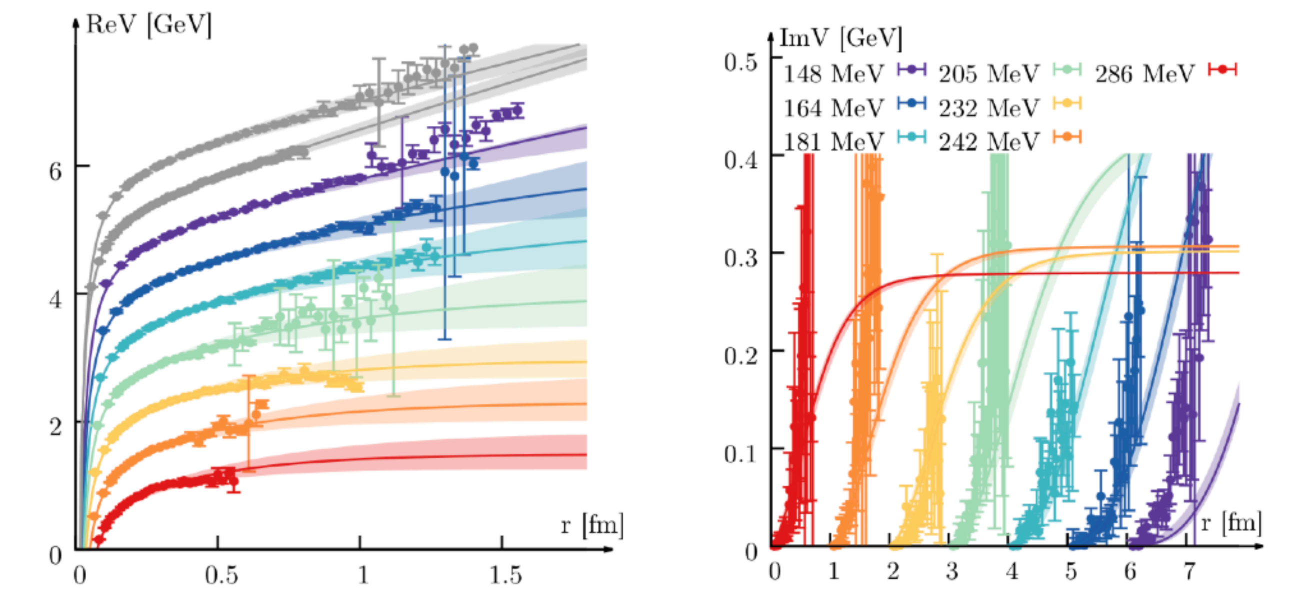
*$M_D$ is temperature dependent Debye screening mass.*

# Potential

## Finite-Temperature




P. Petreczky, J. Phys. G 37 (2010) 094009.




Burnier Y, Kaczmarek O, Rothkopf A. JHEP, 2015, 12:101.

D. Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5, 056010 .

$$F_{Q\bar{Q}}(r,T) = \frac{\sigma}{m_D}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{m_D r}}{2^{3/4}\Gamma(3/4)}K_{1/4}(m_D^2 r^2)\right] - \alpha\left[m_D + \frac{e^{-m_D r}}{r}\right]$$

$U = F + TS.$ *MD is temperature dependent Debye screening mass.*

- *Not only ReV but also ImV.*
- *ReV is close to the color singlet free energies $F_s$*

# N-Body Schroedinger Equation

$$\left( \sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} V_{ij} \right) \Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = E\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N)$$

*Jacobi coordinates :*

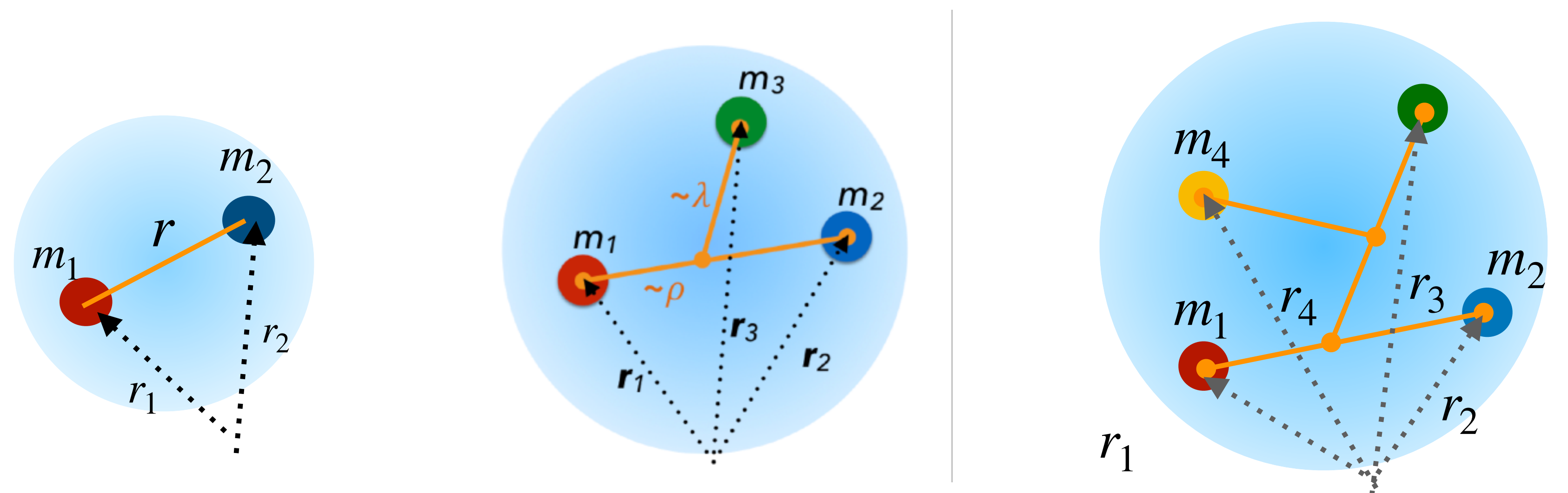
$$\mathbf{R} = \frac{1}{M} \sum_{i=1}^{N} m_i \mathbf{r}_i,$$

*Center of mass coordinate*

$$\mathbf{x_j} = \sqrt{\frac{M_j m_{j+1}}{M_{j+1}\mu}} \left( \mathbf{r}_{j+1} - \frac{1}{M_j} \sum_{i=1}^{j} m_i \mathbf{r}_i \right)$$

*N-1 Relative coordinates*

*j = 1, …, N-1. and* $M_j = \sum_{i=1}^{j} m_i$




...

*Then, factorize the N-body motion into a center-of-mass motion and a relative motion*

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \Theta(\mathbf{R})\Phi(\mathbf{x}_1, \ldots, \mathbf{x}_{N-1}),$$

# N-Body Schroedinger Equation

*Further, N-1 relative coordinates can be transformed to a single hyperradial coordinate and 3N-4 hyperangular coordinates.*

$$(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N-1}) \to (\rho, \alpha_{N-1}, \ldots, \alpha_2, \theta_1, \phi_1, \ldots, \theta_{N-1}, \phi_{N-1})$$

$$\rho = \sqrt{\mathbf{x}_1^2 + \ldots + \mathbf{x}_{N-1}^2} \qquad \sin\alpha_i = x_i/\rho_i \qquad \rho_i = \sqrt{\sum_{j=1}^{i} \mathbf{x}_j^2} \qquad \hat{x}_i = (\theta_i, \phi_i)$$

**N. Barnea, et al. Phys. Rev. C 61.054001(2000)**
**FBS Colloquium. Few-Body System 25, 199-238(1998)**

*The relative motion is controlled by :*

$$\left[\frac{1}{2\mu}\left(-\frac{1}{\rho^{3N-4}}\frac{d}{d\rho}\rho^{3N-4}\frac{d}{d\rho} + \frac{\hat{K}_{N-1}^2}{\rho^2}\right) + V(\rho, \Omega)\right]\Phi(\rho, \Omega) = E_r \Phi(\rho, \Omega),$$

$$\hat{K}_{N-1}^2 = -\frac{\partial^2}{\partial\alpha_{N-1}^2} + \frac{(3N-9) - (3N-5)\cos(2\alpha_{N-1})}{\sin(2\alpha_{N-1})}\frac{\partial}{\partial\alpha_{N-1}} + \frac{1}{\cos^2\alpha_{N-1}}\hat{K}_{N-2}^2 + \frac{1}{\sin^2\alpha_{N-1}}\hat{l}_{N-1}^2,$$

$$\hat{K}_{N-1}^2 \mathcal{Y}_\kappa(\Omega) = K(K+3N-5)\mathcal{Y}_\kappa(\Omega).$$

*hyper-angular momentum operator*

$$\Phi(\rho, \Omega) = \sum_\kappa R_\kappa(\rho)\mathcal{Y}_\kappa(\Omega)$$

*hyper-spherical harmonic function expansion*

$$\to \quad \left[\frac{1}{2\mu}\left(\frac{1}{\rho^{3N-4}}\frac{d}{d\rho}\rho^{3N-4}\frac{d}{d\rho} - \frac{K(K+3N-5)}{\rho^2}\right) + E_r\right]R_\kappa = \sum_{\kappa'} V_{\kappa\kappa'} R_{\kappa'}$$

$$V_{\kappa\kappa'} = \int \mathcal{Y}_\kappa^*(\Omega)V(\rho, \Omega)\mathcal{Y}_{\kappa'}(\Omega)d\Omega.$$

# Parameters

$$V_{Q\bar{Q}}(|\mathbf{r}_{ij}|) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma|\mathbf{r}_{ij}| + \beta e^{-\gamma|\mathbf{r}_{ij}|}\mathbf{s}_i \cdot \mathbf{s}_j$$

$$F_{Q\bar{Q}}(r,T) = \frac{\sigma}{m_D}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{m_D r}}{2^{3/4}\Gamma(3/4)}K_{1/4}(m_D^2 r^2)\right] - \alpha\left[m_D + \frac{e^{-m_D r}}{r}\right]$$

*Quark mass.*

- *The parameters can be fixed by quarkonium mass in vacuum!*

$$\eta_c, J/\psi, \chi_c, \psi', \Upsilon(1S), \chi_b, \Upsilon(2S)$$

- *At finite temperature, there are no-free parameters!*

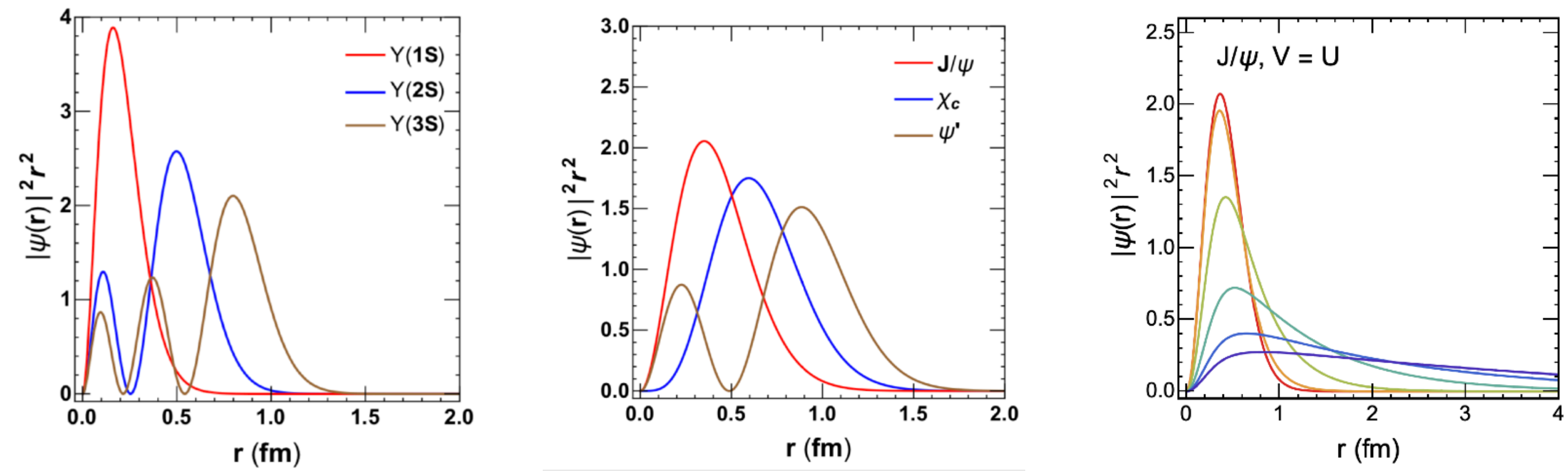
TABLE I: Potential model parameters

| $m_b$ | $m_c$ | $\alpha$ | $\sigma$ | $\gamma$ | $\beta_b$ | $\beta_c$ |
|---|---|---|---|---|---|---|
| 4.7 GeV | 1.29 GeV | 0.308 | 0.15 $\text{GeV}^2$ | 1.982 GeV | 0.239 GeV | 1.545 GeV |

# Results(Two-Body)

$$\left[\frac{1}{2\mu}\left(-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+\frac{l(l+1)}{r^2}\right)+V_{Q\bar{Q}}(r)\right]R(r)=E_rR(r).$$

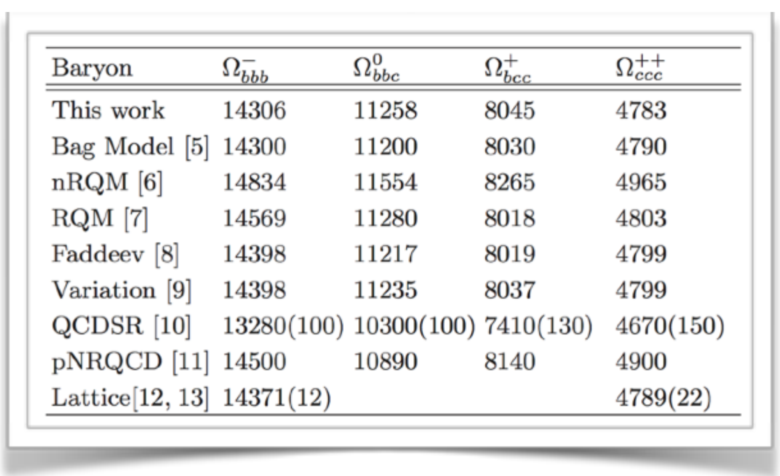
| States | $\eta_c(1S)$ | $J/\psi(1S)$ | $h_c(1P)$ | $\chi_c(1P)$ | $\eta_c(2S)$ | $\psi(2S)$ | $h_c(2P)$ | $\chi_c(2P)$ |
|---|---|---|---|---|---|---|---|---|
| $M_{Exp.}$(GeV) | 2.981 | 3.097 | 3.525 | 3.556 | 3.639 | 3.686 | - | 3.927 |
| $M_{Th.}$(GeV) | 2.967 | 3.102 | 3.480 | 3.500 | 3.654 | 3.720 | 3.990 | 4.000 |
| $\langle r\rangle$(fm) | 0.365 | 0.427 | 0.635 | 0.655 | 0.772 | 0.802 | 0.961 | 0.980 |
| States | $\eta_b(1S)$ | $\Upsilon(1S)$ | $h_b(1P)$ | $\chi_b(1P)$ | $\eta_b(2S)$ | $\Upsilon(2S)$ | $\chi_b(2P)$ | $\Upsilon(3S)$ |
| $M_{Exp.}$(GeV) | 9.398 | 9.460 | 9.898 | 9.912 | 9.999 | 10.023 | 10.269 | 10.355 |
| $M_{Th.}$(GeV) | 9.397 | 9.459 | 9.845 | 9.860 | 9.957 | 9.977 | 10.221 | 10.325 |
| $\langle r\rangle$(fm) | 0.200 | 0.214 | 0.377 | 0.387 | 0.465 | 0.474 | 0.603 | 0.680 |



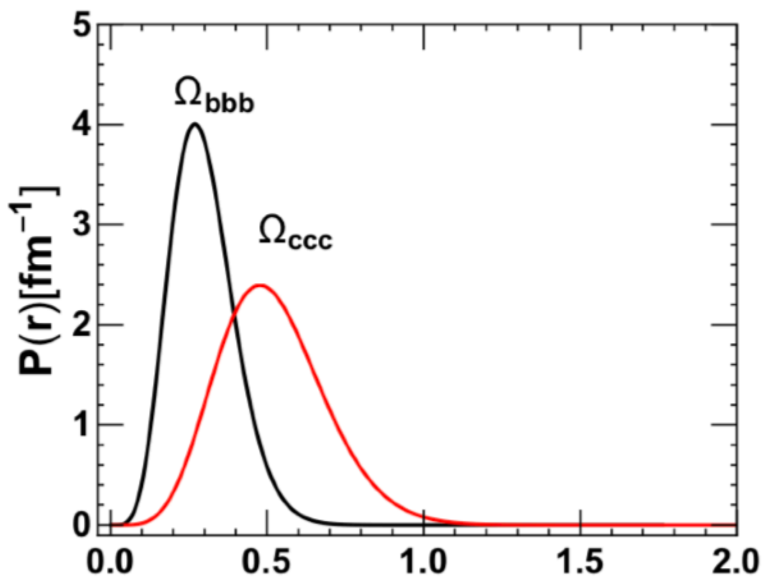

...

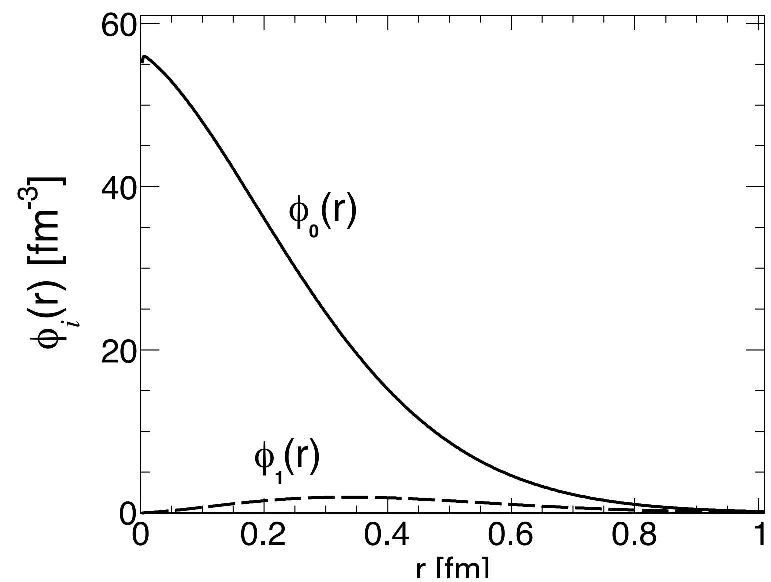
# Results(Three-Body)

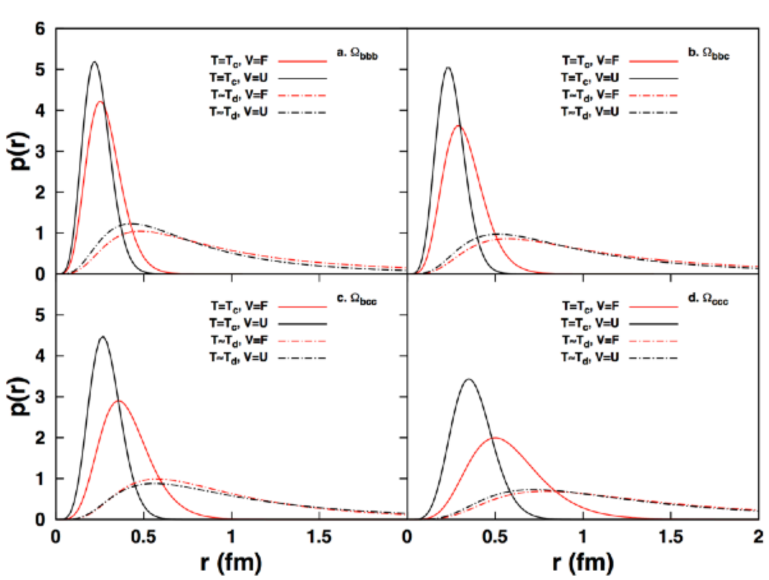
| States | $\Omega_{ccc}$ | $\Omega_{ccb}$ | $\Omega^*_{ccb}$ | $\Omega_{bbc}$ | $\Omega^*_{bbc}$ | $\Omega_{bbb}$ |
|---|---|---|---|---|---|---|
| $J^P$ | $\frac{3}{2}^+$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{3}{2}^+$ |
| $M_{Th.}$(GeV) | 4.797 | 8.143 | 8.207 | 10.920 | 10.953 | 14.363 |
| $r_{rms}$(fm) | 0.289 | 0.200 | 0.211 | 0.171 | 0.175 | 0.153 |

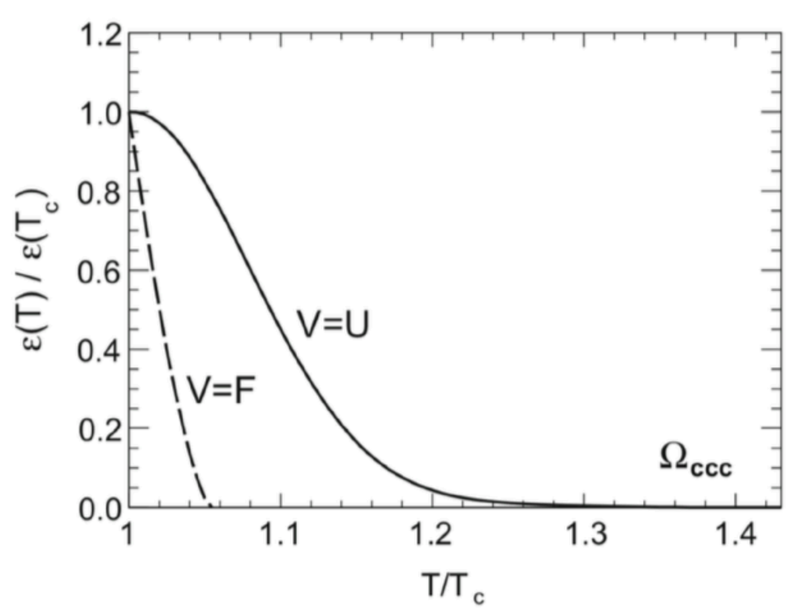


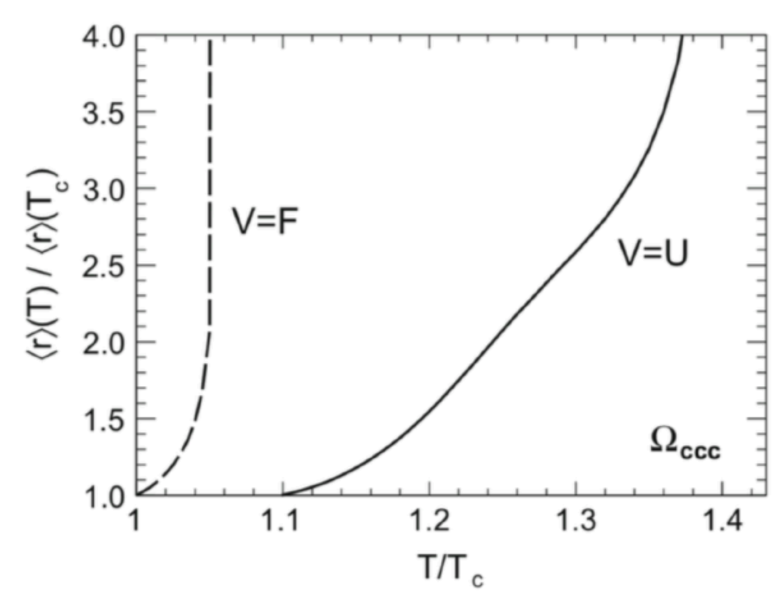
| Baryon | $\Omega^-_{bbb}$ | $\Omega^0_{bbc}$ | $\Omega^+_{bcc}$ | $\Omega^{++}_{ccc}$ |
|---|---|---|---|---|
| This work | 14306 | 11258 | 8045 | 4783 |
| Bag Model [5] | 14300 | 11200 | 8030 | 4790 |
| nRQM [6] | 14834 | 11554 | 8265 | 4965 |
| RQM [7] | 14569 | 11280 | 8018 | 4803 |
| Faddeev [8] | 14398 | 11217 | 8019 | 4799 |
| Variation [9] | 14398 | 11235 | 8037 | 4799 |
| QCDSR [10] | 13280(100) | 10300(100) | 7410(130) | 4670(150) |
| pNRQCD [11] | 14500 | 10890 | 8140 | 4900 |
| Lattice[12, 13] | 14371(12) | | | 4789(22) |



