



From Spin Polarization of Quarks to Global Spin Alignment of ϕ Mesons

Xin-Li Sheng
盛欣力

Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys. Rev. C 97, 034917 (1018).
N. Weickgenannt, XLS, E. Speranza, Q. Wang, and D. H. Rischke, Phys. Rev. D 100, 056018 (2019).
XLS, L. Oliva, and Q. Wang, arXiv: 1910.13684.
XLS, Q. Wang, and X.-G. Huang, arXiv: 2005.00204.

Outline



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- Introduction
- Quark polarization
 - Wigner function
 - Kinetic theory with spin
 - Thermal equilibrium state
- Spin alignment of ϕ
 - Coalescence model
 - Numerical simulations
 - Mean-field of ϕ
- Summary

Spin effects



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Spin

Chiral Magnetic Effect

D.E.Kharzeev, et al.
Nucl.Phys.A803,227 (2008)

Chiral Separation Effect

D.E.Kharzeev, et al.
Prog.Part.Nucl.Phys.88,1 (2016)



$$\begin{aligned} J^\mu &= \xi_B B^\mu + \xi \omega^\mu \\ J_5^\mu &= \xi_{B5} B^\mu + \xi_5 \omega^\mu \end{aligned}$$

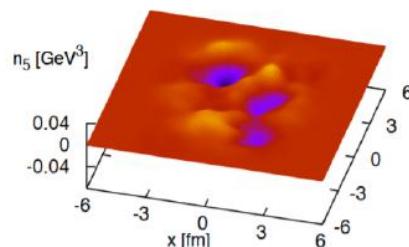
Chiral Vortical Effect

D.T.Son and P.Surowka,
Phys.Rev.Lett.103,191601 (2009)

Axial Chiral Vortical Effect

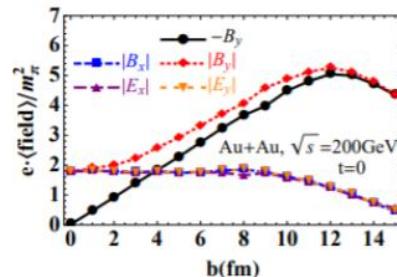
D.E.Kharzeev, et al.
Prog.Part.Nucl.Phys.88,1 (2016)

Spin imbalance



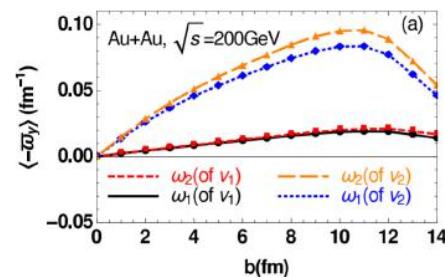
Y.Hirono, et al.
arXiv:1412.0311.

Magnetic field



W.-T.Deng, X.-G.Huang,
Phys.Rev.C85,044907 (2012).

Flow vorticity



W.-T.Deng, et al.
J.Phys.Conf.Ser.779,012070 (2017)

Polarization



$$f_{\pm} \sim \exp [-(E_0 \mp \mathbf{B} \cdot \boldsymbol{\mu})]$$

Polarization through spin-magnetic coupling:

$$P = \frac{f_+ - f_-}{f_+ + f_-} \sim \frac{\mathbf{B} \cdot \boldsymbol{\mu}}{T}$$

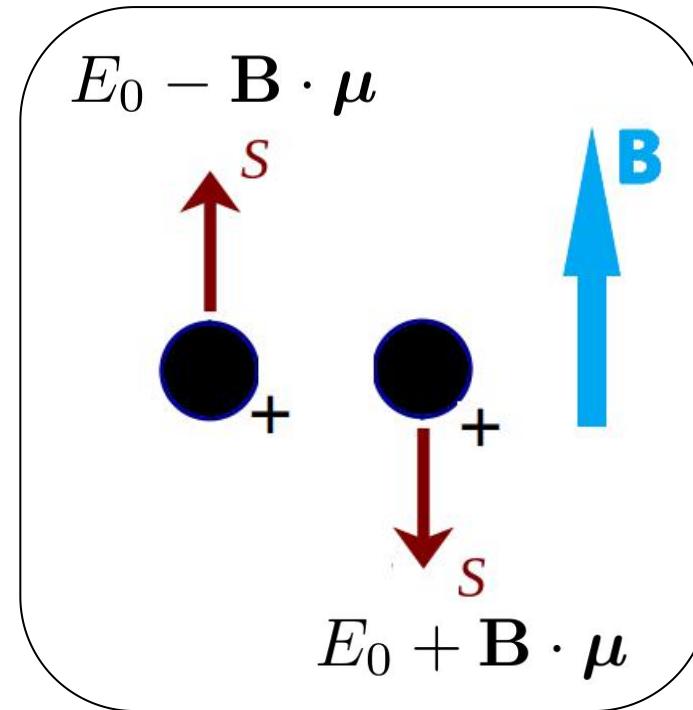
Magnetic moment: $\boldsymbol{\mu} = \frac{Q}{m} \mathbf{S}$

Orbital angular momentum



Spin

Spin-orbital coupling



S. A. Voloshin, arXiv:nucl-th/0410089
Z.-T. Liang, X.-N. Wang, Phys. Rev. Lett. 94, 039901 (2004).
J.-J. Zhang, R.-H. Fang, Q. Wang, and X.-N. Wang,
Phys. Rev. C 100, 064904 (2019)

Spin effects



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Global (or longitudinal) Lambda polarization

STAR Collaboration, Nature 548, 62 (2017).

STAR Collaboration, Phys. Rev. Lett. 123, 132301 (2019).

$$\Lambda \rightarrow p + \pi^-$$

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} \left(1 + \alpha_H \mathcal{P}_\Lambda \frac{\mathbf{n}^* \cdot \mathbf{p}^*}{|\mathbf{p}^*|} \right)$$

