

Recent results on light nuclei production in extended thermal model descriptions

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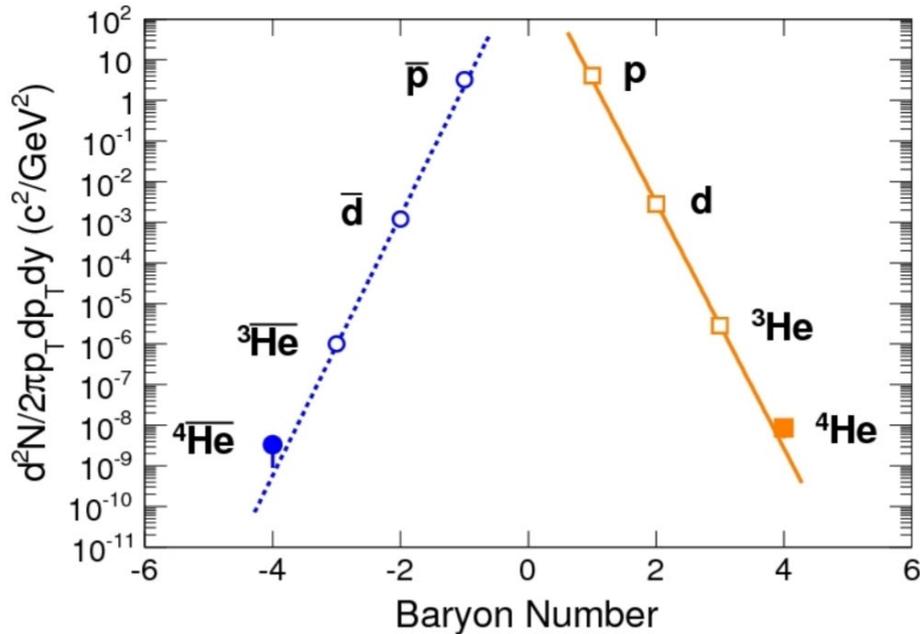


Alexander von Humboldt
Stiftung/Foundation

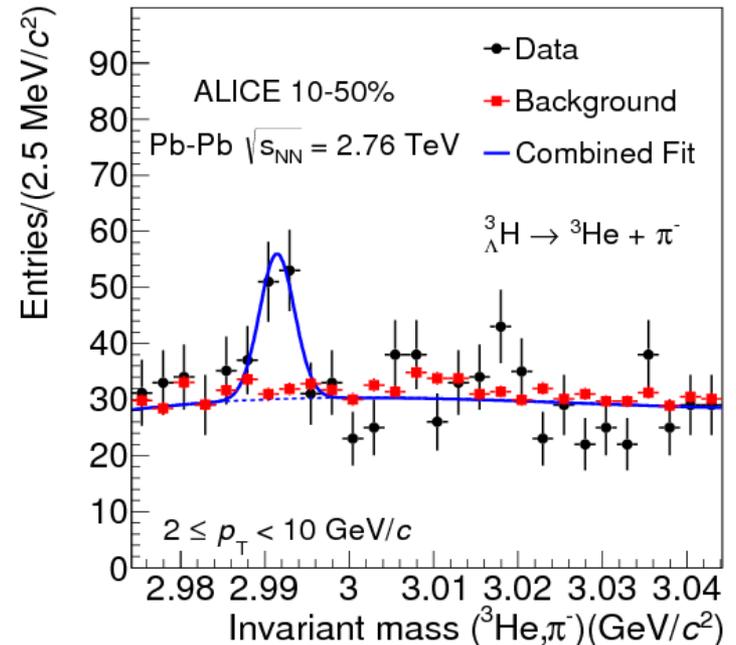
Outline

1. Short intro to thermal model and **Thermal-FIST** 
2. Light nuclei in extended thermal model descriptions
 - Canonical suppression
 - The Saha equation approach
 - Feeddown contributions from excited nuclear states
3. Summary

Loosely-bound objects in heavy-ion collisions



[STAR collaboration, Nature **473**, 353 (2011)]



[ALICE Collaboration, PLB **754**, 360 (2016)]

binding energies: ${}^2\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$, ${}^3_{\Lambda}\text{H}$: 2.22, 7.72, 28.3, 0.130 MeV $\ll T \sim 150$ MeV

“snowballs in hell”

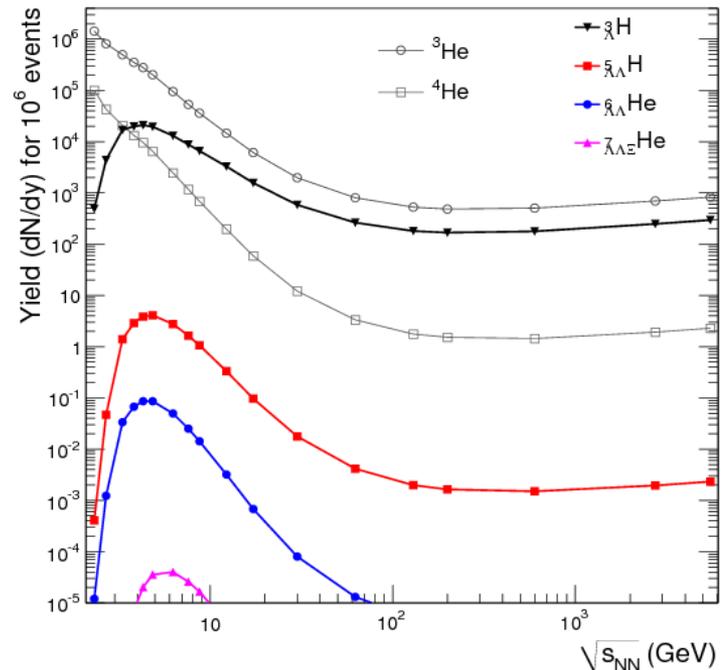
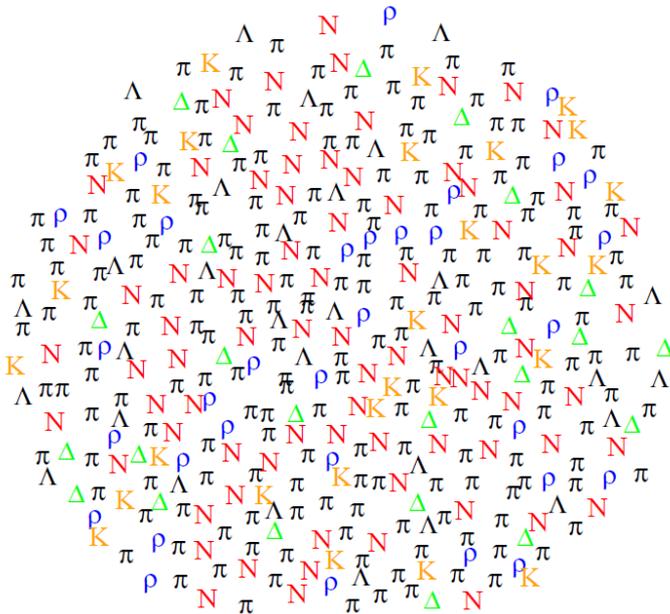
The production mechanism is not established. Common approaches include **thermal** nuclei emission together with hadrons [Andronic et al., PLB '11;...] or final-state **coalescence** of nucleons close in phase-space [Butler, Pearson, PRL '61; Scheibl, Heinz, PRC '99;...]

Hadron resonance gas (HRG) at freeze-out

HRG: Equation of state of hadronic matter as a multi-component (non-)interacting gas of known hadrons, resonances, and *light nuclei*

$$\ln Z \approx \sum_{i \in M, B} \ln Z_i^{id} = \sum_{i \in M, B} \frac{d_i V}{2\pi^2} \int_0^\infty \pm p^2 dp \ln \left[1 \pm \exp \left(\frac{\mu_i - E_i}{T} \right) \right]$$

Grand-canonical ensemble: $\mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$ *chemical equilibrium*



Thermal-FIST

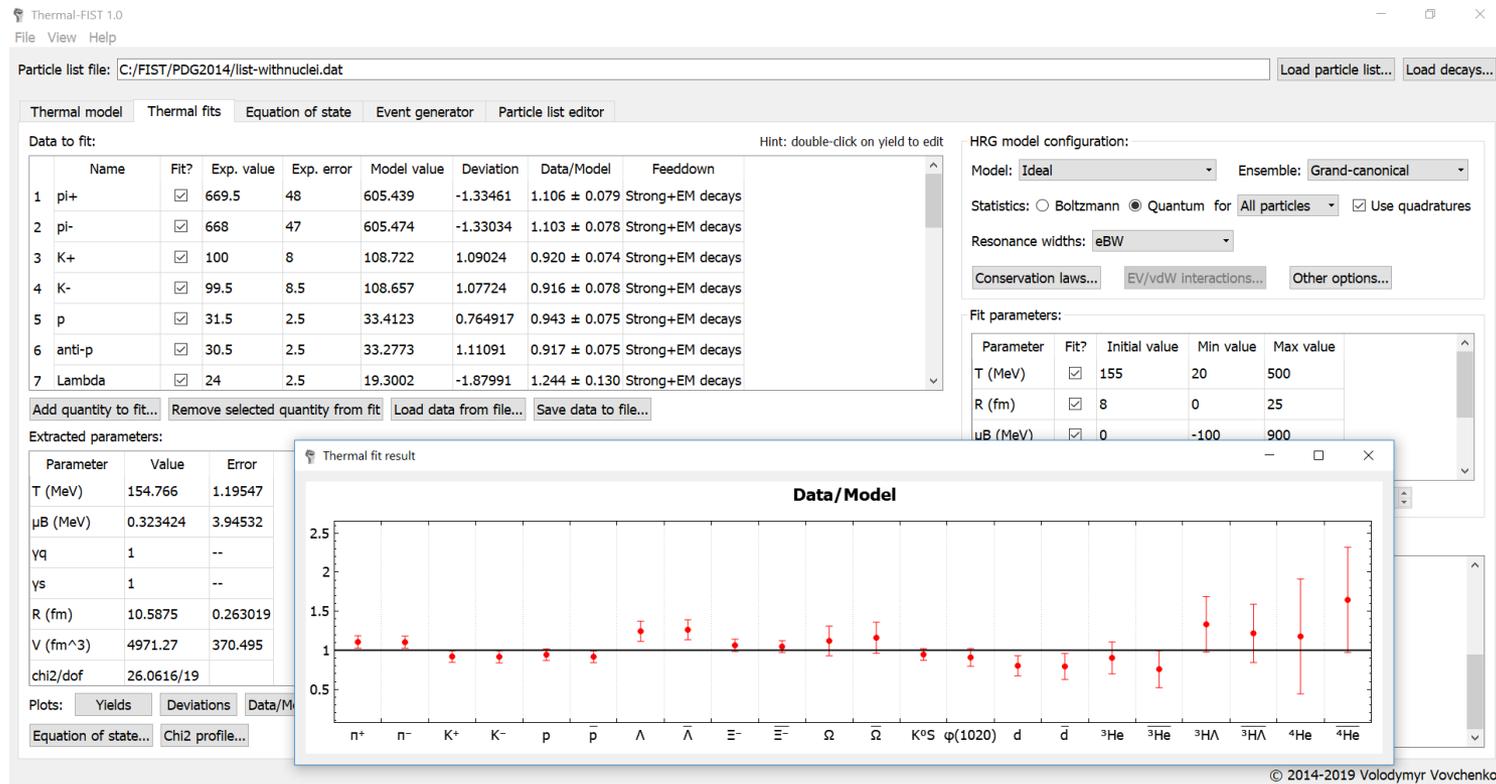


Thermal-FIST* (a.k.a. **FIST** or **FAUST**)

C++/Qt/Jupyter

open source: <https://github.com/vlvovch/Thermal-FIST>

reference: V.V., H. Stoecker, *Computer Physics Communications* **244**, 295 (2019)



A framework for general-purpose statistical-thermal model applications

**Thermal, Fast and Interactive Statistical Toolkit*

Using Thermal-FIST



The package is *cross-platform* (Linux, Mac, Windows, Android)

Installation using **git** and **cmake**

```
# Clone the repository from GitHub
git clone https://github.com/vlvovch/Thermal-FIST.git
cd Thermal-FIST

# Create a build directory, configure the project with cmake
# and build with make
mkdir build
cd build
cmake ../
make

# Run the GUI frontend
./bin/QtThermalFIST

# Run the test calculations from the paper
./bin/examples/cpc1HRGTDep
./bin/examples/cpc2chi2
./bin/examples/cpc3chi2NEQ
./bin/examples/cpc4mcHRG
```

GUI requires free [Qt5 framework](#), the rest of the package has *no external dependencies*

[Quick start guide](#)

[Documentation](#)

[Physics manual](#)

Statistical-thermal model aspects in FIST



- Extensions of the HRG model
 - finite resonance widths
 - repulsive (excluded volume) and van der Waals (*criticality*) interactions
 - particle number fluctuations and correlations
 - chemical non-equilibrium (γ_q, γ_s) a la Rafelski
 - unstable nuclei
- Equation of state
- Canonical statistical model (CSM)
 - (local) (selective) exact conservation of conserved charges
 - canonical suppression of light nuclei
- Monte Carlo generator (Blast-wave, canonical ensemble,...)
- Hadronic phase and dynamical freeze-out
 - partial chemical equilibrium
 - suppression of resonance yields
 - evolution of light nuclei abundances via the Saha equation

Statistical-thermal model aspects in FIST



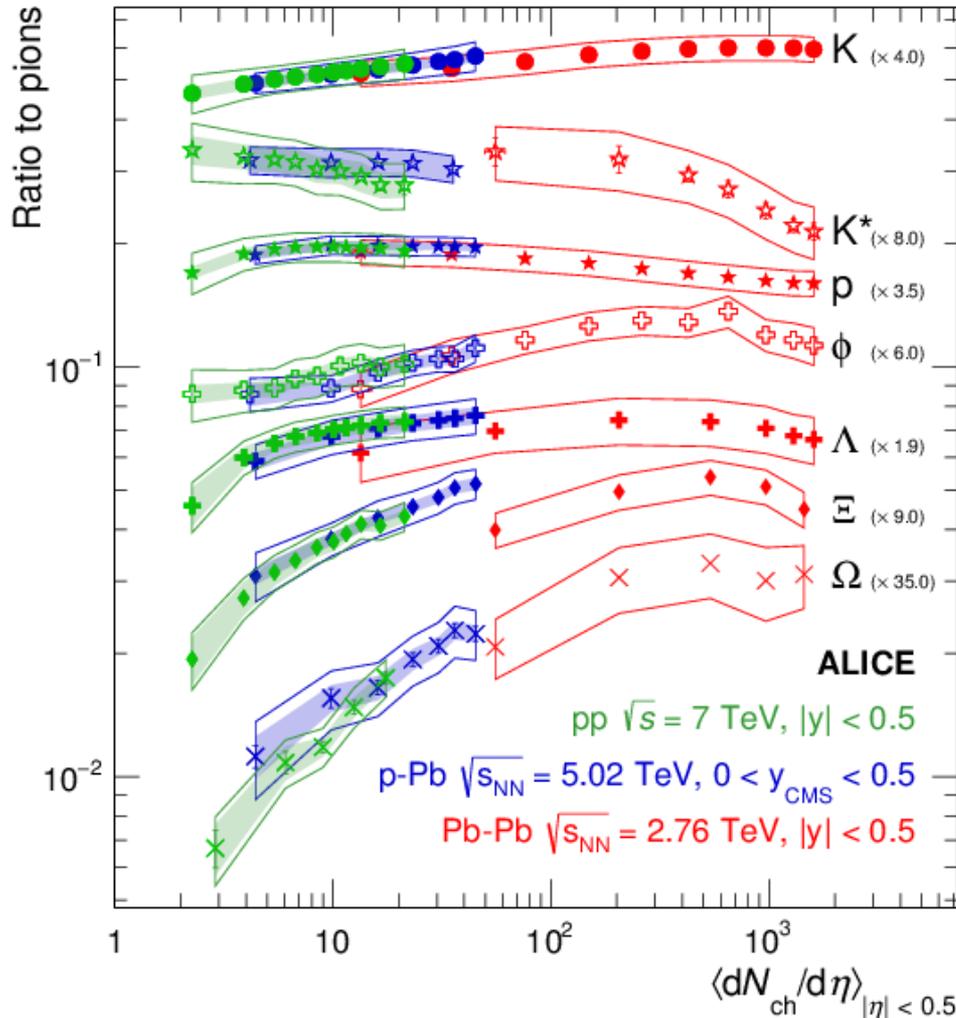
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Canonical suppression of light nuclei at the LHC

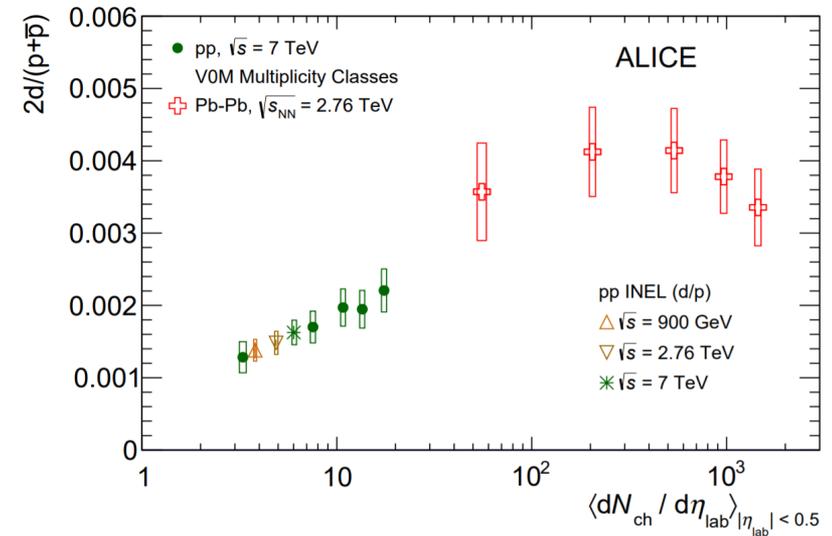
V. Vovchenko, B. Doenigus, H. Stoecker, *Phys. Lett. B* 785, 171 (2018)

source code: <https://github.com/vlvovch/CSM>

Multiplicity dependence of hadrochemistry



[ALICE collaboration, 1807.11321]



[ALICE collaboration, 1902.09290]

- Grand-canonical thermal picture predicts **no multiplicity dependence**
- Apply **canonical** statistical model?