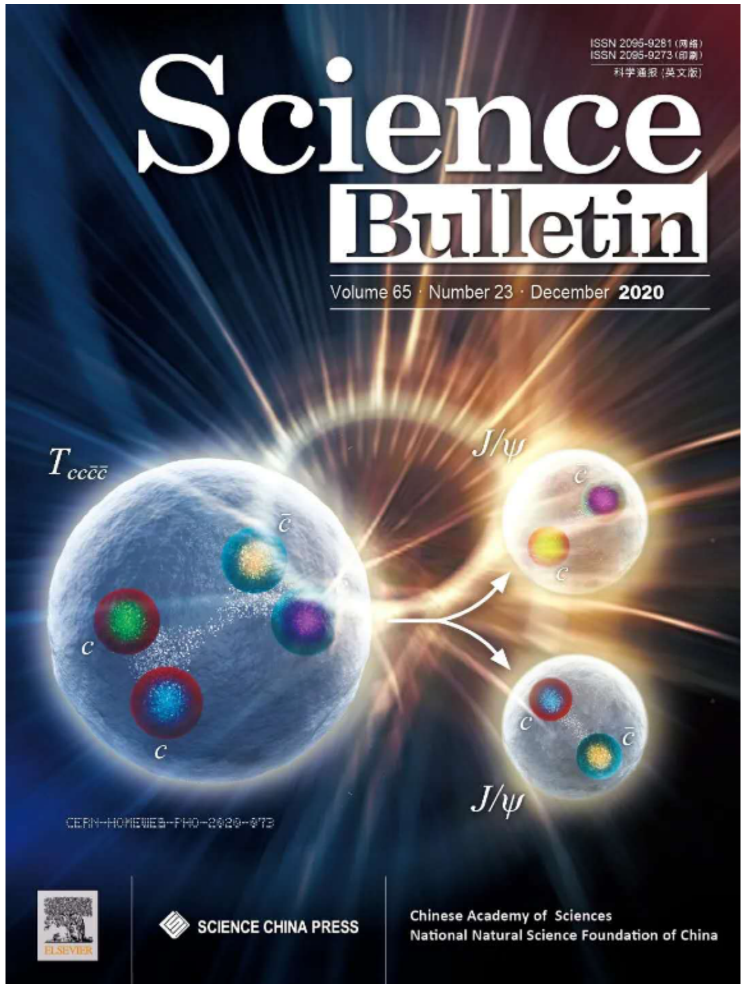



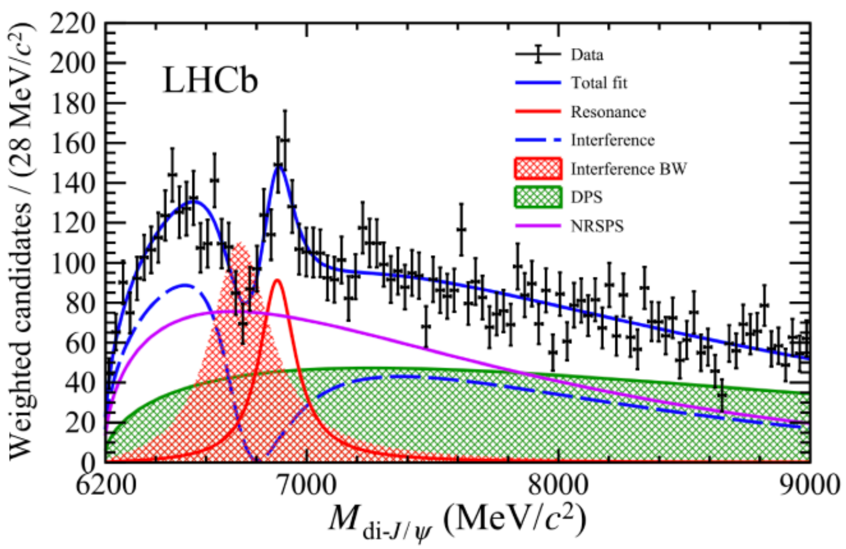



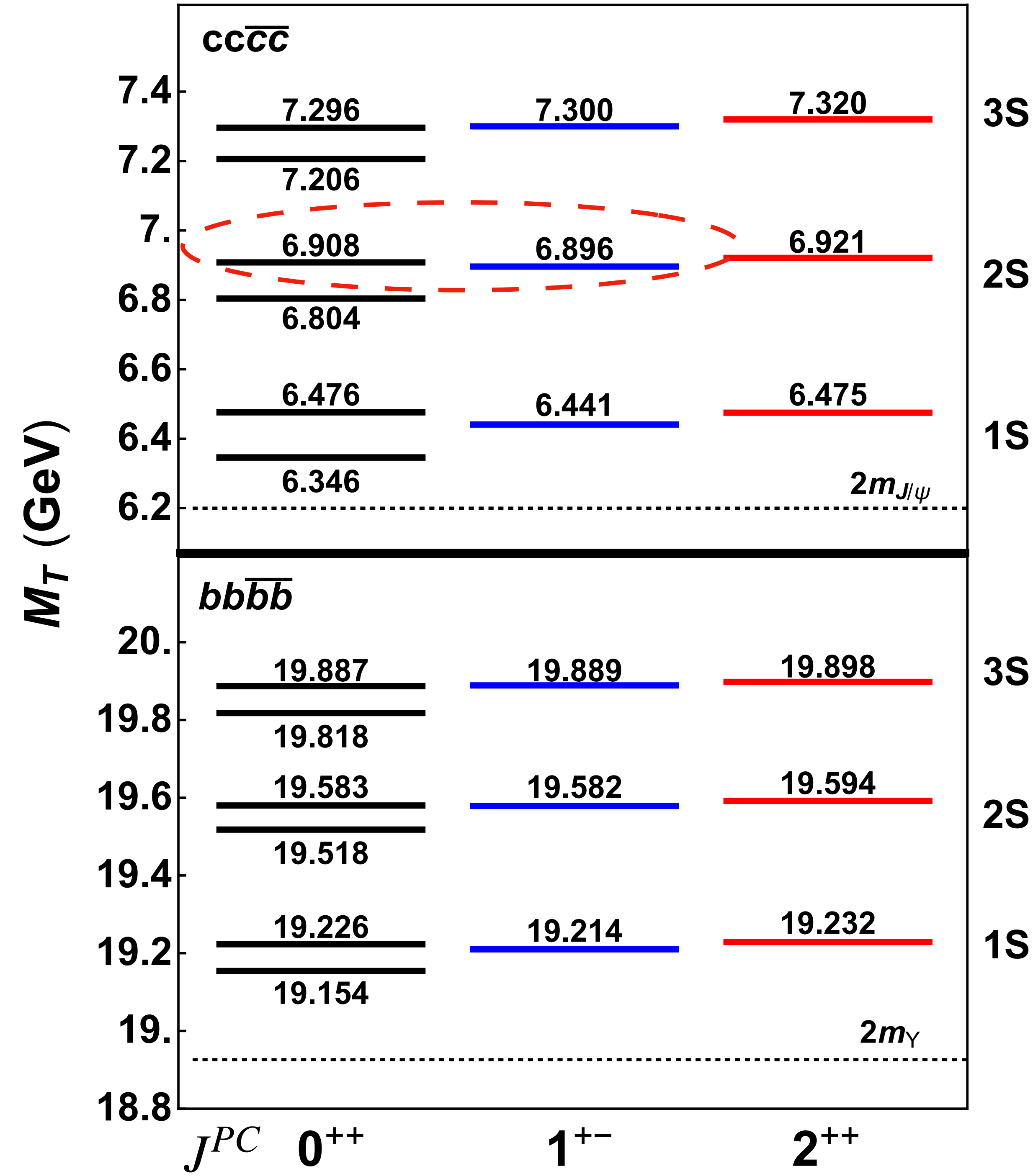
# Results(Four-Body)




$$m[X(6900)] = 6905 \pm 11 \pm 7\ \text{MeV}/c^2$$

$$\Gamma[X(6900)] = 80 \pm 19 \pm 33\ \text{MeV}$$




*LHCb Collaboration, Science Bulletin, 2020, 65(23)1983-1993*

*Please see details in:*

*JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319*

# Results(Four-Body)

JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319

TABLE III: The calculated tetraquark mass $M_T$ and the root-mean-squared radius $r_{\rm rms}$ for the ground and radial-excited states, 1$S$, 2$S$ and 3$S$ of $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with quantum numbers $J^{PC} = 0^{++}$, $1^{+-}$, and $2^{++}$.

| $J^{PC}$ | | $0^{++}$ | | | | | | $1^{+-}$ | | | $2^{++}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| State | | 1$S$ | | 2$S$ | | 3$S$ | | 1$S$ | 2$S$ | 3$S$ | 1$S$ | 2$S$ | 3$S$ |
| $cc\bar{c}\bar{c}$ | $M_T$(GeV) | 6.346 | 6.476 | 6.804 | 6.908 | 7.206 | 7.296 | 6.441 | 6.896 | 7.300 | 6.475 | 6.921 | 7.320 |
| | $r_{\rm rms}$(fm) | 0.323 | 0.351 | 0.445 | 0.457 | 0.550 | 0.530 | 0.331 | 0.446 | 0.547 | 0.339 | 0.452 | 0.552 |
| $bb\bar{b}\bar{b}$ | $M_T$(GeV) | 19.154 | 19.226 | 19.518 | 19.583 | 19.818 | 19.887 | 19.214 | 19.582 | 19.889 | 19.232 | 19.594 | 19.898 |
| | $r_{\rm rms}$(fm) | 0.180 | 0.186 | 0.259 | 0.259 | 0.328 | 0.325 | 0.181 | 0.257 | 0.324 | 0.183 | 0.259 | 0.326 |

# Results(Four-Body)

*JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319*

TABLE III: The calculated tetraquark mass $M_T$ and the root-mean-squared radius $r_{\rm rms}$ for the ground and radial-excited states, $1S$, $2S$ and $3S$ of $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with quantum numbers $J^{PC} = 0^{++}$, $1^{+-}$, and $2^{++}$.

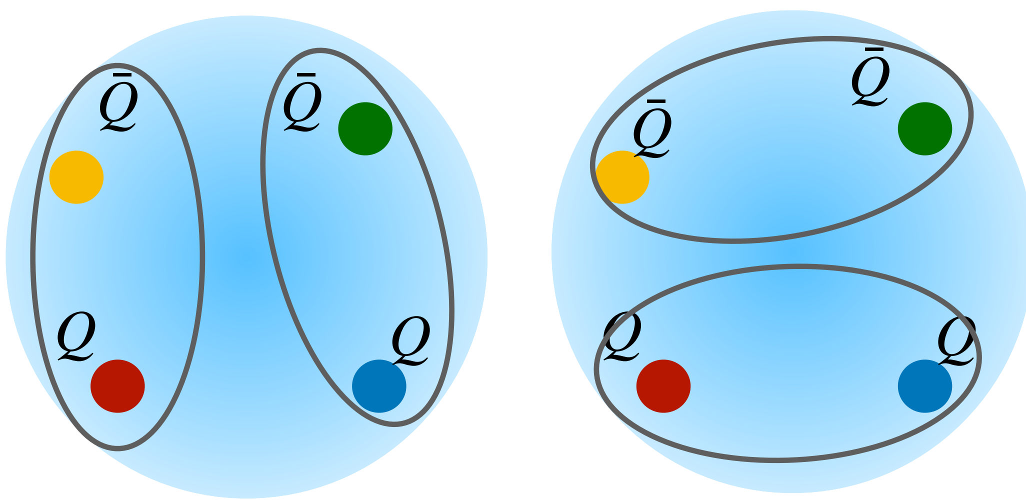
| $J^{PC}$ | | $0^{++}$ | | | | | | $1^{+-}$ | | | $2^{++}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| State | | $1S$ | | $2S$ | | $3S$ | | $1S$ | $2S$ | $3S$ | $1S$ | $2S$ | $3S$ |
| $cc\bar{c}\bar{c}$ | $M_T$(GeV) | 6.346 | 6.476 | 6.804 | 6.908 | 7.206 | 7.296 | 6.441 | 6.896 | 7.300 | 6.475 | 6.921 | 7.320 |
| | $r_{\rm rms}$(fm) | 0.323 | 0.351 | 0.445 | 0.457 | 0.550 | 0.530 | 0.331 | 0.446 | 0.547 | 0.339 | 0.452 | 0.552 |
| $bb\bar{b}\bar{b}$ | $M_T$(GeV) | 19.154 | 19.226 | 19.518 | 19.583 | 19.818 | 19.887 | 19.214 | 19.582 | 19.889 | 19.232 | 19.594 | 19.898 |
| | $r_{\rm rms}$(fm) | 0.180 | 0.186 | 0.259 | 0.259 | 0.328 | 0.325 | 0.181 | 0.257 | 0.324 | 0.183 | 0.259 | 0.326 |

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (3 \otimes 3) \otimes (\bar{3} \otimes \bar{3}) = \underline{\bar{3} \otimes 3} \oplus \underline{6 \otimes \bar{6}} \oplus \bar{3} \otimes \bar{6} \oplus 6 \otimes 3$$

*Diquark-diquark states*

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (3 \otimes \bar{3}) \otimes (3 \otimes \bar{3}) = \underline{1 \otimes 1} \oplus 1 \otimes 8 \oplus 8 \otimes 1 \oplus \underline{8 \otimes 8}$$

*Meson-meson states*



# Results(Four-Body)

*JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319*

TABLE III: The calculated tetraquark mass $M_T$ and the root-mean-squared radius $r_{\rm rms}$ for the ground and radial-excited states, $1S$, $2S$ and $3S$ of $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with quantum numbers $J^{PC} = 0^{++}$, $1^{+-}$, and $2^{++}$.

| $J^{PC}$ | | $0^{++}$ | | | | | | $1^{+-}$ | | | $2^{++}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| State | | $1S$ | | $2S$ | | $3S$ | | $1S$ | $2S$ | $3S$ | $1S$ | $2S$ | $3S$ |
| $cc\bar{c}\bar{c}$ | $M_T$(GeV) | 6.346 | 6.476 | 6.804 | 6.908 | 7.206 | 7.296 | 6.441 | 6.896 | 7.300 | 6.475 | 6.921 | 7.320 |
| | $r_{\rm rms}$(fm) | 0.323 | 0.351 | 0.445 | 0.457 | 0.550 | 0.530 | 0.331 | 0.446 | 0.547 | 0.339 | 0.452 | 0.552 |
| $bb\bar{b}\bar{b}$ | $M_T$(GeV) | 19.154 | 19.226 | 19.518 | 19.583 | 19.818 | 19.887 | 19.214 | 19.582 | 19.889 | 19.232 | 19.594 | 19.898 |
| | $r_{\rm rms}$(fm) | 0.180 | 0.186 | 0.259 | 0.259 | 0.328 | 0.325 | 0.181 | 0.257 | 0.324 | 0.183 | 0.259 | 0.326 |

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (3 \otimes 3) \otimes (\bar{3} \otimes \bar{3}) = \underline{\bar{3} \otimes 3} \oplus \underline{6 \otimes \bar{6}} \oplus \bar{3} \otimes \bar{6} \oplus 6 \otimes 3$$

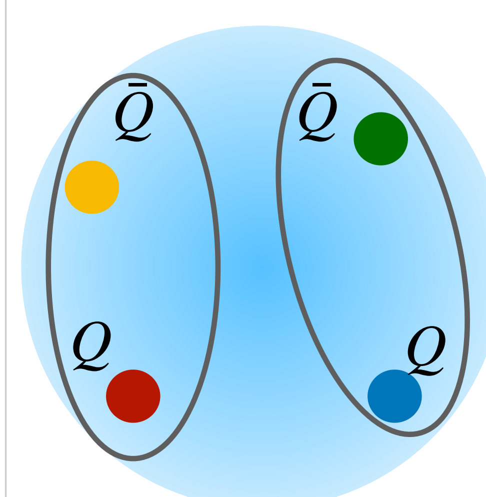
Diquark-diquark states

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (3 \otimes \bar{3}) \otimes (3 \otimes \bar{3}) = \underline{1 \otimes 1} \oplus 1 \otimes 8 \oplus 8 \otimes 1 \oplus \underline{8 \otimes 8}$$

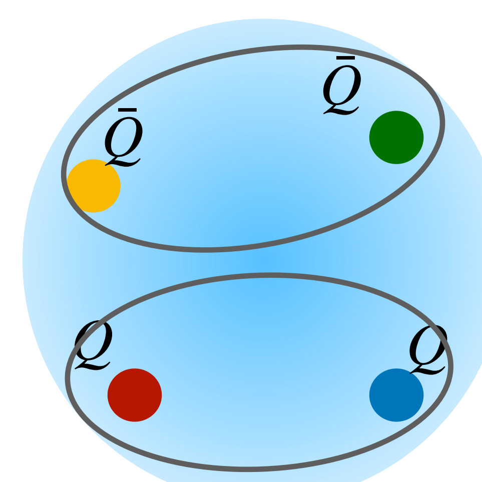
Meson-meson states

TABLE IV: The fraction of tetraquarks $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with $J^{PC} = 0^{++}$ in different color configures.

| | $cc\bar{c}\bar{c}$ | | | | | | $bb\bar{b}\bar{b}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| State | $1S$ | | $2S$ | | $3S$ | | $1S$ | | $2S$ | | $3S$ | |
| $M_T$(GeV) | 6.346 | 6.476 | 6.804 | 6.908 | 7.206 | 7.296 | 19.154 | 19.226 | 19.518 | 19.583 | 19.818 | 19.887 |
| $\lvert\phi_1\rangle$ | 45.0% | 54.2% | 29.8% | 72.0% | 19.9% | 65.6% | 31.5% | 67.7% | 13.4% | 86.9% | 6.2% | 94.1% |
| $\lvert\phi_2\rangle$ | 55.0% | 45.8% | 70.2% | 28.0% | 80.1% | 34.4% | 68.5% | 32.3% | 86.6% | 13.1% | 93.8% | 5.9% |
| $\lvert\phi_3\rangle$ | 96.4% | 6.3% | 89.5% | 21.2% | 81.6% | 39.0% | 97.8% | 3.6% | 88.2% | 16.6% | 79.6% | 25.8% |
| $\lvert\phi_5\rangle$ | 3.6% | 93.7% | 10.5% | 78.8% | 18.4% | 61.0% | 2.2% | 96.4% | 11.7% | 83.4% | 20.4% | 74.2% |
| $\lvert\phi_4\rangle$ | 6.8% | 91.0% | 23.9% | 64.1% | 38.5% | 50.6% | 14.5% | 84.6% | 36.2% | 58.8% | 49.6% | 44.8% |
| $\lvert\phi_6\rangle$ | 93.2% | 9.0% | 76.1% | 35.9% | 61.5% | 49.4% | 85.5% | 15.4% | 63.8% | 41.2% | 50.4% | 55.2% |