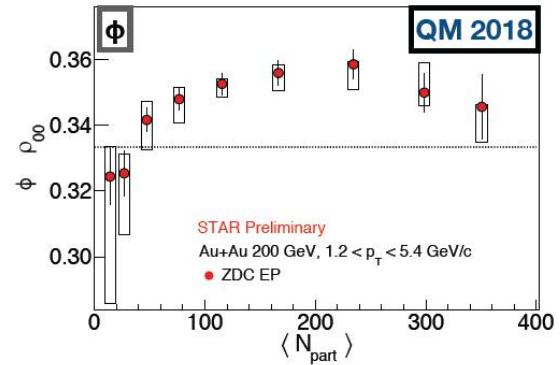
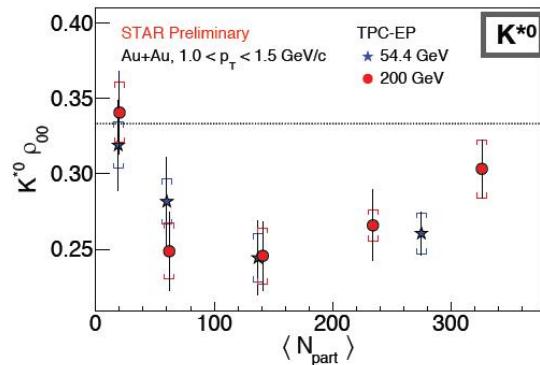
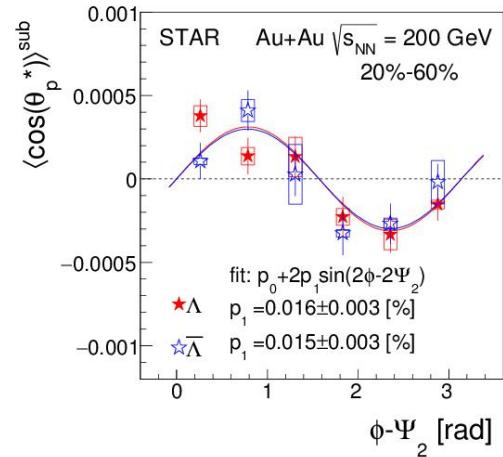
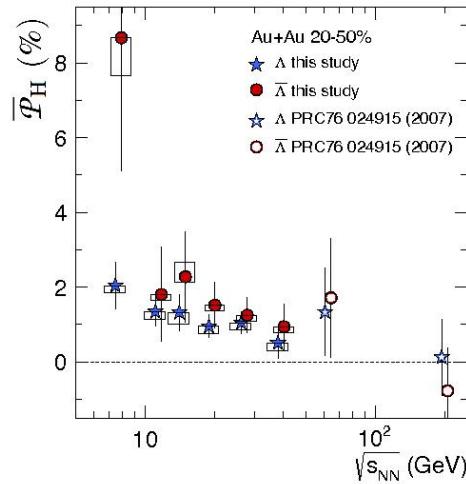
Vector meson spin alignment

Subhash Singha, Quark Matter 2019.

$$K^{*0} \rightarrow K^+ + \pi^- ,$$

$$\phi \rightarrow K^+ + K^-$$

$$\frac{dN}{d\cos\theta} = \frac{3}{4} [(1 - \rho_{00}) + (3\rho_{00} - 1) \cos^2 \theta]$$



- For midcentral collisions

• $K^{*0} \rho_{00} < 1/3$

• $\phi \rho_{00} > 1/3$

Motivation



Initial angular momentum and magnetic field

Kinetic theory with spin

Spin polarization of quarks / anti-quarks

Coalescence model

**Spin polarization of hyperons / spin alignment
of vector mesons**

Angular distribution of daughter particles

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Definition of Wigner function



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Covariant Wigner function:

$$W(x, p) = \int \frac{d^4y}{(2\pi)^4} \exp(-iy^\mu p_\mu) \underbrace{U\left(x + \frac{y}{2}, x - \frac{y}{2}\right)}_{\text{Gauge link}} \langle \Omega | \hat{\psi}\left(x + \frac{y}{2}\right) \otimes \hat{\psi}\left(x - \frac{y}{2}\right) | \Omega \rangle.$$

Fourier transform Gauge link Two-point correlation

Lorentz covariant and gauge invariant

Gauge link:

$$U\left(x + \frac{y}{2}, x - \frac{y}{2}\right) = \exp \left[-iy^\mu \int_{-1/2}^{1/2} ds \mathbb{A}_\mu(x + sy) \right]$$

Straight line



Wigner function: 4×4 complex matrix, with 16 constraints,

$$W^\dagger = \gamma^0 W \gamma^0$$

Expansion in terms of generators of Clifford algebra,

$$\Gamma_i = \{\mathbb{I}_4, i\gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \frac{1}{2}\sigma^{\mu\nu}\}$$

$$W(x, p) = \frac{1}{4} \left(\mathbb{I}_4 \mathcal{F} + i\gamma^5 \mathcal{P} + \gamma^\mu \mathcal{V}_\mu + \gamma^5 \gamma^\mu \mathcal{A}_\mu + \frac{1}{2} \sigma^{\mu\nu} \mathcal{S}_{\mu\nu} \right)$$

Real functions in 8-d phase space

$$\text{Tr}(\Gamma_i W)$$

Physical interpretations



$$W(x, p) = \frac{1}{4} \left(\mathbb{I}_4 \mathcal{F} + i\gamma^5 \mathcal{P} + \gamma^\mu \mathcal{V}_\mu + \gamma^5 \gamma^\mu \mathcal{A}_\mu + \frac{1}{2} \sigma^{\mu\nu} \mathcal{S}_{\mu\nu} \right)$$

	Property	Physical meaning (distribution in phase space)
\mathcal{F}	Scalar	Mass
\mathcal{P}	Pseudoscalar	Pseudoscalar condensate
\mathcal{V}^μ	Vector	Net fermion current
\mathcal{A}^μ	Axial-vector	Polarization (or spin current)
$\mathcal{S}^{\mu\nu}$	Tensor	Electric/magnetic dipole-moment

Net-fermion number current, spin current

$$\mathbb{N}^\mu(x) = \int d^4 p \mathcal{V}^\mu(x, p), \quad \mathbb{N}_5^\mu(x) = \int d^4 p \mathcal{A}^\mu(x, p).$$

Physical interpretations



$$W(x, p) = \frac{1}{4} \left(\mathbb{I}_4 \mathcal{F} + i\gamma^5 \mathcal{P} + \gamma^\mu \mathcal{V}_\mu + \gamma^5 \gamma^\mu \mathcal{A}_\mu + \frac{1}{2} \sigma^{\mu\nu} \mathcal{S}_{\mu\nu} \right)$$

	Property	Physical meaning (distribution in phase space)
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Canonical energy-momentum tensor,
canonical spin angular momentum tensor

$$\mathbb{T}_{\text{mat}}^{\mu\nu}(x) = \int d^4 p p^\nu \mathcal{V}^\mu(x, p), \quad \mathbb{S}_{\text{mat}}^{\rho, \mu\nu}(x) = -\frac{1}{2} \epsilon^{\rho\mu\nu\alpha} \int d^4 p \mathcal{A}_\alpha(x, p).$$

Semi-classical expansion



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- Semi-classical expansion:
Expand Wigner function and its kinetic equations in terms of \hbar
Zeroth order: classical spinless particles
Linear order: spin corrections
- Valid if
 1. $|\hbar\gamma^\mu \partial_{x_\mu} W| \ll m |W|$
 \Rightarrow Compton wave length \ll wave length of macroscopic fluctuations
 2. $|\hbar\gamma^\mu F_{\mu\nu} \partial_\nu^\nu W| \ll m |W| \Rightarrow$ Weak EM fields
 3. $\Delta R \Delta P \gg \hbar = 197 \text{MeV} \cdot \text{fm}$
 ΔR typical spatial scale of EM field
 ΔP typical momentum scale of Wigner function