Canonical statistical model (CSM)

Exact conservation of B , Q , S in a correlation volume V_c

[Rafelski, Danos, et al., PLB '80; Hagedorn, Redlich, ZPC '85]

$$Z(B, Q, S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi} e^{-i(B\phi_B + Q\phi_Q + S\phi_S)} \exp \left[\sum_j z_j^1 e^{i(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)} \right]$$

$$z_j^1 = V_c \int dm \rho_j(m) d_j \frac{m^2 T}{2\pi^2} K_2(m/T)$$

$$\langle N_j^{\text{prim}} \rangle^{\text{ce}} = \frac{Z(B - B_j, Q - Q_j, S - S_j)}{Z(B, Q, S)} \langle N_j^{\text{prim}} \rangle^{\text{gce}}$$

[Becattini et al., ZPC '95, ZPC '97]

Implemented in Thermal-FIST for a full HRG

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Exact conservation around midrapidity, $V_C = kdV/dy$. How large is k ?

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Implemented in Thermal-FIST for a full HRG

Exact conservation around midrapidity, $V_C = kdV/dy$. How large is k ?

Net-proton fluctuations affected by baryon number conservation

[Bzdak, Koch, Skokov, 1203.4529; Braun-Munzinger, Rustamov, Stachel, 1612.00702]

$$\frac{\kappa_2(p - \bar{p})}{\langle p \rangle + \langle \bar{p} \rangle} \simeq 1 - \frac{\langle p \rangle}{k dN_B/dy}$$

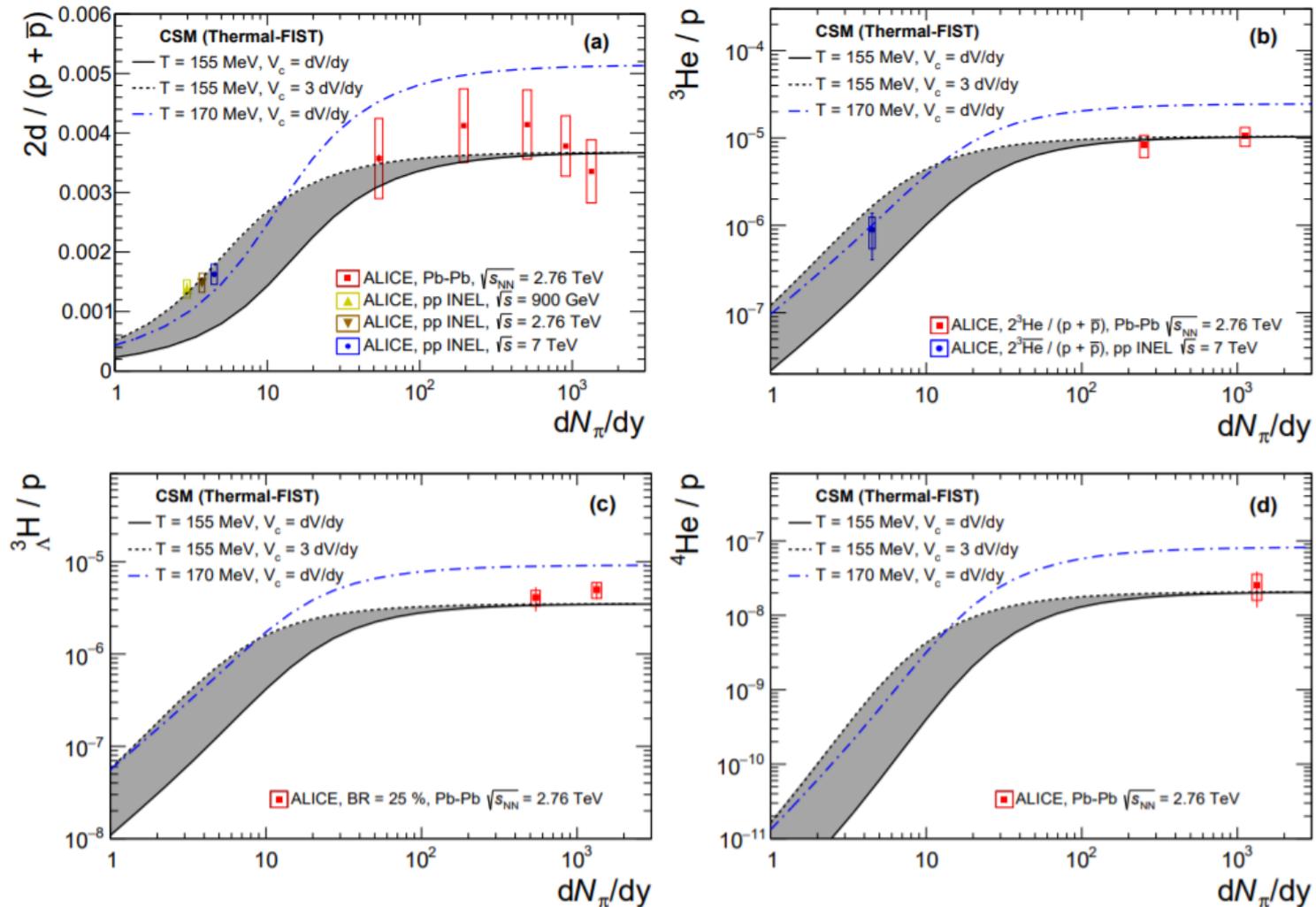
Using ALICE data for net-p fluctuations [1910.14396] one obtains $k \sim 3-4$ for most of the centrality bins in Pb-Pb collisions

[V.V., Dönigus, Stoecker, 1906.03145, PRC '19]

“Vanilla” CSM

$T_{ch} = 155$ MeV, $V_C = 3dV/dy$, multiplicity dependence driven by V_C only

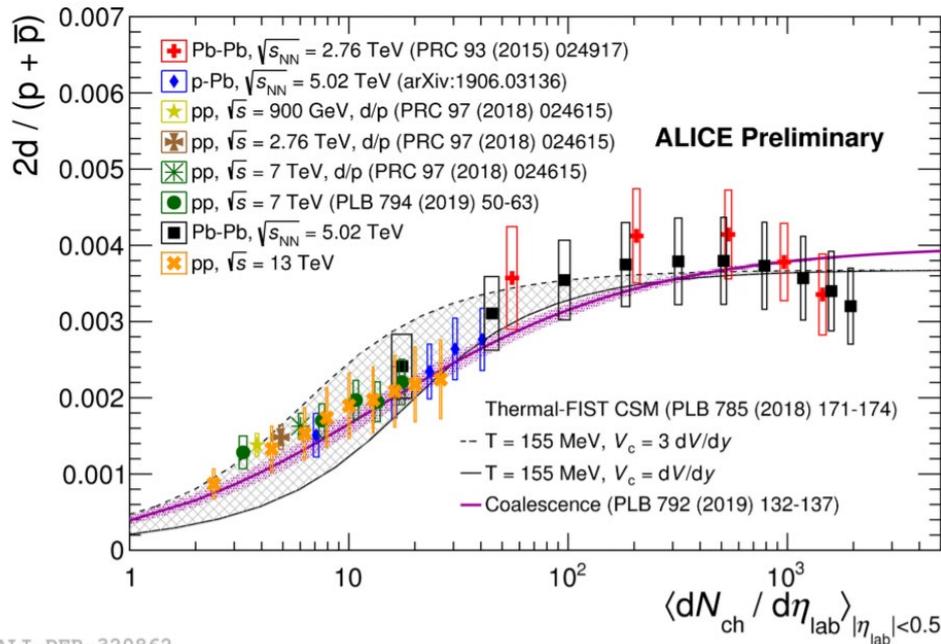
[V.V., Dönigus, Stoecker, 1808.05245, PLB '18]



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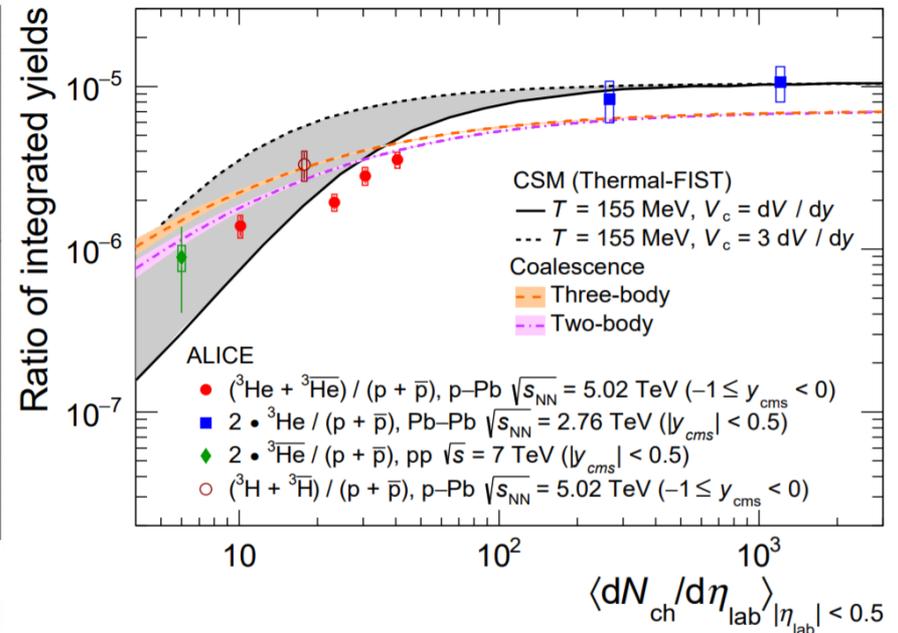
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ALI-DER-320862

[L. Barioglio, QM19]



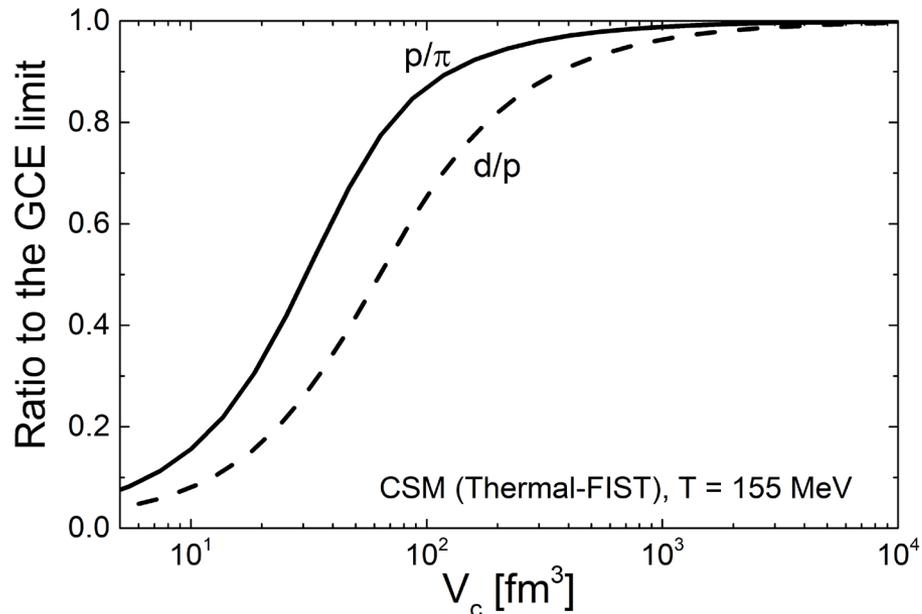
[ALICE collaboration, 1910.14401]

Basic CSM appears to capture trends seen in light nuclei production data

“Vanilla” CSM: nuclei vs p/π ratio

Canonical suppression affects not only nuclei, but also the p/π ratio

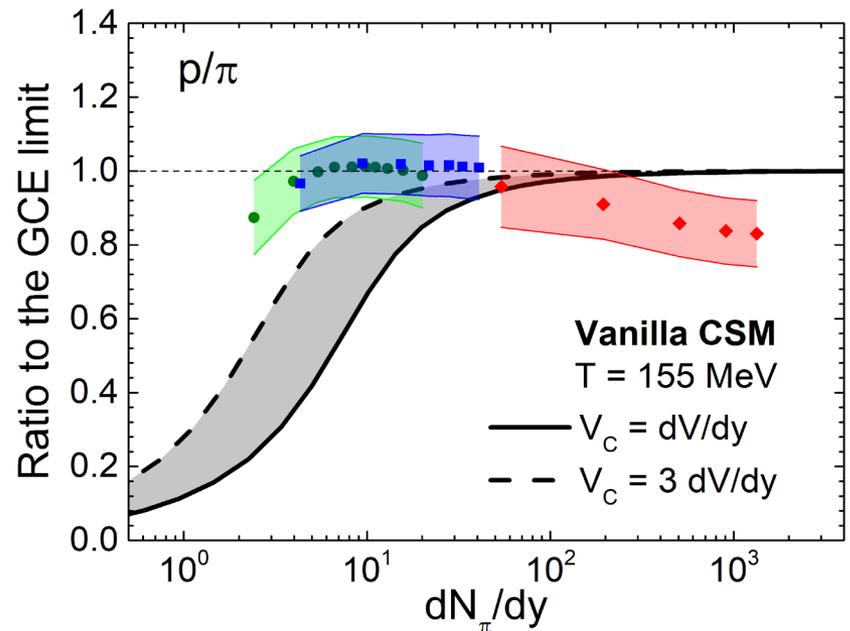
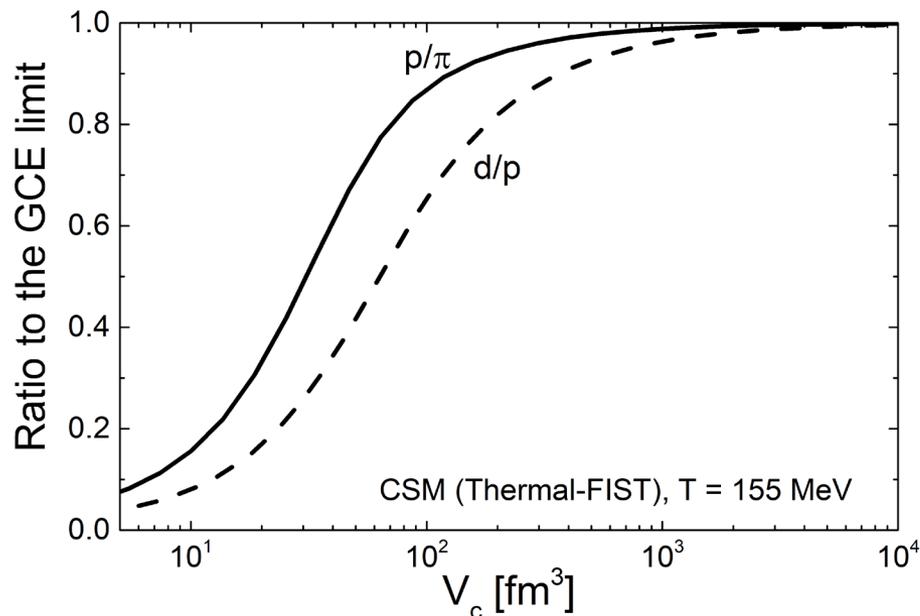
The effect for p/π is generally milder than d/p , but not insignificant



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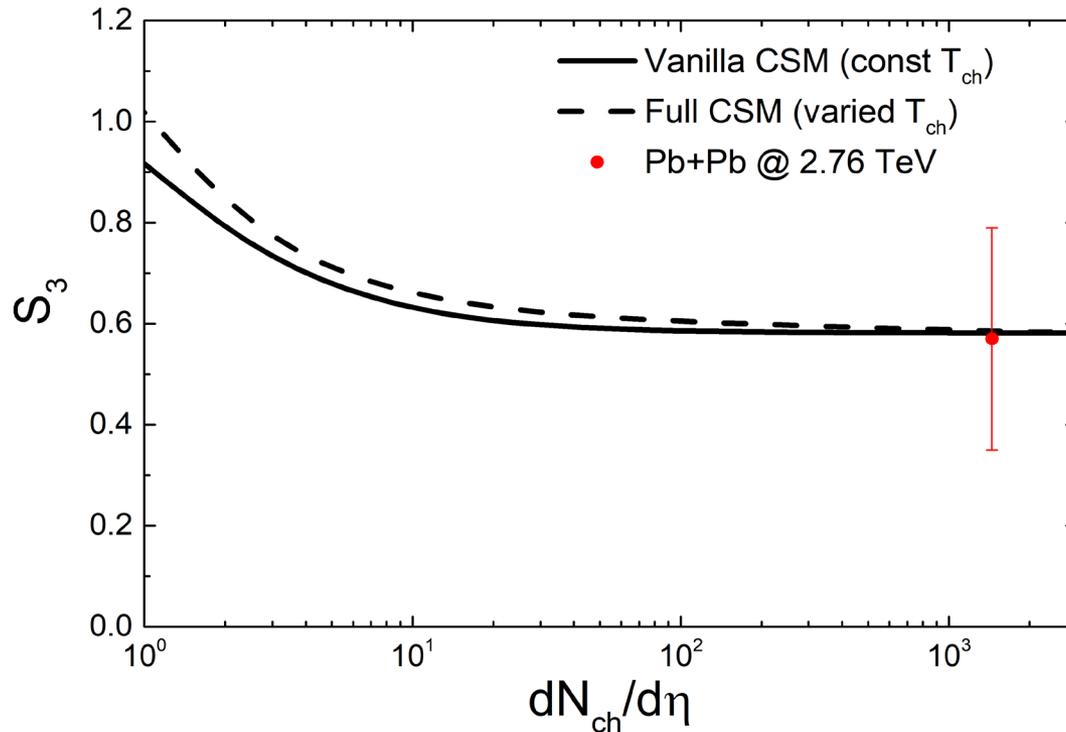
p/π suppression predicted by vanilla CSM not supported by the data