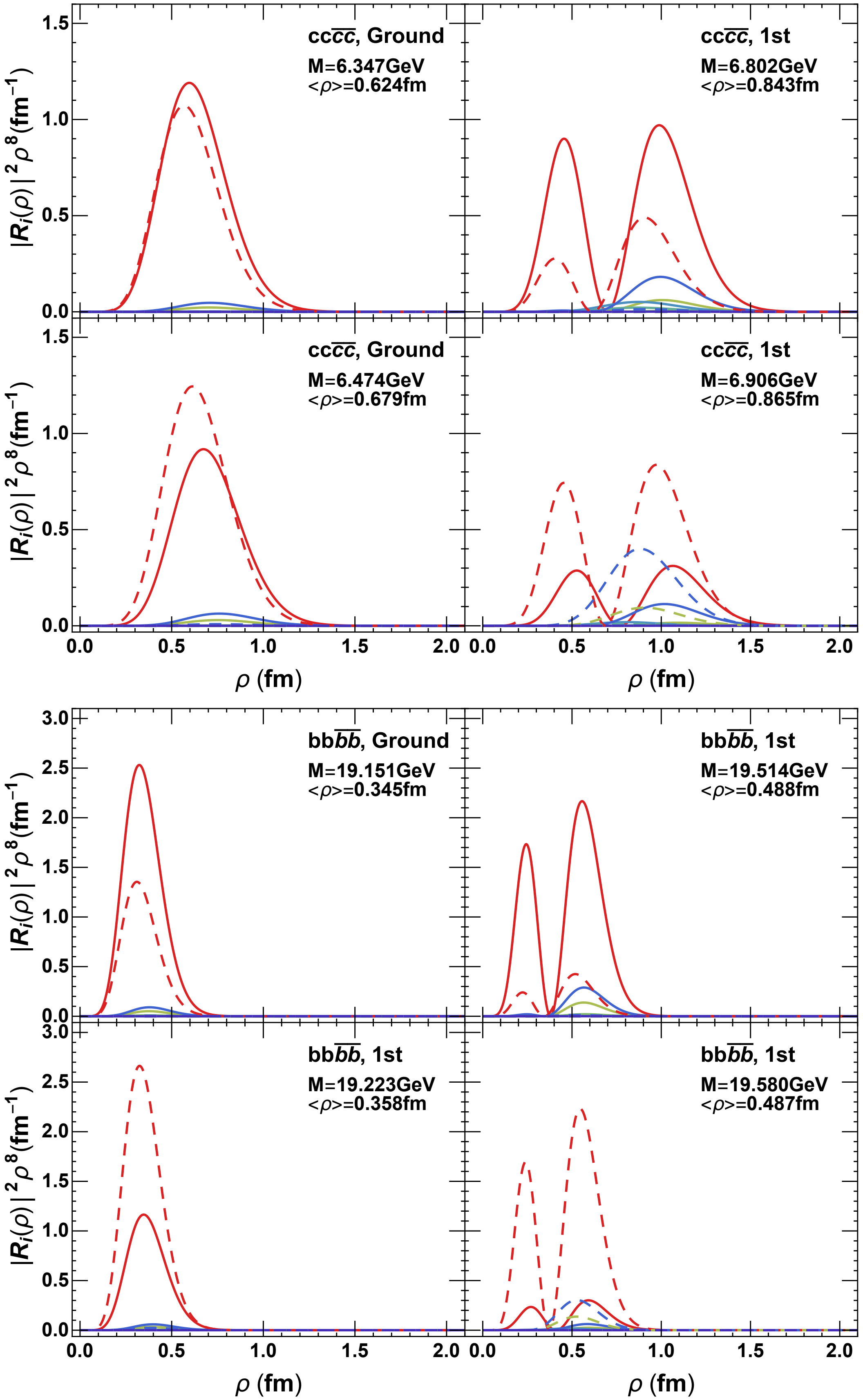
1S

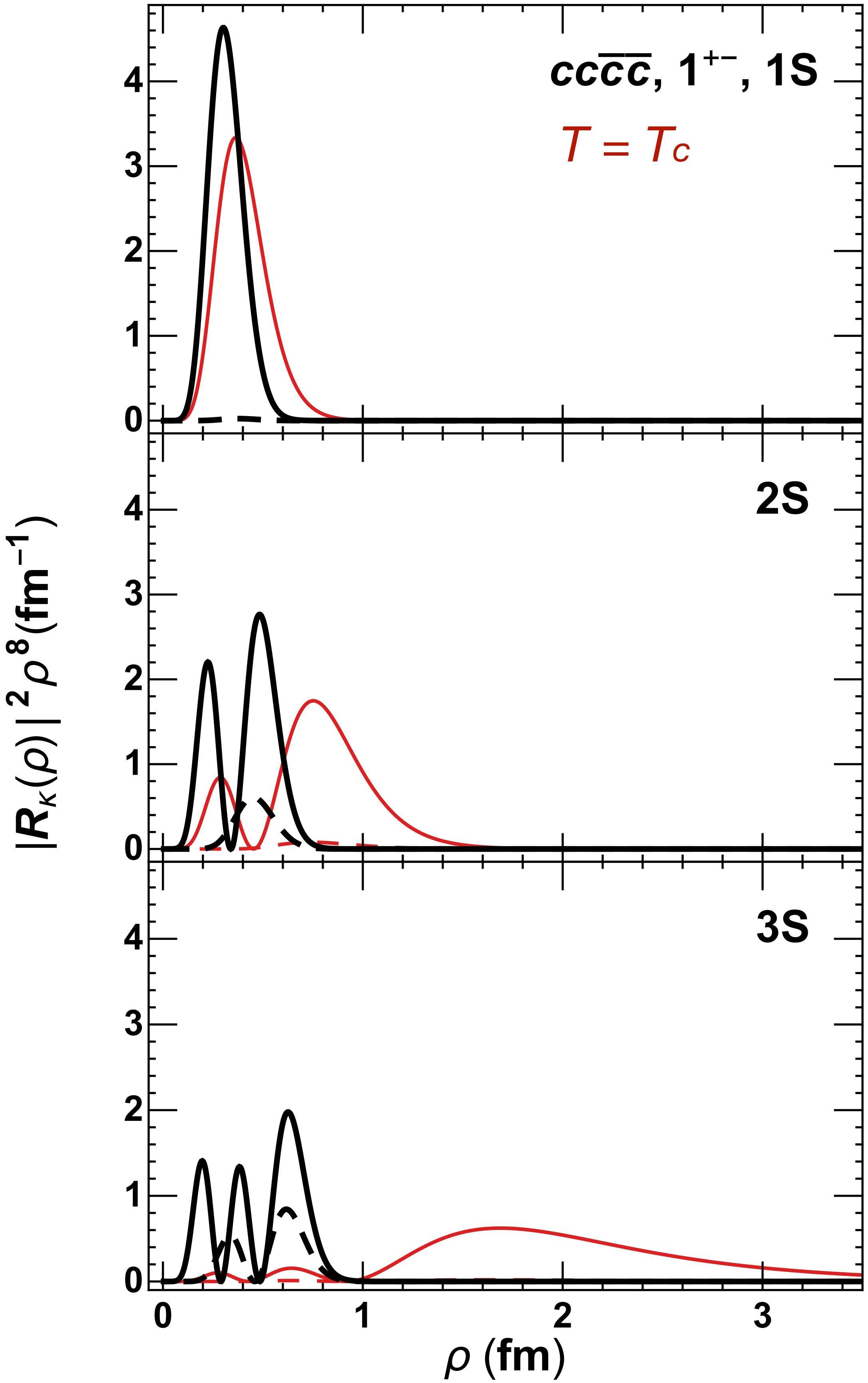


3S

# Results(Four-Body)

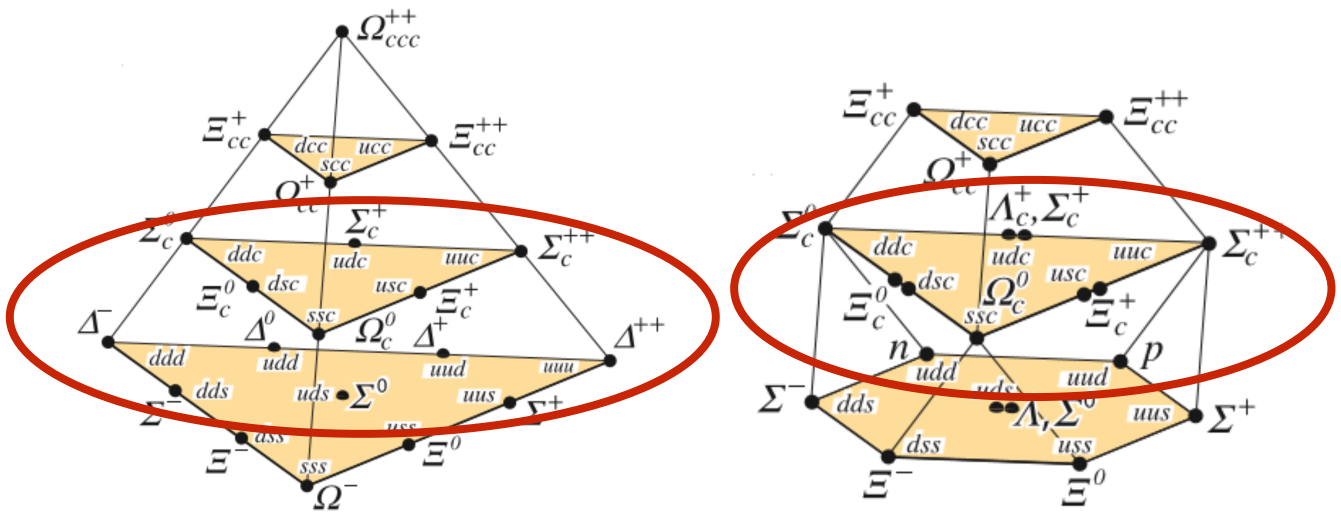
JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319




*Wave function in vacuum and finite-temperature $T_c$ !*

# N-Body Dirac Equation



*Need to include whole relativistic correction: kinematics and spin.*

1. *Schroedinger-like quasipotential equation*

$$\left(\frac{b^2(M)}{2\mu} - \frac{p^2}{2\mu}\right)\Psi(p) = \int \frac{d^3q}{(2\pi)^3} V(p,q;M)\Psi(q) \quad b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}$$

***D. Ebert, Phys. Lett. B635, 93(2006)   D. Ebert, Phys. Rev D66, 014008(2002)***

2. *Bethe-Salpeter equation*

$$G = S_a S_b + S_a S_b K_{ab} G$$

Bound state appear as poles in the Green function

***E. E. Salpeter and H. A. Bethe, Phys. Rev 84, 1232(1951)***

The 3-D truncated BS Equation have been proposed for the relativistic 2-body problem

3. *Two-Body Dirac Equation (TBDE)*

***H. Crater, J. Yoo and C. Wong. PRD 79. 034011(2009)***

Provide a covariant 3-D truncation !

***Covariant Hamiltonian formalism with constraints***

# N-Body Dirac Equation

*Two-body Dirac eq.*

$$\mathcal{S}_1\Psi \equiv \gamma_{51}\left[\gamma_1^{\mu}(p_\mu - A_\mu) + m + S\right]\Psi = 0,$$

$$\mathcal{S}_2\Psi \equiv \gamma_{52}\left[\gamma_2^{\mu}(p_\mu - A_\mu) + m + S\right]\Psi = 0.$$

$\Psi$ *is 16 component wavefunction,*
$A_\mu$ *relativistic four-vector potential,*
$S$ *scalar potential.*

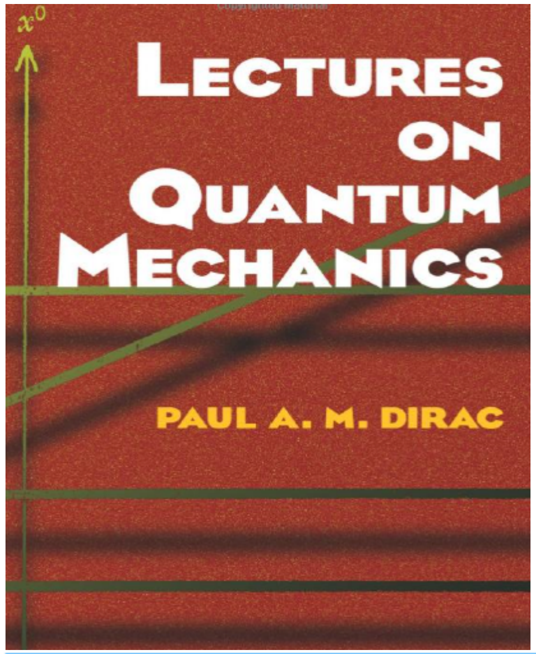
*Taking Pauli reduction and scale transformation in center-of-mass frame, the relative motion can be expressed as a four-component relativistic Schrödinger-like equation:*

**H. Crater, J. Yoo and C. Wong. PRD 79. 034011(2009)**

$$\left[p^2 + \Phi(A(r), S(r), p, P, \omega, \sigma_1, \sigma_2)\right]\psi = b^2\psi$$

$$\begin{aligned}
\Phi_{ij} \;=\;& 2m_{ij}S + S^2 + 2\epsilon_{ij}A - A^2 + \Phi_D + \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\Phi_{SS} \\
+\;& \mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Phi_{SO} + \mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\Phi_{SOD} + i\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\Phi_{SOX} \\
+\;& (\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij})\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Phi_{SOT} + (3(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\Phi_T.
\end{aligned}$$

# N-Body Dirac Equation




CONTENTS



ANNALS OF PHYSICS 148, 57–94 (1983)

## 2BDE & 3BDE by Crater et al.

Two-Body Dirac Equations

HORACE W. CRATER

PHYSICAL REVIEW D 89, 014023 (2014)

**Baryon spectrum analysis using Dirac's covariant constraint dynamics**

Joshua F. Whitney and Horace W. Crater

(Received 10 October 2013; revised manuscript received 16 December 2013; published 30 January 2

We present a relativistic quark model for the baryons that combines three related relativistic formalis
The three-bo
the three pai
state energi
equations of
interactions
quasipotenti
dynamics us
Richardson
and several different algorithms, including a gradient approach, and a Monte Carlo method.

Applications of Two Body Dirac Equations to Hadron and Positronium Spectroscopy arXiv:1403.6466

H. W. Crater, J. Schiermeyer, J. Whitney — The University of Tennessee Space Institute
C. Y. Wong — Oak Ridge National Laboratory

March 27, 2014

*P. A. M. Dirac, Yeshiva University, New York, 1964*
*Sazdjian, J. Math. Phys. 28, 2618(1987)*
*H. Crater, J. Yoo and C. Wong. PRD 79. 034011(2009)*
*S. Shi, X. Guo and Pengfei Zhuang. PRD 88. 014021(2013)*
*Whitney, H. Crater. Phys. Rev. D89, 014023(2014)*
*...*

N-Body Bound State Relativistic Wave Equations

H. SAZDJIAN

*Division de Physique Théorique,* Institut de Physique Nucléaire, Université Paris XI, F-91406, Orsay Cedex, France*

Received July 17, 1988; revised December 21, 1988

The manifestly covariant formalism with constraints is used for the construction of relativistic wave equations to describe the dynamics of $N$ interacting spin 0 and/or spin $\frac{1}{2}$ particles. The total and relative time evolutions of the system are completely determined by means of kinematic-type wave equations. The internal dynamics of the system is $3^{N-1}$-dimensional, in addition to the contribution of the spin degrees of freedom. It is governed by a single dynamical wave equation that determines the eigenvalue of the total mass squared of the system. The interaction is introduced in a closed form by means of two-body potentials. Many-body potentials can also be incorporated. 

# N-Body Dirac Equation

*1. Use the spherical harmonic oscillators basis to expand the relative wf. which not only increase the precision but also can be used to study the excited states.*

With

$$|n_\rho, l_\rho, m_\rho; n_\lambda, l_\lambda, m_\lambda\rangle \equiv \Psi_{n_\rho,l_\rho,m_\rho}(\boldsymbol{\rho})\Psi_{n_\lambda,l_\lambda,m_\lambda}(\boldsymbol{\lambda})$$

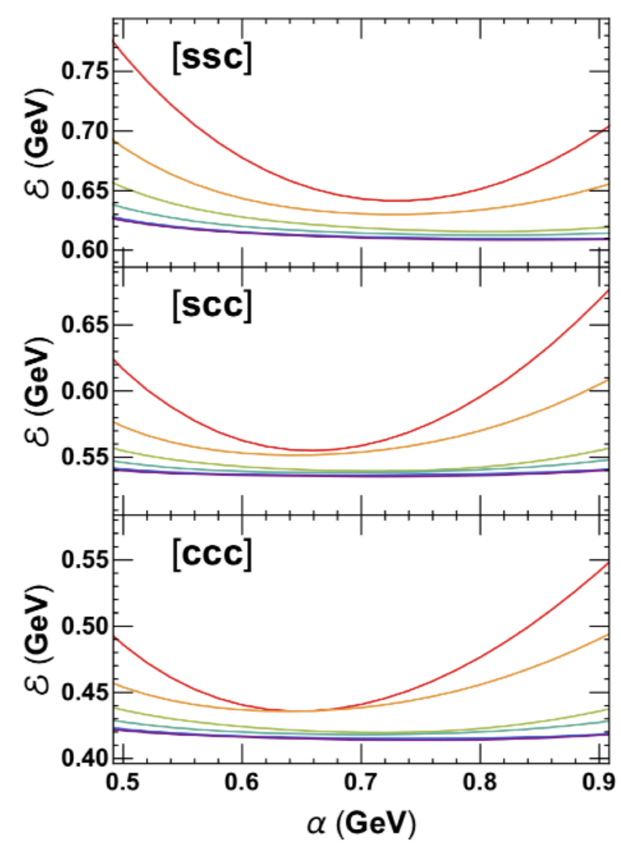
$$\Psi_{n,l,m}(\boldsymbol{r}) \equiv \sqrt{\frac{2\,\Gamma(n+1)}{\Gamma(n+l+3/2)}}\alpha^{l+3/2} r^l e^{-\frac{\alpha^2 r^2}{2}} L_n^{l+1/2}(\alpha^2 r^2) Y_l^m(\theta,\varphi)$$

one obtains the Hamiltonian matrix:

$$\left\langle n'_\rho, l'_\rho, m'_\rho; n'_\lambda, l'_\lambda, m'_\lambda \middle| \widehat{H} \middle| n_\rho, l_\rho, m_\rho; n_\lambda, l_\lambda, m_\lambda \right\rangle$$

then the lowest eigenvalue (ground state) as *E*

minimize energy ***E*** by varying width parameter $\alpha$




*2. Take a universal set of quark mass and coupling parameters for all hadrons!*

$$m_u = m_d = 0.135 \text{ GeV}$$
$$m_s = 0.263 \text{ GeV}$$
$$m_c = 1.400 \text{ GeV}$$
$$m_b = 4.773 \text{ GeV}$$
$$\alpha_{qq} = \alpha_{q\bar{q}}/2.22 = 0.20$$
$$\sigma_{qq} = \sigma_{q\bar{q}}/2.04 = 0.09 \text{ GeV}^2$$

# Results

| Meson | $J^P$ | $M_E$ (GeV) | $M_T$ (GeV) | $D_R$ | $r_{rms}$ (fm) |
|---|---|---|---|---|---|
| $D^0$ | $0^-$ | 1.865 | 1.940 | 4.0% | 0.41 |
| $D^{*0}$ | $1^-$ | 2.007 | 2.066 | 3.0% | 0.47 |
| $D^+$ | $0^-$ | 1.870 | 1.940 | 3.8% | 0.41 |
| $D^{*+}$ | $1^-$ | 2.010 | 2.066 | 2.8% | 0.47 |
| $D_s$ | $0^-$ | 1.968 | 2.028 | 3.1% | 0.40 |
| $D_s^*$ | $1^-$ | 2.112 | 2.157 | 2.1% | 0.45 |
| $\eta_c$ | $0^-$ | 2.984 | 2.990 | 0.2% | 0.32 |
| $\eta_c(2S)$ | $0^-$ | 3.637 | 3.609 | −0.8% | 0.63 |
| $h_{c1}$ | $1^+$ | 3.525 | 3.506 | −0.5% | 0.54 |
| $J/\psi$ | $1^-$ | 3.097 | 3.123 | 0.8% | 0.37 |
| $\psi(2S)$ | $1^-$ | 3.686 | 3.701 | 0.4% | 0.68 |
| $\chi_{c0}$ | $0^+$ | 3.415 | 3.442 | 0.8% | 0.48 |
| $\chi_{c1}$ | $1^+$ | 3.511 | 3.504 | −0.2% | 0.53 |
| $\chi_{c2}$ | $2^+$ | 3.556 | 3.519 | −1.0% | 0.56 |
| $B^-$ | $0^-$ | 5.279 | 5.326 | 0.5% | 0.43 |
| $B^{-*}$ | $1^-$ | 5.325 | 5.371 | 0.9% | 0.46 |
| $B^0$ | $0^-$ | 5.280 | 5.326 | 0.9% | 0.43 |
| $B^{0*}$ | $1^-$ | 5.325 | 5.371 | 0.9% | 0.46 |
| $B_s$ | $0^-$ | 5.367 | 5.408 | 0.8% | 0.41 |
| $B_s^*$ | $1^-$ | 5.415 | 5.458 | 0.8% | 0.44 |
| $\eta_b$ | $0^-$ | 9.399 | 9.378 | −0.2% | 0.18 |
| $\eta_b(2S)$ | $0^-$ | 9.999 | 9.964 | −0.3% | 0.44 |
| $h_{b1}$ | $1^+$ | 9.899 | 9.918 | 0.2% | 0.38 |
| $\Upsilon(1S)$ | $1^-$ | 9.460 | 9.507 | 0.5% | 0.22 |
| $\Upsilon(2S)$ | $1^-$ | 10.023 | 10.025 | 0.0% | 0.47 |
| $\chi_{b0}$ | $0^+$ | 9.859 | 9.878 | 0.2% | 0.35 |
| $\chi_{b1}$ | $1^+$ | 9.893 | 9.912 | 0.2% | 0.37 |
| $\chi_{b2}$ | $2^+$ | 9.912 | 9.929 | 0.2% | 0.38 |

| Baryon | $J^P$ | $M_E$ (GeV) | $M_T$ (GeV) | $D_R$ |
|---|---|---|---|---|
| $\Lambda_c^+$ | $(1/2)^+$ | 2.286 | 2.440 | 6.8% |
| $\Sigma_c^{++}$ | $(1/2)^+$ | 2.454 | 2.413 | −1.6% |
| $\Sigma_c^+$ | $(1/2)^+$ | 2.453 | 2.413 | −1.5% |
| $\Sigma_c^0$ | $(1/2)^+$ | 2.454 | 2.413 | −1.6% |
| $\Xi_c^+$ | $(1/2)^+$ | 2.468 | 2.557 | 3.6% |
| $\Xi_c^0$ | $(1/2)^+$ | 2.471 | 2.557 | 3.5% |
| $\Xi_c^{\prime +}$ | $(1/2)^+$ | 2.577 | 2.566 | −0.4% |
| $\Xi_c^{\prime 0}$ | $(1/2)^+$ | 2.579 | 2.566 | −0.5% |
| $\Omega_c^0$ | $(1/2)^+$ | 2.695 | 2.681 | −0.5% |
| $\Xi_{cc}^{++}$ | $(1/2)^+$ | 3.621 | 3.632 | 0.3% |
| $\Xi_{cc}^+$ | $(1/2)^+$ | 3.619 | 3.632 | 0.4% |
| $\Omega_{cc}^+$ | $(1/2)^+$ |  | 3.745 |  |
| $\Sigma_c^{*++}$ | $(3/2)^+$ | 2.518 | 2.429 | −3.6% |
| $\Sigma_c^{*+}$ | $(3/2)^+$ | 2.518 | 2.429 | −3.6% |
| $\Sigma_c^{*0}$ | $(3/2)^+$ | 2.518 | 2.429 | −3.6% |
| $\Xi_c^{*+}$ | $(3/2)^+$ | 2.646 | 2.567 | −3.0% |
| $\Xi_c^{*0}$ | $(3/2)^+$ | 2.646 | 2.567 | −3.0% |
| $\Omega_c^{*0}$ | $(3/2)^+$ | 2.766 | 2.689 | −2.8% |
| $\Xi_{cc}^{*++}$ | $(3/2)^+$ |  | 3.644 |  |
| $\Xi_{cc}^{*+}$ | $(3/2)^+$ |  | 3.644 |  |
| $\Omega_{cc}^{*+}$ | $(3/2)^+$ |  | 3.754 |  |
| $\Omega_{ccc}^{++}$ | $(3/2)^+$ |  | 4.784 |  |
| $\Lambda_b^0$ | $(1/2)^+$ | 5.620 | 5.793 | 3.1% |
| $\Sigma_b^+$ | $(1/2)^+$ | 5.811 | 5.769 | −0.7% |
| $\Sigma_b^0$ | $(1/2)^+$ |  | 5.769 |  |
| $\Sigma_b^-$ | $(1/2)^+$ | 5.816 | 5.769 | −0.8% |
| $\Xi_b^0$ | $(1/2)^+$ | 5.792 | 5.913 | 2.1% |
| $\Xi_b^-$ | $(1/2)^+$ | 5.795 | 5.913 | 2.0% |
| $\Xi_b^{\prime 0}$ | $(1/2)^+$ | 5.792 | 5.903 | 1.9% |
| $\Xi_b^{\prime -}$ | $(1/2)^+$ | 5.795 | 5.903 | 1.9% |
| $\Omega_b^-$ | $(1/2)^+$ | 6.046 | 6.021 | −0.4% |
| $\Xi_{bb}^-$ | $(1/2)^+$ |  | 10.210 |  |
| $\Xi_{bb}^0$ | $(1/2)^+$ |  | 10.210 |  |
| $\Omega_{bb}^-$ | $(1/2)^+$ |  | 10.319 |  |
| $\Sigma_b^{*+}$ | $(3/2)^+$ | 5.832 | 5.781 | −0.9% |
| $\Sigma_b^{*0}$ | $(3/2)^+$ |  | 5.7 81 |  |
| $\Sigma_b^{*-}$ | $(3/2)^+$ | 5.835 | 5.781 | −0.9% |
| $\Xi_b^{*0}$ | $(3/2)^+$ |  | 5.915 |  |
| $\Xi_b^{*-}$ | $(3/2)^+$ |  | 5.915 |  |
| $\Omega_b^{*-}$ | $(3/2)^+$ |  | 6.033 |  |
| $\Xi_{bb}^{*+}$ | $(3/2)^+$ |  | 10.221 |  |
| $\Xi_{bb}^{*0}$ | $(3/2)^+$ |  | 10.221 |  |
| $\Omega_{bb}^{*-}$ | $(3/2)^+$ |  | 10.331 |  |
| $\Omega_{bbb}^-$ | $(3/2)^+$ |  | 14.499 |  |

|  | Experiment |  | Model |  |  |
|---|---|---|---|---|---|
| Baryon | $J^P$ | $M_E$ (GeV) | $J^P$ | $M_T$ (GeV) | $D_R$ |
| $\Omega_c^0$ | $(1/2)^+$ | 2.695 | $(1/2)^+(1S)$ | 2.681 | −0.5% |
| $\Omega_c^*(2770)^0$ | $(3/2)^+$ | 2.766 | $(3/2)^+(1S)$ | 2.689 | −2.8% |
| $\Omega_c(3000)^0$ |  | 3.000 | $(1/2)^-(1P)$ | 2.990 | −0.3% |
| $\Omega_c(3050)^0$ | ? | 3.050 | $(3/2)^-(1P)$ | 3.052 | 0.1% |
| $\Omega_c(3065)^0$ |  | 3.065 | $(1/2)^-(1P)$ | 3.074 | 0.3% |
| $\Omega_c(3090)^0$ |  | 3.090 | $(3/2)^-(1P)$ | 3.085 | −0.2% |
| $\Omega_c(3120)^0$ |  | 3.119 | $(5/2)^-(1P)$ | 3.252 | 4.3% |

| Baryon | $r_{rms}$ | $\langle r_{12}^2\rangle^{1/2}$ | $\langle r_{13}^2\rangle^{1/2}$ | $\langle r_{23}^2\rangle^{1/2}$ |
|---|---|---|---|---|
| $\Lambda_c^+, \Sigma_c, \Sigma_c^*$ | 0.30 | 0.59 | 0.56 | 0.56 |
| $\Xi_c, \Xi_c^*$ | 0.30 | 0.58 | 0.56 | 0.54 |
| $\Omega_c^0, \Omega_c^{*0}$ | 0.3 | 0.57 | 0.54 | 0.54 |
| $\Xi_{cc}, \Xi_{cc}^*$ | 0.29 | 0.56 | 0.56 | 0.46 |
| $\Omega_{cc}^{++}, \Omega_{cc}^{*++}$ | 0.28 | 0.54 | 0.54 | 0.46 |
| $\Omega_{ccc}^{++}$ | 0.25 | 0.44 | 0.44 | 0.44 |