Linear order solutions



$$\mathcal{F} = m \left[V \delta(p^2 - m^2) - \frac{\hbar}{2} F^{\mu\nu} \Sigma_{\mu\nu} \delta'(p^2 - m^2) \right] + \mathcal{O}(\hbar^2),$$

$$\mathcal{P} = \frac{\hbar}{4m} \epsilon^{\mu\nu\alpha\beta} \nabla_\mu [p_\nu \Sigma_{\alpha\beta} \delta(p^2 - m^2)] + \mathcal{O}(\hbar^2),$$

$$\begin{aligned} \mathcal{V}_\mu &= \delta(p^2 - m^2) \left(p_\mu V + \frac{\hbar}{2} \nabla^\nu \Sigma_{\mu\nu} \right) \\ &\quad - \hbar \left(\frac{1}{2} p_\mu F^{\alpha\beta} \Sigma_{\alpha\beta} + \Sigma_{\mu\nu} F^{\nu\alpha} p_\alpha \right) \delta'(p^2 - m^2) + \mathcal{O}(\hbar^2), \end{aligned}$$

$$A_\mu = -\frac{1}{2} \epsilon_{\mu\nu\alpha\beta} p^\nu \Sigma^{\alpha\beta} \delta(p^2 - m^2) + \hbar \tilde{F}_{\mu\nu} p^\nu V \delta'(p^2 - m^2) + \mathcal{O}(\hbar^2),$$

$$S_{\mu\nu} = \delta(p^2 - m^2) m \left(\Sigma_{\mu\nu} - \frac{\hbar}{2m} p_{[\mu} \nabla_{\nu]} V \right) - m \hbar F_{\mu\nu} V \delta'(p^2 - m^2) + \mathcal{O}(\hbar^2).$$

- Fermion distribution V , Dipole-moment tensor $\Sigma^{\mu\nu}$.
- Spin corrections, off-shell contribution
- Reduce to classical densities for mass, fermion current, spin polarization, and dipole-moment.

Kinetic equations



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Fermion distribution

$$\delta(p^2 - m^2) \left[p^\mu \nabla_\mu V + \frac{\hbar}{4} (\partial_{x\alpha} F_{\mu\nu}) \partial_p^\alpha \Sigma^{\mu\nu} \right] \rightarrow \text{Collisionless Boltzmann Eq.}$$
$$-\delta'(p^2 - m^2) \frac{\hbar}{2} F^{\mu\nu} p^\alpha \nabla_\alpha \Sigma_{\mu\nu} = 0.$$

Dipole-moment tensor

$$\delta(p^2 - m^2) \left[p^\alpha \nabla_\alpha \Sigma_{\mu\nu} - F_{[\mu}^\alpha \Sigma_{\nu]\alpha} + \frac{\hbar}{2} (\partial_{x\alpha} F_{\mu\nu}) \partial_p^\alpha V \right] \rightarrow \text{Bargmann-Michel-Telegdi (BMT) Eq.}$$
$$+\delta'(p^2 - m^2) \hbar F_{\mu\nu} p^\alpha \nabla_\alpha V = 0.$$

where $\nabla^\mu = \partial_x^\mu - F^{\mu\nu} \partial_{p\nu}$.

- Off-shell contributions because of spin-magnetic coupling
- Mathisson force, effects of inhomogeneous fields

N. Weickgenannt, XLS, E. Speranza, Q. Wang, D. H. Rischke, PRD 100,056018 (2019).

Massless limit



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Massive fermions

Rotational group O(3)

Spin in rest frame

Kinetic theory with
spin for massive
fermions

Y. S. Kim and E. P.
Wigner, J. Math. Phys.
28, 1175 (1987)

Infinite Lorentz boost
+ massless limit

Connection?

Massless fermions

2-d Euclidean group E(2)

Helicity

Chiral Kinetic Theory

Non-smooth connection in:

N. Weickgenannt, XLS, E. Speranza, Q. Wang, D. H. Rischke, PRD 100, 056018 (2019).
K. Hattori, Y. Hidaka, D.-L. Yang, PRD 100, 096011 (2019).



Massive: rest frame **Massless: frame** u^μ

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta$$

$$\Sigma^{\mu\nu} = \mathcal{E}^\mu u^\nu - \mathcal{E}^\nu u^\mu - \epsilon^{\mu\nu\alpha\beta} u_\alpha \mathcal{M}_\beta$$

~~~~~  
**dipole moment  
tensor**