Simultaneous description of light nuclei and p/π ratio remains challenging

CSM: S_3

$$S_3 = \binom{3}{\Lambda} \text{H}/\text{He} / (\Lambda/p)$$

[E864 collab., PRC '04; Zhang et al., PLB '10]

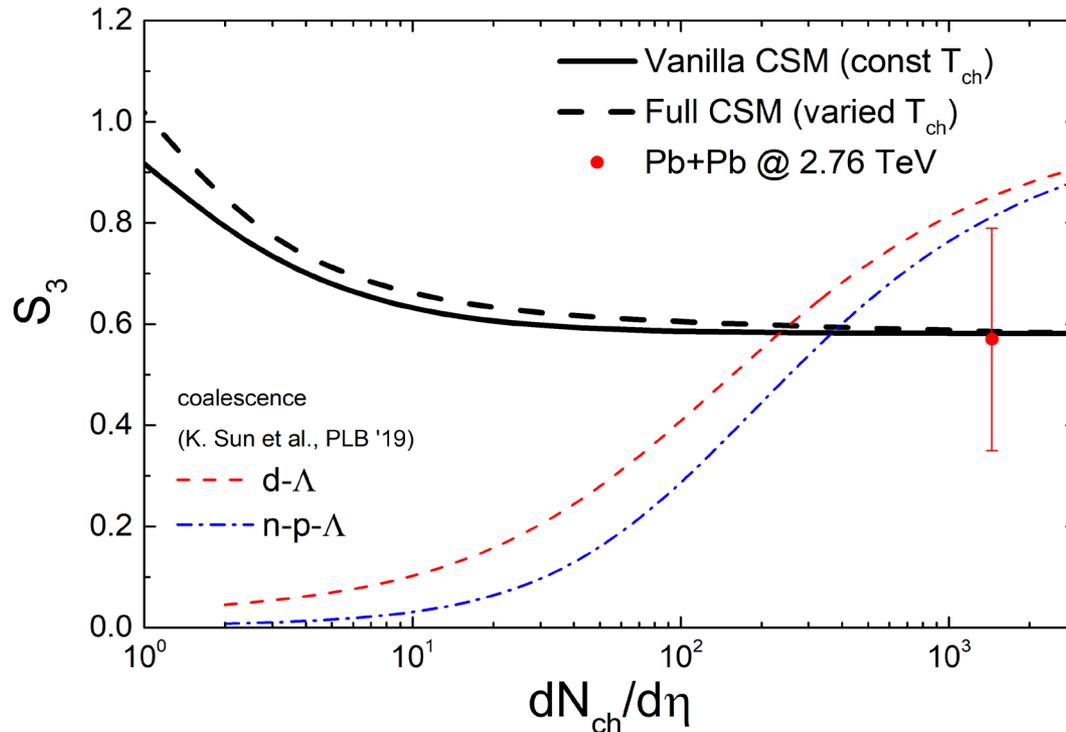


Different versions of CSM give similar predictions, mild increase of S_3 due to baryon and strangeness conservation

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Different versions of CSM give similar predictions, mild increase of S_3 due to baryon and strangeness conservation

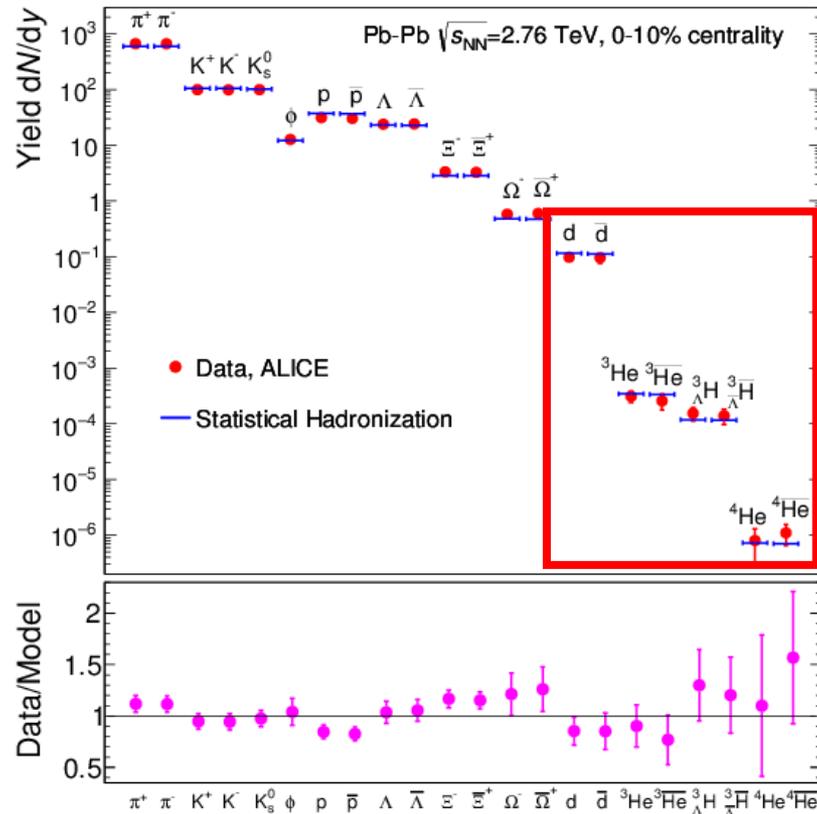
Coalescence [Sun, Dönigus, Ko, PLB '19] predicts **opposite trend**

Hadronic phase and the Saha equation approach to light nuclei production

V.V., K. Gallmeister, J. Schaffner-Bielich, C. Greiner, *Phys. Lett. B* **800**, 135131 (2020)

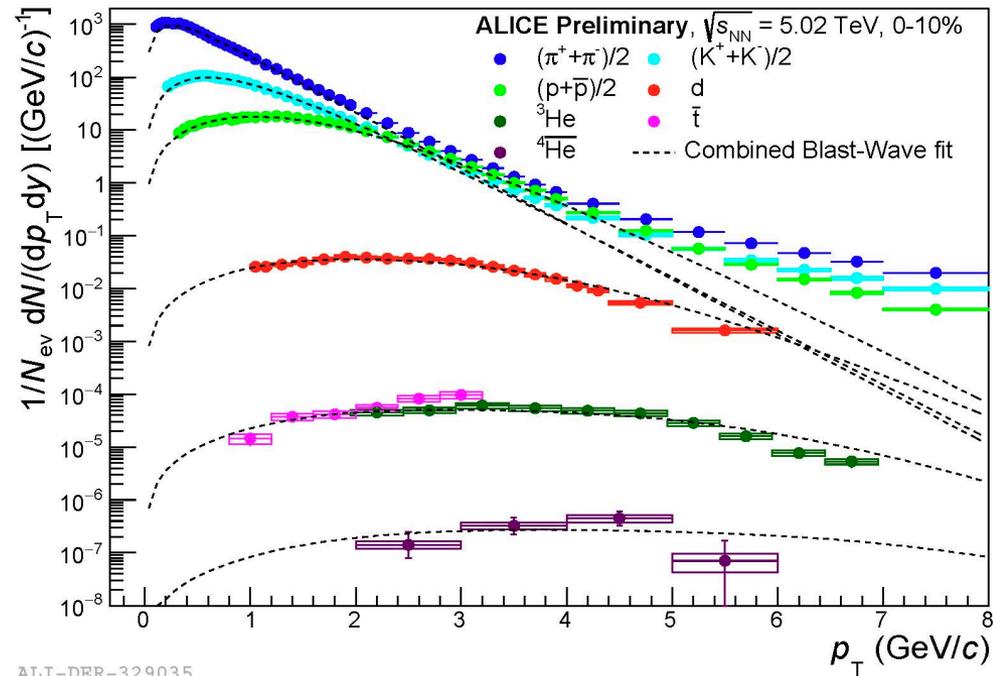
Two experimental observations at the LHC

1. Measured yields are described by thermal model at $T_{ch} \approx 155$ MeV



[A. Andronic et al., Nature **561**, 321 (2018)]

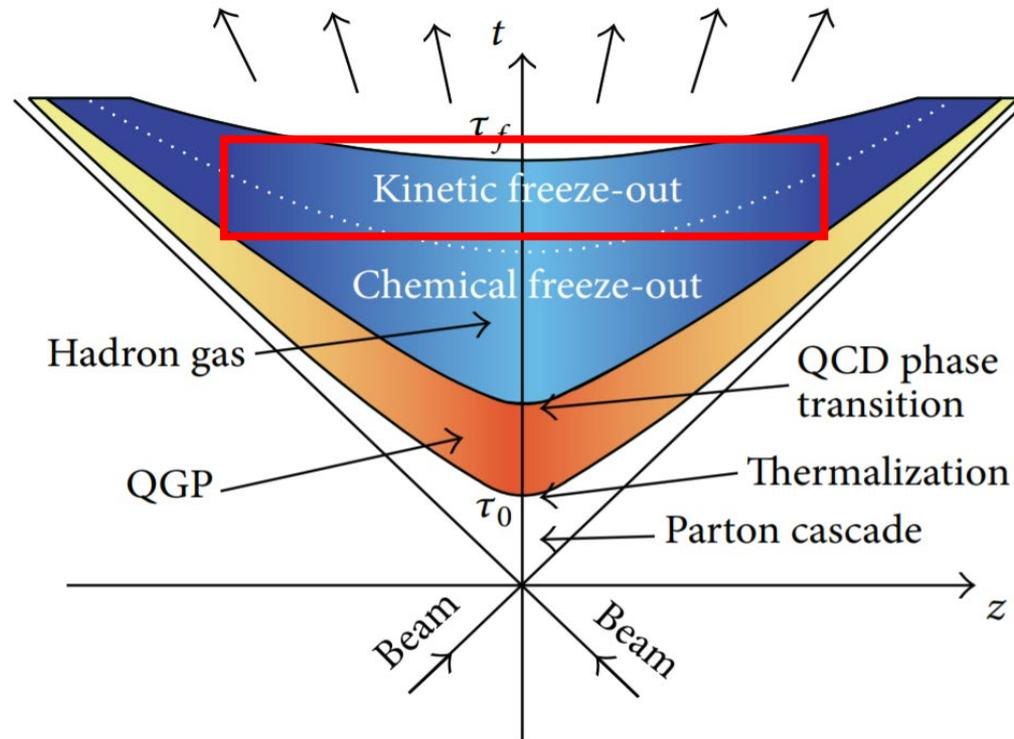
2. Spectra described by blast-wave model at $T_{kin} \approx 100 - 120$ MeV



[E. Bartsch (ALICE Collaboration), QM2019]

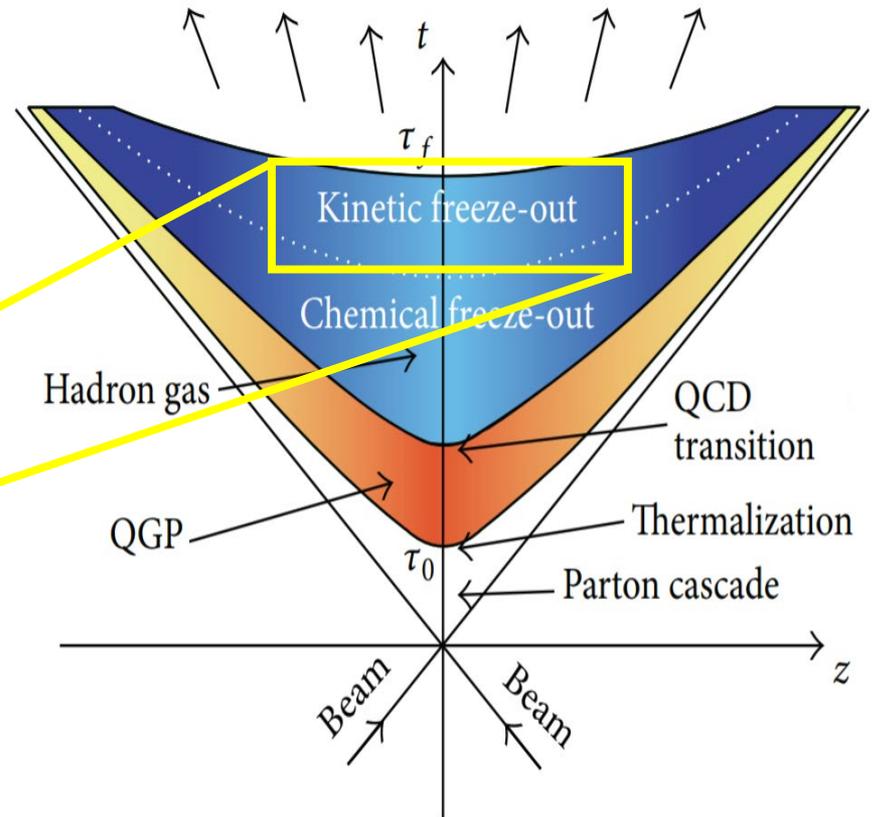
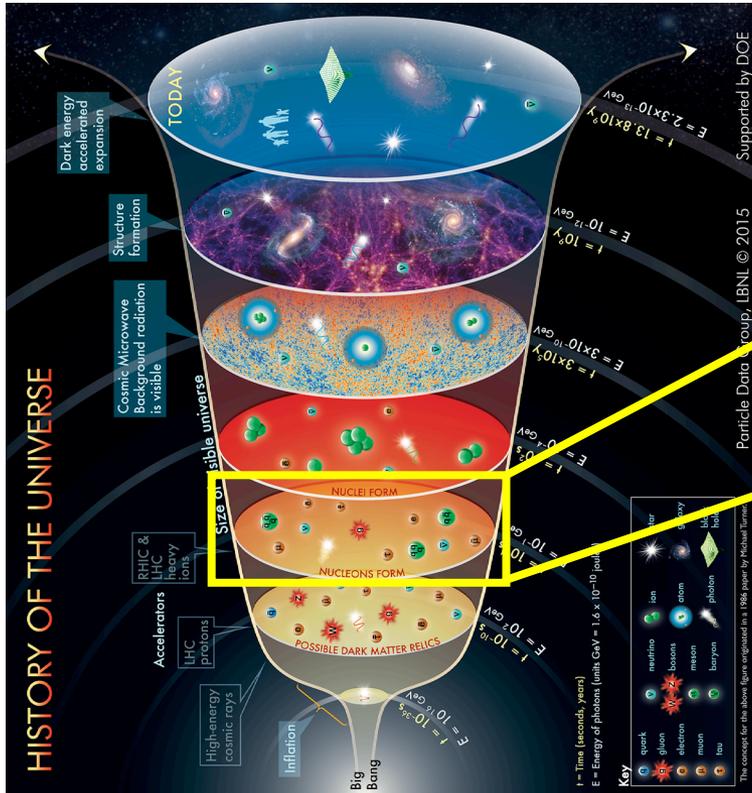
What happens between T_{ch} and T_{kin} ?

Hadronic phase in central HICs



- At $T_{ch} \approx 150 - 160 \text{ MeV}$ inelastic collisions cease, yields of hadrons frozen
- Kinetic equilibrium maintained down to $T_{kin} \approx 100 - 120 \text{ MeV}$ through (pseudo)elastic scatterings

Big Bang vs LHC “Little Bangs”



- Hadrons (nucleons) form and “freeze-out” chemically before nuclei
- Bosons (photons or pions) catalyse nucleosynthesis

$$\text{e.g. } p + n \leftrightarrow d + \gamma \quad \text{vs} \quad p + n + \pi \leftrightarrow d + \pi$$

Saha equation (1920)

- Ionization of a gas (one level)



$$\frac{n_e^2}{n_0} = \frac{2}{\lambda_e^3} \frac{g_1}{g_0} \exp(-\epsilon/T)$$

$$n_1 = n_e$$

λ_e : deBroglie

Megh Nad Saha, Phil. Mag. Series 6 40:238 (1920) 472

- Equivalently, chemical potentials: $\mu_0 = \mu_1 + \mu_e$

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Nuclear equivalent: detailed balance in an expanding system (early universe/HIC)

Deuteron number evolution through $pnX \leftrightarrow dX$, in kinetic equilibrium

$$\frac{dN_d}{d\tau} = \overset{\text{gain}}{\langle \sigma_{dX} v_{rel} \rangle N_d^0 n_x^0 e^{\mu_p/T} e^{\mu_n/T} e^{\mu_X/T}} - \overset{\text{loss}}{\langle \sigma_{dX} v_{rel} \rangle N_d^0 n_x^0 e^{\mu_d/T} e^{\mu_X/T}}$$

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gain \approx loss $\rightarrow \mu_d \approx \mu_p + \mu_n$ **Saha equation**
 = detailed balance
 = law of mass action

Partial chemical equilibrium (PCE)

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92; C.M. Hung, E. Shuryak, PRC '98]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through pseudo-elastic resonance reactions $\pi\pi \leftrightarrow \rho$, $\pi K \leftrightarrow K^*$, $\pi N \leftrightarrow \Delta$, etc.