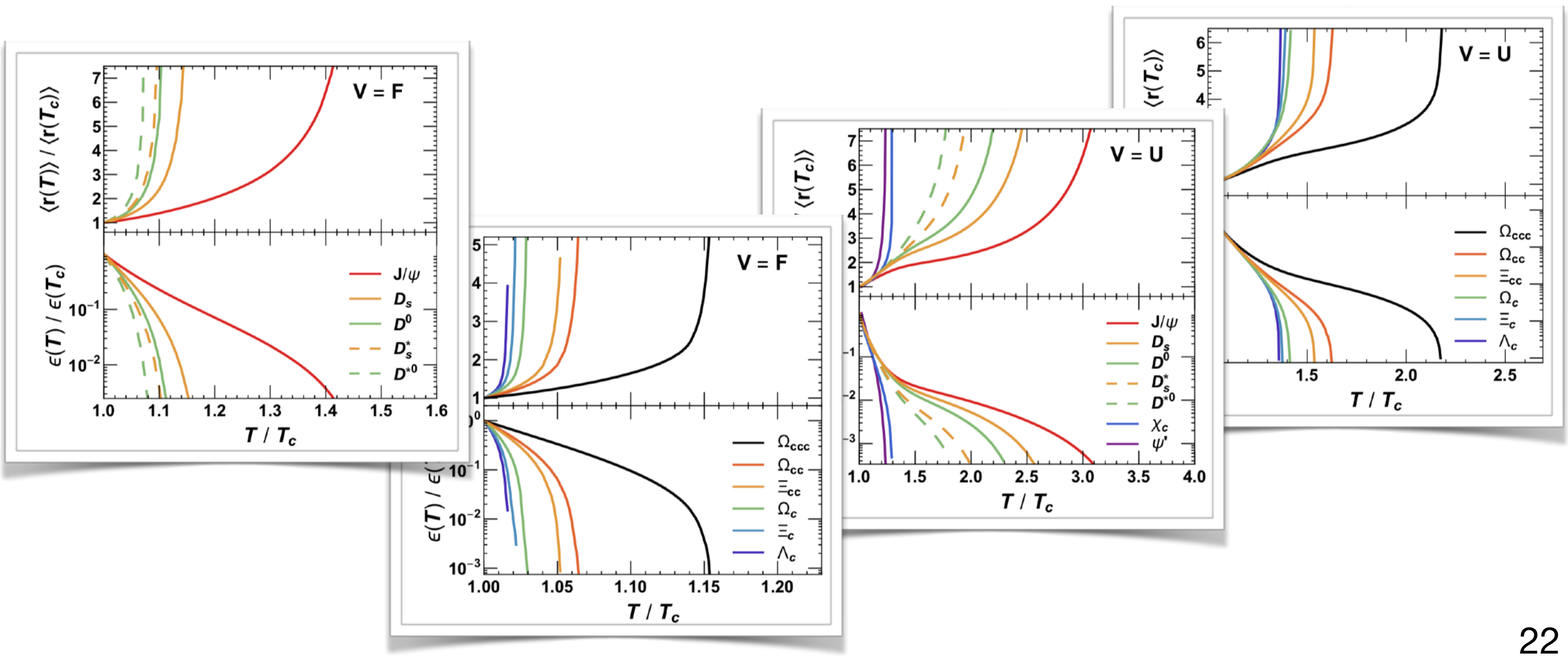
# Results

## 3. We extend to finite temperature case firstly! (important to HIC)

| States | $J/\psi$ | $\chi_c$ | $\psi'$ | $D_s$ | $D_s^*$ | $D^0$ | $D^{*0}$ |
|---|---|---|---|---|---|---|---|
| $T_d/T_c(V=F)$ | 1.42 | <1.0 | <1.0 | 1.14 | 1.10 | 1.10 | 1.08 |
| $T_d/T_c(V=U)$ | 3.09 | 1.30 | 1.24 | 2.50 | 1.98 | 2.35 | 1.80 |
| States | $\Omega_{ccc}$ | $\Omega_{cc}$ | $\Xi_{cc}$ | $\Omega_c$ | $\Xi_c$ | $\Lambda_c$ | |
| $T_d/T_c(V=F)$ | 1.15 | 1.06 | 1.05 | 1.03 | 1.02 | 1.02 | |
| $T_d/T_c(V=U)$ | 2.18 | 1.63 | 1.54 | 1.41 | 1.39 | 1.37 | |

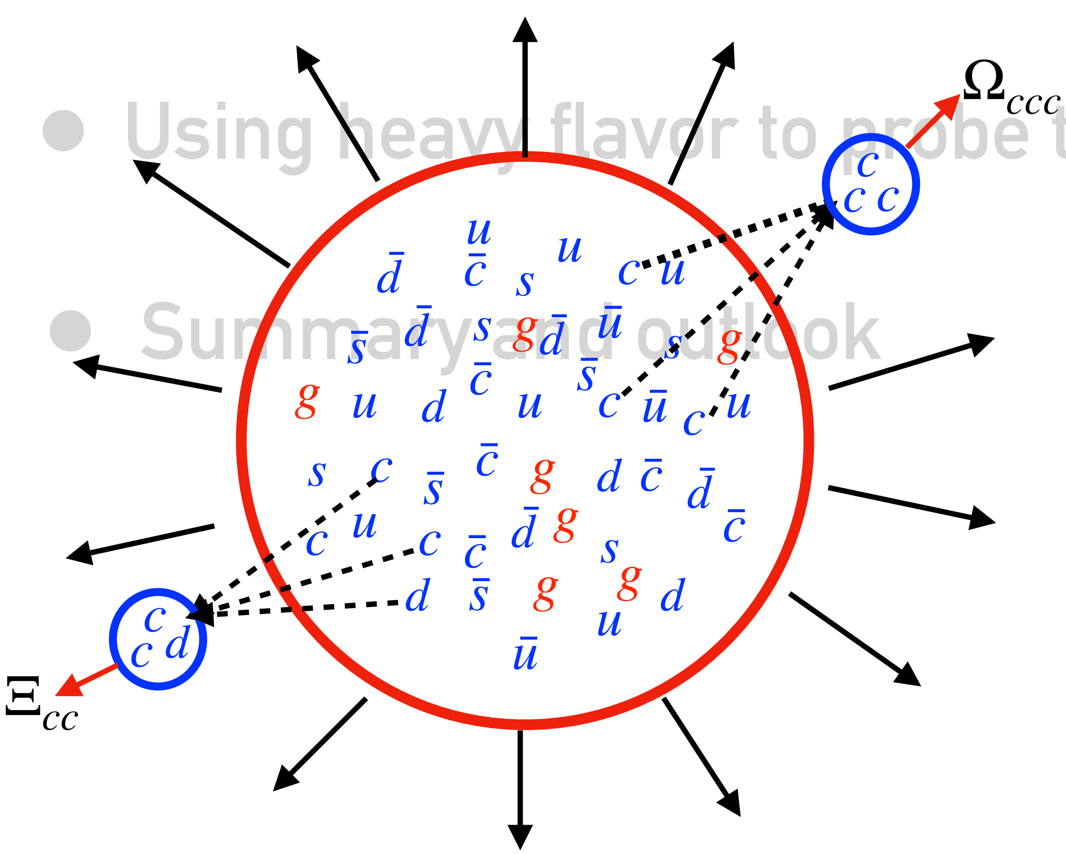
*Binding energy($T_d$) ->0*

*Average size($T_d$) ->infinity*



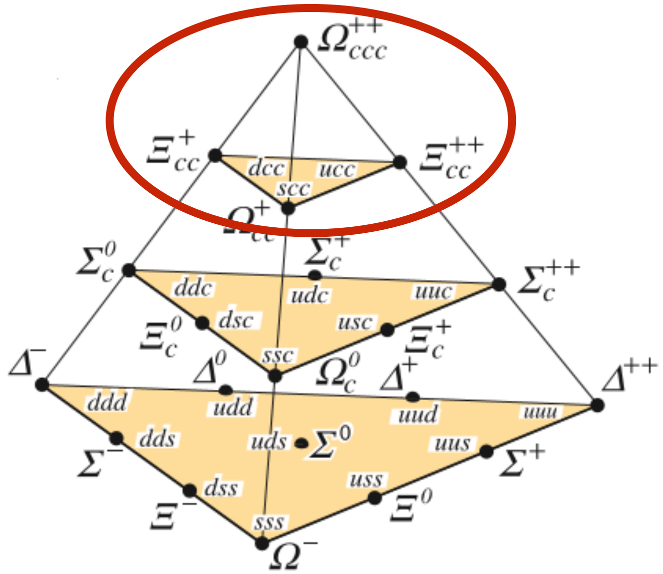
# Outline

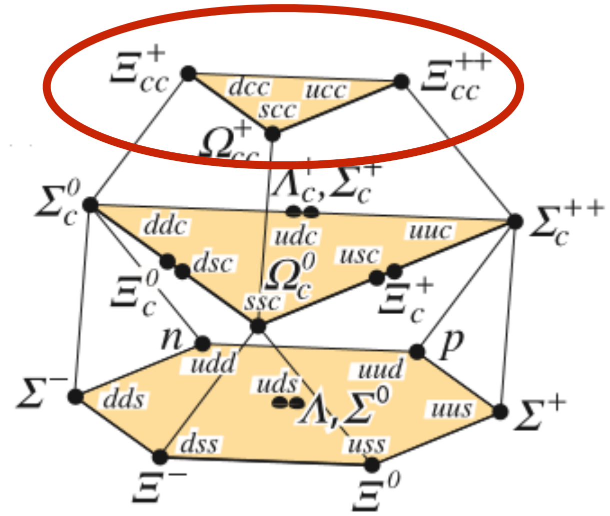
- Introduction to heavy-ion collisions and heavy flavor hadrons
- Static properties of heavy flavor hadrons in vacuum and finite-temp.
- **Dynamic production of heavy flavor hadrons in heavy-ion collisions**
- Using heavy flavor to probe the Hadronization M. and QGP
- Summary and outlook




*Multi-charmed baryons*
*Fully heavy tetraquark states*

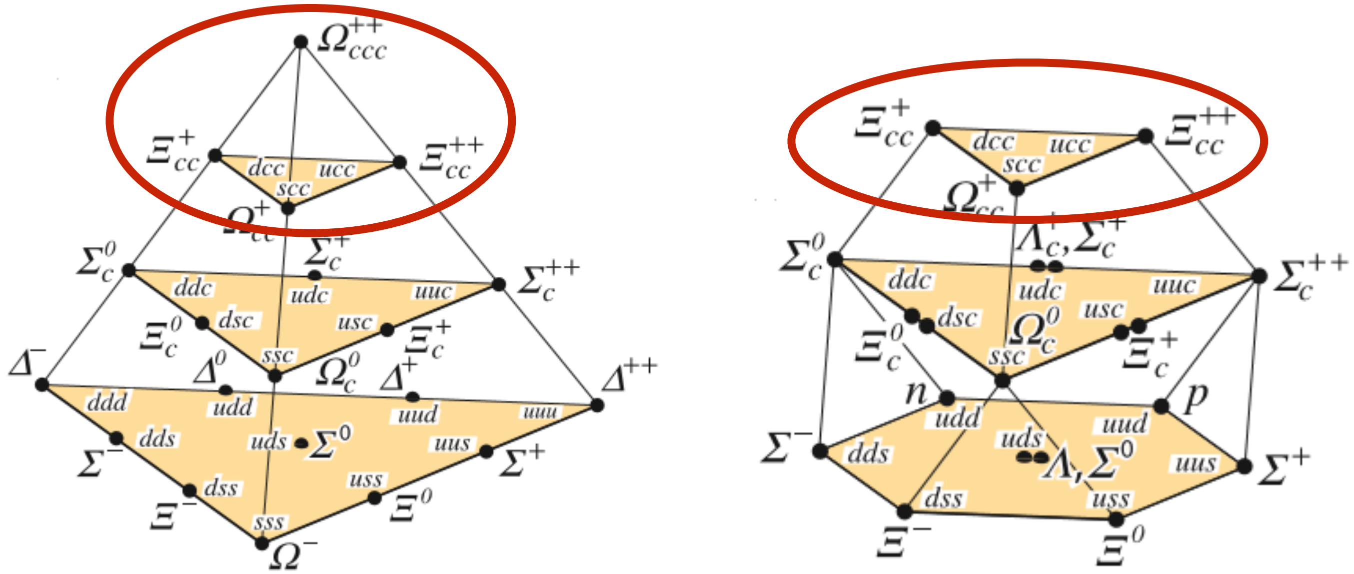
# Multi-charmed baryons




*The flavor SU(4) quark model predicts 22 charmed baryons, but some of them are not yet discovered !*

# Multi-charmed baryons




*The flavor SU(4) quark model predicts 22 charmed baryons, but some of them are not yet discovered !*

*The experimental search for $\Xi_{cc}^{+}$ lasts for decades.*

*SELEX Collaboration, a fixed-target charm hadroproduction experiment at Fermilab, claimed the first observation in 2002*

*FOCUS, BaBar, Belle, LHCb Collaboration*

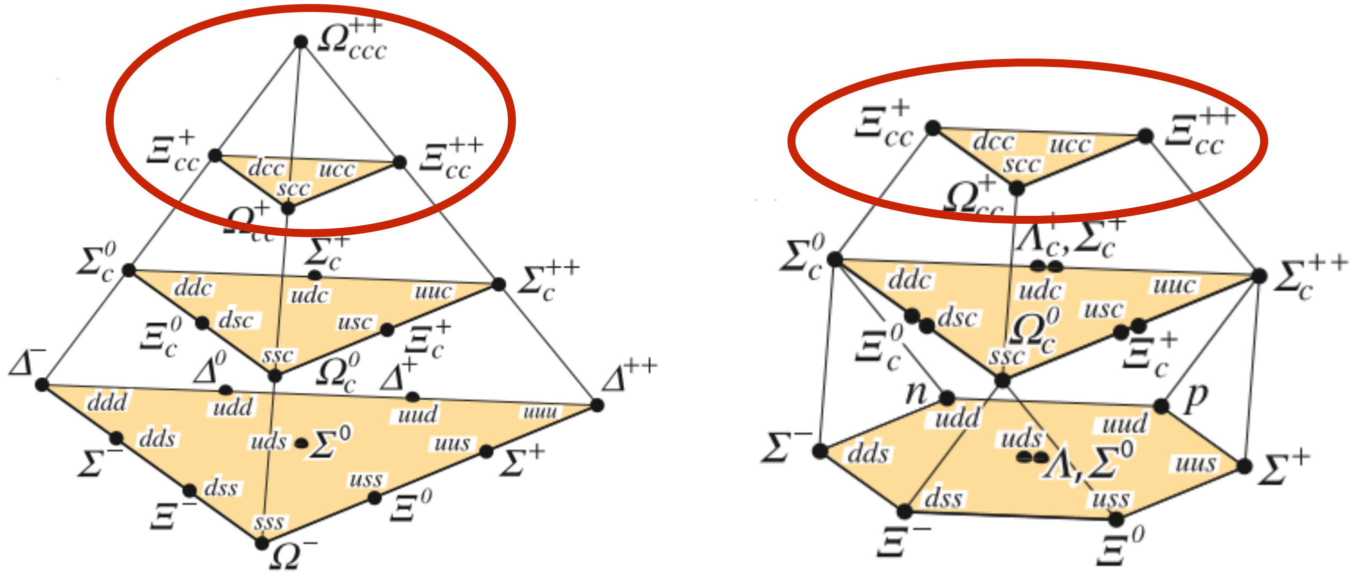

***SELEX Collaboration, Phys. Rev. Lett. 89, 112001(2002).***

***FOCUS Collaboration, Nucl. Phys. Proc. Suppl. 115, 33-36(2003).***

***BaBar Collaboration, Phys. Rev. D. 74, 011103(2006).***

***Belle Collaboration, Phys. Rev. Lett. 97, 162001(2006).***

# Multi-charmed baryons



The flavor SU(4) quark model predicts 22 charmed baryons, but some of them are not yet discovered !

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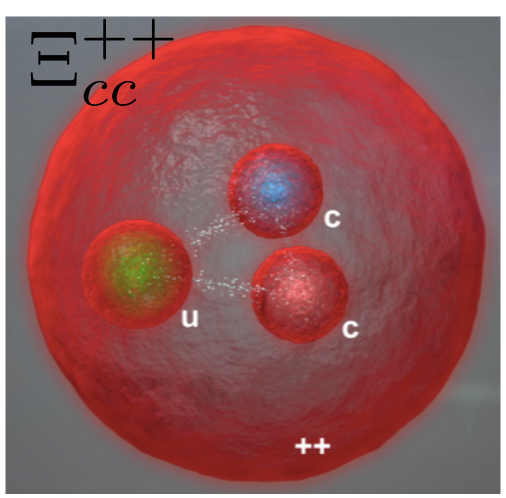
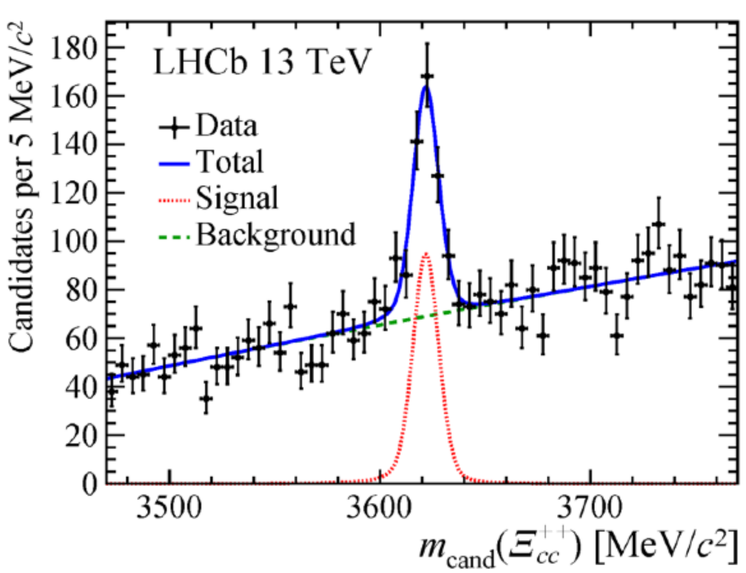
*FOCUS, BaBar, Belle, LHCb Collaboration*

*2017, LHCb results show clear structure at 3620MeV!*





**SELEX Collaboration, Phys. Rev. Lett. 89, 112001(2002).**

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**BaBar Collaboration, Phys. Rev. D. 74, 011103(2006).**

**Belle Collaboration, Phys. Rev. Lett. 97, 162001(2006).**

**LHCb Collaboration, Phys. Rev. Lett. 119, 112001(2017).**

# Multi-charmed baryons

*Need at least two pairs of charm quarks in an event !*

*Elementary particle collisions*

$e^+ + e^- \quad p + p$

- *Rare production of charm quarks*
- *Small cross section from fragmentation*

# Multi-charmed baryons

*Need at least two pairs of charm quarks in an event !*

## Elementary particle collisions

$e^+ + e^-$  $p + p$

- *Rare production of charm quarks*
- *Small cross section from fragmentation*

## Heavy ion collisions

$Au + Au$  $Pb + Pb$

- *Plenty of off-diagonal charm quarks created in heavy ion collisions !*
- *Statistical production(color recombination) in QGP !*

*Their production in high energy nuclear collisions may be largely enhanced !*

Searching for $\Xi_{cc}^+$ in relativistic heavy ion collisions

CrossMark

Jiaxing Zhao, Hang He, Pengfei Zhuang *

Physics Department, Tsinghua University and Collaborative Innovation Center of Quantum Matter, Beijing 100084, China

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Received 13 December 2016
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Editor: W. Haxton

A B S T R A C T

We study the doubly charmed baryon $\Xi_{cc}^+$ structure and production in high energy nuclear collisions. By solving the three-quark Schrödinger equation including relativistic correction and calculating the yield via coalescence mechanism, we find that, the $\Xi_{cc}^+$ created in nuclear collisions is in the quark–diquark state as a consequence of chiral symmetry restoration in hot medium, and the production is extremely enhanced due to the large number of charm quarks.

# Coalescence Mechanism

Shanshan Cao's talk in 123nd HENPIC & Jun Song's talk in 124nd HENPIC

$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C \int \frac{P^\mu d\sigma_\mu(R)}{(2\pi)^3} \frac{d^4 r_x d^4 r_y d^4 p_x d^4 p_y}{(2\pi)^6} F(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{p}_1, \tilde{p}_2, \tilde{p}_3) W(r_x, r_y, p_x, p_y).$$

V. Greco, C. M. Ko and R. Rapp, Phys. Lett. B 595, 202 (2004).
D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).
R. J. Fries, B. Muller, C. Nonaka and S. A. Bass. Phys. Rev. C68, 044902(2004).