~~~~~  
**electric
dipole moment**

~~~~~  
**magnetic  
dipole moment**

$$-\frac{1}{2} F_{\mu\nu} \Sigma^{\mu\nu} = -\mathcal{E}_\mu E^\mu - \mathcal{M}_\mu B^\mu$$



**Massive: rest frame**  $\longrightarrow$  **Massless: frame**  $u^\mu$

$$n^\mu = (u \cdot p) u^\mu \frac{u \cdot n}{u \cdot p} + n_{\parallel} p^{\langle \mu} + n_{\perp}^{\mu}$$

**spin polarization vector**

**axial-charge density**

$$n_{\parallel} = \frac{(u \cdot p)(u \cdot n)}{(u \cdot p)^2 - p^2}$$

**longitudinal / transverse polarization (parallel / perpendicular to momentum in frame  $u^\mu$ )**

$$n_{\perp}^{\mu} = \frac{p^2}{u \cdot p} \mathcal{M}_{\perp}^{\mu} + \frac{\hbar}{2(u \cdot p)} \epsilon^{\mu\nu\alpha\beta} p_{\nu} u_{\alpha} \nabla_{\beta} V$$

**side-jump in massless case**

# Massless limit



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## Axial-vector component of Wigner function

$$\begin{aligned}\mathcal{A}^\mu = & \left[ (u \cdot p) u^\mu + \frac{(u \cdot p)^2}{(u \cdot p)^2 - m^2} p^{\langle \mu} \right] A \delta(p^2 - m^2) \\ & + \frac{\hbar}{2(u \cdot p)} \epsilon^{\mu\nu\alpha\beta} p_\nu u_\alpha (\nabla_\beta V) \delta(p^2 - m^2) + \hbar \tilde{F}_{\mu\nu} p^\nu V \delta'(p^2 - m^2) \\ & + \frac{m^2}{u \cdot p} \mathcal{M}_\perp^\mu \delta(p^2 - m^2).\end{aligned}$$

$\xrightarrow{\text{m} \rightarrow 0}$

Y. Hidaka, S. Pu, and D.-L. Yang, 2017.  
A. Huang, S. Shi, Y. Jiang, J. Liao, and  
P. Zhuang, 2018.  
etc.

### Small-mass behaviors:

(obtained by considering a single-particle wave packet)

$$A = \mathcal{O}(1) + \mathcal{O}(m, m^2, \dots), \quad \mathcal{M}_\perp^\mu = \mathcal{O}(m^{-1}) + \mathcal{O}(1, m, \dots)$$

$$V = \boxed{\mathcal{O}(m^{-1})} + \mathcal{O}(1, m, \dots)$$

**classical case:**  $\mathcal{M} = \frac{Q}{m} \mathbf{S}$

spin-orbital coupling

$$\mathcal{O}(\hbar)$$

XLS, Q. Wang, and X.-G. Huang,  
arXiv:2005.00204.

# Massless limit



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## Mass corrections to CKT

$$0 = \left( p^\mu \nabla_\mu \tilde{V} \right) \delta(p^2 - m^2) + \delta'(p^2 - m^2) \left[ \frac{\hbar}{u \cdot p} \tilde{F}^{\mu\nu} u_\mu p_\nu (p^\alpha \nabla_\alpha A) \right] \\ + \delta(p^2 - m^2) \left[ \frac{\hbar}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \left( \nabla_\mu \frac{u_\alpha}{u \cdot p} \right) (\nabla_\beta A) + \frac{\hbar}{2(u \cdot p)} p_\mu u_\nu (\partial_{x\alpha} \tilde{F}^{\mu\nu}) (\partial_p^\alpha A) \right]$$

**vanish in massless limit**

$$\left\{ \begin{aligned} & + \delta(p^2 - m^2) m^2 \left\{ \frac{\hbar}{2} \epsilon^{\mu\nu\alpha\beta} \left( \nabla_\mu \frac{u_\nu}{u \cdot p} \right) \left[ \nabla_\alpha \left( \frac{1}{u \cdot p} \mathcal{M}_\beta \right) \right] - \frac{\hbar}{2(u \cdot p)} u_\mu (\partial_{x\alpha} \tilde{F}^{\mu\nu}) \left[ \partial_p^\alpha \left( \frac{1}{u \cdot p} \mathcal{M}_\nu \right) \right] \right\} \\ & + \delta'(p^2 - m^2) m^2 \frac{\hbar}{u \cdot p} \tilde{F}^{\mu\nu} u_\mu \left[ p^\alpha \nabla_\alpha \left( \frac{1}{u \cdot p} \mathcal{M}_\nu \right) \right]. \end{aligned} \right.$$

$$0 = (p^\mu \nabla_\mu A) \delta(p^2 - m^2) + \delta'(p^2 - m^2) \left[ \frac{\hbar}{u \cdot p} \tilde{F}^{\mu\nu} u_\mu p_\nu (p^\alpha \nabla_\alpha V) \right] \\ + \delta(p^2 - m^2) \left[ \frac{\hbar}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \left( \nabla_\mu \frac{u_\alpha}{u \cdot p} \right) (\nabla_\beta V) + \frac{\hbar}{2(u \cdot p)} p_\mu u_\nu (\partial_{x\alpha} \tilde{F}^{\mu\nu}) (\partial_p^\alpha V) \right]$$

