DISSIPATIVE SPIN HYDRODYNAMICS

FROM BOTH

MICROSCOPIC & MACROSCOPIC APPROACHES

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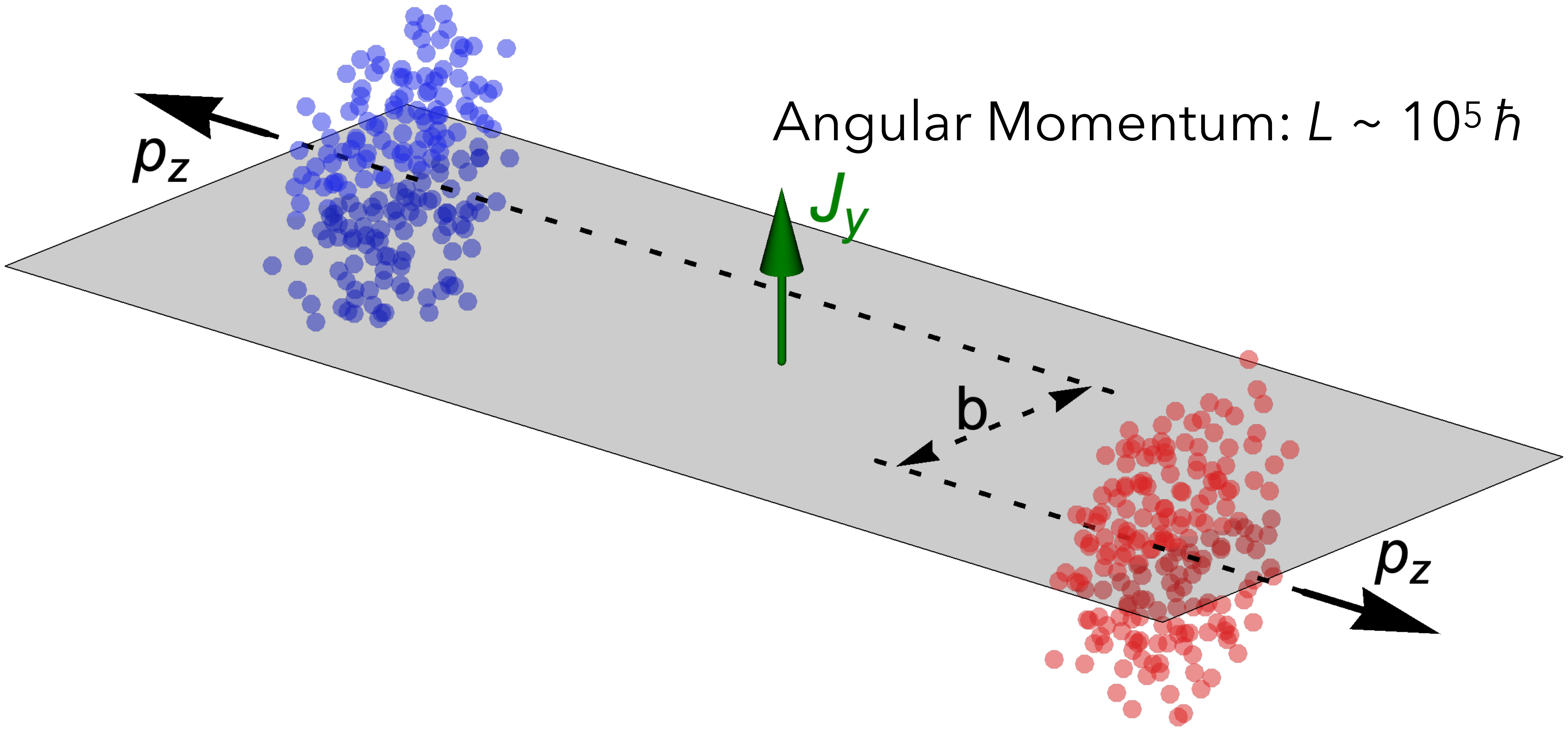
In collaboration with: Charles Gale, Sangyong Jeon

Refs:

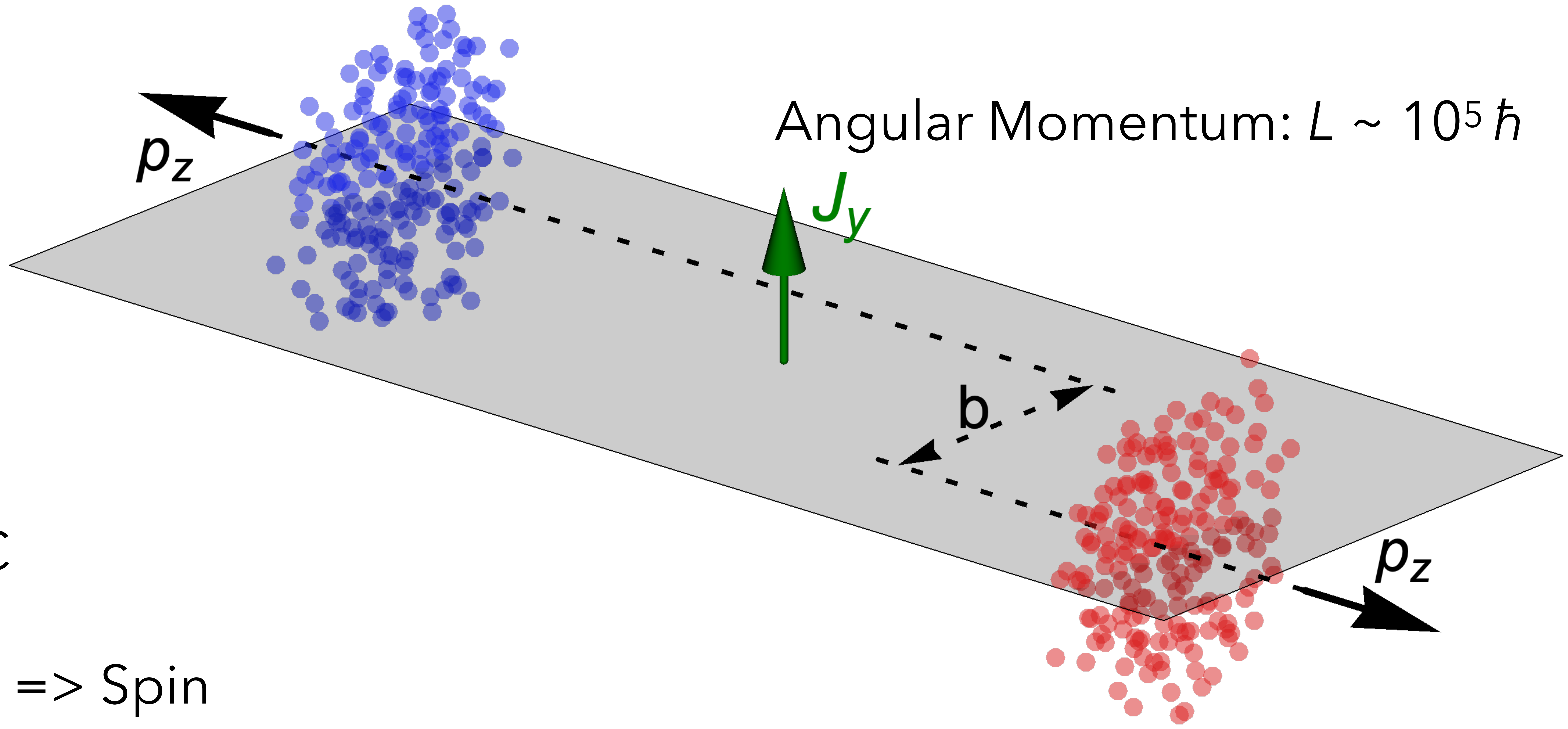
SS, C.Gale, S Jeon, arXiv:2002.01911

SS, C.Gale, S Jeon, in preparation

Angular Momentum In HIC



Angular Momentum In HIC

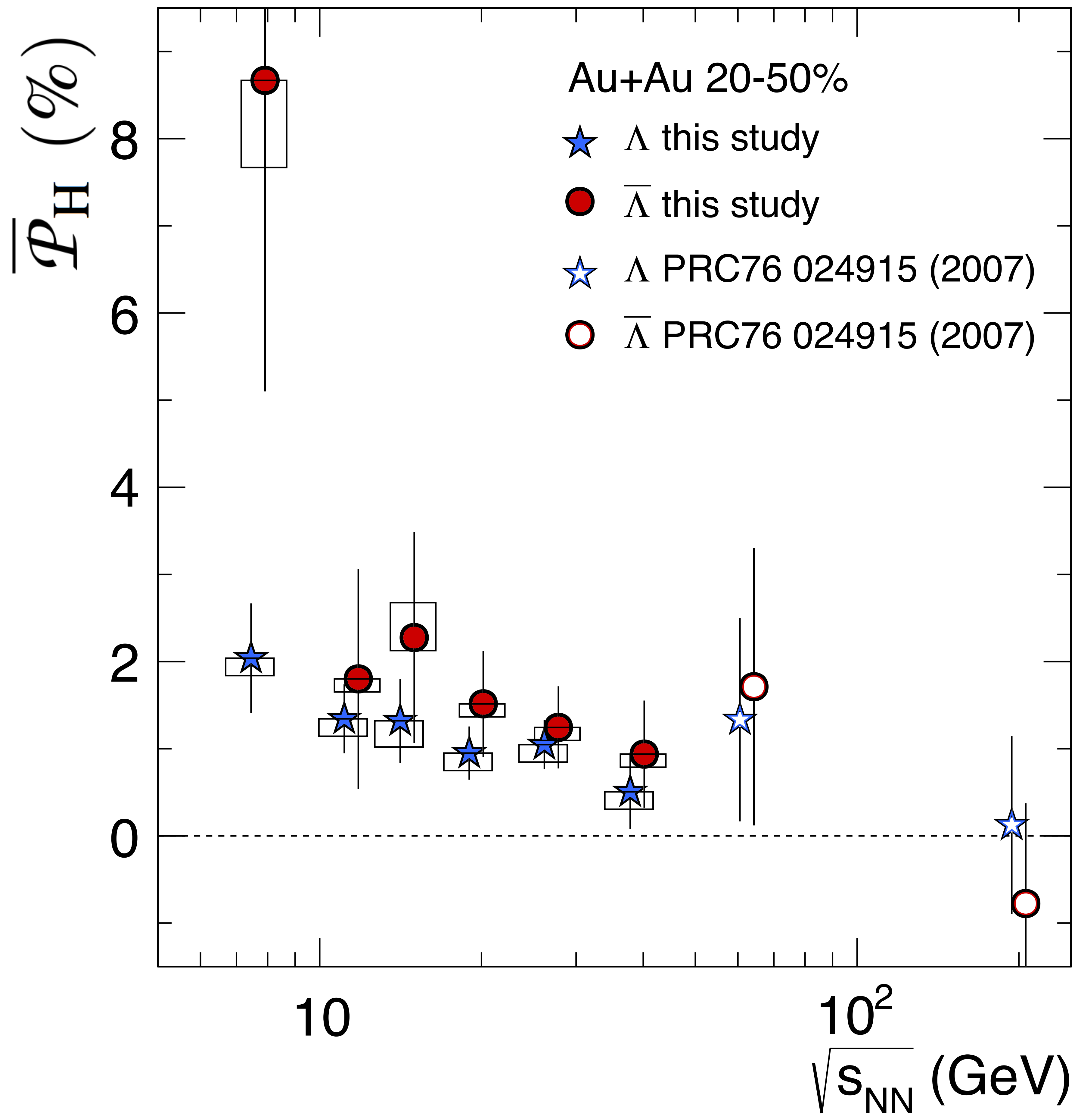


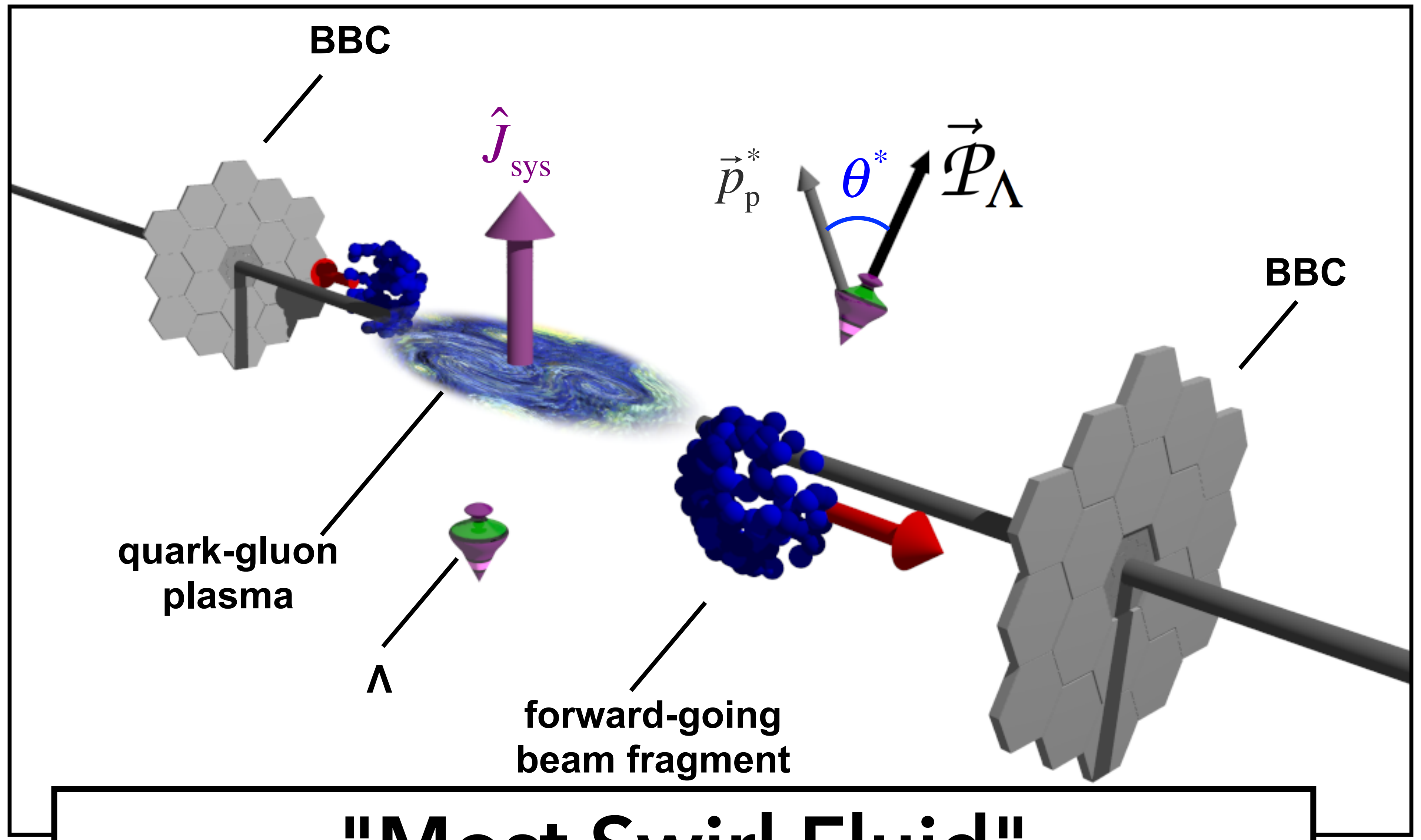
Large initial OAM in HIC

micro. collisions: OAM => Spin

Global Spin Polarization In HIC

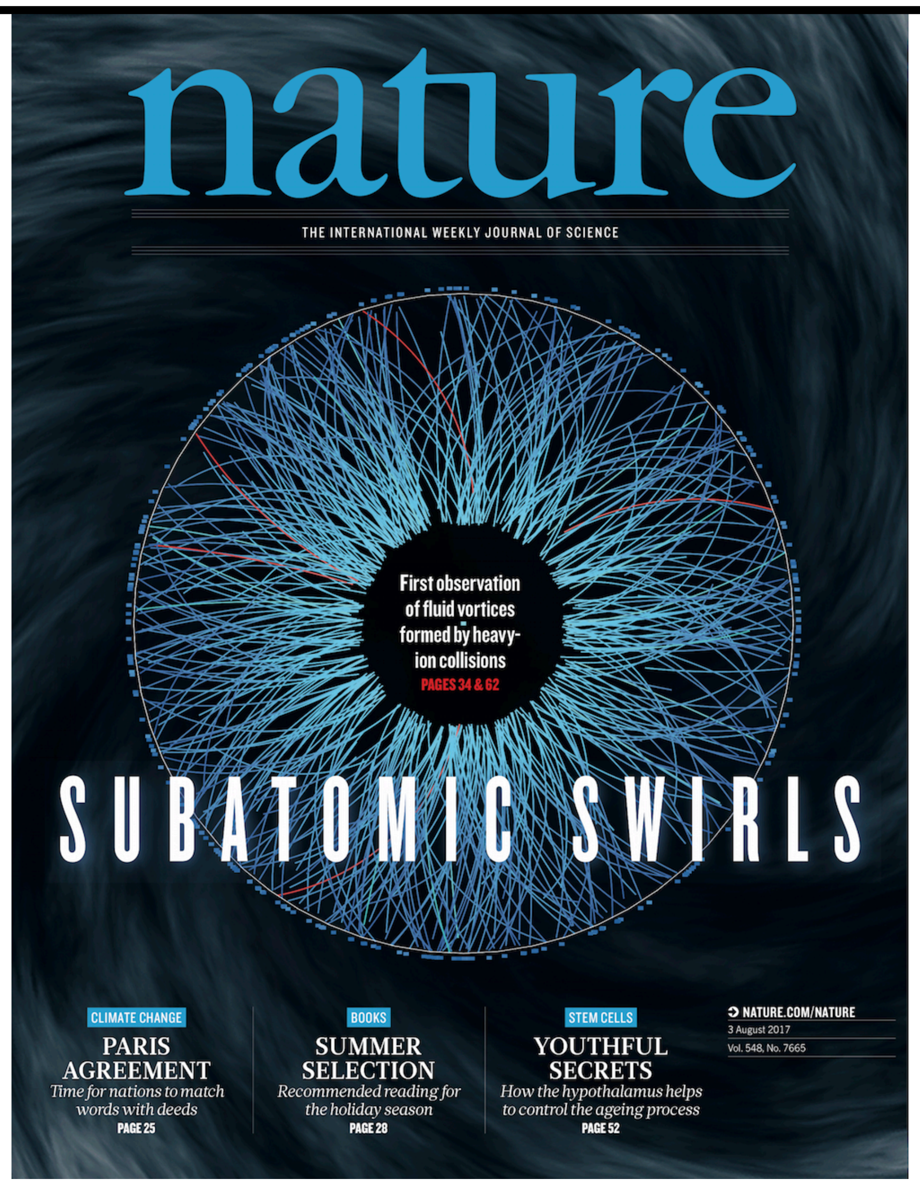
Angular Velocity: $\omega \sim 10^{21}$ Hz