- *The hadronization hypersurface is determined by hydrodynamics.*

$$\partial_\mu T^{\mu v} = 0 \qquad T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$T(\mathbf{x}_T, \tau) = T_c$$

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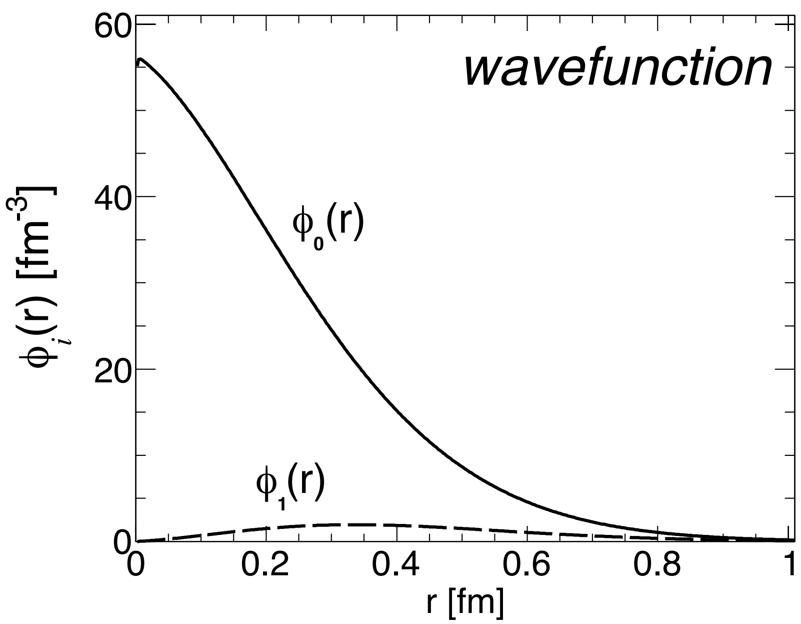


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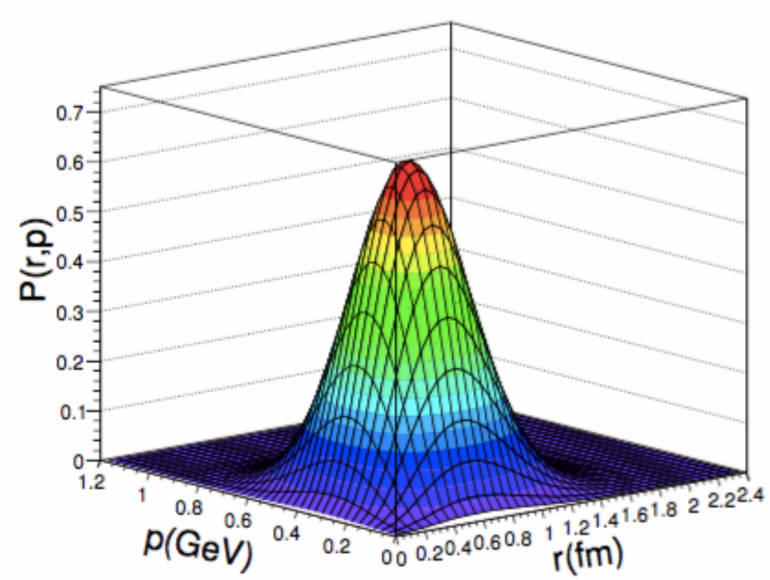
$$T(\mathbf{x}_T, \tau) = T_c$$

- *The Wigner function can self-consistently be determined by the wavefunction. Instead of taking a Gaussian distribution with the width as a free parameter.*

$$W(r,p) = \int d^4y e^{-ipy}\psi(r+\frac{y}{2})\psi(r-\frac{y}{2})$$



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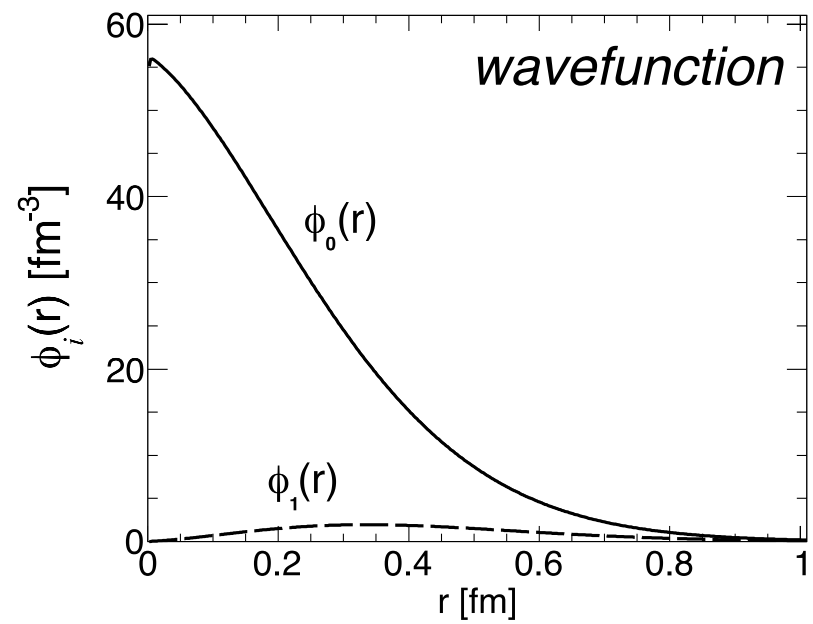


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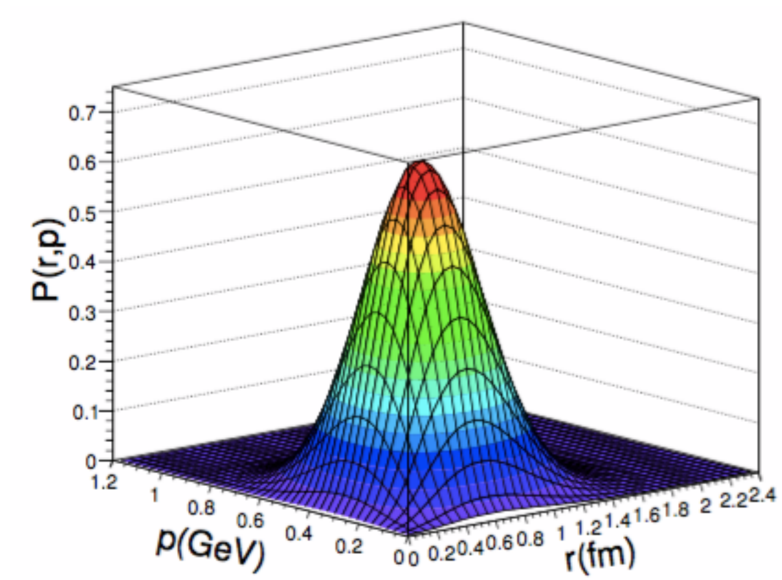
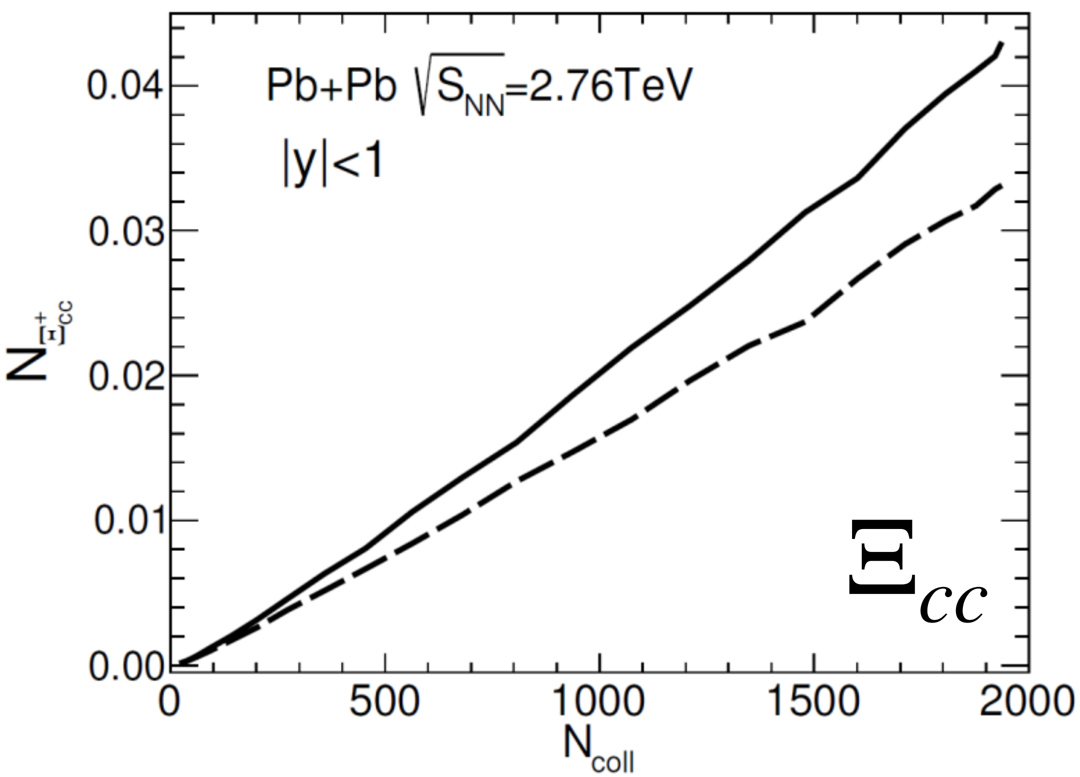
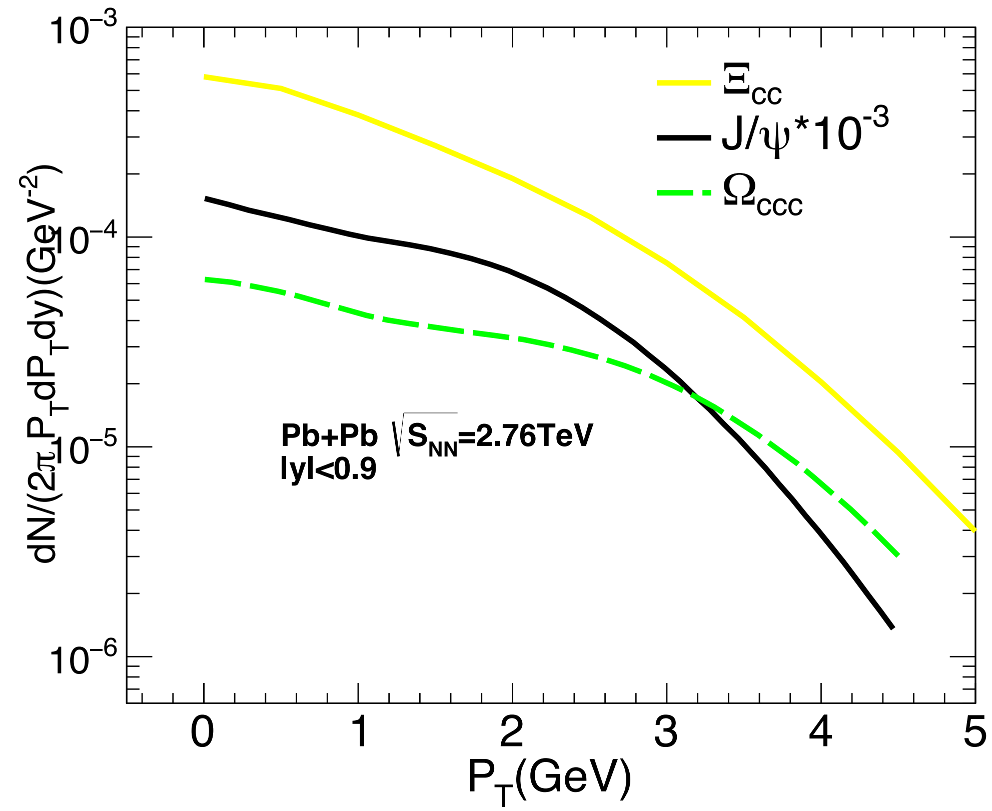



- Light quark: thermal and chemical equilibrium

  Charm quark: thermal and non-chemical equilibrium(approximation)

# Results




*JX Zhao, Hang He and Pengfei Zhuang, Phys. Lett. B771,349(2017).*

$$\sigma_{AA}^{eff} \equiv \frac{N_{AA}}{N_{coll}\Delta\eta}\sigma_{pp}^{inel}$$

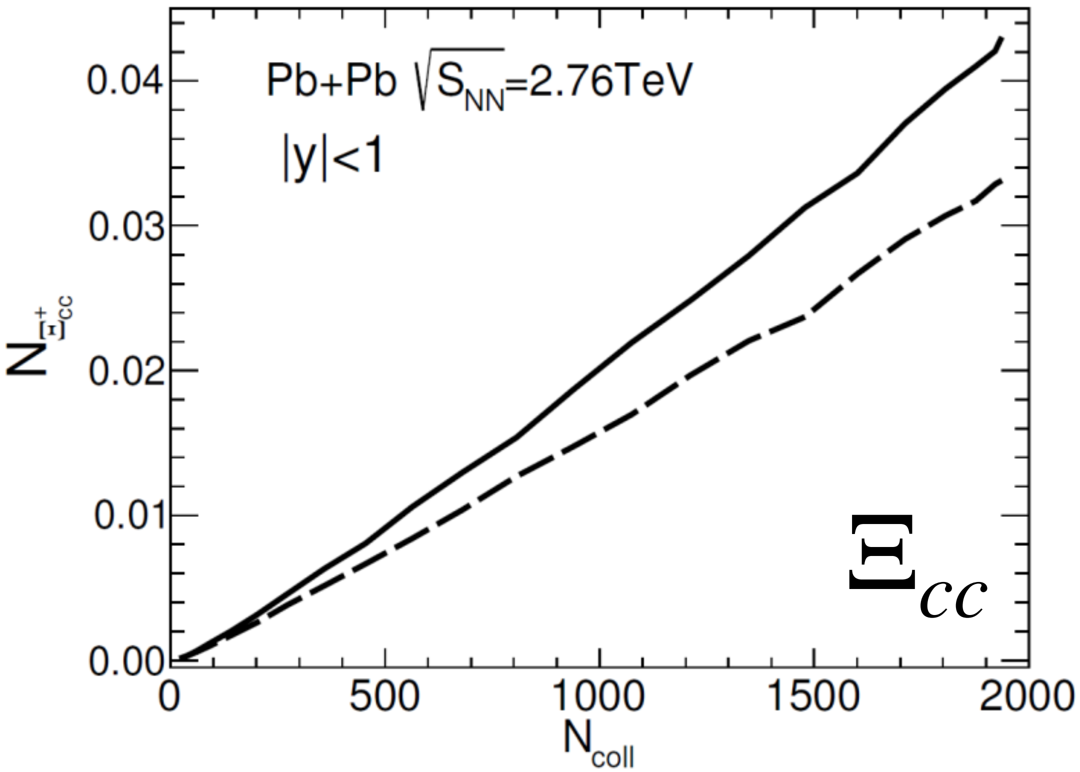
*Inelastic pp cross-section* ($\sigma_{pp}^{inel}$)

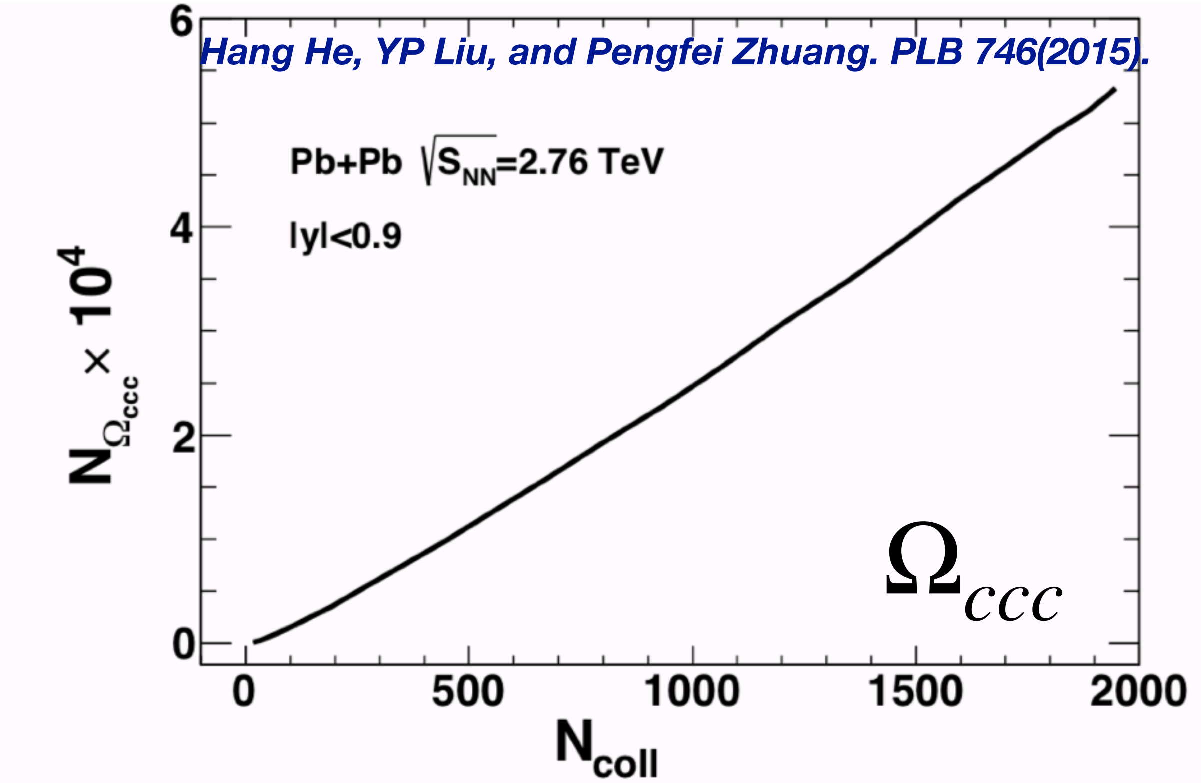
*Number of collisions in AA* ($N_{coll}$)

$\sigma_{AA}^{eff}(\Xi_{cc}) = 412nb$ *is much larger than* $\sigma_{pp}(\Xi_{cc}) = 61nb$

*in PbPb at 2.76TeV* — *in pp at 14TeV*

# Results




$$\frac{\sigma_{AA}^{eff}(\Omega_{ccc})}{\sigma_{pp}(\Omega_{ccc})} : \frac{\sigma_{AA}^{eff}(\Xi_{cc})}{\sigma_{pp}(\Xi_{cc})} : \frac{\sigma_{AA}^{eff}(J/\psi)}{\sigma_{pp}(J/\psi)} \approx 10^2 : 10^1 : 10^0.$$

*JX Zhao and Pengfei Zhuang, Few Body Syst. 58, 100(2017).*

*Due to the combination of uncorrelated charm quarks in the hot medium, the multi-charmed baryon yield are largely enhanced in comparison with p+p collisions!*

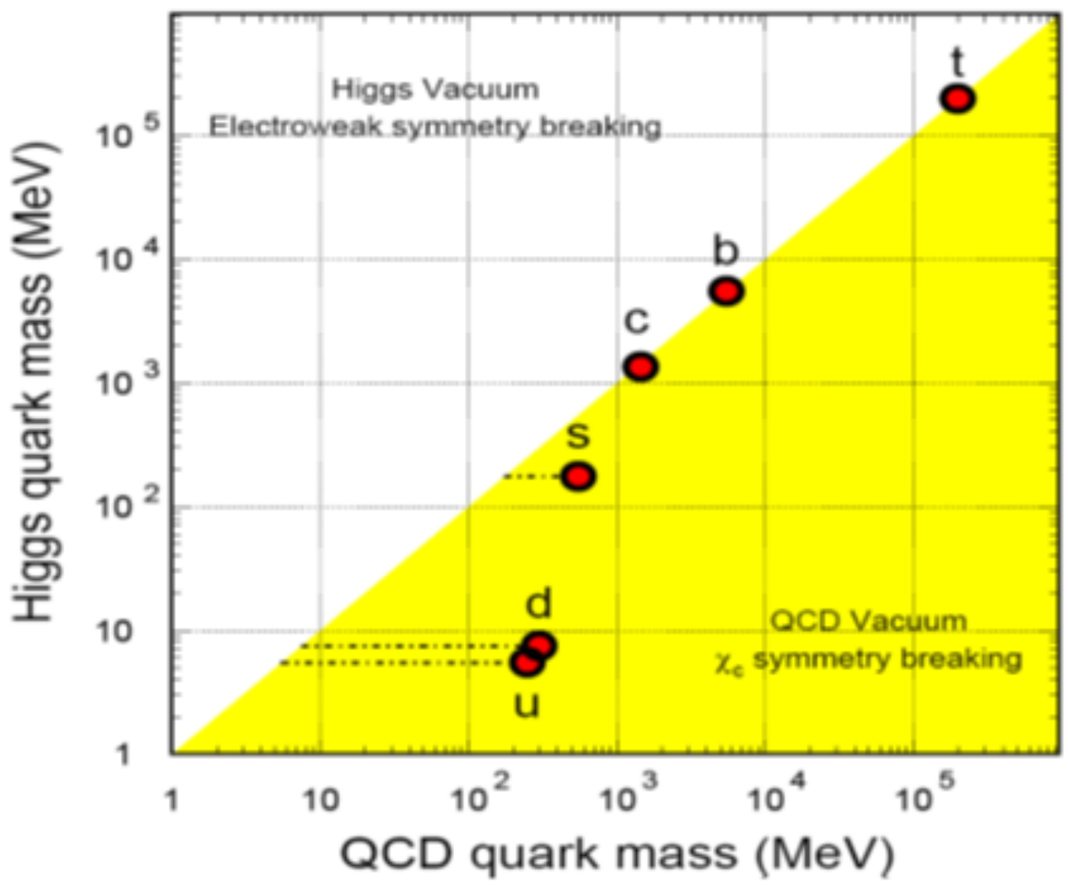
**A next-generation LHC heavy-ion experiment**

D. Adamová (Rez, Nucl. Phys. Inst.), G. Aglieri Rinella (CERN), M. Agnello (INFN, Turin & Turin Polytechnic), Z. Ahammed (IISER, Kolkata), D. Aleksandrov (Kurchatov Inst., Moscow), A. Alici (Enrico Fermi Ctr., Rome & INFN, Bologna), A. Alkin (BITP, Kiev), T. Alt (Frankfurt U., Inst. Kernphys.), I. Altsybeev (St. Petersburg State U.), D. Andreou (CERN) *et al.* Show all 399 authors