**vanish in massless limit**

$$\left\{ \begin{aligned} & - \delta(p^2 - m^2) \frac{m^2}{(u \cdot p)^2} \left[ (p \cdot \nabla u_\mu - F_{\mu\nu} u^\nu) \mathcal{M}^\mu + \frac{\hbar}{2} \epsilon^{\mu\nu\alpha\beta} (\nabla_\mu V) u_\nu \nabla_\alpha u_\beta \right]. \end{aligned} \right.$$

XLS, Q. Wang, and X.-G. Huang,  
arXiv:2005.00204.

# Thermal equilibrium



- Assuming that the thermal equilibrium distribution have the form

$$f_s^{eq} = (e^{g_s} + 1)^{-1}$$

with  $g_s = p \cdot \beta(x) + a_s(x) + s \frac{\hbar}{4} \Omega_{\mu\nu}(x) \Sigma^{(0)\mu\nu}$  constant

$\blacktriangle$  spin  $\uparrow\downarrow$       Lagrange multipliers

Specifies direction of spin polarization

Constraints for the multipliers

$$\partial_{x\mu} \beta_\nu + \partial_{x\nu} \beta_\mu = 0,$$

$$\partial_{x\mu} a_s(x) = F_{\mu\nu} \beta^\nu(x),$$

$$\partial_{x\mu} \Omega_{\lambda\nu}(x) = 0,$$

Thermal vorticity

$$\Omega_{\mu\nu} = \omega_{\mu\nu} \equiv \frac{1}{2} (\partial_{x\mu} \beta_\nu - \partial_{x\nu} \beta_\mu)$$



## Axial-vector current

$$\begin{aligned} \mathcal{A}^\mu = & \frac{2}{(2\pi\hbar)^3} \sum_s \left[ \delta(p^2 - m^2) \left( s m n^{(0)\mu} - \frac{\hbar}{2} \tilde{\omega}^{\mu\nu} p_\nu \frac{\partial}{\partial(\beta E_p)} \right) \right. \\ & + \hbar \tilde{F}^{\mu\nu} p_\nu \delta'(p^2 - m^2) \left. \right] \left[ \theta(p^0) f_s^{(0)+} + \theta(-p^0) f_s^{(0)-} \right] \\ & - \frac{\hbar}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \Xi_{\alpha\beta} \delta(p^2 - m^2) + \mathcal{O}(\hbar^2). \end{aligned}$$

Analogue of axial chiral vortical effect

Analogue of chiral separation effect

Zeroth order distribution

$$f_s^{(0)\pm} = \{\exp [\beta(E_p \mp \mu_s)] + 1\}^{-1}$$

### Massive:

R.-H. Fang, L.-G. Pang, Q. Wang,  
and X.-N. Wang, 2016

### Massless:

- D. T. Son and A. R. Zhitnitsky, 2004.
- M. A. Metlitski and A. R. Zhitnitsky, 2005.
- J. Bhattacharya, S. Bhattacharyya, S. Dutta, R. Loganayagam, and P. Surowka, 2011.
- D. T. Son and P. Surowka, 2009.

# Quark spin

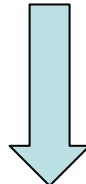


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## Average polarization

$p^\mu = (E_p, \pm \mathbf{p})$  for fermion / antifermion

$$P_\pm^\mu(x, p) = \frac{1}{2m} \left( \tilde{\omega}_{\text{th}}^{\mu\nu} \pm \frac{1}{E_p T} Q \tilde{F}^{\mu\nu} \right) p_\nu [1 - f_{FD}(E_p \mp \mu)]$$



Projecting onto y-direction  
Boltzmann limit

## Average polarization along y-direction

$$\begin{aligned} P_{s/\bar{s}}^y(t, \mathbf{x}, \mathbf{p}_{s/\bar{s}}) &= \frac{1}{2} \omega_{\text{th}}^y \pm \frac{1}{2m_s} \hat{\mathbf{y}} \cdot (\boldsymbol{\varepsilon} \times \mathbf{p}_{s/\bar{s}}) \\ &\pm \frac{Q_s}{2m_s T} B_y + \frac{Q_s}{2m_s^2 T} \hat{\mathbf{y}} \cdot (\mathbf{E} \times \mathbf{p}_{s/\bar{s}}) \end{aligned}$$

**Acceleration**  $\boldsymbol{\varepsilon} = -(1/2)[\partial_t(\beta \mathbf{u}) + \nabla(\beta u^0)]$

# Outline



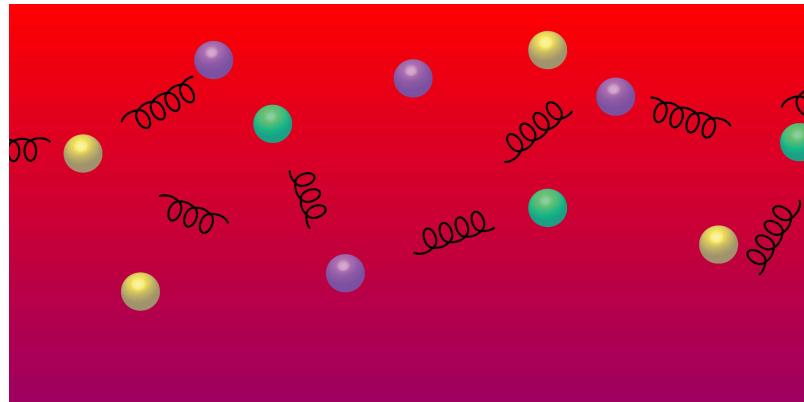
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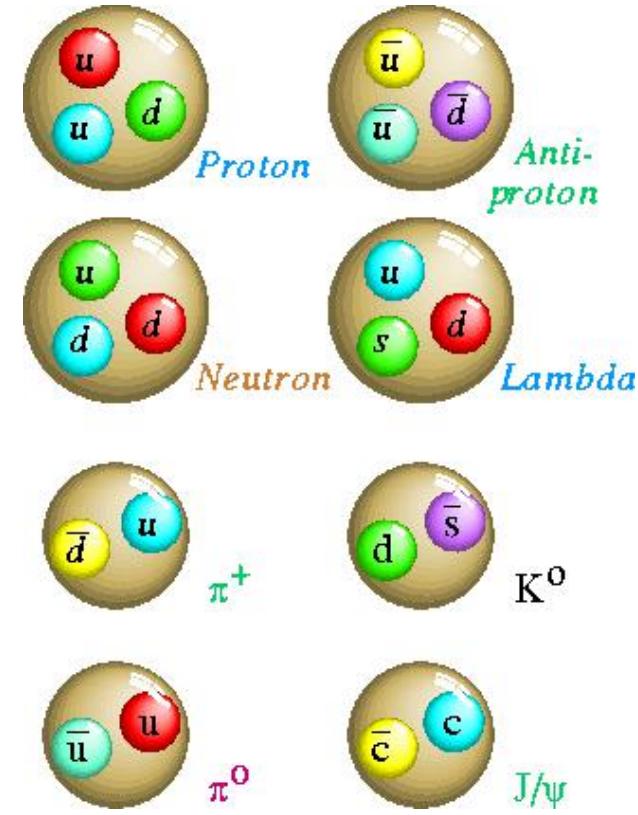
# Coalescence model



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$$qqq \rightarrow B$$



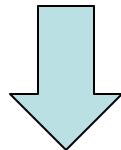
R. C. Hwa and C. B. Yang, Phys. Rev. C 70, 024904 (2004)

R. J. Fries, B. Muller, C. Nonaka, S. A. Bass,

Phys. Rev. C 68, 044902 (2003)

V. Greco, C. M. Ko, P. Lévai, L.W. Chen, Phys. Rev. C 68, 034904 (2003);

L. W. Chen, C. M. Ko, Phys. Rev. C 73, 044904 (2006)



include spin DoF

Yang-Guang Yang, Ren-Hong Fang,  
Qun Wang, and Xin-Nian Wang, Phys.Rev.C 97, 3 (2018).

# Coalescence model



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## Quark-antiquark state

$$|q_1, \bar{q}_2; s_1, s_2; \mathbf{p}_1, \mathbf{p}_2\rangle$$