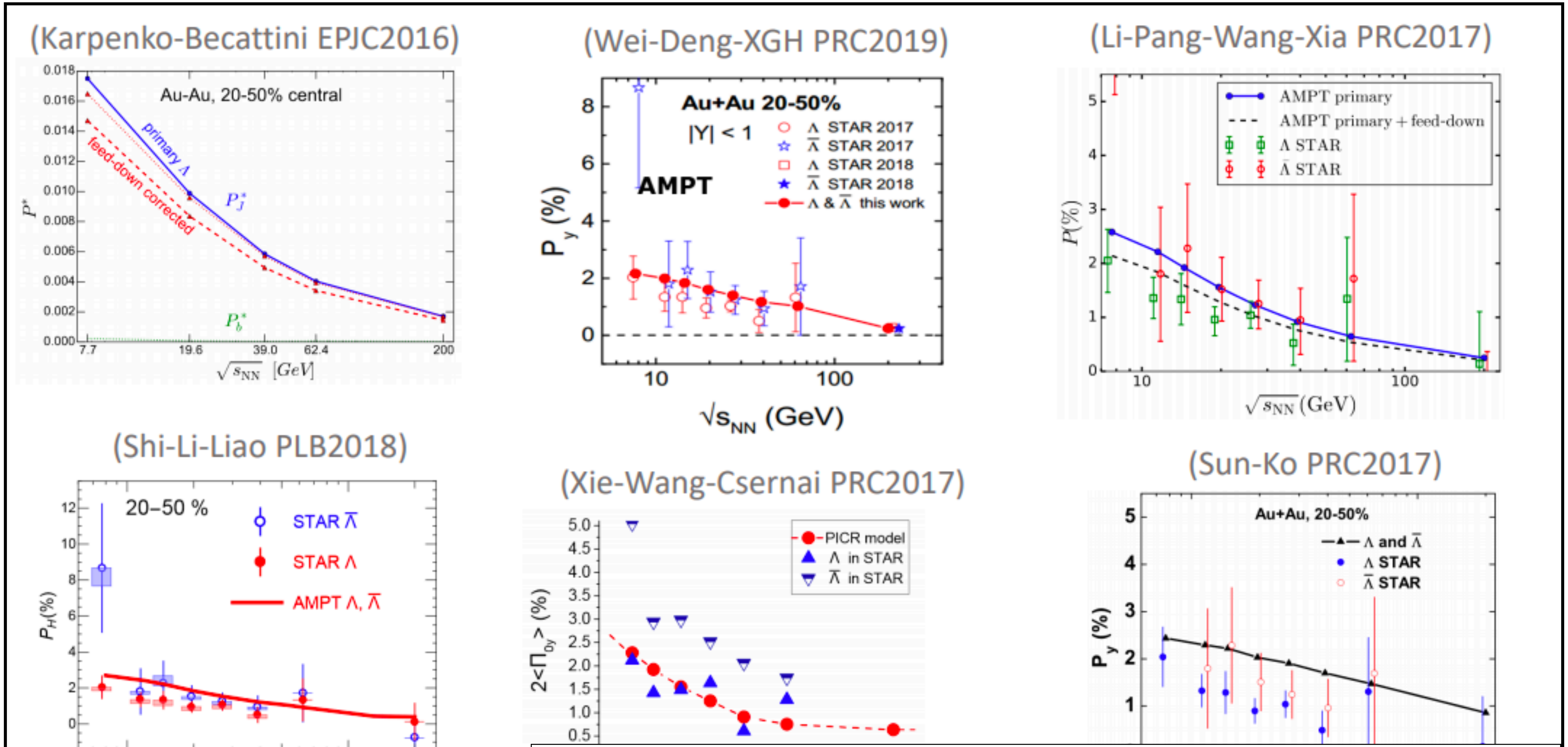


"Most Swirl Fluid"

STAR, Nature 548, 62 (2017)



Global Spin Polarization In HIC



Slide from Xu Guang Huang's QM19 Plenary Talk

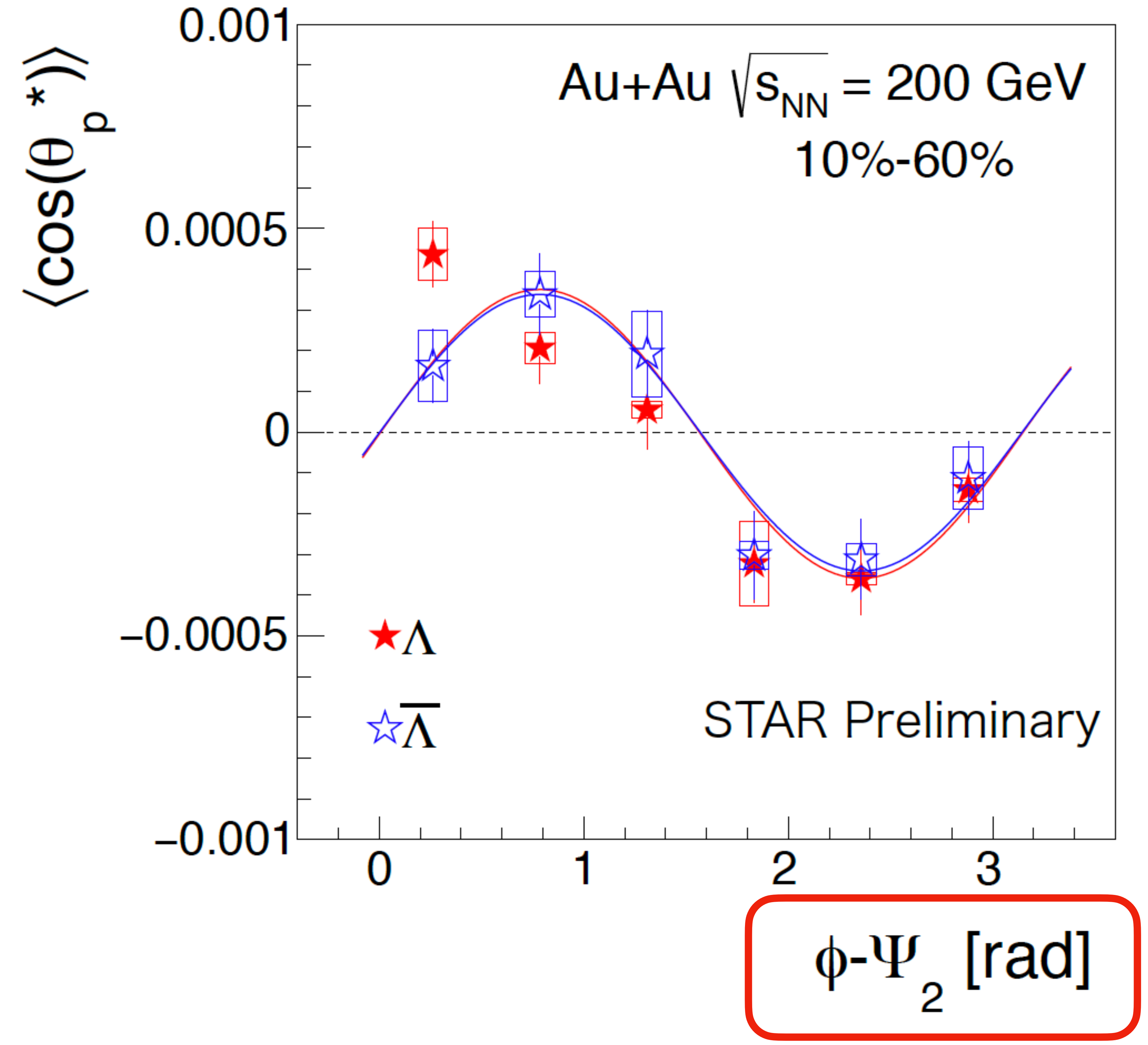
Assuming equilibrium of spin degrees of freedom,

global polarization rate can be well understood by theo. models.

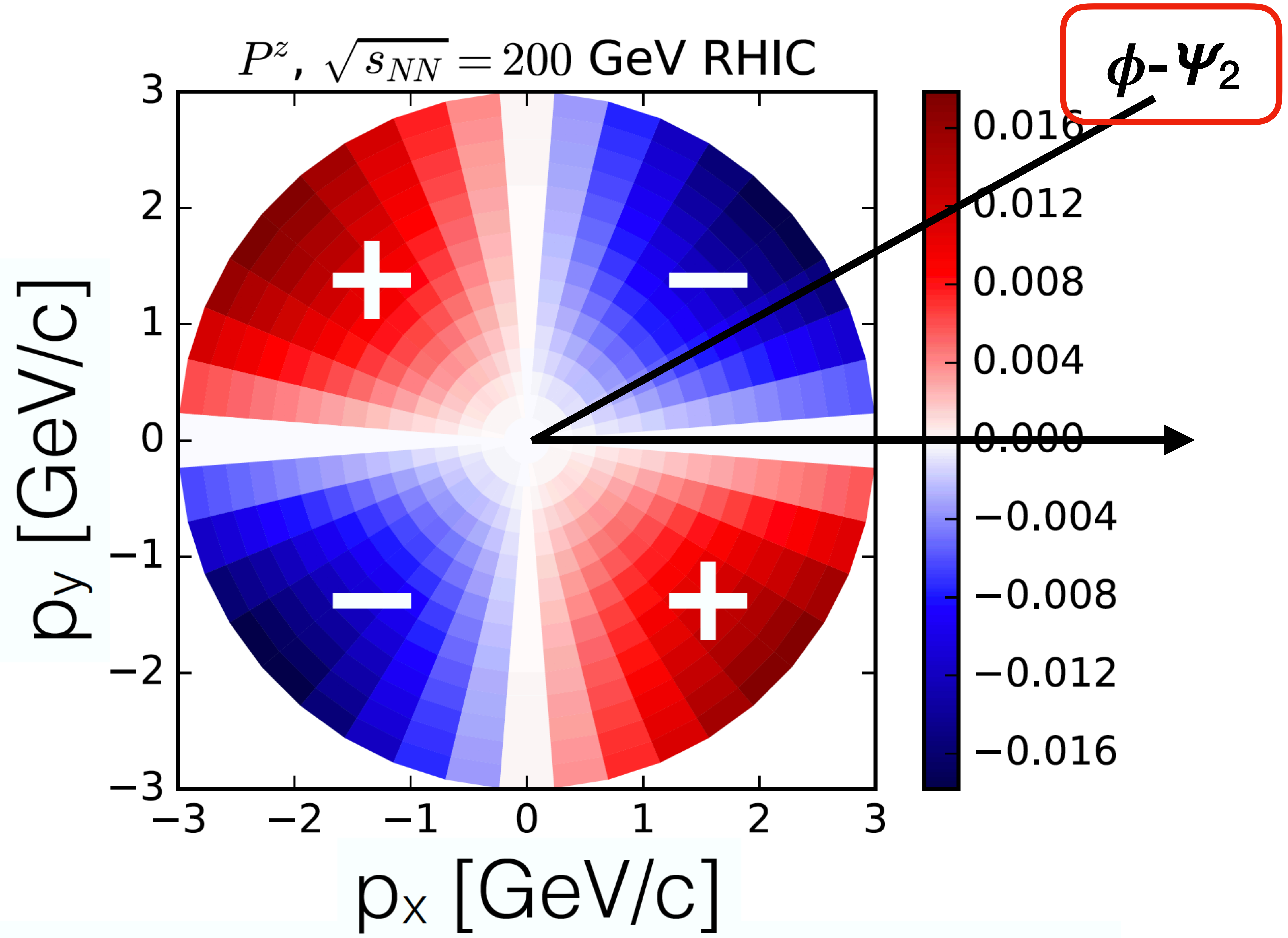
$$\varpi^{\mu\nu} \equiv \frac{1}{2}\left(\partial_\nu \frac{u_\mu}{T} - \partial_\mu \frac{u_\nu}{T}\right)$$