Jan 31, 2019 - 21 pages

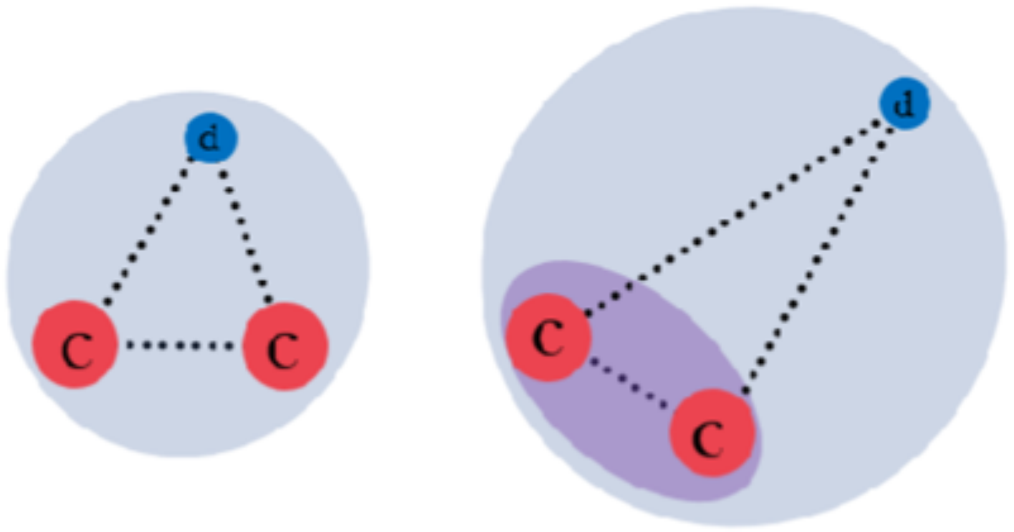
e-Print: arXiv:1902.01211 [physics.ins-det] | PDF

# Results




*the coalescence happens at the hadronization hypersurface where the broken chiral symmetry is restored, and the light quark mass drops down significantly:*

$$m_c(T_c) \simeq m_c(0),$$
$$m_q(T_c) \simeq 100\ \text{MeV} < m_q(0) \simeq 300\ \text{MeV}$$

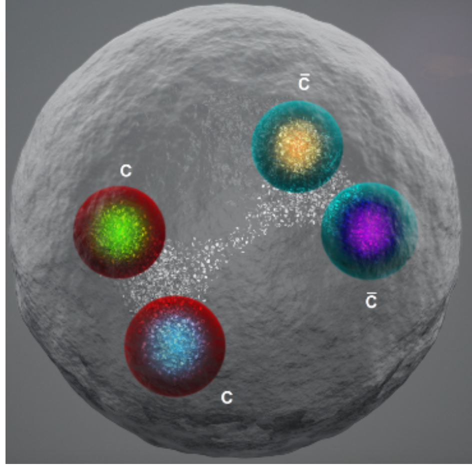



| Average $r_i$(fm) | Model 1 ($m_d$=0.3GeV) | Model 2 ($m_d$=0.1GeV) |
|---|---|---|
| $< r_{cc} >$ | 0.4437 | 0.4620 |
| $< r_{(cc)-q} >$ | 0.5294 | 0.9662 |

*JX Zhao, Hang He and Pengfei Zhuang, Phys. Lett. B771,349(2017).*

$\Xi_{cc}^{+}$ *produced in heavy-ion collisions is more like a quark-diquark state (cc-q)!*

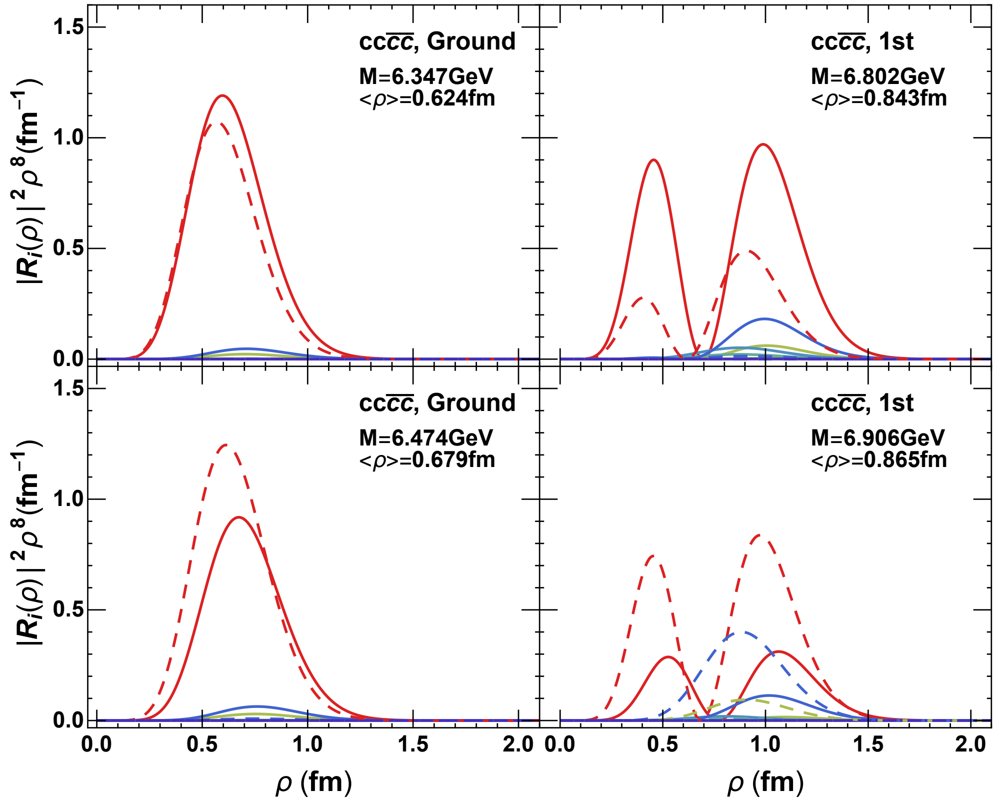
# Fully-heavy tetraquark




*Fully-heavy tetraquark state production in heavy-ion collisions!*

*Base on Coalescence Mechanism*

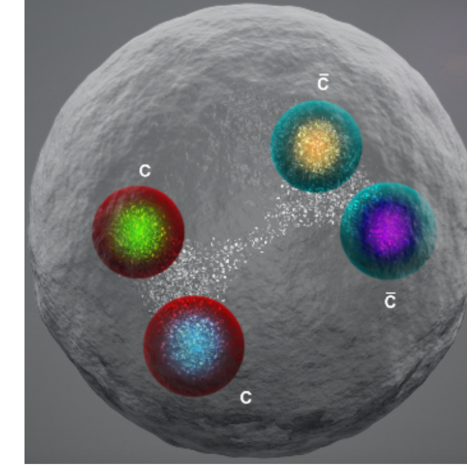
*Coalescence probability = Wigner function*




***JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319***

# Results

*Fully-heavy tetraquark state production in heavy-ion collisions!*

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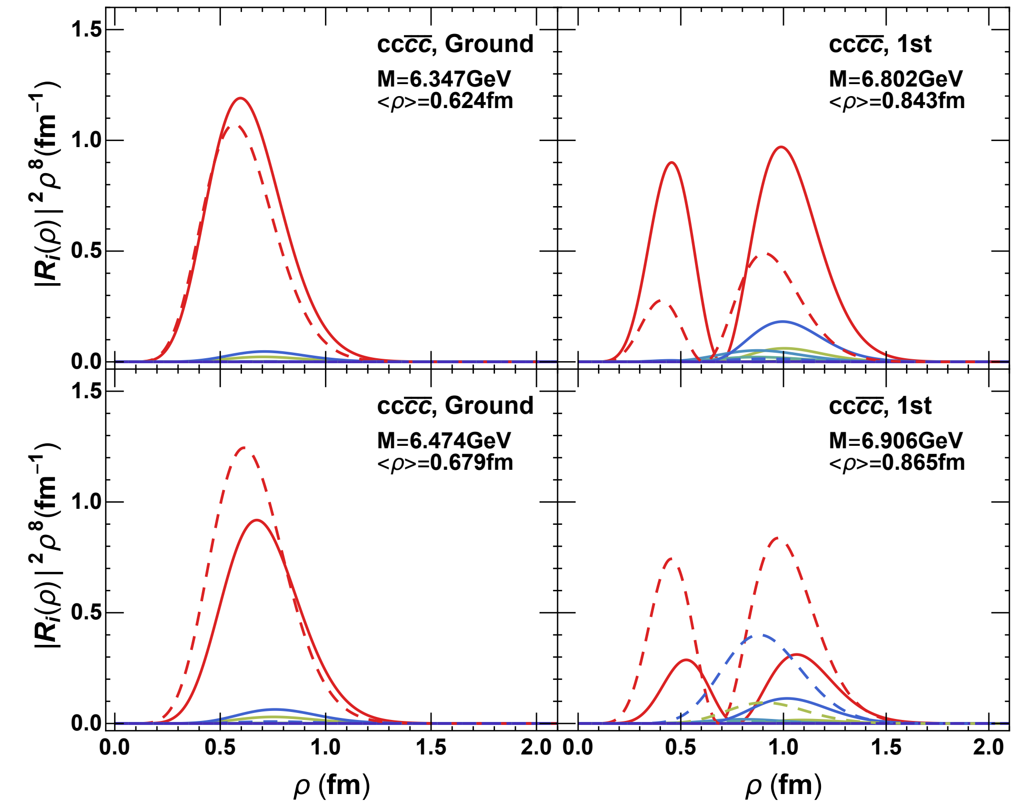
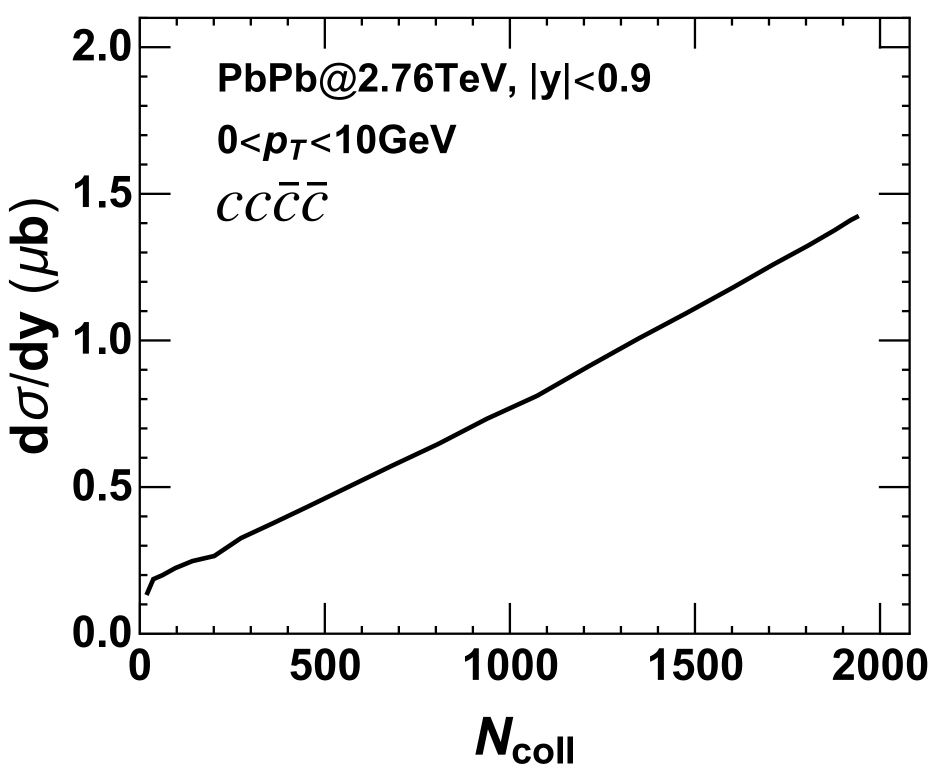



*JX Zhao, Shuzhe shi, and Pengfei Zhuang, arXiv: 2009.10319*

$$\left.\frac{d\sigma}{dy}\right|_{pp} = 78pb$$ *in pp at 7TeV*

*Marek Karliner el al, Phys.Rev.D 95 (2017) 3, 034011.*
*Ruilin Zhu, arXiv: 2010.09082.*

$$\left.\frac{d\sigma}{N_{coll}dy}\right|_{AA} \approx 770pb$$ *in AA at 5.02TeV*




*The four-lepton decay channel*

$$X(cc\bar{c}\bar{c}) \rightarrow l_1^+ l_2^- l_3^+ l_4^-$$

*can be well separated from the bulk back ground and makes it possible to find such exotic states in heavy-ion collision even in low pt region!*

CMS

# Outline

- Introduction to heavy-ion collisions and heavy flavor hadrons
- Static properties of heavy flavor hadrons in vacuum and finite-temp.
- Dynamic production of heavy flavor hadrons in heavy-ion collisions
- Using heavy flavor to probe the Hadronization M. and QGP
- Summary and outlook

# Study coalescence mechanism

*Coalescence mechanism successfully explained the Baryon/Meson Ratio and Quark Number Scaling of Elliptic flow observed in heavy ion collisions!*

$$\frac{dN_h}{d^2P_T d\eta} = C \int P^\mu d\sigma_\mu \prod_{i=1}^{n} \frac{d^4x_i d^4p_i}{(2\pi)^3} f_i(x_i, p_i) \times W_h(x_1, ..., x_i, p_1, ..., p_i).$$

***V. Greco, C. M. Ko and R. Rapp, Phys. Lett. B 595, 202 (2004).***

***D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).***

***R. J. Fries, B. Muller, C. Nonaka and S. A. Bass. Phys. Rev. C68, 044902(2004).***

***R. Fries, V. Greco, P. Sorensen. Anna. Rev. Null. Part. Sci 58(2008)177.***

***1. Energy-momentum conservation ?***

*2—>1; 3—>1*

***L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).***

***L. Ravagli, H. van Hees and R. Rapp, Phys. Rev. C 79, 064902 (2009).***

***Min He, Ralf Rapp, Phys.Rev.Lett. 124 (2020) 4, 042301.***

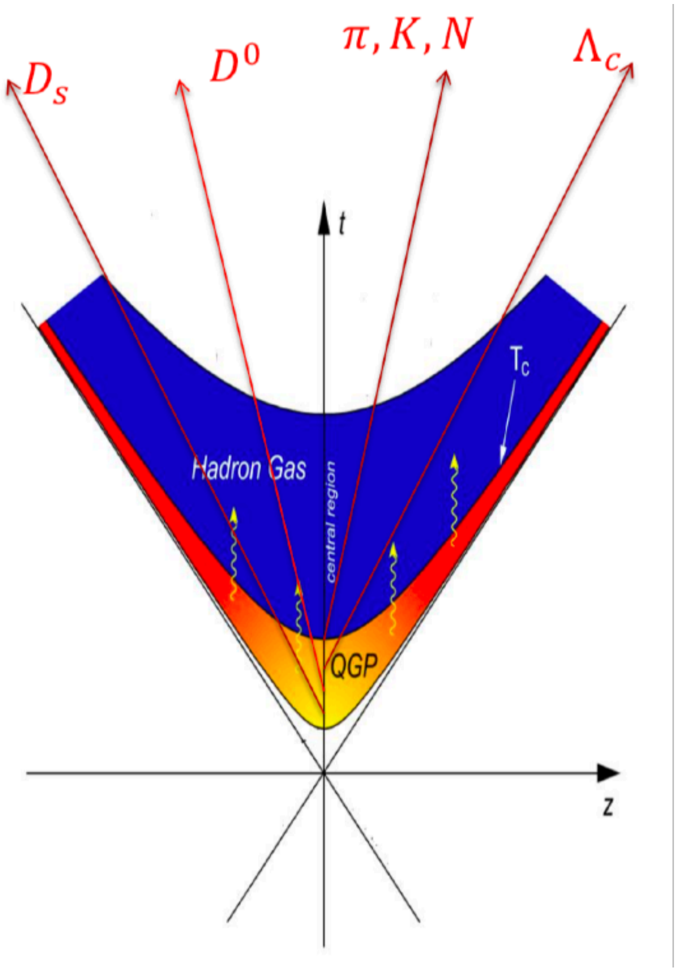
***2. Coalescence probability ?***

*The Wigner function can self-consistently be determined by the wavefunction. No-free parameters.*