~~~~~ ~~~~~ ~~~~~

flavor of spin in a fixed momentum
quark or antiquark quantization
direction

Density operator $\rho = V^2 \sum_{s_1, s_2} \sum_{q_1, \bar{q}_2} \int \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \frac{d^3 \mathbf{p}_2}{(2\pi)^3} w_{q_1, s_1}(\mathbf{p}_1) w_{\bar{q}_2, s_2}(\mathbf{p}_2)$

$$\times |q_1, \bar{q}_2; s_1, s_2; \mathbf{p}_1, \mathbf{p}_2\rangle \langle q_1, \bar{q}_2; s_1, s_2; \mathbf{p}_1, \mathbf{p}_2|,$$

Weighted functions

$$w_{q, \pm 1/2}(\mathbf{p}) = \frac{1}{2} [1 \pm \boxed{\mathcal{P}_q(\mathbf{p})}]$$
$$w_{\bar{q}, \pm 1/2}(\mathbf{p}) = \frac{1}{2} [1 \pm \mathcal{P}_{\bar{q}}(\mathbf{p})]$$

Average polarization along given direction

Yang-Guang Yang, Ren-Hong Fang,
Qun Wang, and Xin-Nian Wang, Phys.Rev.C 97, 3 (2018).

Coalescence model



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Meson state

$$|M; S, S_z, \mathbf{p}\rangle$$

~~~    ~~~~~    ~~  
 type of      spin      momentum  
 meson        status

Yang-Guang Yang, Ren-Hong Fang,  
Qun Wang, and Xin-Nian Wang,  
Phys.Rev.C 97, 3 (2018).

## Spin density matrix element for mesons

$$\begin{aligned} \rho_{S_{z1}S_{z2}}^S(\mathbf{p}) &= \langle M; S, S_{z1}, \mathbf{p} | \rho_{q\bar{q}} | M; S, S_{z2}, \mathbf{p} \rangle \\ &= V^2 \sum_{s_1 s_2} \sum_{q_1 \bar{q}_2} \int \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \frac{d^3 \mathbf{p}_2}{(2\pi)^3} w_{q1,s1}(\mathbf{p}_1) w_{q2,s2}(\mathbf{p}_2) \end{aligned}$$

wave functions in  
momentum space

$$\times |\langle q_1, \bar{q}_2; \mathbf{p}_1, \mathbf{p}_2 | M; \mathbf{p} \rangle|^2$$

$$\times \langle M; S, S_{z1} | q_1, \bar{q}_2; s_1, s_2 \rangle \langle q_1, \bar{q}_2; s_1, s_2 | M; S, S_{z2} \rangle$$

spin-flavor wave functions

**Example:**  $|\langle q_1, \bar{q}_2; \mathbf{p}_1, \mathbf{p}_2 | \phi; \mathbf{p} \rangle|^2 = \frac{(2\pi)^3}{V^2} \delta^{(3)}(\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2) |\varphi_\phi(\mathbf{q})|^2$

$$|\phi, 1, 0\rangle = \frac{1}{\sqrt{2}} |s, +\rangle |\bar{s}, -\rangle + \frac{1}{\sqrt{2}} |s, -\rangle |\bar{s}, +\rangle$$

# Coalescence model



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**Normalized spin density matrix element for  $\phi(1020)$  :  $J^P = 1^-$**

$$\rho_{00}^\phi = \frac{\rho_{00}^{S=1}}{\rho_{11}^{S=1} + \rho_{00}^{S=1} + \rho_{-1,-1}^{S=1}}$$

$$\approx \frac{1}{3} - \frac{4}{9} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} P_s^y(\mathbf{p}) P_{\bar{s}}^y(-\mathbf{p}) |\psi_\phi(\mathbf{p})|^2$$

Yang-Guang Yang, Ren-Hong Fang,  
Qun Wang, and Xin-Nian Wang,  
Phys.Rev.C 97, 3 (2018).

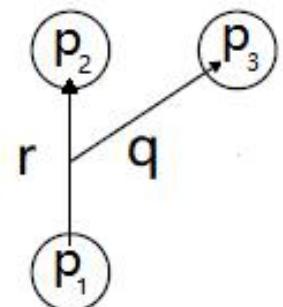
Wave function can be found in quark potential model

## Polarization of Baryons

$$P_B^y = \frac{\rho_{\frac{1}{2}\frac{1}{2}}^B - \rho_{-\frac{1}{2},-\frac{1}{2}}^B}{\rho_{\frac{1}{2}\frac{1}{2}}^B + \rho_{-\frac{1}{2},-\frac{1}{2}}^B} \longrightarrow$$

Calculated using quark polarization and baryon wave function