Local Spin Polarization In HIC

Local longitudinal polarization due to collective flow



STAR, Preliminary

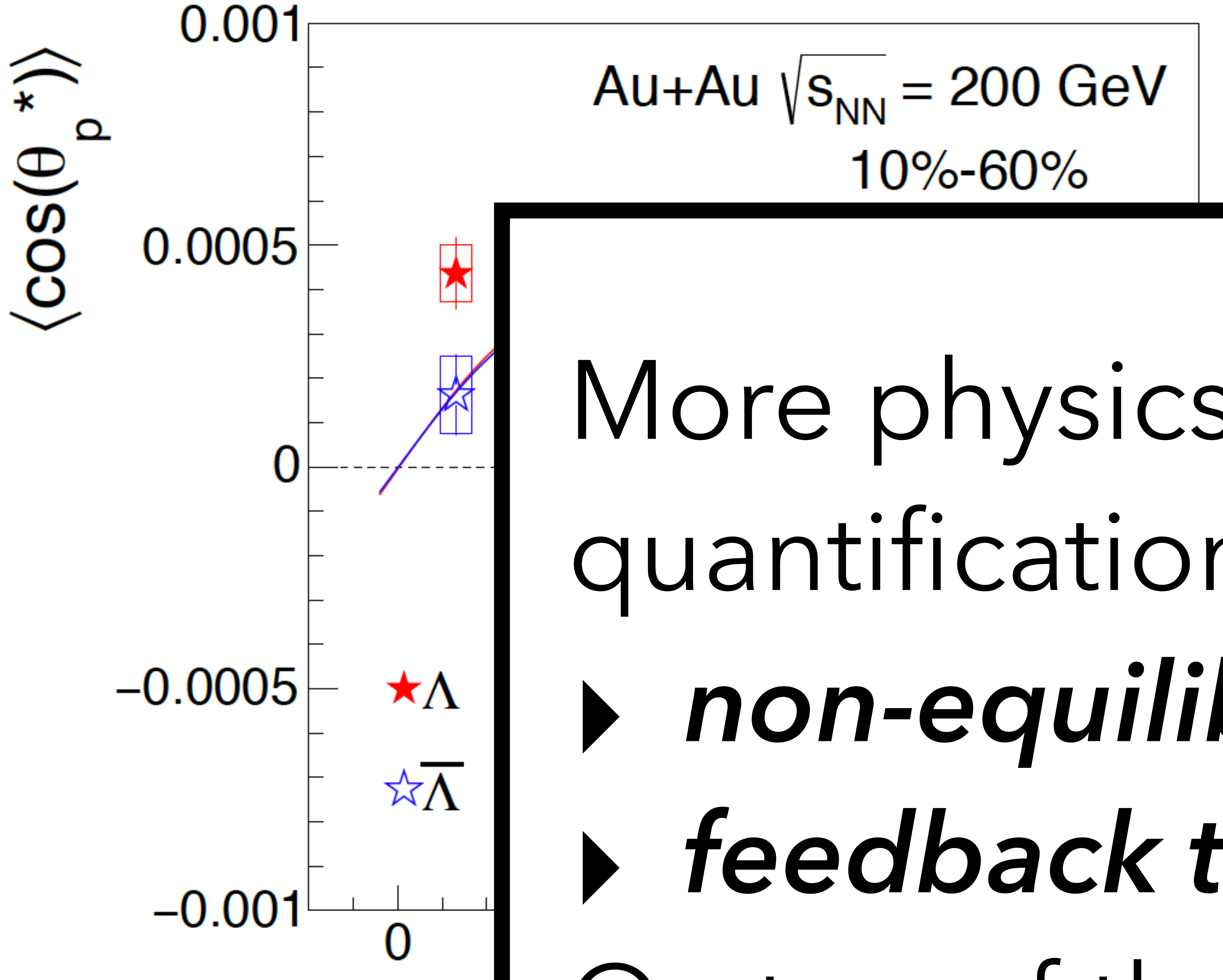


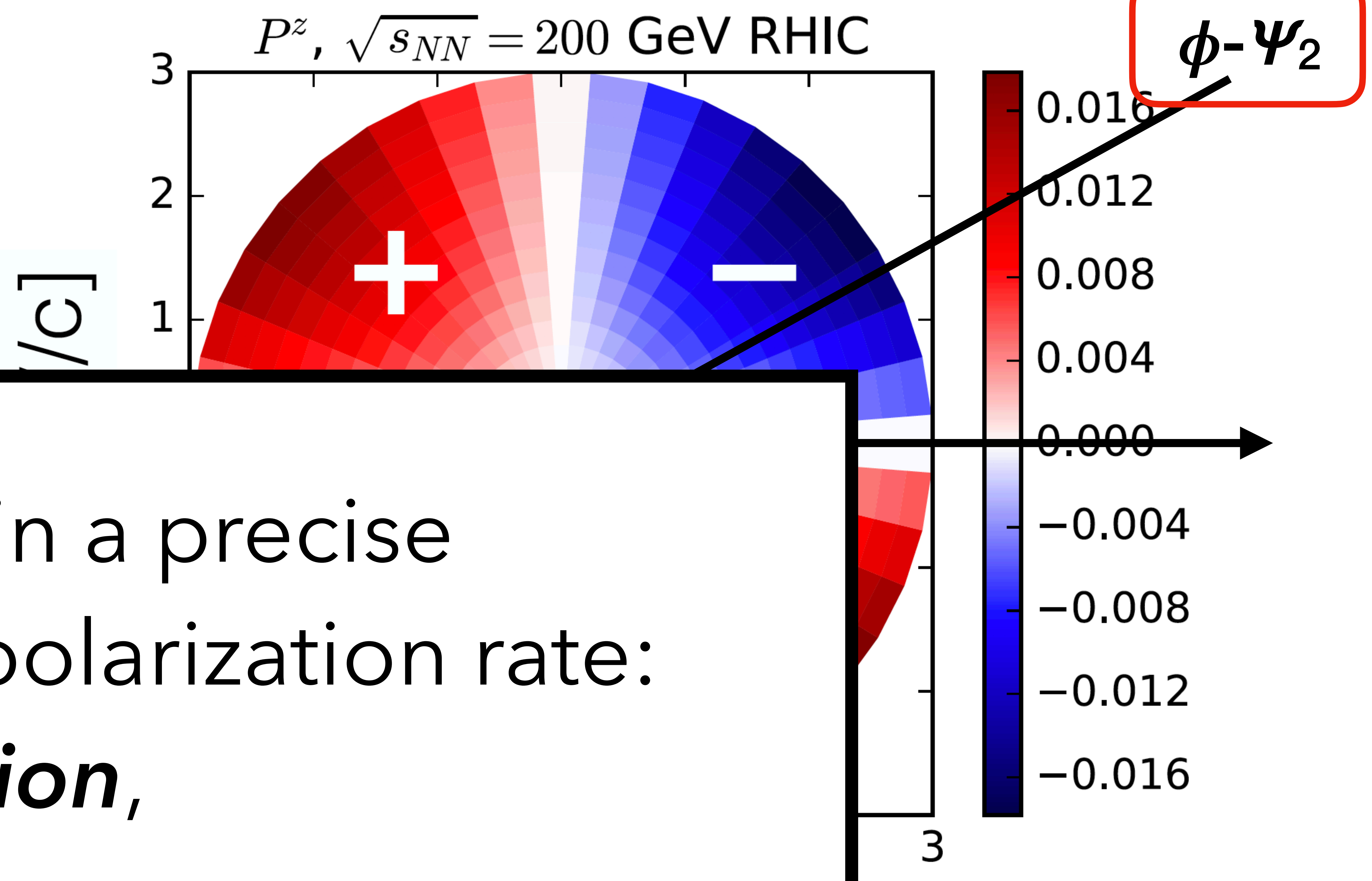
- (Hydro) F. Becattini & I. Karpenko, PRL 2018

Similarly in other models

Local Spin Polarization In HIC

Local longitudinal polarization due to collective flow





STA

, PRL 2018

More physics are needed in a precise quantification of the spin polarization rate:

- ***non-equilibrium correction***,
- ***feedback to the fluid***.

On top of that, the theory should correctly reflect ***microscopic properties*** of the system.