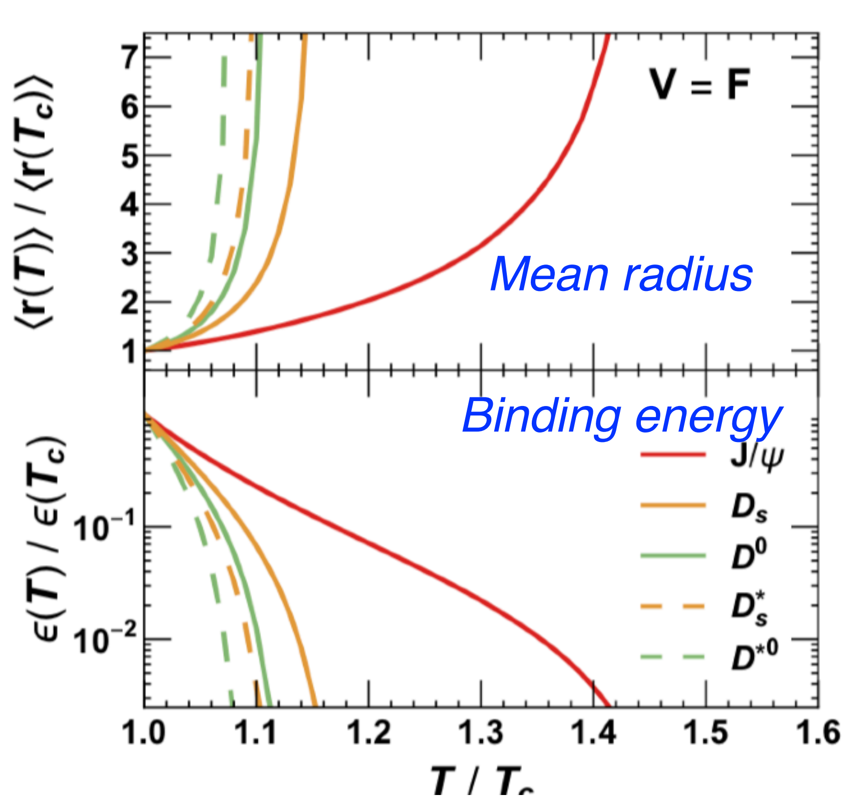
***3. Hadronization Sequence ?***

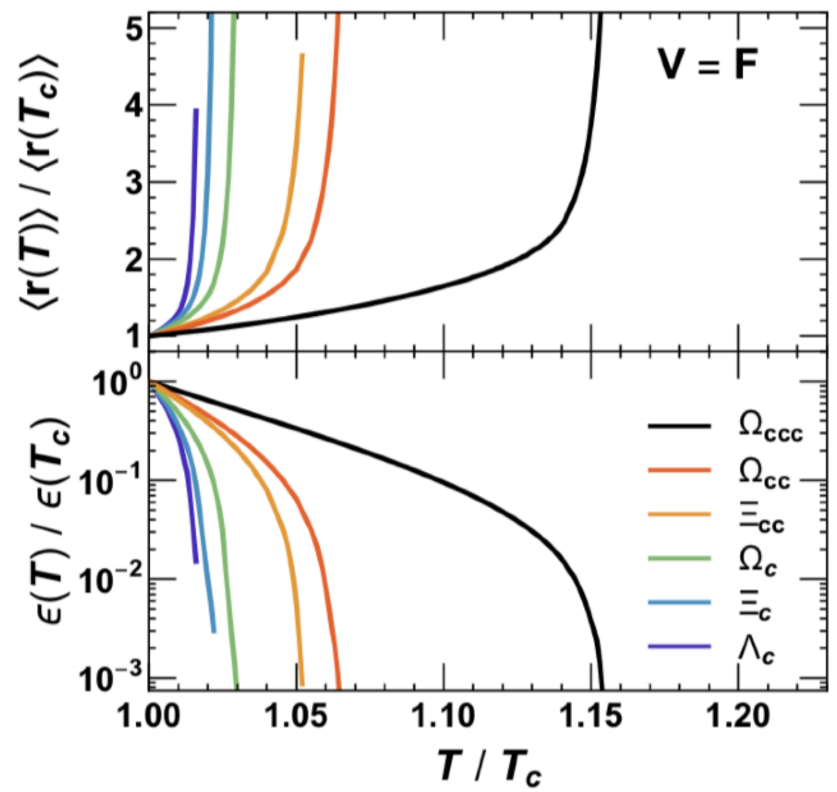
***4. Charm quark conservation ?***

***…***



# Hadronization Sequence & Charm conservation

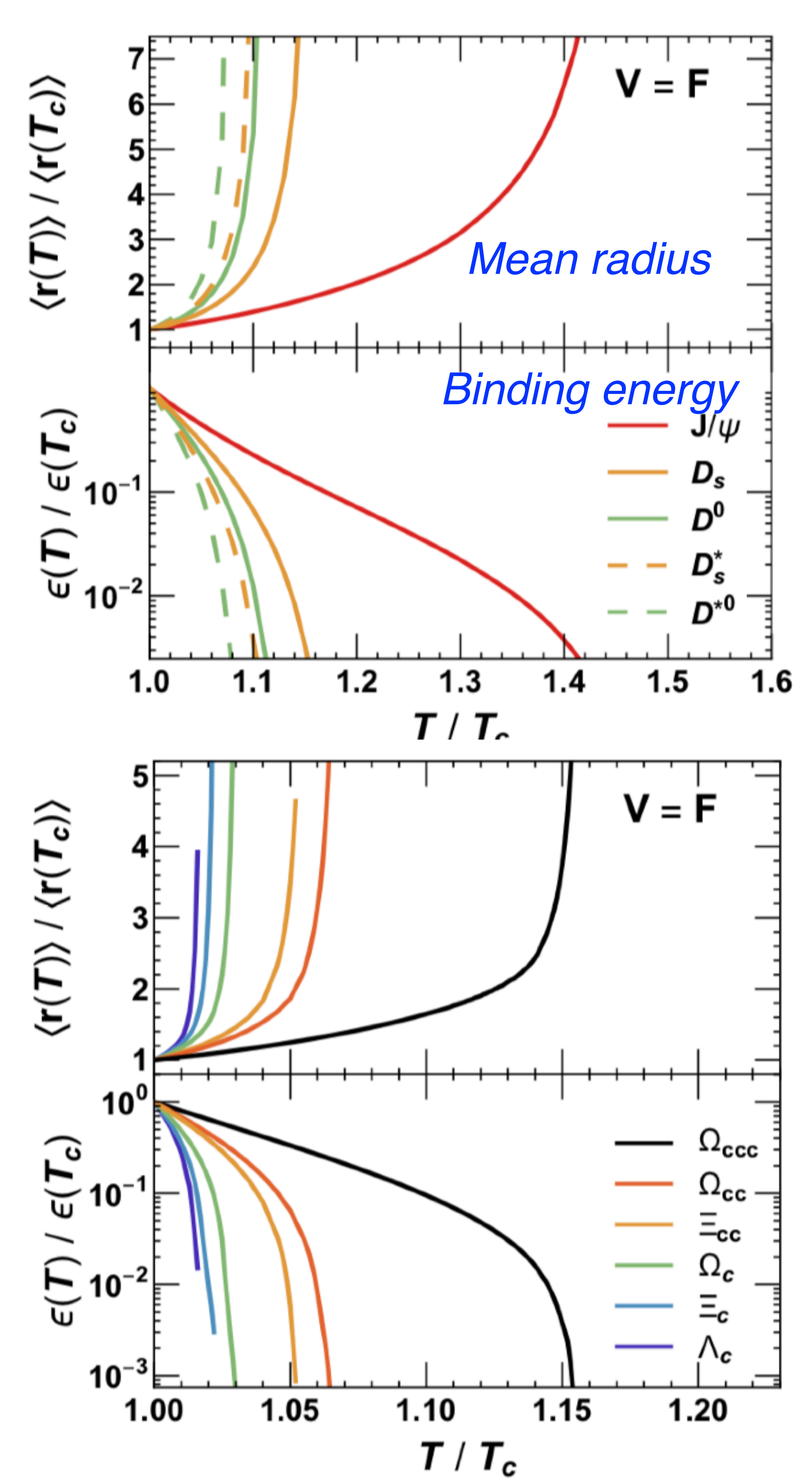


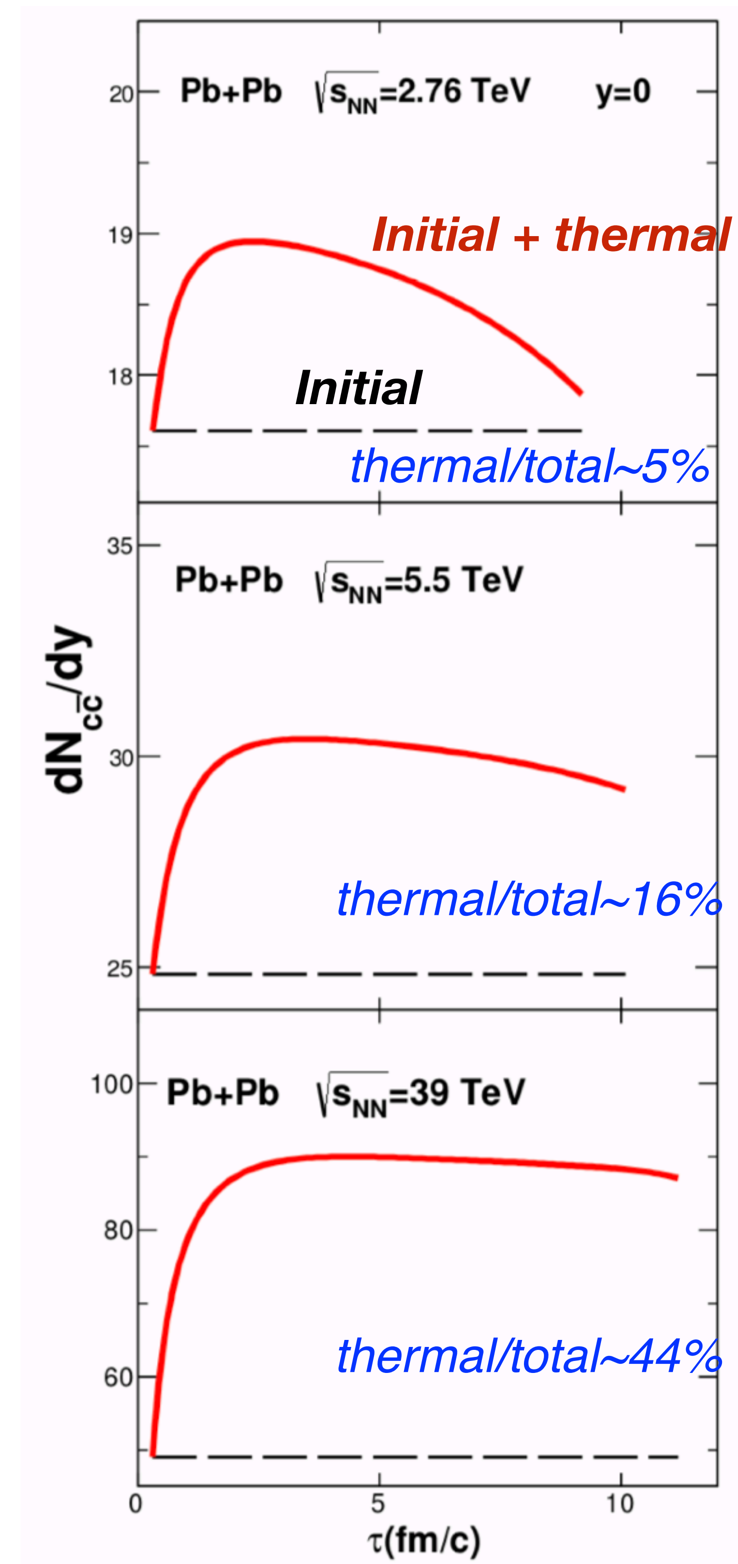




*Shuzhe Shi, JX Zhao, and Pengfei Zhuang, Chin.Phys.C 44 (2020) 8, 084101.*

$$T_{J/\psi} > T_{D_s} > T_{D^0} > T_{\Lambda_c} > T_{\pi,K,N}$$

# Hadronization Sequence & Charm conservation




Shuzhe Shi, JX Zhao, and Pengfei Zhuang, Chin.Phys.C 44 (2020) 8, 084101.

K. Zhou, Z.Chen, C.Greiner and P. Zhuang. Phys. Lett. B 758, 434 (2016)

$$T_{J/\psi} > T_{D_s} > T_{D^0} > T_{\Lambda_c} > T_{\pi,K,N}$$

Charm conserved at RHIC, almost conserved at LHC!

# Sequential Coalescence Mechanism

$$\frac{dN_h}{d^2\mathbf{P}_T d\eta} = C \int \frac{P^\mu d\sigma_\mu(R)}{(2\pi)^3} \prod_{i=1}^{n-1} \frac{d^4 r_i d^4 q_i}{(2\pi)^3} \prod_{i=1}^{n} f_i(\tilde{x}_i, \tilde{p}_i) W_h(x_1, \ldots, x_i, p_1, \ldots, p_i),$$

- $T_{J/\psi} > T_{D_s} > T_{D^0} > T_{\Lambda_c} > T_{\pi,K,N}$

*The hadronization hypersurface is determined by hydrodynamics and dissociation temperature: $T_d$*  $T(\mathbf{x}_T, \tau) = T_d$

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  *The hadronization hypersurface is determined by hydrodynamics and dissociation temperature: Td* $T(\mathbf{x}_T,\tau) = T_d$

- *Charm conservation*

$$f_c(x,p) = r_h \rho_c(x)\left[\alpha f_{th}(p) + \beta f_{pp}(p)\right]$$

$f_{th}(p)$ *FD distribution* $f_{pp}(p)$ *PYTHIA distribution*

$r_h$ *Charm conservation factor!*

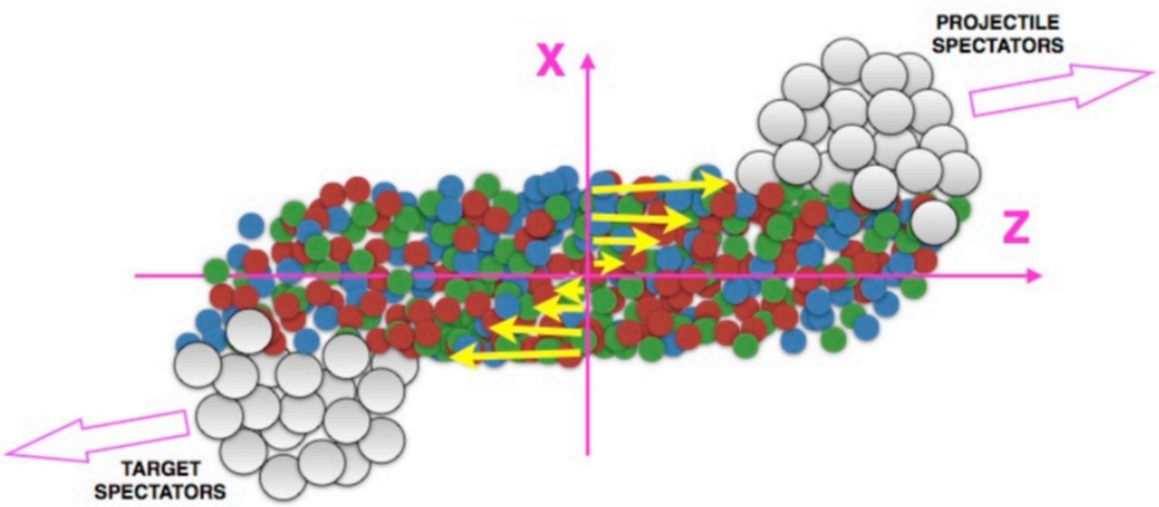
$$r_h = \frac{involved\ charm\ quarks}{total\ charm\ quarks\ N_c} = \begin{cases} 1 & for\ \ h = D_s \\ 1-\frac{N_{D_s}}{N_c}\ \ (\sim 90\%) & for\ \ h = D^0 \\ 1-(N_{D_s}+N_{D^0})/N_c\ \ (\sim 60\%) & for\ \ h = \Lambda_c \end{cases}$$

*If charmed hadrons are sequentially produced, more charm quarks are involved in the earlier production and less in the later production. Enhancement for earlier produced hadrons and a suppression for later produced hadrons.*

*JX. Zhao, S. Shi, Nu Xu and Pengfei Zhuang, arXiv:1805.10858.*

*JX. Zhao, S. Shi, Nu Xu and P. Zhuang, EPJ Web Conf. 202 (2019) 06004.*

# *Probe initial longitudinal profile in heavy-ion collisions*




*a huge orbital angular momentum (OAM) is produced in non-central heavy-ion collisions.*

*inhomogeneous expansion will generate nonzero vorticity which leads to the global polarization of hadrons.*

***Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005).***
***Jiang Y, Lin Z W, Liao J. Phys. Rev. C94(4):044910(2016).***
***Wei-Tian Deng, Xu-Guang Huang, Phys.Rev.C 93 (2016) 6, 064907.***
***F. Becattini, el al. Eur.Phys.J.C 75 (2015) 9, 406.***
***H. Li, L. G. Pang, Q. Wang and X. L. Xia, Phys. Rev. C 96, no.5, 054908 (2017).***
***Y. Sun and C. M. Ko, Phys. Rev. C 96, no.2, 024906 (2017).***

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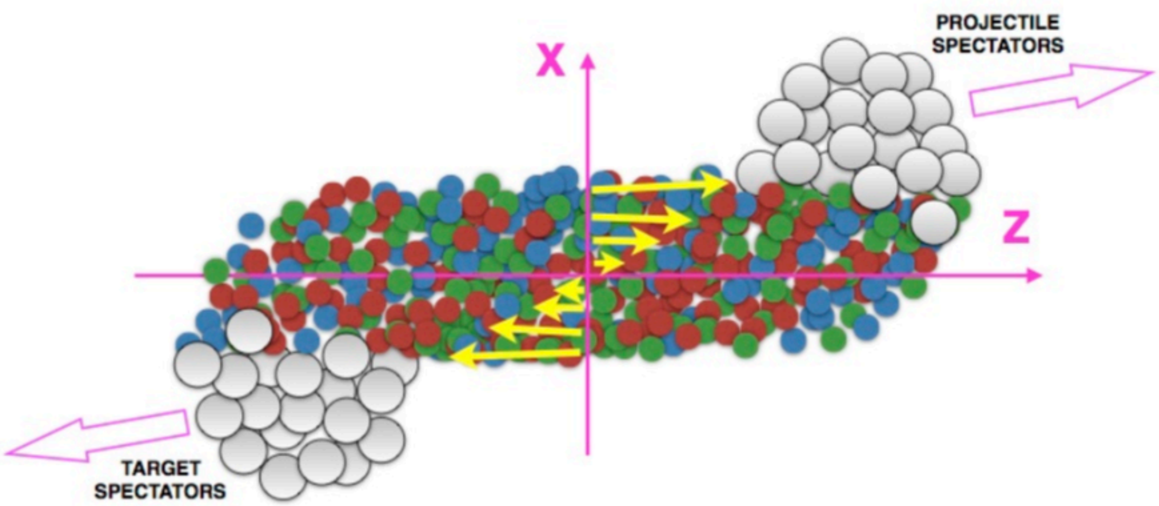


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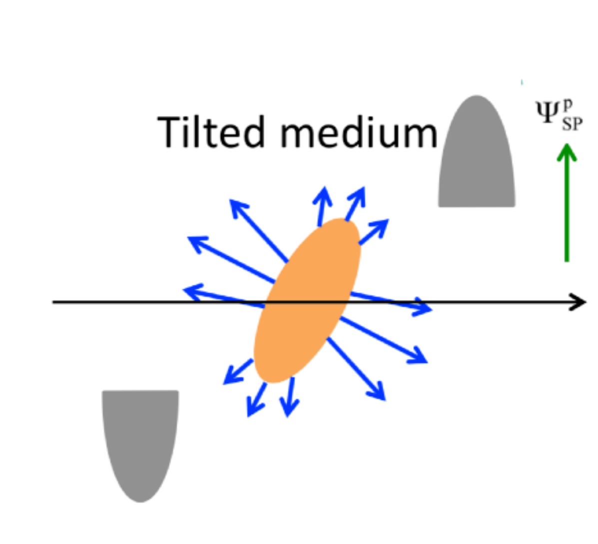
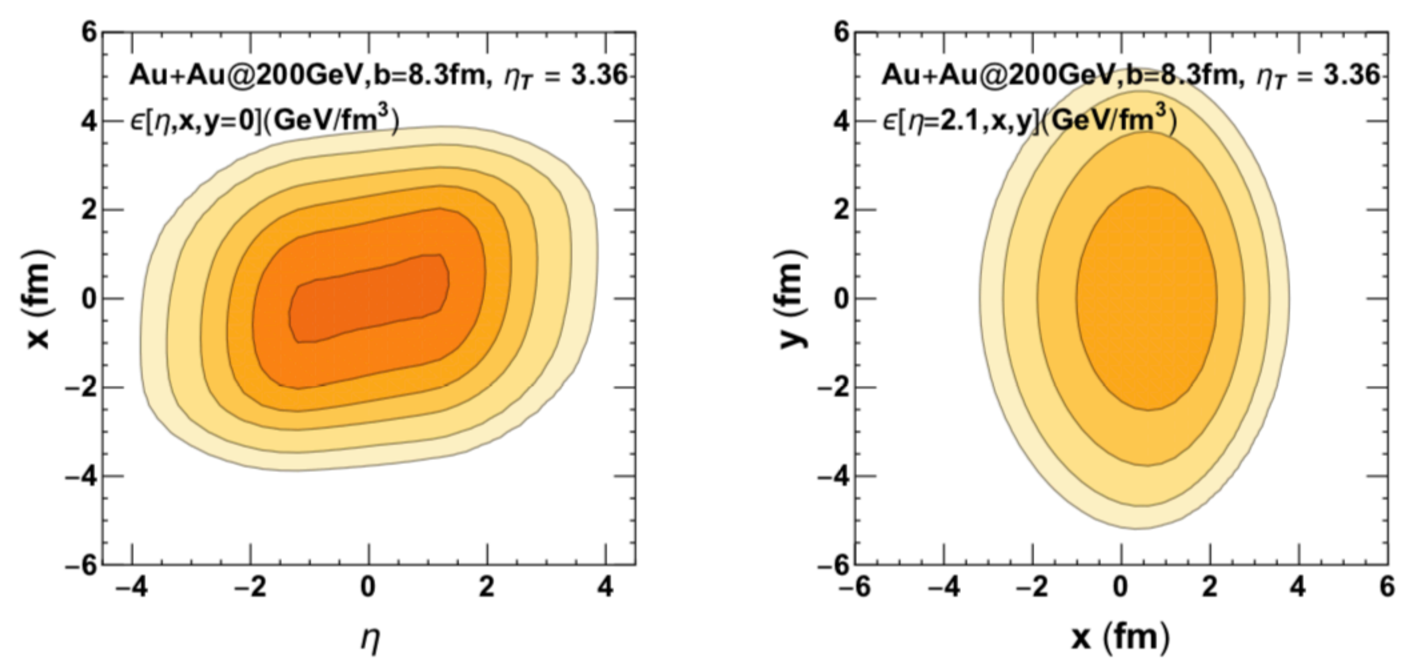
Y. Sun and C. M. Ko, Phys. Rev. C 96, no.2, 024906 (2017).


*vorticity field can be described by tilted hydro !*





P. Bozek, I. Wyskiel, Phys. Rev. C 81 (2010) 054902.
S. Chatterjee, P. Bozek, Phys. Rev. Lett. 120 (2018) 192301.



Baoyi Chen, Maoxin Hu, Huanyu Zhang, JX Zhao. Phys.Lett.B 802 (2020) 135271.

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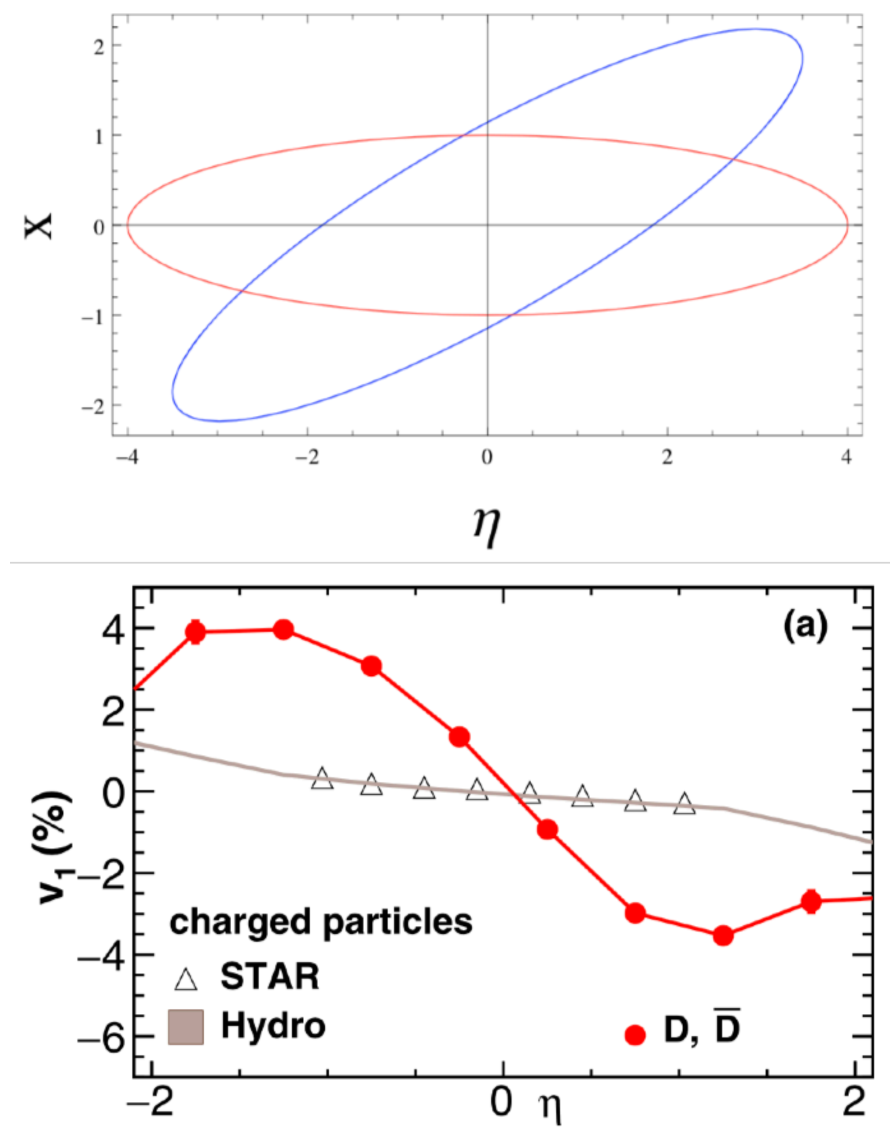


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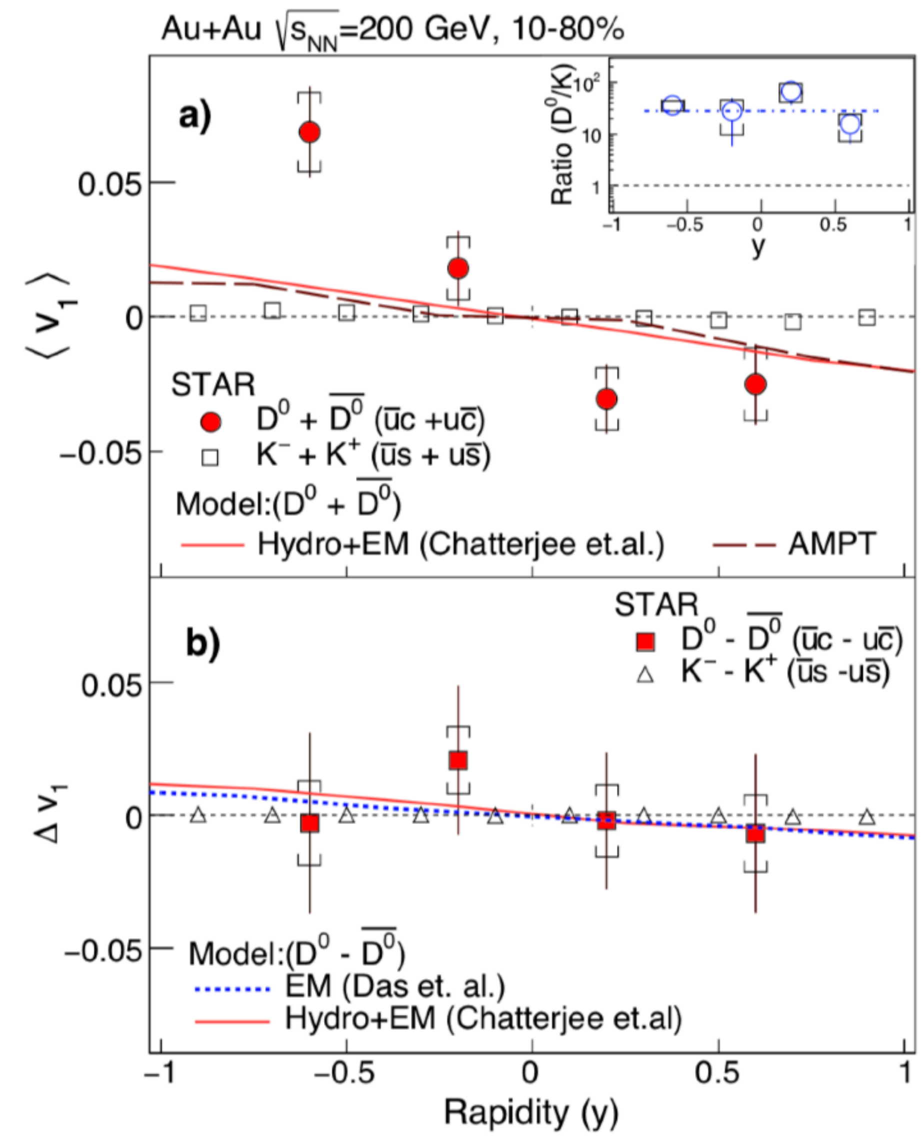