$$P_\Lambda^y(\mathbf{p}) \simeq \frac{1}{3} \int \frac{d^3 \mathbf{r}}{(2\pi)^3} \frac{d^3 \mathbf{q}}{(2\pi)^3} |\psi_\Lambda(\mathbf{r}, \mathbf{q})|^2 [P_s^y(\mathbf{p}_1) + P_s^y(\mathbf{p}_2) + P_s^y(\mathbf{p}_3)]$$



# Polarizations



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## Polarization of $s$ and $\bar{s}$

### Acceleration

$$\boldsymbol{\varepsilon} = -(1/2)[\partial_t(\beta \mathbf{u}) + \nabla(\beta u^0)]$$

$$P_{s/\bar{s}}^y(t, \mathbf{x}, \mathbf{p}_{s/\bar{s}}) = \frac{1}{2}\omega_{\text{th}}^y \pm \frac{Q_s}{2m_s T} B_y \pm \frac{1}{2m_s} \hat{\mathbf{y}} \cdot (\boldsymbol{\varepsilon} \times \mathbf{p}_{s/\bar{s}}) + \frac{Q_s}{2m_s^2 T} \hat{\mathbf{y}} \cdot (\mathbf{E} \times \mathbf{p}_{s/\bar{s}})$$

## Polarization of $\Lambda$ hyperon

$$P_{\Lambda/\bar{\Lambda}}^y(t, \mathbf{x}) = \frac{1}{2}\omega_{\text{th}}^y \pm \frac{Q_s}{2m_s T} B_y$$

## 00-element of spin density matrix for $\phi$ mesons

$$\begin{aligned} \rho_{00}^\phi &\approx \frac{1}{3} - \frac{4}{9} \langle P_\Lambda^y P_\Lambda^y \rangle - \frac{1}{27m_s^2} \langle \mathbf{p}^2 \rangle_\phi \langle \varepsilon_z^2 + \varepsilon_x^2 \rangle \\ &+ \frac{e^2}{243m_s^4 T_{\text{eff}}^2} \langle \mathbf{p}^2 \rangle_\phi \langle E_z^2 + E_x^2 \rangle, \end{aligned}$$

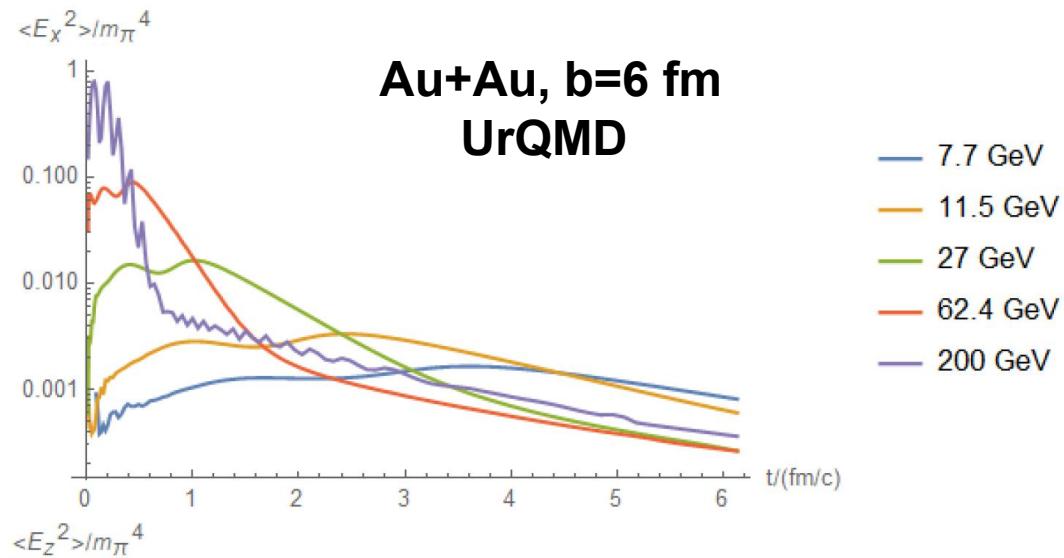
Average over  
fireball volume and  
finite polarization  
time

XLS, L. Oliva, and Q. Wang, arXiv:1910.13684

# Electric field

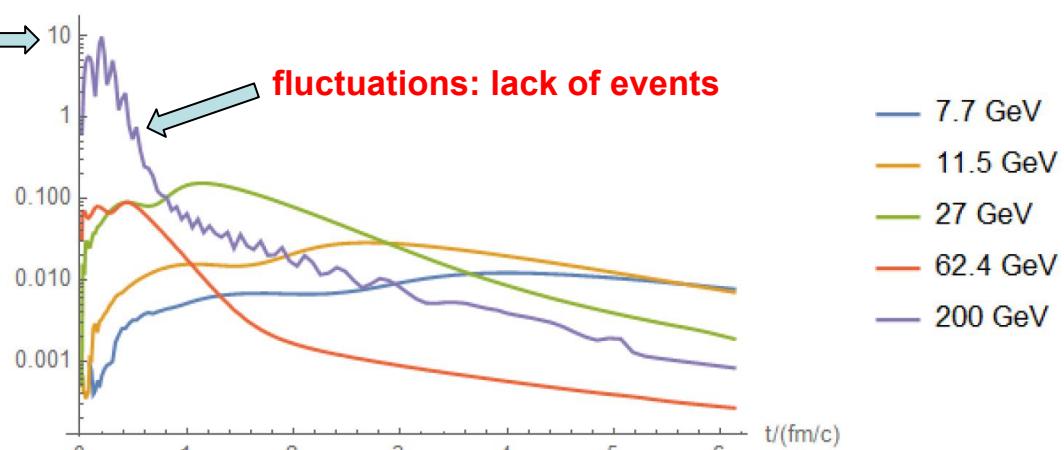


Average of  $E_x^2$  and  $E_z^2$   
over the fireball volume



Peak value

$$\langle E_z^2 + E_x^2 \rangle \ll 10m_\pi^2$$



# Acceleration



**CLVisc**

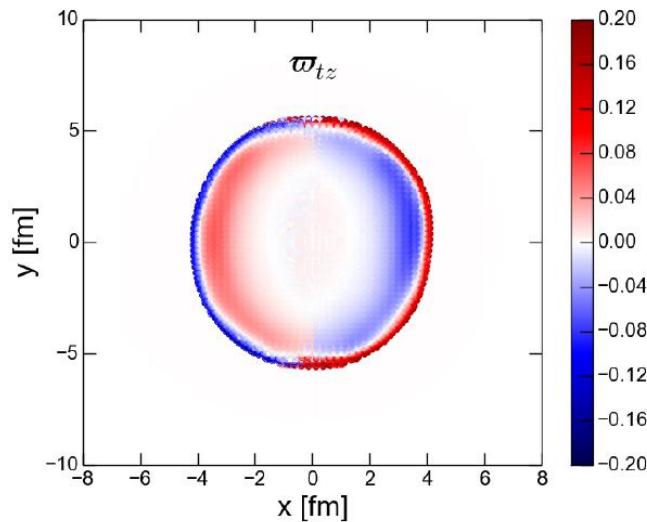
L. Pang, Q. Wang, and X.-N. Wang, Phys. Rev. C 86, 024911 (2012)  
L.-G. Pang, H. Petersen, and X.-N. Wang, Phys. Rev. C 97, 064918 (2018)

Average over the freezeout hypersurface:

$$\langle \varepsilon_z^2 + \varepsilon_x^2 \rangle \sim 10^{-4}$$

I. Karpenko and F. Becattini,  
Eur. Phys. J. C 77, 4 (2017)

freezeout hypersurface,  
projected onto x-y plane



# Mean-field



$$\begin{aligned}\rho_{00}^\phi \approx & \frac{1}{3} - \frac{4}{9} \langle P_\Lambda^y P_\Lambda^y \rangle - \frac{1}{27m_s^2} \langle \mathbf{p}^2 \rangle_\phi \langle \varepsilon_z^2 + \varepsilon_x^2 \rangle \\ & + \frac{e^2}{243m_s^4 T_{\text{eff}}^2} \langle \mathbf{p}^2 \rangle_\phi \langle E_z^2 + E_x^2 \rangle,\end{aligned}$$

**Can not be significantly larger / smaller than 1/3**

**Au+Au 200GeV**

$$\frac{4}{9} \langle P_\Lambda^y P_\Lambda^y \rangle \sim 10^{-5}$$

$$\left\{ \begin{array}{l} P_\Lambda \approx (1.08 \pm 0.15 \pm 0.11)\% \\ P_{\bar{\Lambda}} \approx (1.38 \pm 0.30 \pm 0.13)\% \end{array} \right.$$

$$\frac{1}{27m_s^2} \langle \mathbf{p}^2 \rangle_\phi \langle \varepsilon_z^2 + \varepsilon_x^2 \rangle \sim 10^{-8}$$

$$\left\{ \begin{array}{l} \langle \varepsilon_z^2 + \varepsilon_x^2 \rangle \sim 10^{-4} \\ \langle \mathbf{p}^2 \rangle_\phi \approx 0.18 \text{ GeV}^2 \\ m_s \sim 450 \text{ MeV} \\ T_{\text{eff}} \sim 100 - 300 \text{ MeV} \\ e^2 \langle E_z^2 + E_x^2 \rangle \ll (10m_\pi^2)^2 \end{array} \right.$$