Outline (& Summary)

- Microscopic:

  Microscopic Theory: Chiral Kinetic Theory

  - Thermal Equilibrium

    Ideal Spin Hydrodynamics

  - … + Non-Equilibrium Correction

    Viscous Spin Hydrodynamics

- Macroscopic:

  Macroscopic Principle: conservation laws & 2nd law of thermodynamics

  Extra terms found compared to previous literature

Microscopic Theory

Chiral Kinetic Theory

Wigner function: quantum distribution function

$$W_{ab}(x,p) \equiv \left\langle \int \mathrm{d}^4 y\, e^{\frac{i}{\hbar} p\cdot y} \widehat{\bar{\psi}}_b(x+\frac{y}{2}) \widehat{\psi}_a(x-\frac{y}{2}) \right\rangle$$

a 4×4 matrix

Chiral Kinetic Theory

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a 4×4 matrix, decomposed in Clifford basis

$$W \equiv \frac{1}{4}\Big(\mathcal{F} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\mathcal{L}_{\mu\nu}\sigma^{\mu\nu}\Big)$$

Hydrodynamic Quantities:

$$J^\mu \equiv \langle\bar{\psi}\gamma^\mu\psi\rangle = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\mathcal{V}^\mu\,,$$

$$T^{\mu\nu} \equiv \langle\bar{\psi}(i\gamma^\mu\partial^\nu)\psi\rangle = \int \frac{\mathrm{d}^4p}{(2\pi)^4}p^\nu\mathcal{V}^\mu\,,$$

Chiral Kinetic Theory

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$$J_5^\mu \equiv \langle \bar{\psi}\gamma^\mu \gamma^5 \psi\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathcal{A}^\mu ,$$

$$S^{\lambda\mu\nu} \equiv \frac{1}{4}\langle \bar{\psi}\{\gamma^\lambda, \sigma^{\mu\nu}\}\psi\rangle = \frac{1}{2}\epsilon^{\lambda\mu\nu\sigma} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathcal{A}_\sigma ,$$

Chiral Kinetic Theory

Dirac equation => Equation of Motion

$$\gamma_\mu(p^\mu + \frac{1}{2}i\hbar\partial^\mu)W(x,p) = mW(x,p)$$

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Equation of Motion for Clifford components:

$$\partial_\alpha \begin{pmatrix} \mathcal{V}^\mu \\ \mathcal{A}^\mu \\ \mathcal{L}^{\mu\nu} \\ \mathcal{F} \\ \mathcal{P} \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \mathcal{V}^\mu \\ \mathcal{A}^\mu \\ \mathcal{L}^{\mu\nu} \\ \mathcal{F} \\ \mathcal{P} \end{pmatrix}$$

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Massless Limit

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Chiral Kinetic Equation (up to ħ-order):

$$\left[p^\mu \partial_\mu \pm \hbar \left(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u \cdot p}\right) \partial_\nu\right] f_\pm = 0$$

$$\mathcal{J}^\mu_\pm \equiv \frac{1}{2}(\mathcal{V}^\mu \pm \mathcal{A}^\mu) = \left(p^\mu \pm \hbar \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u \cdot p} \partial_\nu\right) f_\pm$$

Distribution Function

[Refs: Hidaka-Pu-Yang PRD2017&2018;
Huang-SS-Jiang-Liao-Zhuang PRD2018;
Liu-Gao-Mameda-Huang PRD2019]

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Quantum Corrections

from Microscopic to Macroscopic

Thermal Equilibrium Distribution

Chiral Kinetic Equation $$\Big[p^\mu\partial_\mu \pm \hbar\Big(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma}{2u\cdot p}\Big)\partial_\nu\Big] f_\pm = 0$$

Thermal Equilibrium Distribution:

[Ref: Liu-Gao-Mameda-Huang PRD2019]

$$f_{\mathrm{eq},\pm}(p) = \frac{1}{\exp[\frac{p\cdot u - \mu_\pm}{T} \pm \hbar \frac{\epsilon^{\mu\nu\rho\sigma}\varpi_{\mu\nu}n_\rho p_\sigma}{4\,n\cdot p}] + 1}$$

Thermal Equilibrium Distribution

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Ideal Chiral Hydrodynamics:

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

$$J^\mu_{\mathrm{eq},\pm} = \frac{\mu_\pm}{6}\left(T^2 + \frac{\mu_\pm^2}{\pi^2}\right)u^\mu \underline{\pm \frac{\hbar}{2}\left(\frac{T^2}{6} + \frac{\mu_\pm^2}{2\pi^2}\right)\omega^\mu}$$

chiral-vortical current

$$T^{\mu\nu}_{\mathrm{eq}} = \left(\frac{7\pi^2 T^4}{45} + \frac{2T^2(\mu_V^2 + \mu_A^2)}{3} + \frac{\mu_V^4 + 6\mu_V^2\mu_A^2 + \mu_A^4}{3\pi^2}\right)(u^\mu u^\nu - g^{\mu\nu}/4)$$
$$\underline{+ \frac{\hbar\,\mu_A}{12}\left(T^2 + \frac{3\mu_V^2 + \mu_A^2}{\pi^2}\right)(8\omega^\mu u^\nu + T\epsilon^{\mu\nu\sigma\lambda}\varpi_{\sigma\lambda})}$$

feedback to the medium

Non-Equilibrium Correction

14+6 Moment Expansion:

λ_X: polynomials of $(u \cdot p)$

$$f^{\pm} \equiv f^{\pm}_{\rm eq} + f^{\pm}_{\rm eq}(1 - f^{\pm}_{\rm eq})\left[\lambda^{\pm}_{\Pi}\Pi + \lambda^{\pm}_{\nu}\nu^{\mu}_{\pm}p_{\mu} + \lambda^{\pm}_{\pi}\pi^{\mu\nu}p_{\mu}p_{\nu} \mp \hbar\lambda^{\pm}_{\Omega}\frac{\Omega^{\pm}\cdot p}{u\cdot p}\right]$$

non-eq. correction to polarization

Non-Equilibrium Correction

14+6 Moment Expansion:

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$$f^{\pm} \equiv f_{\rm eq}^{\pm} + f_{\rm eq}^{\pm}(1 - f_{\rm eq}^{\pm})\left[\lambda_{\Pi}^{\pm}\Pi + \lambda_{\nu}^{\pm}\nu_{\pm}^{\mu}p_{\mu} + \lambda_{\pi}^{\pm}\pi^{\mu\nu}p_{\mu}p_{\nu} \mp \hbar\lambda_{\Omega}^{\pm}\frac{\Omega^{\pm}\cdot p}{u \cdot p}\right]$$

non-eq. correction to polarization

Dissipative Quantities:

$\delta f^{\pm} \equiv f^{\pm} - f_{\rm eq}^{\pm}$

$$\Pi \equiv -\frac{1}{3}\int_p \Delta^{\mu\nu}p_{\mu}p_{\nu}(\delta f_+ + \delta f_-)$$

$$\nu_{\pm}^{\mu} \equiv \int_p \Delta_{\alpha}^{\mu}p^{\alpha}\delta f_{\pm}$$

$$\pi^{\mu\nu} \equiv \int_p \Delta_{\alpha\beta}^{\mu\nu}p^{\alpha}p^{\beta}(\delta f_+ + \delta f_-)$$

$$\Omega_{\pm}^{\mu} \equiv \pm\int_p \Delta_{\alpha}^{\mu}p^{\alpha}(u \cdot p)\delta f_{\pm}$$

Non-Equilibrium Correction

14+6 Moment Expansion:

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$$f^{\pm} \equiv f^{\pm}_{\rm eq} + f^{\pm}_{\rm eq}(1-f^{\pm}_{\rm eq})\left[\lambda^{\pm}_{\Pi}\Pi + \lambda^{\pm}_{\nu}\nu^{\mu}_{\pm}p_{\mu} + \lambda^{\pm}_{\pi}\pi^{\mu\nu}p_{\mu}p_{\nu} \mp \hbar\lambda^{\pm}_{\Omega}\frac{\Omega^{\pm}\cdot p}{u\cdot p}\right]$$

non-eq. correction to polarization

EoM for Dissipative Quantities:

$\delta f^{\pm} \equiv f^{\pm} - f^{\pm}_{\rm eq}$

$$\hat{\rm d}\Pi \equiv -\frac{1}{3}\int_p \Delta^{\mu\nu}p_{\mu}p_{\nu}(\hat{\rm d}\delta f_{+} + \hat{\rm d}\delta f_{-})$$

$$\Delta^{\mu}_{\alpha}\hat{\rm d}\nu^{\alpha}_{\pm} \equiv \int_p \Delta^{\mu}_{\alpha}p^{\alpha}\hat{\rm d}\delta f_{\pm}$$

$[\hat{\rm d} \equiv u\cdot\partial]$

$$\Delta^{\mu\nu}_{\alpha\beta}\hat{\rm d}\pi^{\alpha\beta} \equiv \int_p \Delta^{\mu\nu}_{\alpha\beta}p^{\alpha}p^{\beta}(\hat{\rm d}\delta f_{+} + \hat{\rm d}\delta f_{-})$$

$$\Delta^{\mu}_{\alpha}\hat{\rm d}\Omega^{\alpha}_{\pm} \equiv \pm\int_p \Delta^{\mu}_{\alpha}p^{\alpha}(u\cdot p)\hat{\rm d}\delta f_{\pm}$$

Chiral Kinetic Equation

$$\left[p^{\mu}\partial_{\mu} \pm \hbar\left(\partial_{\mu}\frac{\epsilon^{\mu\nu\rho\sigma}p_{\rho}u_{\sigma}}{2u\cdot p}\right)\partial_{\nu}\right]f_{\pm} = C_{\pm}[f_{+},f_{-}]$$

Viscous Spin Hydrodynamics

Spin-Hydro with Non-Equilibrium Correction:

$$J^\mu_\pm = n_\pm u^\mu + \nu^\mu_\pm \pm \hbar\Big(\frac{3J^\pm_{1,1}}{2T} - \frac{3\Pi}{2m^2}\Big)\omega^\mu \pm \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho\partial_\sigma\Big(\frac{G^{(1),\pm}_{4,1}}{D^\pm_{3,1}}\nu_{\pm,\lambda}\Big)$$
$$\pm \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho\sigma_\sigma{}^\xi\Big(\frac{J^\pm_{2,2}}{2J^\pm_{4,2}}\pi_{\lambda\xi}\Big) \mp \frac{\hbar}{2}\omega_\lambda\Big(\frac{J^\pm_{2,2}}{2J^\pm_{4,2}}\pi^{\mu\lambda}\Big)$$

"normal" viscous hydro

CVE in eq.