E.g.: $\pi + 2\rho + 3\omega + \dots = const$, $N + \Delta + N^* + \dots = const$, $K + K^* + \dots = const$

Effective chemical potentials:

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \mu_i, \quad \langle n_i \rangle_j - \text{mean number of hadron } i \text{ from decays of hadron } j, \quad j \in \text{HRG}$$

Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j n_j(T, \tilde{\mu}_j) V = N_i(T_{ch}), \quad i \in \text{stable}$$

numerical solution

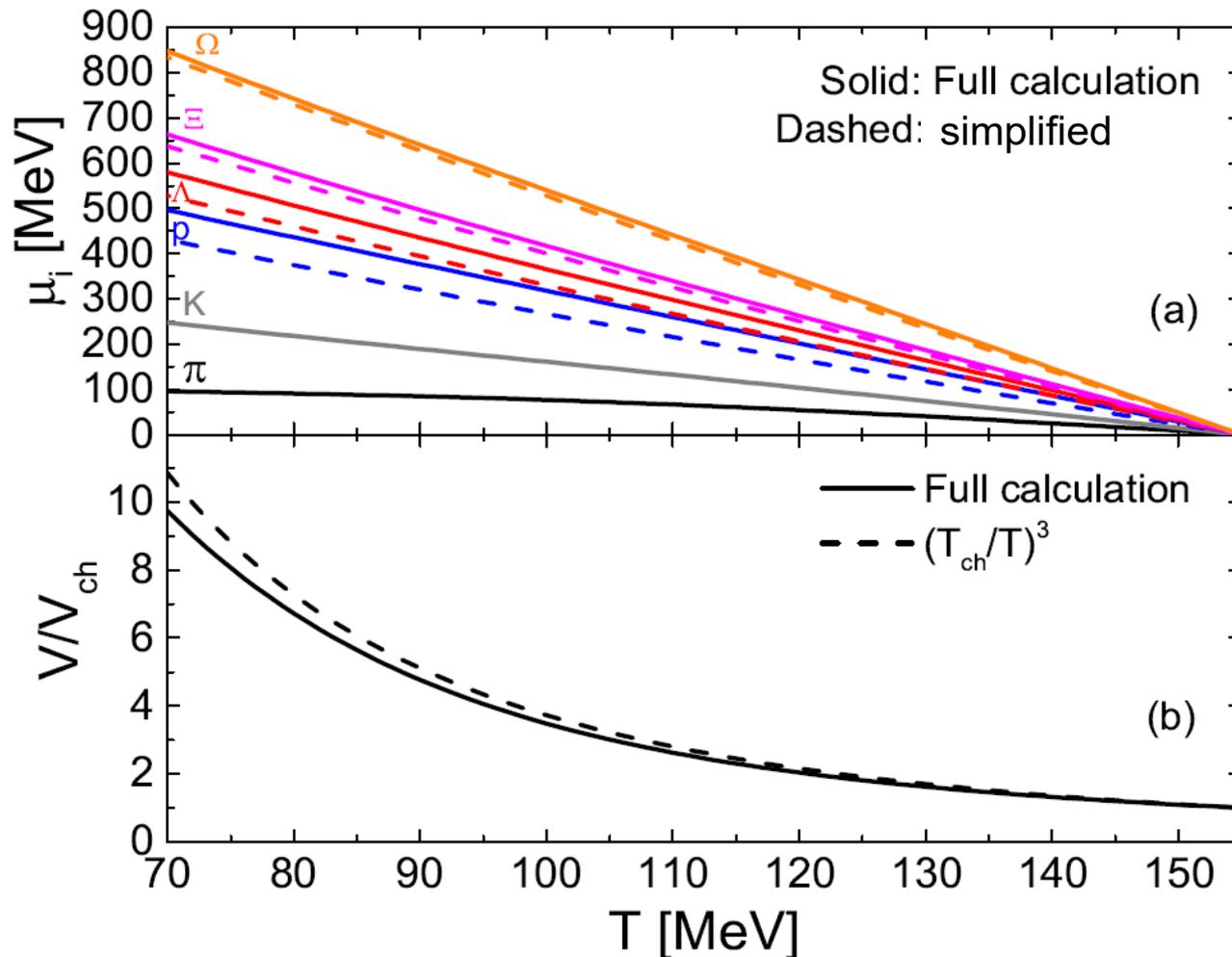

$$\{\mu_i(T)\}, V(T)$$
$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) V = S(T_{ch})$$

Numerical implementation of PCE in **Thermal-FIST**

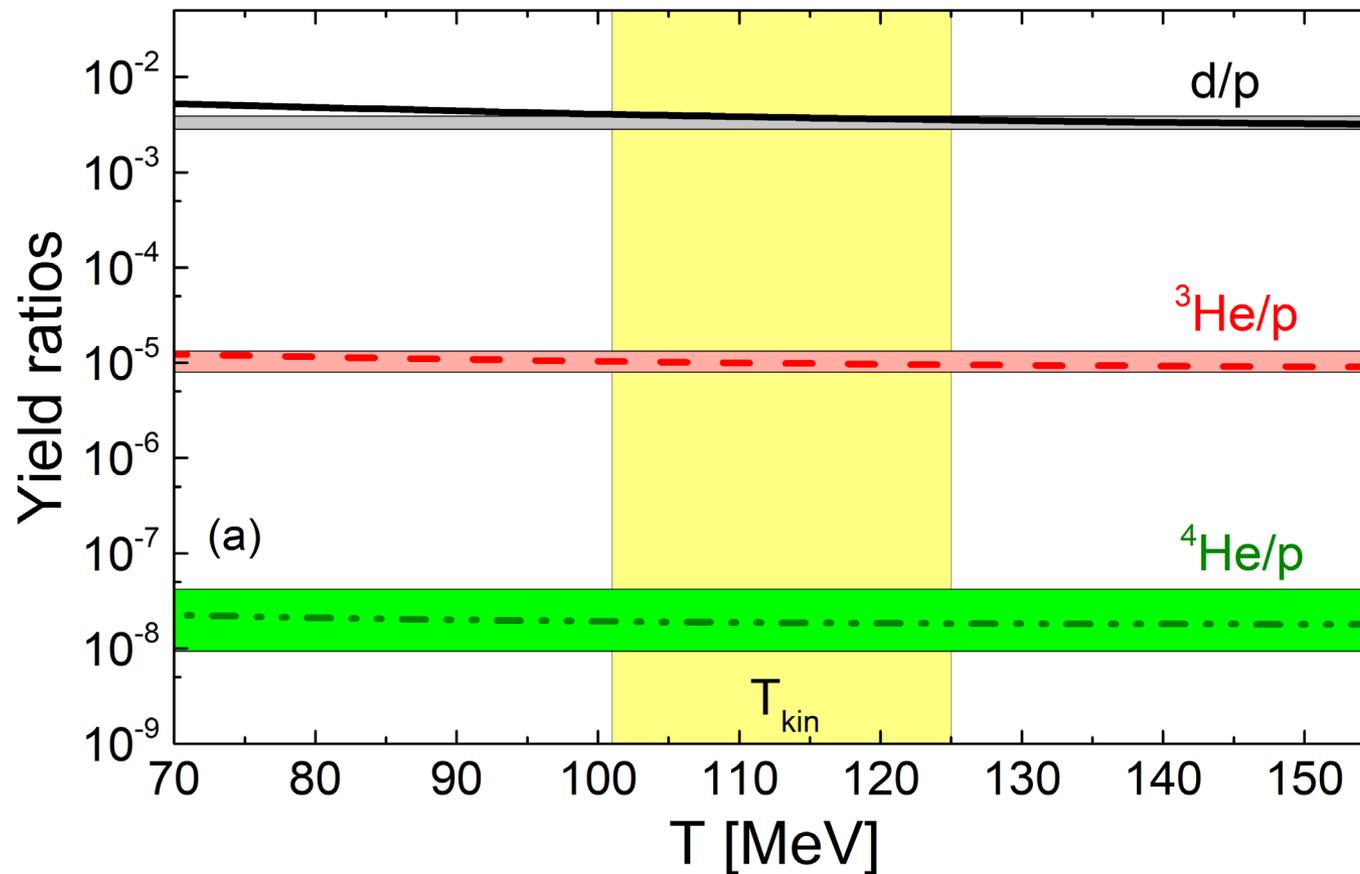
Full calculation: parameters

“Initial conditions”: $T_{ch} = 155$ MeV, $V_{ch} = 4700$ fm³ (chemical freeze-out)

values from V.V., Gorenstein, Stoecker, 1807.02079



Full calculation: nuclei

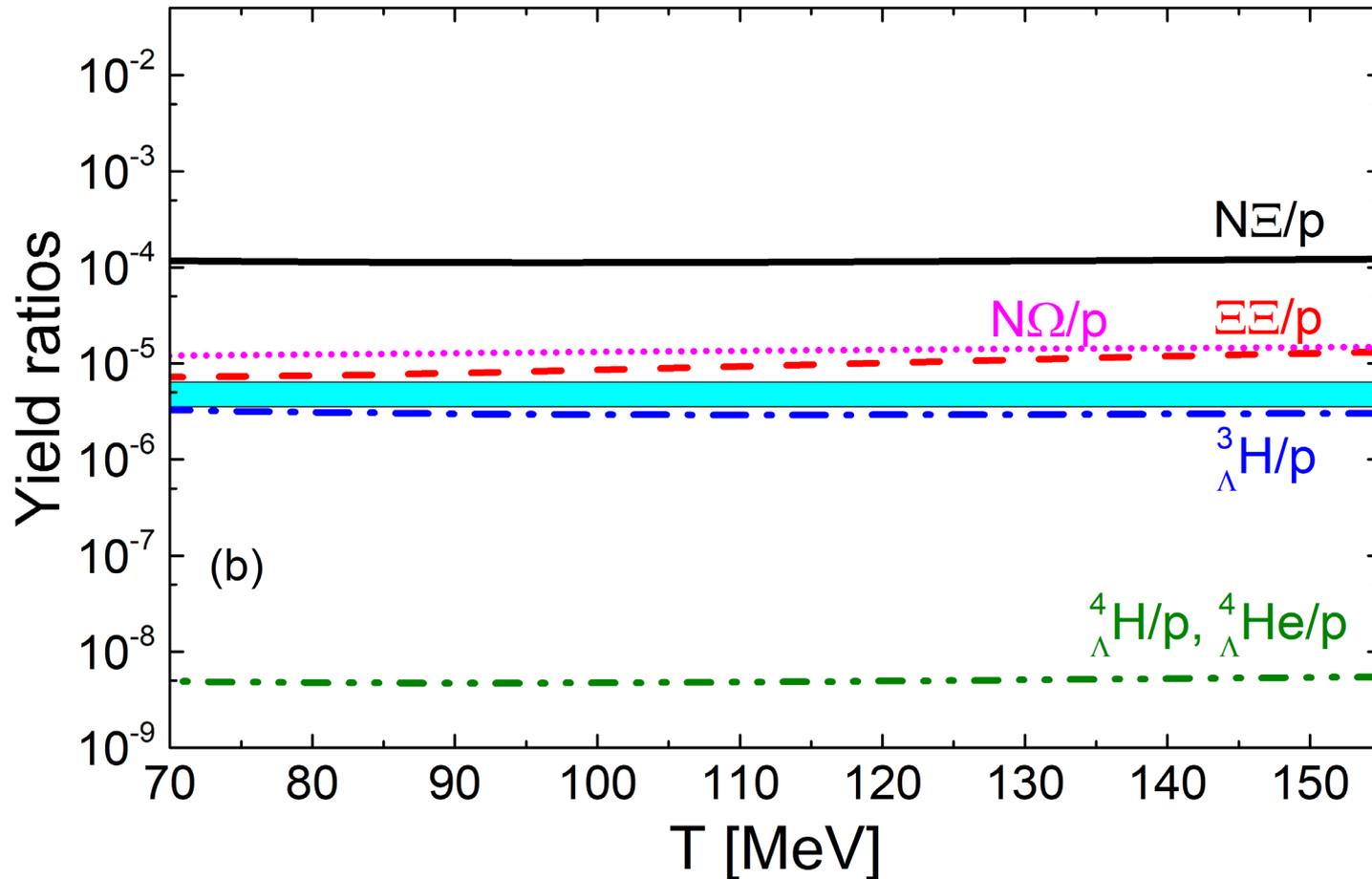


Deviations from thermal model predictions are moderate despite significant cooling and dilution. Is this the reason for why thermal model works so well?

Echoes earlier transport model conclusions for d [D. Oliinychenko, et al., PRC 99, 044907 (2019)]

For $T = T_{kin}$ similar results reported in [X. Xu, R. Rapp, EPJA 55, 68 (2019)]

Saha equation: hypernuclei

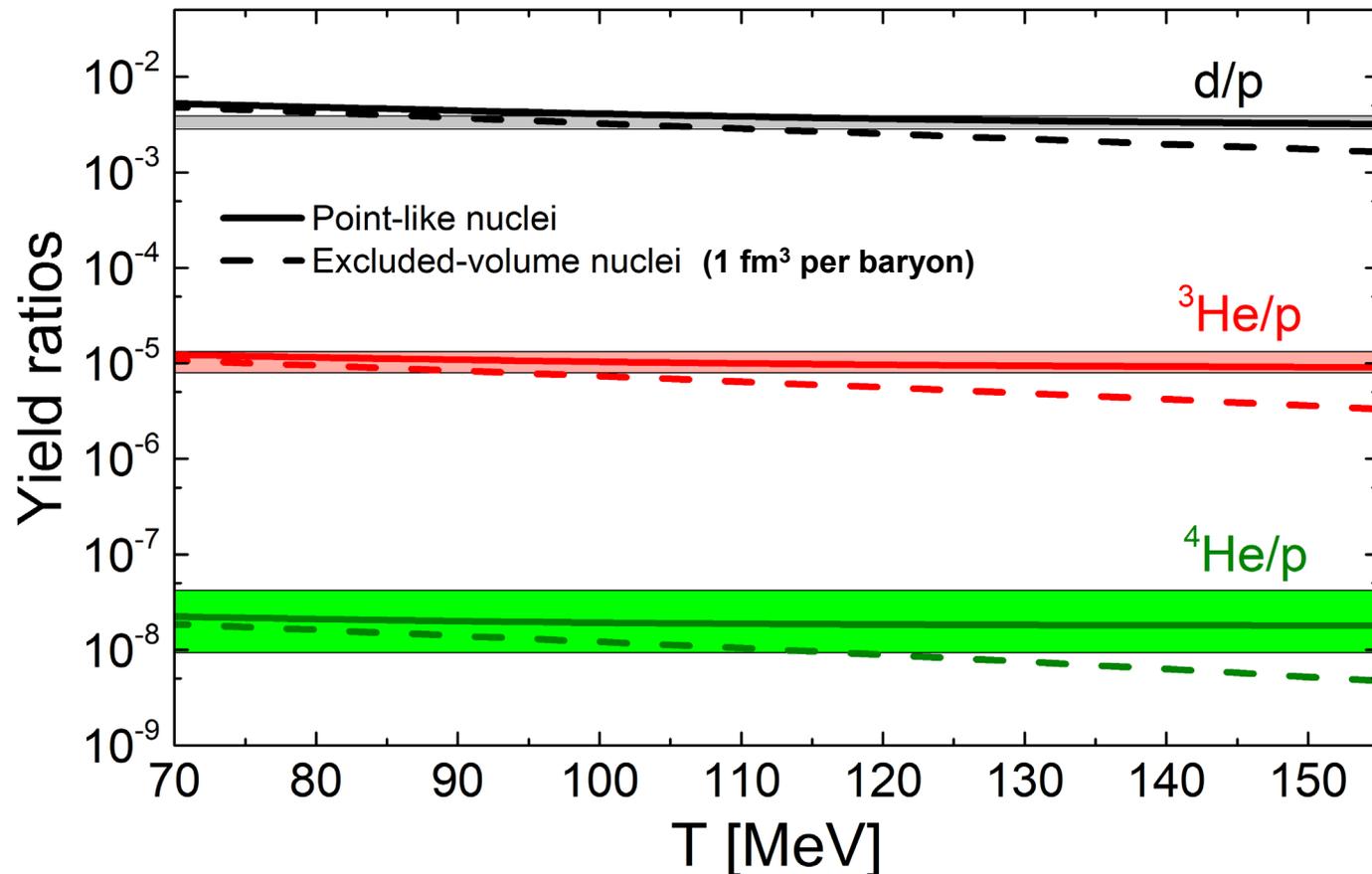


Hypernuclei stay close to the thermal model prediction. An exception is a hypothetical $\Xi\Xi$ state ← *planned measurement in Runs 3 & 4 at the LHC*

[LHC Yellow Report, 1812.06772]

Saha equation and excluded volume effects

Eigen volumes: effective mechanism for nuclei suppression at large densities



Excluded-volume effects go away as the system dilutes.

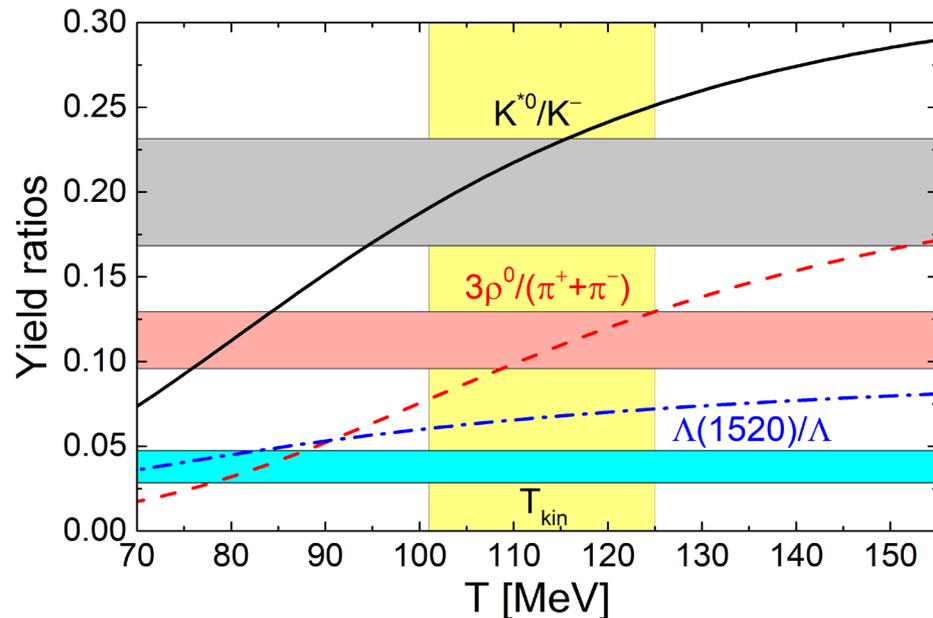
At $T \cong 100$ MeV agrees with the point-particle model and describes data.