$$\frac{e^2}{243m_s^2 T_{\text{eff}}^2} \langle \mathbf{p}^2 \rangle_\phi \langle E_z^2 + E_x^2 \rangle \sim 10^{-6}$$

# Vector meson field



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## Strong interaction mediated by a scalar field and a vector field

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \sum_j \bar{\psi}_j (i \not{\partial} - m_j + g_{\sigma j} \sigma - g_{V j} \not{V}) \psi_j \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu\end{aligned}$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \quad \longrightarrow \quad \begin{aligned}\mathbf{E}_V &= -\nabla V_0 - \frac{\partial}{\partial t} \mathbf{V} \\ \mathbf{B}_V &= \nabla \times \mathbf{V}\end{aligned}$$

### Proposed in

L. P. Csernai, J. I. Kapusta, and T. Welle, Phys. Rev. C99, 021901 (2019)  
to explain difference in polarizations of  $\Lambda$  and  $\bar{\Lambda}$

# Vector meson field



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## Mean field of $\phi$

$$F_{\phi}^{\mu\nu} = \partial^{\mu}\phi^{\nu} - \partial^{\nu}\phi^{\mu} \quad \phi^{\mu} \approx -(g_{\phi}/m_{\phi}^2)J_s^{\mu}$$

## Strangeness current

$$J_s^{\mu}(t, \mathbf{x}) = (\rho_s, \mathbf{J}_s) = (\rho_s, j_s^{(x)}, j_s^{(y)}, j_s^{(z)})$$

$$\partial_{\mu} J_s^{\mu} = 0 \quad \int d^3 \mathbf{x} \rho_s(t, \mathbf{x}) = 0$$

## Contribution to $\rho_{00}^{\phi}$

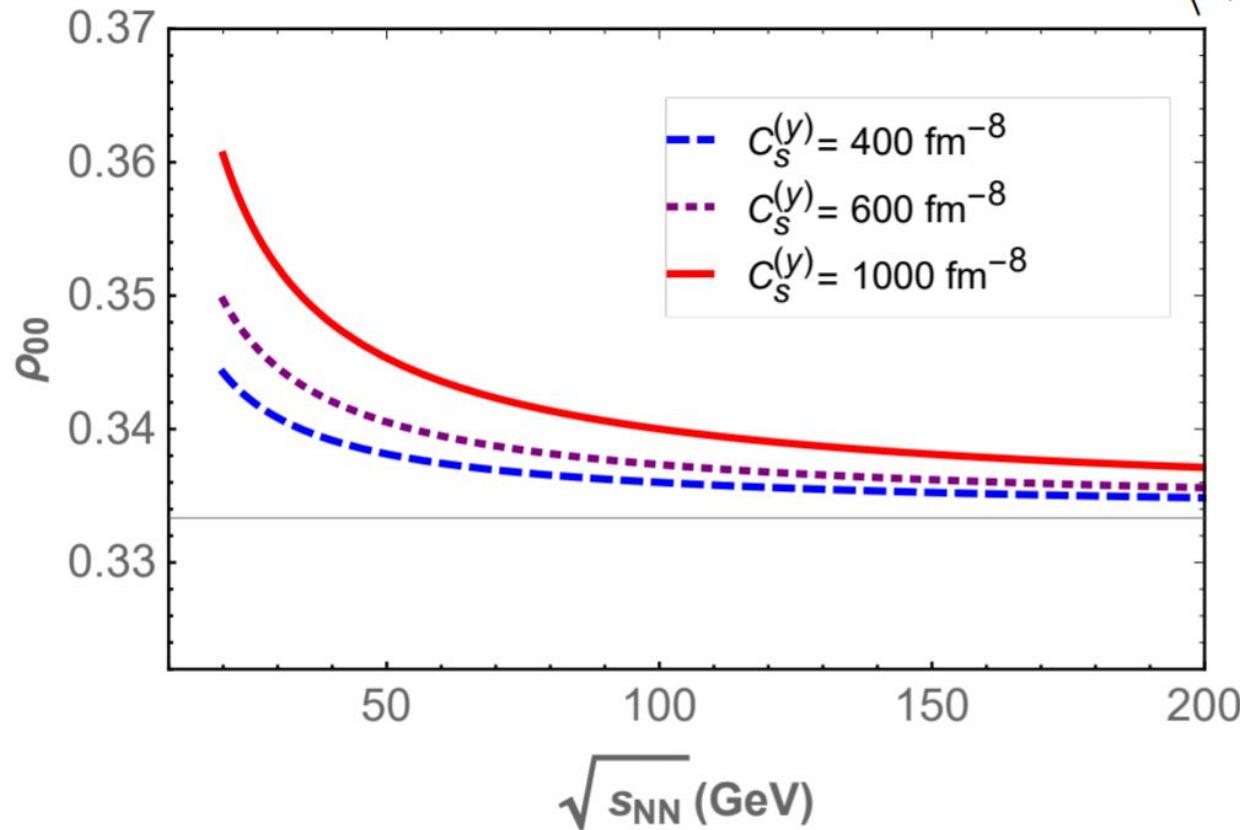
$$c_{\phi} \equiv \frac{g_{\phi}^4}{27m_s^4 m_{\phi}^4 T_{\text{eff}}^2} \langle \mathbf{p}^2 \rangle_{\phi} \left\langle \left( \tilde{E}_{\phi}^{(z)} \right)^2 + \left( \tilde{E}_{\phi}^{(x)} \right)^2 \right\rangle \text{ Positive!}$$

$$\tilde{E}_{\phi}^{(i)} \equiv \partial \rho_s / \partial z + \partial j_s^{(i)} / \partial t$$

# Mean-field



Beam-energy dependence of  $\rho_{00}^\phi$   $C_s^{(y)} \equiv g_\phi^4 \left\langle \left( \tilde{E}_\phi^{(z)} \right)^2 + \left( \tilde{E}_\phi^{(x)} \right)^2 \right\rangle$



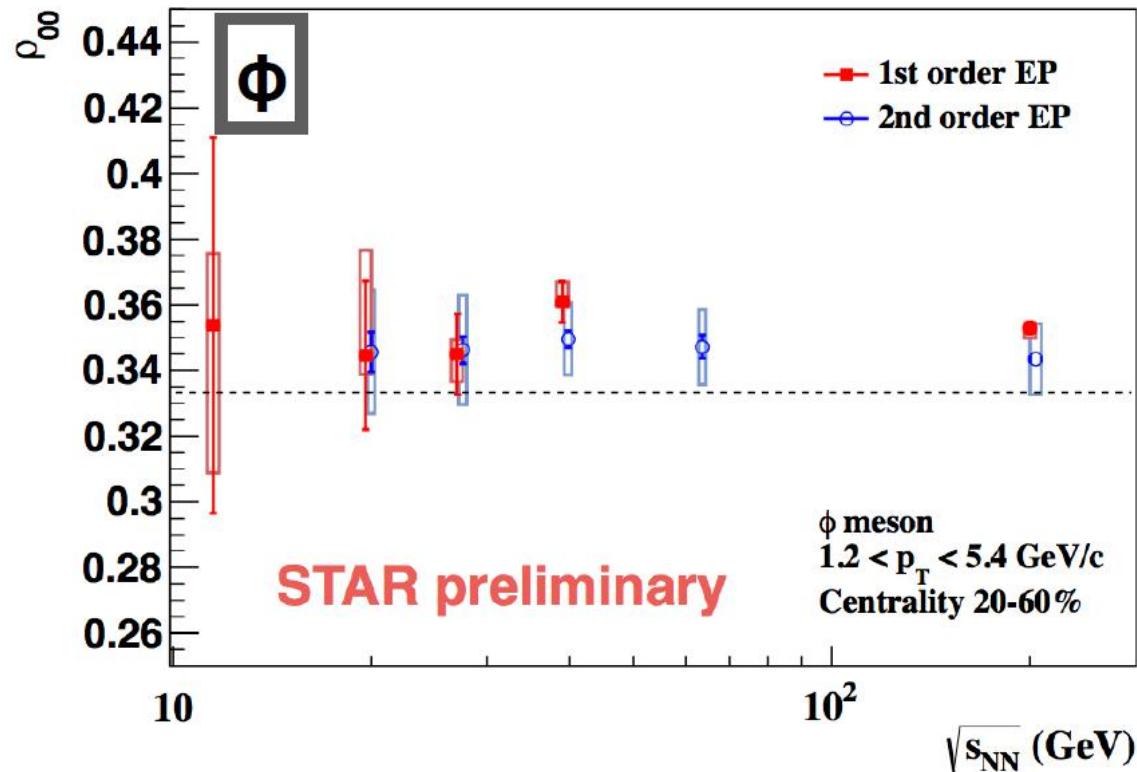
XLS, L. Oliva, and Q. Wang, arXiv:1910.13684

# Mean-field



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## Results shown at QM 2018



# Outline



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- Introduction
- Quark polarization
  - Wigner function
  - Kinetic theory with spin
  - Thermal equilibrium state
- Spin alignment of  $\phi$ 
  - Coalescence model
  - Numerical simulations
  - Mean-field of  $\phi$
- Summary

# Summary



- Derived kinetic equations with spin corrections for massive fermions
- Found smooth connection between kinetic theories for massive fermions and for massless fermions
- Studied thermal equilibrium with spin, reproduced polarization of fermions in electromagnetic field and vortical field

# Summary



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- Derived 00-element of spin density matrix for  $\phi$  mesons, in which electric field and acceleration also contribute
- A significant derivation from 1/3 for  $\rho_{00}^\phi$  maybe result of a mean field of  $\phi$ , instead of electromagnetic field or vorticity

Thanks for your attention !