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (P_0 + \Pi)(u^\mu u^\nu - g^{\mu\nu}) + \pi^{\mu\nu} + \hbar\frac{J^+_{2,1} - J^-_{2,1}}{2T}(u^\mu\omega^\nu + u^\nu\omega^\mu) + \hbar(I^+_{1,0} - I^-_{1,0})\omega^\mu u^\nu$$
$$+ \hbar(u^\mu\Omega^\nu_+ + u^\nu\Omega^\mu_+) - \hbar(u^\mu\Omega^\nu_- + u^\nu\Omega^\mu_-)$$
$$+ \frac{\hbar}{2}\Big(\frac{J^+_{3,2}}{2J^+_{4,2}} - \frac{J^-_{3,2}}{2J^-_{4,2}}\Big)\epsilon^{\mu\rho\sigma\lambda}u^\nu u_\rho(\partial_\sigma u^\xi)\pi_{\lambda\xi} + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho\partial_\sigma\Big(\Big(\frac{J^+_{3,2}}{2J^+_{4,2}} - \frac{J^-_{3,2}}{2J^-_{4,2}}\Big)\pi^\nu_\lambda\Big)$$
$$+ \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho(\partial_\sigma u^\nu)(K_+\nu_{+,\lambda} - K_-\nu_{-,\lambda}) + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\nu}u_\rho(\partial_\sigma u^\lambda)(K_+\nu_{+,\lambda} - K_-\nu_{-,\lambda})$$
$$+ \hbar\omega^\mu(K_+\nu^\nu_+ - K_-\nu^\nu_-) + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho u^\nu\partial_\sigma(\nu_{+,\lambda} - \nu_{-,\lambda}) + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\nu}u_\rho u^\lambda\partial_\sigma(\nu_{+,\lambda} - \nu_{-,\lambda})$$

$$K_\pm \equiv 1 + \frac{J^\pm_{3,1}J^\pm_{3,2} - J^\pm_{2,2}J^\pm_{4,1}}{D^\pm_{3,1}}$$

Viscous Spin Hydrodynamics

Relaxation Equations for Dissipative Quantities:

$$\hat{\mathrm{d}}\Pi + \tau_\Pi^{-1}\Pi = -\frac{\zeta\theta}{\tau_\Pi} + [\text{2nd order terms}]$$

$$\Delta^{\mu\nu}_{\alpha\beta}\hat{\mathrm{d}}\pi^{\alpha\beta} + \tau_\pi^{-1}\pi^{\mu\nu} = \frac{2\eta\sigma^{\mu\nu}}{\tau_\pi} + [\text{2nd order terms}]$$

$$\Delta^\mu_\alpha\hat{\mathrm{d}}\nu^\alpha_\pm + \tau_{\nu,\pm}^{-1}\nu^\mu_\pm = \frac{\sigma}{\tau_{\nu,\pm}}\partial^\mu\frac{\mu_\pm}{T} + M_{\nu,\pm}\nu^\mu_\mp + C_{\Omega,\pm}\Omega^\mu_+ + C_{\Omega,\mp}\Omega^\mu_- + [\text{2nd order terms}]$$

$$\Delta^\mu_\alpha\hat{\mathrm{d}}\Omega^\alpha_\pm + \tau_{\Omega,\pm}^{-1}\Omega^\mu_\pm = M_{\Omega,\pm}\Omega^\mu_\mp + [\text{2nd order terms}]$$

Ω^μ: non-equilibrium correction of spin polarization vector.

τ: Relaxation Time; *M* & *C*: Mixing Terms

τ, *M* & *C* are integrals of collision kernel, and reflect the ***microscopic*** properties

The hydrodynamic theory is a macroscopic theory.

<= **Conserv. Laws & 2nd Law of Thermodynamics**

In Ref.[1], the (1st order) viscous spin hydrodynamics theory is obtained from such macroscopic principles.

A Natural Question: Comparison of these results?

Macroscopic Theory

[1] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya *Phys.Lett.B* 795 (2019) 100-106, arXiv:1901.06615.

Macroscopic Quantities & Principles

Definition	Conservation/Production	
$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - P\,\Delta^{\mu\nu} + \Pi^{\mu\nu}\,,$	$0 = \partial_\mu T^{\mu\nu}\,,$	Energy-Momentum
$N^\mu_f = n_f u^\mu + \nu^\mu_f\,,$	$0 = \partial_\mu N^\mu_f\,,$	Conserved Charges
$\mathcal{S}^{\mu\nu\lambda} = u^\mu S^{\nu\lambda} + \sigma^{\mu\nu\lambda}\,,$	$0 = \partial_\mu \mathcal{M}^{\mu\nu\lambda}\,,$	Total Angular Momentum
$S^\mu = s\,u^\mu + \sigma^\mu\,,$	$0 \leq \partial_\mu S^\mu\,,$	Entropy Current

$$0 = u_\nu \partial_\mu T^{\mu\nu} - \sum_f \mu_f \partial_\mu N^\mu_f - \mathcal{W}_{\nu\lambda}\partial_\mu \mathcal{M}^{\mu\nu\lambda}$$

Macroscopic Quantities & Principles

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$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - P\,\Delta^{\mu\nu} + \Pi^{\mu\nu}\,,$	$0 = \partial_\mu T^{\mu\nu}\,,$	Energy-Momentum
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$$0 = u_\nu \partial_\mu T^{\mu\nu} - \sum_f \mu_f \partial_\mu N_f^\mu - \mathcal{W}_{\nu\lambda}\partial_\mu \mathcal{M}^{\mu\nu\lambda}$$

$$\Rightarrow \quad T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + \phi_{\alpha\beta}\Phi^{\alpha\beta} + h_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha \frac{1}{T}) - T\sum_f \nu_f^\mu \nabla_\mu \alpha_f - T\,\sigma^{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

2nd law of thermodynamics is fulfilled if only **quadratic** terms appear on the RHS.

From Conservation/Production Laws To Viscous Hydro

$$T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + \phi_{\alpha\beta}\Phi^{\alpha\beta} + h_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T}) - T\sum_f \nu_f^\mu\nabla_\mu\alpha_f - T\,\sigma^{\mu\nu\lambda}\nabla_\mu\Big(\frac{\mathcal{W}_{\nu\lambda}}{T}\Big)$$

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$$S^\mu \equiv s\,u^\mu - \sum_f \alpha_f\nu_f^\mu + \frac{\Pi^{\mu\nu}u_\nu}{T} - \frac{\mathcal{W}_{\nu\lambda}}{T}\sigma^{\mu\nu\lambda}$$

$$\Pi = \zeta\cdot\theta\,,$$

$$\pi^{\mu\nu} = \eta\cdot\sigma^{\mu\nu}\,,$$

$$\phi^{\mu\nu} = \lambda_\perp\cdot\Delta^\mu_\alpha\Delta^\nu_\beta\Phi^{\alpha\beta} - \lambda_\parallel\cdot(u^\mu\Delta^\nu_\beta - u^\nu\Delta^\mu_\beta)u_\alpha\Phi^{\alpha\beta}\,,$$

$$\nu_f^\mu = \sigma_f\cdot\nabla^\mu\alpha_f\,,$$

$$h^\mu = \kappa\cdot\Big(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\Big)\,,$$

$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\frac{\mathcal{W}^{\nu\lambda}}{T}\,,$$

From Conservation/Production Laws To Viscous Hydro

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$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\frac{\mathcal{W}^{\nu\lambda}}{T}\,,$$

anti-symmetric part of stress-tensor:

$$\Phi_{\mu\nu} \equiv 2\mathcal{W}_{\mu\nu} - \frac{T}{2}\left(\partial_\nu\frac{u_\mu}{T} - \partial_\mu\frac{u_\nu}{T}\right)$$

$$= 2\mathcal{W}_{\mu\nu} - T\,\varpi_{\mu\nu}$$

From Conservation/Production Laws To Viscous Hydro

$$T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + \phi_{\alpha\beta}\Phi^{\alpha\beta} + h_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T}) - T\sum_f \nu_f^\mu\nabla_\mu\alpha_f - T\,\sigma^{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

2nd law of thermodynamics is fulfilled if only **quadratic** terms appear on the RHS.

$$S^\mu \equiv s\,u^\mu - \sum_f \alpha_f\nu_f^\mu + \frac{\Pi^{\mu\nu}u_\nu}{T} - \frac{\mathcal{W}_{\nu\lambda}}{T}\sigma^{\mu\nu\lambda}$$

$$\Pi = \zeta\cdot\theta\,,$$

$$\pi^{\mu\nu} = \eta\cdot\sigma^{\mu\nu}\,,$$

$$\phi^{\mu\nu} = \lambda_\perp\cdot\Delta^\mu_\alpha\Delta^\nu_\beta\Phi^{\alpha\beta} - \lambda_\parallel\cdot(u^\mu\Delta^\nu_\beta - u^\nu\Delta^\mu_\beta)u_\alpha\Phi^{\alpha\beta}\,,$$

$$\nu_f^\mu = \sigma_f\cdot\nabla^\mu\alpha_f\,,$$

$$h^\mu = \kappa\cdot\left(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\right),$$

$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\frac{\mathcal{W}^{\nu\lambda}}{T}\,,$$

anti-symmetric part of stress-tensor:

$$\Phi_{\mu\nu} \equiv 2\mathcal{W}_{\mu\nu} - \frac{T}{2}\left(\partial_\nu\frac{u_\mu}{T} - \partial_\mu\frac{u_\nu}{T}\right) = 2\mathcal{W}_{\mu\nu} - T\,\varpi_{\mu\nu}$$

From CKT:

$$\phi^{\mu\nu} \propto \Delta^\mu_\alpha\Delta^\nu_\beta\epsilon^{\alpha\beta\rho\sigma}\varpi_{\rho\sigma} - \Delta^\alpha_\rho\Delta^\beta_\sigma\epsilon^{\mu\nu\rho\sigma}\varpi_{\alpha\beta}$$

From Conservation/Production Laws To Viscous Hydro

$$T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + \phi_{\alpha\beta}\Phi^{\alpha\beta} + h_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T}) - T\sum_f \nu_f^\mu\nabla_\mu\alpha_f - T\,\sigma^{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

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From CKT:

$$\phi^{\mu\nu} \propto \Delta^\mu_\alpha\Delta^\nu_\beta\epsilon^{\alpha\beta\rho\sigma}\varpi_{\rho\sigma} - \Delta^\alpha_\rho\Delta^\beta_\sigma\epsilon^{\mu\nu\rho\sigma}\varpi_{\alpha\beta}$$

one possibility:

In Ref.[1], $\mathcal{W}_{\mu\nu}$ is the undetermined spin potential.