At $T = T_{ch}$ does not describe data

Resonance suppression in hadronic phase

Yields of **resonances** are *not* conserved in partial chemical equilibrium

E.g. K^* yield dilutes during the cooling through reactions $\pi K \leftrightarrow K^*$

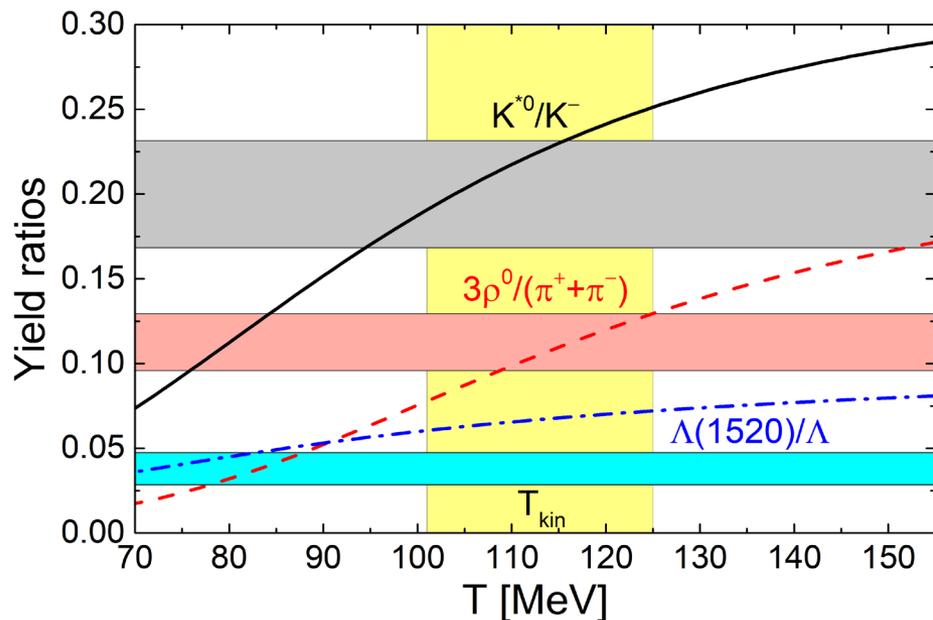


[V.V., Gallmeister, Schaffner-Bielich, Greiner, 1903.10024]

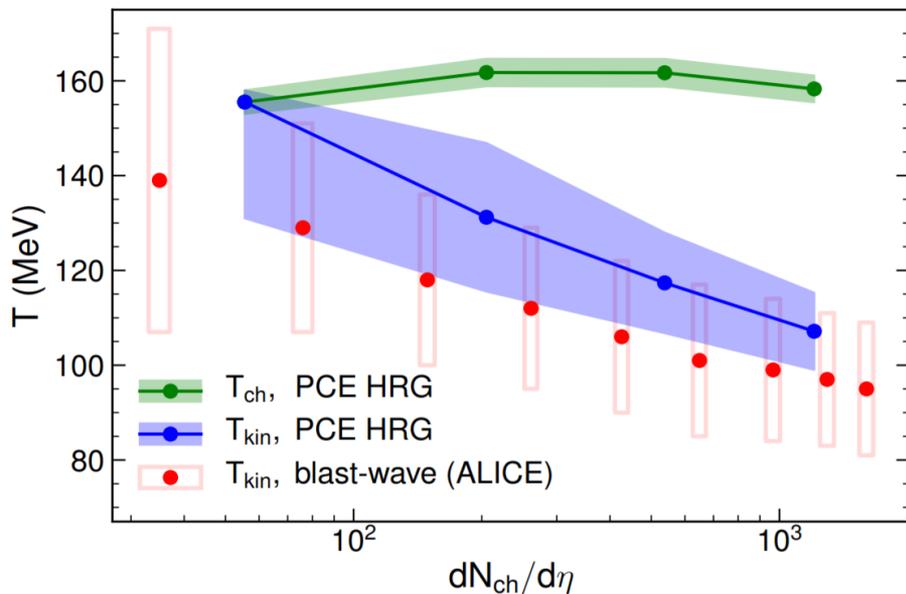
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[V.V., Gallmeister, Schaffner-Bielich, Greiner, 1903.10024]



[Motornenko, V.V., Greiner, Stoecker, 1908.11730]

Suppressed resonance yields are consistent with existence of hadronic phase

Fitting the yields of **short-lived resonances** is a new way to extract the **kinetic freeze-out temperature**

Saha equation vs rate equations

with D. Oliinychenko and V. Koch, *to appear*

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$$\frac{dN_d}{d\tau} = \underbrace{\langle \sigma_{dX} v_{rel} \rangle}_{\text{small}} \underbrace{N_d^0 n_x^0}_{\text{big}} e^{\mu_p/T} e^{\mu_n/T} e^{\mu_X/T} - \underbrace{\langle \sigma_{dX} v_{rel} \rangle}_{\text{small}} \underbrace{N_d^0 n_x^0}_{\text{big}} e^{\mu_d/T} e^{\mu_X/T}$$

gain **loss**

gain \approx loss

\rightarrow

$$\mu_d \approx \mu_p + \mu_n$$

Saha equation

= detailed balance

= law of mass action

Saha equation vs rate equations

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small ~~*big*~~ ~~*big*~~

~~gain \approx loss \rightarrow $\mu_d \approx \mu_p + \mu_n$ **Saha equation**
 = detailed balance
 = law of mass action~~

Relax the assumption of equilibrium for $AX \leftrightarrow \sum_i A_i X$ reactions

Saha equation vs rate equations

- Pion catalysis of light nuclei reactions. **Destruction** through $A\pi \rightarrow \sum_i A_i\pi$ and **creation** through $\sum_i A_i\pi \rightarrow A\pi$. **Detailed balance principle respected** but *relative chemical equilibrium not enforced*
- Bulk hadron matter evolves in partial chemical equilibrium, unaffected by light nuclei

$$\frac{dN_A}{d\tau} = \langle \sigma_{A\pi}^{\text{in}} v_{\text{rel}} \rangle n_{\pi}^{\text{pce}} (N_A^{\text{saha}} - N_A)$$

Static fireball: $n_{\pi}^{\text{pce}}, N_A^{\text{saha}}, \langle \sigma_{A\pi}^{\text{in}} v_{\text{rel}} \rangle = \text{const}$

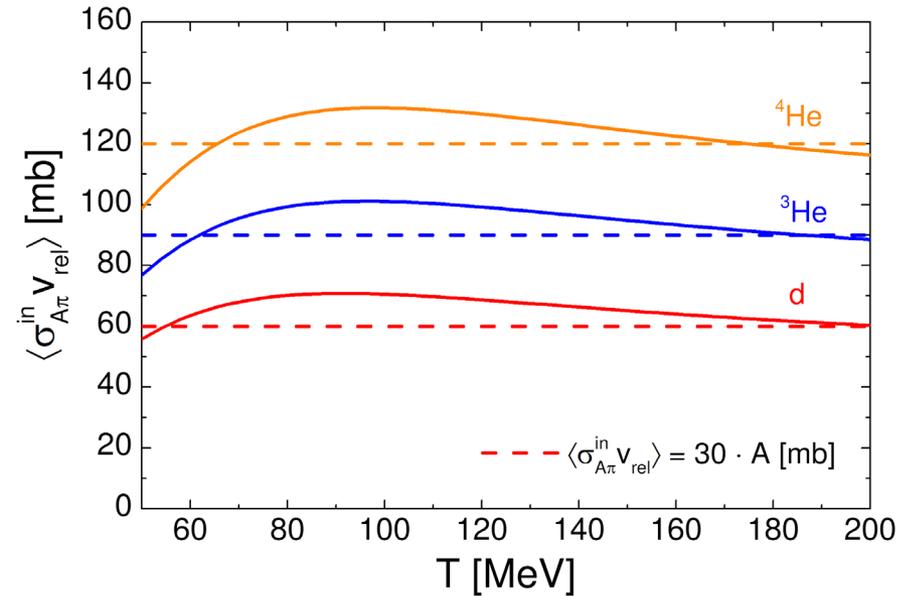
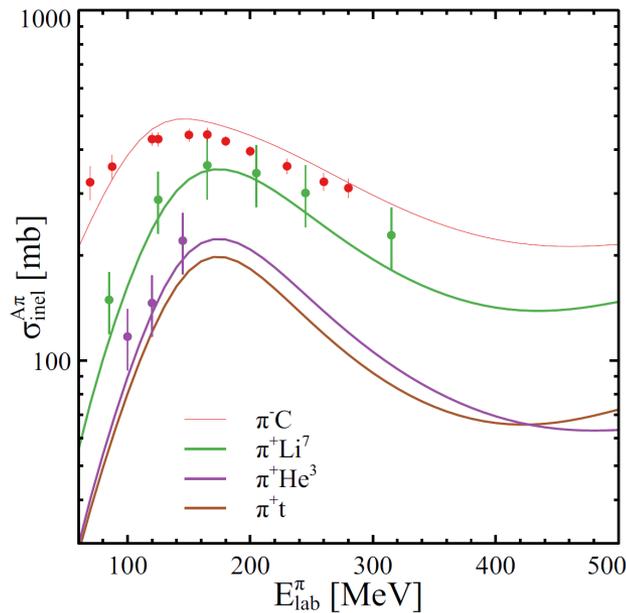
$$N_A(\tau) = N_A^{\text{saha}} + (N_A(\tau_0) - N_A^{\text{saha}}) e^{-\frac{\tau - \tau_0}{\tau_{\text{eq}}}}, \quad \tau_{\text{eq}} = \frac{1}{\langle \sigma_{A\pi}^{\text{in}} v_{\text{rel}} \rangle n_{\pi}^{\text{pce}}}$$

Saha limit: $\tau_{\text{eq}} \rightarrow 0$ ($\sigma_{A\pi}^{\text{in}} \rightarrow \infty$)

Model input

- Cross sections**

Optical model for $\sigma_{A\pi}^{\text{in}}$ [J. Eisenberg, D.S. Koltun, '80]



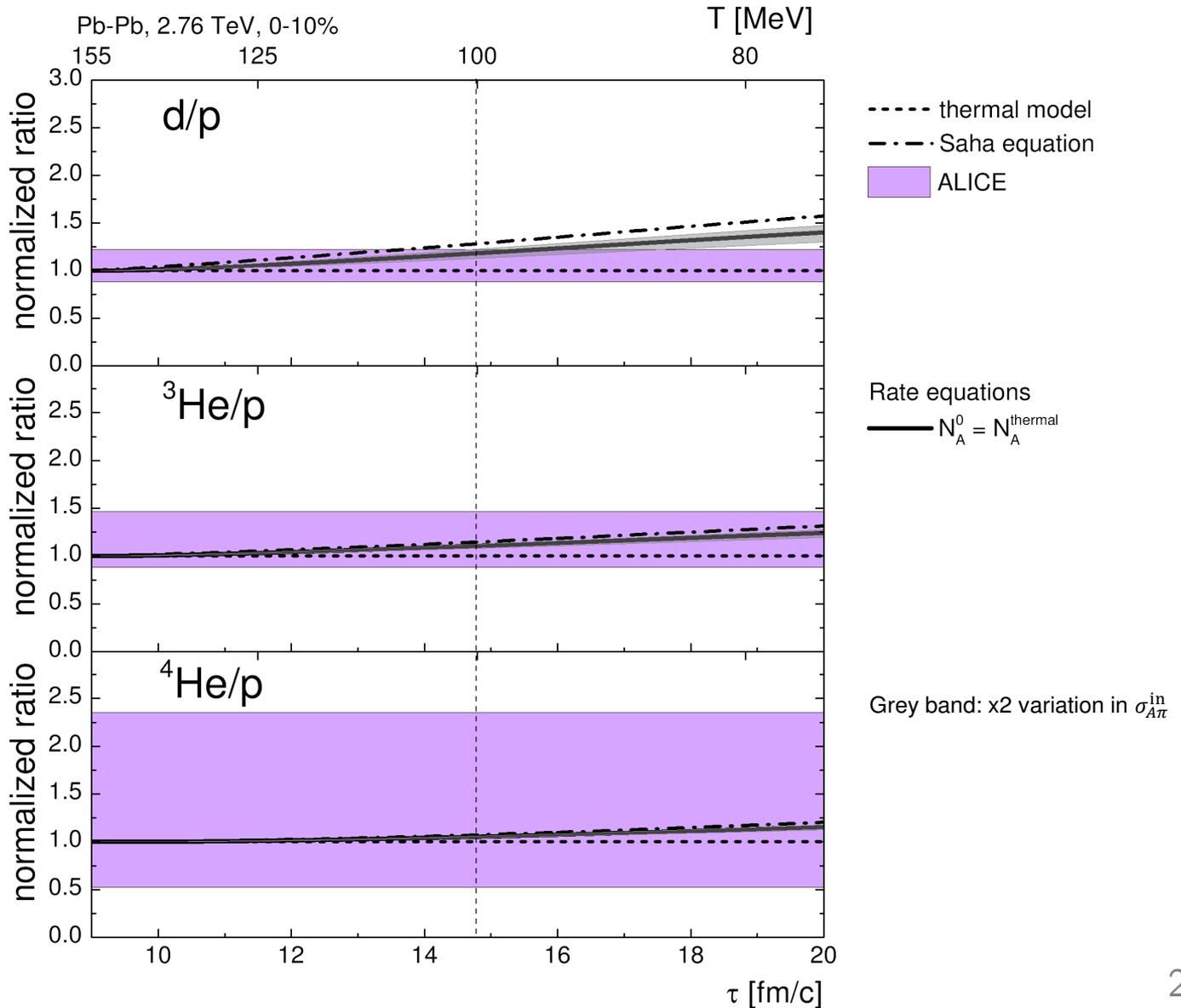
Being implemented in SMASH [D. Oliinychenko]

- Expansion** (both transverse and longitudinal)

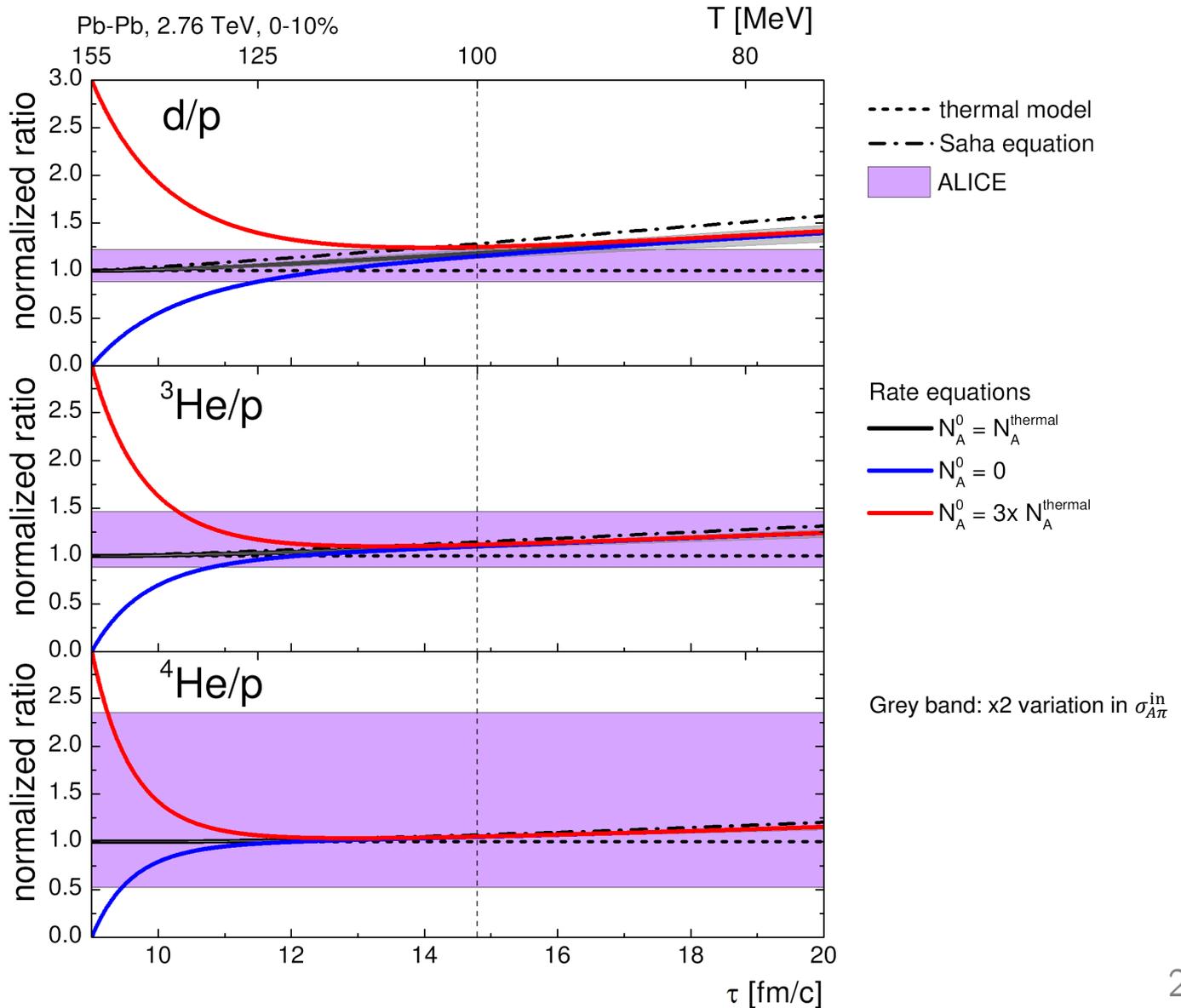
$$\frac{V}{V_{\text{ch}}} = \frac{\tau}{\tau_{\text{ch}}} \frac{\tau_{\perp}^2 + \tau^2}{\tau_{\perp}^2 + \tau_{\text{ch}}^2},$$