*STAR Collaboration. Phys.Rev.Lett. 123 (2019) 16, 162301.*

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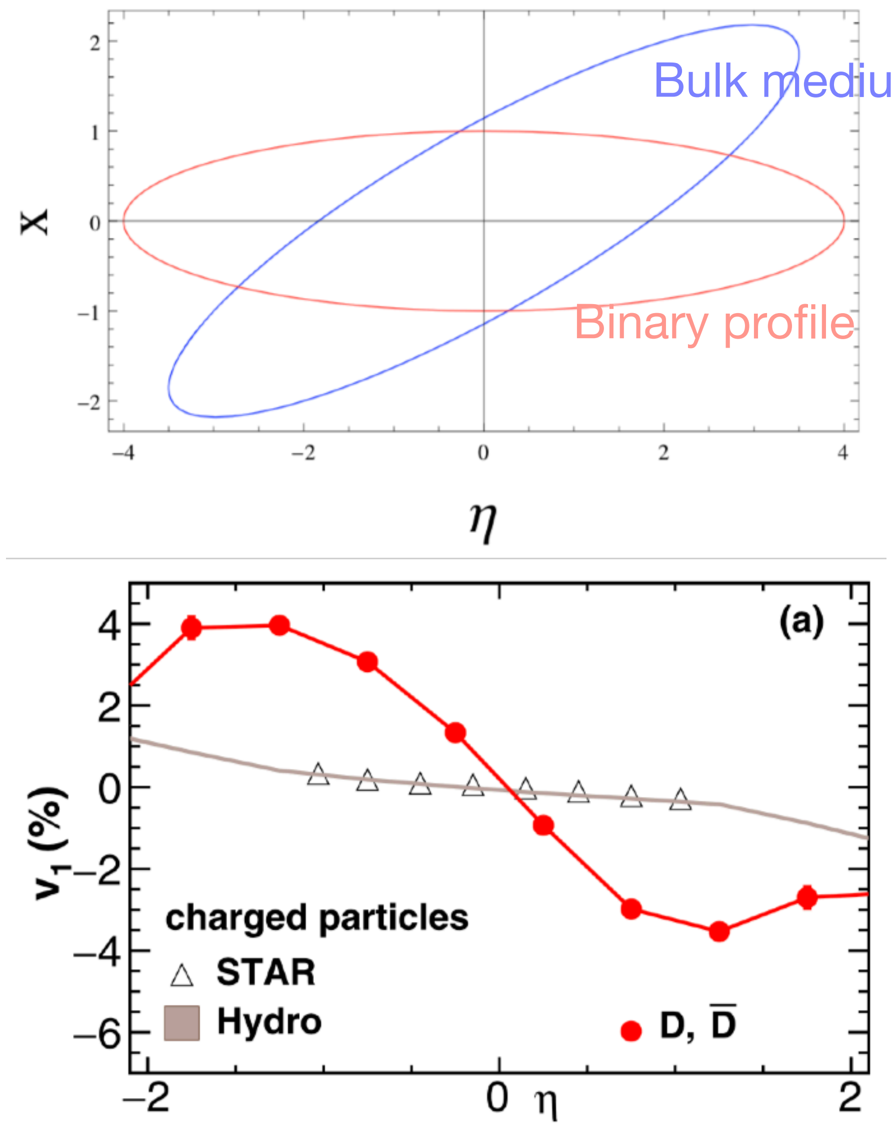
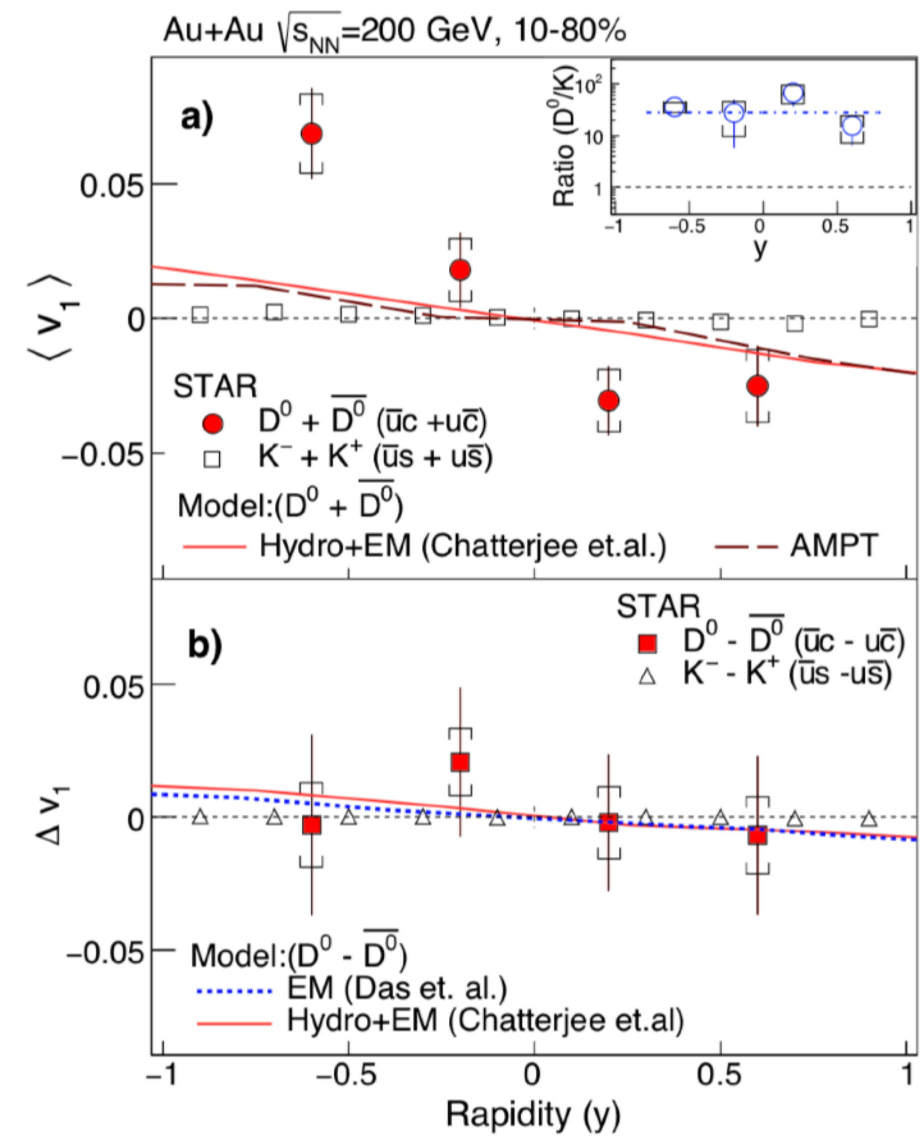



*S. Chatterjee, P. Bozek, Phys. Rev. Lett. 120 (2018) 192301.*




*STAR Collaboration. Phys.Rev.Lett. 123 (2019) 16, 162301.*

## Charmonium:

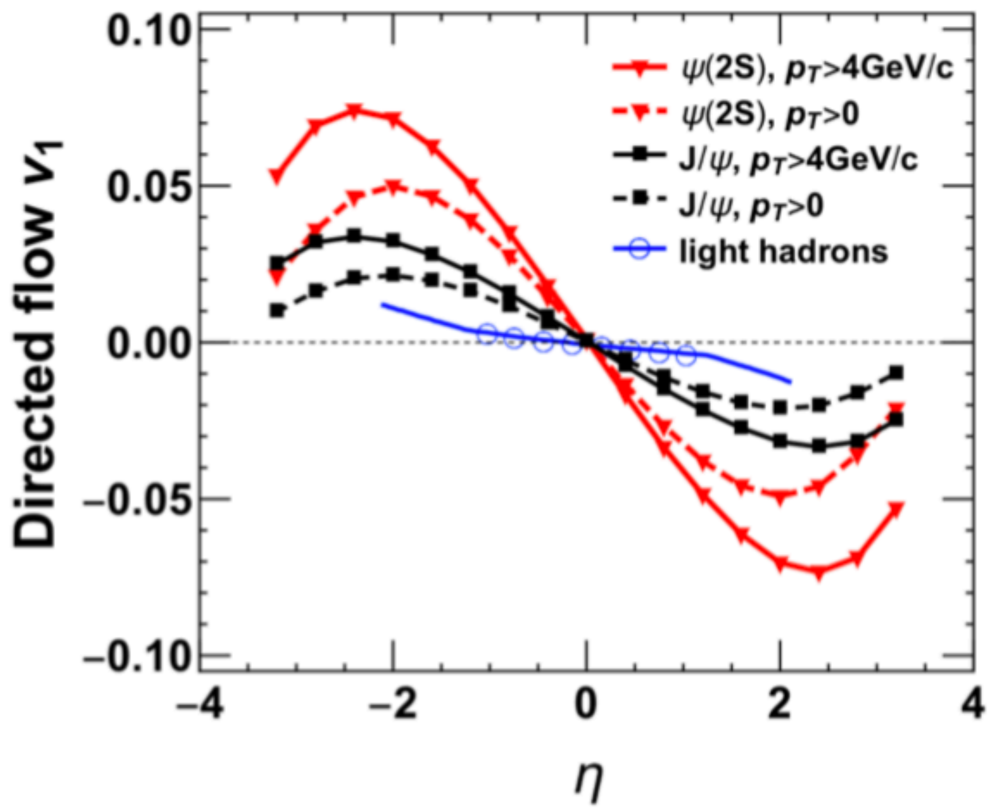
- *Produced at the early stage of nuclear collisions*
- *Dissociated by the initial QGP with high T and less affected by the following QGP expansion*
- *Charm quark diffusion coeff. is under debate*
- *Charmonium $v_1$ is not affected by EM field*

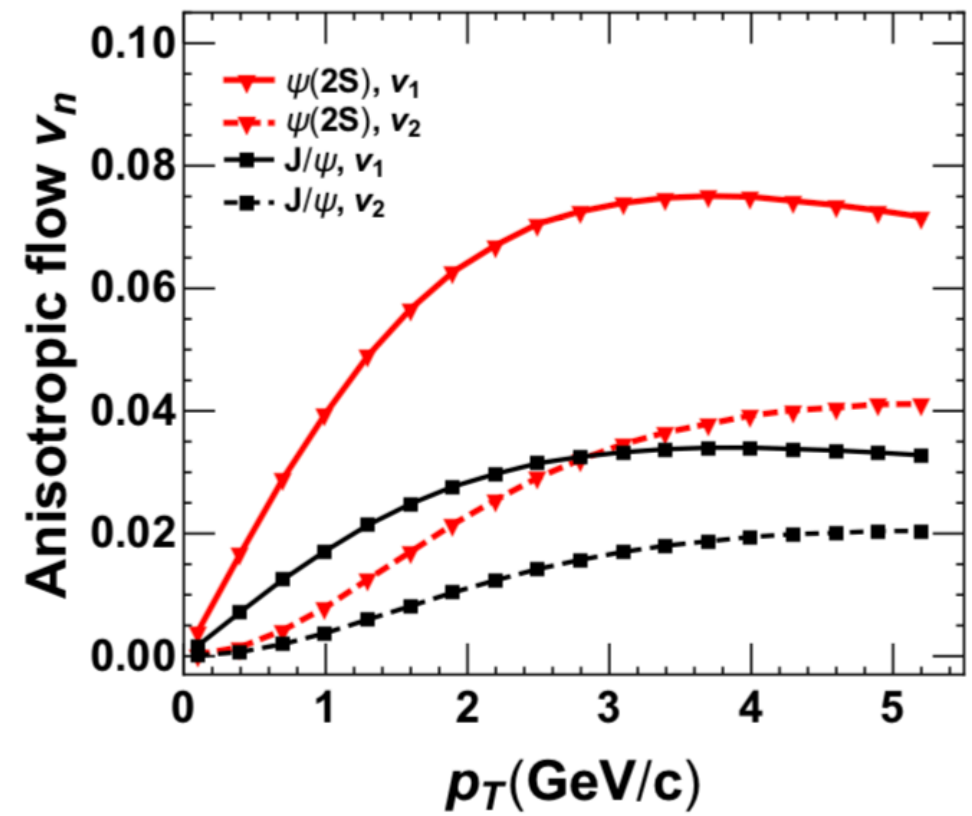
# Probe initial longitudinal profile in heavy-ion collisions

*Baoyi Chen, Maoxin Hu, Huanyu Zhang, JX Zhao. Phys.Lett.B 802 (2020) 135271.*

*The dissociation and regeneration of charmonium in heavy-ion collision is described by transport approach (THU model).*

*Boltzmann equation:* $$\left[\cosh(y-\eta)\frac{\partial}{\partial\tau}+\frac{\sinh(y-\eta)}{\tau}\frac{\partial}{\partial\eta}+\mathbf{v}_T\cdot\nabla_T\right]f_\psi=-\alpha f_\psi+\beta.$$




...

*Different mechanisms for $v_1$:*

*(1) Open charm quarks (D meson) and light hadrons: coupled with QGP motion.*
*(2) Charmonium: biased dissociations.*

*Charmonium are more sensitive to initial longitudinal profile!*

# Outline

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- Summary and outlook

# Summary

- *We systematically studied the static properties of heavy flavor hadrons in vacuum and finite temperature with improved N-body Schroedinger equation and N-body Dirac equation!*

- *We proposed a way to search for multi-charmed baryon and fully-heavy tetraquark states in experiment. Due to the combination of uncorrelated charm quarks in QGP, the yield of multi-charmed baryon and fully-heavytetra quark states is significantly enhanced in comparison with the production in p+p !*

- *We built a framework to realize sequential hadronization with charm conservation in HIC!*
  *Hadronization sequence and coalescence probability of charmed hadrons are determined by 2&3-body Dirac equation.*
  *Charm conservation leads to an enhancement for earlier produced hadrons and a suppression for later produced hadrons.*

- *We used heavy flavor hadrons to probe the s initial longitudinal profile in heavy-ion collisions.*

# Read more

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Review

# Heavy flavors under extreme conditions in high energy nuclear collisions

Jiaxing Zhao [a], Kai Zhou [b], Shile Chen [a], Pengfei Zhuang [a]

Show more

https://doi.org/10.1016/j.ppnp.2020.103801

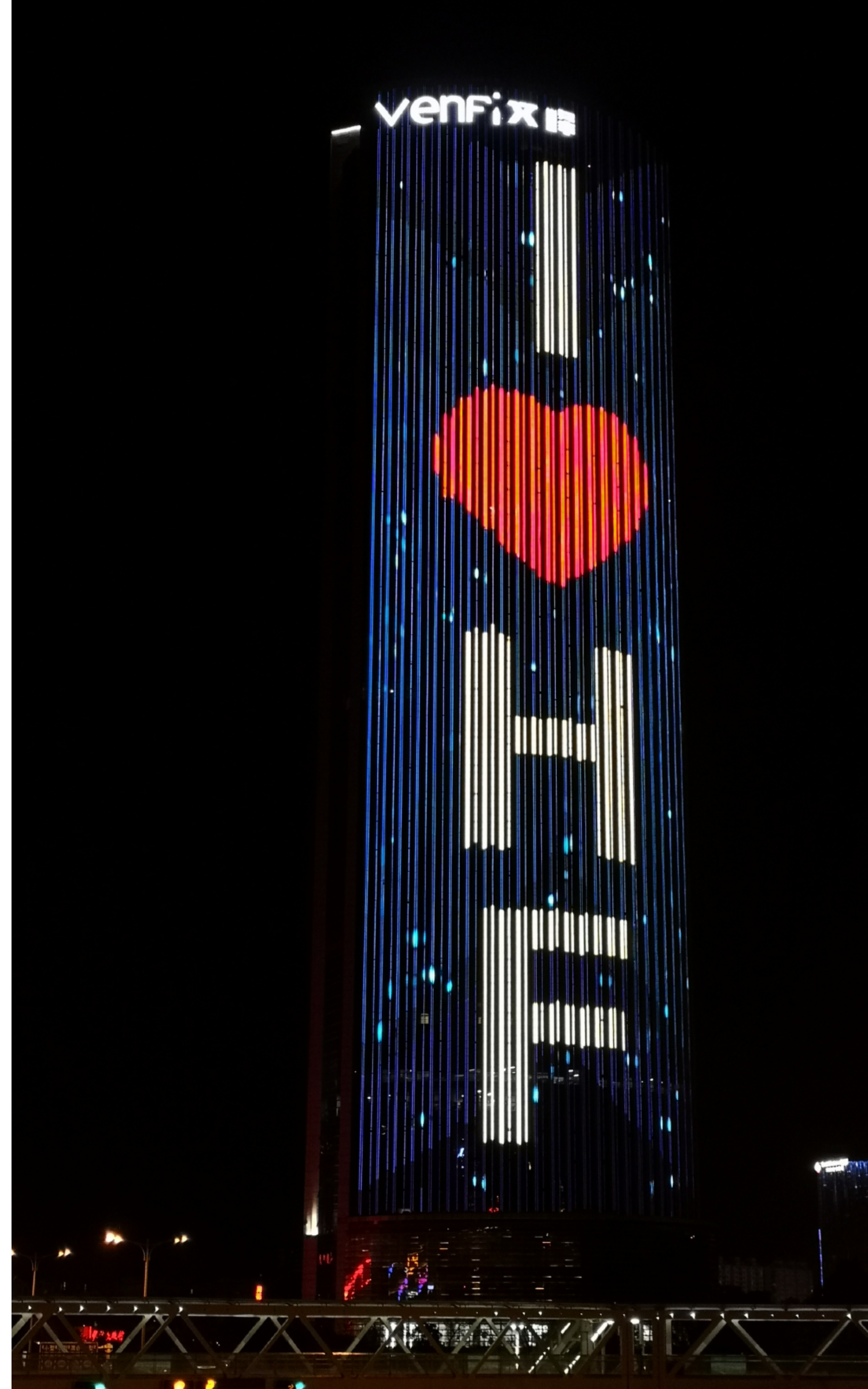


***arXiv: 2005.08277.***

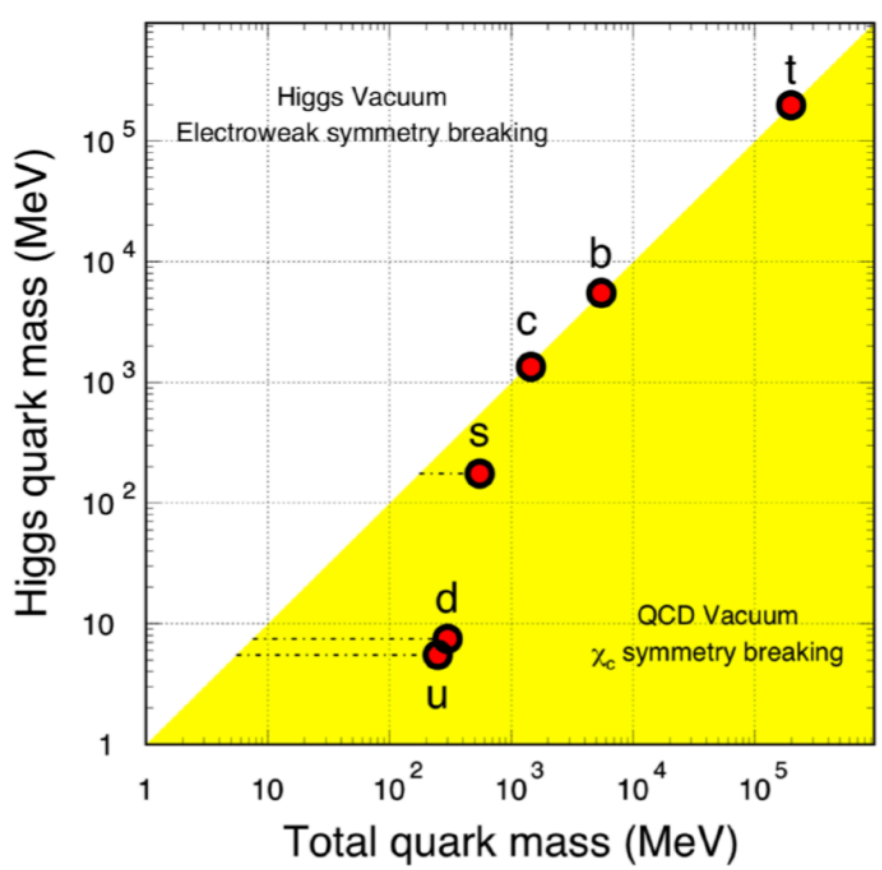
*Many thanks to my collaborators:*

Pengfei Zhuang, Nu Xu, Zhe Xu, Lianyi He,
Kia Zhou, Baoyi Chen, Hang He, Shuzhe Shi,
Guojun Huang, Shile Chen,···

*Thanks for your attention!*

# *Heavy quark as a probe*




- *Mass not change in QGP medium, number conserved.*
- *Mc, Mb >>* $\Lambda_{QCD}$, *produced by initial hard scattering and can be described by pQCD.*
- *Probe strong and short lived EM field and vorticity field*
- *Clean and easy to distinguish*

# N-Body Dirac Equation

*Two-body Dirac eq.*

$$\mathcal{S}_1\Psi \equiv \gamma_{51}\left[\gamma_1^{\mu}(p_\mu - A_\mu) + m + S\right]\Psi = 0,$$

$$\mathcal{S}_2\Psi \equiv \gamma_{52}\left[\gamma_2^{\mu}(p_\mu - A_\mu) + m + S\right]\Psi = 0.$$

*$\Psi$ is 16 component wavefunction,*
*$A_\mu$ relativistic four-vector potential,*
*$S$ scalar potential.*

*Taking Pauli reduction and scale transformation in center-of-mass frame, the relative motion can be expressed as a four-component relativistic Schrödinger-like equation:*

**H. Crater, J. Yoo and C. Wong. PRD 79. 034011(2009)**

$$\left[p^2 + \Phi(A(r), S(r), p, P, \omega, \sigma_1, \sigma_2)\right]\psi = b^2\psi$$

*Three-body Dirac eq.*

$$\mathcal{H}_i\psi = (p_i^2 + m_i^2 + \Phi_i)\psi = 0,$$

$$[\mathcal{H}_i, \mathcal{H}_j]\psi = 0, (i, j = 1, 2, \ldots, N).$$

↓

$$\sum_{i=1}^{N}\left[-\epsilon_i^2 + N\frac{p_{i\perp}^2/(2\epsilon_i)}{\sum_{j=1}^{N}1/(2\epsilon_j)} + m_i^2\right]\psi = 0,$$

**Whitney, H. Crater. Phys. Rev. D89, 014023(2014)**

↓

$$\left[\sum_{i=1}^{3}\frac{\mathbf{p}_i^2}{2\epsilon_i} + \sum_{i<j}^{3}\frac{\epsilon_i + \epsilon_j}{2\epsilon_i\epsilon_j}\Phi_{ij}\right]\Psi = E\Psi,$$

$$\begin{aligned}\Phi_{ij} &= 2m_{ij}S + S^2 + 2\epsilon_{ij}A - A^2 + \Phi_D + \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\Phi_{SS} \\ &+ \mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Phi_{SO} + \mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\Phi_{SOD} + i\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\Phi_{SOX} \\ &+ (\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij})\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Phi_{SOT} + (3(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\Phi_T.\end{aligned}$$

# Wavefunction analysis

*Pauli exclusion principle requires the wave-function to be anti-symmetric when exchanging two identical fermions*

$$\Psi = \psi \phi_f \chi_s \phi_c$$

*Spin space :*

$$2 \otimes 2 = 1 \oplus 3 \qquad 2 \otimes 2 \otimes \ldots = \ldots$$

*SU(2) Direct product decomposition*

*Color space :*

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = \underline{\bar{3} \otimes 3} \oplus \underline{6 \otimes \bar{6}} \oplus \bar{3} \otimes \bar{6} \oplus 6 \otimes 3$$

$$3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3} = [\ldots] \oplus \underline{3 \otimes \bar{3}} \oplus \underline{3 \otimes \bar{3}} \oplus \underline{3 \otimes \bar{3}}$$

*SU(3) Direct product decomposition*