Comparison with CKT can fix the form of $\mathcal{W}_{\mu\nu}$.

From Conservation/Production Laws To Viscous Hydro

$$T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + \phi_{\alpha\beta}\Phi^{\alpha\beta} + h_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T}) - T\sum_f \nu_f^\mu\nabla_\mu\alpha_f - T\,\sigma^{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

2nd law of thermodynamics is fulfilled if only **quadratic** terms appear on the RHS.

$$S^\mu \equiv s\,u^\mu - \sum_f \alpha_f\nu_f^\mu + \frac{\Pi^{\mu\nu}u_\nu}{T} - \frac{\mathcal{W}_{\nu\lambda}}{T}\sigma^{\mu\nu\lambda}$$

$$\Pi = \zeta\cdot\theta\,,$$

$$\pi^{\mu\nu} = \eta\cdot\sigma^{\mu\nu}\,,$$

$$\phi^{\mu\nu} = \lambda_\perp\cdot\Delta^\mu_\alpha\Delta^\nu_\beta\Phi^{\alpha\beta} - \lambda_\parallel\cdot(u^\mu\Delta^\nu_\beta - u^\nu\Delta^\mu_\beta)u_\alpha\Phi^{\alpha\beta}\,,$$

$$\nu_f^\mu = \sigma_f\cdot\nabla^\mu\alpha_f\,,$$

$$h^\mu = \kappa\cdot\left(\widehat{\mathrm{D}}u^\mu + T\nabla^\mu\frac{1}{T}\right),$$

$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\frac{\mathcal{W}^{\nu\lambda}}{T}\,,$$

anti-symmetric part of stress-tensor:

$$\Phi_{\mu\nu} \equiv 2\mathcal{W}_{\mu\nu} - \frac{T}{2}\left(\partial_\nu\frac{u_\mu}{T} - \partial_\mu\frac{u_\nu}{T}\right) = 2\mathcal{W}_{\mu\nu} - T\,\varpi_{\mu\nu}$$

From CKT:

$$\phi^{\mu\nu} \propto \Delta^\mu_\alpha\Delta^\nu_\beta\epsilon^{\alpha\beta\rho\sigma}\varpi_{\rho\sigma} - \Delta^\alpha_\rho\Delta^\beta_\sigma\epsilon^{\mu\nu\rho\sigma}\varpi_{\alpha\beta}$$

another possibility:

Whether the macroscopic solution is complete?

It has been shown by Son&Surowka[PRL103,191601(2009)] that CVE&CME currents can be introduced, without violating conservation/production laws.

Extra Terms In The Macro. Solution?

$$T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + (\phi_{\alpha\beta} - X_{\alpha\beta})\Phi^{\alpha\beta} + (h_\alpha - c_h\omega_\alpha)(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T})$$

$$- T\sum_f (\nu_f^\mu - c_{\nu,f}\omega^\mu)\nabla_\mu\alpha_f - T\,(\sigma^{\mu\nu\lambda} - Y_{\mu\nu\lambda})\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

$$S^\mu \equiv s\,u^\mu - \sum_f \alpha_f\nu_f^\mu + \frac{\Pi^{\mu\nu}u_\nu}{T} - \frac{\mathcal{W}_{\nu\lambda}}{T}\sigma^{\mu\nu\lambda} + c_S\omega^\mu + \left(\frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\right)$$

Prerequisite:

new terms shall cancel in the prod. eq.

$$T\partial_\mu\left(c_S\omega^\mu + \frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\right)$$

$$= -X_{\alpha\beta}\Phi^{\alpha\beta} - c_h\omega_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T})$$

$$+ T\sum_f c_{\nu,f}\omega^\mu\nabla_\mu\alpha_f + T\,Y_{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

Extra Terms In The Macro. Solution?

$$T\partial_\mu S^\mu = \Pi\,\theta + \frac{1}{2}\pi_{\alpha\beta}\sigma^{\alpha\beta} + (\phi_{\alpha\beta} - X_{\alpha\beta})\Phi^{\alpha\beta} + (h_\alpha - c_h\omega_\alpha)(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha \frac{1}{T})$$
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$$S^\mu \equiv s\,u^\mu - \sum_f \alpha_f \nu_f^\mu + \frac{\Pi^{\mu\nu}u_\nu}{T} - \frac{\mathcal{W}_{\nu\lambda}}{T}\sigma^{\mu\nu\lambda} + c_S\omega^\mu + \left(\frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\right)$$

$$\Pi = \zeta\cdot\theta\,,$$
$$\pi^{\mu\nu} = \eta\cdot\sigma^{\mu\nu}\,,$$
$$\phi^{\mu\nu} = \lambda_\perp\cdot\Delta^\mu_\alpha\Delta^\nu_\beta\Phi^{\alpha\beta} - \lambda_\parallel\cdot(u^\mu\Delta^\nu_\beta - u^\nu\Delta^\mu_\beta)u_\alpha\Phi^{\alpha\beta} + X^{\mu\nu}\,,$$
$$\nu_f^\mu = \sigma_f\cdot\nabla^\mu\alpha_f + c_{\nu,f}\omega^\mu\,,$$
$$h^\mu = \kappa\cdot\left(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\right) + c_h\omega^\mu\,,$$
$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\frac{\mathcal{W}^{\nu\lambda}}{T} + Y^{\mu\nu\lambda}\,,$$

$$X^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\left[c_{\phi,\omega}\omega_{\rho\sigma} + \frac{c_{\phi,T}}{T}u_\rho\partial_\sigma T + \sum_f c_{\phi,f}u_\rho\partial_\sigma\alpha_f\right].$$

Prerequisite:
new terms shall cancel in the prod. eq.

$$T\partial_\mu\left(c_S\omega^\mu + \frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\right)$$
$$= -X_{\alpha\beta}\Phi^{\alpha\beta} - c_h\omega_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T})$$
$$+ T\sum_f c_{\nu,f}\omega^\mu\nabla_\mu\alpha_f + T\,Y_{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

Constrain equation for coefficients:
$Y^{\mu\nu\lambda}$, $Z^{\mu\nu\lambda}$, c_x

Constraining Extra Terms

Constrain equations:

$$T\partial_\mu\left(c_S\omega^\mu + \frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\right)$$
$$= -X_{\alpha\beta}\Phi^{\alpha\beta} - c_h\omega_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T})$$
$$+ T\sum_f c_{\nu,f}\omega^\mu\nabla_\mu\alpha_f + T\,Y_{\mu\nu\lambda}\nabla_\mu\left(\frac{\mathcal{W}_{\nu\lambda}}{T}\right)$$

Constraining Extra Terms

$$0 = \Big[-2c_S + T\frac{\partial c_S}{\partial T} + \frac{3c_{\phi,\omega} - c_{\phi,T}}{T}\Big]\omega^\mu \nabla_\mu T$$
$$+ T\sum_f \Big[\frac{\partial c_S}{\partial \alpha_f} - c_{\nu,f} - c_{\phi,f} + \frac{2n_f}{\varepsilon + P}\Big(c_{\phi,\omega} + \frac{c_h}{2} - c_S T\Big)\Big]\omega^\mu \nabla_\mu \alpha_f$$
$$+ \Big[2X^{\mu\nu} + \partial_\lambda\Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P}\omega^\lambda S^{\mu\nu}\Big)\Big]\mathcal{W}_{\mu\nu}$$

Constrain equations:

$$T\partial_\mu\Big(c_S\omega^\mu + \frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\Big)$$
$$= -X_{\alpha\beta}\Phi^{\alpha\beta} - c_h\omega_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T})$$
$$+ T\sum_f c_{\nu,f}\omega^\mu\nabla_\mu\alpha_f + T\,Y_{\mu\nu\lambda}\nabla_\mu\Big(\frac{\mathcal{W}_{\nu\lambda}}{T}\Big)$$

Constraining Extra Terms

$$0 = \Big[- 2c_S + T\frac{\partial c_S}{\partial T} + \frac{3c_{\phi,\omega} - c_{\phi,T}}{T}\Big]\omega^\mu \nabla_\mu T$$
$$+ T\sum_f \Big[\frac{\partial c_S}{\partial \alpha_f} - c_{\nu,f} - c_{\phi,f} + \frac{2n_f}{\varepsilon + P}\Big(c_{\phi,\omega} + \frac{c_h}{2} - c_S T\Big)\Big]\omega^\mu \nabla_\mu \alpha_f$$
$$+ \Big[2X^{\mu\nu} + \partial_\lambda\Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P}\omega^\lambda S^{\mu\nu}\Big)\Big]\mathcal{W}_{\mu\nu}$$