$$\tau_{\text{ch}} = 9 \text{ fm}, \quad \tau_{\perp} = 6.5 \text{ fm}$$

Rate equations at LHC



Rate equations at LHC



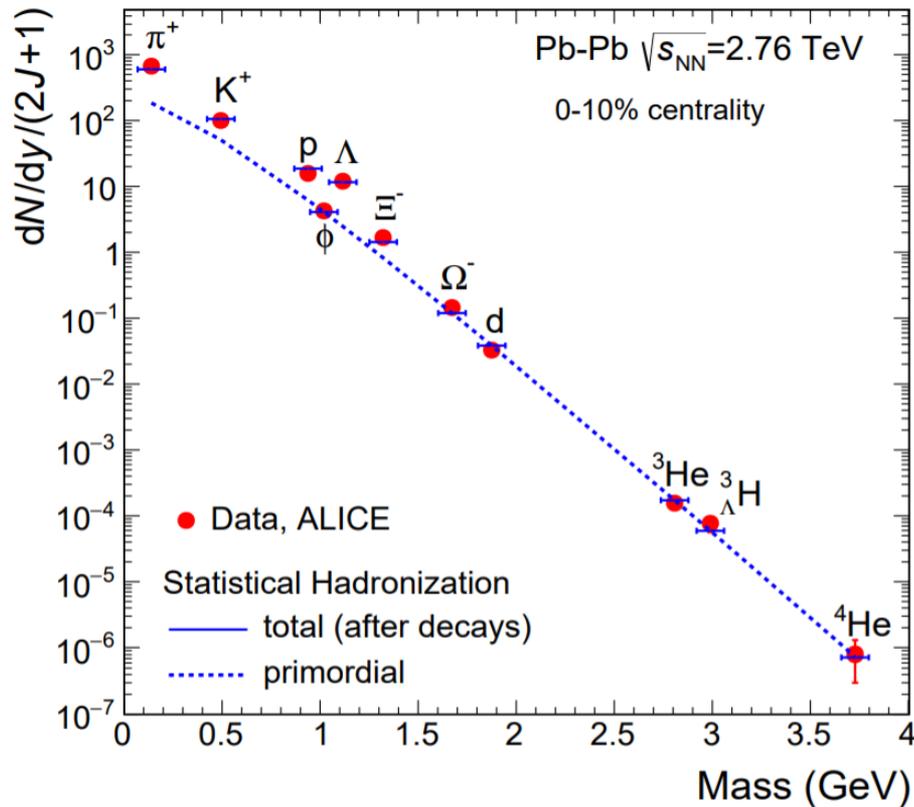
Feeddown contributions from decays of unstable nuclei

V.V., B. Dönigus, B. Kardan, M. Lorenz, H. Stoecker, [arXiv:2004.04411](#)

Feeddown in thermal model

Is well-known to be important for hadron yields

$$N_i^{\text{tot}} = N_i^{\text{prim}} + \sum_j BR(j \rightarrow i) N_j^{\text{prim}}$$



[Andronic et al., Nature (2018)]

Production of p

Primordial density = $0.0028648 \text{ fm}^{-3}$

$T = 155 \text{ MeV}$

Primordial yield = 11.4594

Total yield = 31.4347

Primordial + strong decays = 31.4347

Primordial + strong + EM decays = 31.4347

Primordial + strong + EM + weak decays = 48.5857

| Source | Multiplicity | Fraction (%) |
|--------------------------------------|--------------|--------------|
| Primordial | 11.4594 | 36.4545 |
| Decays from primordial Delta(1232)++ | 4.86466 | 15.4755 |
| Decays from primordial Delta(1232)+ | 3.24327 | 10.3175 |
| Decays from primordial Delta(1232)0 | 1.62139 | 5.15797 |
| Decays from primordial N(1520)0 | 0.5628 | 1.79038 |
| Decays from primordial Delta(1600)++ | 0.540859 | 1.72058 |
| Decays from primordial N(1520)+ | 0.436374 | 1.38819 |
| Decays from primordial N(1440)0 | 0.412215 | 1.31134 |
| Decays from primordial Delta(1600)+ | 0.3931 | 1.25053 |
| Decays from primordial N(1440)+ | 0.367071 | 1.16773 |
| Decays from primordial N(1675)+ | 0.362324 | 1.15263 |
| Decays from primordial N(1680)0 | 0.352206 | 1.12044 |

[V.V., Stoecker, CPC (2019)]

Feeddown to yields of light nuclei seldom considered in HICs

Feeddown from excited nuclei

${}^4\text{He}$

| E_x (MeV) | J^π | Decay |
|----------------|---------|--------------------|
| g.s. | 0^+ | |
| 20.21 | 0^+ | p |
| 21.01 | 0^- | p, n |
| 21.84 | 2^- | p, n |
| 23.33 | 2^- | p, n |
| 23.64 | 1^- | p, n, (γ) |
| 24.25 | 1^- | p, n, d |
| 25.28 | 0^- | p, n |
| 25.95 | 1^- | p, n, γ |
| 27.42 | 2^+ | p, n, d |
| 28.31 | 1^+ | p, n, d |
| 28.37 | 1^- | (p, n), d |
| 28.39 | 2^- | (p, n), d |
| 28.64 | 0^- | d |
| 28.67 | 2^+ | d, γ |
| 29.89 | 2^+ | (p, n), d |

${}^4\text{H}$

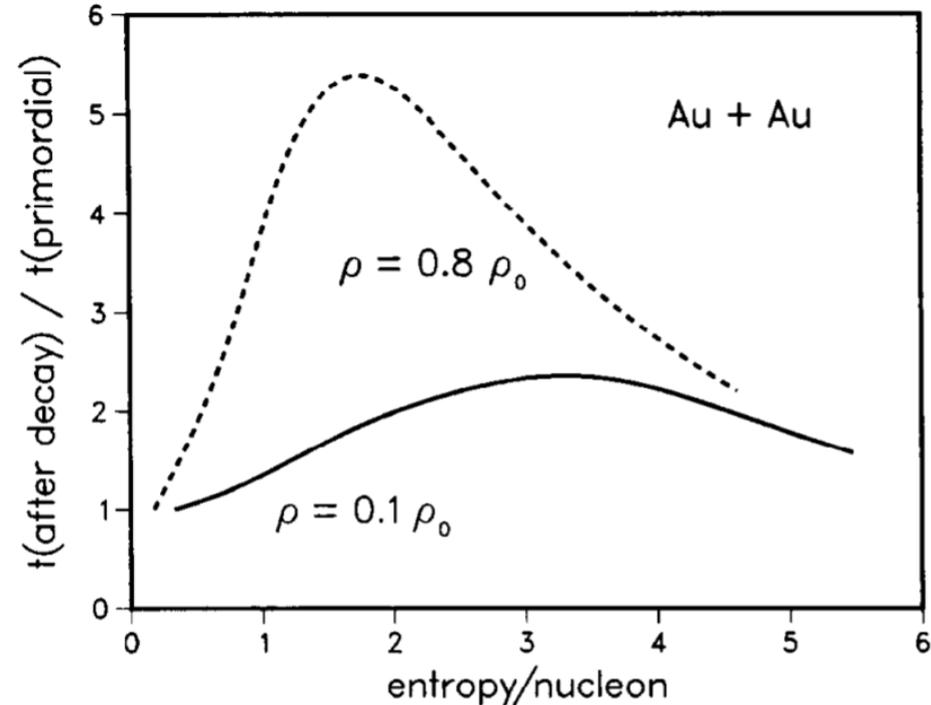
| E_x (MeV) | J^π | Decay |
|-------------------|---------|-------------------|
| g.s. ^a | 2^- | n, ${}^3\text{H}$ |
| 0.31 | 1^- | n, ${}^3\text{H}$ |
| 2.08 | 0^- | n, ${}^3\text{H}$ |
| 2.83 | 1^- | n, ${}^3\text{H}$ |

${}^4\text{Li}$

| E_x (MeV) | J^π | Decay |
|-------------------|---------|--------------------|
| g.s. ^a | 2^- | p, ${}^3\text{He}$ |
| 0.32 | 1^- | p, ${}^3\text{He}$ |
| 2.08 | 0^- | p, ${}^3\text{He}$ |
| 2.85 | 1^- | p, ${}^3\text{He}$ |

[Tilley, Weller, Hale, NPA '92]

See also <https://www.nndc.bnl.gov/>



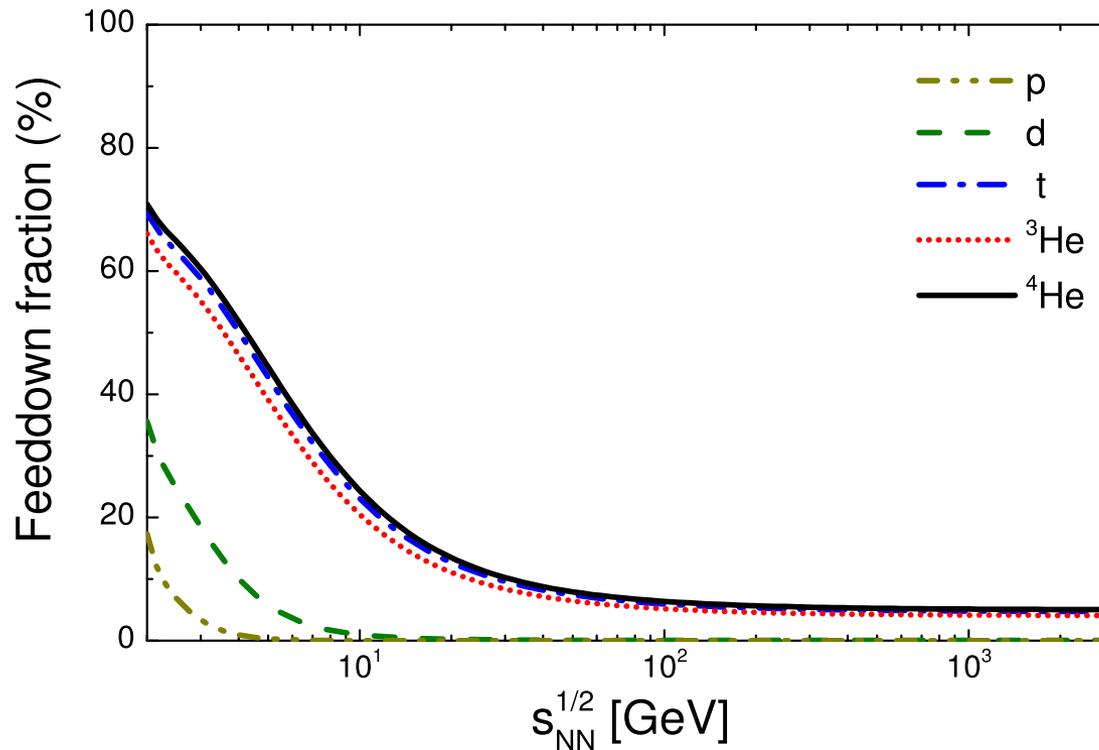
[Hahn, Stöcker, NPA '88]

In what follows feeddown from known $A = 4$ and significant $A = 5$ unstable nuclei included. Nuclei are modeled as point particles.

Relevance of excited ${}^4\text{He}$ states also recently pointed out in a baryon preclustering study [Torres-Rincon, Shuryak, 1910.08119]

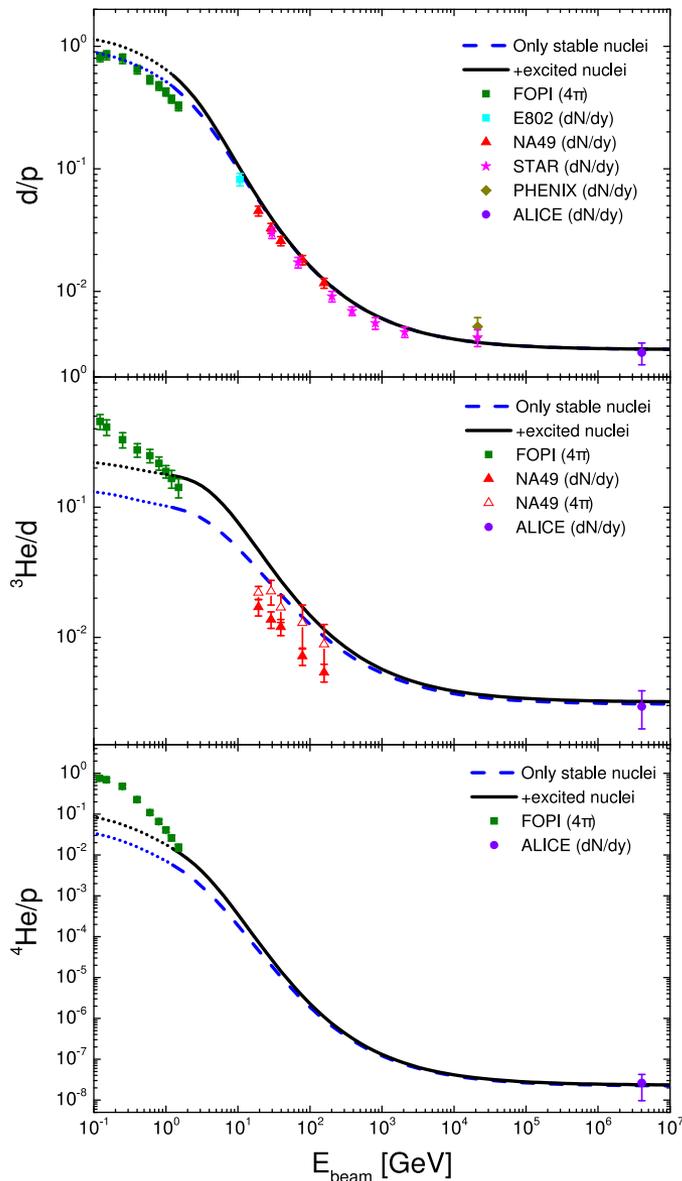
Feeddown from excited nuclei

Feeddown fraction along the phenomenological freeze-out curve



- **LHC:** 5% effect. Can be measured through p- ^3He , p- ^4He correlation?
- **RHIC/SPS:** 10-40% effect
- **GSI-HADES/FAIR:** Feeddown accounts for more than half of t, ^3He , ^4He

Feeddown from excited nuclei vs data



- **NA49** data on ${}^3\text{He}/d$ overestimated by both versions of thermal model. Large differences in dN/dy and 4π data (rapidity dependence?)

- Low energy **FOPI** data on ${}^3\text{He}/d$ and ${}^4\text{He}/d$ support nuclear feeddown but d/p data do not.

NB: chemical freeze-out curve is an extrapolation in FOPI range

- Preliminary thermal fits to HADES data favor the scenario with feeddown.
[M. Lorenz (HADES) @ EMMI workshop (Wroclaw, 2019)]

More data to come!

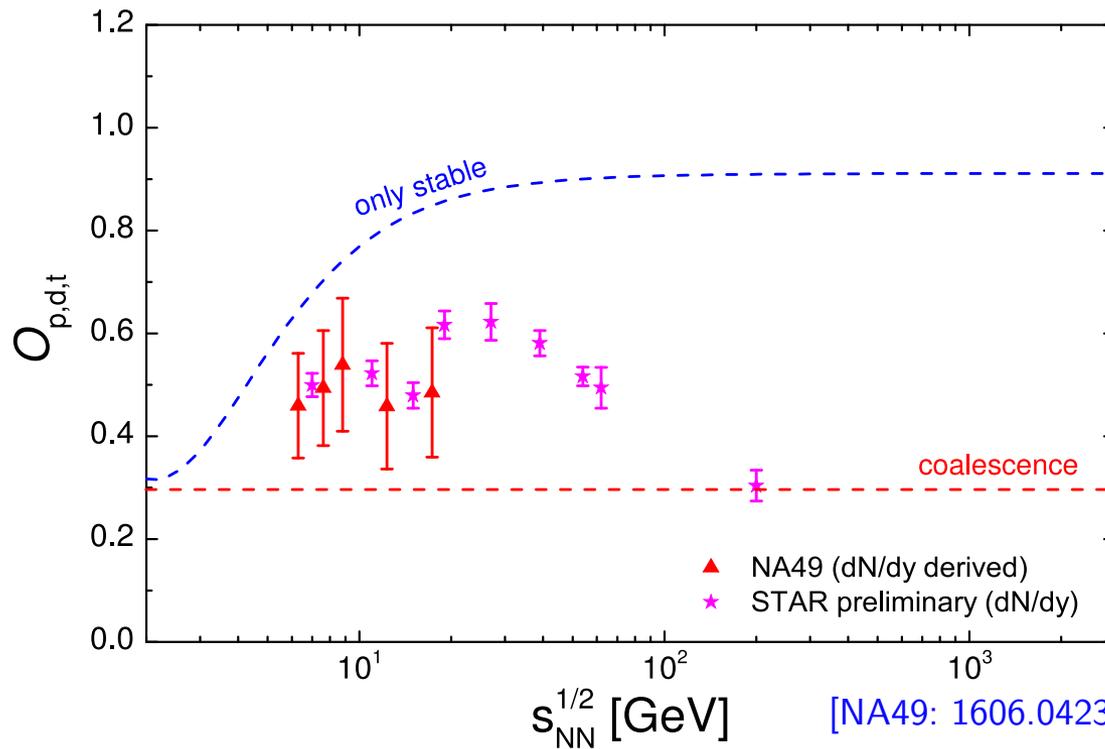
Feeddown from excited nuclei: $O_{t,p,d}$