Constrain equations:

$$T\partial_\mu\Big(c_S\omega^\mu + \frac{\mathcal{W}_{\nu\lambda}}{T}Z^{\mu\nu\lambda}\Big)$$
$$= -X_{\alpha\beta}\Phi^{\alpha\beta} - c_h\omega_\alpha(\widehat{\mathrm{D}}u^\alpha + T\,\nabla^\alpha\frac{1}{T})$$
$$+ T\sum_f c_{\nu,f}\omega^\mu \nabla_\mu \alpha_f + T\,Y_{\mu\nu\lambda}\nabla_\mu\Big(\frac{\mathcal{W}_{\nu\lambda}}{T}\Big)$$

shall be valid for arbitrary $T, \alpha_f, \mathcal{W}_{\mu\nu}$

$$0 = \Big[- 2c_S + T\frac{\partial c_S}{\partial T} + \frac{3c_{\phi,\omega} - c_{\phi,T}}{T}\Big]$$
$$0 = \Big[\frac{\partial c_S}{\partial \alpha_f} - c_{\nu,f} - c_{\phi,f} + \frac{2n_f}{\varepsilon + P}\Big(c_{\phi,\omega} + \frac{c_h}{2} - c_S T\Big)\Big]$$
$$0 = \Big[2X^{\mu\nu} + \partial_\lambda\Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P}\omega^\lambda S^{\mu\nu}\Big)\Big]$$

Constraining Extra Terms

$$0 = \Big[- 2c_S + T\frac{\partial c_S}{\partial T} + \frac{3c_{\phi,\omega} - c_{\phi,T}}{T} \Big]$$

$$0 = \Big[\frac{\partial c_S}{\partial \alpha_f} - c_{\nu,f} - c_{\phi,f} + \frac{2n_f}{\varepsilon + P}\Big(c_{\phi,\omega} + \frac{c_h}{2} - c_S T \Big) \Big]$$

$$0 = \Big[2X^{\mu\nu} + \partial_\lambda \Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P} \omega^\lambda S^{\mu\nu} \Big) \Big]$$

Req#1:

$$c_S = T^2 f[\alpha_f] \,, \qquad \Rightarrow \qquad c_{\phi,T} = 3c_{\phi,\omega} \,,$$

Req#2:

$$X^{\mu\nu} = -\frac{1}{2}\partial_\lambda \Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P} \omega^\lambda S^{\mu\nu} \Big)$$

a full divergence term

Other unconstrained d.o.f. shall be, and can be determined by microscopic theories.

Side Note: terms obtained from CKT fulfill these requirements.

Summary

- Chiral Kinetic Theory ⊕ (14+6) moment expansion ⇒
    - Viscous-Spin Hydrodynamics (for Massless Fermions)
    - Accounting for:
        - Non equilibrium correction to spin polarization
        - Back-reaction to the orbital motion of fluid
- Compared to macroscopically derived viscous spin hydro:
    - Extra CVE-like terms could be introduced in macro. solution
    - Unconstrained coefficients in the macro. solution shall be, and can be determined by microscopic theories.

Back-Up Slides

Chiral Kinetic Theory

Dirac equation => Equation of Motio

$$\gamma_\mu (p^\mu + \frac{1}{2}$$

Equation of Motion for Clifford comp

$$\partial_\alpha \begin{pmatrix} \mathcal{V}^\mu \\ \mathcal{A}^\mu \\ \mathcal{L}^{\mu\nu} \\ \mathcal{F} \\ \mathcal{P} \end{pmatrix}$$

u^μ

- time-like auxiliary field;
- reflects the ***ambiguity*** in spin-tensor;

  (a.k.a. side-jump effect, see e.g.

  [Chen-Son-Stephanov PRL2015],

  [Hidaka-Pu-Yang PRD2017&2018],

  [Huang-Shi-Jiang-Liao-Zhuang PRD2018])
- physical observables are ***independent*** of *u^μ*;
- *u^μ* = flow velocity in this research. By doing so, the "*f*" is to the distribution function in fluid co-moving frame.

Chiral Kinetic Equation (up to ħ-orde

$$\left[p^\mu \partial_\mu \pm \hbar \left(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u \cdot p}\right) \partial_\nu \right] f_\pm = 0$$

$$\mathcal{J}^\mu_\pm \equiv \frac{1}{2}(\mathcal{V}^\mu \pm \mathcal{A}^\mu) = \left(p^\mu \pm \hbar \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u \cdot p} \partial_\nu \right) f_\pm$$

about the Symmetry of Stress Tensor

In this work, we take the canonical definition of energy-momentum tensor:

$$T^{\mu\nu}_{\rm can} \equiv \langle \bar{\psi}(i\gamma^\mu \partial^\nu)\psi \rangle = \int \frac{{\rm d}^4 p}{(2\pi)^4} p^\nu \mathcal{V}^\mu$$

It could be asymmetric at quantum level, e.g. if taking the eq. distribution:

$$T^{\mu\nu}_{\rm eq} = \Big(\frac{7\pi^2 T^4}{45} + \frac{2T^2(\mu_V^2 + \mu_A^2)}{3} + \frac{\mu_V^4 + 6\mu_V^2\mu_A^2 + \mu_A^4}{3\pi^2} \Big)(u^\mu u^\nu - g^{\mu\nu}/4)$$
$$+ \underline{\frac{\hbar\, \mu_A}{12}\Big(T^2 + \frac{3\mu_V^2 + \mu_A^2}{\pi^2}\Big)(8\omega^\mu u^\nu + T\epsilon^{\mu\nu\sigma\lambda}\varpi_{\sigma\lambda})}$$

In principle, one could symmetrize $T^{\mu\nu}$ by performing "pseudo-gauge" transformation:

$$\widetilde{T}^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda}), \qquad \Phi^{\lambda,\mu\nu} + \Phi^{\lambda,\nu\mu} = 0\,.$$

see e.g. [W. Florkowski, R. Ryblewski, A. Kumar, Prog.Part.Nucl.Phys.108(2019)103709]

Constraining Extra Terms: Frame Dependence?

$$0 = \Big[-2c_S + T\frac{\partial c_S}{\partial T} + \frac{3c_{\phi,\omega} - c_{\phi,T}}{T} \Big]$$

$$0 = \Big[\frac{\partial c_S}{\partial \alpha_f} - c_{\nu,f} - c_{\phi,f} + \frac{2n_f}{\varepsilon + P}\Big(c_{\phi,\omega} + \frac{c_h}{2} - c_S T \Big) \Big]$$

$$0 = \Big[2X^{\mu\nu} + \partial_\lambda \Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P} \omega^\lambda S^{\mu\nu} \Big) \Big]$$

velocity can be redefined:

$$u^\mu \to u^\mu + c_{u,\omega}\, \omega^\mu \,,$$

$$c_h \to c_h + (\varepsilon + P)\, c_{u,\omega} \,,$$
$$c_{\nu,f} \to c_{\nu,f} + n_f\, c_{u,\omega} \,,$$
$$c_S \to c_S \,.$$

constrain eqs. unchanged

Req#1:

$$c_S = T^2 f[\alpha_f] \,, \qquad \Rightarrow \qquad c_{\phi,T} = 3c_{\phi,\omega} \,,$$

Req#2:

$$X^{\mu\nu} = -\frac{1}{2}\partial_\lambda \Big(Y^{\lambda\mu\nu} - \frac{2c_{\phi,\omega} + c_h}{\varepsilon + P} \omega^\lambda S^{\mu\nu} \Big)$$

a full divergence term

Other unconstrained d.o.f. shall be, and can be determined by microscopic theories.

Thermodynamic Relation For Systems With Spin

In Ref.[1], the spin correction to the thermodynamic property is assumed to be:

$$\varepsilon = T\,s - p + \sum_f \mu_f\,n_f + \mathcal{W}_{\mu\nu}\,S^{\mu\nu}\,,$$

$$\mathrm{d}\varepsilon = T\,\mathrm{d}s + \sum_f \mu_f\,\mathrm{d}n_f + \mathcal{W}_{\mu\nu}\,\mathrm{d}S^{\mu\nu}\,.$$

$S_{\mu\nu}$ is the spin polarization density

$\mathcal{W}_{\mu\nu}$ is the corresponding "chemical potential"

[1] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya *Phys.Lett.B* 795 (2019) 100-106, arXiv:1901.06615.

Notations

projectors	$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ $\Delta^{\mu\nu}_{\alpha\beta} \equiv (1/2)\Delta^\mu_\alpha \Delta^\nu_\beta + (1/2)\Delta^\mu_\beta \Delta^\nu_\alpha - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$
derivatives	$\theta \equiv \partial \cdot u$ $\sigma^{\mu\nu} \equiv \Delta^{\mu\nu}_{\alpha\beta}\partial^\alpha u^\beta$ $\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$ $\varpi_{\mu\nu} \equiv \frac{1}{2}\Big(\partial_\nu \frac{u_\mu}{T} - \partial_\mu \frac{u_\nu}{T}\Big)$
thermal integrals	$I_{n,q} \equiv \frac{1}{(2q+1)!!}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 E_p}(u\cdot p)^{n-2q}\big(-\Delta^{\mu\nu}p_\mu p_\nu\big)^q f_0$ $J_{n,q} \equiv \frac{1}{(2q+1)!!}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 E_p}(u\cdot p)^{n-2q}\big(-\Delta^{\mu\nu}p_\mu p_\nu\big)^q f_0(1-f_0)$ $G^{(q)}_{n,m} \equiv J_{n,q}J_{m,q} - J_{n-1,q}J_{m+1,q}$ $G_{n,m} \equiv G^{(0)}_{n,m} = J_{n,0}J_{m,0} - J_{n-1,0}J_{m+1,0}$ $D_{n,q} \equiv J_{n+1,q}J_{n-1,q} - J^2_{n,q}$