$O_{t,p,d} = N_t N_p / (N_d)^2$ suggested as a possible probe of critical behavior

[K.J. Sun et al., PLB '17, PLB '18; H. Liu et al., PLB '20]

coalescence: $O_{p,d,t} = 1/(2\sqrt{3}) \approx 0.29$ **thermal/Saha:** $O_{p,d,t} = 1/(2\sqrt{3}) \times (1 + Res \rightarrow p)$

+excited nuclei: $O_{p,d,t} = 1/(2\sqrt{3}) \times (1 + Res \rightarrow p)(1 + Res \rightarrow t)/(1 + Res \rightarrow d)^2$



[NA49: 1606.04234; STAR: 2002.10677]

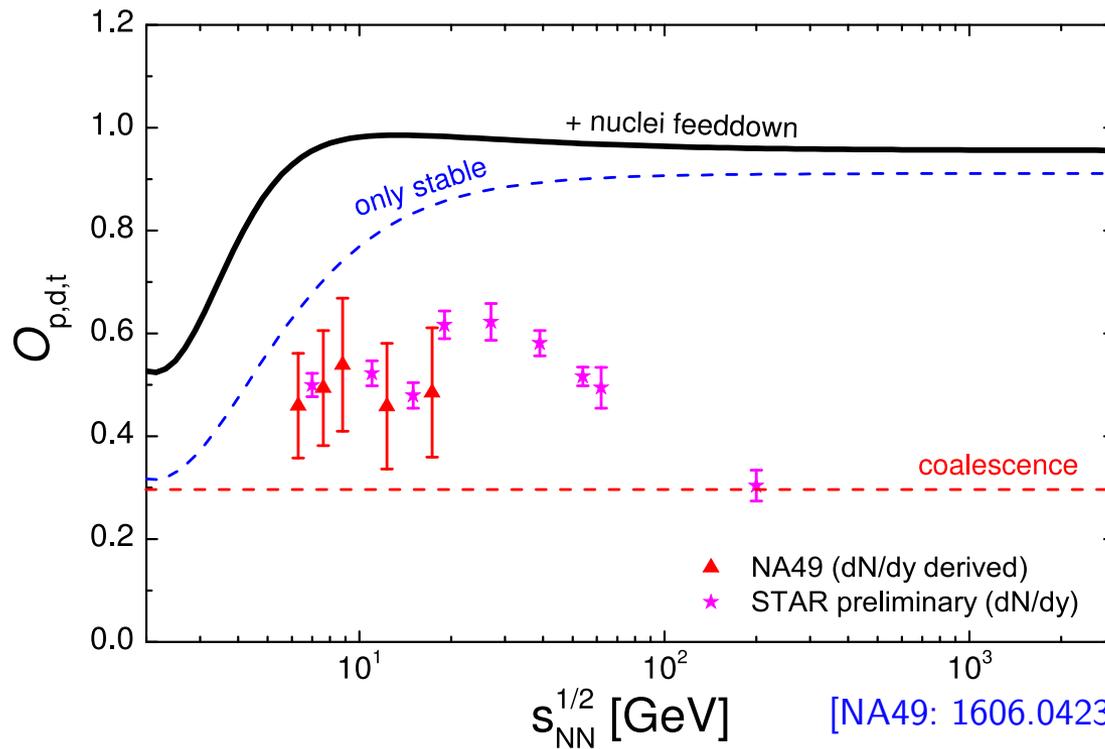
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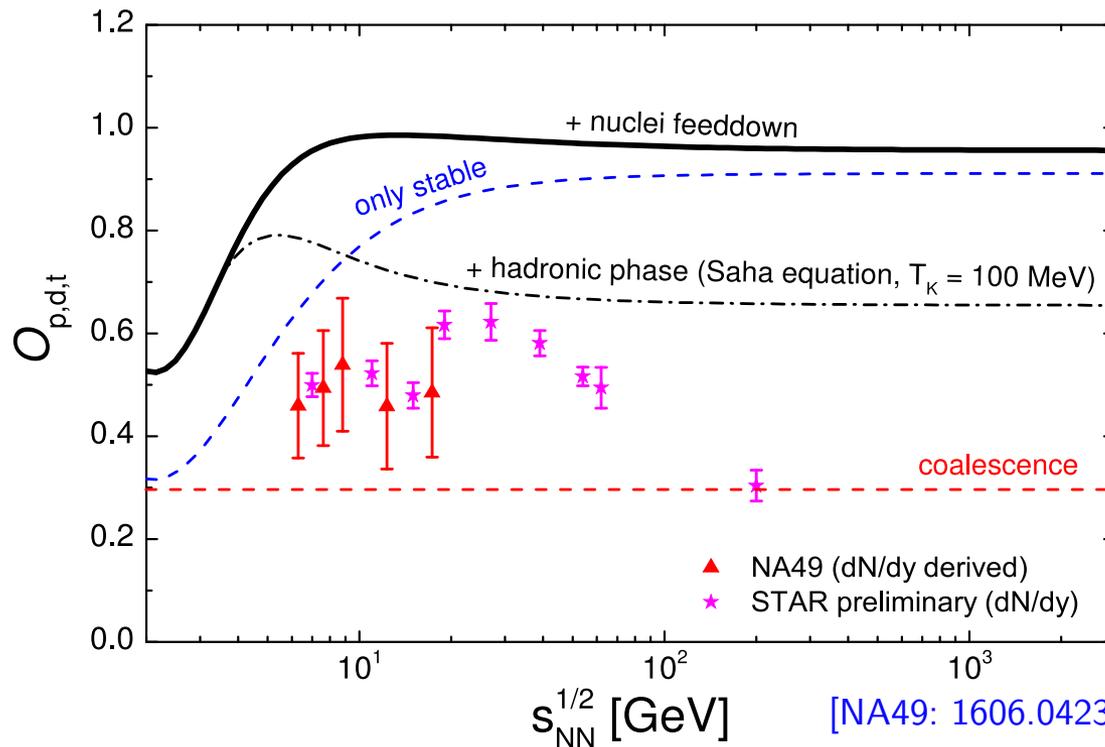
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[NA49: 1606.04234; STAR: 2002.10677]

Possible to obtain a non-monotonic behavior of $O_{t,p,d}$ in an ideal gas picture

Summary and outlook

- **Multiplicity dependence** of light nuclei abundances at the LHC is consistent with basic **canonical suppression** considerations (CSM), but no simultaneous description of the p/π ratio is achieved.
- The **Saha equation** extends the thermal approach down to the **kinetic freeze-out**, offers possible explanation why the thermal model for point-like nuclei works so well. Kinetic theory (**rate equations**) agree with the Saha equation, for *all* nuclei up to ^4He .
- Feeddown from **unstable nuclei** is **sizable** for yields of t , ^3He , ^4He at small and intermediate energies.

Summary and outlook

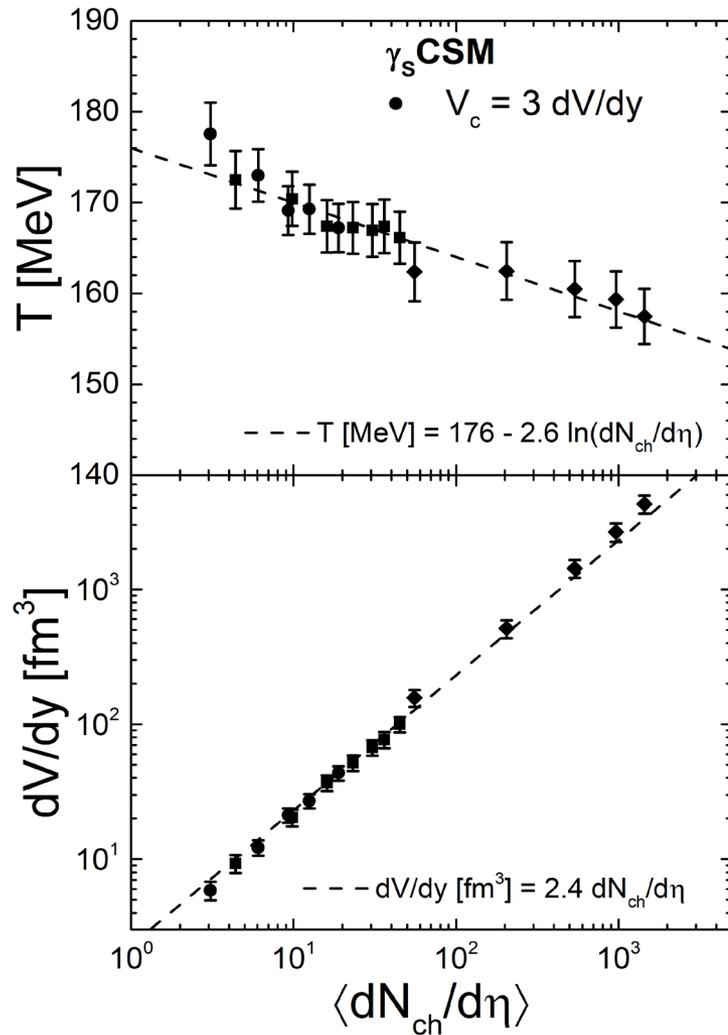
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Thanks for your attention!

Backup slides

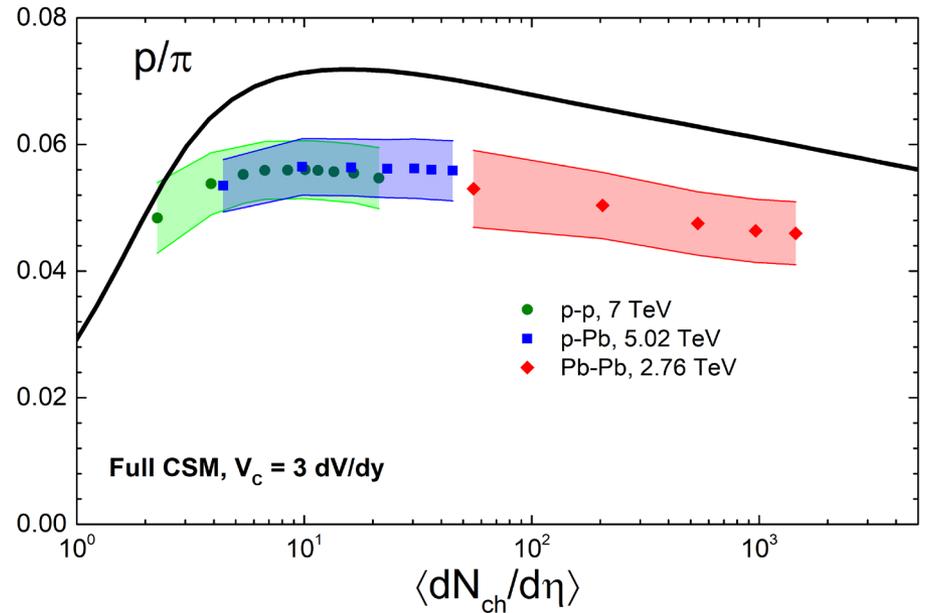
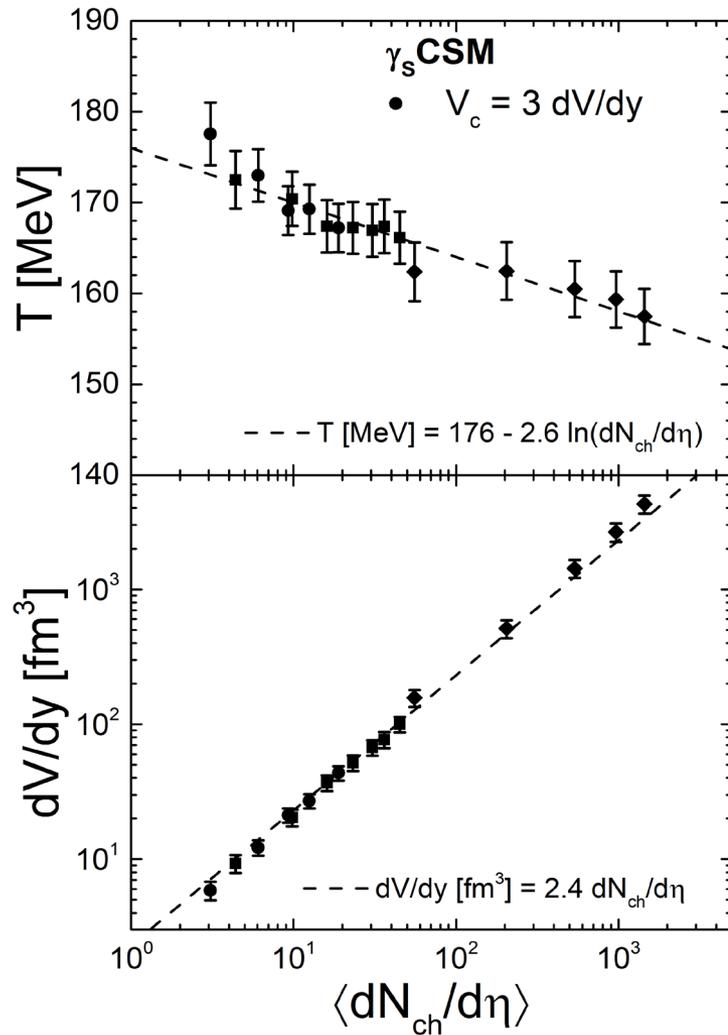
Full CSM

Full CSM: allow for **multiplicity-dependent** T_{ch} [V.V., Dönigus, Stoecker, 1906.03145, PRC '19]



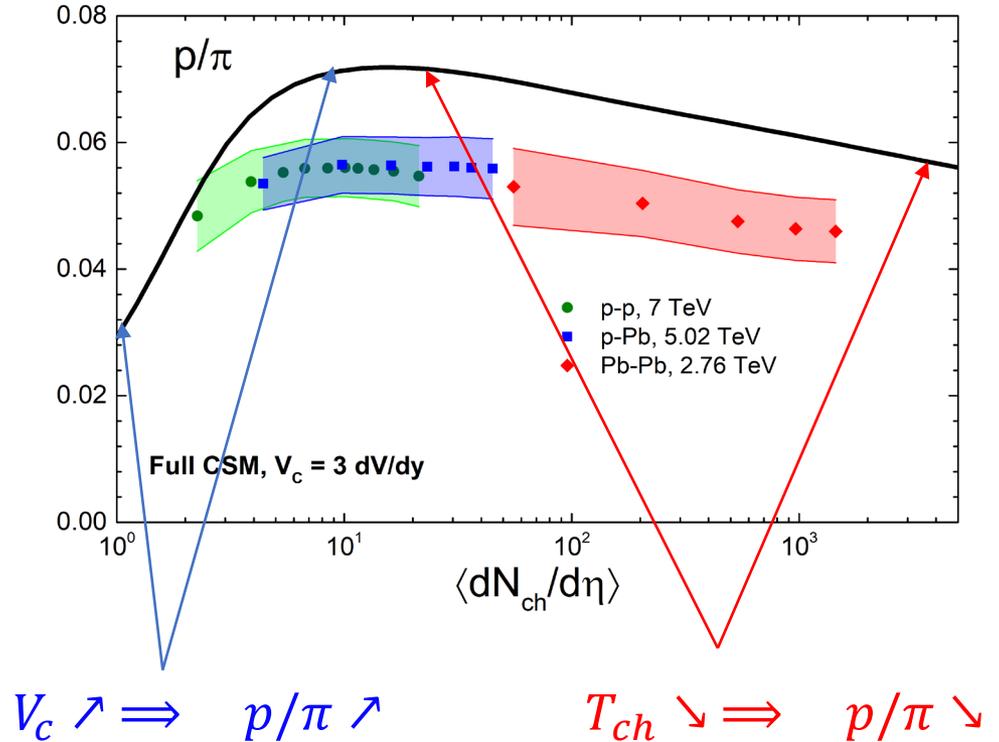
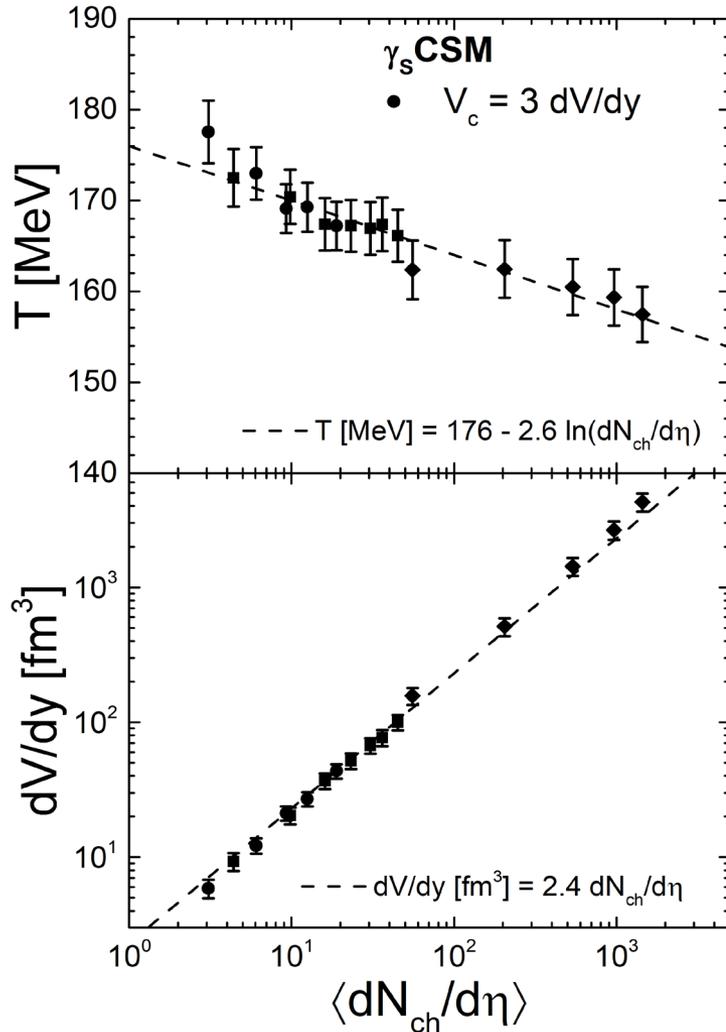
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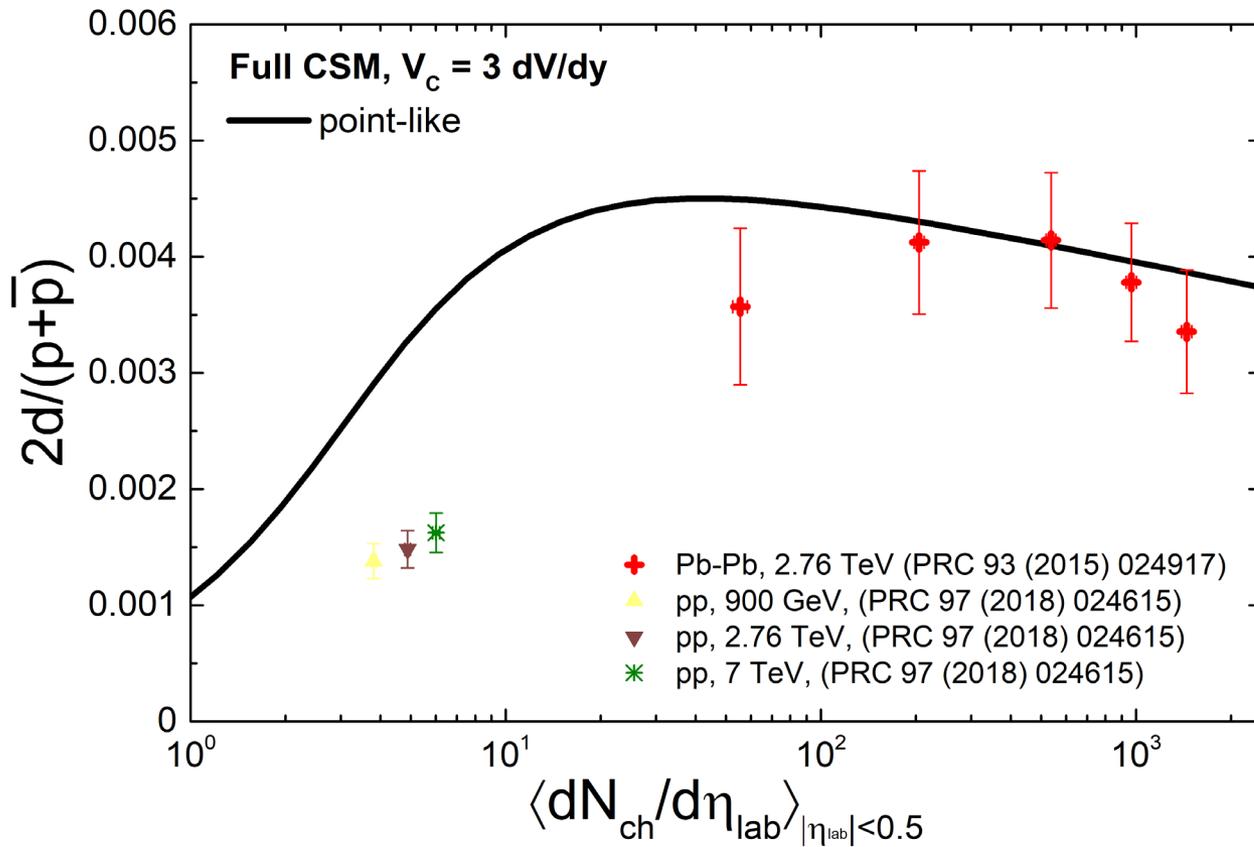


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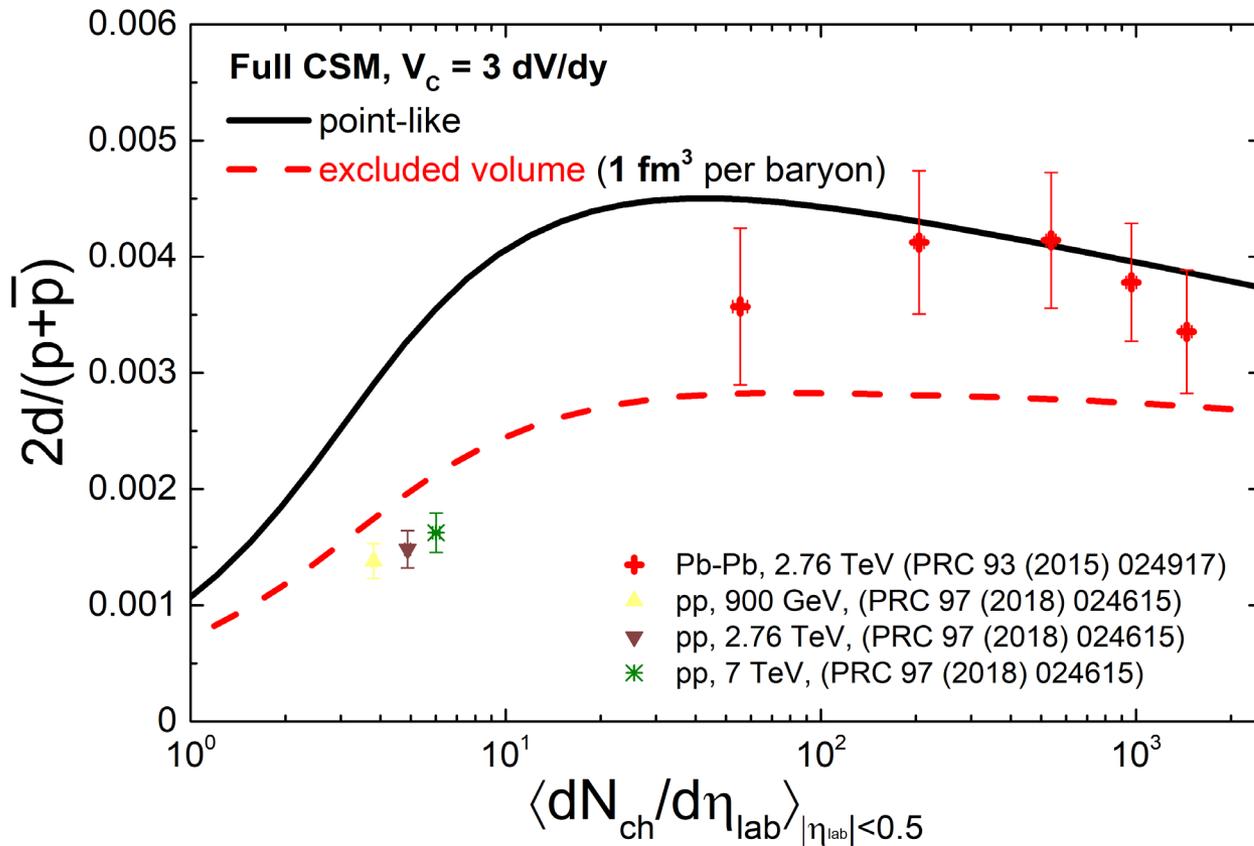


Full CSM: d/p



$T_{ch} \searrow \Rightarrow p/\pi \searrow$

Full CSM: d/p



$$T_{ch} \searrow \Rightarrow p/\pi \searrow$$

Excluded volume (schematic): $N_i \rightarrow N_i \exp\left(-\frac{v_i p}{T}\right) \Rightarrow d/p \searrow$

Simultaneous description of light nuclei and p/π ratio remains challenging

Big Bang vs LHC nucleosynthesis

Similarities:

- Inelastic nucleonic reactions freeze-out before nuclei formation
- Isentropic expansion of boson-dominated matter (photons in BBN vs mesons in HIC), baryon-to-boson ratio: $\eta_{BBN} \sim 10^{-10}$, $\eta_{LHC} \sim 0.05$
- Strong nuclear formation and regeneration reactions → **Saha equation**

Differences:

- Time scales: 1-100 s in BBN vs $\sim 10^{-22}$ s in HIC
- Temperatures: $T_{BBN} < 1$ MeV vs $T_{HIC} \sim 100$ MeV
- Binding energies, proton-neutron mass difference, and neutron lifetime important in BBN, less so in HICs
- $\mu_B \approx 0$ at the LHC, $\mu_B \neq 0$ in BBN
- Resonance feeddown important at LHC, irrelevant in BBN

LHC nucleosynthesis: BBN-like setup

- Chemical equilibrium lost at $T_{ch} = 155$ MeV, abundances of nucleons are frozen and acquire effective fugacity factors: $n_i = n_i^{eq} e^{\mu_N/T}$
- Isentropic expansion driven by effectively massless mesonic d.o.f.

$$\frac{V}{V_{ch}} = \left(\frac{T_{ch}}{T}\right)^3, \quad \mu_N \simeq \frac{3}{2} T \ln\left(\frac{T}{T_{ch}}\right) + m_N \left(1 - \frac{T}{T_{ch}}\right)$$

- Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{eq}}{\prod_i n_{A_i}^{eq}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \quad \text{e.g. } \mu_d = \mu_p + \mu_n, \mu_{3\text{He}} = 2\mu_p + \mu_n, \dots$$

Saha equation



$$X_A = d_A \left[(d_M)^{A-1} \zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{3+A}{2}} \right] A^{5/2} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03 \quad \text{fixed at } T_{ch}$$

$$\text{BBN: } X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}} \right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

[E. Kolb, M. Turner, "The Early Universe" (1990)]

(BBN-like) Saha equation vs thermal model

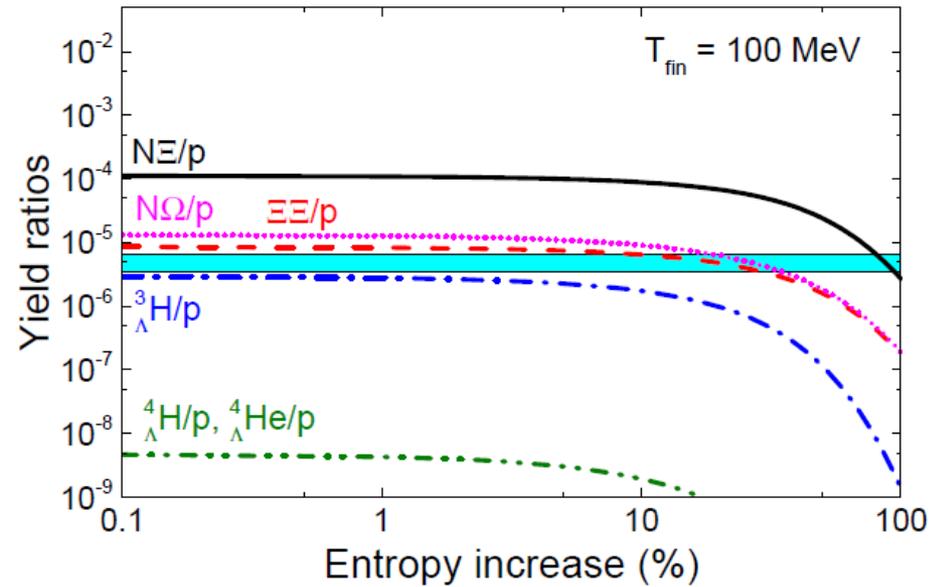
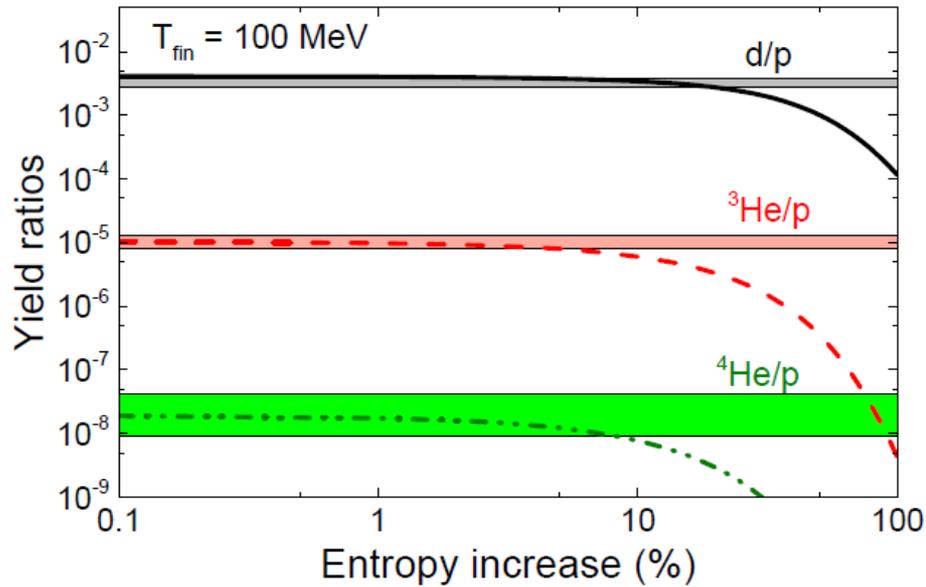
Saha equation:
$$\frac{N_A(T)}{N_A(T_{\text{ch}})} \simeq \left(\frac{T}{T_{\text{ch}}}\right)^{\frac{3}{2}(A-1)} \exp \left[B_A \left(\frac{1}{T} - \frac{1}{T_{\text{ch}}} \right) \right] \quad B_A \ll T$$

Thermal model:
$$\left[\frac{N_A(T)}{N_A(T_{\text{ch}})} \right]_{\text{eq.}} \simeq \left(\frac{T}{T_{\text{ch}}}\right)^{-\frac{3}{2}} \exp \left[-m_A \left(\frac{1}{T} - \frac{1}{T_{\text{ch}}} \right) \right] \quad m_A \gg T$$

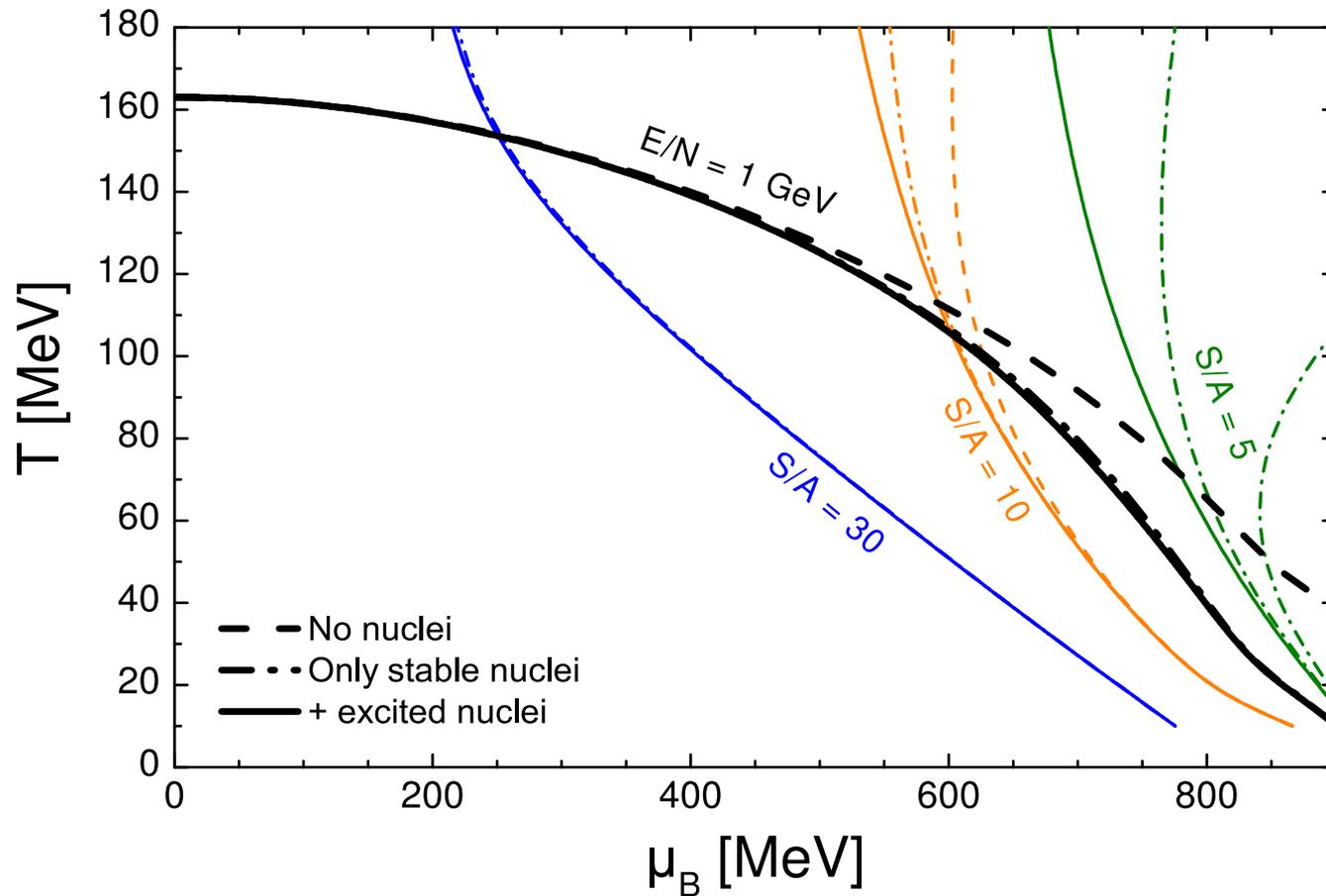
Strong exponential dependence on the temperature is eliminated in the Saha equation approach

Further, quantitative applications require numerical treatment of full spectrum of *massive* mesonic and baryonic resonances

Saha equation: Entropy production effect



Feeddown from nuclei: Isentropes



Feeddown from nuclei: Rapidity dependence

Fireballs at midrapidity: $\mu_B(y_S) \approx \mu_B(0) + b y_S^2$

[Becattini et al., 0709.2599]

